2+1D simulations of pre-equilibrium stage with QCD kinetic theory

Aleksas Mazeliauskas

Institut für Theoretische Physik
Universität Heidelberg

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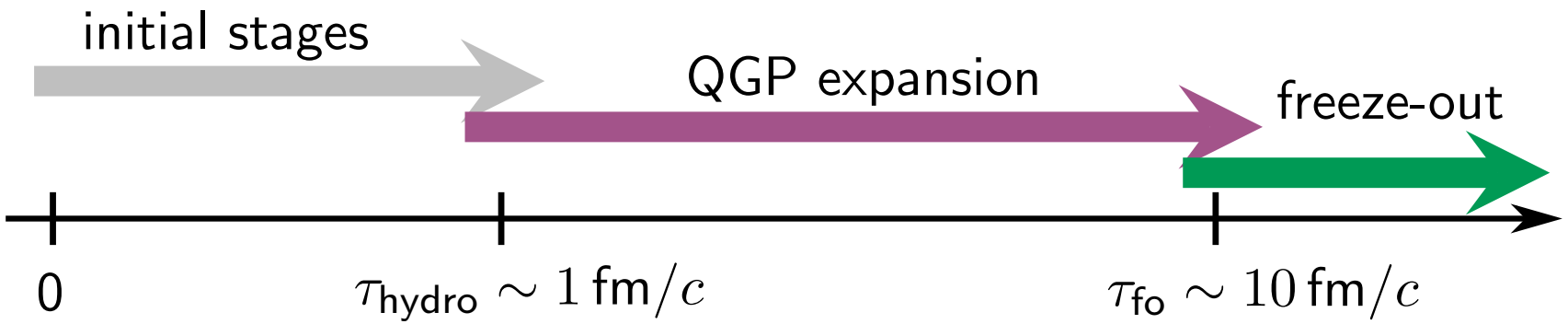
A. Kurkela, AM, J.-F. Paquet, S. Schlichting and D. Teaney, arXiv:1805.01604, 1805.00961

Motivation



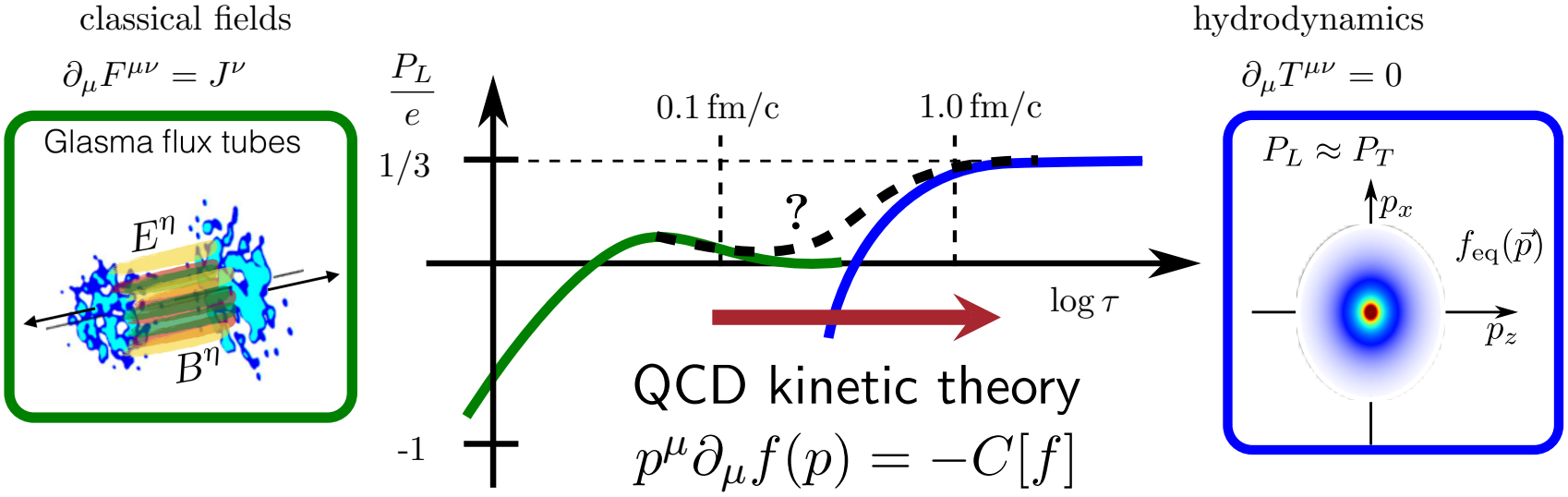
Why is the correct modeling of initial stages important?

- Initial collision geometry ⇒ particle correlations, e.g. flow harmonics.
  Pre-equilibrium dynamics smears the geometry, creates flow.
- Consistency in physics pictures ⇒ independence of the crossover time
  Need "hydrodynamization" of initial conditions to justify hydro.
- Small collision systems (eg. p-p, p-Pb) ⇒ short life-time.
  Initial stages can be a significant part of the total evolution.

Initial stages at very high collision energy

Consider a weak coupling limit at $\sqrt{s} \to \infty$

- Initial state – classical gluonic fields, Yang-Mills evolution, anisotropic
- Final state – viscous hydrodynamics, close to equilibrium



Use QCD kinetic theory to connect early particle production to hydrodynamic simulation.

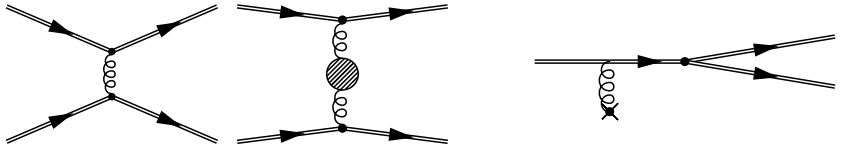
Thermalization in QCD effective kinetic theory

"Bottom-up" thermalization scenario — Baier, Mueller, Schiff, and Son (2001)[1]

QCD effective kinetic theory — Arnold, Moore, Yaffe (2003)[2]

Weakly coupled quark and gluon quasi-particles in a soft background.

$$\partial_\tau f + \frac{\mathbf{p}}{|p|}\cdot\nabla f - \frac{p_z}{\tau}\partial_{p_z} f = -\underbrace{\mathcal{C}_{2\leftrightarrow 2}[f]}_{} - \underbrace{\mathcal{C}_{1\leftrightarrow 2}[f]}_{}$$



- elastic $2 \leftrightarrow 2$ and *inelastic* $1 \leftrightarrow 2$ scatterings (with LPM suppression)

the same QCD processes as in jet quenching

- only parameter — the coupling constant $\lambda = 4\pi\alpha_s N_c$.

Caveats: in practice need to extrapolate λ to realistic values ($\alpha_s \approx 0.3$),

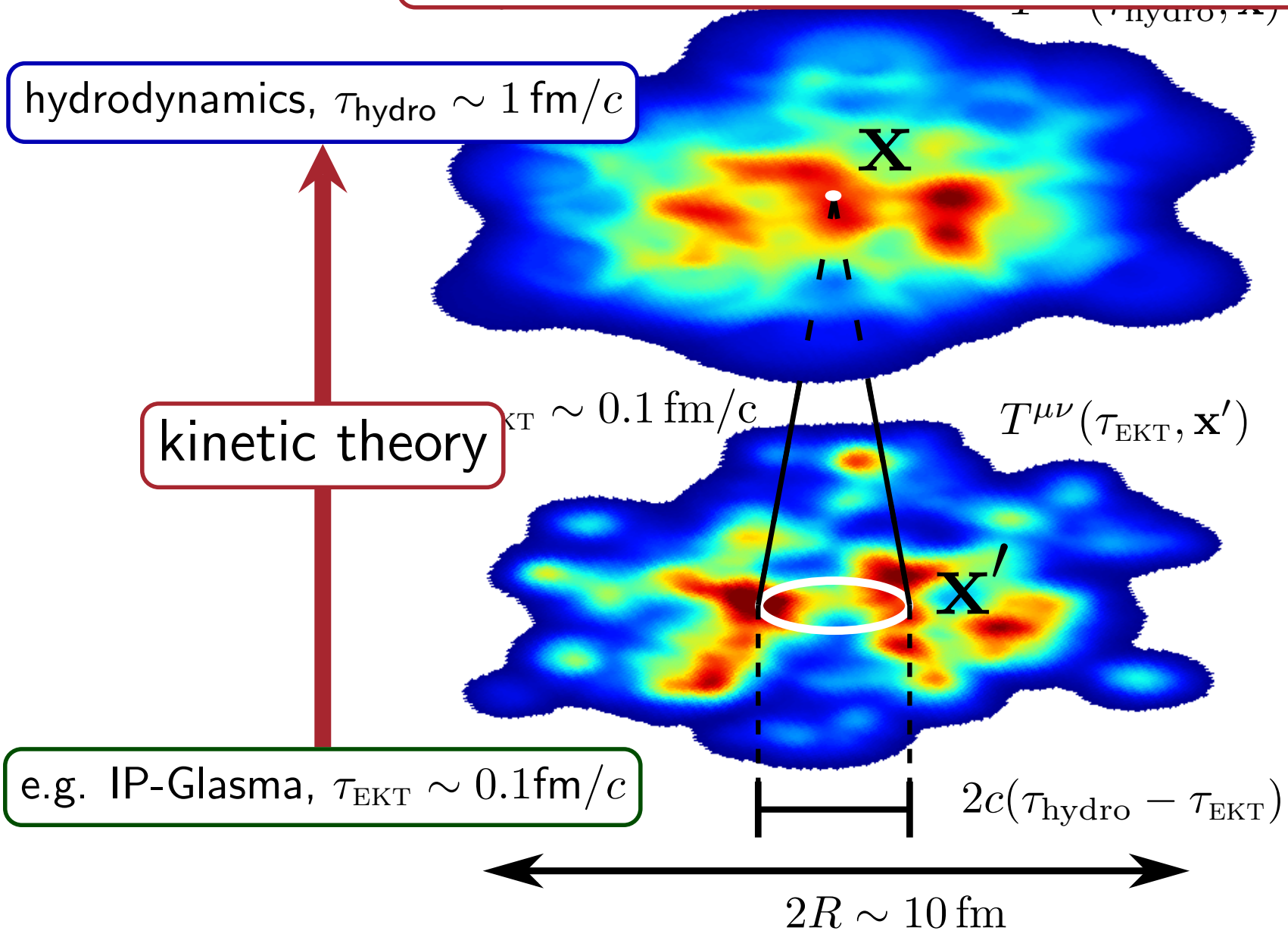
for comparison with strong coupling approach, see Keegan, Kurkela, Romatschke, Schee and Zhu (2015) [3]

isotropic screening (avoids plasma instabilities)

including them does not change dynamics, see Berges, Boguslavski, Schlichting, Venugopalan (2014)[4],

