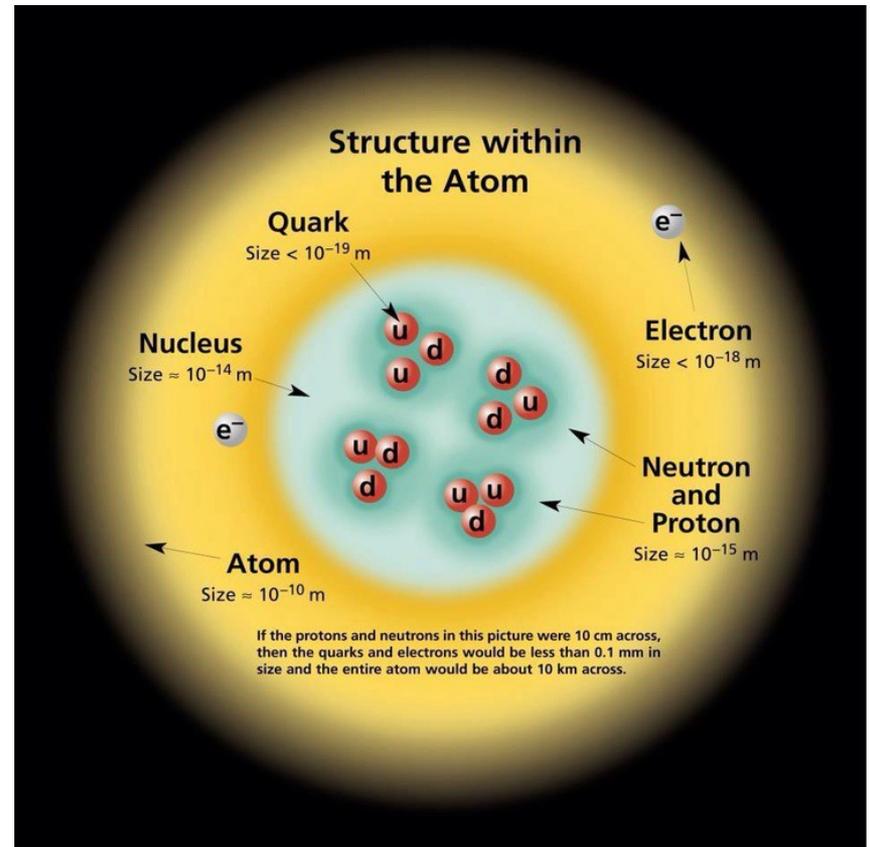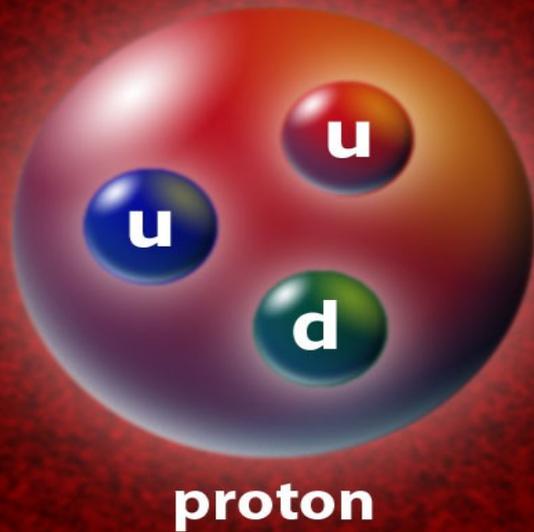# Low x QCD

Jamal Jalilian-Marian

Baruch College, City University of New York
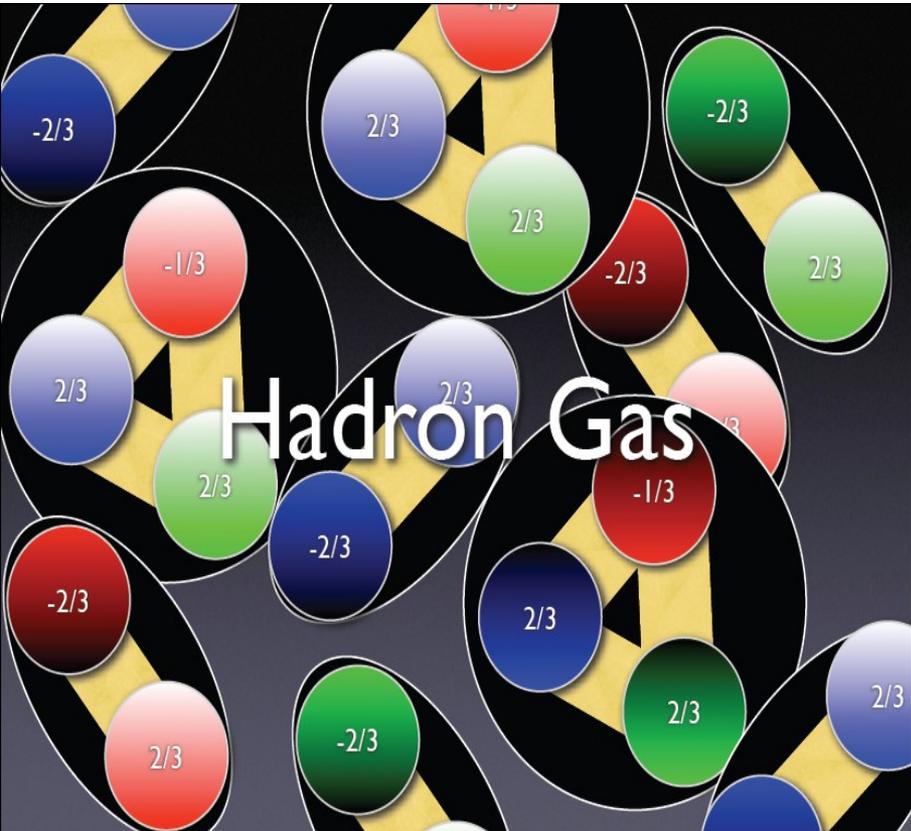
New York NY

Quantum ChromoDynamics (QCD)

strong force confining quarks inside a proton
(and keeping protons inside a nucleus)

Low T

High T

Hadron Gas

Quark-Gluon Plasma

Quantum ChromoDynamics (QCD)

theory of interactions between quarks and gluons

$$SU(N_c)$$

with $N_c = 3$

$$\mathcal{L} = -\frac{1}{4} G^a_{\mu\nu}\, G^{\mu\nu}_a + \overset{f}{\Sigma}\, \bar{\Psi}^{\alpha}_i \left[i\slashed{D} - m_f \right]^{ij}_{\alpha\beta} \Psi^{\beta}_j$$

$$G^a_{\mu\nu}(x) \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - \textcolor{red}{gf^{abc} A^b_\mu A^c_\nu}$$

$$a, b, c = 1, \cdots, 8$$

color index:
$$\alpha, \beta = 1, 2, 3$$

f^{abc} *group structure constants*

Lorentz index: $\mu, \nu = 0, 1, 2, 3$

$\slashed{D} \equiv D_\mu \gamma^\mu$ *with* $\{\gamma^\mu, \gamma^\nu\} = 2\, g^{\mu\nu}$

