LHC reduced lattice

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Contents

Motivation - Use

- Motivation
  - Construction of a simplified non-linear lattice that:
    1. is small enough in order to speed up the tracking process
    2. reproduces all the linear properties of the LHC (tune, natural chromatisity)
    3. capture the global non-linear behavior of the LHC (tune shift with amplitude, DA, etc.)
    4. is flexible enough in order to use it for different operational scenarios
- The reduced non-linear lattice can be used for
  - Studies that are not interested in the element by element dynamics.
  - Fast multi-particle tracking for beam-beam effects, noise, etc. Fast tracking of distributions.

Symplectify a truncated Taylor series

- Use a truncated Taylor series (TTS) up to 3^{rd} order (up to octupolar contribution).

$$\mathbf{Z}_j = \Delta\mathbf{Z}_j + \sum_{k=1}^{6} \mathbf{R}_{jk}\mathbf{Z}_k + \sum_{k=1}^{6}\sum_{n=1}^{6} \mathbf{T}_{jkn}\mathbf{Z}_k\mathbf{Z}_n + \sum_{k=1}^{6}\sum_{n=1}^{6}\sum_{m=1}^{6} \mathbf{Y}_{jknm}\mathbf{Z}_k\mathbf{Z}_n\mathbf{Z}_m$$

- Symplectify the TTS according to the symplectic condition $\mathbf{J}^T\mathbf{S}\mathbf{J} = \mathbf{S}$

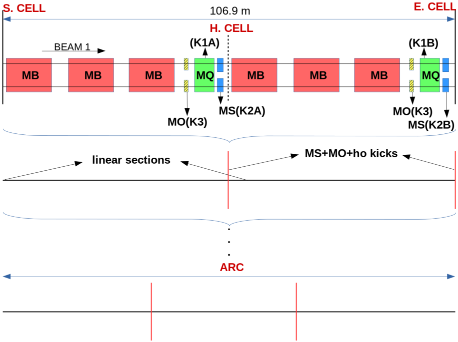
Potential benefit

- get a compact map that includes the necessary information after each revolution

Why TTS is not used

- not trivial symplectification processes (a large set of algebrical equations must be satisfied simultaneously)
- difficult to give a physical interpretation to the element of the tensors in order to adjust the map for different operational scenario (need to recompute the map for different settings)

Use of Lie algebra



- Simplify the LHC lattice
- Extensive use of the Baker-Campbell-Hausdorff (BCH) formula and of the following relations:

$$e^{:f:} g(h) = g\left(e^{:f:} h\right)$$

$$e^{:g:} e^{:f:} = e^{:f:} exp\left[: e^{-:f:} g :\right]$$

Potential benefit

- symplecticity and flexibility of the Lie operators, elegant formalism, etc.

Why this simplification is not used

- extensive use of approximations that lead to a low order integrator
- the complexity of the non-linear kicks is increased very fast (presence of higher multipoles that increase the computational time)

Effective lattice - contraction

Constraction steps

- Since we are interested in reproducing the LHC properties, a new linear lattice with similar optical functions (β, α, D, D') and the same tunes (Q_X, Q_Y) is assumed.
- We are not interested in the actual structure of this new lattice so the linear sections are described by rotations.
- This lattice is divided in two parts. The phase advance of the first part is equal to the one from IP1 to IP5 and for the second part it is equal to the phase advance from IP5 to IP1.
- Non-linear kicks are added to the linear lattice. The β function at the kicks is the same as in the full machine.
- The phase advance difference between kicks of the same family is equal to $\pi + n2\pi$ in order to mitigate the resonances as in the LHC.

Effective lattice - rotation map

$$
\begin{pmatrix}
\sqrt{\frac{{}_f\beta}{{}_i\beta}}\ \left(Cos(\psi) + {}_i\alpha\ Sin(\psi)\right) & \sqrt{{}_f\beta\,{}_i\beta}\ Sin(\psi) & M13 \\
\frac{({}_i\alpha - {}_f\alpha)\ Cos(\psi) - (1 + {}_i\alpha\ \ {}_f\alpha)\ Sin(\psi)}{\sqrt{{}_f\beta\,{}_i\beta^i}} & \sqrt{\frac{{}_i\beta}{{}_f\beta}}\ \left(Cos(\psi) - {}_f\alpha\ Sin(\psi)\right) & M23 \\
0 & 0 & 1
\end{pmatrix}
$$

$$
M13 = {}_fD - {}_iD'\sqrt{{}_i\beta\,{}_f\beta}\ Sin(\psi) - {}_iD\sqrt{\frac{{}_f\beta}{{}_i\beta}}\ \left(Cos(\psi) + {}_i\alpha\ Sin(\psi)\right)
$$

