Generalized fragmentation functions for fractal jet observables

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Outline

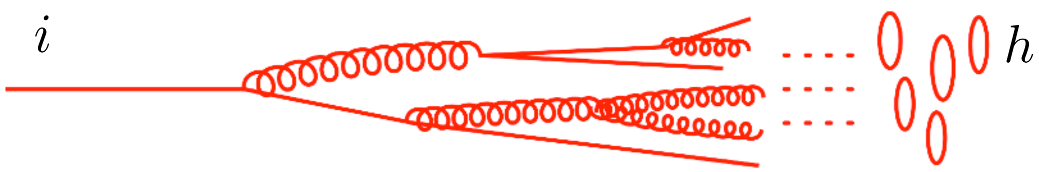
- Standard vs generalized fragmentation: single-hadron production vs jet observables probing subsets of hadrons defined by non-kinematic quantum numbers
- Generalized fragmentation functions (GFFs): definition and RGE
- Fractal jet observables from recursive binary clustering trees
- GFF for fractal observables: extraction, evolution, applications to q/g discrimination
- Resummed calculation: track thrust at NLL
- Conclusions and Outlook

Elder, Procura, Thaler, Waalewijn and Zhou, JHEP 2017

Chang, Procura, Thaler and Waalewijn, PRL 2013 and PRD 2013

Single-hadron fragmentation

- Fragmentation function (FF) $D_i^h(x,\mu)$ is the number density of hadrons of type h carrying the momentum fraction x of the parent parton i



- Final-state counterpart of PDF, universal ingredient in factorization theorems for single-inclusive hadron production (long-distance physics). At leading power

$$\frac{1}{\sigma^{(0)}}\frac{\mathrm{d}\sigma}{\mathrm{d}x}(e^+e^- \to hX) = \sum_i \int_x^1 \frac{\mathrm{d}z}{z} \boxed{C_i(z,s,\mu)} \boxed{D_i^h(x/z,\mu)} \,, \qquad x = 2E_h/\sqrt{s}$$

- For any parton flavor i, FFs satisfy the momentum conservation sum rule

$$\sum_h \int_0^1 \mathrm{d}x\, x\, D_i^h(x,\mu) = 1$$

Single-hadron fragmentation

- Field theoretic definition of the bare unpolarised quark FF

Collins and Soper (1982)

$$D_i^h(x,\mu) = \frac{1}{x}\int \mathrm{d}^2 p_h^\perp \int \frac{\mathrm{d}y^+ \mathrm{d}^2 y_\perp}{2(2\pi)^3} e^{ip^- y^+/2} \sum_X \frac{1}{2N_C} \mathrm{Tr}\left[\frac{\gamma^-}{2}\langle 0|\psi_i(y^+,0,y_\perp)|hX\rangle\langle hX|\overline{\psi}_i(0)|0\rangle\right]$$

for vanishing quark transverse momentum, for $A^- = 0$ and with $p_h^- = x\,p^-$

- Evolution of FFs

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}D_i^h(x,\mu) = \sum_j \int_x^1 \frac{\mathrm{d}z}{z}\frac{\alpha_s(\mu)}{\pi} P_{ji}(z) D_j^h(x/z,\mu) \quad \text{with} \quad P_{ji}(z) = P_{ji}^{(0)}(z) + \frac{\alpha_s}{2\pi}P_{ji}^{(1)}(z) + \dots$$

Single-hadron fragmentation

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for vanishing quark transverse momentum, for $A^- = 0$ and with $p_h^- = x\,p^-$

- Evolution of FFs at leading order in α_s, with $P_{ji}^{(0)}(z) \equiv P_{i\to jk}(z)$, can be written as

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}D_i^h(x,\mu) = \frac{1}{2}\sum_{j,k}\int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2 \int_0^1 \mathrm{d}z\,\frac{\alpha_s(\mu)}{\pi}\,P_{i\to jk}(z)$$
$$\times\left(D_j^h(x_1,\mu)\;\delta[x - zx_1] + D_k^h(x_2,\mu)\;\delta[x-(1-z)x_2]\right)$$

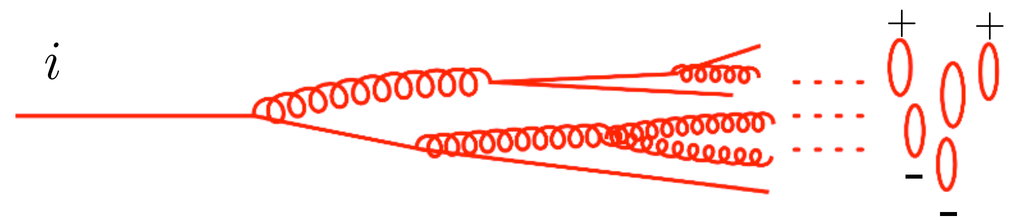
FF evolution requires information on only one of the two branches in the splitting

Generalized fragmentation

Elder, Procura, Thaler, Waalewijn and Zhou, JHEP 2017

- Consider a soft-safe, collinear-unsafe observable x carried by a subset S of collinear hadrons defined by non-kinematic quantum numbers (like electric charge), irrespective of their multiplicities.
  A generalized FF (GFF) is the probability density for the particles in S to yield a value of x from jets initiated by the parton i.
  Example: the distribution in the energy fraction x of all tracks ("track function")

Chang, MP, Thaler and Waalewijn (2013)



- Fundamental difference between GFF and multi-hadron FF: in GFFs the number of identified final-state hadrons is not fixed

Generalized fragmentation

- Field theoretic definition of the bare unpolarised quark GFF

$$\mathcal{F}_i(x,\mu) = \int \mathrm{d}y^+ \mathrm{d}^2 y_\perp e^{ip^- y^+/2} \frac{1}{2N_C} \sum_{S,X} \delta[x - \tilde{x}(p^-, S)] \mathrm{Tr}\left[\frac{\gamma^-}{2} \langle 0|\psi_i(y^+, 0, y_\perp)|SX\rangle\langle SX|\overline{\psi}_i(0)|0\rangle\right]$$

- RG evolution for a generic GFF very complicated. For a sufficiently simple numerical evaluation, we consider what we call fractal observables, whose RGE at LO is

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \mathcal{F}_i(x,\mu) = \frac{1}{2} \sum_{j,k} \int \mathrm{d}z \, \mathrm{d}x_1 \, \mathrm{d}x_2 \, \frac{\alpha_s(\mu)}{\pi} P_{i\to jk}(z) \, \mathcal{F}_j(x_1,\mu) \, \mathcal{F}_k(x_2,\mu) \, \delta[x - \hat{x}(z, x_1, x_2)]$$

which involves sampling over both branches: nonlinear evolution.
A fractal observable x depends on the momentum of the parent parton only through the momentum sharing z.
A broad class of fractal observables built using recursive binary clustering trees.

Generalized fragmentation

- Field theoretic definition of the bare unpolarised quark GFF

$$\mathcal{F}_i(x,\mu) = \int \mathrm{d}y^+ \mathrm{d}^2 y_\perp e^{ip^- y^+/2} \frac{1}{2N_C} \sum_{S,X} \delta[x - \tilde{x}(p^-, S)] \mathrm{Tr}\left[\frac{\gamma^-}{2} \langle 0|\psi_i(y^+,0,y_\perp)|SX\rangle\langle SX|\overline{\psi}_i(0)|0\rangle\right]$$

- RG evolution for a GFF of a fractal observable at higher orders:

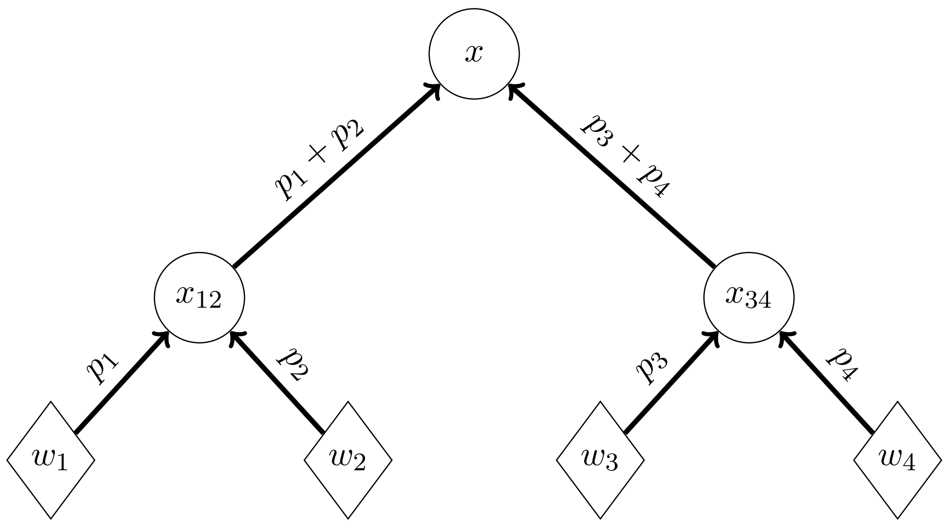
$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu}\mathcal{F}_i = \frac{\alpha_s}{\pi} P_{i\to jk} \otimes \mathcal{F}_j \otimes \mathcal{F}_k + \left(\frac{\alpha_s}{\pi}\right)^2 P_{i\to jk\ell} \otimes \mathcal{F}_j \otimes \mathcal{F}_k \otimes \mathcal{F}_\ell + \ldots$$

still self-similar but, starting at NLO, it depends on how partons are clustered into a binary tree.