spinor index: $i, j = 1, 2, 3, 4$

$D_\mu \equiv \partial_\mu + \textcolor{red}{igA_\mu}$ *covariant derivative*

quarks:
fermions, spin 1/2
4x1 spinor, come in N_c colors
6 flavors (up, down, ..., top)
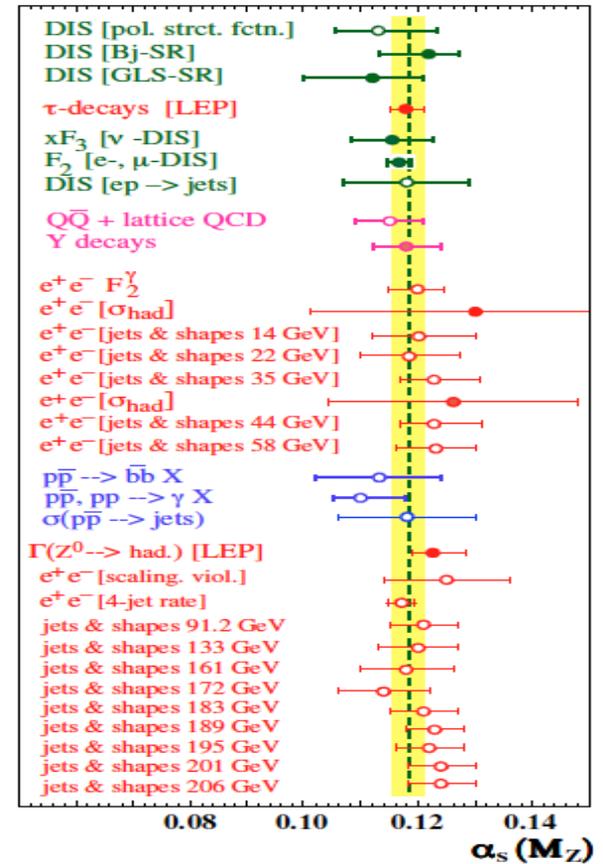*carry electric charge*
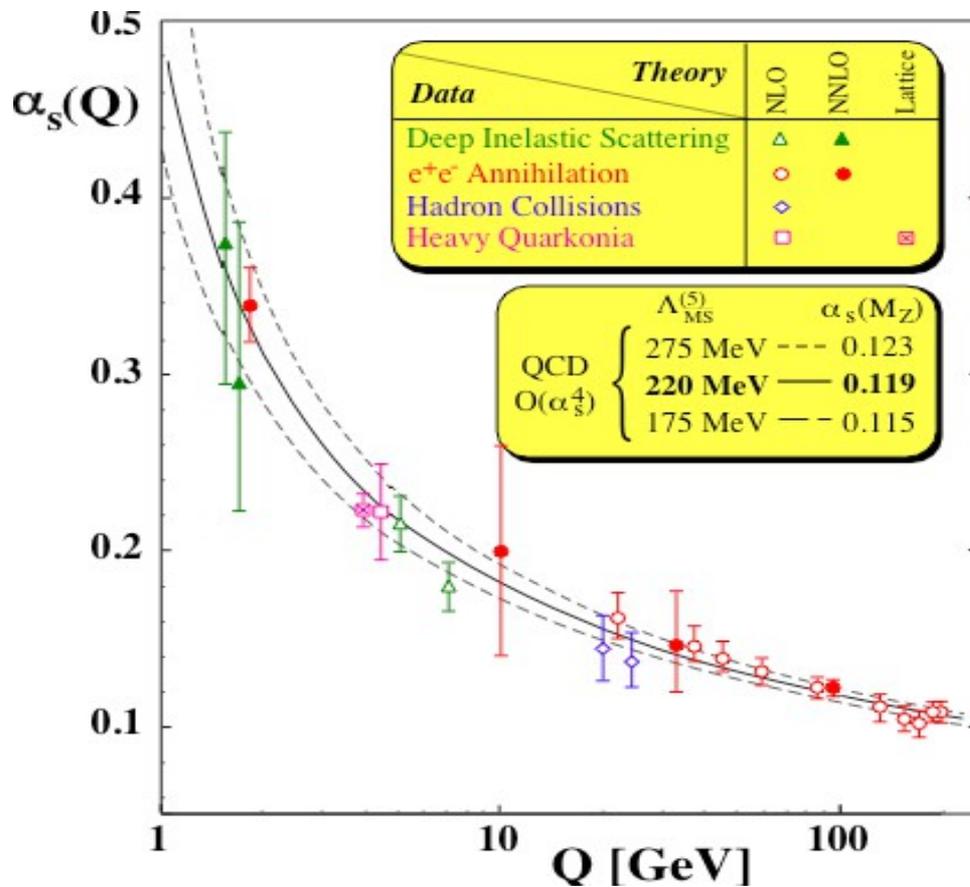
gluons:
bosons, spin 1
come in $N_c^2 - 1$ colors
flavor blind
have no electric charge
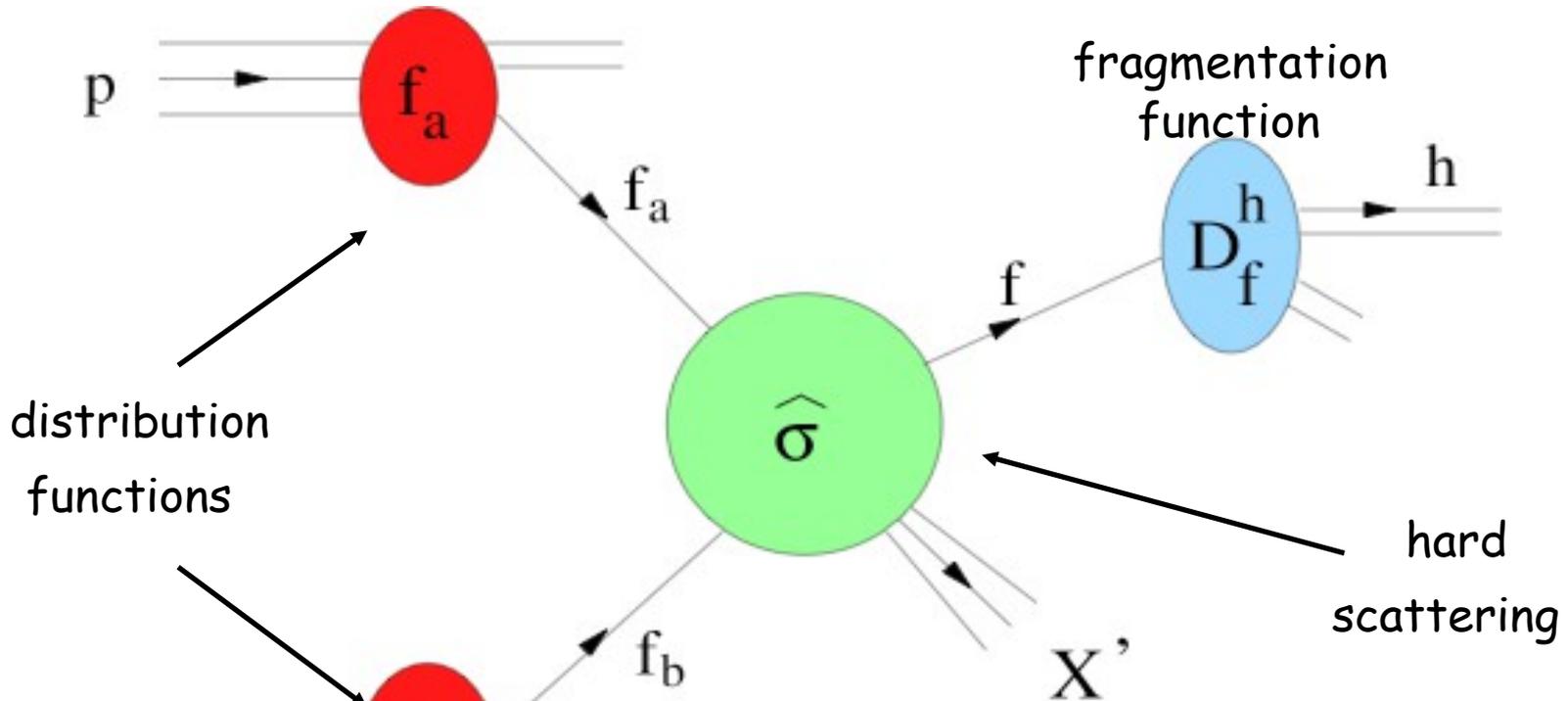
Perturbative QCD

running of the coupling constant

expansion in powers of the coupling $\qquad \alpha_s \ll 1$

pQCD in pp Collisions

collinear factorization: separation of soft (long distance) and hard (short distance)

fragmentation function

distribution functions

hard scattering

$$x \equiv \frac{p}{P}$$

$$\frac{d\sigma^{pp \to h\, X}}{d^2 p_t\, dy} \sim f_a(x_1) \otimes f_b(x_2) \otimes \hat{\sigma} \otimes D_f^h(z) + \cdots\cdots$$

power corrections
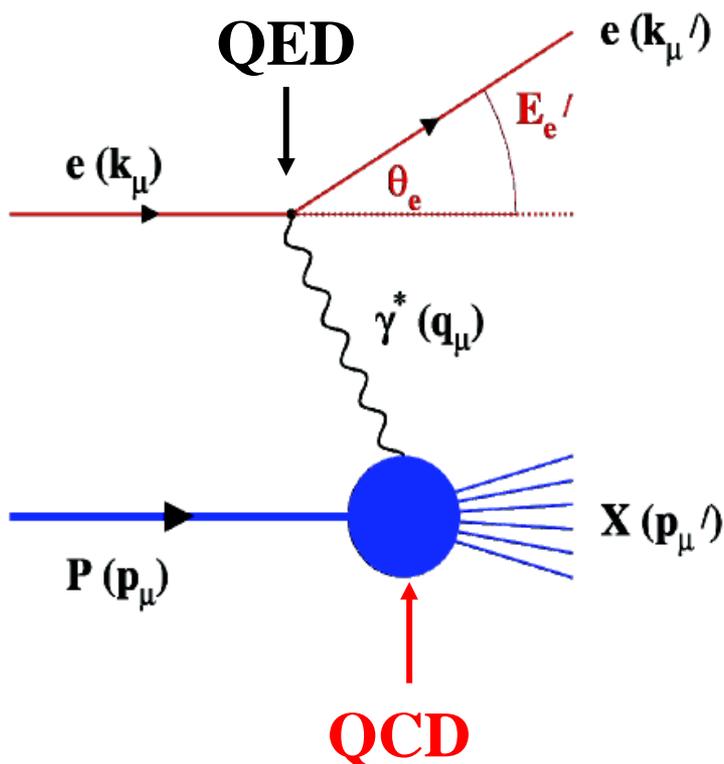
Deep Inelastic Scattering (DIS)
probing hadron structure