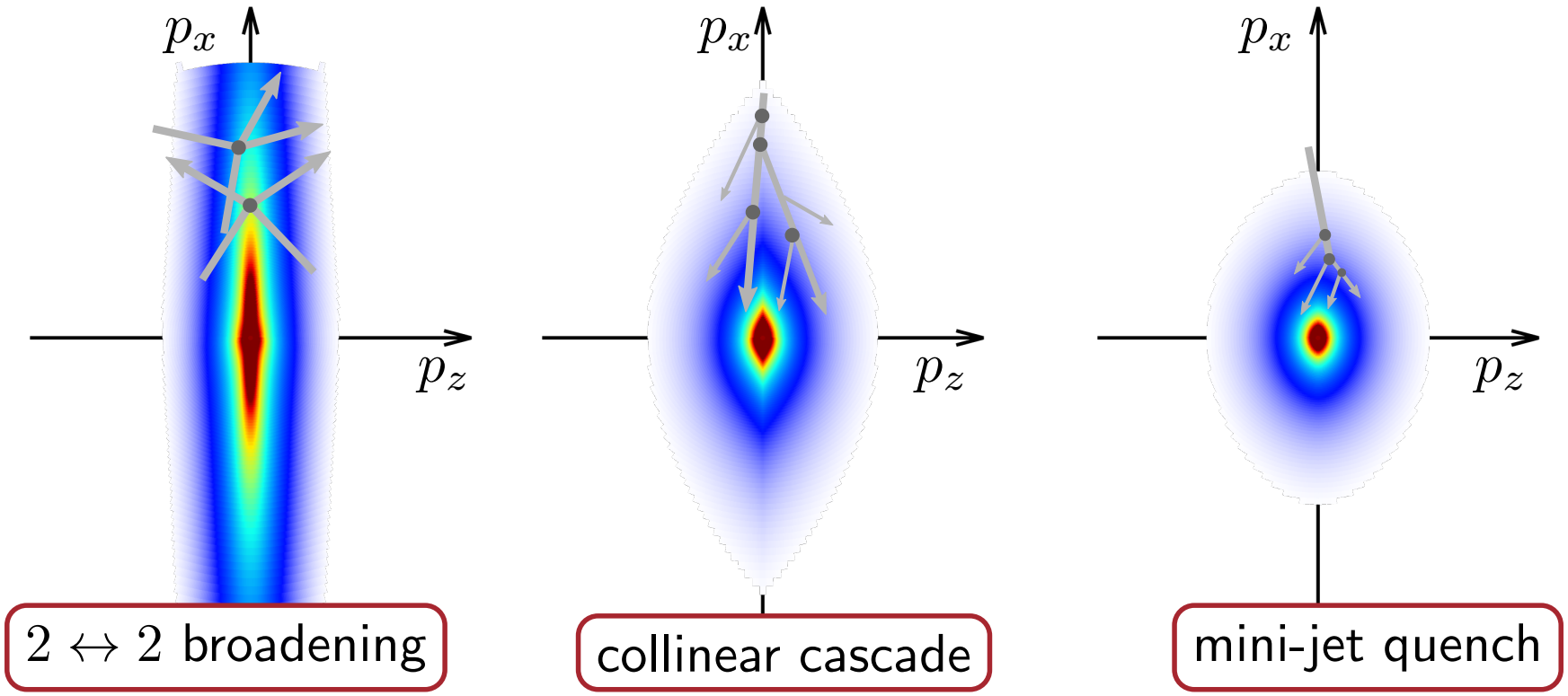
Separation of scales: background and perturbations

$$T^{\mu\nu}(\tau_{\text{hydro}}, \mathbf{x}) = \bar{T}^{\mu\nu}(\tau_{\text{hydro}}) + \delta T^{\mu\nu}(\tau_{\text{hydro}}, \mathbf{x})$$



Thermalization stages in kinetic theory

Evolution of gluon distribution function

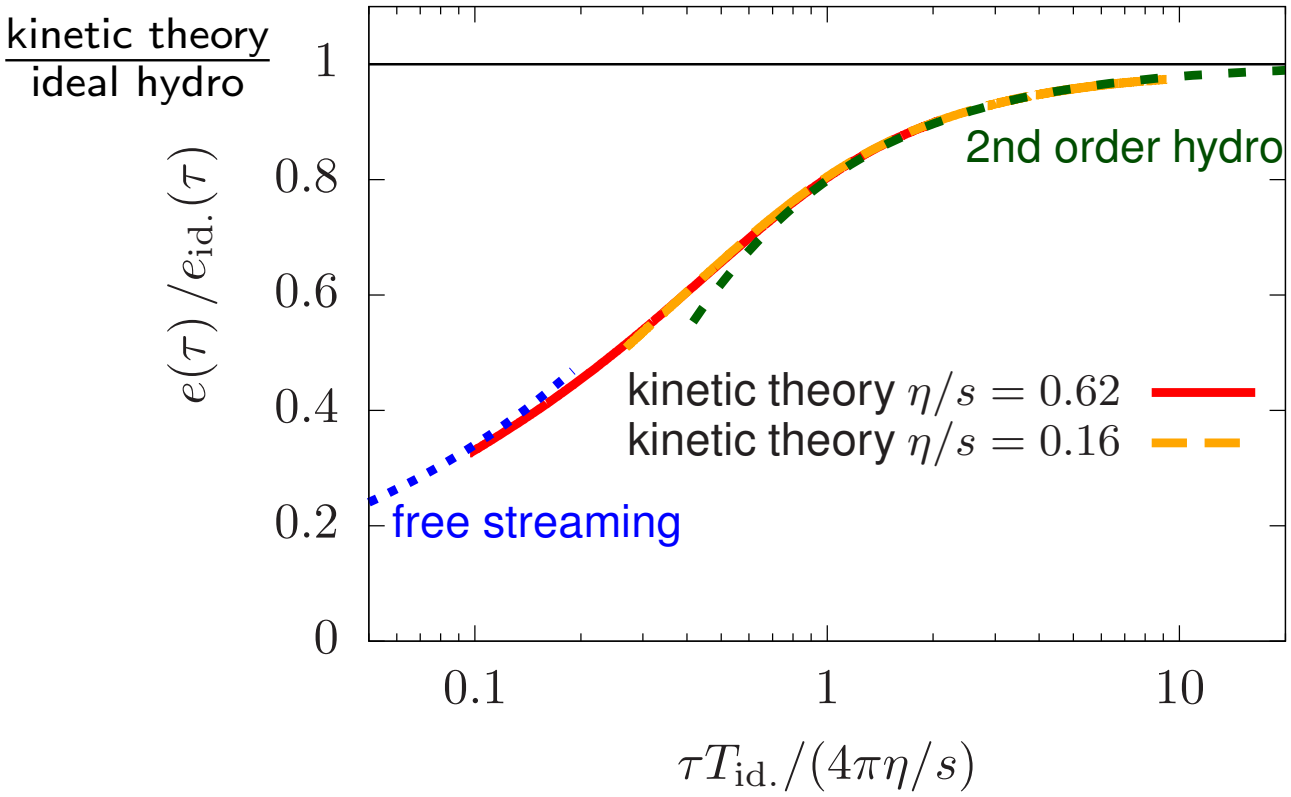


The equilibration time governed by the coupling constant

$$\alpha_s \quad \Longrightarrow \quad \eta/s \quad \Longrightarrow \quad \tau_R(\tau) = (4\pi\eta/s)/T$$

From kinetic theory to hydrodynamics

scaling in hydro: $$e(\tau) = \underbrace{\nu_g \frac{\pi^2}{30} T_{\text{ld.}}^4}_{\text{“ideal” temp.}} \left(\underbrace{1}_{\text{ideal}} - \underbrace{\frac{8}{3}\frac{\eta/s}{\tau T_{\text{ld.}}}}_{\text{viscous}} + \underbrace{C_2 \left(\frac{\eta/s}{\tau T_{\text{ld.}}} \right)^2}_{\text{2nd order hydro}} \right)$$



Kinetic equilibration becomes <u>universal</u> for scaled time $\dfrac{\tau}{\tau_R(\tau)} = \dfrac{\tau T(\tau)}{4\pi\eta/s}$

