

Nuclear PDFs

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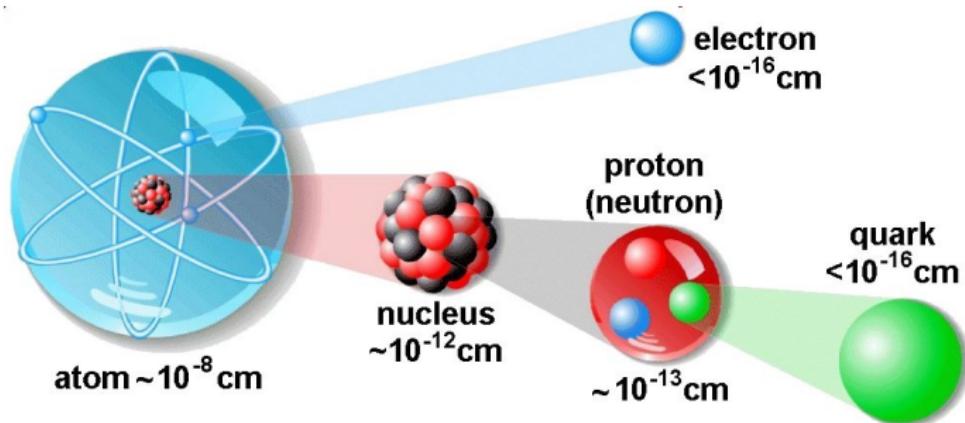


Outline

- ▶ Introduction
- ▶ Proton PDFs and global QCD analysis
- ▶ Nuclear PDFs
- ▶ nCTEQ results
- ▶ LHC heavy quark(onium) data
- ▶ Summary

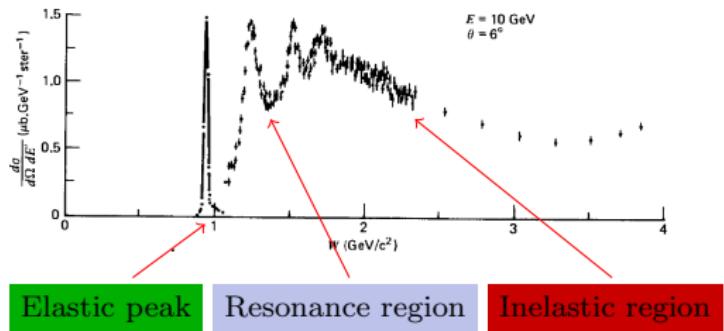
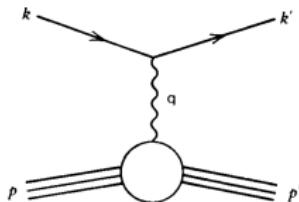
Structure of matter

- ▶ Structure of matter depends on the resolution scale at which it is observed



Structure of matter

- ▶ Until 1960's protons and neutrons were regarded as elementary particles and basic constituents of matter.
- ▶ The first evidence for their inner structure came from inelastic collisions.

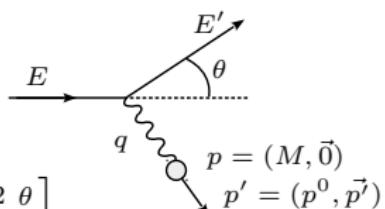


Elastic scattering (spin-1/2 relativistic point-like particle):

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \frac{E'}{E} \left[\cos^2 \frac{\theta}{2} - \frac{q^2}{2M^2} \sin^2 \frac{\theta}{2} \right]$$

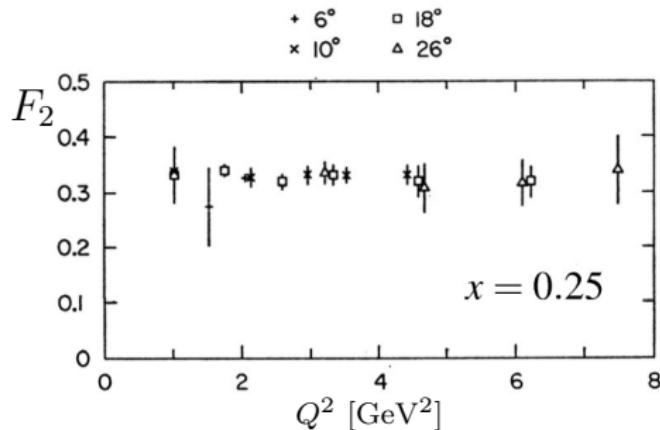
Deep Inelastic Scattering (DIS) :

$$\frac{d\sigma}{dE' d\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \left[\frac{F_2(x, Q^2)}{E - E'} \cos^2 \frac{\theta}{2} + \frac{2F_1(x, Q^2)}{M} \sin^2 \frac{\theta}{2} \right]$$



Structure of matter

- ▶ Until 1960's protons and neutrons were regarded as elementary particles and basic constituents of matter.
- ▶ The first evidence for their inner structure came from inelastic collisions.
- ▶ From the observation of **Bjorken scaling** (cross section becomes approximately independent on the energy scale if proton is made up from point-like particles) in SLAC in late 60's.

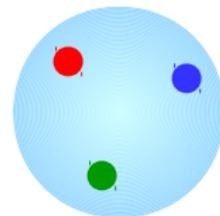


- ▶ Explanation given by Feynman in **parton model** (partons later identified with quarks from Gell-Mann model).

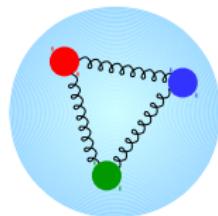
Parton Distribution Functions (PDFs)

PDF $[f_{a/p}(x, \mu)]$: probability that a parton a carries fraction x of proton's momentum (valid at leading-order of QCD).

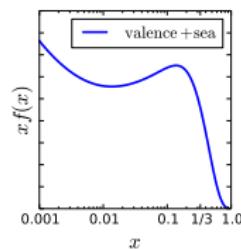
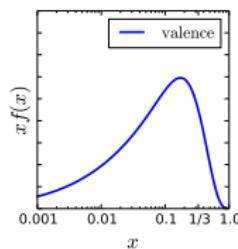
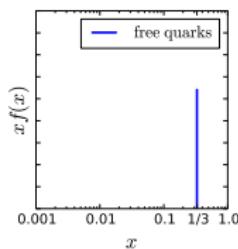
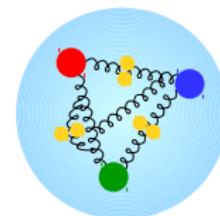
Free quarks



Bound quarks



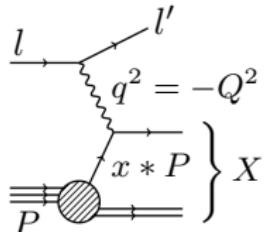
Bound quarks + QCD effects



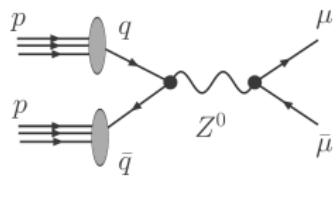
$$x = \frac{\text{longitudinal parton momentum}}{\text{longitudinal nucleon momentum}} = \frac{p_{\text{parton}}^+}{p_{\text{nucleon}}^+}, \quad \text{where} \quad p^\pm = (p^0 \pm p^3)/\sqrt{2}$$

PDFs and QCD Factorization

- **Factorization** in case of **Deep Inelastic Scattering** (DIS)

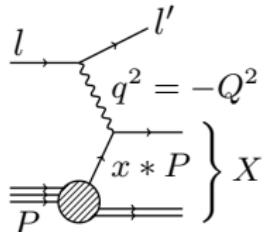

$$\frac{d^2\sigma}{dxdQ^2} = \sum_{i=q,\bar{q},g} \int_x^1 \frac{dz}{z} f_i(z, \mu) d\hat{\sigma}_{il \rightarrow l'X} \left(\frac{x}{z}, \frac{Q}{\mu} \right) + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{Q^2}\right)$$

- **Factorization** in case of **Drell-Yan lepton pair production** (DY)

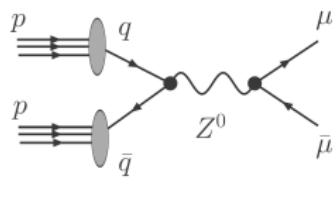

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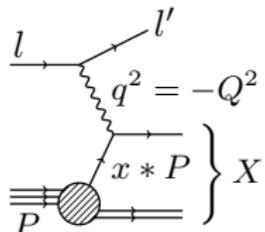
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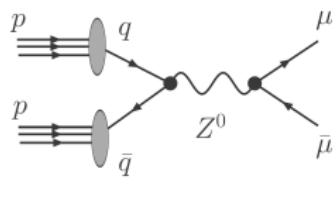
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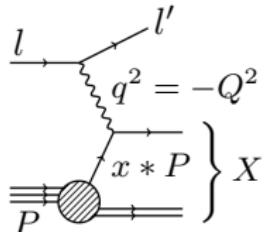

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PDFs are **UNIVERSAL** – do not depend on the process!!!

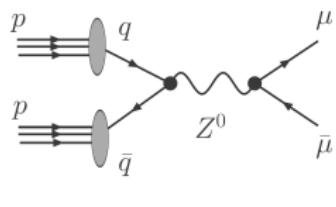
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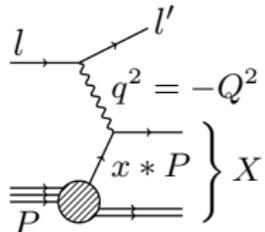
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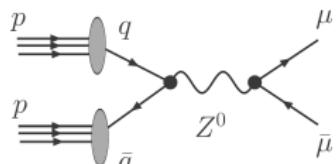
- $f_i(z, \mu)$ – proton PDFs of parton i (**non-perturbative**).
- $\hat{\sigma}$ – parton level matrix element (**calculable in pQCD**).
- $\mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^2}{Q^2}\right)$ – non-leading terms defining accuracy of factorization formula.

PDFs and QCD Factorization

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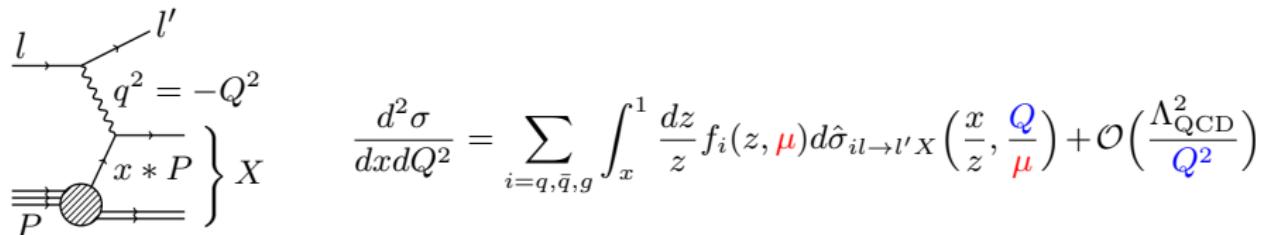
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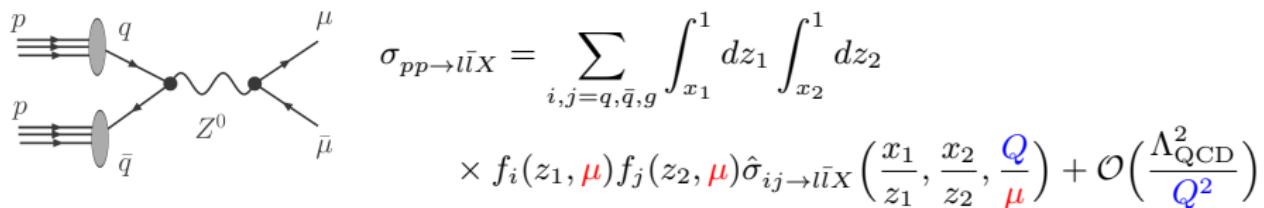
- Q – characteristic energy scale (DIS: 4-momentum transfer $-q^2$, DY: mass of produced Z/γ boson).

PDFs and QCD Factorization

- **Factorization** in case of **Deep Inelastic Scattering** (DIS)


$$q^2 = -Q^2$$
$$\left. \begin{array}{c} l \\ \text{---} \\ l' \\ \text{---} \\ q^2 = -Q^2 \\ \text{---} \\ x * P \\ \text{---} \\ P \end{array} \right\} X$$
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- **Factorization** in case of **Drell-Yan lepton pair production** (DY)


$$p \quad q \quad \mu$$
$$p \quad \bar{q} \quad Z^0 \quad \bar{\mu}$$
$$\sigma_{pp \rightarrow l\bar{l}X} = \sum_{i,j=q,\bar{q},g} \int_{x_1}^1 dz_1 \int_{x_2}^1 dz_2 f_i(z_1, \mu) f_j(z_2, \mu) d\hat{\sigma}_{ij \rightarrow l\bar{l}X} \left(\frac{x_1}{z_1}, \frac{x_2}{z_2}, \frac{Q}{\mu} \right) + \mathcal{O} \left(\frac{\Lambda_{\text{QCD}}^2}{Q^2} \right)$$

- Q – characteristic energy scale (DIS: 4-momentum transfer $-q^2$, DY: mass of produced Z/γ boson).
- μ – factorization scale, naturally set to be of order $\sim Q$ (set to be the same as renormalization scale).