Kinematic Invariants

QED

$e\,(k_\mu')$

E_e'

$e\,(k_\mu)$

θ_e

$\gamma^*\,(q_\mu)$

$\mathbf{X\,(p_\mu')}$

$\mathbf{P\,(p_\mu)}$

QCD

(structure functions)

$$Q^2 = -q^2 = -(k_\mu - k_\mu')^2$$

$$Q^2 = 4E_e E_e' \sin^2\!\left(\frac{\theta_e'}{2}\right)$$

$$y = \frac{pq}{pk} = 1 - \frac{E_e'}{E_e}\cos^2\!\left(\frac{\theta_e'}{2}\right)$$

$$x = \frac{Q^2}{2pq} = \frac{Q^2}{sy}$$

$$\mathbf{s} \equiv (\mathbf{p} + \mathbf{k})^2$$

Measure of resolution power

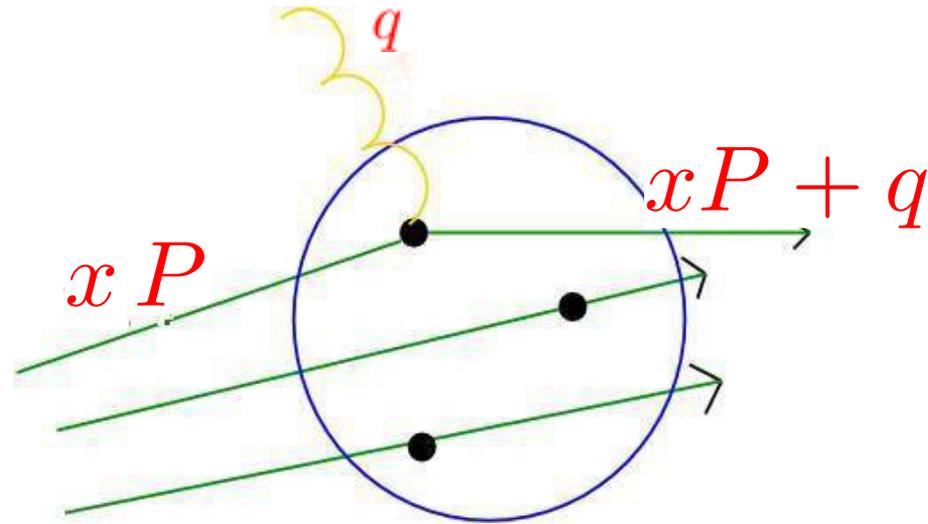Measure of inelasticity

Measure of momentum fraction of struck quark

what is inside a hadron: parton model

Bjorken: $Q^2, S \to \infty$ but $x_{Bj} = \dfrac{Q^2}{S}$ fixed (scaling limit)

structure functions
depend only on x_{Bj}

Feynman:

parton constituents of
proton are "free" on time scale
$1/Q << 1/\Lambda$ (interaction
time scale between partons)

q

xP

$xP + q$

x_{Bj} *fraction of hadron momentum carried by a parton =* x

general structure of the QCD corrections [O(α_s)]

using small quark/gluon mass as a regulator:

$$\frac{d^2\hat{\sigma}}{dx\,dQ^2}\Big|_{F_2} \equiv \hat{F}_2^q$$

$$= e_q^2 x \left[\underset{\text{LO}}{\delta(1-x)} + \frac{\alpha_s(\mu_r)}{4\pi}\left[P_{qq}(x)\ln\frac{Q^2}{m_g^2} + C_2^q(x) \right]\right]$$
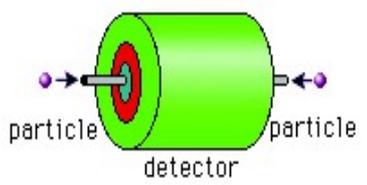
large logarithms
(collinear emission)

finite
coefficients

$$\frac{d^2\hat{\sigma}}{dx\,dQ^2}\Big|_{F_2} \equiv \hat{F}_2^g$$

$$= \sum_q e_q^2 x \left[0 + \frac{\alpha_s(\mu_r)}{4\pi}\left[P_{qg}(x)\ln\frac{Q^2}{m_q^2} + C_2^g(x) \right]\right]$$

divergences absorbed in pdf

$$\mathbf{F_2(x, Q^2)} \equiv \overset{f}{\sum} \mathbf{e_f^2 \, x [q_f(x, Q^2) + \bar{q}_f(x, Q^2)]}$$

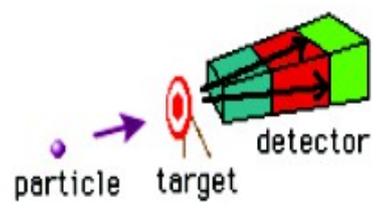# DGLAP "evolution" equation

$$\frac{d}{d\ln\mu}\begin{pmatrix} q(x,\mu) \\ g(x,\mu) \end{pmatrix} = \int_x^1 \frac{dz}{z} \begin{pmatrix} \mathcal{P}_{qq} & \mathcal{P}_{qg} \\ \mathcal{P}_{gq} & \mathcal{P}_{gg} \end{pmatrix}_{(z,\alpha_s)} \cdot \begin{pmatrix} q(x/z,\mu) \\ g(x/z),\mu) \end{pmatrix}$$

☐ Collider experiment: Electron-Proton collisions at HERA (DESY, Hamburg, Germany)
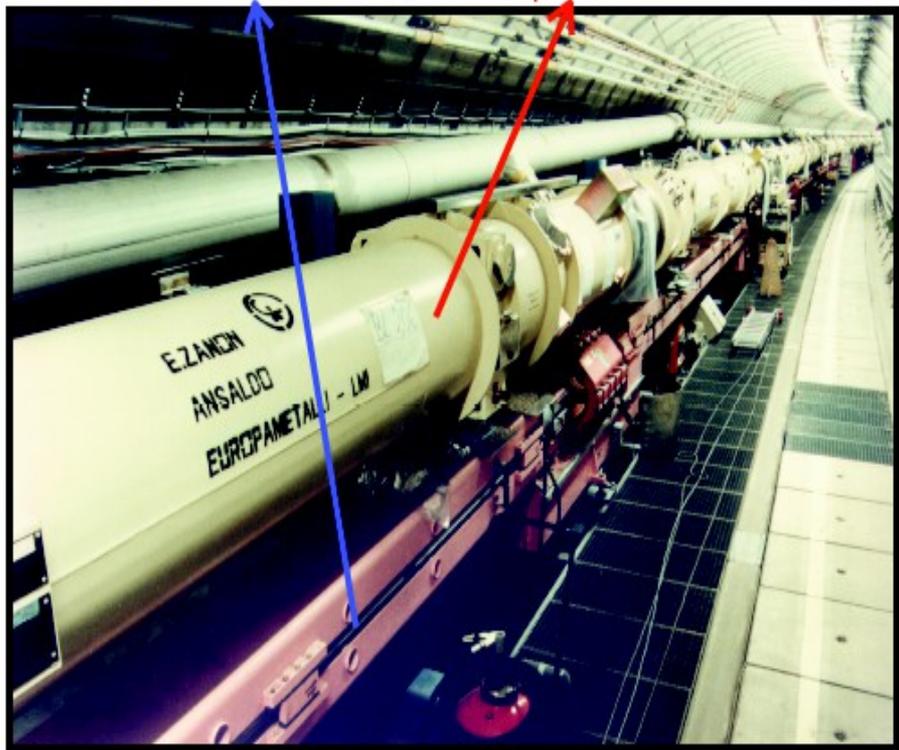