$$
M23 = {}_fD' + {}_iD'\sqrt{\frac{{}_i\beta}{{}_f\beta}}\ \left({}_f\alpha\ Sin(\psi) - Cos(\psi)\right) + {}_iD\left[\frac{({}_f\alpha - {}_i\alpha)\ Cos(\psi) + (1 + {}_f\alpha\,{}_i\alpha)\ Sin(\psi)}{\sqrt{{}_f\beta\,{}_i\beta}}\right]
$$

$$
\psi_j = \mu + G\ \delta
$$

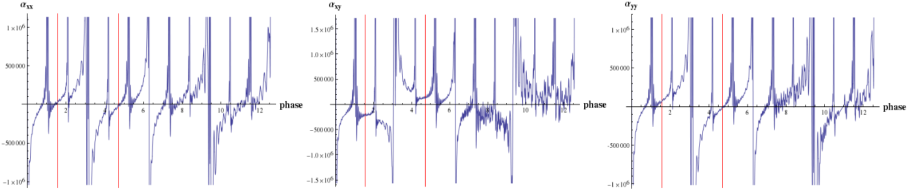
where i is the initial and f the final value

Effective lattice - non-linear map

$$
\begin{aligned}
{}_fX &= {}_iX \\
{}_fP_x &= {}_iP_x - K_S\,({}_iX^2 - {}_iY^2) - K_O\left(\frac{{}_iX^3}{3} - {}_iX\,{}_iY^2\right) \\
{}_fY &= {}_iY \\
{}_fP_y &= {}_iP_y + K_S\;{}_iX\,{}_iY + K_O\left({}_iX^2\,{}_iY - \frac{{}_iY^3}{3}\right) \\
{}_f\delta &= {}_i\delta
\end{aligned}
$$

Effective lattice - Tune shift with amplitude cased by the sextupoles

- For the LHC, the tune shift with amplitude (TSA) caused by the sextupoles is close to zero. This can be reproduced tuning the phase advance difference ($\Delta\psi$) between the kicks of same family.
- A combination of π and 3π for $\Delta\psi$ values is used. This also preserve the $-\mathcal{I}$ transformation.
- $TSA_{S,m} = \alpha_{mm} J_m + \alpha_{xy} J_n$, m=x,y n=x,y $m \neq n$.



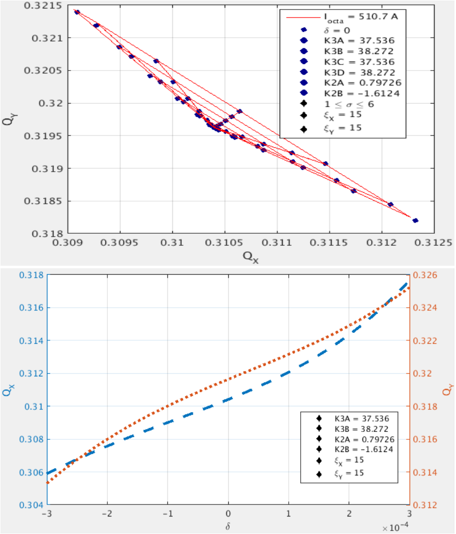
Different versions

- Various versions, according to the number of the non-linear kicks, are studied.
- The following results refer to the version 1 (V1) which is the fastest and to the V2 (with the largest number of kicks) which is less affected by the non-linear resonances.

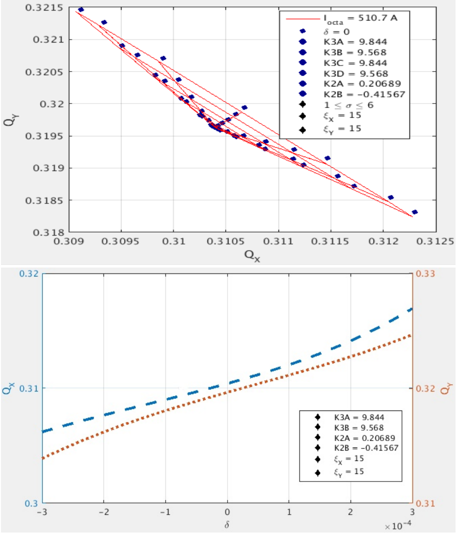
Version	Total number of elements	Number of kicks	Number of rotations
V1	34	16	18
V2	132	64	66