Properties of PDFs

- ▶ **Number sum rules** – connect partons to quarks from SU(3) flavour symmetry of hadrons; proton (uud), neutron (udd). For protons:

$$\int_0^1 dx \underbrace{[f_u(x) - f_{\bar{u}}(x)]}_{u-\text{valence distr.}} = 2 \quad \int_0^1 dx \underbrace{[f_d(x) - f_{\bar{d}}(x)]}_{d-\text{valence distr.}} = 1$$
$$\int_0^1 dx [f_s(x) - f_{\bar{s}}(x)] = \int_0^1 dx [f_c(x) - f_{\bar{c}}(x)] = 0$$

- ▶ **Momentum sum rule** – momentum conservation connecting all flavours

$$\sum_{i=q,\bar{q},g} \int_0^1 dx x f_i(x) = 1$$

Momentum carried by **up** and **down** quarks is only around half of the total proton momentum the rest of the momentum is carried by **gluons** and small amount by **sea** quarks. In case of CT14NLO PDFs ($\mu = 1.3$ GeV):

$$\int_0^1 dx x [f_u(x) + f_d(x)] \simeq 0.51$$

$$\int_0^1 dx x f_g(x) \simeq 0.40$$

Scale dependence of PDFs $f_i(x, \mu)$

- ▶ x -**dependence** of PDFs is NOT calculable in pQCD
- ▶ μ^2 -**dependence** is calculable in pQCD – given by **DGLAP**
(Dokshitzer-Gribov-Lipatov-Altarelli-Parisi) evolution equations

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DGLAP evolution equations

$$\frac{df_q(x, \mu^2)}{d \log \mu^2} = \frac{\alpha_S(\mu^2)}{2\pi} \int_x^1 \frac{dy}{y} \left[P_{qq}\left(\frac{x}{y}\right) f_q(y, \mu^2) + P_{qg}\left(\frac{x}{y}\right) f_g(y, \mu^2) \right]$$

$$\frac{df_g(x, \mu^2)}{d \log \mu^2} = \frac{\alpha_S(\mu^2)}{2\pi} \int_x^1 \frac{dy}{y} \left[P_{gg}\left(\frac{x}{y}\right) f_g(y, \mu^2) + P_{gq}\left(\frac{x}{y}\right) f_q(y, \mu^2) \right]$$

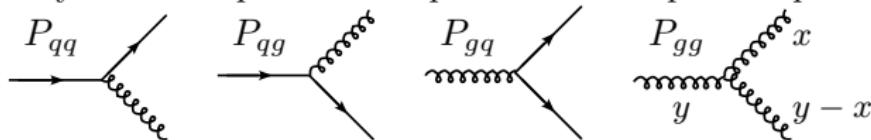
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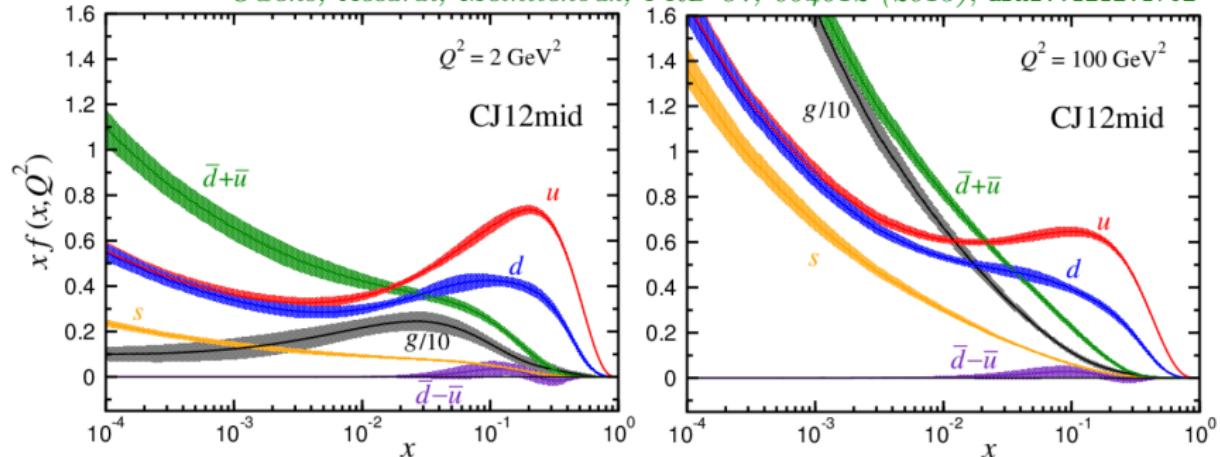
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- ▶ Different PDFs mix – set of $(2n_f + 1)$ coupled integro-differential equations.
- ▶ Initial conditions obtained from fitting experimental data.
- ▶ Splitting functions are calculable in pQCD
 $P_{ij}(z) = P_{ij}^{(0)}(z) + \frac{\alpha_S}{2\pi} P_{ij}^{(1)}(z) + \dots$
they have interpretation as probabilities of parton splittings:



Scale dependence of PDFs

Owens, Accardi, Melnitchouk, PRD 87, 094012 (2013), arXiv:1212.1702



- ▶ u and d have a characteristic valence bump at $x \sim 1/3$; they dominate at large x and $u > d$; at low x , $u \simeq d$.
- ▶ g dominates at low- x and falls steeply at large x .
- ▶ Gluon radiation and creation of $q\bar{q}$ pairs – QCD evolution – transfer of momentum from large to small $x \rightarrow$ gluon and quark distributions get steeper at small x .

Determination of PDFs

- ▶ **Problem:**

In order to calculate any high-energy cross-section in pQCD we need PDFs but we don't know how to compute them.

- ▶ **Solution:**

We will determine initial conditions for DGLAP equations by fitting experimental data in a process of *global QCD analysis*.

Schematics of Global Analysis

1. Choose experimental data (e.g. DIS, DY, inclusive jet prod., etc.)
2. Parametrize PDFs at low initial scale $\mu = Q_0 = 1.3\text{GeV}$:

$$f_i(x, Q_0) = f_i(x; a_0, a_1, \dots) = a_0 x^{a_1} (1-x)^{a_2} P(x; a_3, \dots)$$

3. Use DGLAP equation to evolve $f_i(x, \mu)$ from $\mu = Q_0$ to $\mu = Q_{\max}$.
4. Calculate theory predictions corresponding to the data (σ_{DIS} , σ_{DIS} , etc.).
5. Calculate appropriate χ^2 function – compare data and theory

$$\chi^2(\{a_i\}) = \sum_{\text{experiments}} w_n \chi_n^2(\{a_i\})$$

$$\chi_n^2(\{a_i\}) = \sum_{\text{data points}} \left(\frac{\text{data} - \text{theory}(\{a_i\})}{\text{uncertainty}} \right)^2$$

(by default $w_n = 1$)

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2. Parametrize PDFs at low initial scale $\mu = Q_0 = 1.3\text{GeV}$:

$$f_i(x, Q_0) = f_i(x; a_0, a_1, \dots) = a_0 x^{a_1} (1-x)^{a_2} P(x; a_3, \dots)$$

3. Use DGLAP equation to evolve $f_i(x, \mu)$ from $\mu = Q_0$ to $\mu = Q_{\max}$.
4. Calculate theory predictions corresponding to the data (σ_{DIS} , σ_{DIS} , etc.).
5. Calculate appropriate χ^2 function – compare data and theory

$$\chi^2(\{a_i\}) = \sum_{\text{experiments}} w_n \chi_n^2(\{a_i\})$$

$$\chi_n^2(\{a_i\}) = \sum_{\text{data points}} \left(\frac{\text{data} - \text{theory}(\{a_i\})}{\text{uncertainty}} \right)^2$$

(by default $w_n = 1$)

6. Minimize χ^2 function with respect to parameters a_0, a_1, \dots

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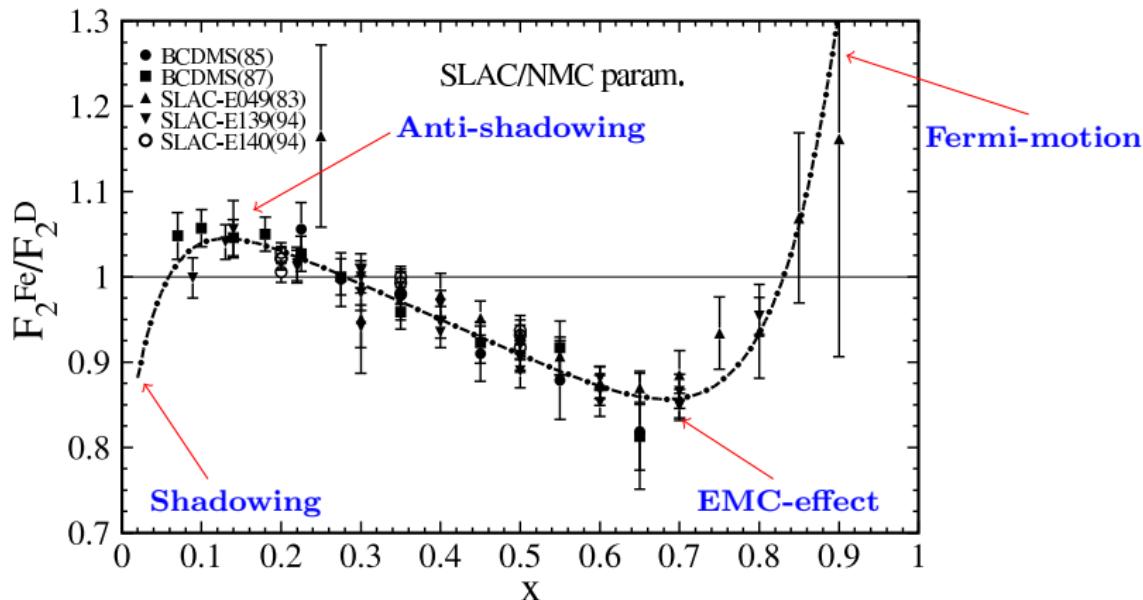
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Nuclear PDFs

Motivation

- ▶ Cross-sections in nuclear collisions are modified

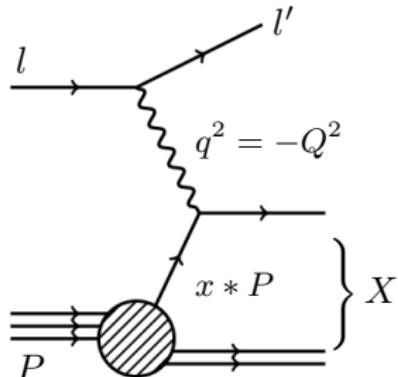
$$F_2^A(x) \neq ZF_2^p(x) + NF_2^n(x)$$



- ▶ Can we translate these modifications into **universal nuclear PDFs?**

Motivation

- ▶ Factorization in case of deep inelastic scattering (DIS)

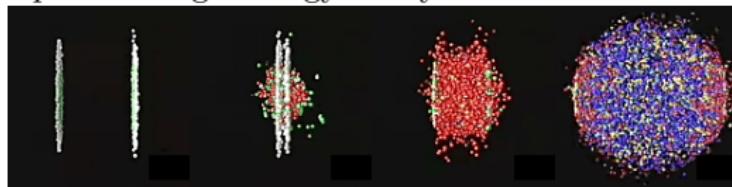


$$\frac{d^2\sigma}{dxdQ^2} = \sum_i f_i(x, Q^2) \otimes d\hat{\sigma}_{il \rightarrow l'X}$$

- ▶ We assume that the nuclear effects can be absorbed into the universal nPDFs.
- ▶ Do not consider any cold nuclear matter effects (e.g. energy loss).

Motivations: Why do we need nuclear PDFs?

- ▶ Information on the structure of nucleus.
- ▶ Description of high-energy heavy ion collisions at the **LHC** and **RHIC**.



Key ingredient to use perturbative probes of QGP
(separate non QGP effects in pA , AA collisions).

- ▶ Computation of prompt atmospheric neutrino flux.
- ▶ Differentiate flavors in free-proton PDFs (e.g. strange)

charged lepton DIS

$$F_2^{l\pm} \sim \left(\frac{1}{3}\right)^2 [d + s] + \left(\frac{2}{3}\right)^2 [u + c]$$

neutrino DIS

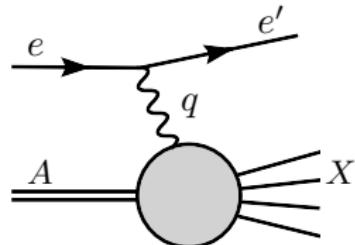
$$F_2^\nu \sim [d + s + \bar{u} + \bar{c}]$$

$$F_2^{\bar{\nu}} \sim [\bar{d} + \bar{s} + u + c]$$

$$F_3^\nu \sim 2[d + s - \bar{u} - \bar{c}]$$

$$F_3^{\bar{\nu}} \sim 2[u + c - \bar{d} - \bar{s}]$$

Variables: DIS of nuclear target $eA \rightarrow e'X$



- DIS variables in case on nucleons

in nucleus $\begin{cases} Q^2 \equiv -q^2 \\ x_A \equiv \frac{Q^2}{2 p_A \cdot q} \end{cases}$

- p^A – nucleus momentum
- $x_A \in (0, 1)$ – analog of Bjorken variable
(fraction of the nucleus momentum carried by a nucleon)

- Analogue variables for partons:

- $p_N = \frac{p_A}{A}$ – average nucleon momentum
- $x_N \equiv \frac{Q^2}{2 p_N \cdot q} = A x_A$ – parton momentum fraction with respect to the average nucleon momentum p_N
- $x_N \in (0, A)$ – parton can carry more than the average nucleon momentum p_N .