particle — detector — particle

Colliding-Beam Experiments

Equivalent to fixed target of

$E_e = 50600$ GeV:

particle → target — detector

Fixed-Target Experiments

$E_e = 27.5$ GeV $E_p = 920$ GeV

E.ZANON ANSALDO EUROPAMETALLI – LMI

Circumference: 6.3km

HERA

PETRA

A DIS event

$Q^2 = 25030 \text{ GeV}^2, \quad y = 0.56, \quad x=0.50$

proton

e$^+$

e$^+$

jet

e$^+$

Q^2

X

proton

jet

H1   Run 122145   Event 69506
Date 19/09/1995

z

Deep Inelastic Scattering

$$F_2 \equiv \sum_{f=q,\bar{q}} e_f^2 \, xq(x, Q^2)$$

QCD: _scaling violations_

early experiments (SLAC,...):
scale invariance of hadron structure

$$x = \frac{p^+}{P^+}$$

x is the fraction of hadron energy carried by a parton

What drives the growth of parton distributions?

Splitting functions at leading order $O(\alpha_s^0)$ $\qquad$ $(x \neq 1)$
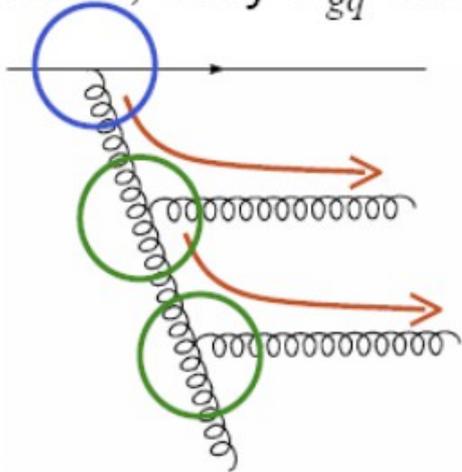
$$P_{qq}^{(0)}(x) = C_F \frac{1+x^2}{1-x}$$

$$P_{qg}^{(0)}(x) = \frac{1}{2}\left[x^2 + (1-x)^2\right]$$

$$P_{gq}^{(0)}(x) = C_F \frac{1+(1-x)^2}{x}$$

$$P_{gg}^{(0)}(x) = 2C_A\left[\frac{x}{1-x} + \frac{1-x}{x} + x(1-x)\right]$$
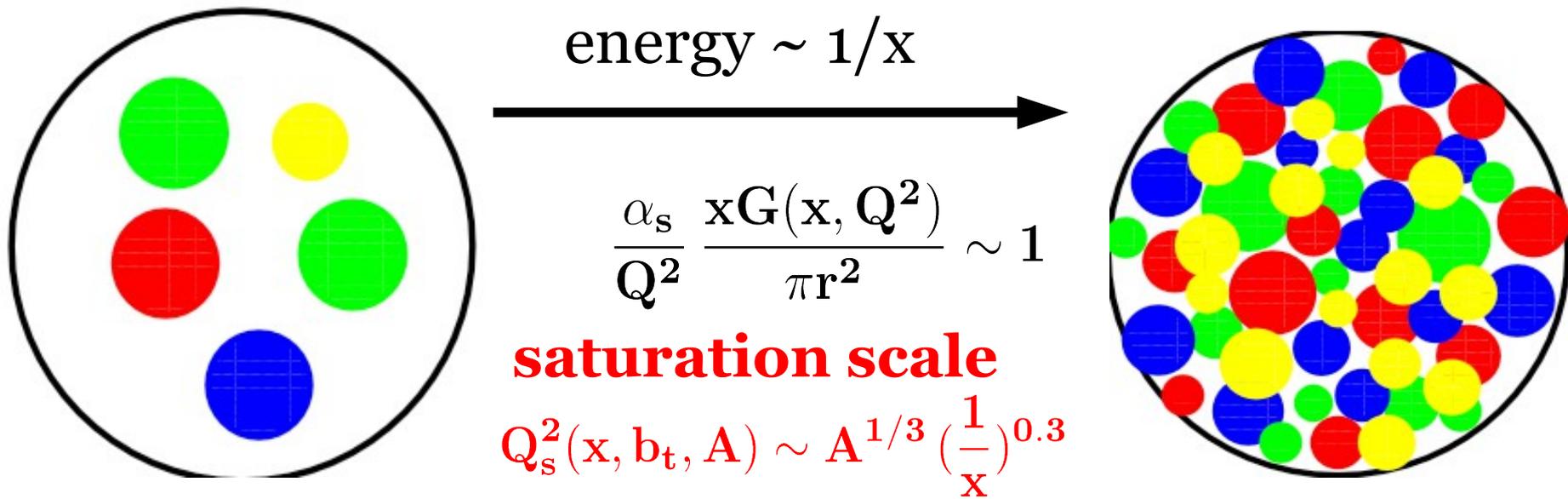
At small x, only P_{gq} and P_{gg} are relevant.

→ Gluon dominant at small x!

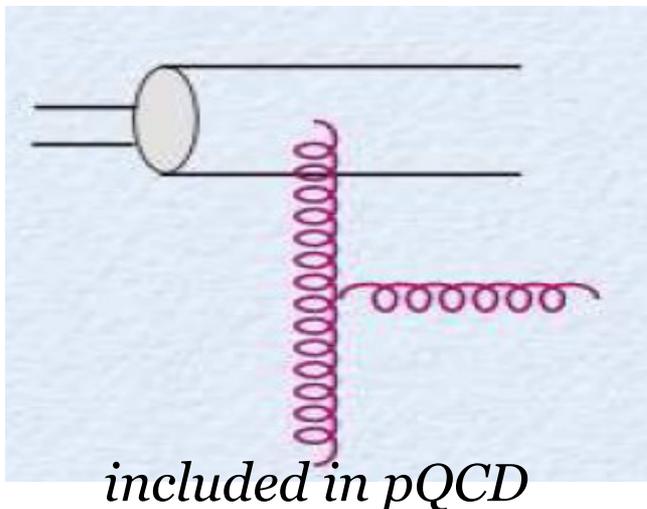The double log approximation (DLA) of DGLAP is easily solved.

-- increase of gluon distribution at small x

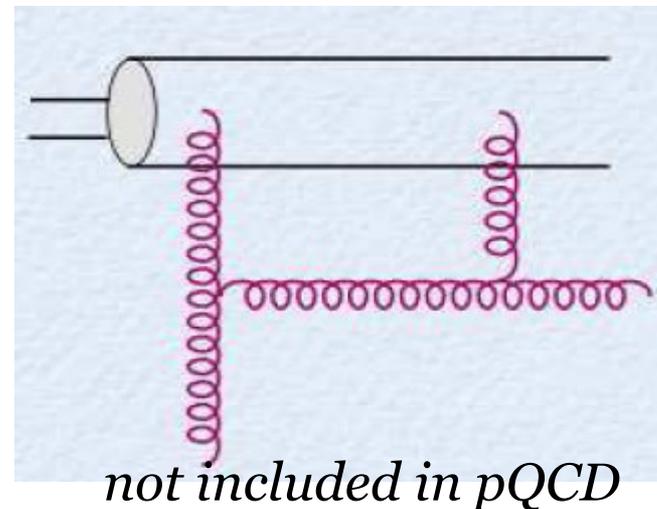$$xg(x, Q^2) \sim e^{\sqrt{\alpha_s\,(\log 1/x)\,(\log Q^2)}}$$

energy ~ 1/x

$$\frac{\alpha_s}{Q^2} \frac{xG(x, Q^2)}{\pi r^2} \sim 1$$

saturation scale

$$Q_s^2(x, b_t, A) \sim A^{1/3} \left(\frac{1}{x}\right)^{0.3}$$

<u>collinear factorization breaks down at small x</u>

"attractive" bremsstrahlung   vs.   "repulsive" recombination

$$S \to \infty, \ Q^2 \ fixed$$

$$x_{Bj} \equiv \frac{Q^2}{S} \to 0$$

included in pQCD

not included in pQCD

Low x QCD:
many-body dynamics of universal gluonic matter (CGC)

How does this happen ?

How do correlation functions of these evolve ?

Are there scaling laws ?