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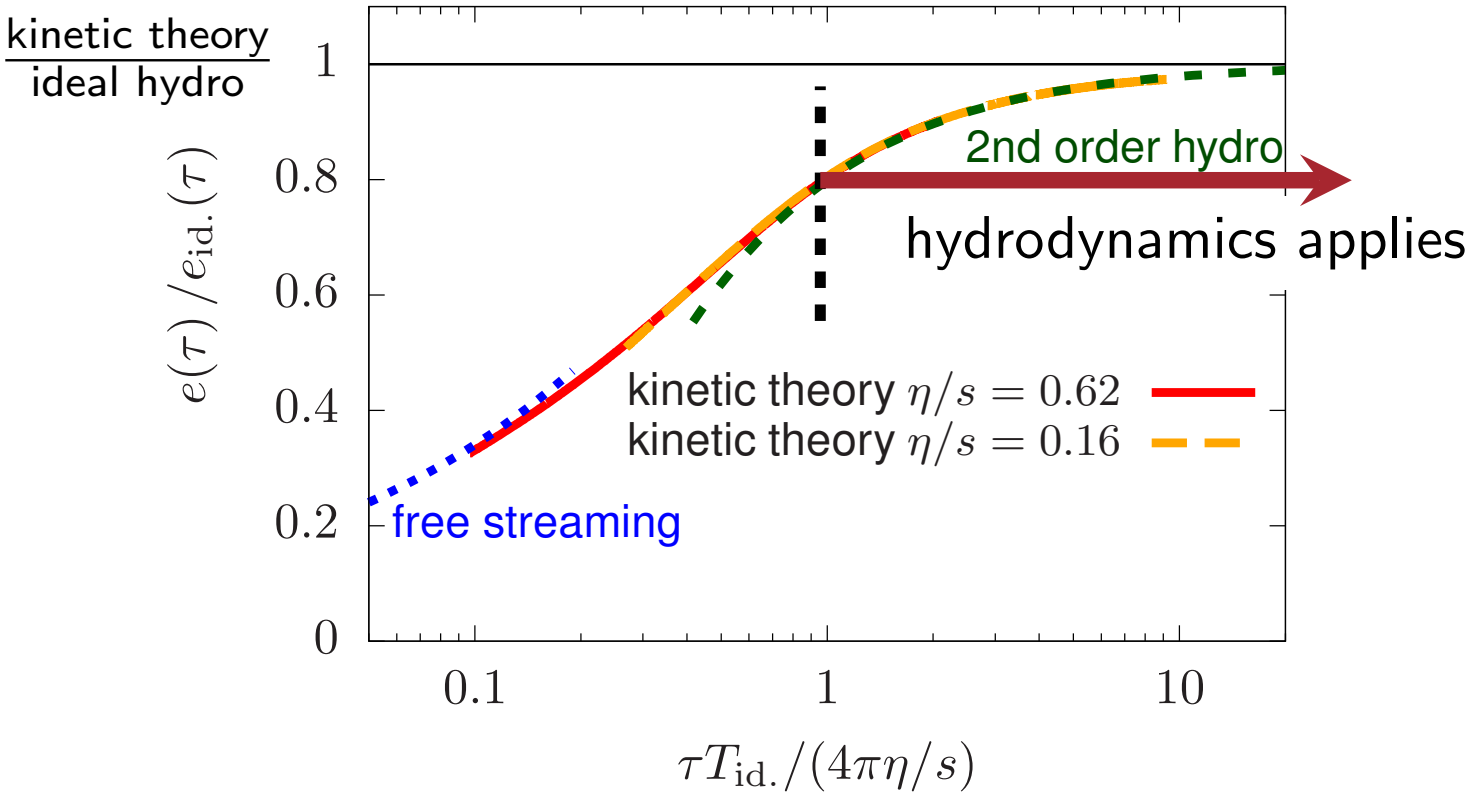
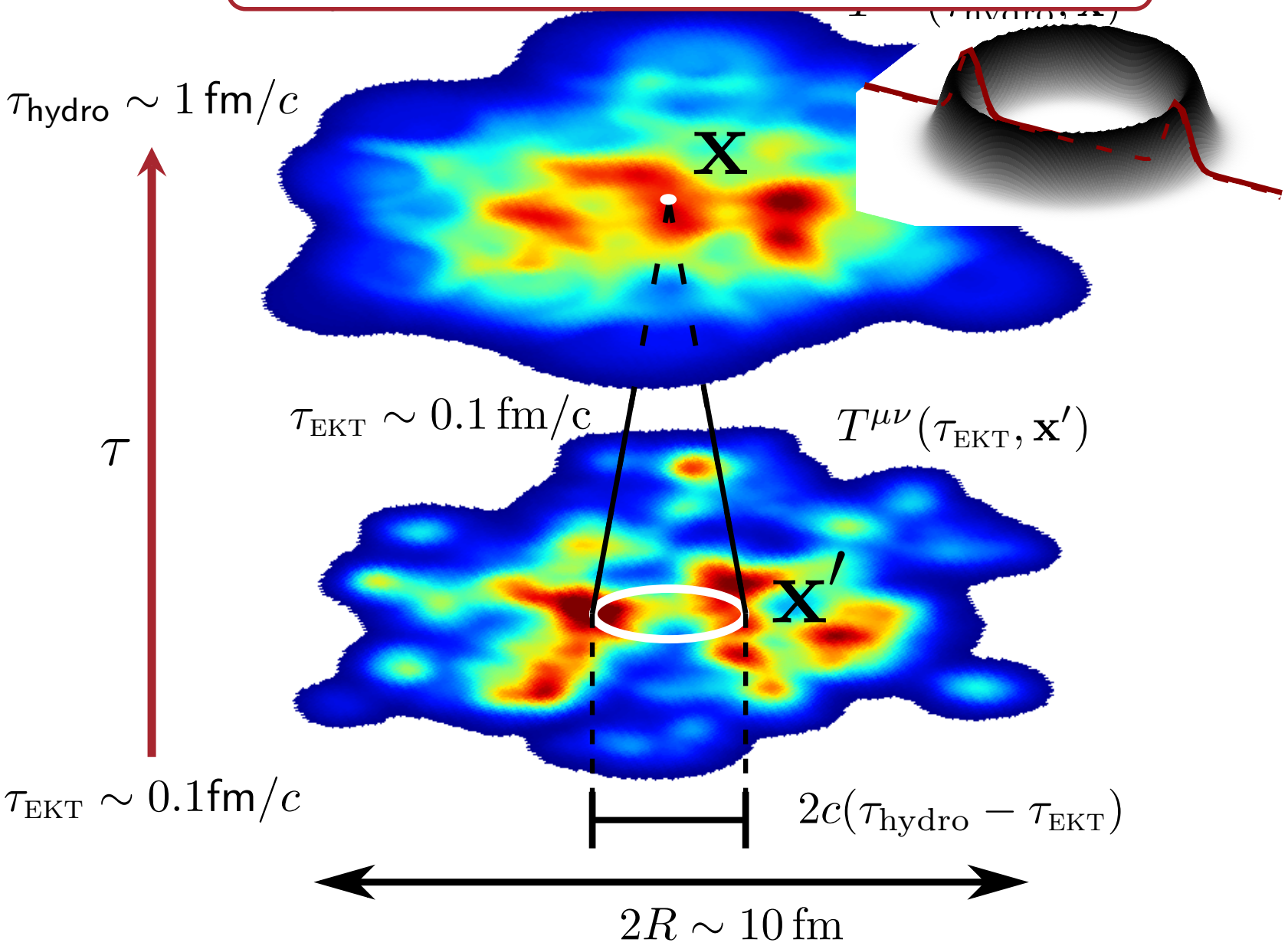


Kinetic equilibration becomes <u>universal</u> for scaled time $\dfrac{\tau}{\tau_R(\tau)} = \dfrac{\tau T(\tau)}{4\pi\eta/s}$

Event-by-event pre-equilibrium evolution with kinetic theory

$$T^{\mu\nu}(\tau_{\text{hydro}}, \mathbf{x}) = \bar{T}^{\mu\nu}(\tau_{\text{hydro}}) + \delta T^{\mu\nu}(\tau_{\text{hydro}}, \mathbf{x})$$



Plane wave perturbations in transverse plane

Linearized gluon distribution function at initial time

$$f(\tau, \mathbf{p}, \mathbf{x}_\perp) \quad = \quad \underbrace{\bar{f}_{\mathbf{p}}}_{\text{uniform background}} \quad + \quad \underbrace{\delta f_{\boldsymbol{k}_\perp, \mathbf{p}} e^{i\boldsymbol{k}_\perp \cdot \mathbf{x}_\perp}}_{\text{transverse perturbations}} \quad .$$

Coupled evolution of background and perturbations

$$(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z})\bar{f}_{\mathbf{p}} = -\mathcal{C}[\bar{f}] \qquad \text{background}$$

$$(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z} + \frac{i\mathbf{p}_\perp \cdot \boldsymbol{k}_\perp}{p})\delta f_{\boldsymbol{k}_\perp, \mathbf{p}} = -\delta\mathcal{C}[\bar{f}, \delta f] \qquad \boldsymbol{k}_\perp \text{ perturbation}$$

Extract response functions for δf corresponding to initial energy $(\delta T^{\tau\tau})$ and momentum ($\delta T^{\tau i}$) perturbations

$$\underbrace{\delta T^{\mu\nu}_{\mathbf{x}}(\tau_{\text{hydro}}, \mathbf{x}')}_{\text{goes into hydro}} = \int d^2\mathbf{x}' \; \underbrace{G^{\mu\nu}_{\alpha\beta}\left(\mathbf{x} - \mathbf{x}', \tau_{\text{hydro}}, \tau_{\text{EKT}}\right)}_{\text{linear response function}} \underbrace{\delta T^{\alpha\beta}_{\mathbf{x}}(\tau_{\text{EKT}}, \mathbf{x}')}_{\text{initial}} .$$

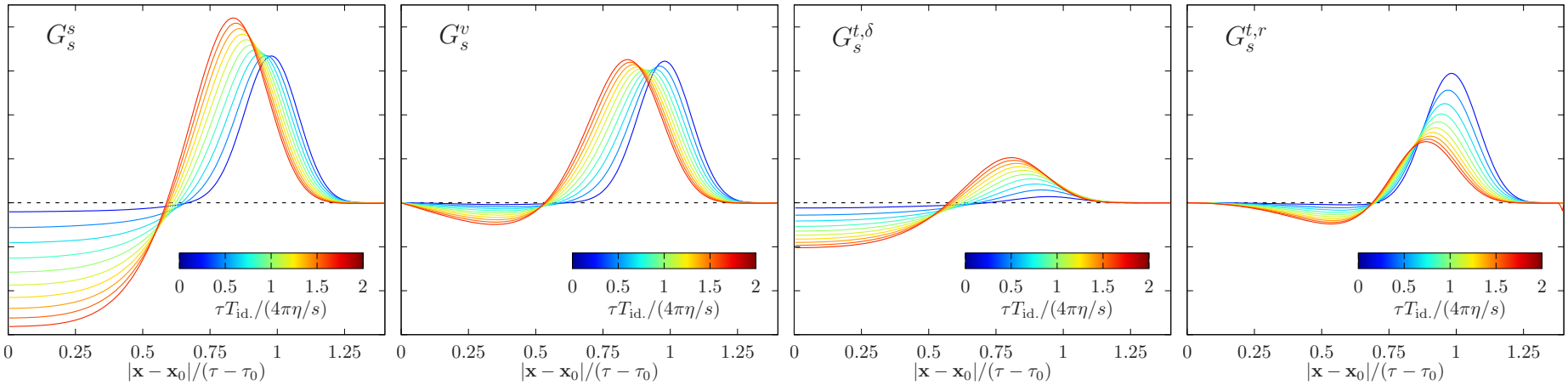
Kinetic theory response functions

Invariant form of non-equilibrium response functions

$$G^{\mu\nu}\left(\tau, \tau_0, |\mathbf{x}-\mathbf{x}_0|, e(\tau_0), \lambda\right) \Rightarrow G^{\mu\nu,\mathrm{univ}}\left(\frac{\tau T_{\mathrm{ld.}}}{\eta/s}, \frac{|\mathbf{x}-\mathbf{x}_0|}{(\tau-\tau_0)}\right)$$