Assumptions entering the nuclear PDF analysis

1. Factorization & DGLAP evolution

- ▶ allow for definition of **universal PDFs**
- ▶ make the formalism **predictive**
- ▶ needed even if it is broken

2. Isospin symmetry $\begin{cases} u^{n/A}(x) = d^{p/A}(x) \\ d^{n/A}(x) = u^{p/A}(x) \end{cases}$ $f_i^{(A, Z)} = \frac{Z}{A} f_i^{p/A} + \frac{A - Z}{A} f_i^{n/A}$
3. The *bound proton* PDFs have the *same evolution equations* and sum rules as the free proton PDFs *provided we neglect any contributions from the region $x > 1$* (which is expected to have negligible contribution [PRC 73, 045206 (2006), [arXiv:hep-ph/0509241](https://arxiv.org/abs/hep-ph/0509241)])

Then observables \mathcal{O}^A can be calculated as:

$$\mathcal{O}^A = Z \mathcal{O}^{p/A} + (A - Z) \mathcal{O}^{n/A}$$

With the above assumptions we can use the free proton framework to analyze nuclear data

Schematics of Global Analysis

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Uncertainties in global analysis

- ▶ **Experimental errors** (included in PDFs error analysis)
- ▶ **Theoretical uncertainties** (e.g. HF schemes; not included)
- ▶ “**Details**” of Global Fits
(e.g. parametrization; not included)

Propagating experimental errors to PDFs:

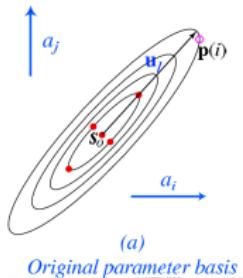
- ▶ Hessian Method
 - ▶ Eigenvector PDFs
 - ▶ Quadratic approximation
 - ▶ Simple computation of correlations
- ▶ Lagrange Multipliers
- ▶ Monte Carlo Methods
 - ▶ generate N data samples by varying data within errors;
 - ▶ perform N fits to the samples → PDF replicas
 - ▶ estimate uncertainty by calculating moments of PDF replicas

Hessian method

[JHEP 07 (2002) 012, arXiv:hep-ph/0201195]

- ▶ **Expand** χ^2 function around minimum, $\{a_i^0\}$,

$$\chi^2 = \chi_0^2 + \sum_{ij} \frac{1}{2} (a_i - a_i^0)(a_j - a_j^0) \left(\frac{\partial^2 \chi^2}{\partial a_i \partial a_j} \right)_0$$

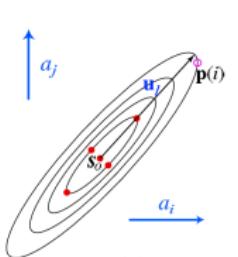


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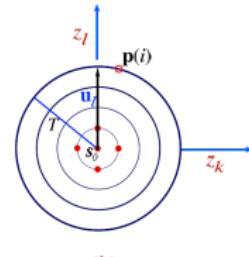
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- ▶ **Expand** χ^2 function around minimum, $\{a_i^0\}$, and **diagonalize**

$$\chi^2 = \chi_0^2 + \sum_{ij} \frac{1}{2} (a_i - a_i^0)(a_j - a_j^0) \left(\frac{\partial^2 \chi^2}{\partial a_i \partial a_j} \right)_0 = \chi_0^2 + \sum_i \lambda_i z_i^2$$



Original parameter basis

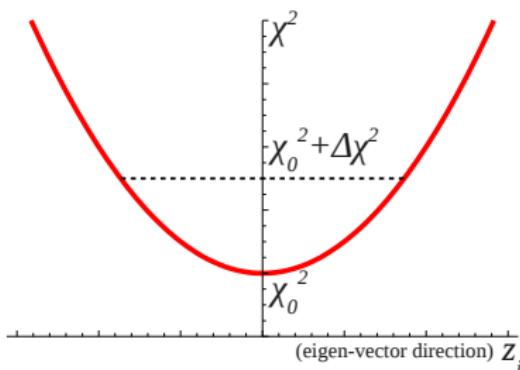


Orthonormal eigenvector basis

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- ▶ Choose tolerance criteria $\Delta\chi^2 = \chi^2 - \chi_0^2$ value (defining $1-\sigma$ uncertainty),
 - ▶ ideal case $\Delta\chi^2 = 1$
 - ▶ realistic global analysis $\Delta\chi^2 \sim 1 - 100$



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 - ▶ ideal case $\Delta\chi^2 = 1$
 - ▶ realistic global analysis $\Delta\chi^2 \sim 1 - 100$
- ▶ Construct error PDFs corresponding to each eigenvector direction:

$$f_i^\pm = f(\{z_i\}) = f(0, \dots, z_i = \pm\sqrt{\Delta\chi^2}, \dots, 0)$$

$$z_i = \pm\sqrt{\Delta\chi^2}$$

- ▶ Calculate errors of observable X :

$$\Delta X = \sqrt{\sum_i \left(\frac{\partial X}{\partial z_i} \times \delta z_i \right)^2} \simeq \frac{1}{2} \sqrt{\sum_i \left[X(f_i^+) - X(f_i^-) \right]^2}$$

Differences with the free-proton PDFs

- ▶ Theoretical status of Factorization
- ▶ Parametrization – more parameters to model A -dependence
- ▶ Different data sets – much less data:

- ▶ Less data \rightarrow less constraining power \rightarrow more assumptions (fixing) about a_i parameters
- ▶ Assumptions limit/replace uncertainties!

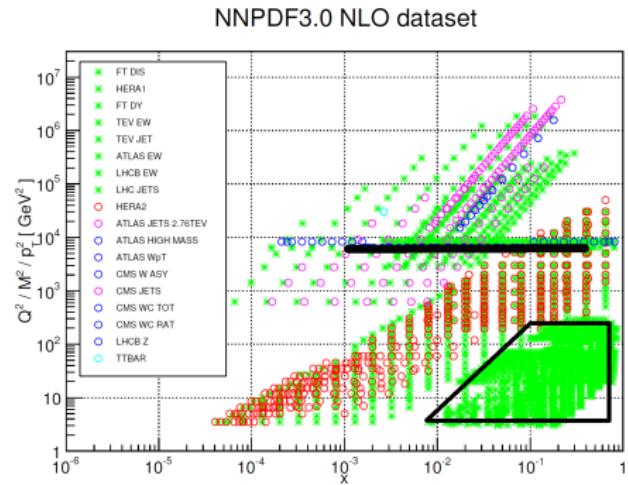
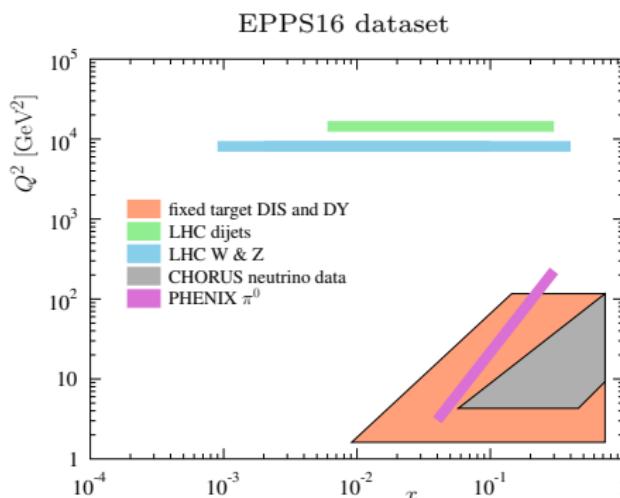
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Summary of available nuclear PDFs

Available nuclear PDFs

► Multiplicative nuclear correction factors

$$f_i^{\text{p/A}}(x_N, \mu_0) = R_i(x_N, \mu_0, A) f_i^{\text{free proton}}(x_N, \mu_0)$$

- **HKN:** Hirai, Kumano, Nagai [PRC 76, 065207 (2007)]
- **DSSZ:** de Florian, Sassot, Stratmann, Zurita [PRD 85, 074028 (2012)]
- **EPS09:** Eskola, Paukkunen, Salgado [JHEP 04 (2009) 065]
- **EPPS16:** Eskola, Paakkinen, Paukkunen, Salgado [EPJC 77 (2017) 163]
- **KT16:** Khanpour, Tehrani [PRD 93, 014026 (2016)]

► Convolution relation

$$f_i^{\text{p/A}}(x_N, Q_0^2) = \int_{x_N}^A \frac{dy}{y} W_i(y, A, Z) f_i^{\text{free proton}}\left(\frac{x_N}{y}, Q_0^2\right)$$

- **DS04:** de Florian, Sassot [PRD 69, 074028 (2004)]
- Native nuclear PDFs

$$\begin{aligned} f_i^{\text{p/A}}(x_N, \mu_0) &= f_i(x_N, A, \mu_0) \\ f_i(x_N, A = 1, \mu_0) &\equiv f_i^{\text{free proton}}(x_N, \mu_0) \end{aligned}$$

- **nCTEQ15:** Kovarik, Kusina, Jezo, Clark, Keppel, Lyonnet, Morfin, Olness, Owens, Schienbein, Yu [PRD 93, 085037 (2016), arXiv:1509.00792]

nPDF framework

► Parametrization

- PDF of nucleus (A - mass, Z - charge)

$$f_i^{(A,Z)}(x, Q) = \frac{Z}{A} f_i^{p/A}(x, Q) + \frac{A-Z}{A} f_i^{n/A}(x, Q)$$

- bound neutron PDFs, $f_i^{n/A}$, constructed assuming iso-spin symmetry
(Note that $f_i^{p/A}$, $f_i^{n/A}$ are not physical objects just convenient way of parameterizing f_i^A)
- bound proton PDFs parametrized:

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$$x f_i^{p/A}(x, Q_0) = x^{c_1} (1-x)^{c_2} e^{c_3 x} (1 + e^{c_4 x})^{c_5}$$

$$c_k \rightarrow c_k(\textcolor{red}{A}) \equiv c_{k,0} + c_{k,1} \left(1 - \textcolor{red}{A}^{-c_{k,2}} \right)$$

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EPPS16 [[arXiv:1612.05741](https://arxiv.org/abs/1612.05741)]

$$x f_i^{p/A}(x, Q_0) = x^{c_1} (1-x)^{c_2} e^{c_3 x} (1+e^{c_4 x})^{c_5} \quad f_i^{p/A}(x, Q) = R_i^A(x, Q) f_i^p(x, Q),$$

$$c_k \rightarrow c_k(\textcolor{red}{A}) \equiv c_{k,0} + c_{k,1} \left(1 - \textcolor{red}{A}^{-c_{k,2}} \right)$$

$$R_i^A(x, Q_0) = \begin{cases} a_0 + a_1(x - x_a)^2 & x \leq x_a \\ b_0 + b_1 x^\alpha + b_2 x^{2\alpha} + b_3 x^{3\alpha} & x_a \leq x \leq x_e \\ c_0 + (c_1 - c_2 x)(1-x)^{-\beta} & x_e \leq x \end{cases}$$

$$d_i \rightarrow d_i(\textcolor{red}{A}) = d_i(A_{\text{ref}}) \left(\frac{\textcolor{red}{A}}{A_{\text{ref}}} \right)^{\gamma_i [d_i(A_{\text{ref}})-1]},$$

with $d_i = a_i, b_i, \dots$ and $A_{\text{ref}} = 12$

nPDF framework

► Parametrization

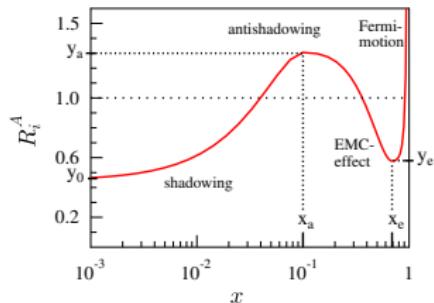
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Comparison of available nPDFs

	EPPS16	EPS09	nCTEQ15	DSSZ12	HKN07	KA15
FT e-DIS	✓	✓	✓	✓	✓	✓
FT ν -DIS	✓	✗	✗ [#]	✓	✗	✗
FT Drell-Yan	✓	✓	✓	✓	✓	✓
RHIC π^0	✓	✓	✓	✗	✗	✗
LHC W/Z	✗	✗	✗*	✗	✗	✗
LHC dijet	✗	✗	✗	✗	✗	✗
QCD order	NLO	LO & NLO	NLO	NLO	LO & NLO	NNLO
Kinematic cuts	$Q > 1.3\text{GeV}$	$Q > 1.3\text{GeV}$	$Q > 2\text{GeV}$ $W > 3.5\text{GeV}$	$Q > 1\text{GeV}$	$Q > 1\text{GeV}$	$Q > 1\text{GeV}$
No data points	1811	929	740	1579	1241	1479
No free param.	20	15	16	25	12	16
χ^2/dof	1.00	0.79	0.81	0.99	1.21	1.15
Error analysis	Hessian	Hessian	Hessian	Hessian	Hessian	Hessian
Tolerance $\Delta\chi^2$	52	50	35	30	13.7	1?
Proton baseline	CT14NLO	CTEQ6.1	CTEQ6.1-like	MSTW2008	MRST1998	JR09
Heavy-quark eff.	✓	✗	✓	✓	✗	✗
Flavour sep.	✓	✗	✓ (val.)	✗	✗	✗
Reference	[1612.05741]	[0902.4154]	[1509.00792]	[1112.6324]	[0709.3038]	[1601.00939]