- Here we consider only LO evolution (1:1 correspondence with pairwise clustering)

Fractal observables

- Constructing fractal observables: weights, recursion relation, binary clustering



- We used generalized-k_t clustering algorithms: $$d_{ij} = \min[E_i^{2p}, E_j^{2p}]\,\Omega_{ij}^2$$

- Requirements on the functional form of the recursion relation $\hat{x}(z, x_1, x_2)$:

  Symmetry under $z \leftrightarrow 1-z,\ x_1 \leftrightarrow x_2$ and

$$\lim_{z\to 1} \hat{x}(z, x_1, x_2) = x_1, \qquad \lim_{z\to 0} \hat{x}(z, x_1, x_2) = x_2$$

which implies that the contribution from an entire soft branch is suppressed.

Goals

- Most of analytic jet physics studies are based either on single-hadron fragmentation or IRC-safe jet shapes. We introduce a theoretical framework for QCD calculations in the intermediate scenario of soft-safe and collinear-unsafe observables defined on subsets of hadrons.

- We can calculate the evolution of GFF for fractal observables and produce systematically improvable results with reliable theory uncertainties, both a fixed order and with logarithmic resummation.

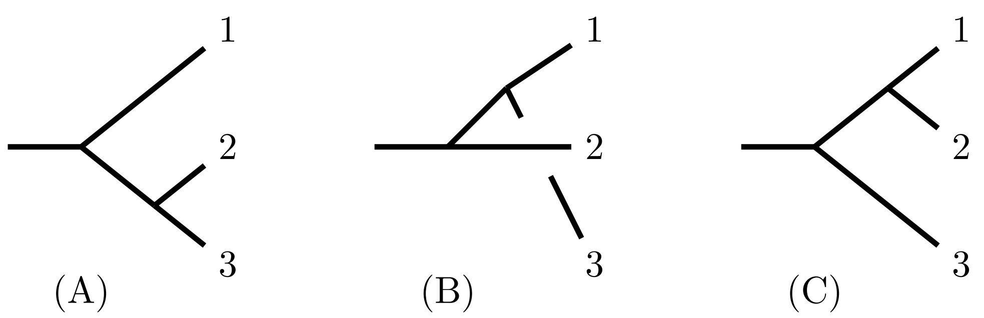
- Studying parton fragmentation in terms of observables sensitive to the clustering tree: novel aspects of jet substructure?

Fractal observables: WEF

- Weighted energy fractions are obtained from the recursion relation

$$\hat{x}(z, x_1, x_2) = x_1\, z^{\kappa} + x_2\, (1-z)^{\kappa} \qquad (\kappa > 0)$$

- Independent of the binary clustering tree (associative observable) :



$$x_A = x_B = x_C$$

On a jet :

$$x = \sum_{a \in \text{jet}} w_a\, z_a^{\kappa}, \qquad z_a \equiv \frac{E_a}{E_{\text{jet}}}$$

- Includes weighted jet charge (w_a = electric charges of the hadrons in the jet),
  track energy fraction ($\kappa = 1$ and $w_a = 1$ for charged and 0 for neutral),
  jet p_T^D ($\kappa = 2$ and $w_a = 1$ for all particles)

Fractal observables: node products

- Node product observables are obtained from the recursion relation

$$\hat{x} = x_1 \, z^\kappa + x_2 \, (1-z)^\kappa + (4z(1-z))^{\kappa/2}$$

- Non-associative (depend on the clustering tree) except for $\kappa = 2$

On a jet, these observables simplifies into a sum over leaves and nodes

$$x = \sum_{a \in \mathrm{jet}} w_a z_a^\kappa + \sum_{\mathrm{nodes}} (4 z_L z_R)^{\kappa/2}, \qquad z_{L,R} = \frac{E_{L,R}}{E_{\mathrm{jet}}}$$

Intermediate complexity between WEFs and full-tree observables

Fractal observables: full tree

- Full-tree observables from the recursion relation

$$\hat{x} = \left(z^{\kappa} x_1 + (1-z)^{\kappa} x_2\right) e^{\xi z(1-z)}$$

- Non-associative variants of WEFs (recovered for $\xi = 0$)

- We calculated these observables on a jet by performing the full tree traversal. They depend on the generalized-k_t exponent p, on the starting weights w_a, on the parameters κ and ξ in the recursion relation.

NUMERICAL ANALYSIS

- Extraction of GFFs for WEFs, node products and full-tree fractal observables
- Leading-order RG evolution of GFFs vs parton showers
- Applications of fractal observables to quark/gluon discrimination

Extraction of GFFs

- Extraction of LO GFFs from differential cross section in the fractal observable

$$\frac{1}{\sigma_i}\frac{d\sigma_i}{dx} = \mathcal{F}_i^{\rm LO}(x)$$

more involved but straightforward also at NLO (matching coefficients are known)

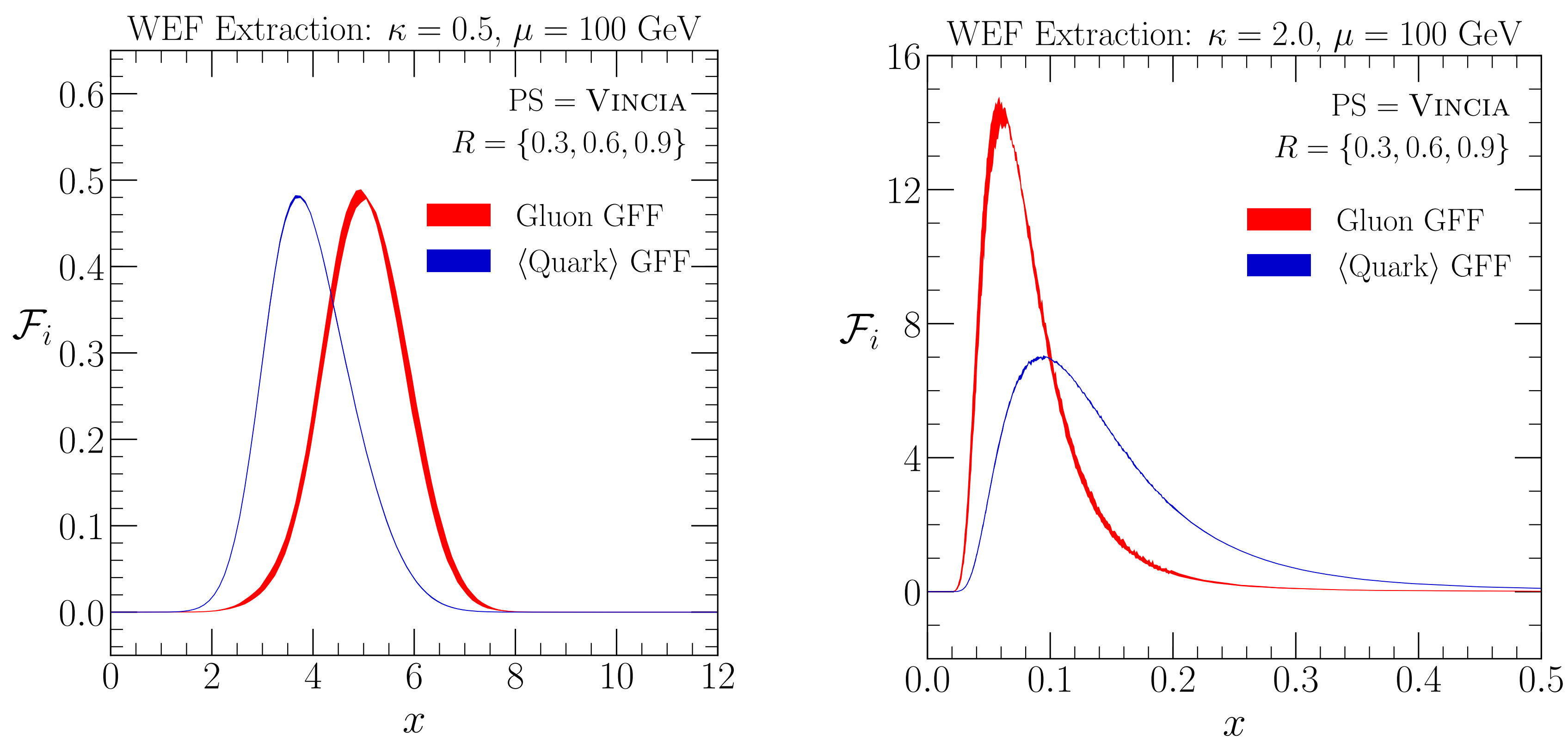
- Pure quark and gluon jet samples from $e^+e^- \to \gamma/Z^* \to q\bar{q}$ and $e^+e^- \to H^* \to gg$ processes in PYTHIA 8.215 (+ FastJet)

