Correction of β-beating due to beam-beam for the LHC and its impact on dynamic aperture

WEOAB2

Luis Medina[1,2], R. Tomás[2], J. Barranco[3], X. Buffat[1], Y. Papaphilippou[1], T. Pieloni[3]

[1] *Universidad de Guanajuato, León, Mexico*
[2] *CERN-BE-ABP, Geneva, Switzerland*
[3] *EPFL, Laussane, Switzerland*
`lmedinam@cern.ch`

8th International Particle Accelerator Conference
17 May 2017

This work is supported by the European Circular Energy-Frontier Collider Study, H2020 programme under grant agreement no. 654305, by the Swiss State Secretariat for Education, Research and Innovation SERI, and by the Beam project (CONACYT), Mexico.

Introduction

Beam-beam effects

Beam-beam effects

When the bunches of two beams of a particle collider come into proximity, they interact electromagnetically and give rise to **beam-beam (BB) effects**

When the bunches of two beams of a particle collider come into proximity, they interact electromagnetically and give rise to **beam-beam (BB) effects**

- Tune shift

Beam-beam effects

When the bunches of two beams of a particle collider come into proximity, they interact electromagnetically and give rise to **beam-beam (BB) effects**

- Tune shift
- Tune spread

Beam-beam effects

When the bunches of two beams of a particle collider come into proximity, they interact electromagnetically and give rise to **beam-beam (BB) effects**

- Tune shift
- Tune spread
- **β-beating**

Beam-beam effects

When the bunches of two beams of a particle collider come into proximity, they interact electromagnetically and give rise to **beam-beam** (BB) **effects**

- Tune shift
- Tune spread
- **β-beating**
- Beam stability and dynamic aperture

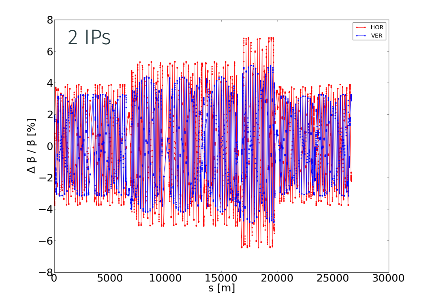
Beam-beam effects

When the bunches of two beams of a particle collider come into proximity, they interact electromagnetically and give rise to **beam-beam** (BB) **effects**

- Tune shift
- Tune spread
- **β-beating**
- Beam stability and dynamic aperture
- Etc.

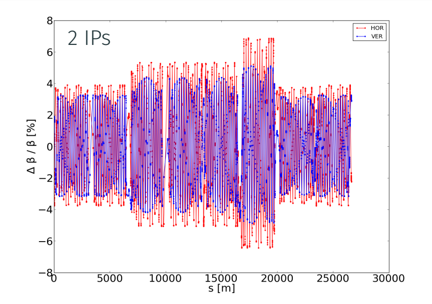
Motivation: beam-beam effects in the LHC and HL-LHC

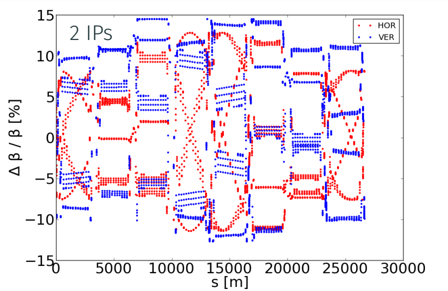
Motivation: beam-beam effects in the LHC and HL-LHC



LHC: $\xi_{bb} = 0.01$ (total)
8 % β-beating

Motivation: beam-beam effects in the LHC and HL-LHC

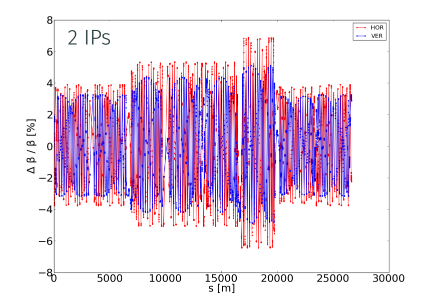


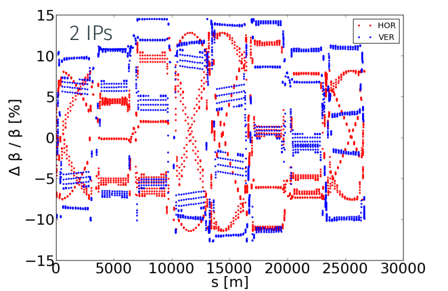


LHC: $\xi_{bb} = 0.01$ (total)
8 % β-beating

HL-LHC: $\xi_{bb} = 0.02 - 0.03$ (total)
15 % to 23 % β-beating

Motivation: beam-beam effects in the LHC and HL-LHC





LHC: $\xi_{bb} = 0.01$ (total)
8 % β-beating

HL-LHC: $\xi_{bb} = 0.02 - 0.03$ (total)
15 % to 23 % β-beating

- Impact on **performance**
  - $\pm 9\,\%$ β^* change for HL-LHC
  - Direct repercussion on luminosity → **luminosity imbalance** between the main experiments
- Impact on **protection system**

Compensation techniques

Compensation techniques

- Other compensation techniques:

Compensation techniques

- Other compensation techniques:
  - Electron beam lens

Compensation techniques

- Other compensation techniques:
  - Electron beam lens
  - Current-bearing wires

Compensation techniques

- Other compensation techniques:
  - Electron beam lens
  - Current-bearing wires
- Correction of β-beating by compensation of the **BB linear kick** with local **magnets**

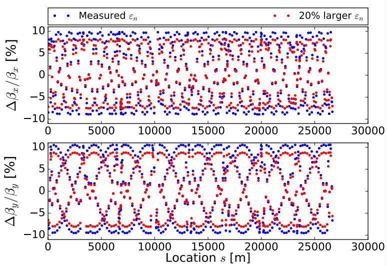
Compensation techniques

- Other compensation techniques:
  - Electron beam lens
  - Current-bearing wires

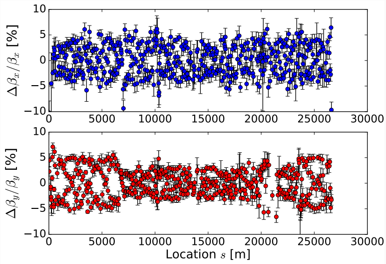
- Correction of β-beating by compensation of the **BB linear kick** with local **magnets**
  - First step for a correction scheme involving higher multipoles in view of the **HL-LHC**

Compensation techniques

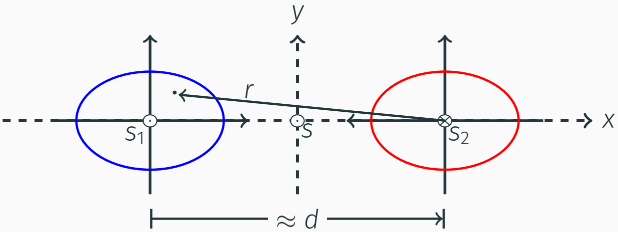
- Other compensation techniques:
  - Electron beam lens
  - Current-bearing wires
- Correction of β-beating by compensation of the **BB linear kick** with local **magnets**
  - First step for a correction scheme involving higher multipoles in view of the **HL-LHC**
  - **First measurements** and **preliminary test** in the LHC (**P. Gonçalves** *et. al.*, **TUPVA030**)