[#] In a separate dedicated analysis [PRL106, 122301, (2011), 1012.0286; PRD80, 094004, (2009), 0907.2357]

* See a reweighting study [EPJC77, 488 (2017), 1610.02925]

nCTEQ15 fit details

Fit properties:

- ▶ fit @NLO
- ▶ $Q_0 = 1.3\text{GeV}$
- ▶ using ACOT heavy quark scheme
- ▶ kinematic cuts:
 $Q > 2\text{GeV}$, $W > 3.5\text{GeV}$
 $p_T > 1.7 \text{ GeV}$
- ▶ 708 (DIS & DY) + 32 (single π^0) = 740 data points after cuts
- ▶ 16+2 free parameters
 - ▶ 7 gluon
 - ▶ 7 valence
 - ▶ 2 sea
 - ▶ 2 pion data normalizations
- ▶ $\chi^2 = 587$, giving $\chi^2/\text{dof} = 0.81$

Error analysis:

- ▶ use Hessian method

$$\chi^2 = \chi_0^2 + \frac{1}{2} H_{ij} (a_i - a_i^0)(a_j - a_j^0)$$
$$H_{ij} = \frac{\partial^2 \chi^2}{\partial a_i \partial a_j}$$

- ▶ tolerance $\Delta\chi^2 = 35$ (every nuclear target within 90% C.L.)
- ▶ eigenvalues span 10 orders of magnitude → require numerical precision
- ▶ use noise reducing derivatives

nCTEQ15 fit details

Kinematic cuts

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nCTEQ:

$$\begin{cases} Q > 2 \text{ GeV} \\ W > 3.5 \text{ GeV} \end{cases}$$

EPS: $Q > 1.3 \text{ GeV}$

HKN: $Q > 1 \text{ GeV}$

DSSZ: $Q > 1 \text{ GeV}$

$$^{TT}_{ij} = \frac{\partial a_i}{\partial a_j}$$

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$$- a_i^0)(a_j - a_j^0)$$

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- 708 (DIS data pair)
- 16+2 free
 ► 7 gJ
- 7 via
- 2 sea
- 2 pion
- $\chi^2 = 587$

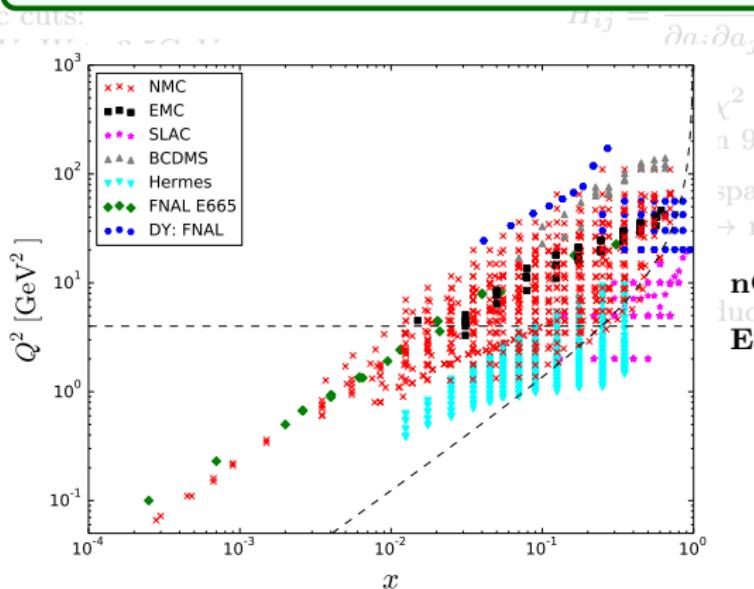
nCTEQ:

$$\begin{cases} Q > 2 \text{ GeV} \\ W > 3.5 \text{ GeV} \end{cases}$$

EPS: $Q > 1.3 \text{ GeV}$

HKN: $Q > 1 \text{ GeV}$

DSSZ: $Q > 1 \text{ GeV}$



$\chi^2 = 35$ (every nuclear
at 90% C.L.)

span 10 orders of
→ require numerical

nCTEQ: 740 data points
including derivatives
EPS09: 929 data points

nCTEQ15 fit details

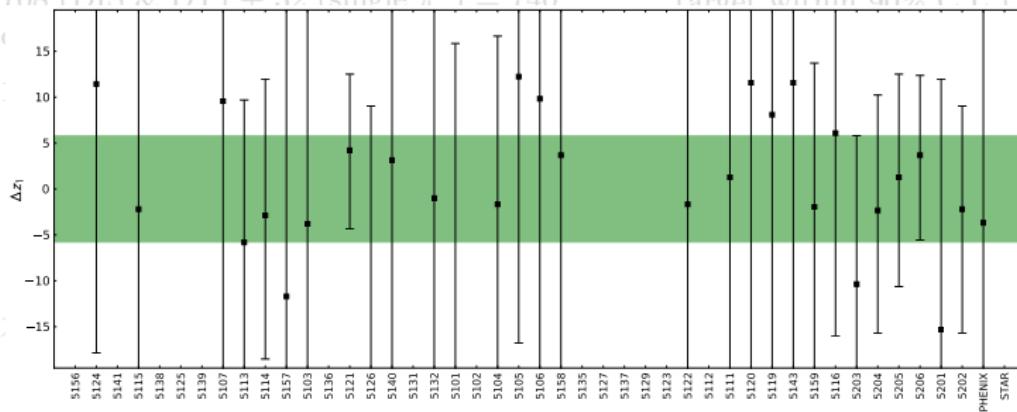
Fit details

Hessian method

- ▶ choice of tolerance: $T = 35$
[PRD65 (2001) 014012,
[arXiv:hep-ph/0101051](https://arxiv.org/abs/hep-ph/0101051)]
- ▶ quadratic approximation

$p_T > 1.7$ GeV

- ▶ 708 (DIS & DY) + 32 (single π^0) = 740



Error analysis:

- ▶ use Hessian method

$$\chi^2 = \chi_0^2 + \frac{1}{2} H_{ij} (a_i - a_i^0)(a_j - a_j^0)$$
$$H_{ij} = \frac{\partial^2 \chi^2}{\partial a_i \partial a_j}$$

- ▶ tolerance $\Delta\chi^2 = 35$ (every nuclear target within 90% C.L.)

s of
numerical
lives

nCTEQ15 fit details

Fitter details

Hessian method

- ▶ choice of tolerance: $T = 35$
- ▶ quadratic approximation

► kinematic cuts:

$Q > 2\text{GeV}$, $W > 3.5\text{GeV}$

$p_T > 1.7 \text{ GeV}$

► 708 (DIS & DY) + 32 (single π^0) = 740 data points after cuts

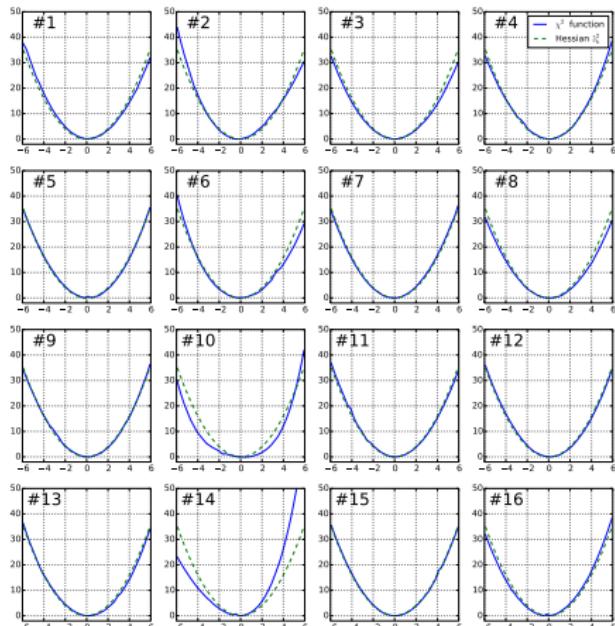
► 16+2 free parameters

- ▶ 7 gluon
- ▶ 7 valence
- ▶ 2 sea
- ▶ 2 pion data normalizations

► $\chi^2 = 587$, giving $\chi^2/\text{dof} = 0.81$

Error analysis:

- ▶ use Hessian method

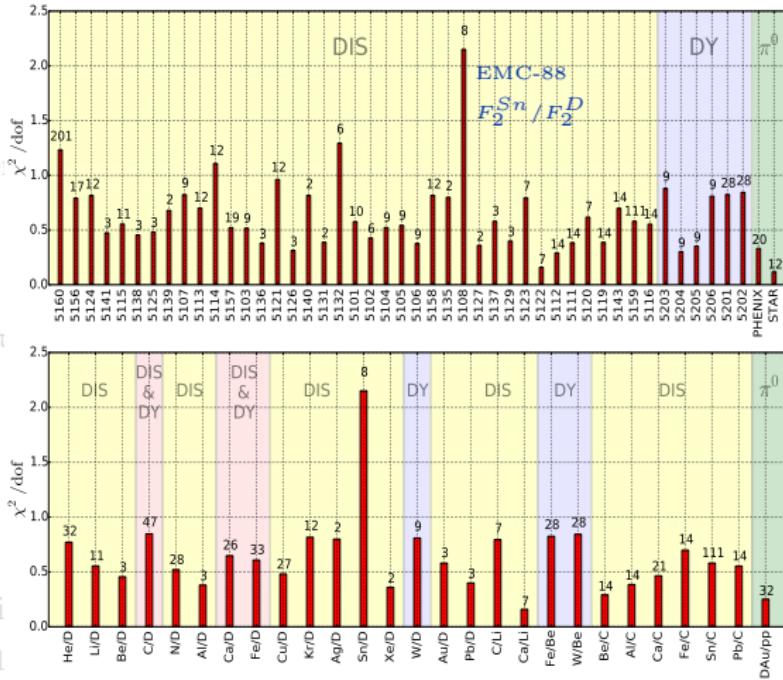


nCTEQ15 fit details

Fit quality

► $\chi^2/dof = 0.81$

- ▶ kinematic cuts:
 $Q > 2\text{GeV}$, $W > 3.5\text{GeV}$
 $p_T > 1.7 \text{ GeV}$
- ▶ 708 (DIS & DY) + 32 (single π) data points after cuts
- ▶ 16+2 free parameters
 - ▶ 7 gluon
 - ▶ 7 valence
 - ▶ 2 sea
 - ▶ 2 pion data normalization
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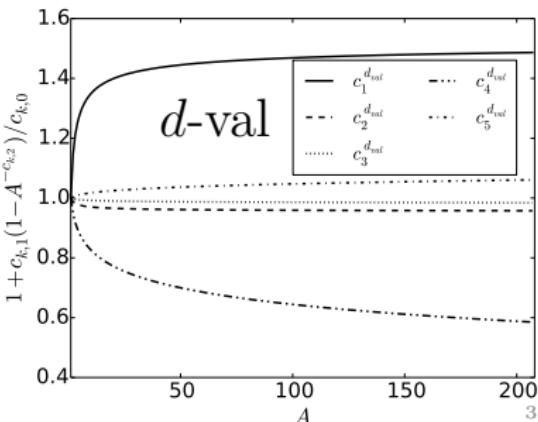
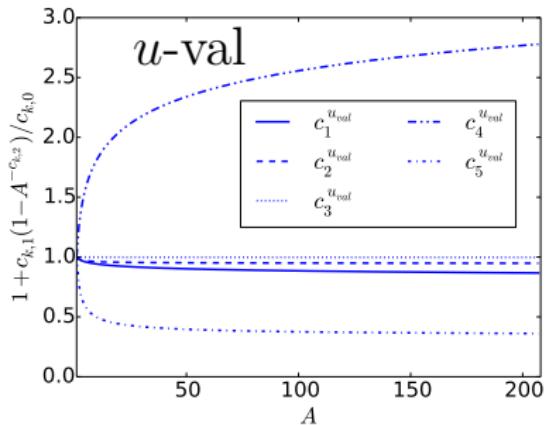
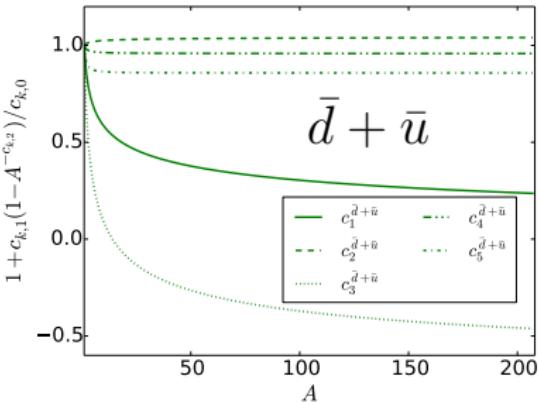
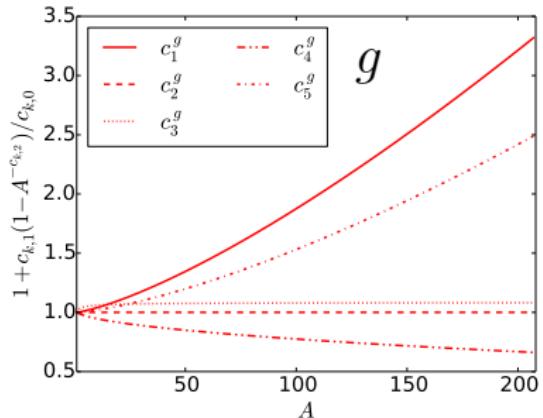


nCTEQ RESULTS

[PRD 93, 085037 (2016), arXiv:1509.00792]