- For jets from e^+e^- annihilation we expect that the GFF characteristic scale is $\mu = E_{\rm jet}R$ which is confirmed by our numerics

Extraction of GFFs using PYTHIA

- Quark singlet $\mathcal{S}(x,\mu) = \frac{1}{2n_f} \sum_{i\in\{u,\bar{u},d,\ldots\bar{b}\}} \mathcal{F}_i(x,\mu)$ and gluon GFFs for WEF with $w_a = 1$

$$\hat{x}(z, x_1, x_2) = x_1\, z^\kappa + x_2\,(1-z)^\kappa \quad \longrightarrow \quad x = \sum_{a\in \text{jet}} w_a\, z_a^\kappa\,, \qquad z_a \equiv \frac{E_a}{E_{\text{jet}}}$$



- Varying R and keeping $\mu = E_{\text{jet}} R$ fixed: appropriate scale for the GFF extraction

Extraction of GFFs using PYTHIA

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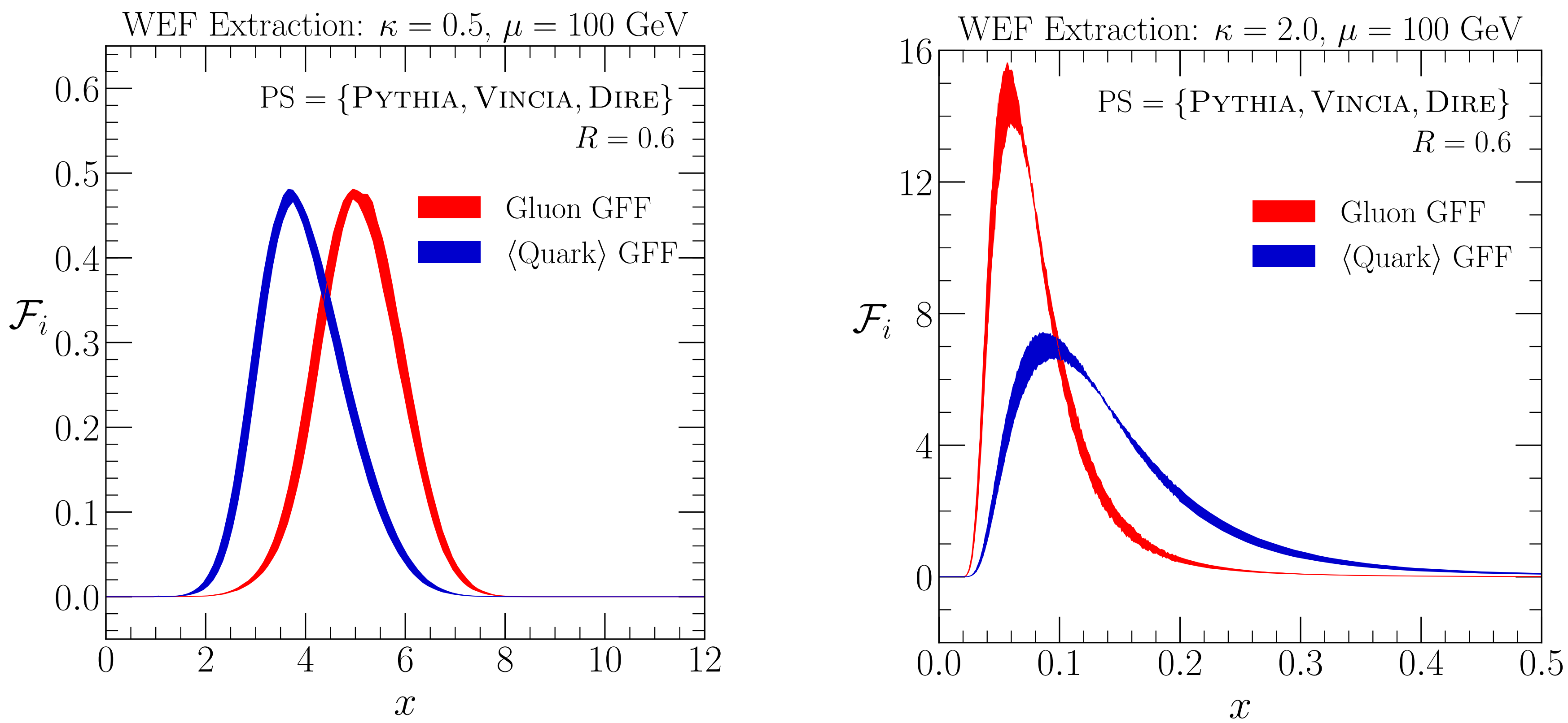


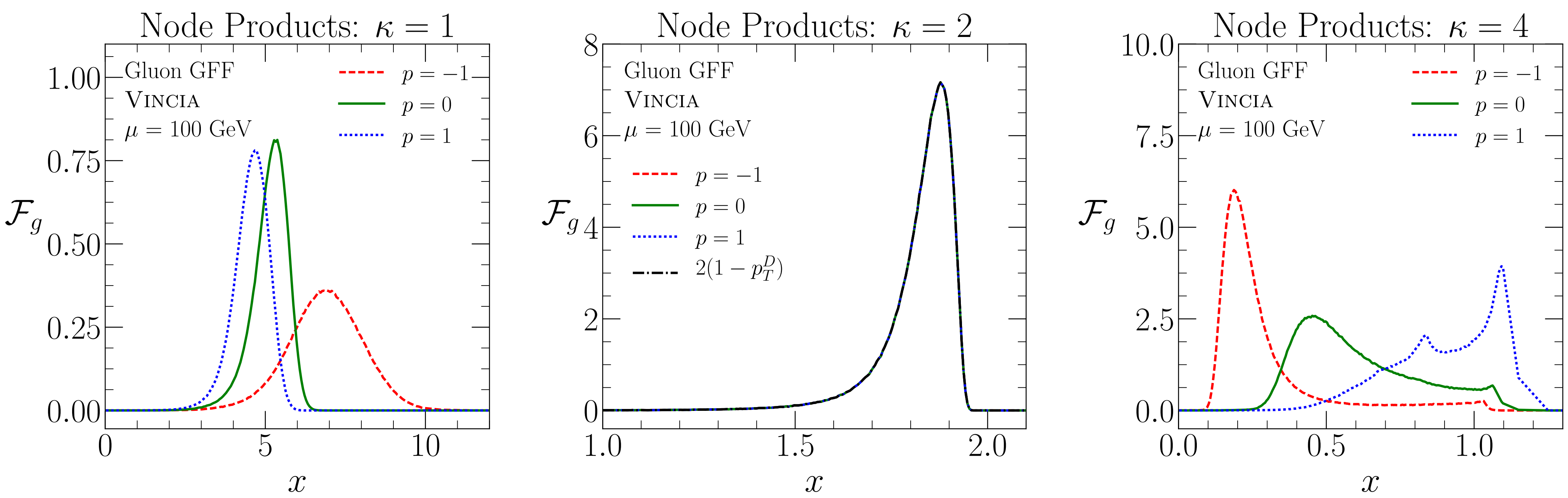
- $\mu = E_{\rm jet} R$ appropriate scale for the GFF extraction