Simulation



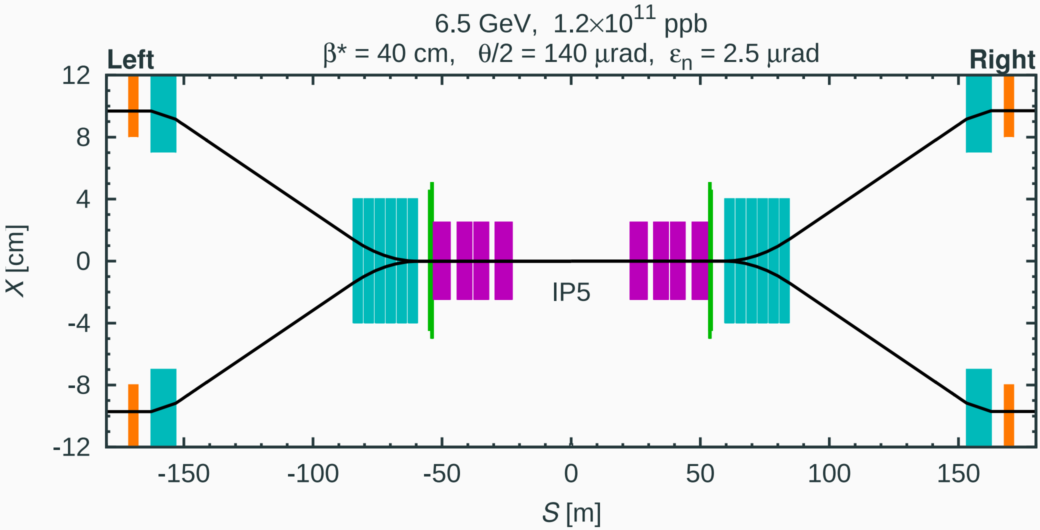
Measurement



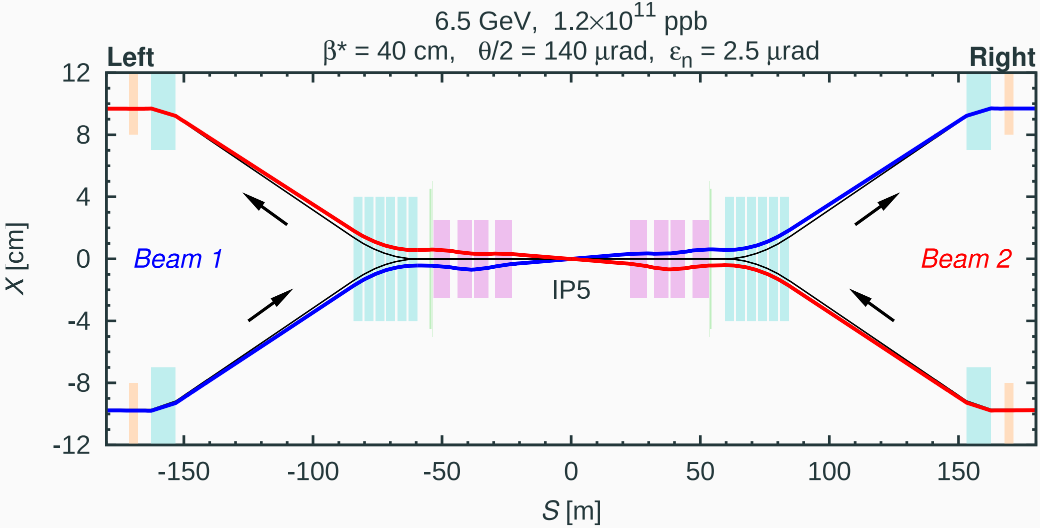
$$\left\{\begin{array}{c}\Delta x' \\ \Delta y'\end{array}\right\} = -\frac{2Nr_0}{\gamma}\frac{1}{r^2}\left\{\begin{array}{c}x \\ y\end{array}\right\}\left[1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)\right]$$

- r Radial distance from the test particle to the center of the opposite beam, $r = \sqrt{x^2 + y^2}$
- σ Beam size (assumed round)
- N Bunch population
- r_0 Classical particle radius
- γ Relativistic Lorentz factor
- d Beam separation