Can CGC explain aspects of HIC ?
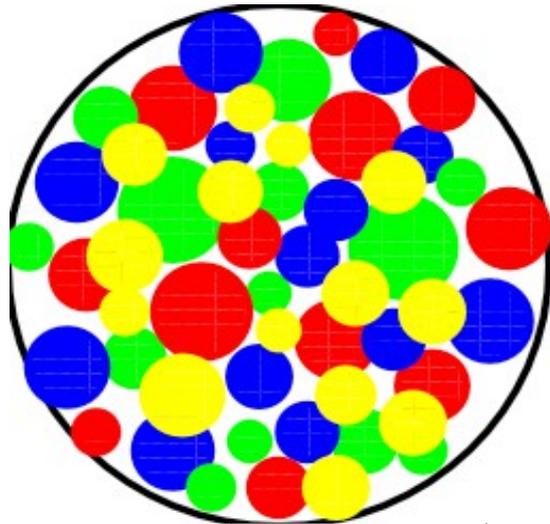
Initial conditions for hydro?
Thermalization ?
Long range rapidity correlations ?
Azimuthal angular correlations ?
Nuclear modification factor ?

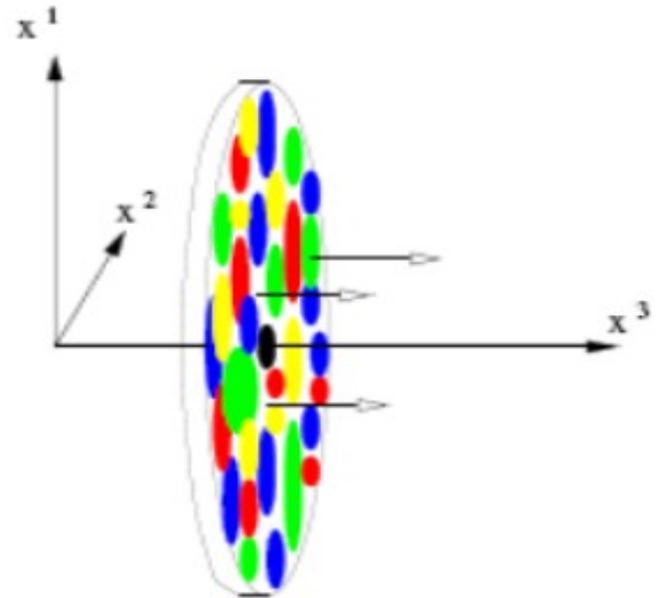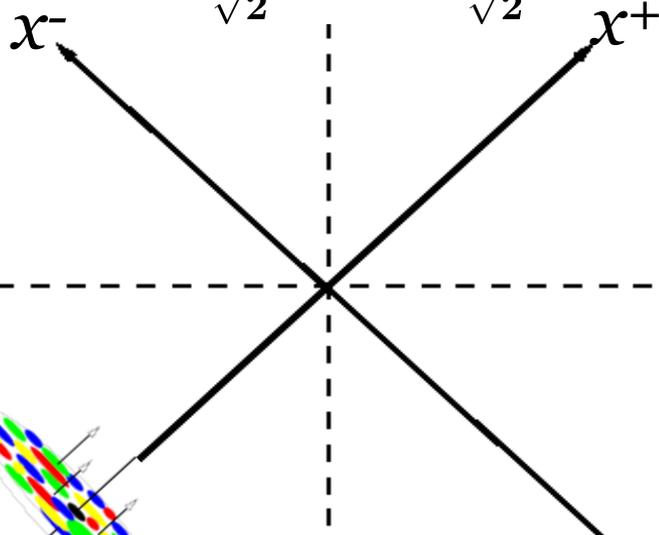# a very large nucleus at high energy: MV model

boost

$$R \rightarrow \frac{R}{\gamma}$$
$$\gamma \sim 100 \quad \text{RHIC}$$
$$\gamma \sim 2500 \quad \text{LHC}$$

$$x^+ \equiv \frac{t+z}{\sqrt{2}} \qquad x^- \equiv \frac{t-z}{\sqrt{2}}$$

sheet of color charge moving along x^+ and sitting at $x^- = 0$

$$\boxed{\mathbf{J}_\mathbf{a}^\mu(\mathbf{x}) \equiv \delta^{\mu\,+}\,\delta(\mathbf{x}^-)\,\rho_\mathbf{a}(\mathbf{x_t})}$$

color current   color charge

$$\mathbf{A_i^a}(\mathbf{x}^-,\mathbf{x_t}) = \theta(\mathbf{x}^-)\,\alpha_\mathbf{i}^\mathbf{a}(\mathbf{x_t})$$

with $\quad \partial_i\,\alpha_i^a = g\,\rho^a$
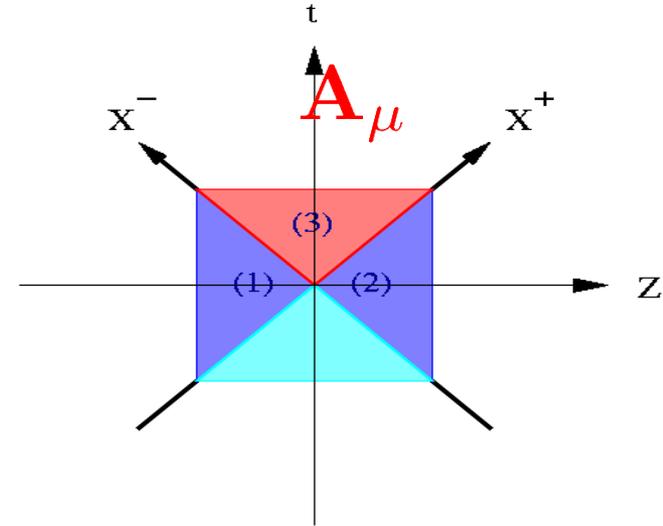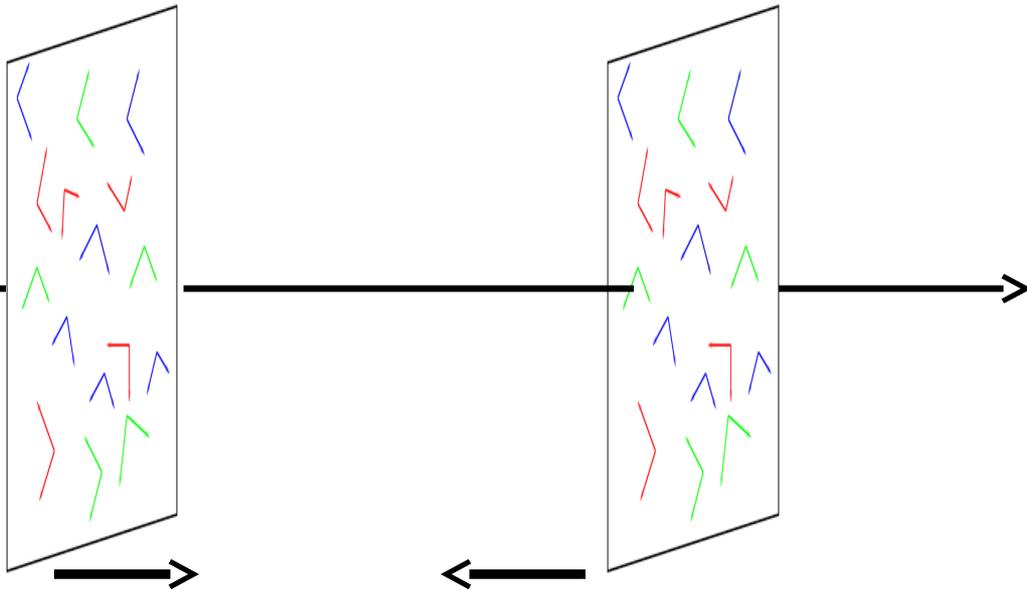
a very large nucleus at high energy: MV model