Fitting parameters A -dependence: $c_k(A) = c_{k,0} + c_{k,1}(1 - A^{-c_{k,2}})$

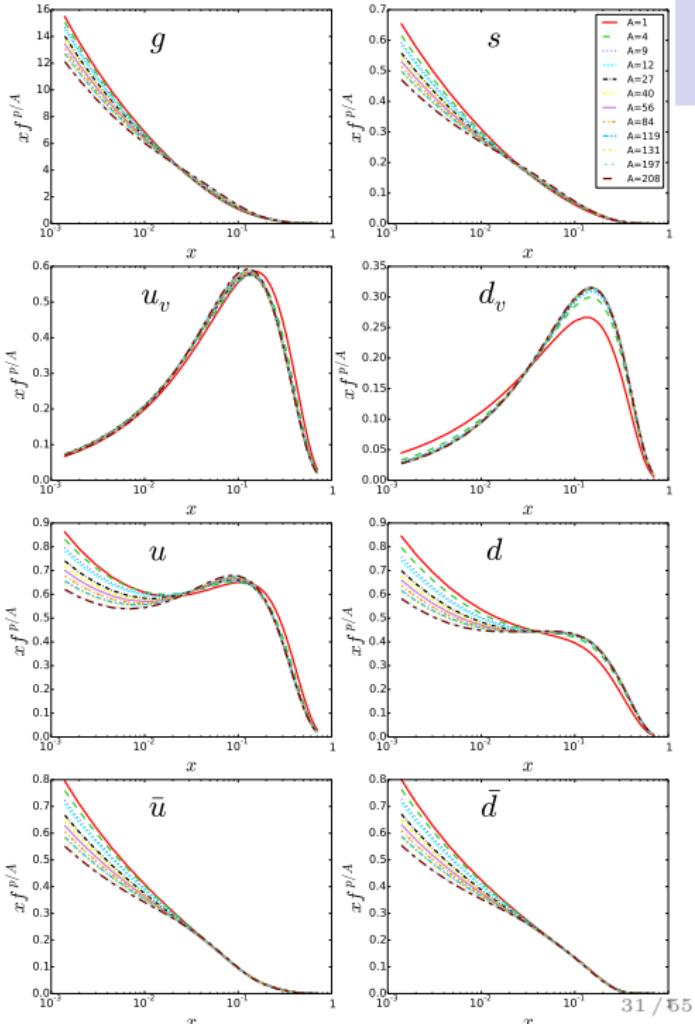
$$x f_i^{p/A}(x, Q_0) = x^{c_1} (1-x)^{c_2} e^{c_3 x} (1 + e^{c_4 x})^{c_5}$$



nCTEQ results

Nuclear PDFs A -dependence
($Q = 10\text{GeV}$)

$$x f_i^{p/A}(x, Q)$$

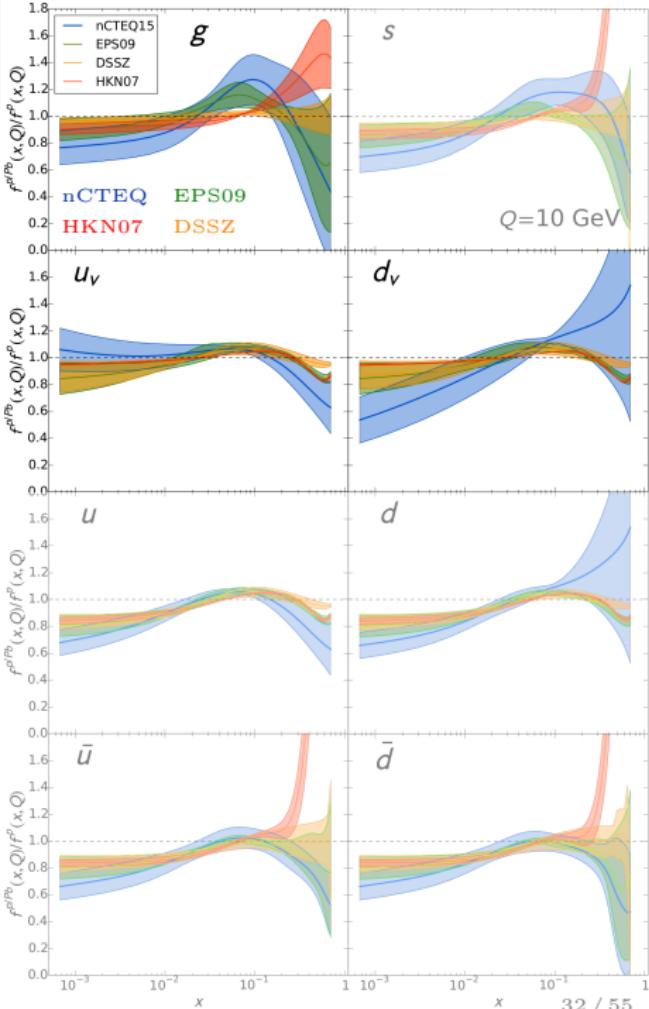


nCTEQ results

Nuclear correction factors ($Q = 10$ GeV)

$$R_i^{\text{Pb}} = \frac{f_i^{p/\text{Pb}}(x, Q)}{f_i^p(x, Q)}$$

- ▶ different solution for d -valence & u -valence compared to EPS09 & DSSZ
- ▶ sea quark nuclear correction factors similar to EPS09
- ▶ nuclear correction factors depend largely on underlying proton baseline

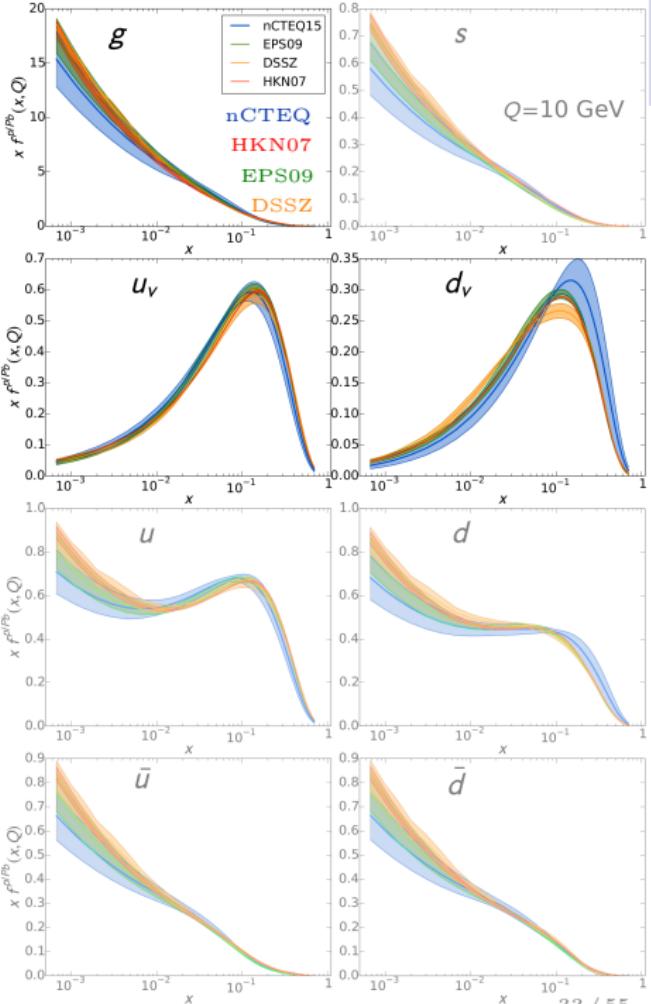


nCTEQ results

Bound proton PDFs ($Q = 10\text{ GeV}$)

$$x f_i^{p/\text{Pb}}(x, Q)$$

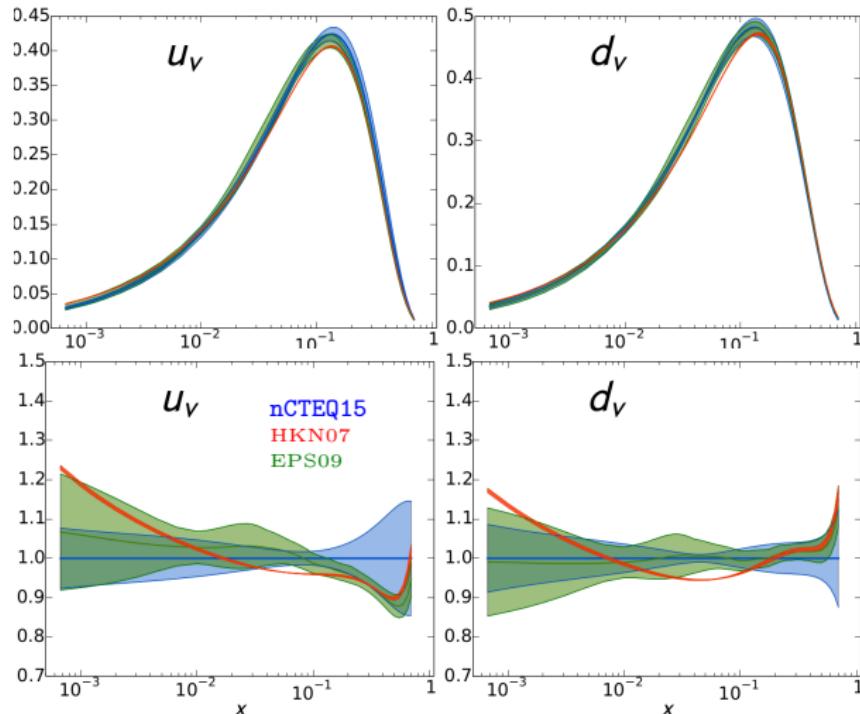
- ▶ nCTEQ features larger uncertainties than previous nPDFs
- ▶ better agreement between different groups (nPDFs don't depend on proton baseline)



Valence nuclear distributions

Full lead nucleus distribution:

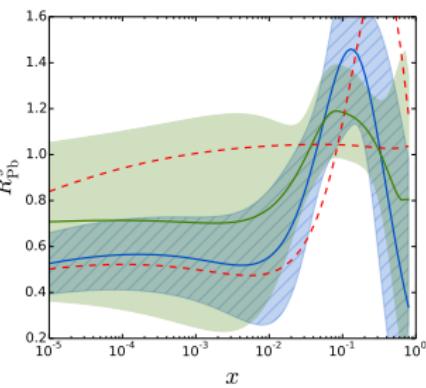
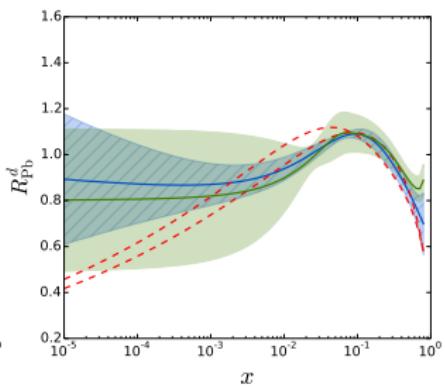
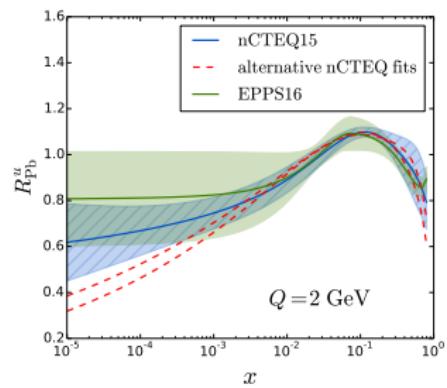
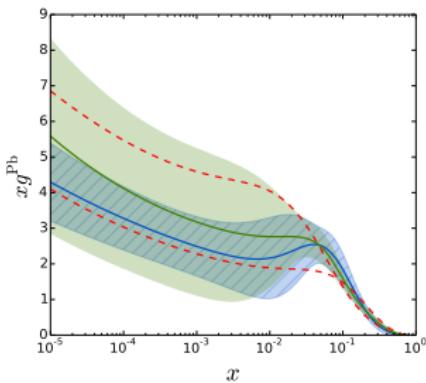
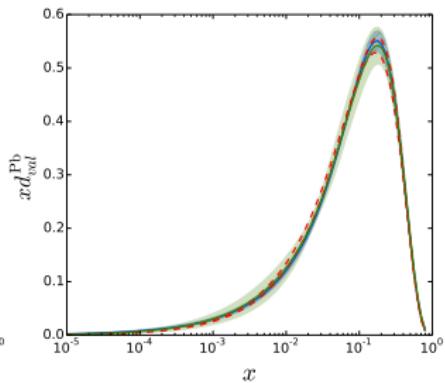
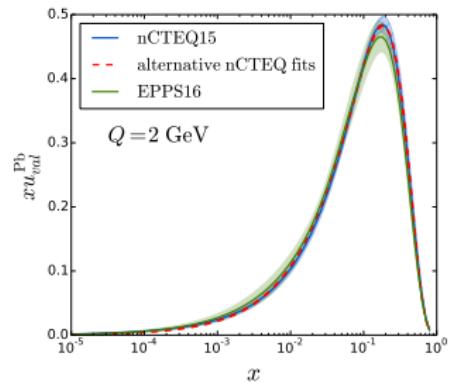
$$f^{\text{Pb}} = \frac{82}{208} f^{p/\text{Pb}} + \frac{208 - 82}{208} f^{n/\text{Pb}}$$