All components of energy-momentum tensor from kinetic response

energy T^{00} | velocity T^{0i} | shear-stress T^{ij}



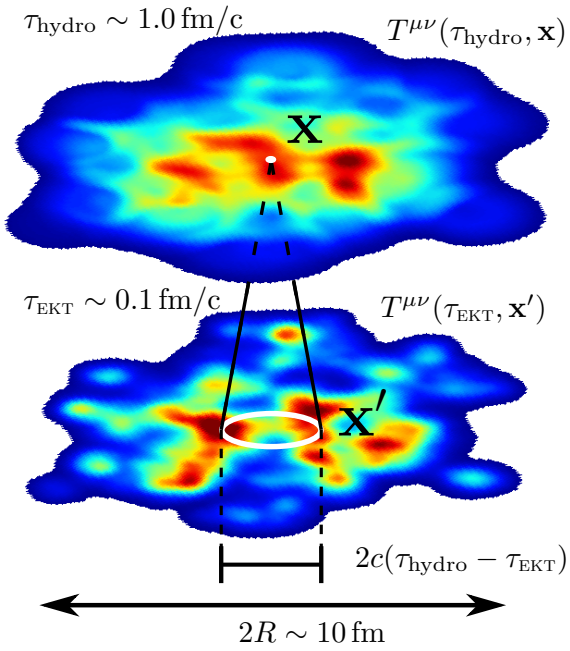
- long wavelengths & late times $\Rightarrow$ hydrodynamic response
  cf. universal velocity response Vredevoogd and Pratt (2008) [5], Shee, Romatschke and Pratt (2013) [6]
- short wavelengths & early times $\Rightarrow$ free streaming response
  Broniowski, Florkowski and Chojnacki, and Kisiel (2008) [7] Liu, Shen and Heinz (2015) [8]

Practical event-by-event kinetic pre-equilibrium: KøMPøST

- Input non-equilibrium $T^{\mu\nu}(\tau_{\rm EKT}, \mathbf{x})$, i.e. $P_L \approx 0$ (e.g. IP-Glasma)
- Equilibrate background according to the scaling curve.

$$\overline{T}^{\mu\nu}(\tau_{\rm EKT}) \approx \text{diag}\,(e, e/2, e/2, 0) \Rightarrow \overline{T}^{\mu\nu}(\tau_{\text{hydro}}) = \text{diag}\,(e, P_T, P_T, P_L)$$

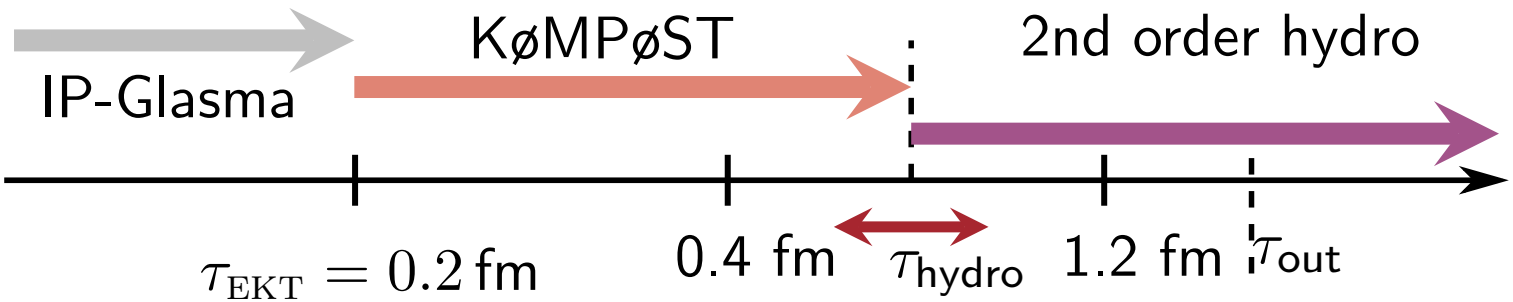
- Propagated linearized energy ($\delta T^{\tau\tau}$) and momentum ($\delta T^{\tau i}$) perturbations with Green's functions
- Pass total $T^{\mu\nu}(\tau_{\text{hydro}}) = \bar{T}^{\mu\nu}(\tau_{\text{hydro}}) + \delta T^{\mu\nu}(\tau_{\text{hydro}})$ to hydro

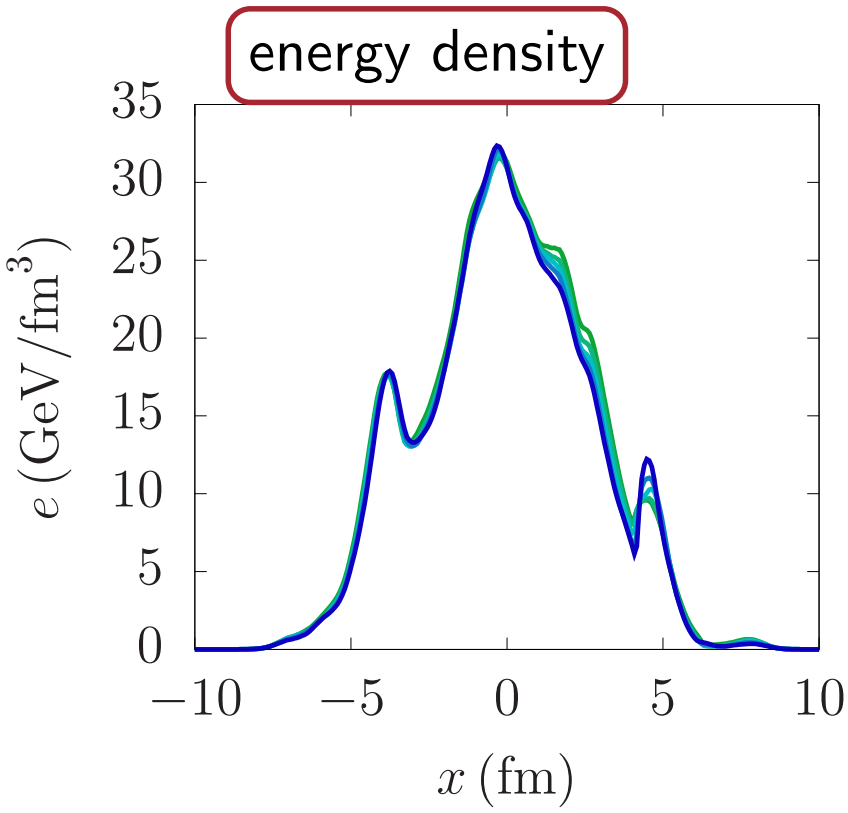


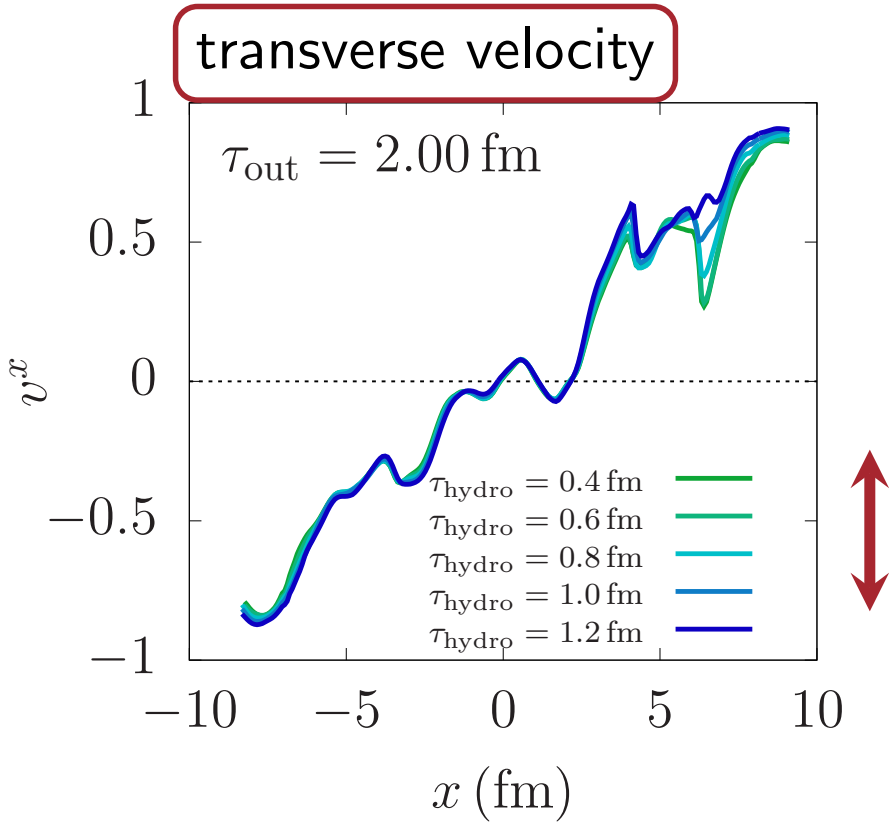
KøMPøST code publicly available at
github.com/KMPST/KoMPoST

Kurkela, AM, Paquet, Schlichting and Teaney (2018)[9, 10]

Results: event-by-event initial stage matching



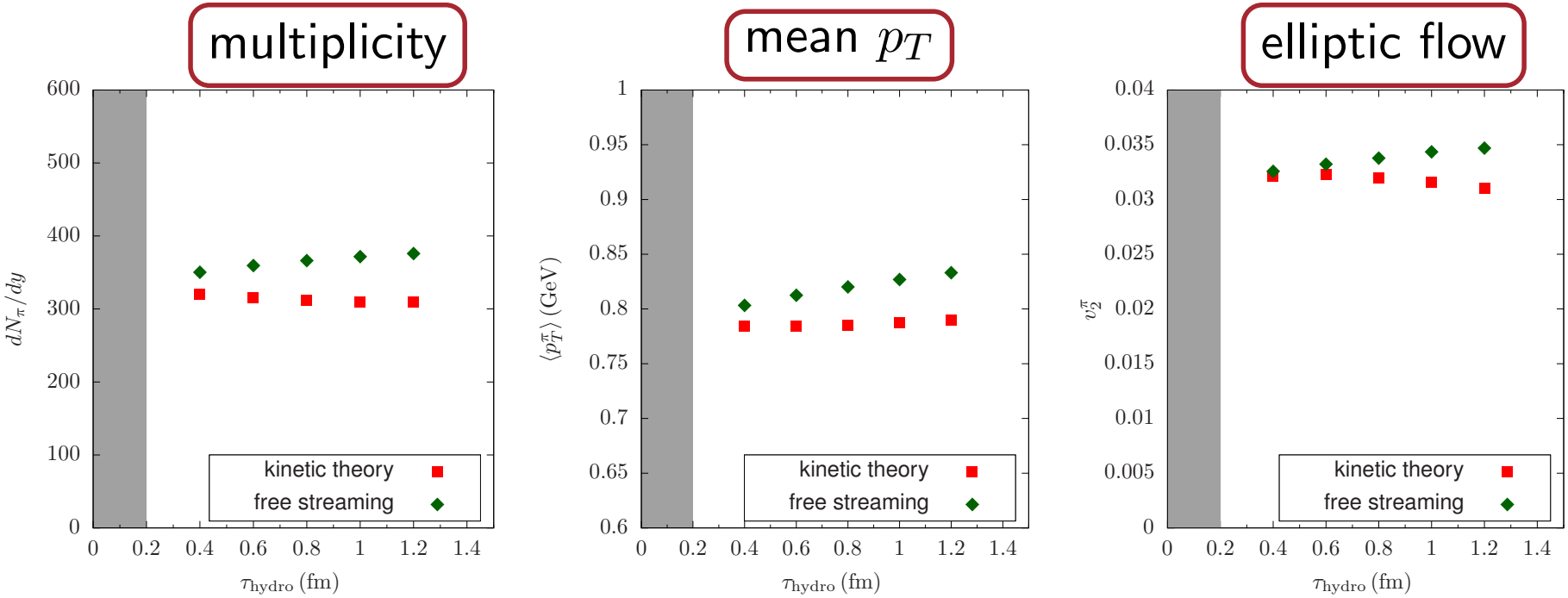




Smooth matching between kinetic phase and hydrodynamics ⇒ independence of τ_{hydro}.