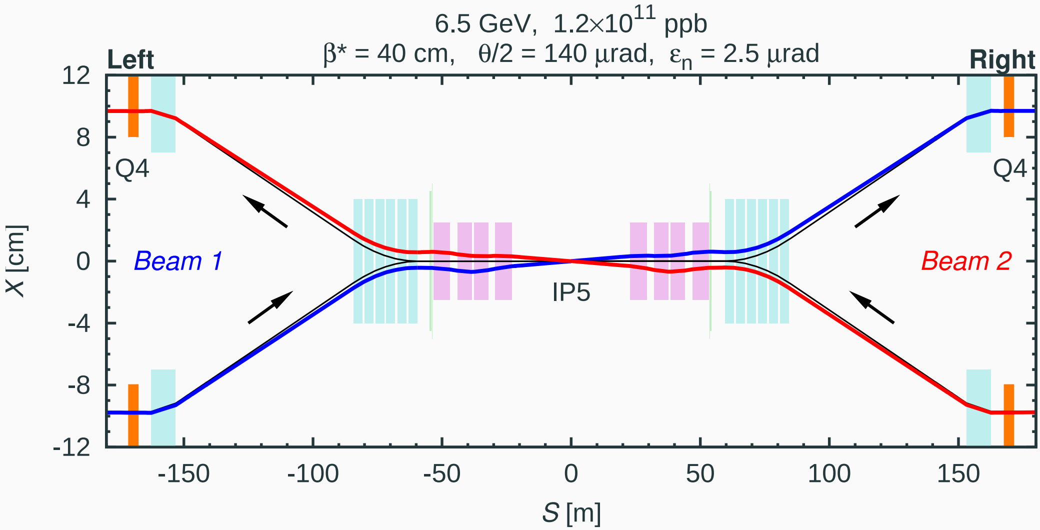
Example: LHC interaction region



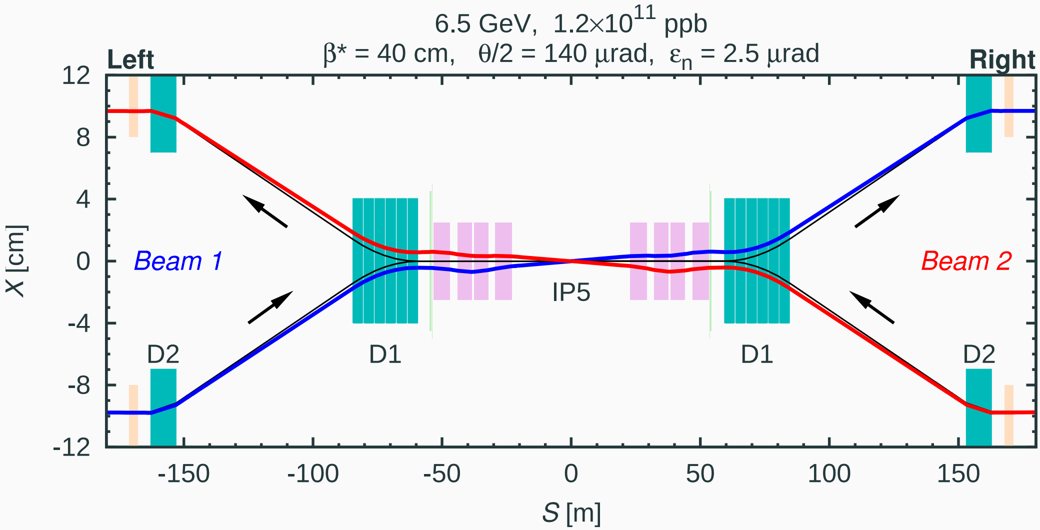
Example: LHC interaction region



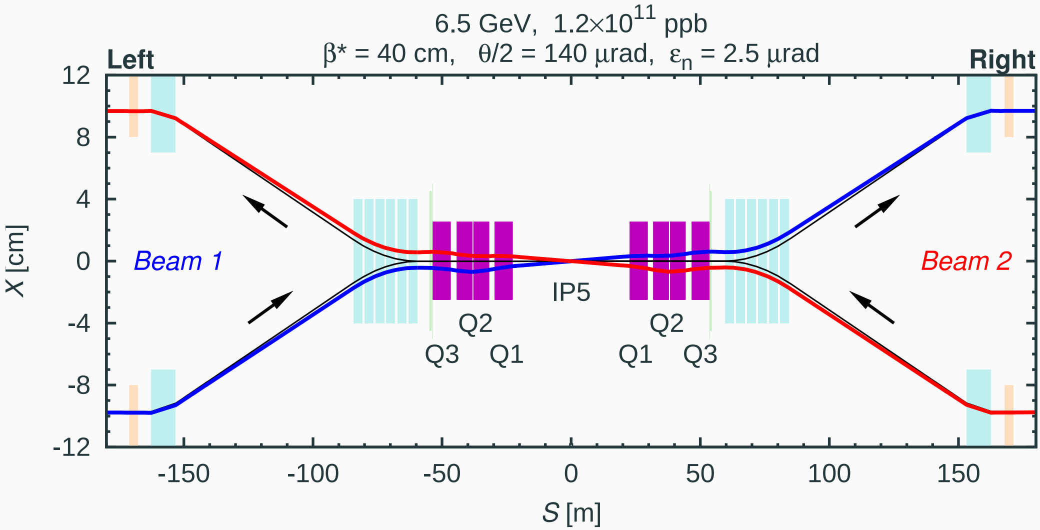
Example: LHC interaction region



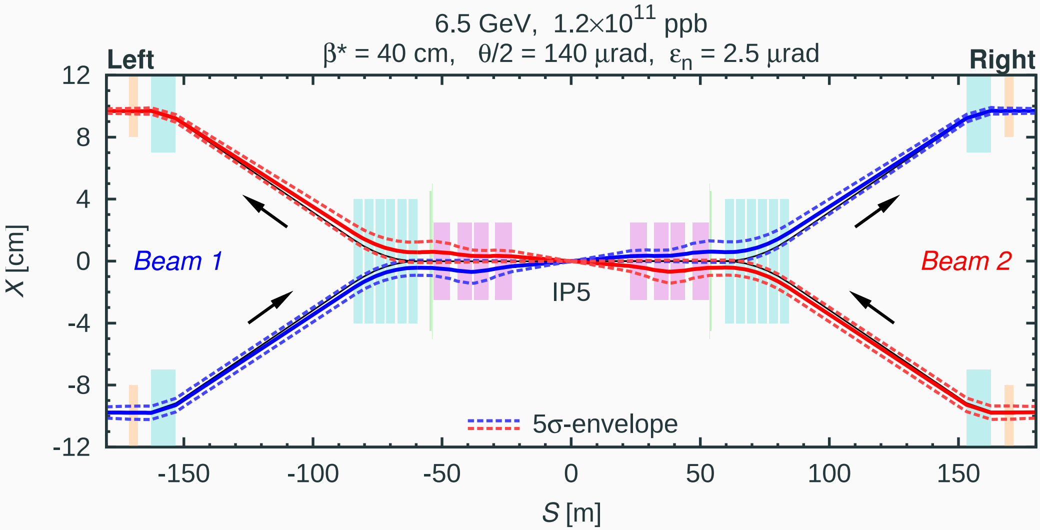
Example: LHC interaction region



Example: LHC interaction region



Example: LHC interaction region

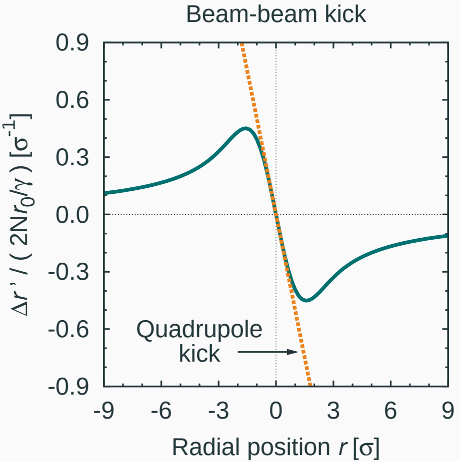


Head-on and long-range beam-beam expansion

Head-on (HO) beam-beam

- **Linearisation** of kick for **small amplitudes**:

$$\left\{ \begin{array}{c} \Delta x'|_{r\to 0} \\ \Delta y'|_{r\to 0} \end{array} \right\} = -\frac{N r_0}{\gamma \sigma^2} \left\{ \begin{array}{c} x \\ y \end{array} \right\}$$

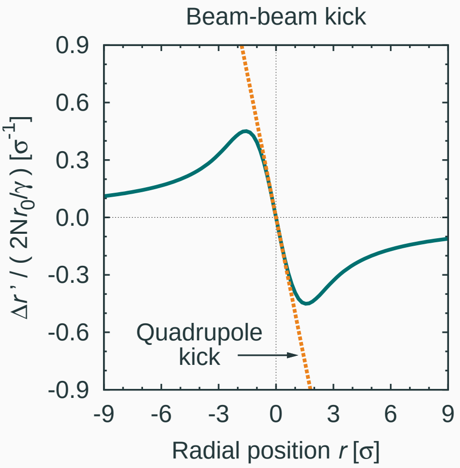


Head-on (HO) beam-beam

- **Linearisation** of kick for **small amplitudes**:

$$\begin{Bmatrix} \Delta x'|_{r\to 0} \\ \Delta y'|_{r\to 0} \end{Bmatrix} = -\frac{N r_0}{\gamma\sigma^2} \begin{Bmatrix} x \\ y \end{Bmatrix}$$

  - Same effect on both planes



Head-on (HO) beam-beam

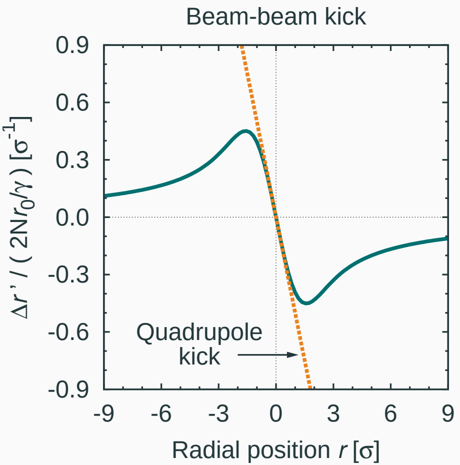
- **Linearisation** of kick for **small amplitudes**:

$$\begin{Bmatrix} \Delta x'|_{r\to 0} \\ \Delta y'|_{r\to 0} \end{Bmatrix} = -\frac{N r_0}{\gamma \sigma^2} \begin{Bmatrix} x \\ y \end{Bmatrix}$$

  - Same effect on both planes

- **Beam-beam parameter** as a measure of the induced **tune shift**:

$$\xi_{bb} \equiv \frac{\mathrm{d}(\Delta r')}{\mathrm{d}r} \frac{\beta^*}{4\pi} = \frac{N r_0 \beta^*}{4\pi\gamma\sigma^2}$$



- **Linearisation** of kick for **small amplitudes**:

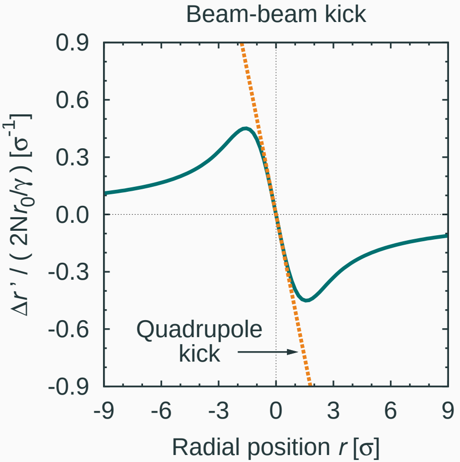
$$\left\{ \begin{array}{c} \Delta x'|_{r\to 0} \\ \Delta y'|_{r\to 0} \end{array} \right\} = -\frac{Nr_0}{\gamma\sigma^2} \left\{ \begin{array}{c} x \\ y \end{array} \right\}$$

  - Same effect on both planes

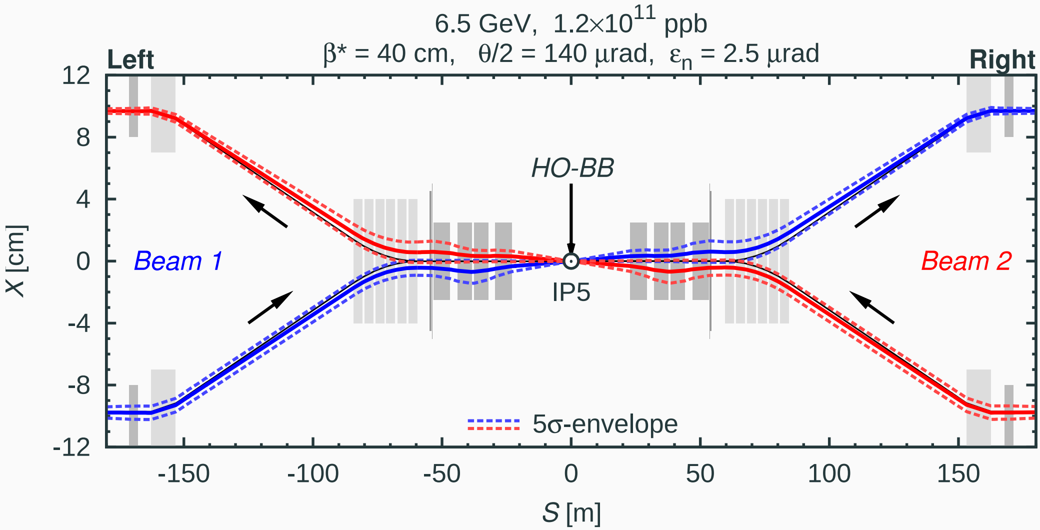
- **Beam-beam parameter** as a measure of the induced **tune shift**:

$$\xi_{bb} \equiv \frac{\mathrm{d}(\Delta r')}{\mathrm{d}r} \frac{\beta^*}{4\pi} = \frac{Nr_0\beta^*}{4\pi\gamma\sigma^2}$$