Current nPDFs

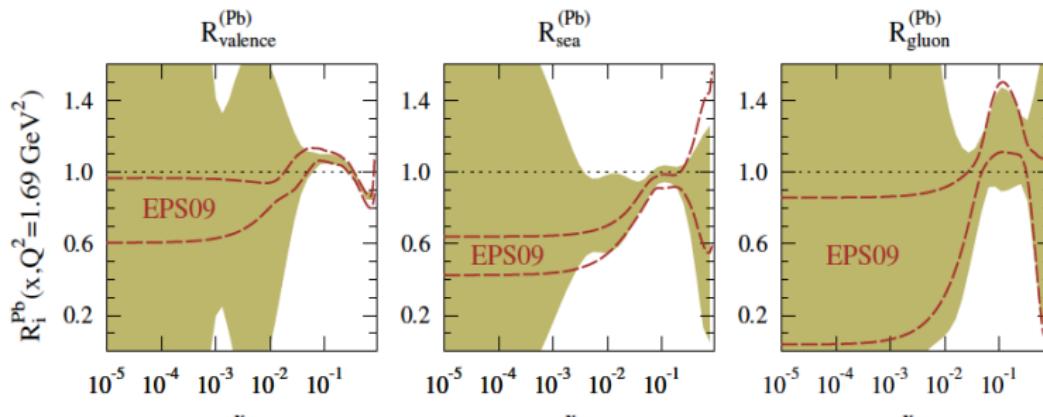
[arXiv:1509.00792, arXiv:1012.1178]

[arXiv:1612.05741]



New fit framework:

The baseline fit using the new fit functions: no control over small x !



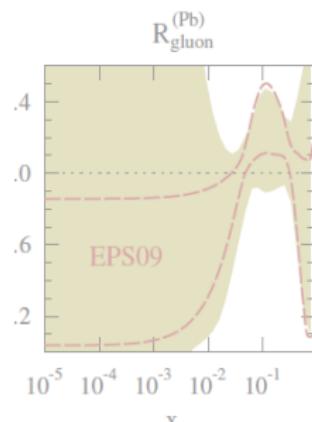
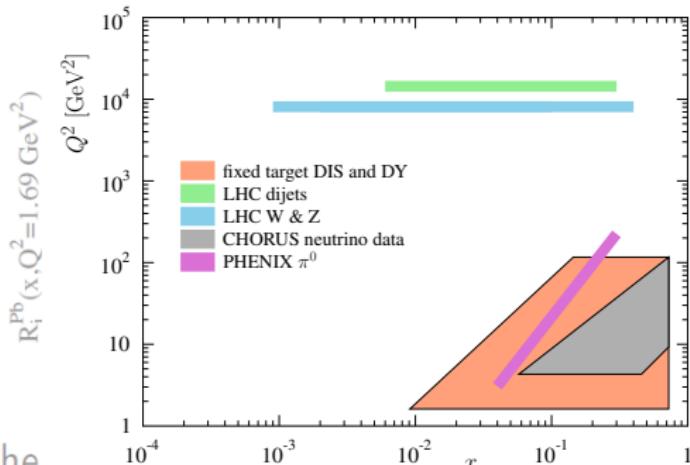
The lower bound restricted here by $F_L(Q^2 = 2 \text{ GeV}^2, x > 10^{-5}) > 0$

Maybe against “physical intuition” (small- x theory predicts shadowing, $R_i < 1$), but consistent with the data.

E.g. in EPS09, small- x shadowing was essentially built in

New fit framework:

The baseline fit using the new fit functions: no control over small x !



The $\int_{x=10^{-5}}^{x=10^{-1}} R_i^{Pb}(x, Q^2=1.69 \text{ GeV}^2, x > 10^{-5}) > 0$

Maybe Gluon nPDFs is particularly badly known (LHC dijet, RHIC π^0)
 $R_i < 1$), but consistent with the data.

E.g. in EPS09, small- x shadowing was essentially built in

Heavy flavor LHC data

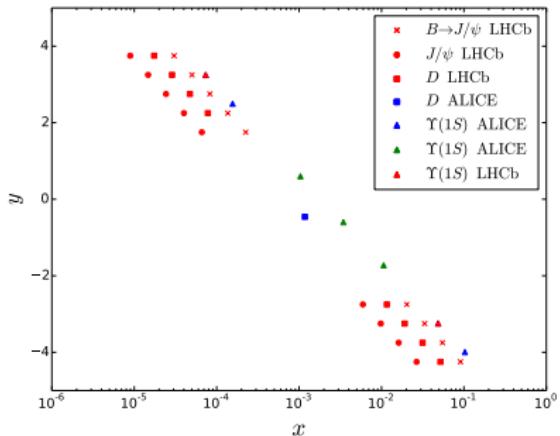
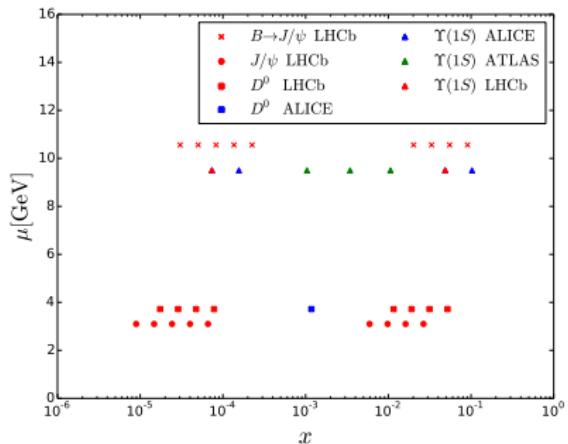
[arXiv:1712.07024]

Available heavy-flavor pPb LHC data

	D^0	J/ψ	$B \rightarrow J/\psi$	$\Upsilon(1S)$
μ_0	$\sqrt{4M_{D^0}^2 + P_{T,D^0}^2}$	$\sqrt{M_{J/\psi}^2 + P_{T,J/\psi}^2}$	$\sqrt{4M_B^2 + \left(\frac{M_B}{M_{J/\psi}} P_{T,J/\psi}\right)^2}$	$\sqrt{M_{\Upsilon(1S)}^2 + P_{T,\Upsilon(1S)}^2}$
$p+p$ data	LHCb [1]	LHCb [2,3]	LHCb [2,3]	ALICE [4], ATLAS [5], CMS [6], LHCb [7,8]
R_{pPb} data	ALICE [9], LHCb [15]	ALICE [10,11], LHCb [16,12]	LHCb [12]	ALICE [13], ATLAS [14], LHCb [17]

- [1] LHCb, R. Aaij et al., JHEP 06, 147 (2017), 1610.02230.
- [2] LHCb, R. Aaij et al., Eur. Phys. J. C71, 1645 (2011), 1103.0423.
- [3] LHCb, R. Aaij et al., JHEP 06, 064 (2013), 1304.6977.
- [4] ALICE, B. B. Abelev et al., Eur. Phys. J. C74, 2974 (2014), 1403.3648.
- [5] ATLAS, G. Aad et al., Phys. Rev. D87, 052004 (2013), 1211.7255.
- [6] CMS, S. Chatrchyan et al., Phys. Lett. B727, 101 (2013), 1303.5900.
- [7] LHCb, R. Aaij et al., Eur. Phys. J. C72, 2025 (2012), 1202.6579.
- [8] LHCb, R. Aaij et al., JHEP 11, 103 (2015), 1509.02372.
- [9] ALICE, B. B. Abelev et al., Phys. Rev. Lett. 113, 232301 (2014), 1405.3452.
- [10] ALICE, J. Adam et al., JHEP 06, 055 (2015), 1503.07179.
- [11] ALICE, B. B. Abelev et al., JHEP 02, 073 (2014), 1308.6726.
- [12] LHCb, R. Aaij et al., (2017), 1706.07122.
- [13] ALICE, B. B. Abelev et al., Phys. Lett. B740, 105 (2015), 1410.2234.
- [14] The ATLAS collaboration, (2015), ATLAS-CONF-2015-050.
- [15] LHCb, R. Aaij et al., JHEP 1710 (2017) 090, 1707.02750.
- [16] LHCb, R. Aaij et al., JHEP 02, 072 (2014), 1308.6729.
- [17] LHCb, R. Aaij et al., JHEP 07, 094 (2014), 1405.5152.

Kinematic reach of the data



Expected nuclear effects on heavy quark(onium) production in pA collisions

- ▶ Nuclear modification of PDFs: initial-state effect
- ▶ Energy loss (w.r.t. pp collisions): initial-state or final-state effect
- ▶ Break up of the quarkonium in the nuclear matter: final-state effect
- ▶ Break up by comoving particles: final-state effect
- ▶ Colour filtering of intrinsic QQ pairs: initial-state effect
- ▶ ...

► We assume leading twist factorization is valid – ONLY modifications of PDFs are present → “shadowing-only” hypothesis.

Theoretical predictions

- ▶ Theory calculations for heavy quark are done using a data driven method [[PRL107, 082002 \(2011\)](#), [1105.4186](#); [EPJC77, 1 \(2017\)](#), [1610.05382](#)]
 - ▶ partonic matrix elements $|A|^2$ are determined from fits to pp data

$$|\mathcal{A}|^2 = \frac{\lambda^2 \kappa s x_1 x_2}{M_{\mathcal{H}}^2} \exp \left(-\kappa \frac{\min(P_{T,\mathcal{H}}^2, \langle P_T \rangle^2)}{M_{\mathcal{H}}^2} \right) \\ \times \left[1 + \theta \left(P_{T,\mathcal{H}}^2 - \langle P_T \rangle^2 \right) \frac{\kappa}{n} \frac{P_{T,\mathcal{H}}^2 - \langle P_T \rangle^2}{M_{\mathcal{H}}^2} \right]^{-n}$$

with $\kappa, \lambda, \langle P_T \rangle$ and n being fit parameters.

- ▶ Predictions for D^0 and $B \rightarrow J/\psi$ have been validated against available perturbative QCD calculations (FONLL, GMVFNS).
- ▶ Additional features:
 - ✓ uncertainty in pp collision is well controlled by the data
 - ✓ removes model dependence
 - ✓ fast to generate events
 - ✗ currently limited to probes produced in $2 \rightarrow 2$ partonic processes dominated by single partonic channel ($gg, q\bar{q}, \dots$)
→ In our case ($D^0, J/\psi, B \rightarrow J/\psi, \Upsilon(1S)$ production) **gg dominated**.