J.D. Jackson
Classical Electrodynamics

x^+

$x^- = 0$

*random **color Electric & Magnetic fields** in the plane of the fast moving nucleus*

at rest

boost factor 3

Electric field *of a point charge at high energy*

$$F_a^{+i} \sim \delta(x^-)\,\alpha_a^i(x_t)$$

high energy nucleus-nucleus collisions:
colliding sheets of Color Glass Condensates

before the collision:

$$A^+ = A^- = 0$$

$$A_1^i = \theta(x^-)\,\theta(-x^+)\,\alpha_1^i$$

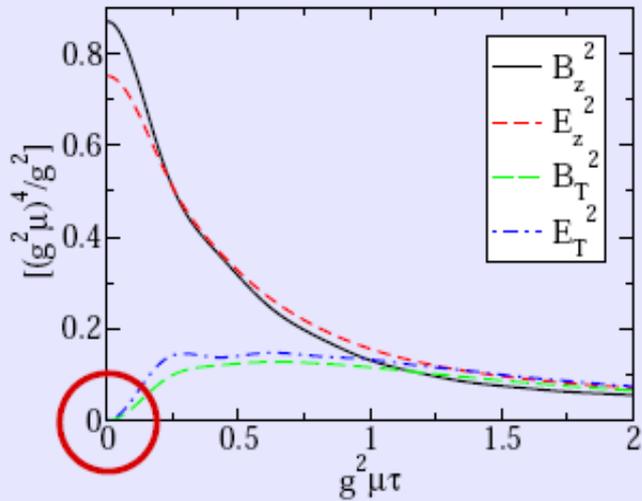$$A_2^i = \theta(-x^-)\,\theta(x^+)\,\alpha_2^i$$

$$A^i = A_1^i + A_2^i$$

after the collision:

solve for $\mathbf{A}_\mu$

in the forward LC

GLASMA:

gluon fields produced after collision of two sheets of color glass

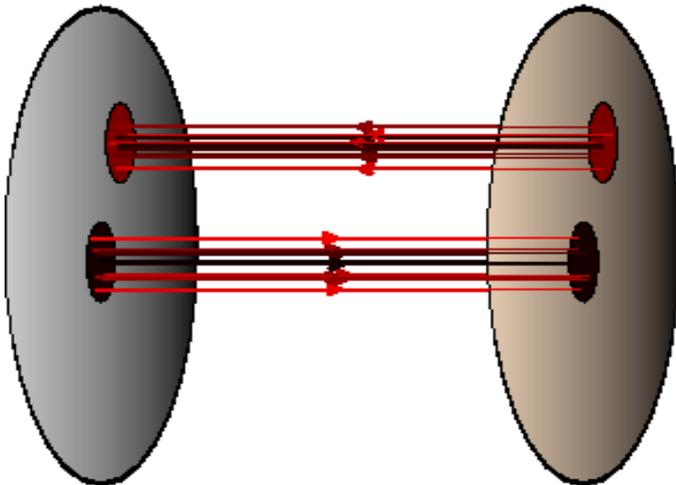early on glasma fields (E and B) are longitudinal

classical solutions are boost invariant
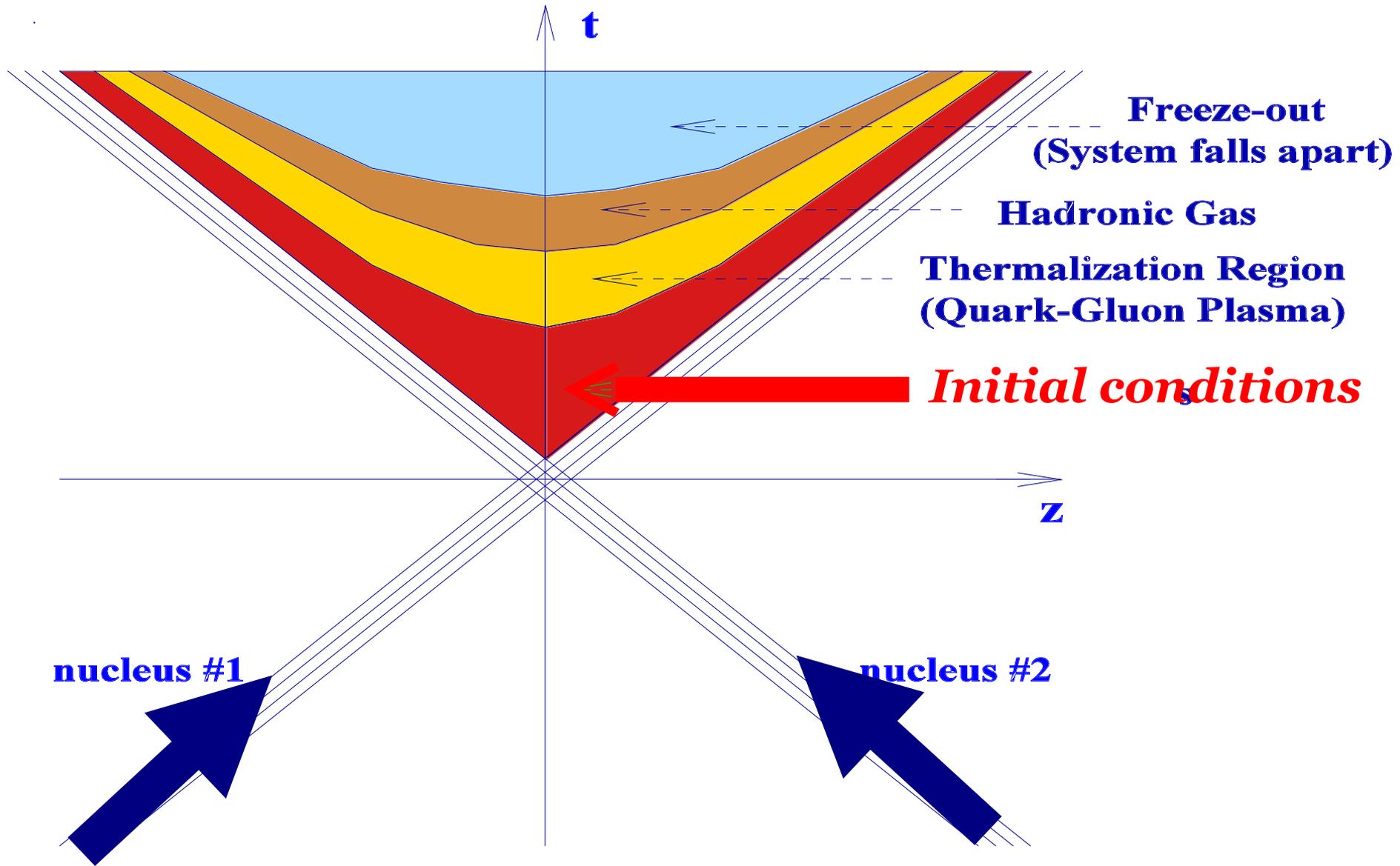
transverse size of the flux tubes is $\sim \dfrac{1}{\mathbf{Q_s}}$

$$\frac{1}{A_\perp}\frac{dN}{d\eta} = \frac{0.3}{g^2}Q_s^2$$

$$\frac{1}{A_\perp}\frac{dE_\perp}{d\eta} = \frac{0.25}{g^2}Q_s^3$$

Space-Time History of a Heavy Ion Collision

eliminate/minimize medium effects
*dilute-dense collisions (**proton-nucleus**)*

Eikonal approximation

$$J_a^\mu \simeq \delta^{\mu-} \rho_a$$

$$D_\mu J^\mu \;=\; D_- J^- = 0$$

$$\partial_- J^- \;=\; 0 \qquad \text{(in A}^+ = 0 \text{ gauge)}$$

does not depend on x⁻

solution to classical EOM:
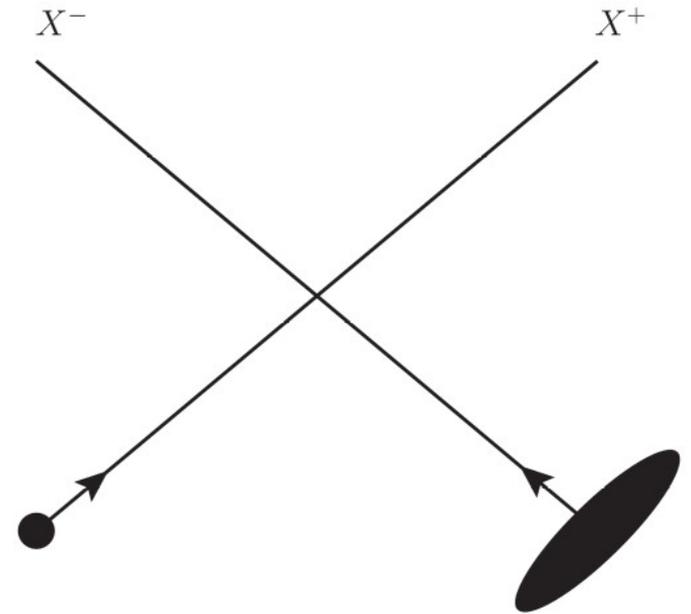
$$A_a^-(x^+, x_t) \equiv n^- S_a(x^+, x_t)$$