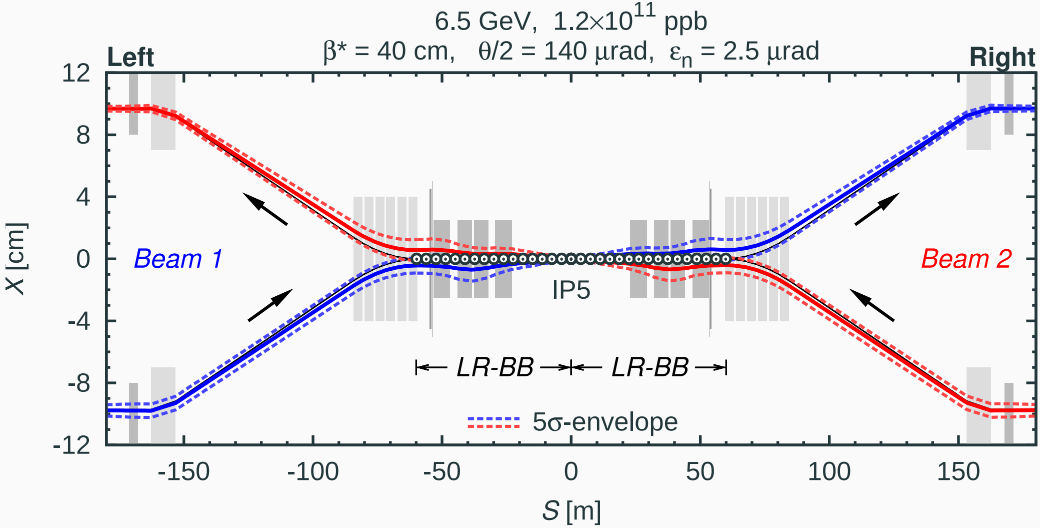
  - Horizontal and vertical



Head-on (HO) beam-beam: LHC



Long-range (LR) beam-beam: LHC (16 collisions per IP side)



Long-range (LR) beam-beam

- **Taylor expansions** up to second order around $(d, 0)$ (horizontal crossing):

Long-range (LR) beam-beam

- **Taylor expansions** up to second order around $(d, 0)$ (horizontal crossing):

$$
\begin{aligned}
\Delta x' &= K_0 + (K_1 + K_1')\Delta x + (K_2 + K_2')(\Delta x)^2 - K_2(\Delta y)^2,\\
\Delta y' &= \qquad\quad -K_1\Delta y \qquad\qquad\quad -2K_2\Delta x\Delta y,
\end{aligned}
$$

Long-range (LR) beam-beam

- **Taylor expansions** up to second order around $(d, 0)$ (horizontal crossing):

$$\begin{aligned}\Delta x' &= K_0 + (K_1 + K_1')\Delta x + (K_2 + K_2')(\Delta x)^2 - K_2(\Delta y)^2,\\ \Delta y' &= -K_1 \Delta y - 2K_2 \Delta x \Delta y,\end{aligned}$$

where K_i and K_i' are functions of

$$E_d \equiv \exp\left(-\frac{d^2}{2\sigma^2}\right) \tag{1}$$

(See Appendix A)

- **Taylor expansions** up to second order around $(d, 0)$ (horizontal crossing):

$$\Delta x' = K_0 + (K_1 + K_1')\Delta x + (K_2 + K_2')(\Delta x)^2 - K_2(\Delta y)^2,$$
$$\Delta y' = -K_1 \Delta y - 2K_2 \Delta x \Delta y,$$

where K_i and K_i' are functions of

$$E_d \equiv \exp\left(-\frac{d^2}{2\sigma^2}\right) \quad (1)$$

(See Appendix A)

Long-range (LR) beam-beam

- **Taylor expansions** up to second order around $(d, 0)$ (horizontal crossing):

$$\begin{aligned}\Delta x' &= K_0 + (K_1 + K_1')\Delta x + (K_2 + K_2')(\Delta x)^2 - K_2(\Delta y)^2, \\ \Delta y' &= \qquad -K_1 \Delta y \qquad\qquad -2K_2 \Delta x \Delta y,\end{aligned}$$

where K_i and K_i' are functions of

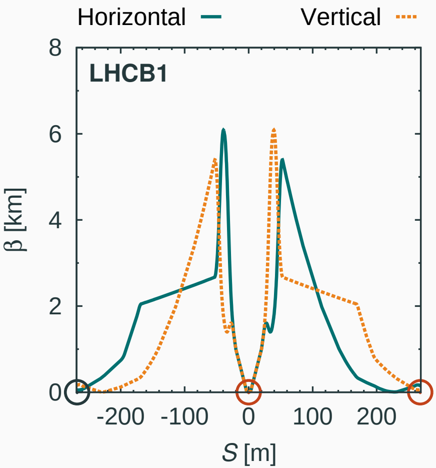
$$E_d \equiv \exp\left(-\frac{d^2}{2\sigma^2}\right) \tag{1}$$

(See Appendix A)

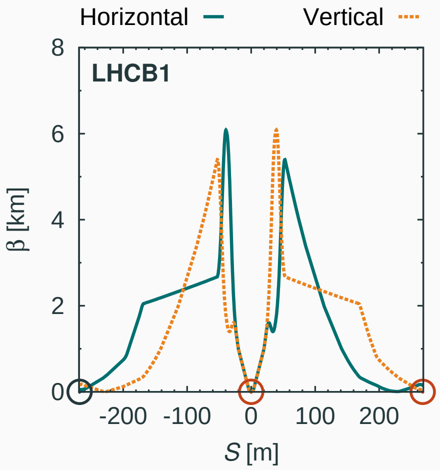
Procedure and results

Procedure

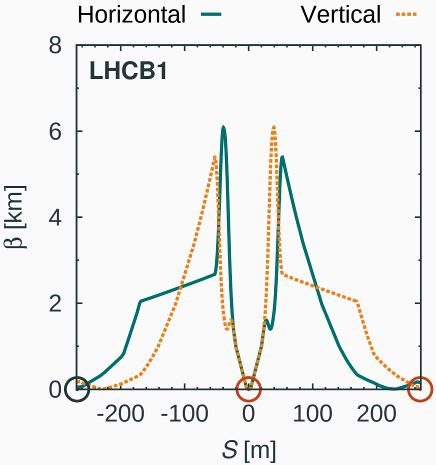
- **Re-matching** of optics ($\beta_{x,y}$, $\alpha_{x,y}$) at the start / IP / end of each IR (separately)



- **Re-matching** of optics ($\beta_{x,y}$, $\alpha_{x,y}$) at the start / IP / end of each IR (separately)
    - **Eight degrees of freedom** per beam per IP



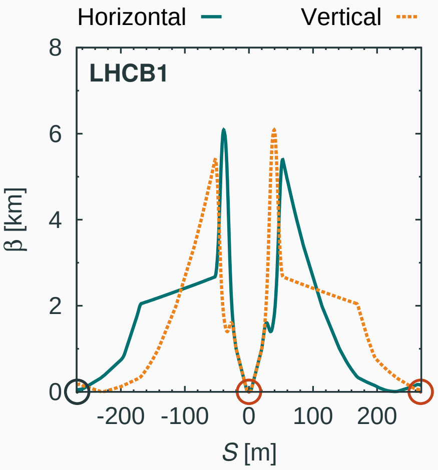
- **Re-matching** of optics ($\beta_{x,y}$, $\alpha_{x,y}$) at the start / IP / end of each IR (separately)
    - **Eight degrees of freedom** per beam per IP
    - Eight variables: **4 left-right pairs of magnets**



Procedure

- **Re-matching** of optics ($\beta_{x,y}$, $\alpha_{x,y}$) at the start / IP / end of each IR (separately)
    - **Eight degrees of freedom** per beam per IP
    - Eight variables: **4 left-right pairs of magnets**
- Re-matching of

  Tunes to **(64.31, 59.32)**
  Chromaticities to **2**



Choice of magnets

Choice of magnets

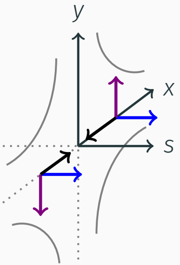
- Correction in **both beams**

Choice of magnets

- Correction in **both beams**
- Magnet strengths for counter-rotating beams: $K_n \to (-1)^n K_n$ (0: dipole, 1: quad, etc.)