- Larger bands: effects of different parton showers (native PYTHIA, VINCIA, DIRE)

Extraction of GFFs using PYTHIA 8

- Non-associative observables: gluon GFFs for node products with $w_a = 0$

$$\hat{x} = x_1\, z^\kappa + x_2\,(1-z)^\kappa + (4z(1-z))^{\kappa/2} \;\longrightarrow\; x = \sum_{a\in \text{jet}} w_a z_a^\kappa + \sum_{\text{nodes}} (4 z_L z_R)^{\kappa/2}\,, \quad z_{L,R} = \frac{E_{L,R}}{E_{\text{jet}}}$$

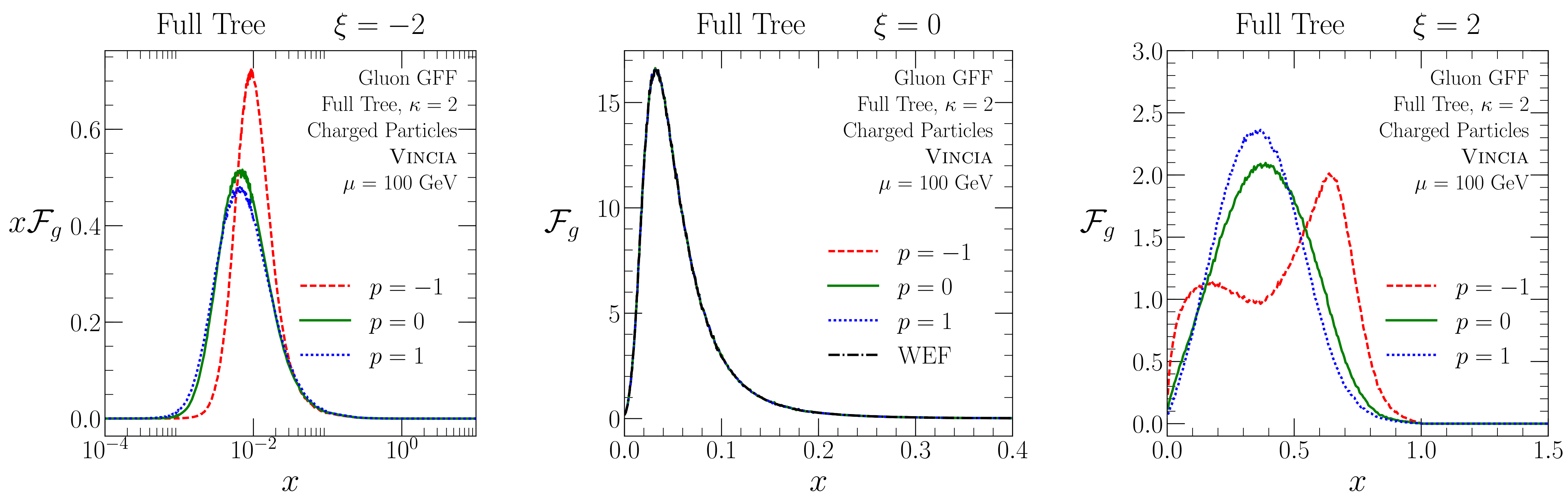


- For $\kappa = 2$ associative. Clear dependence on the clustering algorithm for $\kappa = 1$, $\kappa = 4$

Extraction of GFFs using PYTHIA 8

- Non-associative observables: gluon GFFs for full tree with $w_a = 1$ for charged particles and $w_a = 0$ for neutral ones

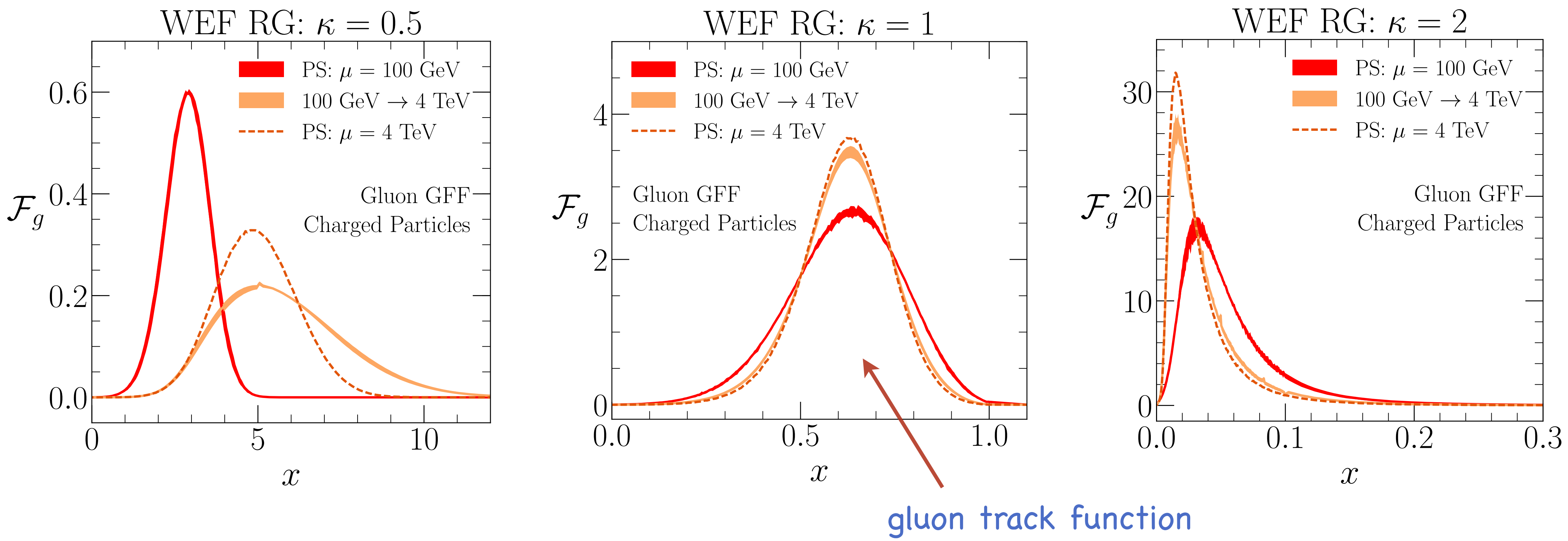
$$\hat{x} = \left(z^{\kappa} x_1 + (1-z)^{\kappa} x_2\right) e^{\xi z(1-z)}$$



Evolution of GFFs for fractal observables

- LO RGE for GFFs vs parton shower (using GFFs extracted at $\mu = 100\,\mathrm{GeV}$ as boundary condition)

- Evolution of gluon GFFs for WEF with $w_a = 1$ charged and $w_a = 0$ neutral:

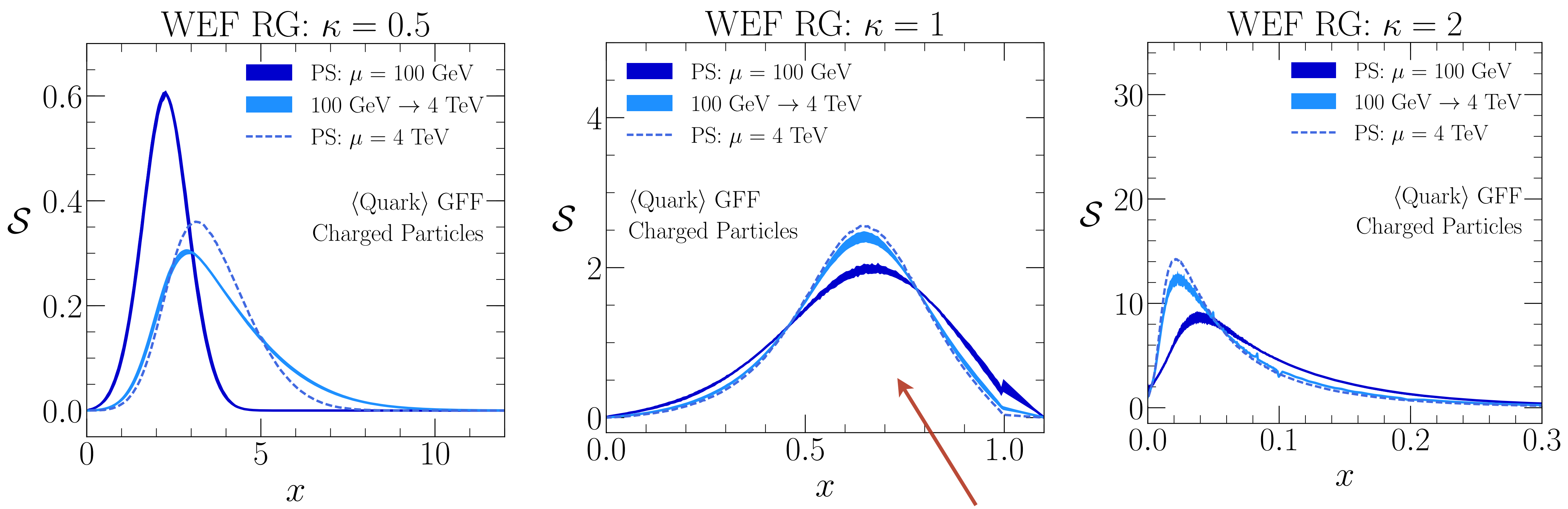


- Better agreement with parton showers for $\kappa \geq 1$

Evolution of GFFs for fractal observables

- LO RGE for GFFs vs parton shower (using GFFs extracted at $\mu = 100\,\mathrm{GeV}$ as boundary condition)

- Evolution of quark-singlet GFFs for WEF with $w_a = 1$ charged and $w_a = 0$ neutral:

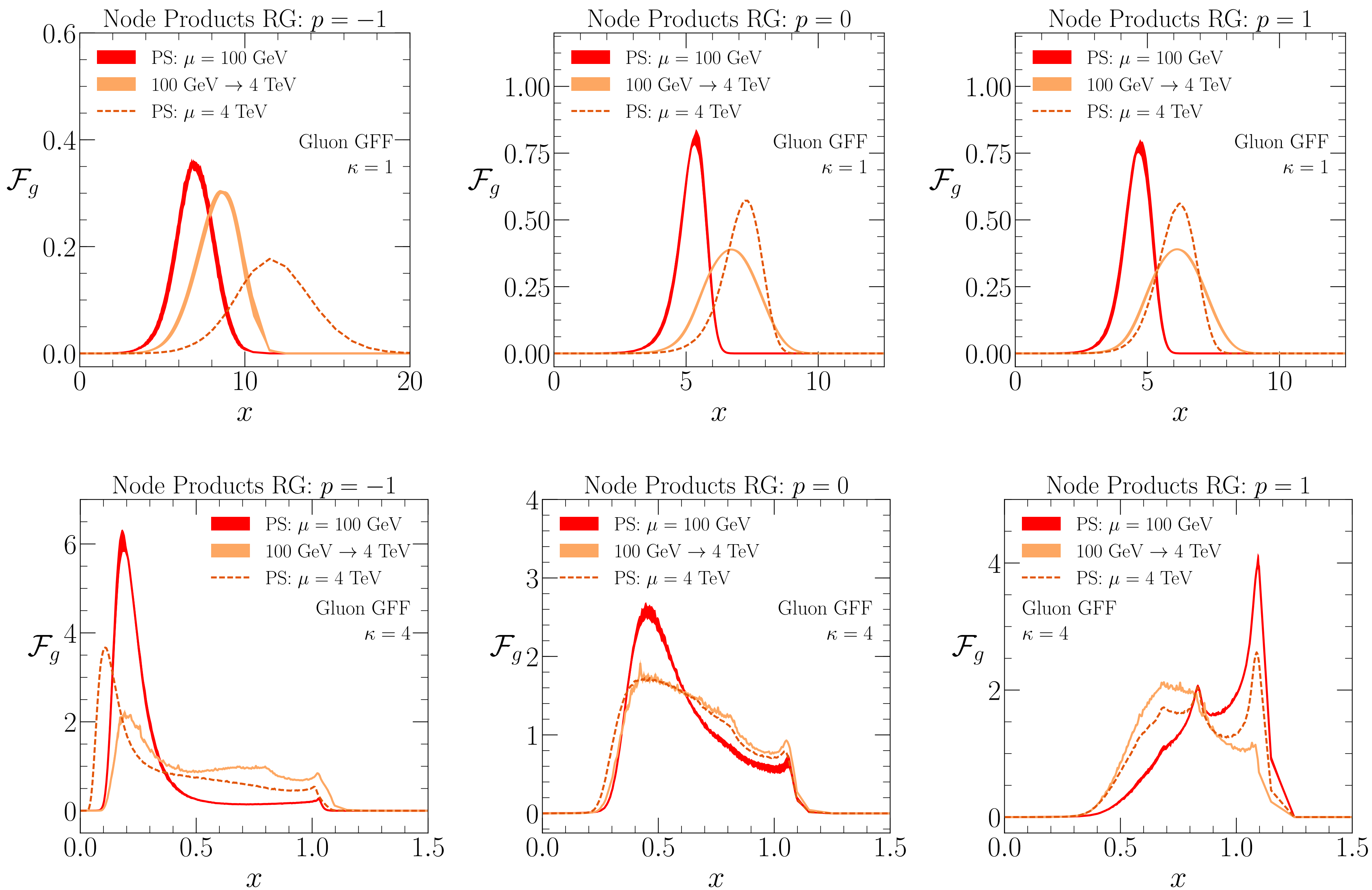


quark-singlet track function

- Better agreement with parton showers for $\kappa \geq 1$

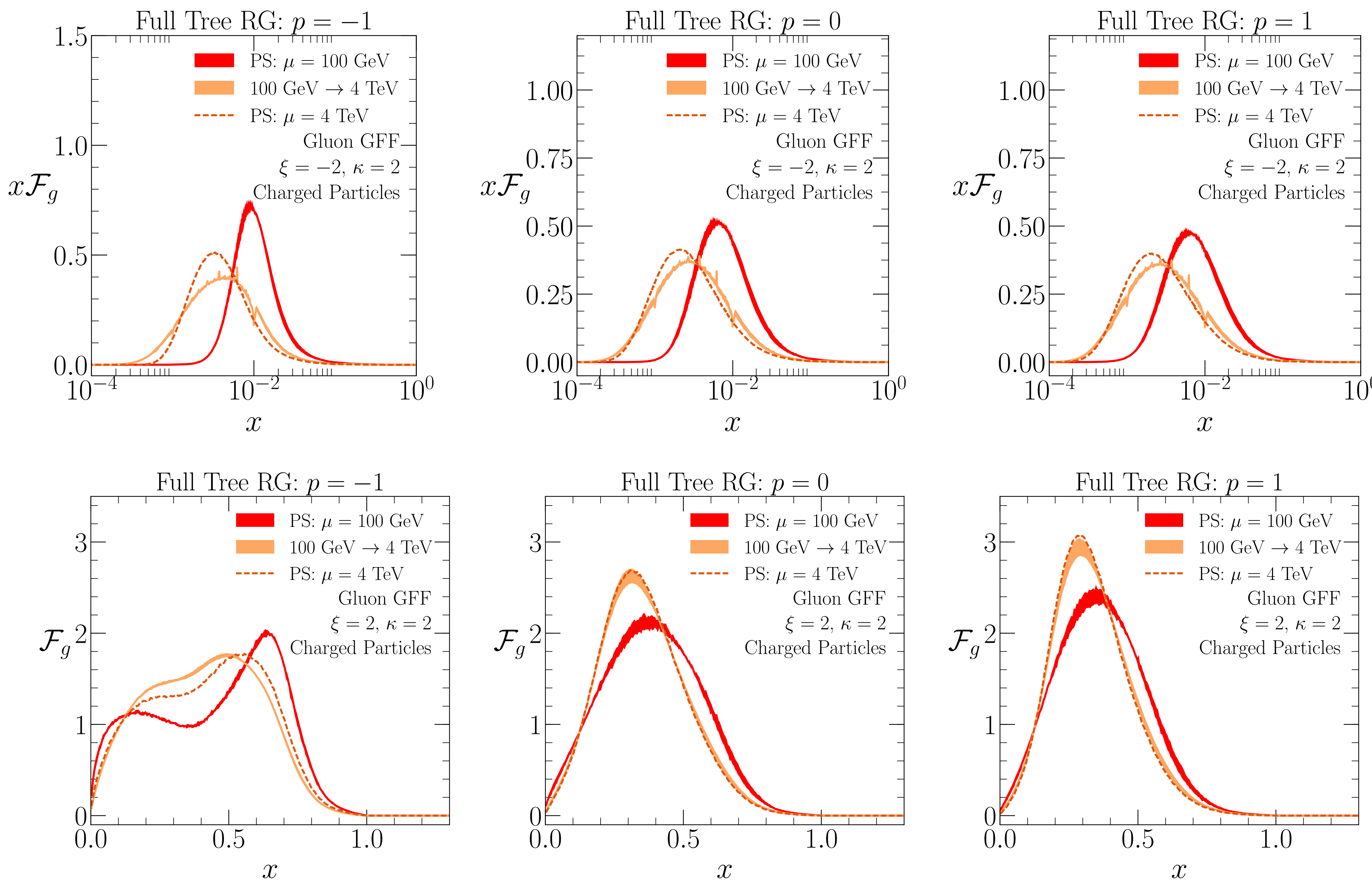
Evolution of GFFs for fractal observables