with
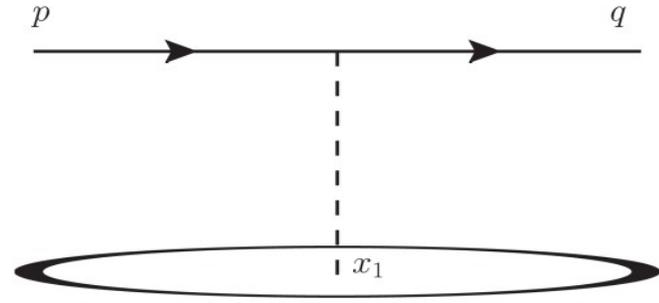
$$n^\mu \;=\; (n^+ = 0, n^- = 1, n_\perp = 0)$$

$$n^2 \;=\; 2\, n^+ n^- - n_\perp^2 = 0$$
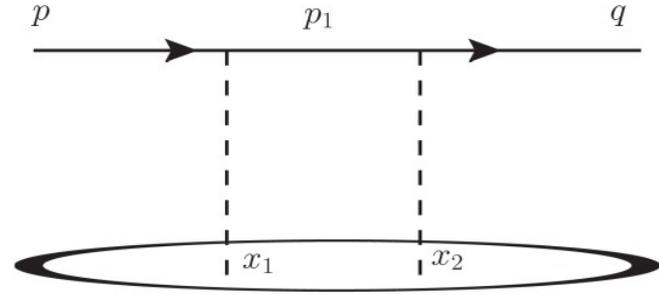
scattering of a quark from background color field $\qquad A_a^-(x^+, x_t)$

X^- $\qquad X^+$

$$i\mathcal{M}_1 = (ig) \int d^4x_1 \, e^{i(q-p)x_1} \, \bar{u}(q) \left[\not{n} \, S(x_1) \right] u(p)$$

$$= (ig)(2\pi)\delta(p^+ - q^+)\int d^2x_{1t} \, dx_1^+ \, e^{i(q^- - p^-)x_1^+} \, e^{-i(q_t - p_t)x_{1t}}$$

$$\bar{u}(q) \left[\not{n} \, S(x_1^+, x_{1t}) \right] u(p)$$

$$i\mathcal{M}_2 = (ig)^2 \int d^4x_1 \, d^4x_2 \int \frac{d^4p_1}{(2\pi)^4} \, e^{i(p_1-p)x_1} \, e^{i(q-p_1)x_2}$$

$$\bar{u}(q) \left[\not{n} \, S(x_2) \frac{i\not{p}_1}{p_1^2 + i\epsilon} \, \not{n} \, S(x_1) \right] u(p)$$
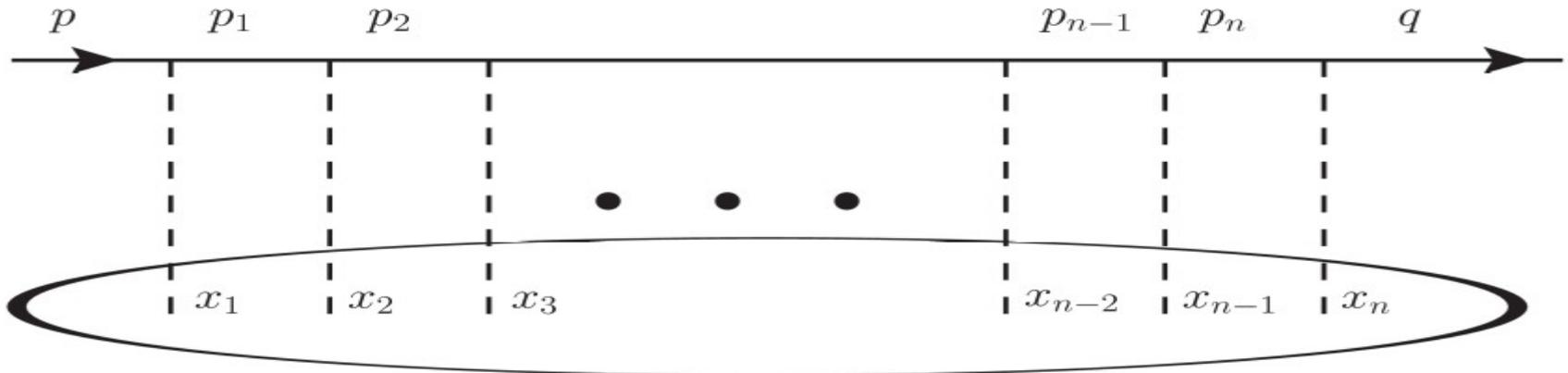

$$\int \frac{dp_1^-}{(2\pi)} \frac{e^{ip_1^-(x_1^+ - x_2^+)}}{2p^+ \left[p_1^- - \frac{p_{1t}^2 - i\epsilon}{2p^+} \right]} = \frac{-i}{2p^+} \, \theta(x_2^+ - x_1^+) \, e^{i\frac{p_{1t}^2}{2p^+}(x_1^+ - x_2^+)}$$

contour integration over the pole leads to
path ordering of scattering

ignore all terms: $\quad O(\frac{p_t}{p^+}, \frac{q_t}{q^+}) \quad$ and use $\quad \not{n} \frac{\not{p}_1}{2n \cdot p} \not{n} = \not{n}$

$$i\mathcal{M}_2 = (ig)^2 \, (-i)(i) \, 2\pi\delta(p^+ - q^+) \int dx_1^+ \, dx_2^+ \, \theta(x_2^+ - x_1^+) \int d^2x_{1t} \, e^{-i(q_t - p_t)\cdot x_{1t}}$$

$$\bar{u}(q) \left[S(x_2^+, x_{1t}) \not{n} \, S(x_1^+, x_{1t}) \right] u(p)$$

$$iM_n = 2\pi\delta(p^+ - q^+)\,\bar{u}(q)\,\slashed{n}\int d^2x_t\, e^{-i(q_t - p_t)\cdot x_t}$$

$$\left\{ (ig)^n\,(-i)^n\,(i)^n \int dx_1^+\, dx_2^+ \cdots dx_n^+\, \theta(x_n^+ - x_{n-1}^+)\cdots\theta(x_2^+ - x_1^+) \right.$$

$$\left. \left[S(x_n^+, x_t)\,S(x_{n-1}^+, x_t)\cdots S(x_2^+, x_t)S(x_1^+, x_t) \right] \right\} u(p)$$

sum over all scatterings
$$iM = \sum_n i\,M_n$$

$$iM(p,q) = 2\pi\delta(p^+ - q^+)\,\bar{u}(q)\,\slashed{n}\int d^2x_t\, e^{-i(q_t - p_t)\cdot x_t}\,[V(x_t) - 1]\,u(p)$$

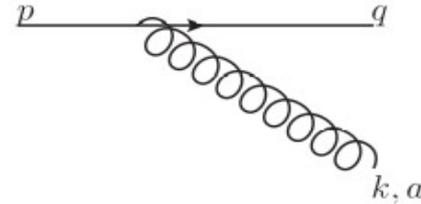with $V(x_t) \equiv \hat{P}\exp\left\{ ig\int_{-\infty}^{+\infty} dx^+\, n^-\, S_a(x^+, x_t)\, t_a \right\}$

$$\frac{d\sigma^{q\,T \to q\,X}}{d^2p_t\, dy} \sim |iM|^2 \sim F.T.\ {\color{red}< Tr\, V(x_t)\, V^\dagger(y_t) >}$$