Choice of magnets

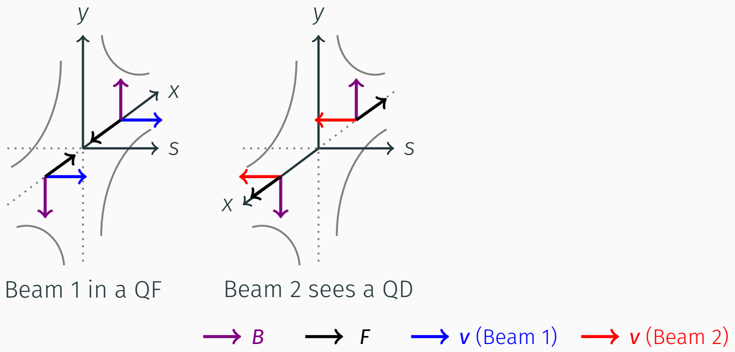
- Correction in **both beams**
- Magnet strengths for counter-rotating beams: $K_n \to (-1)^n K_n$ (0: dipole, 1: quad, etc.)



Beam 1 in a QF

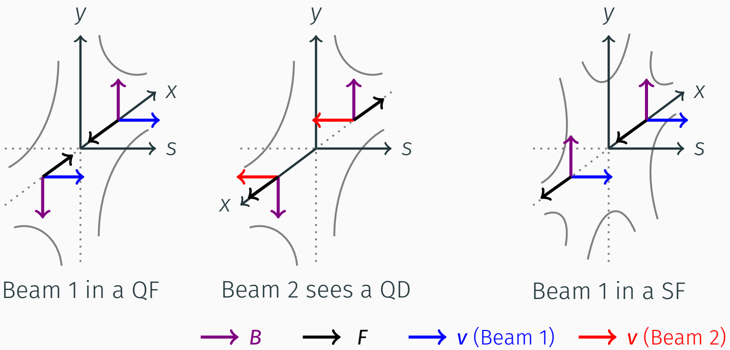
Choice of magnets

- Correction in **both beams**
- Magnet strengths for counter-rotating beams: $K_n \to (-1)^n K_n$ (0: dipole, 1: quad, etc.)



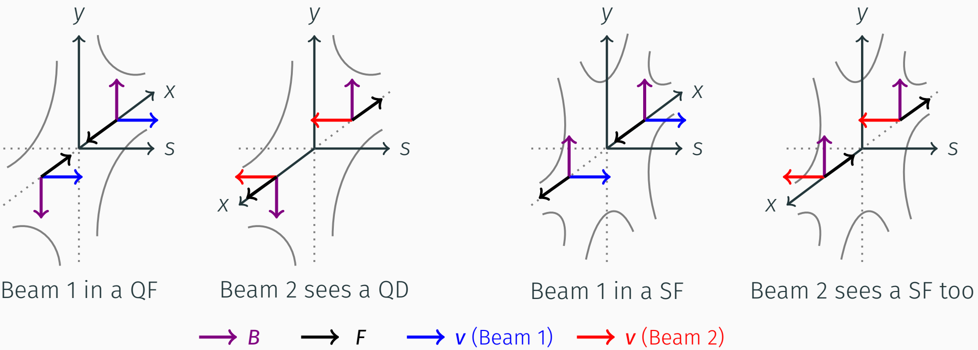
Choice of magnets

- Correction in **both beams**
- Magnet strengths for counter-rotating beams: $K_n \to (-1)^n K_n$ (0: dipole, 1: quad, etc.)

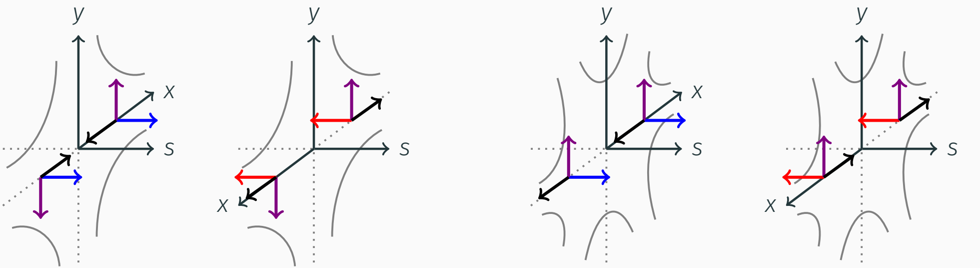


Choice of magnets

- Correction in **both beams**
- Magnet strengths for counter-rotating beams: $K_n \to (-1)^n K_n$ (0: dipole, 1: quad, etc.)



- Correction in **both beams**
- Magnet strengths for counter-rotating beams: $K_n \to (-1)^n K_n$ (0: dipole, 1: quad, etc.)