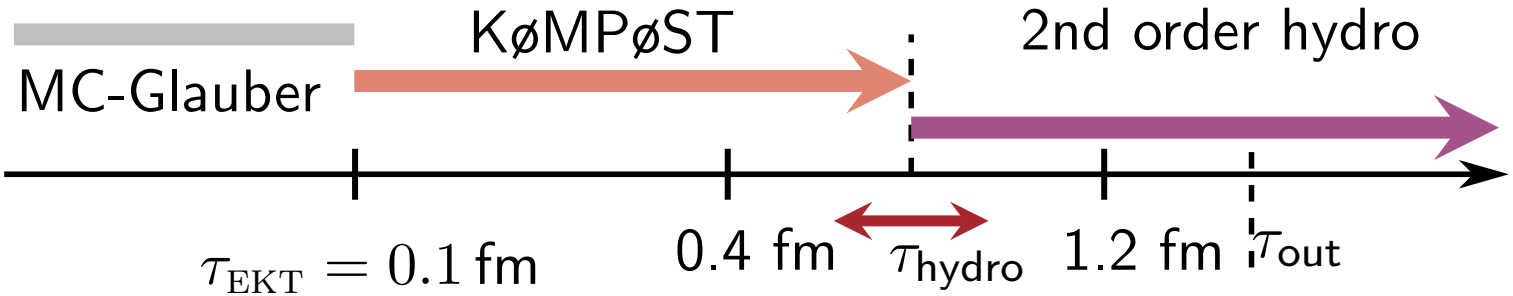
Final hadronic observables

Thermal pions at freeze-out $T_{\text{fo}} = 145\,\text{MeV}$



Approximate independence of τ_{hydro} for kinetic pre-equilibrium evolution!
No need to re-adjust initial conditions with KøMPøST

Pre-equilibrium evolution with MC-Glauber initial conditions



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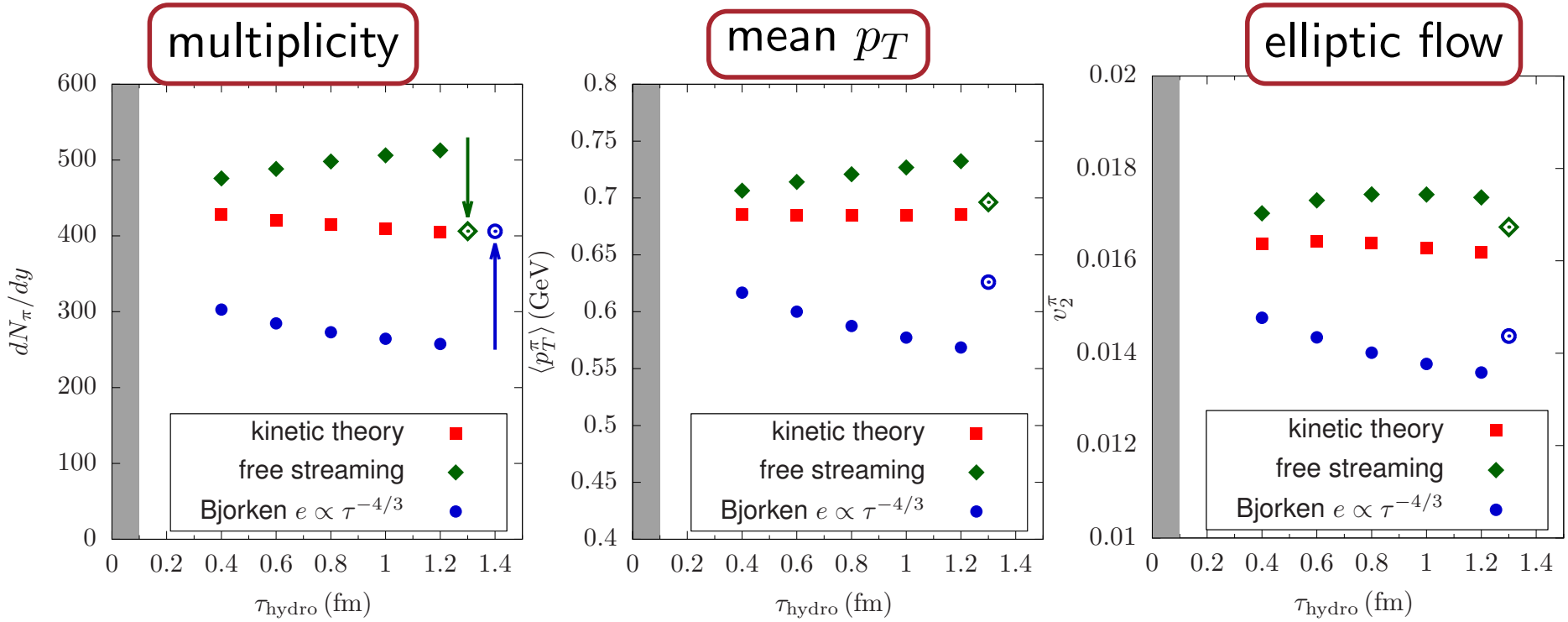

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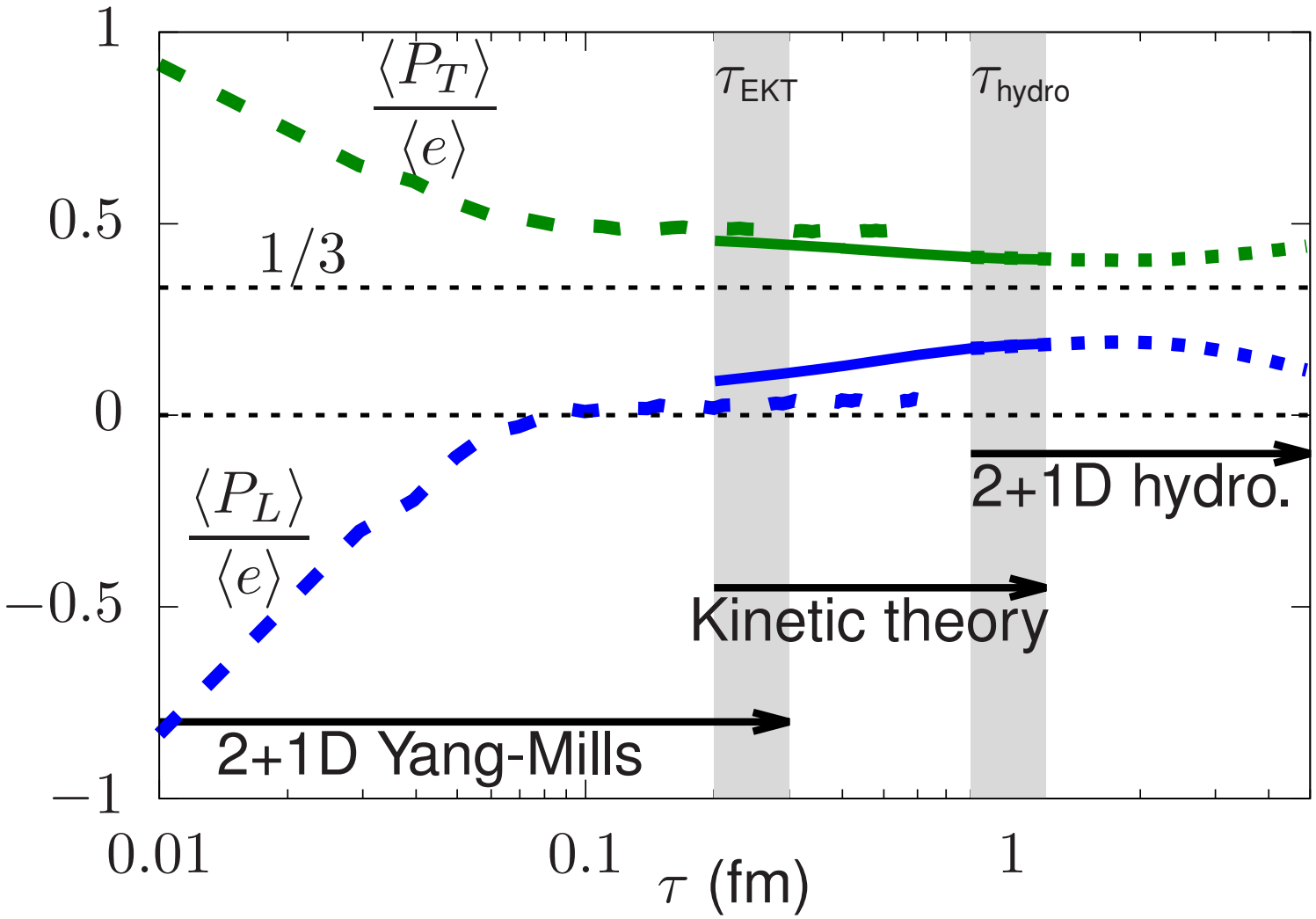
Overlapping descriptions of initial stages in heavy ion collisions



No longer a cartoon picture.

Summary

KøMPøST—linearized kinetic theory propagator of heavy ion initial conditions. github.com/KMPST/KoMPoST, Kurkela, AM, Paquet, Schlichting and Teaney (2018)[9, 10]

- Equilibration of background given by kinetic theory scaling curve in units of $\tau T_{\text{ld.}}/(\eta/s)$.
- Energy $(\delta T^{\tau\tau})$ and momentum $(\delta T^{\tau i})$ perturbations propagated by kinetic Green's functions $G^{\mu\nu}(\tau T_{\text{ld.}}/(\eta/s), r/(\tau - \tau_0))$
- Final observables are largely insensitive to hydro starting time τ_{hydro} with kinetic pre-equilibrium stage.

Smooth and consistent matching between classical field and hydrodynamic descriptions using kinetic theory.