1-loop correction: energy dependence

basic ingredient: soft radiation vertex (LC gauge)

$$g\,\bar{u}(q)\,t^a\,\gamma_\mu\,u(p)\,\epsilon^\mu_{(\lambda)}(k) \longrightarrow 2\,g\,t^a\,\frac{\epsilon_{(\lambda)}\cdot k_t}{k_t^2}$$
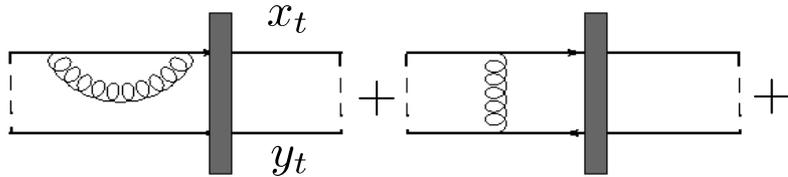
coordinate space:

$$\int \frac{d^2k_t}{(2\pi)^2}e^{ik_t\cdot(x_t-z_t)}2\,g\,t^a\,\frac{\epsilon_{(\lambda)}\cdot k_t}{k_t^2} = \frac{2\,i\,g}{2\pi}\,t^a\,\frac{\epsilon_{(\lambda)}\cdot(x_t-z_t)}{(x_t-z_t)^2}$$
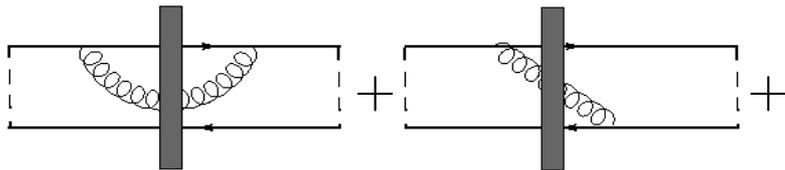
x_t, z_t are transverse coordinates of the quark and gluon

virtual corrections:

$$\longrightarrow Tr\,V(x_t)\,V^\dagger(y_t) \qquad \textcolor{red}{\textit{a dipole}}$$

real corrections:
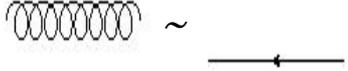

$$\longrightarrow Tr\,V(x_t)\,V^\dagger(z_t)\,Tr\,V(z_t)\,V^\dagger(y_t)$$

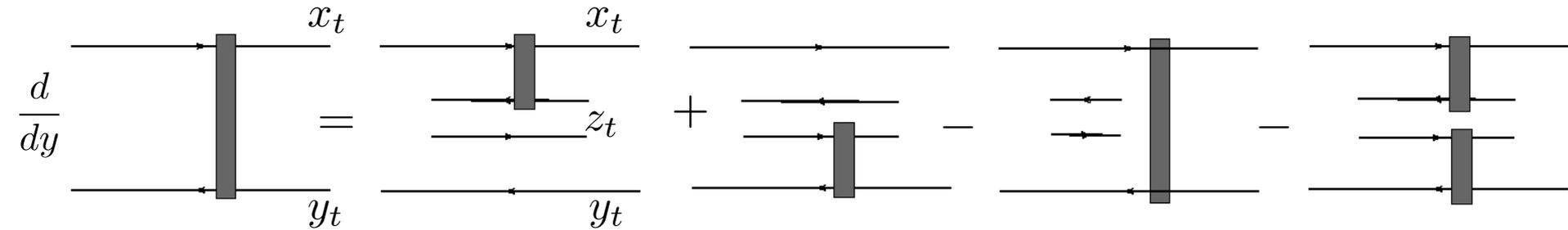$$\frac{1}{(x_t-z_t)^2} \qquad \frac{(x_t-z_t)\cdot(y_t-z_t)}{(x_t-z_t)^2(y_t-z_t)^2}$$

the S matrix
$$S(x_t,y_t)\equiv\frac{1}{N_c}Tr\,V(x_t)\,V^\dagger(y_t)$$

1-loop correction: BK eq.

at large N_c

$$3 \otimes \bar{3} = 8 \oplus 1 \simeq 8$$

$$\frac{d}{dy}T(x_t, y_t) = \frac{N_c \alpha_s}{2\pi^2} \int d^2 z_t \frac{(x_t - y_t)^2}{(x_t - z_t)^2(y_t - z_t)^2} \left[T(x_t, z_t) + T(z_t, y_t) - T(x_t, y_t) - T(x_t, z_t)T(z_t, y_t) \right]$$

$$T \equiv 1 - S$$

$$\tilde{T}(p_t) \sim \frac{1}{p_t^2} \left[\frac{Q_s^2}{p_t^2} \right] \qquad Q_s^2 \ll p_t^2$$

$$\tilde{T}(p_t) \sim \log \left[\frac{Q_s^2}{p_t^2} \right] \qquad Q_s^2 \gg p_t^2$$

$$\tilde{T}(p_t) \sim \frac{1}{p_t^2} \left[\frac{Q_s^2}{p_t^2} \right]^\gamma \qquad Q_s^2 < p_t^2$$

nuclear modification factor

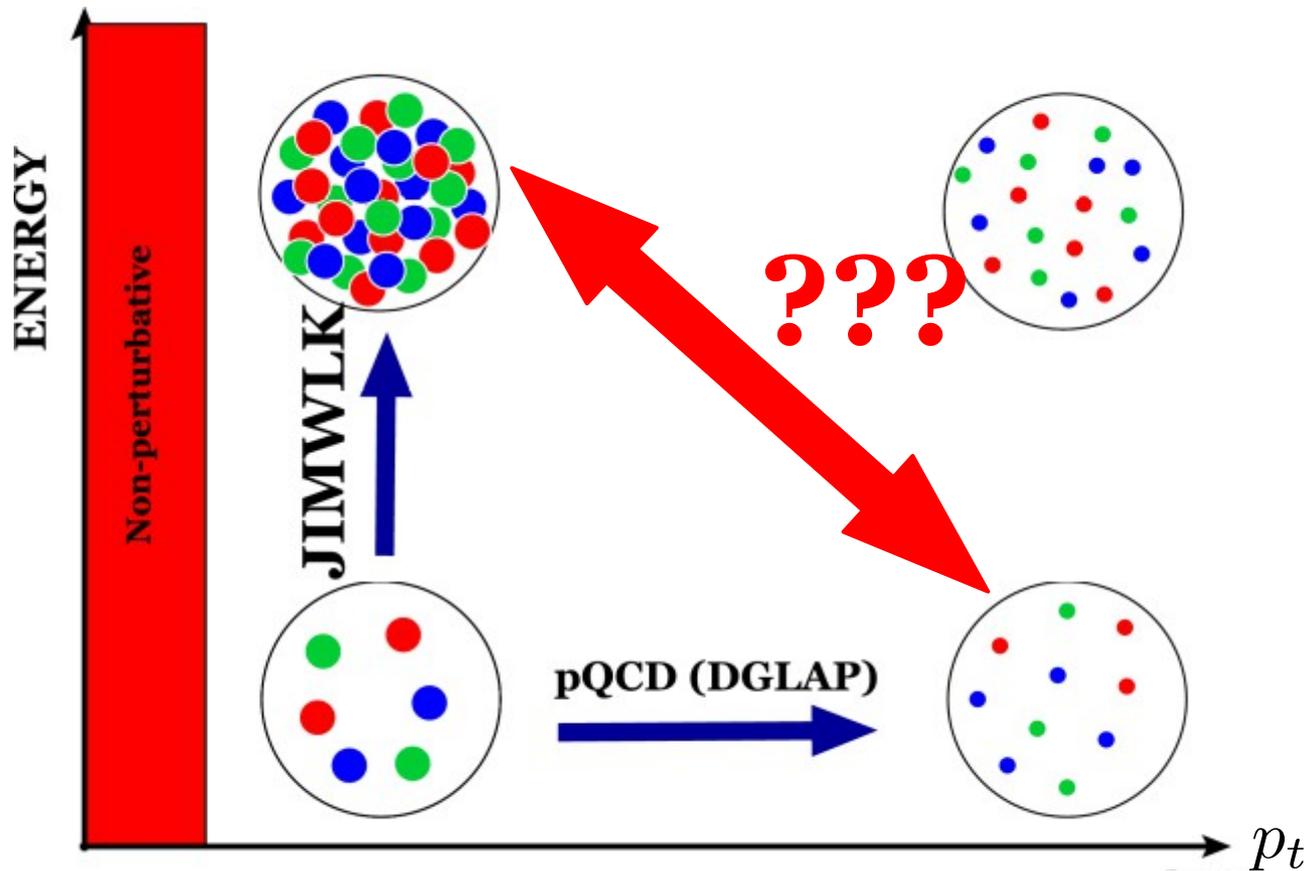$$R_{pA} \equiv \frac{\frac{d\sigma^{pA}}{d^2 p_t \, dy}}{A^{1/3} \frac{d\sigma^{pp}}{d^2 p_t \, dy}}$$

suppression of p_t spectra
nuclear shadowing
centrality dependence
....

QCD kinematic phase space

unifying saturation with large x/high p_t physics ?

jet physics
partially/fully coherent energy loss
interactions of UHE neutrinos, ...