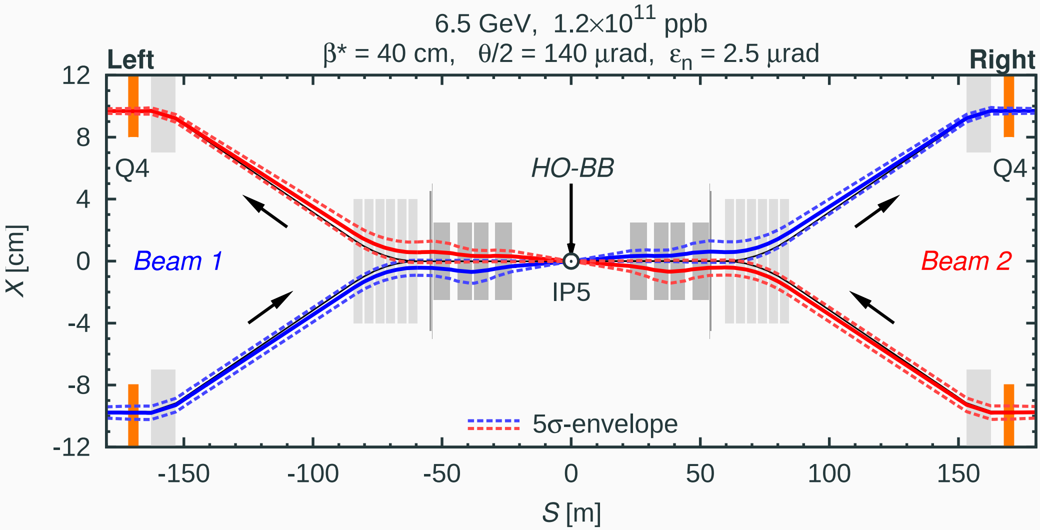


Beam 1 in a QF | Beam 2 sees a QD | Beam 1 in a SF | Beam 2 sees a SF too

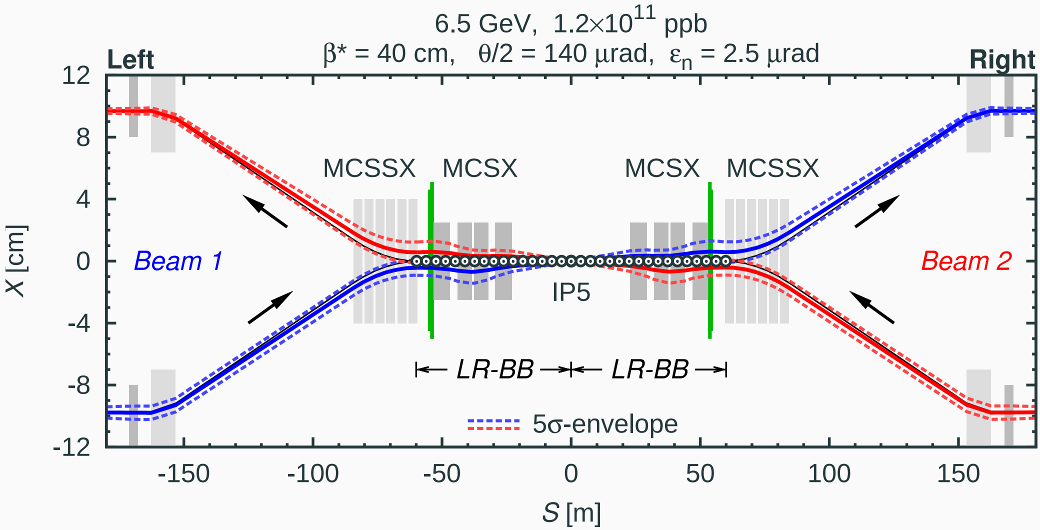
→ B → F → $\mathbf{v}$ (Beam 1) → $\mathbf{v}$ (Beam 2)

- **Quadrupole**, **octupole**, etc. components of the BB **cannot** be directly compensated for both beams using **common magnets**.

Choice of magnets: Matching quadrupoles for HO

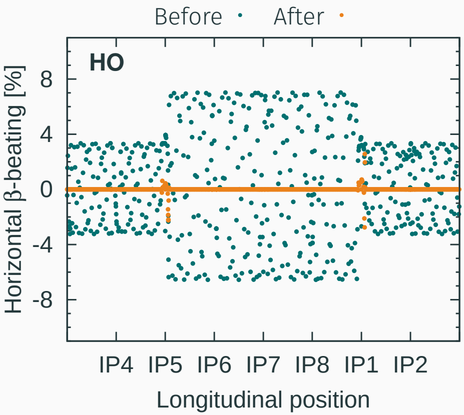


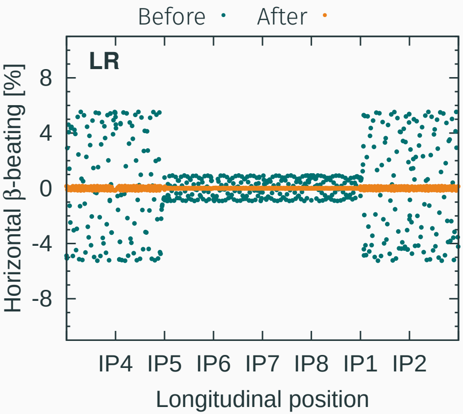
Choice of magnets: Common sextupoles for LR



Reduction of RMS β-beating due to HO-BB or LR-BB

Reduction of RMS β-beating due to HO-BB or LR-BB





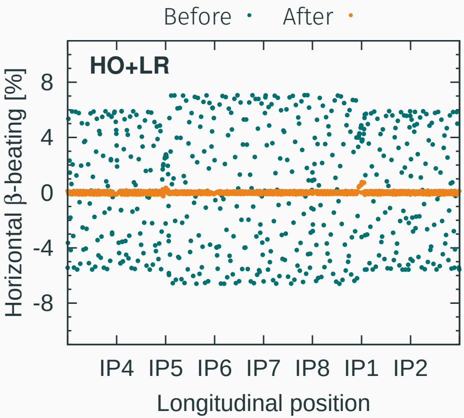
HO: from 3.67 % / 1.91 % (Hor./Ver.)
to **0.30 % / 0.15 %**

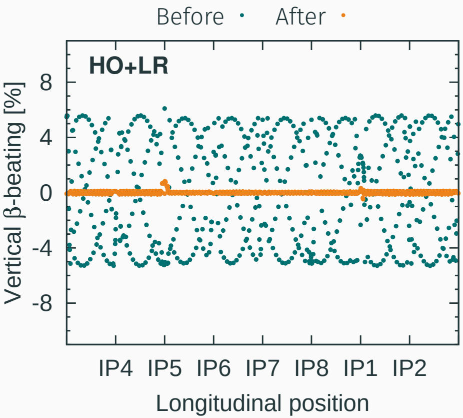
LR: from 2.69 % / 3.84 %
to **0.04 % / 0.04 %**

Reduction of RMS β-beating due to HO-BB <u>and</u> LR-BB

Reduction of RMS β-beating due to HO-BB and LR-BB

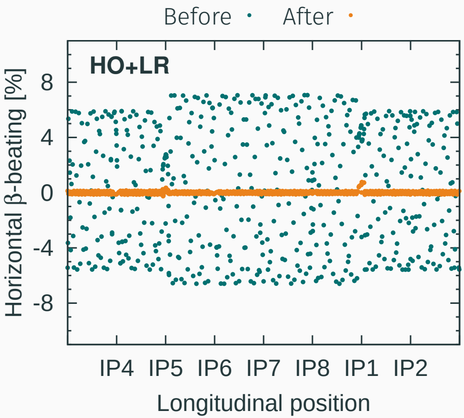
- Reduction of **RMS β-beating to** $< 0.15\,\%$

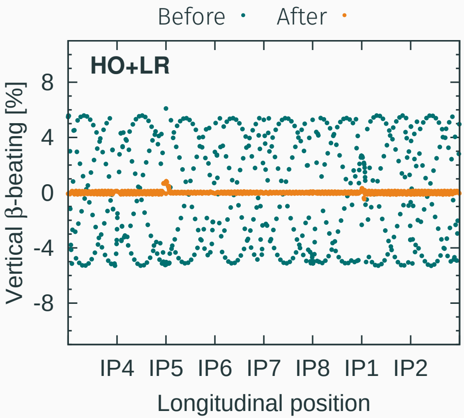




Reduction of RMS β-beating due to HO-BB and LR-BB

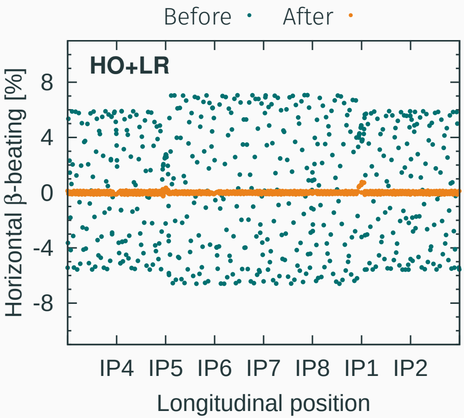
- Reduction of **RMS β-beating to** $< 0.15\,\%$
- Tunes reduced by 0.01, chromaticities increased by 2 units $\rightarrow$ Re-matched to nominal

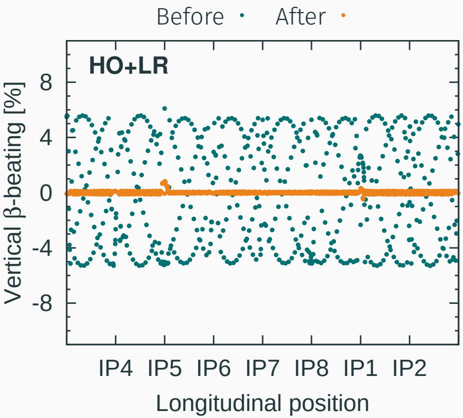




Reduction of RMS β-beating due to HO-BB <u>and</u> LR-BB

- Reduction of **RMS β-beating to** $< 0.15\,\%$
- Tunes reduced by 0.01, chromaticities increased by 2 units $\rightarrow$ Re-matched to nominal
- Correction with an identical process for the **opposite beam** $\rightarrow$ **Similar results**





Stability of the HO-BB and LR-BB correction

Stability of the HO-BB and LR-BB correction

- Correcting sextupole strengths have opposite sign to the sextupolar term of the BB kick.