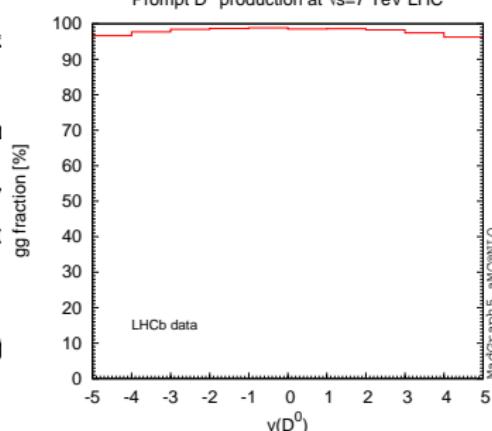
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Impact on nPDFs: reweighting analysis

Reweighting for Hessian PDFs [arXiv:1310.1089, arXiv:1402.6623]

1. Convert Hessian error PDFs into replicas

$$f_k = f_0 + \sum_i^N \frac{f_i^{(+)} - f_i^{(-)}}{2} R_{ki},$$

2. Calculate weights for each replica

$$w_k = \frac{e^{-\frac{1}{2}\chi_k^2/T}}{\frac{1}{N_{\text{rep}}} \sum_i^{N_{\text{rep}}} e^{-\frac{1}{2}\chi_k^2/T}}, \quad \chi_k^2 = \left(\frac{1-f_N}{\sigma^{\text{norm}}} \right)^2 + \sum_j^{N_{\text{data}}} \frac{(f_N D_j - T_j^k)^2}{\sigma_j^2}$$

3. Calculate observables with new (reweighted) PDFs

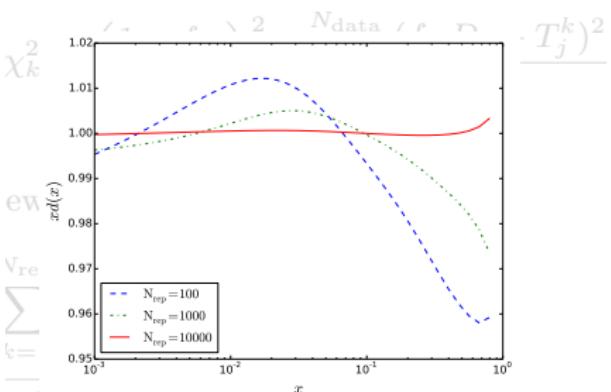
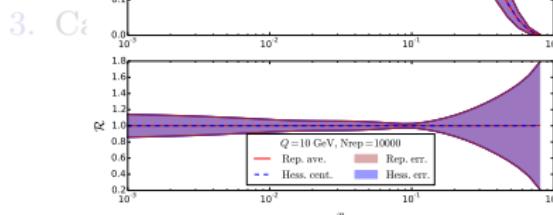
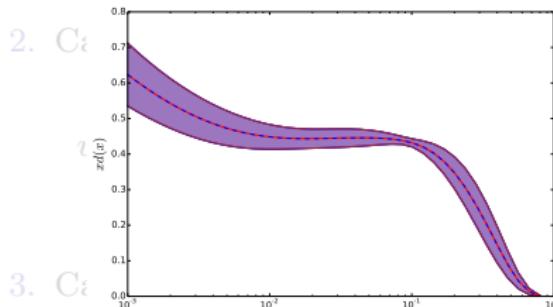
$$\langle \mathcal{O} \rangle_{\text{new}} = \frac{1}{N_{\text{rep}}} \sum_{k=1}^{N_{\text{rep}}} w_k \mathcal{O}(f_k),$$

$$\delta \langle \mathcal{O} \rangle_{\text{new}} = \sqrt{\frac{1}{N_{\text{rep}}} \sum_{k=1}^{N_{\text{rep}}} w_k (\mathcal{O}(f_k) - \langle \mathcal{O} \rangle)^2}.$$

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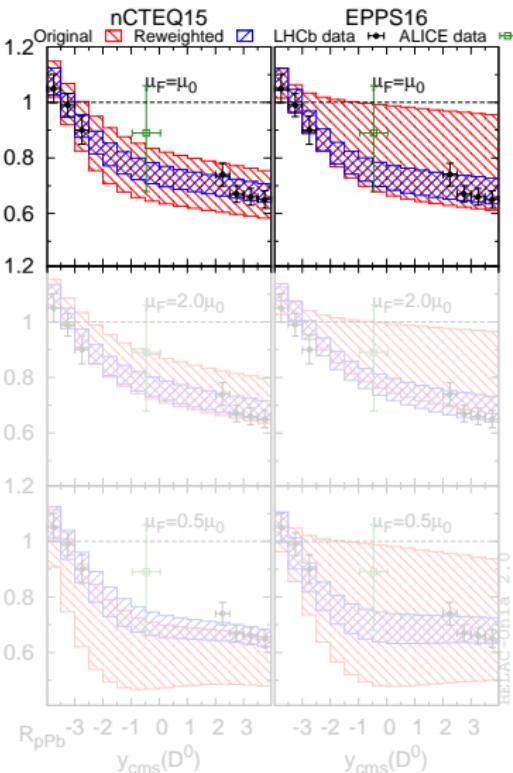
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Reweighting with D^0 data

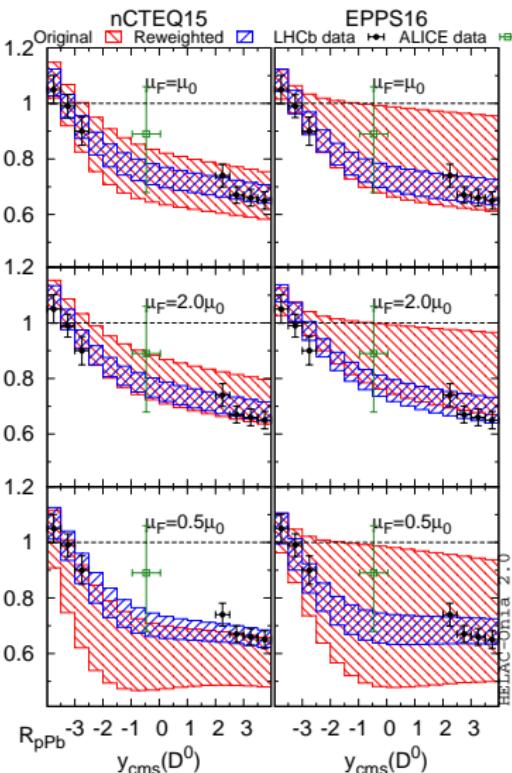


LHCb [JHEP 1710 (2017) 090, 1707.02750]

ALICE [PRL113, 232301 (2014), 1405.3452]

- ▶ Initial description of data is good for both nCTEQ15 and EPPS16.
- ▶ Substantial reduction of uncertainty especially for EPPS16.

Reweighting with D^0 data

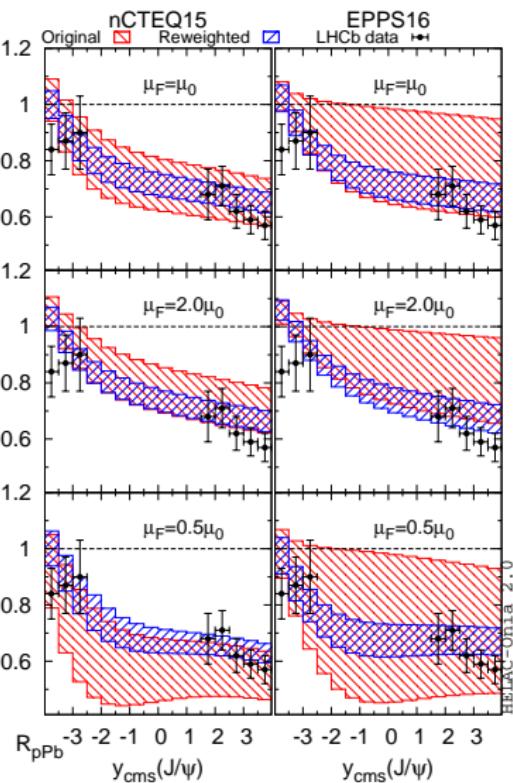


LHCb [JHEP 1710 (2017) 090, 1707.02750]

ALICE [PRL113, 232301 (2014), 1405.3452]

- ▶ Initial description of data is good for both $n\text{CTEQ15}$ and EPPS16 .
- ▶ Substantial reduction of uncertainty especially for EPPS16 .
- ▶ If we include factorization scale uncertainty errors increase and it can become the dominant uncertainty.

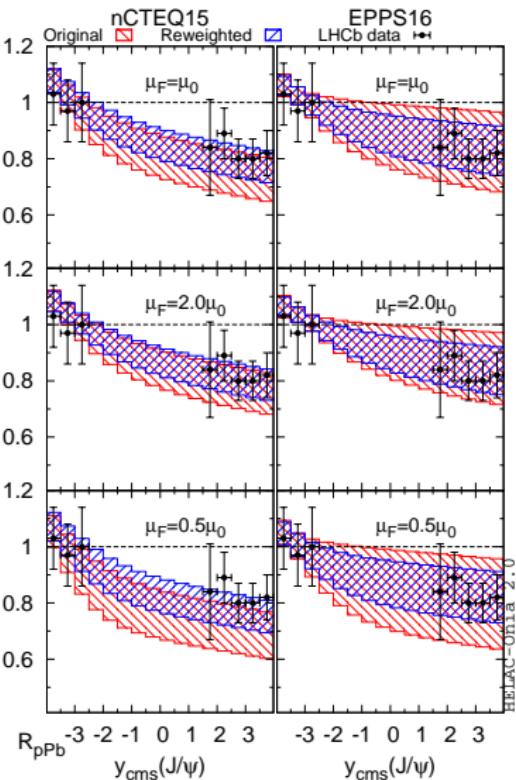
Reweighting with J/ψ data



LHCb [JHEP 02, 072 (2014), 1308.6729; PLB 774 (2017) 159, 1706.07122]
ALICE [JHEP 06, 055 (2015), 1503.07179; JHEP 02, 073 (2014), 1308.6726]

- ▶ Again we observe good agreement with the data; the scale uncertainty becomes important.

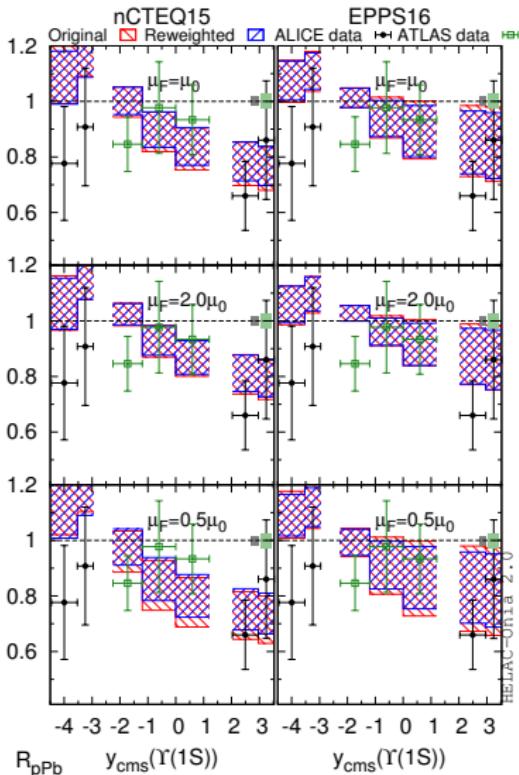
Reweighting with $B \rightarrow J/\psi$ data



LHCb [PLB 774 (2017) 159, 1706.07122]

- ▶ Scale uncertainty is reduced compared to the D^0 and J/ψ case.
- ▶ Data are not yet precise enough to give substantial constraints on nPDFs (but if the precision rises there is big potential).

Reweighting with $\Upsilon(1S)$ data

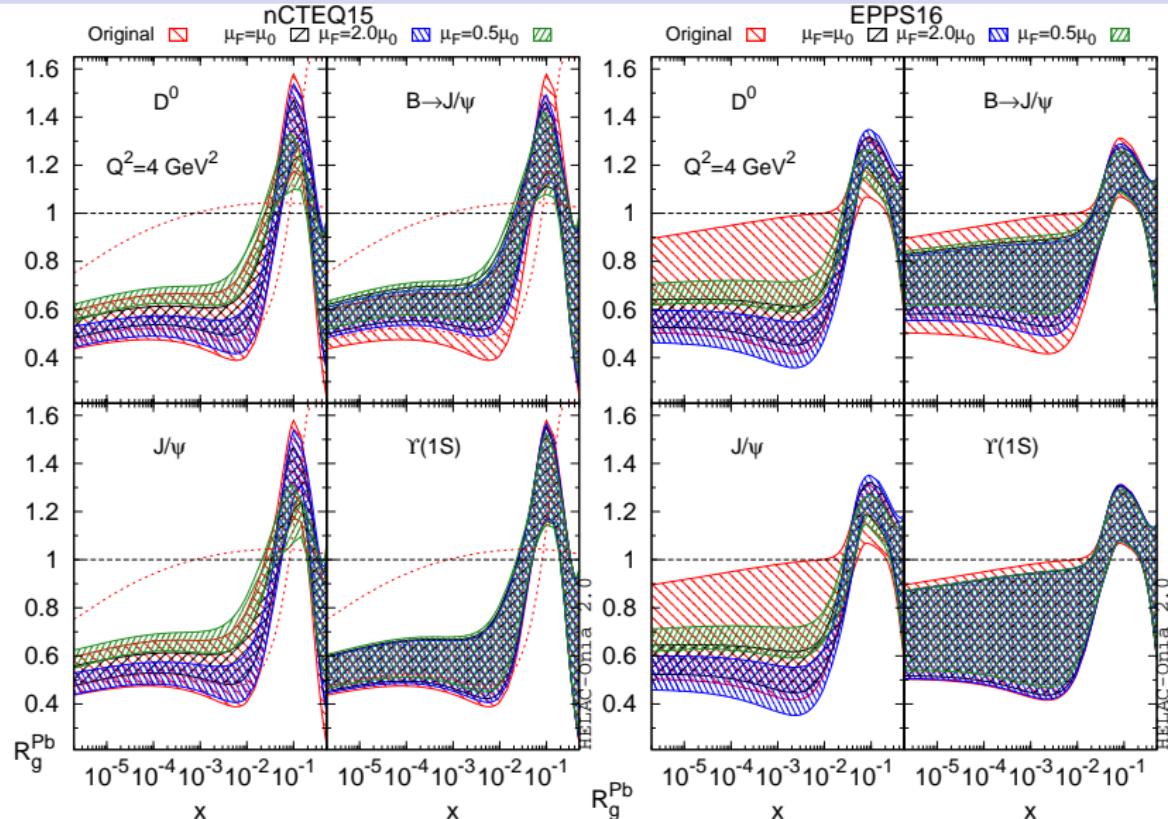


ALICE [PLB 740, 105 (2015), 1410.2234]

ATLAS [ATLAS-CONF-2015-050 (updated in: 1709.03089)]

- With the current precision we don't get any additional constraints on the nPDFs.