Tune shift with amplitude & non-linear cromaticity plots

V1

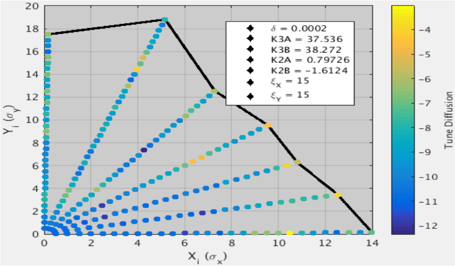


V2

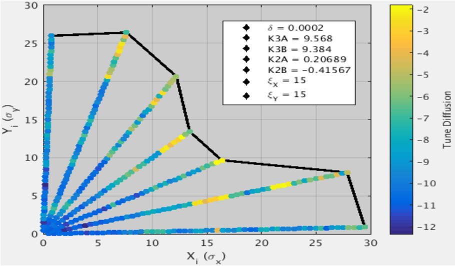


DA plots

V1



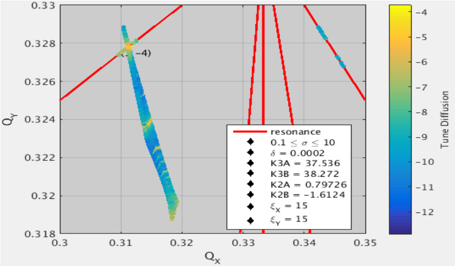
V2

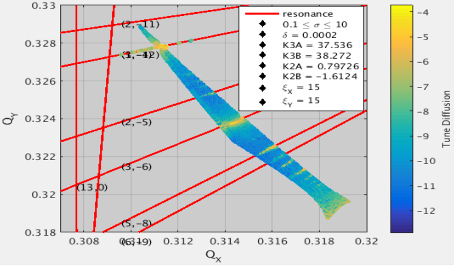


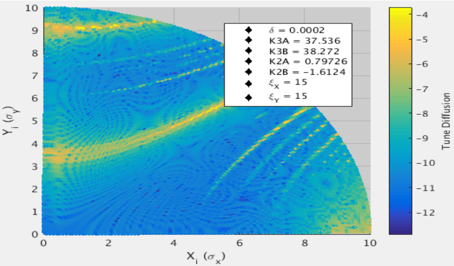
$$TuneDiffusion = log10\left[\sqrt{({}_fQ_X - {}_iQ_X)^2 + ({}_fQ_Y - {}_iQ_Y)^2}\right]$$

Large dynamic aperture in the shadow.

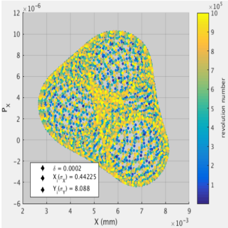
V1 FMA plots

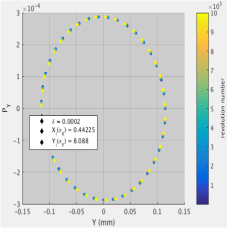




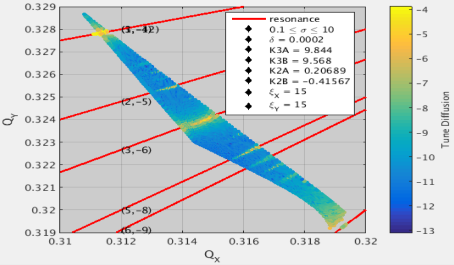


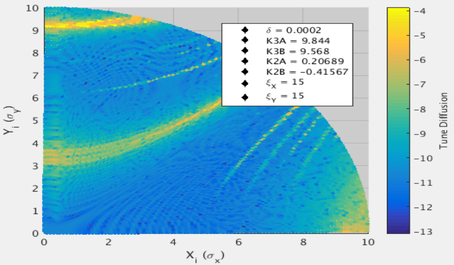
Phase space plots





V2 FMA plots


resonance
$0.1 \leq \sigma \leq 10$
$\delta = 0.0002$
K3A = 9.844
K3B = 9.568
K2A = 0.20689
K2B = -0.41567
$\xi_x = 15$
$\xi_Y = 15$
(2,-5)
(3,-6)
(5,-8)
(6,-9)
Q_Y
Q_X
Tune Diffusion


$\delta = 0.0002$
K3A = 9.844
K3B = 9.568
K2A = 0.20689
K2B = -0.41567
$\xi_x = 15$
$\xi_Y = 15$
Y_i (σ_y)
X_i (σ_x)
Tune Diffusion

Conclusions

- We have a generic method to produce reduced lattices for the LHC and potentially for other machines.

The final reduced lattice:

- reproduces the linear properties of the LHC
- describes very well the global non-linear properties of the LHC
- calibration of the octupole strength for various octupole currents to reproduce different detuning with amplitude
- all the strengths of the nonlinear elements are automatically calculated, giving us the flexibility to study different operational scenarios (different tune, chromatisity, octupole current, etc.)
- is very compact and guarantees a very fast tracking process

thank you

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