- Evolution of gluon GFFs for node products with $w_a = 0$



Evolution of GFFs for fractal observables

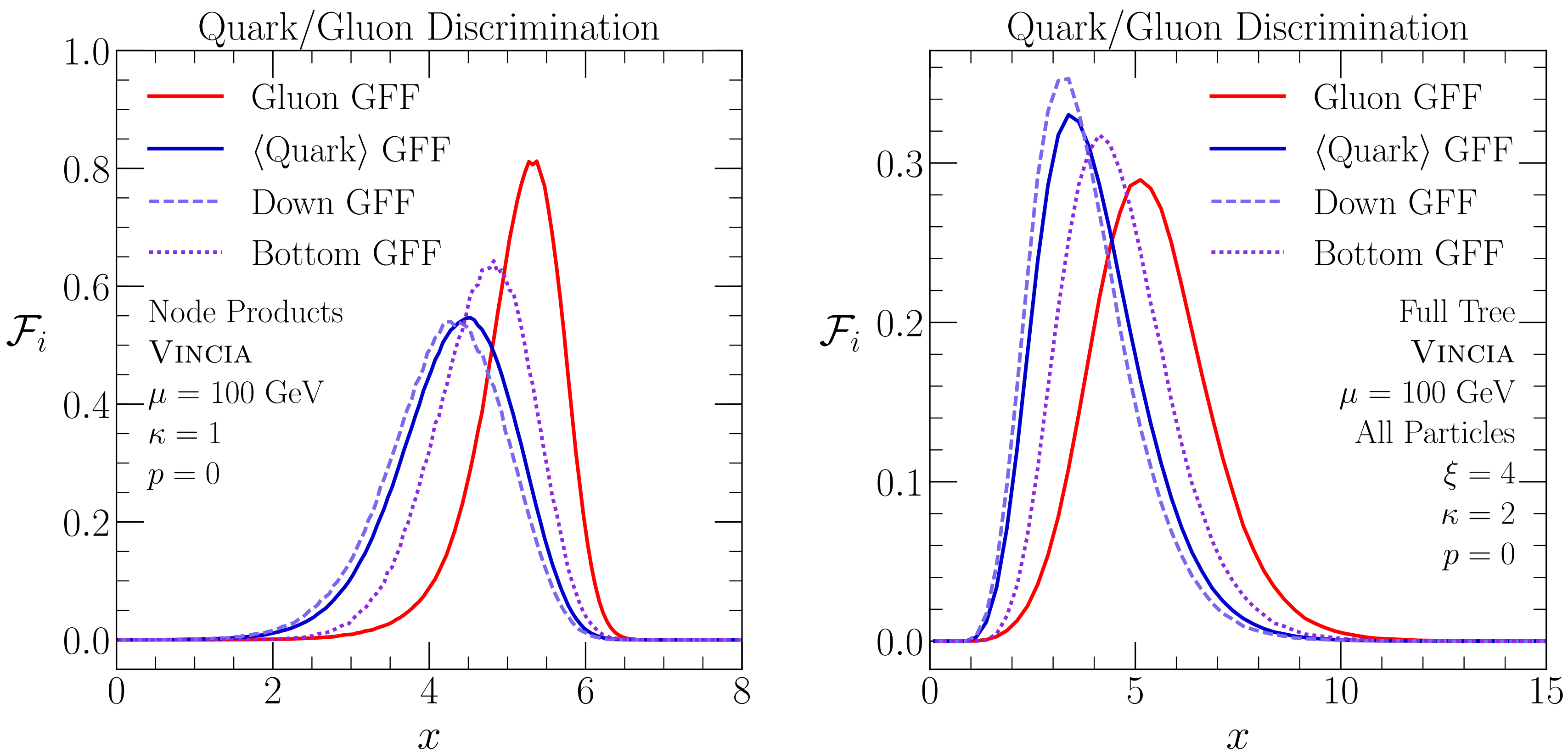
- Evolution of gluon GFFs for full-tree observables



Application to quark/gluon discrimination

- WEFs are already used as tools to discriminate quark- from gluon-initiated jets (e.g. jet p_T^D used by CMS for quark/gluon likelihood analysis).

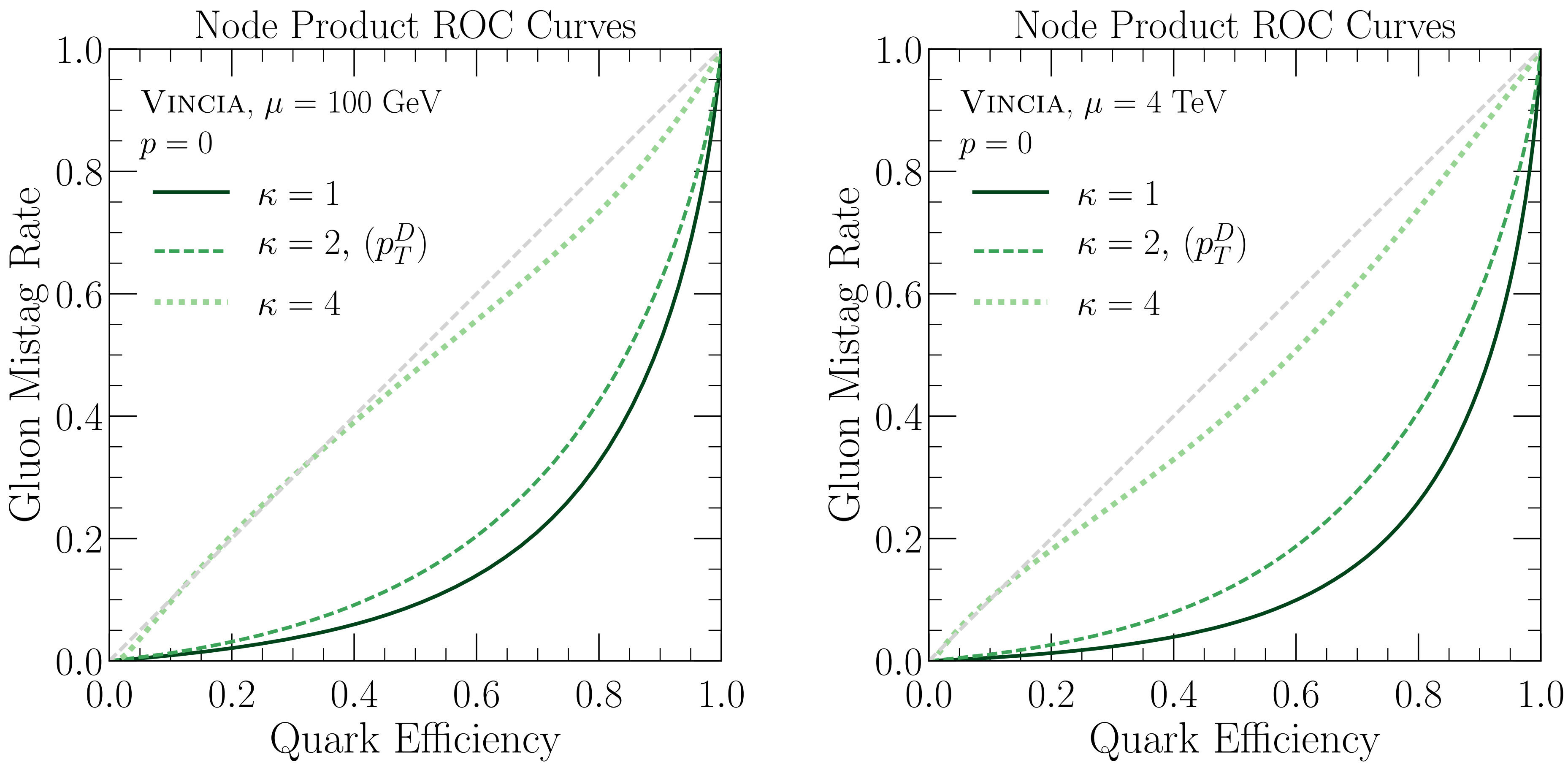
- We studied the potential discrimination power of non-associative observables