Reweighting results: $R_g^{\text{Pb}} = f_g^{\text{Pb}} / f_g^p$



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Comments

- ▶ We observe global coherence of the data constraints: necessary condition to assume a shadowing-only approach.
- ▶ First clear experimental observation on gluon SHADOWING at low x ; visible reduction of the EPPS16 uncertainties; confirmation of the extrapolation done in nCTEQ15.
- ▶ Confirmation of the existence of a gluon anti-shadowing: $R_g(0.05 \lesssim x \lesssim 0.1) > 1$.
- ▶ The scale ambiguity for D^0 and J/ψ production is now the dominant uncertainty.
- ▶ Non-prompt J/ψ are really promising if improved data can be obtained.

Consistency with other data

We checked the consistency of the reweighted (nCTEQ15) nPDFs with other data sets entering global analysis:

- ▶ DIS data (the most precise set NMC Sn/C [[NPB 481 \(1996\) 23](#)]).
- ▶ LHC W/Z boson production data [[EPJC 77, \(2017\) 488](#)].
- ▶ PHENIX J/ψ $R_{d\text{Au}}$ data [[PRL 107 \(2011\) 142301](#); [PRC 87, \(2013\) 034904](#)].

This is very non-trivial and further confirms the “shadowing-only” hypothesis of leading twist factorization is valid within the current data precision!

Where do you get the
PDFs/nPDFs from?

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LHAPDF 6.2.1

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LHAPDF Documentation

Introduction

LHAPDF is a general purpose C++ interpolator, used for evaluating PDFs from discretised data files. Previous versions of LHAPDF were written in Fortran 77/90 and are documented at <http://lhapdf.hepforge.org/lhapdf5/>.

LHAPDF6 vastly reduces the memory overhead of the Fortran LHAPDF (from gigabytes to megabytes!), entirely removes restrictions on numbers of concurrent PDFs, allows access to single PDF members without needing to load whole sets, and separates a new standardised PDF data format from the code library so that new PDF sets may be created and released easier and faster. The C++ LHAPDF6 also permits arbitrary parton contents via the standard PDG ID code scheme, is computationally more efficient (particularly if only one or two flavours are required at each phase space point, as in PDF reweighting), and uses a flexible metadata system which fixes many fundamental metadata and concurrency bugs in LHAPDF5.

Compatibility routines are provided as standard for existing C++ and Fortran codes using the LHAPDF5 and PDFLIB legacy interfaces, so you can keep using your existing codes. But the new interface is much more powerful and pleasant to work with, so we think you'll want to switch once you've used it!

LHAPDF6 is documented in more detail in <http://arxiv.org/abs/1412.7420>

Installation

The source files can be downloaded from <https://www.hepforge.org/downloads/lhapdf>

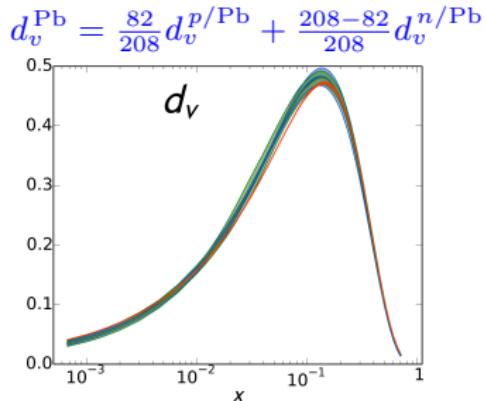
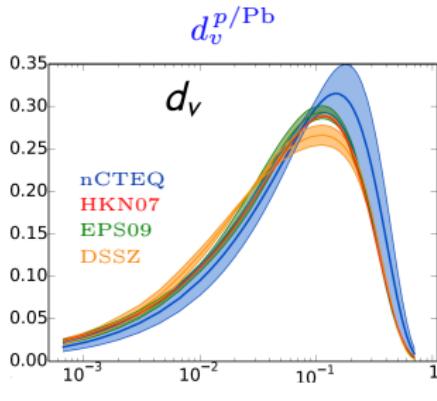
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Summary

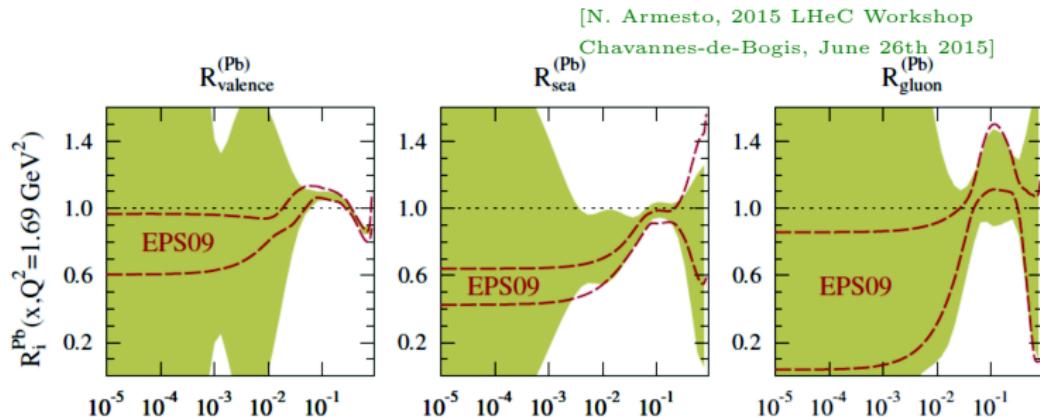
Summary

- ▶ Nuclear PDFs are determined by fitting experimental data following the road paved by the proton PDF global analyses.
- ▶ There are several sets of nPDFs available
 - ▶ EPPS16
 - ▶ nCTEQ15
 - ▶ ...
- ▶ Bound proton PDFs, $f_i^{p/A}$, are only **effective** means of parameterizing the full nPDFs f_i^A



Summary

- ▶ Not all flavours are currently reliably determined in particular constraints on the nuclear **gluon** and **strange** are very mild.
- ▶ nPDFs errors still substantially underestimated especially at low- x (no data below $x \lesssim 10^{-3}$) and large- x .



Summary

- ▶ The LHC data can help to constrain the low- x distributions
 - ▶ W/Z production and di-jet production already used by EPPS16
 - ▶ heavy quarks (D^0 , J/ψ , $B \rightarrow J/\psi$ and $\Upsilon(1S)$) can help with gluon
 - ▶ other possibilities: isolated photons, Drell-Yan
- ▶ Potential future experiments
 - ▶ EIC (electron ion collider)
 - ▶ LHeC
 - ▶ AFTER@LHC (fixed-target with LHC beam)
- ▶ It is crucial to include more data in the fits in order to remove assumptions and make error estimates more reliable.
- ▶ This will help answer questions like:
 - ▶ are nuclear effects truly universal?
 - ▶ basis for including final state nuclear effects,
 - ▶ centrality dependence.



Energy range

7 TeV proton beam on a fixed target

c.m.s. energy: $\sqrt{s} = \sqrt{2m_N E_p} \approx 115 \text{ GeV}$

Boost: $\gamma = \sqrt{s} / (2m_N) \approx 60$

Rapidity shift:

$$y_{c.m.s.} = 0 \rightarrow y_{lab} = 4.8$$

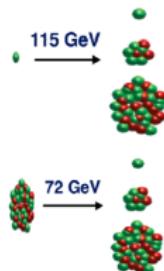
2.76 TeV Pb beam on a fixed target

c.m.s. energy: $\sqrt{s_{NN}} = \sqrt{2m_N E_{\text{Pb}}} \approx 72 \text{ GeV}$

Boost: $\gamma \approx 40$

Rapidity shift:

$$y_{c.m.s.} = 0 \rightarrow y_{lab} = 4.3$$



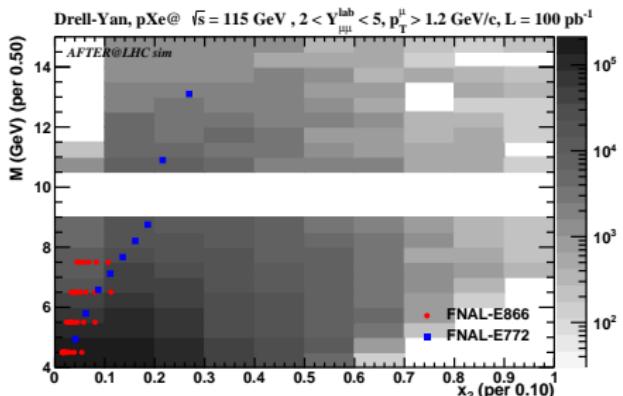
Such \sqrt{s} allow, for the first time, for systematic studies of W boson, bottomonia, p_T spectra, associated production, ..., in the fixed target mode

Effect of boost :

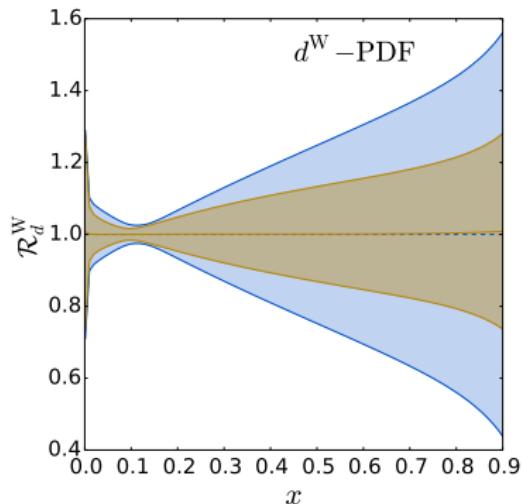
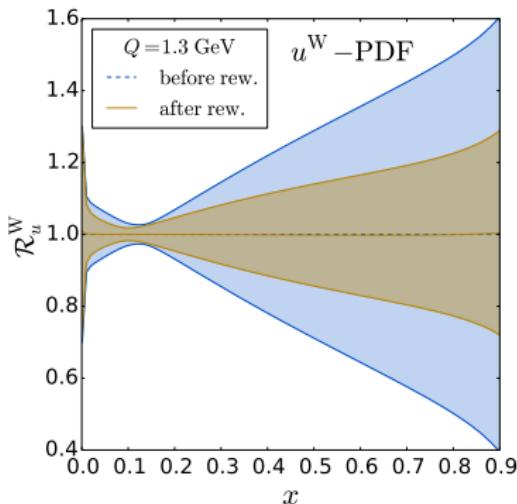
[particularly relevant for high energy beams]

- LHCb and the ALICE muon arm become **backward detectors** $[y_{c.m.s.} < 0]$
- With the reduced \sqrt{s} , their acceptance for physics grows and nearly covers half of the backward region for most probes $[-1 < x_F < 0]$
- Allows for backward physics up to high x_{target} ($\equiv x_2$)
 [uncharted for proton-nucleus; most relevant for p-p \uparrow with large x_2^{\uparrow}]

- ▶ Would be certainly very useful for PDF/nPDF determination
 - ▶ allow to collect data on different targets: Pb, W, Xe,...
 - ▶ e.g. Drell-Yan lepton pair production



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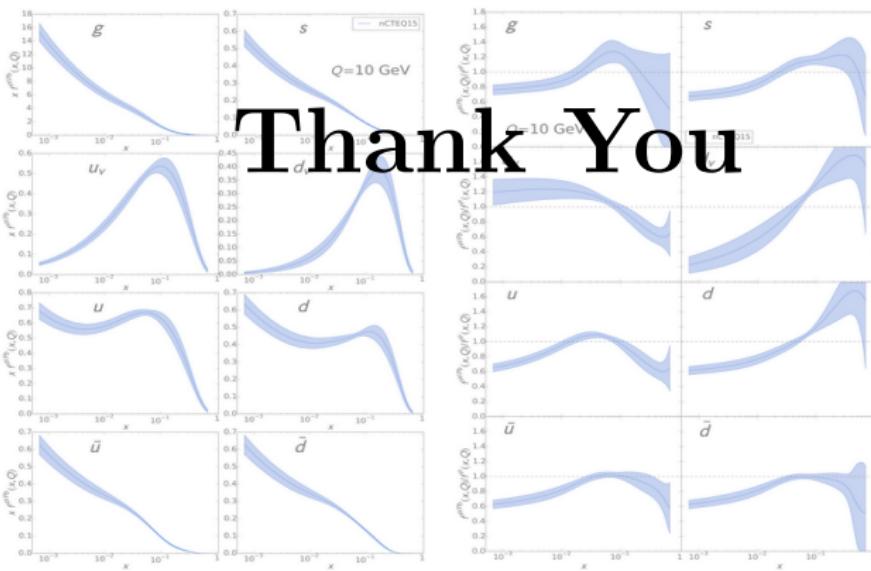


nCTEQ

nuclear parton distribution functions

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nCTEQ project is an extension of the CTEQ collaborative effort to determine parton distribution functions inside of a free proton. It generalizes the free-proton PDF framework to determine densities of partons in bound protons (hence nCTEQ which stands for nuclear CTEQ). All details on the framework and the first complete results can be found in arXiv:15????? [hep-ph]. The effects of the nuclear environment on the parton densities can be shown as modified parton densities or nuclear correction factors (for example for lead as shown below)



Thank You