Stability of the HO-BB and LR-BB correction

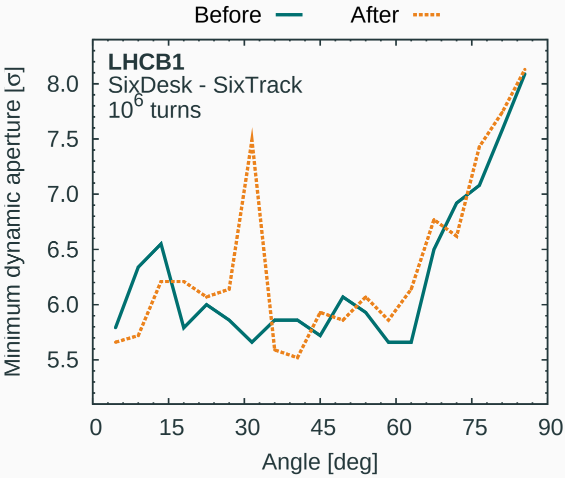
- Correcting sextupole strengths have opposite sign to the sextupolar term of the BB kick.
- **Non-linear** elements

Stability of the HO-BB and LR-BB correction

- Correcting sextupole strengths have opposite sign to the sextupolar term of the BB kick.
- **Non-linear** elements
  - **Long-term stability**?

Stability of the HO-BB and LR-BB correction

- Correcting sextupole strengths have opposite sign to the sextupolar term of the BB kick.
- **Non-linear** elements
    - **Long-term stability**?
- Dynamic aperture (DA), via single-particle tracking.

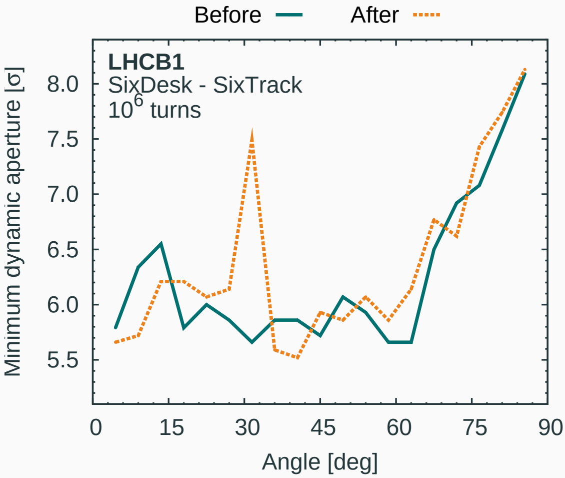


$I_{\mathrm{oct}} = 0\,\mathrm{A}$
2 units of chromaticity

Stability of the HO-BB and LR-BB correction

- Correcting sextupole strengths have opposite sign to the sextupolar term of the BB kick.
- **Non-linear** elements
  - **Long-term stability**?
- Dynamic aperture (DA), via single-particle tracking.
- Little impact on DA $> \mathbf{5.5\sigma}$ for all angles

$I_{\text{oct}} = 0$ A
2 units of chromaticity



Conclusions and outlook

Conclusions and Outlook

- **Beam-beam** interactions can limit the machine **performance**.

Conclusions and Outlook

- **Beam-beam** interactions can limit the machine **performance**.
  - Luminosity imbalance, machine protection

Conclusions and Outlook

- **Beam-beam** interactions can limit the machine **performance**.
  - Luminosity imbalance, machine protection
- Induced **β-beating** can be corrected, at least partially, by matching local magnet strenghts to the multipolar terms of the BB kick expansion.

Conclusions and Outlook

- **Beam-beam** interactions can limit the machine **performance**.
    - Luminosity imbalance, machine protection
- Induced **β-beating** can be corrected, at least partially, by matching local magnet strenghts to the multipolar terms of the BB kick expansion.
- Successful application to the current **LHC** optics (**RMS beating** $< 1\%$)

Conclusions and Outlook

- **Beam-beam** interactions can limit the machine **performance**.
  - Luminosity imbalance, machine protection
- Induced **β-beating** can be corrected, at least partially, by matching local magnet strenghts to the multipolar terms of the BB kick expansion.
- Successful application to the current **LHC** optics (**RMS beating** $< 1\%$)
  - Linear HO corrected with matching quadrupoles

Conclusions and Outlook

- **Beam-beam** interactions can limit the machine **performance**.
  - Luminosity imbalance, machine protection

- Induced **β-beating** can be corrected, at least partially, by matching local magnet strenghts to the multipolar terms of the BB kick expansion.

- Successful application to the current **LHC** optics (**RMS beating $< 1\,\%$**)
  - Linear HO corrected with matching quadrupoles
  - LR quadrupolar term corrected via sextupole feed-down

Conclusions and Outlook

- **Beam-beam** interactions can limit the machine **performance**.
  - Luminosity imbalance, machine protection
- Induced **β-beating** can be corrected, at least partially, by matching local magnet strenghts to the multipolar terms of the BB kick expansion.
- Successful application to the current **LHC** optics (**RMS beating** $< 1\,\%$)
  - Linear HO corrected with matching quadrupoles
  - LR quadrupolar term corrected via sextupole feed-down
- Compensation scheme involving common sextupoles has **negligible impact on DA**.

Conclusions and Outlook

- **Beam-beam** interactions can limit the machine **performance**.
    - Luminosity imbalance, machine protection
- Induced **β-beating** can be corrected, at least partially, by matching local magnet strenghts to the multipolar terms of the BB kick expansion.
- Successful application to the current **LHC** optics (**RMS beating** $< 1\,\%$)
    - Linear HO corrected with matching quadrupoles
    - LR quadrupolar term corrected via sextupole feed-down
- Compensation scheme involving common sextupoles has **negligible impact on DA**.
- **First measurements** and **test** of correction in LHC $\rightarrow$ anyalsis on-going

Conclusions and Outlook

- **Beam-beam** interactions can limit the machine **performance**.
    - Luminosity imbalance, machine protection
- Induced **β-beating** can be corrected, at least partially, by matching local magnet strenghts to the multipolar terms of the BB kick expansion.
- Successful application to the current **LHC** optics (**RMS beating** $< 1\,\%$)
    - Linear HO corrected with matching quadrupoles
    - LR quadrupolar term corrected via sextupole feed-down
- Compensation scheme involving common sextupoles has **negligible impact on DA**.
- **First measurements** and **test** of correction in LHC $\rightarrow$ anyalsis on-going
- Extension to higher orders, and to the **HL-LHC**:

Conclusions and Outlook

- **Beam-beam** interactions can limit the machine **performance**.
  - Luminosity imbalance, machine protection
- Induced **β-beating** can be corrected, at least partially, by matching local magnet strenghts to the multipolar terms of the BB kick expansion.
- Successful application to the current **LHC** optics (**RMS beating $< 1\,\%$**)
  - Linear HO corrected with matching quadrupoles
  - LR quadrupolar term corrected via sextupole feed-down
- Compensation scheme involving common sextupoles has **negligible impact on DA**.
- **First measurements** and **test** of correction in LHC $\rightarrow$ anyalsis on-going
- Extension to higher orders, and to the **HL-LHC**:
  - Compensation of beam-beam octupolar component via feed-down from decapoles (not present in the LHC)

Thank you

Appendix A: Long-range beam-beam kick expansion

LR-BB kick expansion

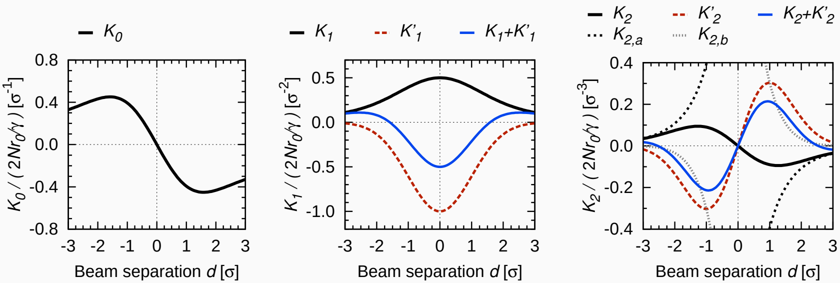
- Horizontal crossing
- **Taylor expansions** up to second order around $(d, 0)$ (**horizontal** crossing):

$$\Delta x' = K_0 + (K_1 + K_1')\Delta x + (K_2 + K_2')(\Delta x)^2 - K_2(\Delta y)^2,$$
$$\Delta y' = \qquad -K_1 \Delta y \qquad - 2K_2 \Delta x \Delta y,$$

where

$$K_0 = -\frac{2Nr_0}{\gamma}\left(\frac{1-E_d}{d}\right), \qquad E_d \equiv \exp\left(-\frac{d^2}{2\sigma^2}\right)$$
$$K_1 = +\frac{2Nr_0}{\gamma}\left(\frac{1-E_d}{d^2}\right), \qquad K_1' = -\frac{2Nr_0}{\gamma}\frac{E_d}{\sigma^2},$$
$$K_2 = -\frac{2Nr_0}{\gamma}\left(\frac{1-E_d}{d^3} - \frac{E_d}{2\sigma^2 d}\right) \equiv K_{2,a} + K_{2,b}, \qquad K_2' = +\frac{2Nr_0}{\gamma}\frac{E_d d}{2\sigma^4}$$

LR-BB kick expansion



Dipolar (*left*), quadrupolar (*center*), and sextupolar (*right*) terms in the LR kick multipolar expansion.