these observables outperform variants using charged particles only

Application to quark/gluon discrimination

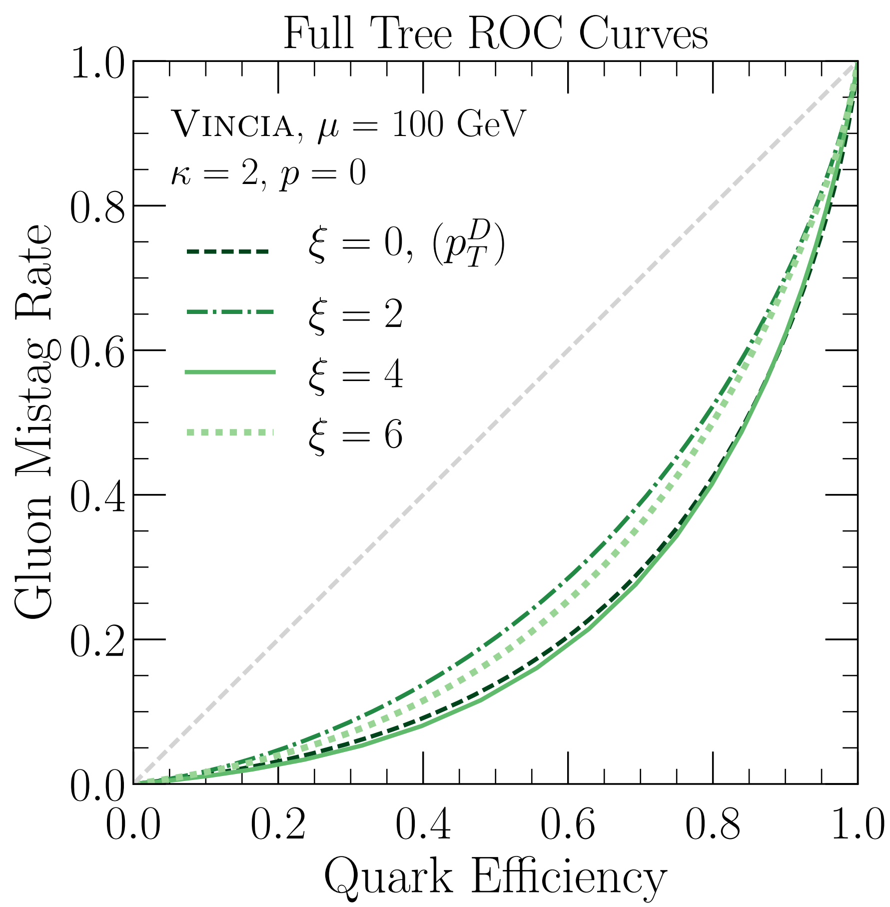
- In terms of receiver operating characteristic (ROC) curves

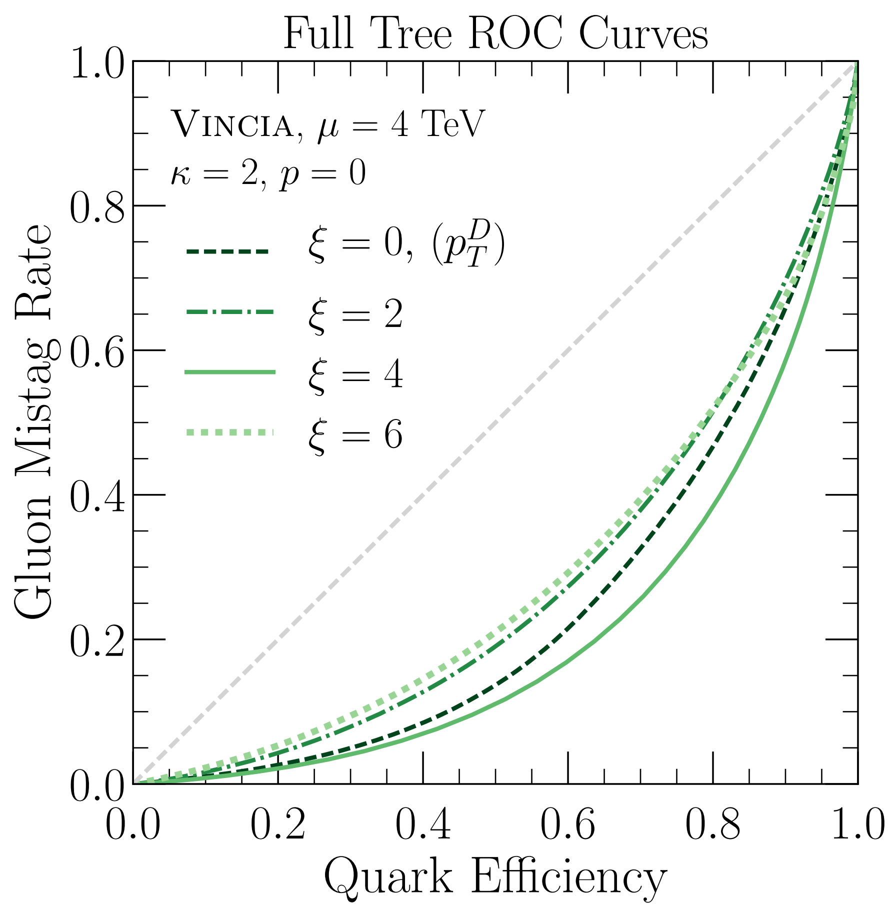


- Some of these observables outperform p_T^D

Application to quark/gluon discrimination

- In terms of receiver operating characteristic (ROC) curves





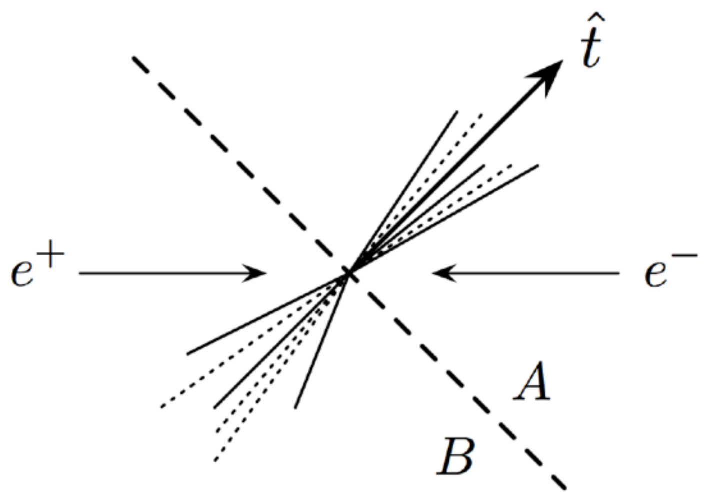
- Some of these observables outperform p_T^D