Outlook

- Going beyond linear reponse (important in small collision systems).
- Chemical equilibration and hydrodynamization with QCD kinetic theory

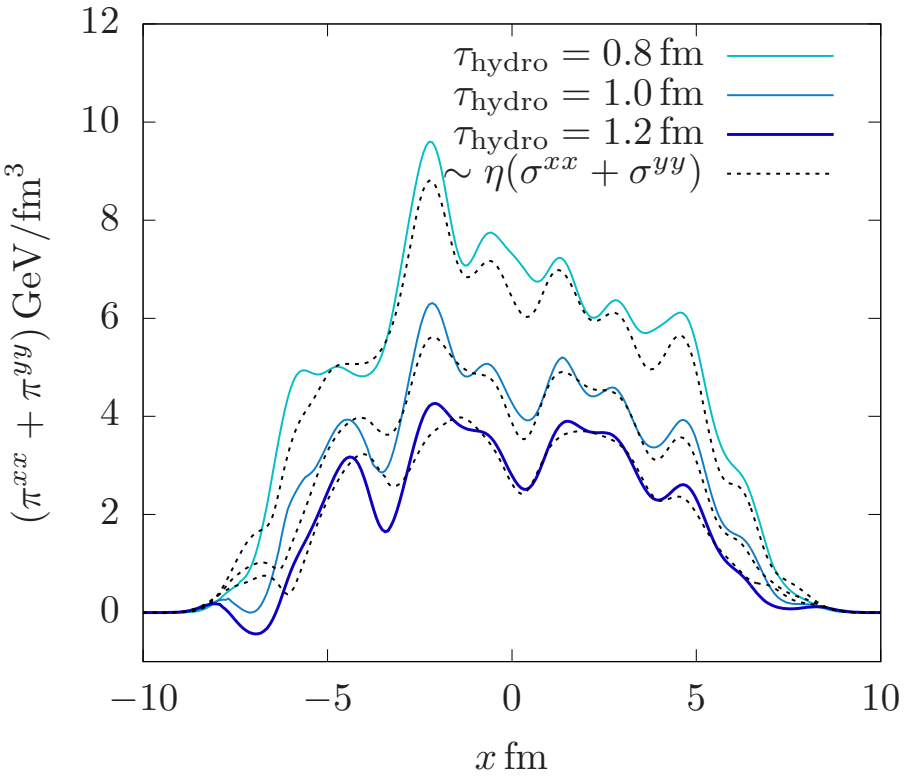
Kurkela, AM, *in preparation*

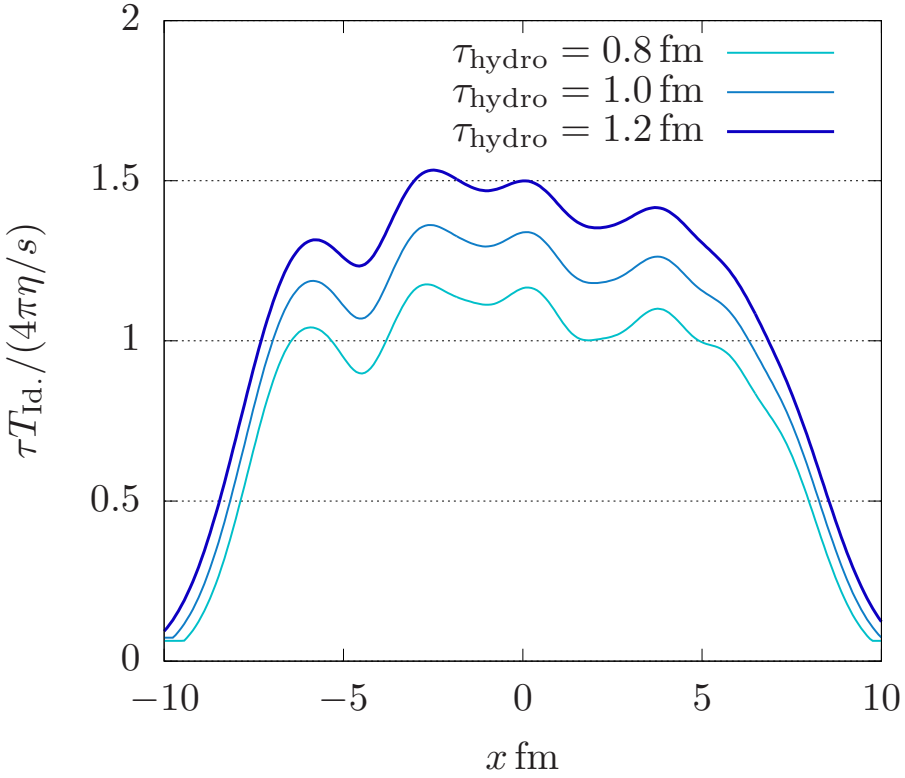
Backup

Shear-stress tensor from KøMPøST evolution

Kinetic evolution automatically satisfies constitutive equations.

Note: MC-Glauber initial conditions





Expect hydro to apply for $\tau T_{id.}/(4\pi\eta/s) > 1$

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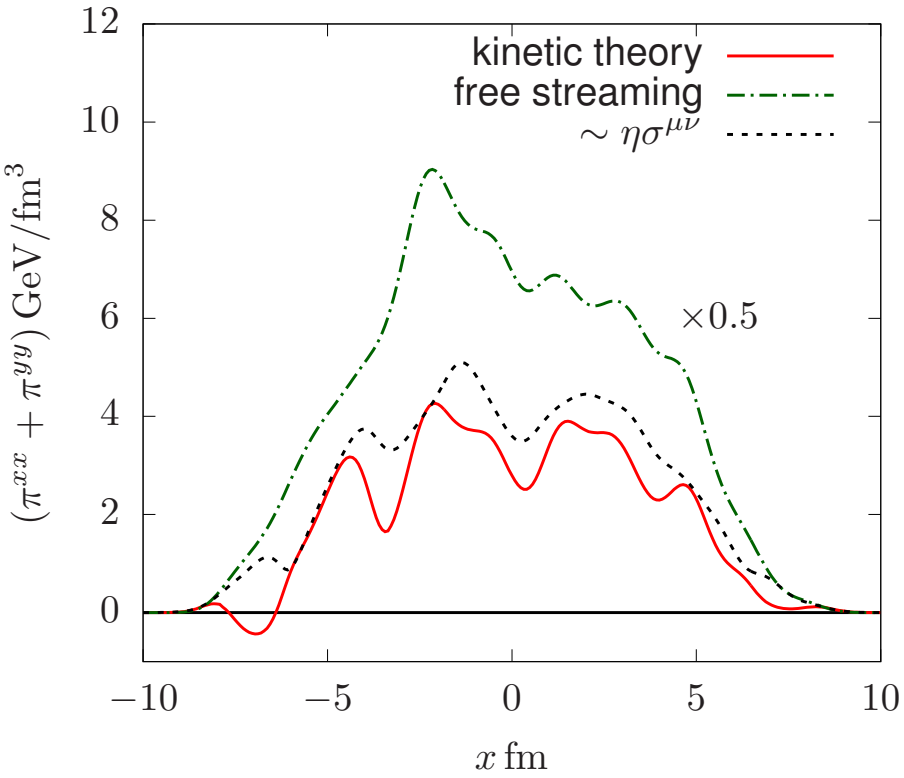
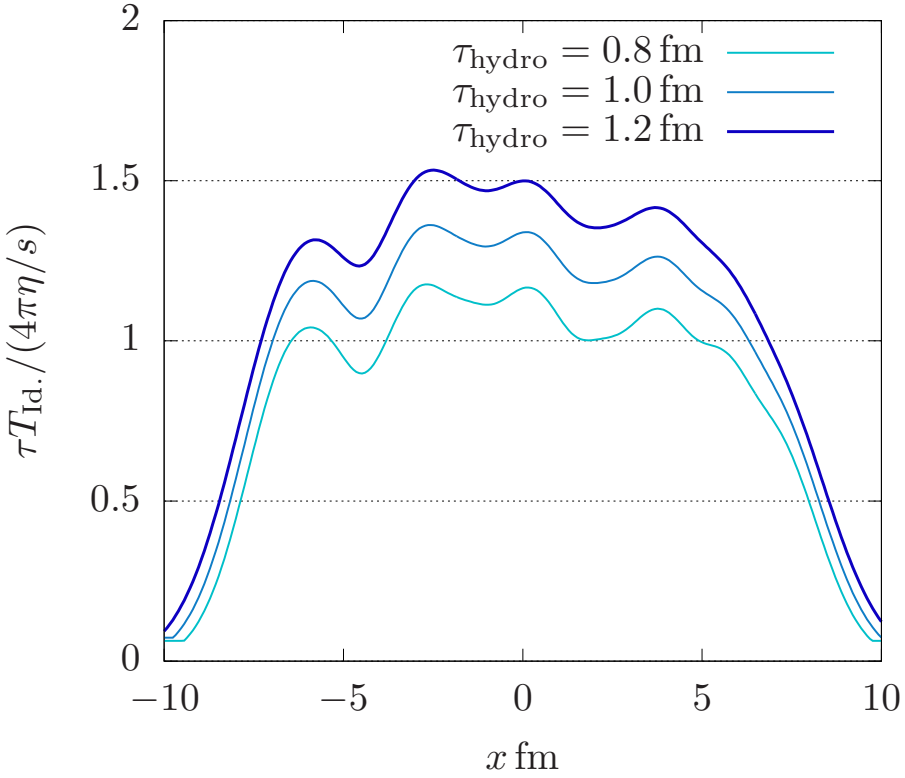




Expect hydro to apply for $\tau T_{id.}/(4\pi\eta/s) > 1$

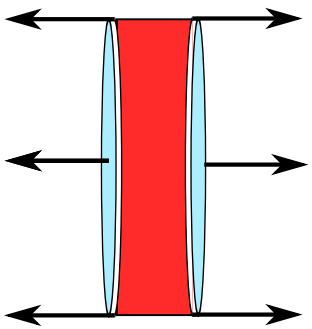
System size and hydrodynamization time

Will hydrodynamically flowing QGP be formed for a given system size?