- **Taylor expansions** up to second order around $(0, d)$ (**vertical** crossing):

$$\Delta x' = -K_1 \Delta x - 2K_2 \Delta x \Delta y,$$

$$\Delta y' = K_0 + (K_1 + K_1') \Delta y - K_2 (\Delta x)^2 + (K_2 + K_2')(\Delta y)^2$$

LR-BB kick expansion: large separation

- Horizontal crossing
- **Taylor expansions** up to second order around $(d, 0)$ (**horizontal** crossing):

$$\begin{aligned}\Delta x' &= K_0 + K_1 \Delta x + K_2 (\Delta x)^2 - K_2 (\Delta y)^2, \\ \Delta y' &= \qquad -K_1 \Delta y \qquad - 2K_2 \Delta x \Delta y,\end{aligned}$$

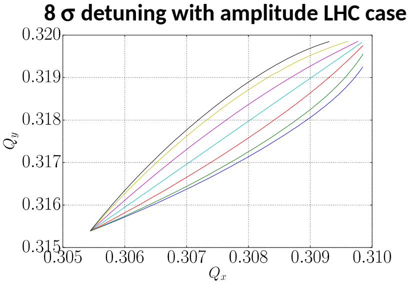
where

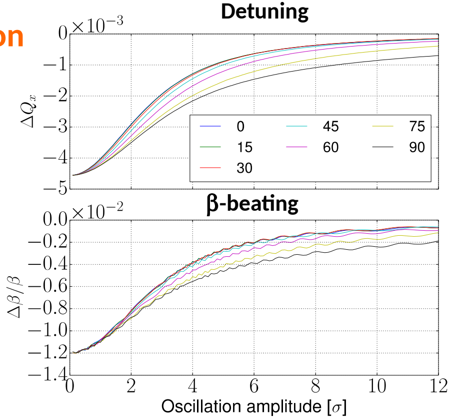
$$\begin{aligned}K_0 &= -\frac{2Nr_0}{\gamma}\left(\frac{1-E_d}{d}\right), \qquad\qquad E_d \equiv \exp\left(-\tfrac{d^2}{2\sigma^2}\right) \\ K_1 &= +\frac{2Nr_0}{\gamma}\left(\frac{1-E_d}{d^2}\right), \\ K_2 &= -\frac{2Nr_0}{\gamma}\left(\frac{1-E_d}{d^3}\right) = K_{2,a}\end{aligned}$$

Appendix B: Amplitude-dependent non-linear β-beating

Appendix B: Amplitude-dependent non-linear β-beating (head-on collision)

HEAD-ON Beam-Beam Interaction





The non-linear beta-beating vanish asymptotically with the particle amplitude (halo particles effect negligible)

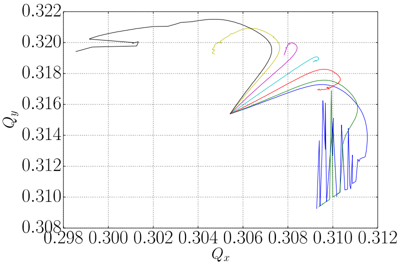
→ Similar behavior as detuning with amplitude, can be used to increase Lumi

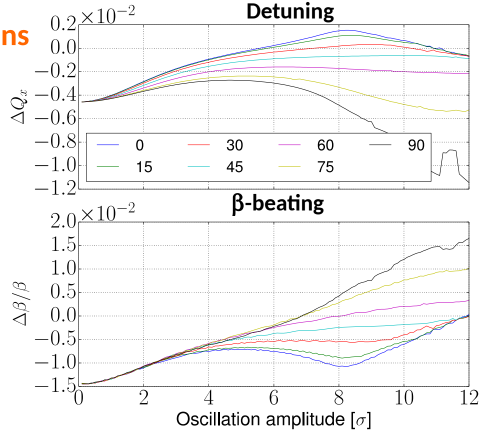
Relevant for performances !

Appendix B: Amplitude-dependent non-linear β-beating (head-on collision)

HEAD-ON + Long-Range Interactions

8 σ detuning with amplitude LHC case





The **non-linear beta-beating does NOT vanish asymptotically with the particle amplitude** (core particles see mainly HO)

→ If β-beating of particles at amplitudes < 6 σ approaches tolerances of collimation system → **Cleaning Efficiency could be affected!**

Resources

Resources -- i

- W. Herr and T. Pieloni, “Beam-beam effects”, in Proc. CAS (Advanced Accelerator Physics), edited by W. Herr, Trondheim, Norway, Aug. 2013, CERN-2014-009 (CERN, Geneva, 2014) `arXiv:1601.05235 [physics.acc-ph]`, `doi:10.5170/CERN-2014-009.431`
- D. Neuffer and S.G. Pegg, “Beam-beam tune shifts and spreads in the SSC -- Head on, long range, and PACMAN conditions”, SSC-063, 1986. `http://lss.fnal.gov/archive/other/ssc/ssc-63.pdf`
- J. Shi, L. Jin, and O. Kheawpum, “Multipole compensation of long-range beam-beam interactions with minimization of nonlinearities in Poincaré maps of a storage-ring collider”, Phys. Rev. E, vol. 69, issue 3, p. 036502, Mar. 2004. `doi:10.1103/PhysRevE.69.036502`, `https://link.aps.org/doi/10.1103/PhysRevE.69.036502`
- T. Pieloni *et al.*, “Dynamic beta and beta-beating effects in the presence of the beam-beam interactions”, in *HB'16*, Malmö, Sweden, Jun. 2016, paper MOPR027, pp. 136--139, 2016 `doi:10.18429/JACoW-HB2016-MOPR027`, `http://jacow.org/hb2016/papers/mopr027.pdf`
- R. Tomás *et al.*, “Record low β beating in the LHC”, *Phys. Rev. ST Accel. Beams*, vol. 15, issue 9, p. 091001, Sep. 2012. `https://link.aps.org/doi/10.1103/PhysRevSTAB.15.091001`, `doi:10.1103/PhysRevSTAB.15.091001`

Resources -- ii

- P. Gonçalves Jorge *et. al.*, "Measurement of beta-beating due to strong head-on beam-beam interactions in the LHC", presented at the 8th IPAC'17, Copenhagen, Denmark, May 2017, paper TUPVA030, to be published.
- S. Fartoukh, A. Valishev, Y. Papaphilippou, and D. Shatilov, "Compensation of the long-range beam-beam interactions as a path towards new configurations for the high luminosity LHC", *Phys. Rev. ST Accel. Beams*, vol. 18, issue 12, p. 121001, Dec. 2015. `doi:10.1103/PhysRevSTAB.18.121001`, `https://link.aps.org/doi/10.1103/PhysRevSTAB.18.121001`
- M. Pivi, "Beam-beam effects in particle colliders", USPAS, Hampton, VA, USA, 2011. `http://uspas.fnal.gov/materials/11ODU/Beam-Beam.pdf`
- LHC Optics Web: LHC Run II pp physics - Collision (0.4m) optics, `http://lhc-optics.web.cern.ch/lhc-optics/www/opt2016/coll400/index.html`
- MAD - Methodical Accelerator Design, `http://mad.web.cern.ch/mad/`
- SixDesk, `https://github.com/SixTrack/SixDesk/`
- SixTrack -- 6D Tracking Code, `http://sixtrack.web.cern.ch/SixTrack/`