- Quite universally for longitudinally expanding systems

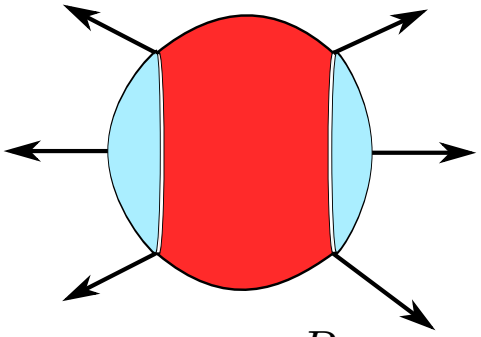
also in holography, see e.g. Heller, Kurkela, and Spalinksi (2017) [11]

$$\frac{\tau_{\text{hydro}} T(\tau_{\text{hydro}})}{4\pi\eta/s} \approx 1$$



$\tau < R$

$T \sim \tau^{-1/3}$



$\tau > R$

$T \sim \tau^{-1}$

- Long lived hydrodynamic phase can form if $\tau_{\text{hydro}} \ll R$

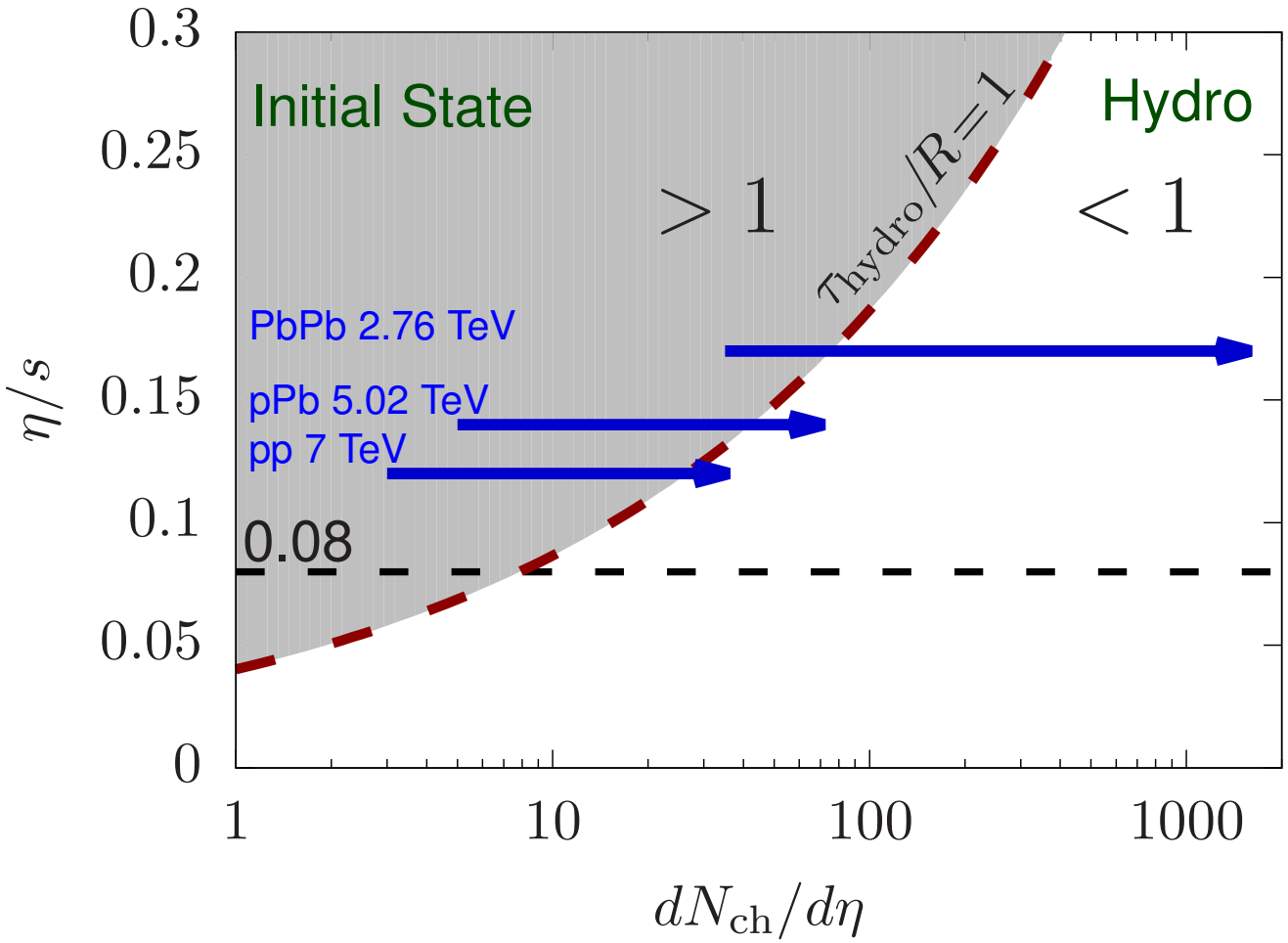
Kurkela, AM, Paquet, Schlichting and Teaney (2018)[9]

$$\frac{\tau_{\text{hydro}}}{R} \approx \left(\frac{4\pi(\eta/s)}{2}\right)^{\frac{3}{2}} \left(\frac{dN_{\text{ch}}/d\eta}{63}\right)^{-\frac{1}{2}} \left(\frac{S/N_{\text{ch}}}{7}\right) \left(\frac{\nu_{\text{eff}}}{40}\right)^{\frac{1}{2}}$$

cf. earlier estimates Basar and Teaney (2013) [12], Schlichting and Tribedy (2016)[13]

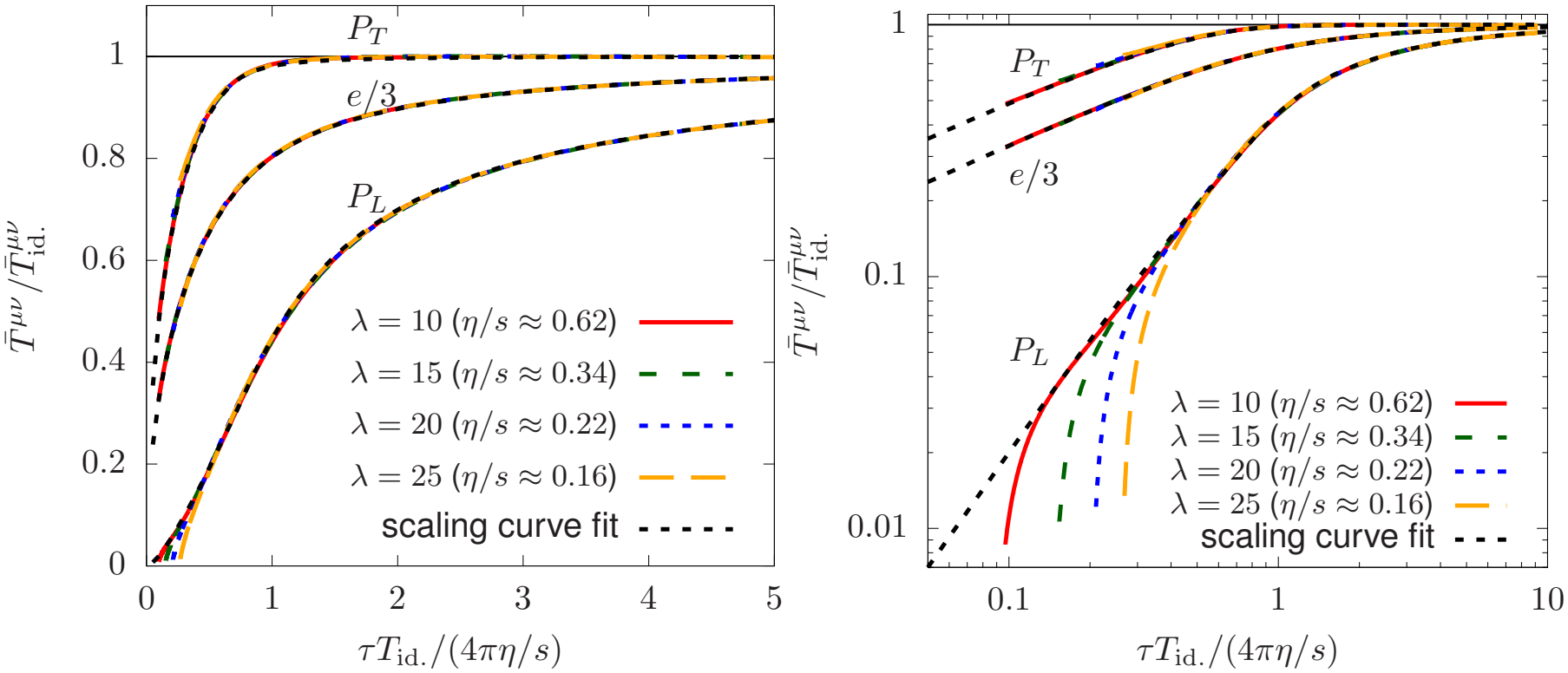
Conformal scaling and system size

$$\frac{\tau_{\text{hydro}}}{R} \approx \left(\frac{4\pi(\eta/s)}{2}\right)^{\frac{3}{2}} \left(\frac{dN_{\text{ch}}/d\eta}{63}\right)^{-\frac{1}{2}} \left(\frac{S/N_{\text{ch}}}{7}\right) \left(\frac{\nu_{\text{eff}}}{40}\right)^{\frac{1}{2}}$$



Little time for hydrodynamization in small collision systems.

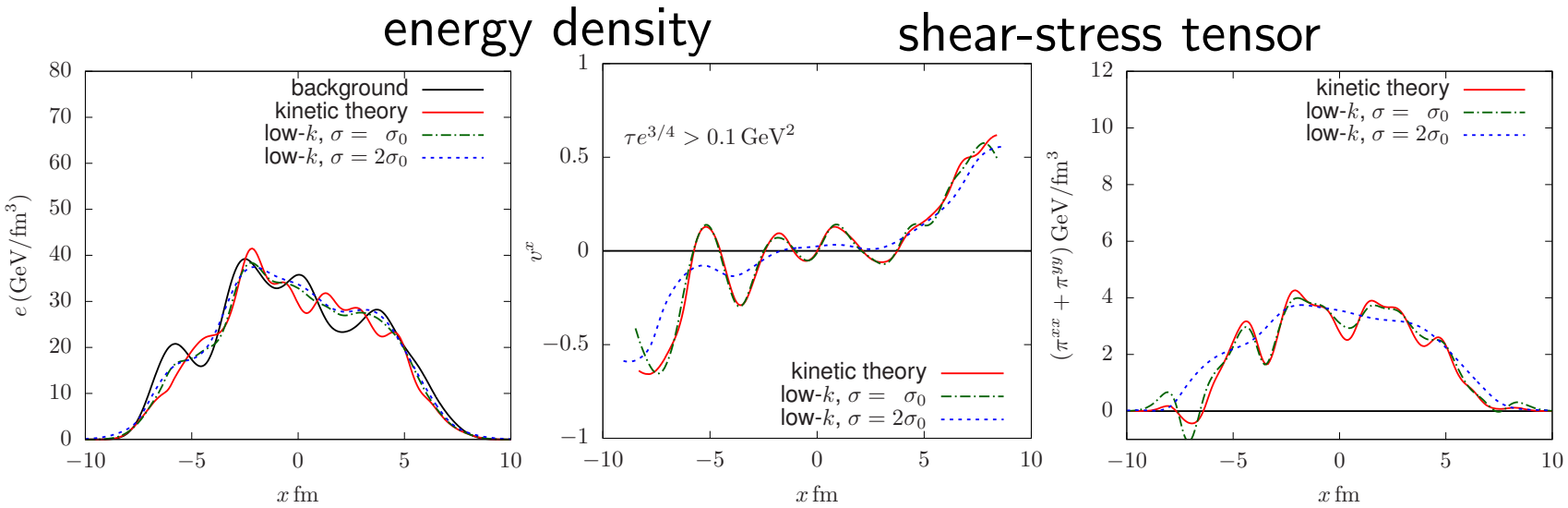
Energy momentum tensor evolution in kinetic theory



Free streaming and low-$|\boldsymbol{k}|$ limits of kinetic theory

Extreme limits of kinetic response:

- small-$|\boldsymbol{k}|$ expansion à la Pratt, cf. [5, 6]
- free streaming response functions, cf.[7, 8]



Note: MC-Glauber initial conditions

Free streaming and low-$|\boldsymbol{k}|$ limits of kinetic theory

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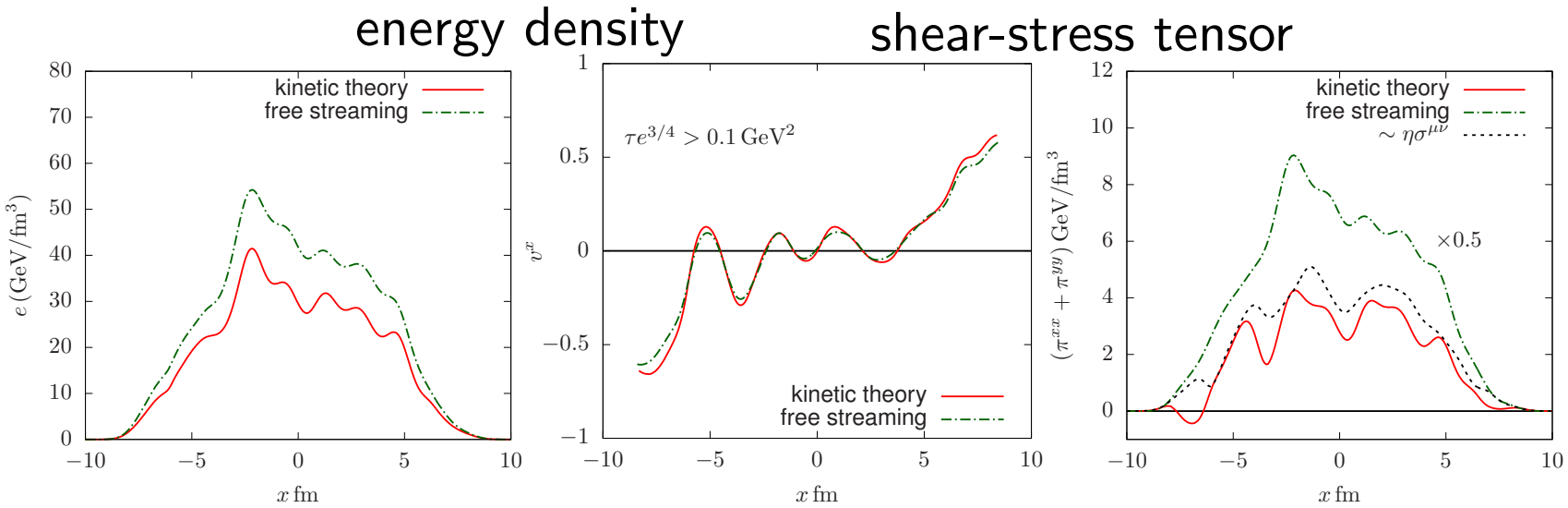


Note: MC-Glauber initial conditions

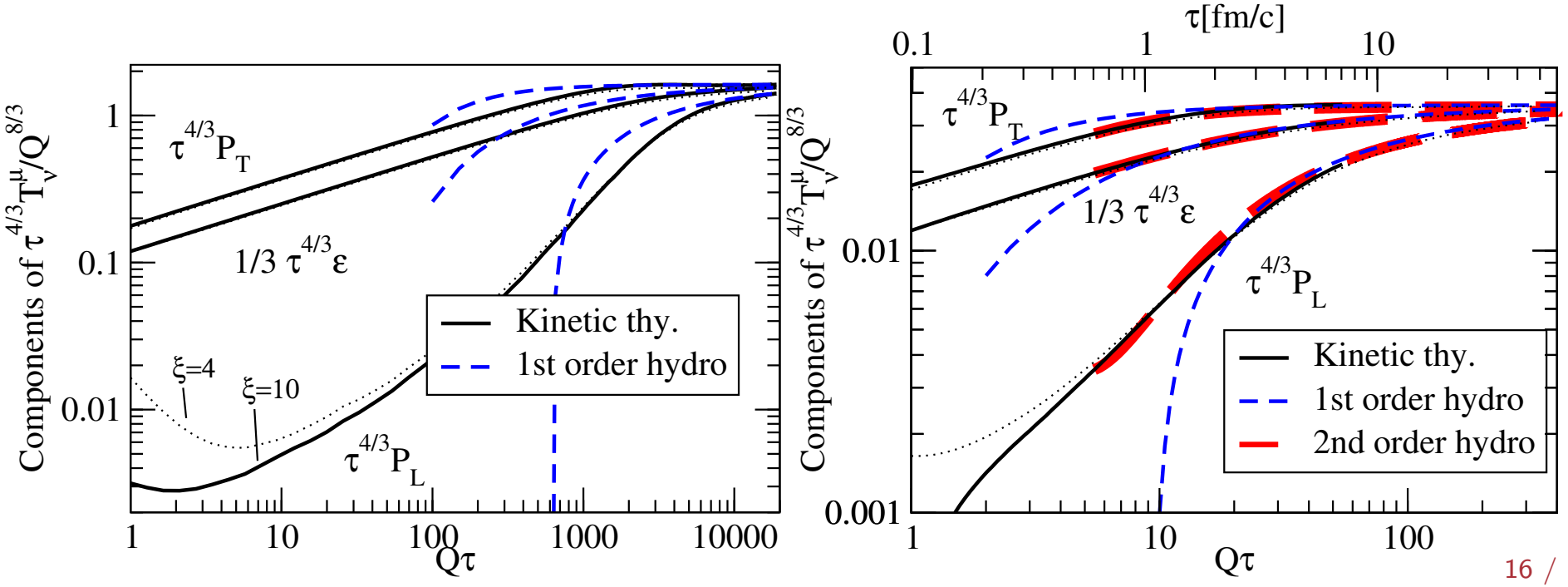
Initial gluon distribution

Initial gluon distribution parametrization at $\tau_0 = 1/Q_s$ Kurkela and Zhu (2015)[15]

$$f_0(p_z, p_\perp) = \frac{2A}{\lambda} \frac{Q_0}{\sqrt{p_\perp^2 + \xi^2 p_z^2}} e^{-\frac{2}{3}\frac{(p_\perp^2 + \xi^2 p_z^2)}{Q_0^2}},$$

where A and Q_0 reproduce co-moving gluon energy and typical momentum from classical lattice simulations Lappi (2011) [14]

$$\tau_0 e \approx \langle p_T \rangle_{\text{lattice}} \frac{dN_{\text{lattice}}}{d^2\mathbf{x}_\perp dy} \qquad \langle p_\perp^2 \rangle \approx \langle p_T \rangle^2_{\text{lattice}},$$



Bibliography I

[1] R. Baier, Alfred H. Mueller, D. Schiff, and D. T. Son.
'Bottom up' thermalization in heavy ion collisions.
Phys. Lett., B502:51–58, 2001, hep-ph/0009237.

[2] Peter Brockway Arnold, Guy D. Moore, and Laurence G. Yaffe.
Effective kinetic theory for high temperature gauge theories.
JHEP, 01:030, 2003, hep-ph/0209353.

[3] Liam Keegan, Aleksi Kurkela, Paul Romatschke, Wilke van der Schee, and Yan Zhu.
Weak and strong coupling equilibration in nonabelian gauge theories.
JHEP, 04:031, 2016, 1512.05347.

[4] Juergen Berges, Kirill Boguslavski, Soeren Schlichting, and Raju Venugopalan.
Universal attractor in a highly occupied non-Abelian plasma.
Phys. Rev., D89(11):114007, 2014, 1311.3005.

[5] Joshua Vredevoogd and Scott Pratt.
Universal Flow in the First Stage of Relativistic Heavy Ion Collisions.
Phys. Rev., C79:044915, 2009, 0810.4325.

[6] Wilke van der Schee, Paul Romatschke, and Scott Pratt.
Fully Dynamical Simulation of Central Nuclear Collisions.
Phys. Rev. Lett., 111(22):222302, 2013, 1307.2539.

Bibliography II

[7] Wojciech Broniowski, Wojciech Florkowski, Mikolaj Chojnacki, and Adam Kisiel.
Free-streaming approximation in early dynamics of relativistic heavy-ion collisions.
Phys. Rev., C80:034902, 2009, 0812.3393.

[8] Jia Liu, Chun Shen, and Ulrich Heinz.
Pre-equilibrium evolution effects on heavy-ion collision observables.
Phys. Rev., C91(6):064906, 2015, 1504.02160.
[Erratum: Phys. Rev.C92,no.4,049904(2015)].

[9] Aleksi Kurkela, Aleksas Mazeliauskas, Jean-François Paquet, Sören Schlichting, and Derek Teaney.
Matching the non-equilibrium initial stage of heavy ion collisions to hydrodynamics with QCD kinetic theory.
2018, 1805.01604.

[10] Aleksi Kurkela, Aleksas Mazeliauskas, Jean-François Paquet, Sören Schlichting, and Derek Teaney.
Effective kinetic description of event-by-event pre-equilibrium dynamics in high-energy heavy-ion collisions.
2018, 1805.00961.

[11] Michal P. Heller, Aleksi Kurkela, and Michal Spalinski.
Hydrodynamization and transient modes of expanding plasma in kinetic theory.
2016, 1609.04803.

Bibliography III

[12] Gökçe Başar and Derek Teaney.
Scaling relation between pA and AA collisions.
Phys. Rev., C90(5):054903, 2014, 1312.6770.

[13] Sören Schlichting and Prithwish Tribedy.
Collectivity in Small Collision Systems: An Initial-State Perspective.
Adv. High Energy Phys., 2016:8460349, 2016, 1611.00329.

[14] T. Lappi.
Gluon spectrum in the glasma from JIMWLK evolution.
Phys. Lett., B703:325–330, 2011, 1105.5511.

[15] Aleksi Kurkela and Yan Zhu.
Isotropization and hydrodynamization in weakly coupled heavy-ion collisions.
Phys. Rev. Lett., 115(18):182301, 2015, 1506.06647.