Calculating track-based observables using track functions

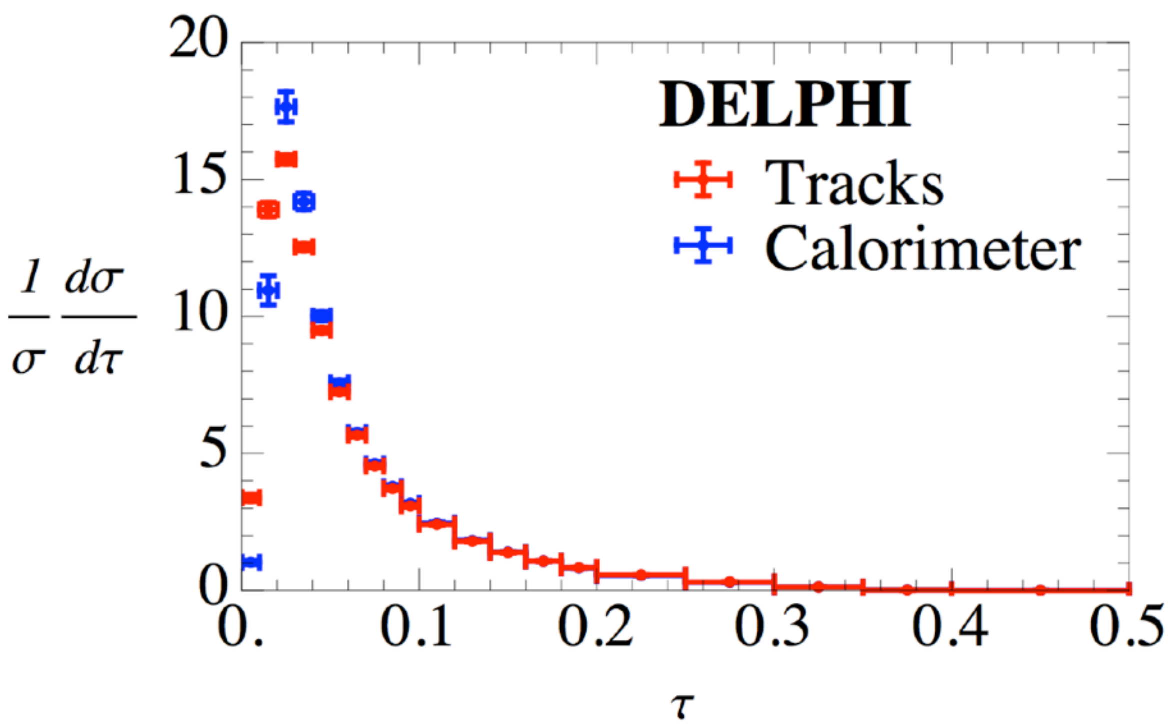
Track functions $T_i(x, \mu)$ are generalized fragmentation functions for the energy fraction carried by charged particles (fractal observable).
Formalism for QCD calculations of observables depending on the kinematics of charged particles alone: track thrust at NLL

Chang, Procura, Thaler and Waalewijn, PRL+PRD 2013

Event shape track thrust



$$1 - \tau = T = \max_{\hat{t}} \frac{\sum_i |\hat{t} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|}$$



- "Calorimeter" thrust = sum over CHARGED and NEUTRAL particles τ
- Track thrust = sum over all CHARGED particles $\bar{\tau}$
- Using track functions we can analytically understand track/calo similarity

Track thrust factorization theorem

- As $\tau \to 0$ we need to resum large double logarithms in the thrust distribution

- Leading-power factorization theorem for track thrust:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\bar{\tau}} = \sigma_0\, H(Q^2,\mu) \int_0^\infty \mathrm{d}\bar{k}\, \mathrm{d}\bar{s}_A\, \mathrm{d}\bar{s}_B \int_0^1 \mathrm{d}x_A\, \mathrm{d}x_B \times \bar{S}(\bar{k},\mu) \bar{J}(\bar{s}_A, x_A, \mu) \bar{J}(\bar{s}_B, x_B, \mu) \delta\Big[\bar{\tau} - \frac{2}{(x_A + x_B)Q}\Big(\frac{\bar{s}_A}{Q} + \frac{\bar{s}_B}{Q} + \bar{k}\Big)\Big]$$

- Track-based jet functions depend on $\bar{s}_i = s_i^{\mathrm{tracks}}/x_i$ and on x_i

- Focus on the TAIL region of the distribution

$\mu_{\mathrm{soft}} \simeq \tau Q \gg \Lambda_{\mathrm{QCD}}$ Soft radiation: perturbation theory + series of power correction parameters

Track thrust soft function

- At NLO single soft gluon emission, simple matching onto track functions

$$\bar{S}^{(1)}(\bar{k},\mu) = \int_0^\infty \mathrm{d}k\, S^{(1)}(k,\mu) \int_0^1 \mathrm{d}x\, T_g(x,\mu)\, \delta(\bar{k}-xk)$$
$$= \delta(\bar{k}) + \frac{\alpha_s C_F}{2\pi}\left[-\frac{8}{\mu}\left(\frac{\ln(k/\mu)}{k/\mu}\right)_+ + \frac{8g_1^L}{\mu}\left(\frac{1}{k/\mu}\right)_+ + \left(\frac{\pi^2}{6} - 4g_2^L\right)\delta(\bar{k})\right]$$

which depends on logarithmic moments of T_g :

$$g_n^L(\mu) \equiv \int_0^1 \mathrm{d}x\, T_g(x,\mu) \ln^n x$$

- Hadronization effects get exponentiated

$$\gamma_{\bar{S}}^{\mathrm{noncusp}}[\alpha_s] = -\frac{4\alpha_s C_F}{\pi}\, g_1^L$$

but still

$$\gamma_H^{\mathrm{noncusp}} + 2\gamma_{\bar{J}}^{\mathrm{noncusp}} + \gamma_{\bar{S}}^{\mathrm{noncusp}} = 0$$

Simplifications at NLL

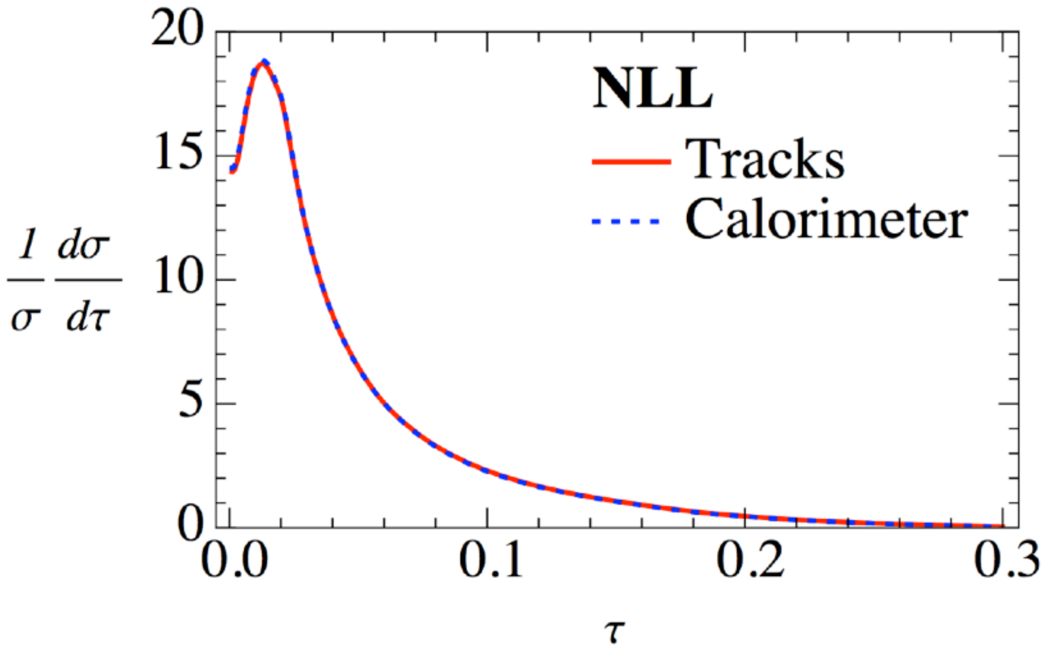
- For the cumulative distributions $\bar{\Sigma}$ (tracks) and Σ (calo, i.e. all particles) at NLL

$$\bar{\Sigma}(\bar{\tau}^c) \equiv \int_0^{\bar{\tau}^c} \mathrm{d}\bar{\tau}\, \frac{\mathrm{d}\sigma}{\mathrm{d}\bar{\tau}} \approx \Sigma(\bar{\tau}^c) \times \ln(3\bar{\tau}^c)^{\Delta}$$

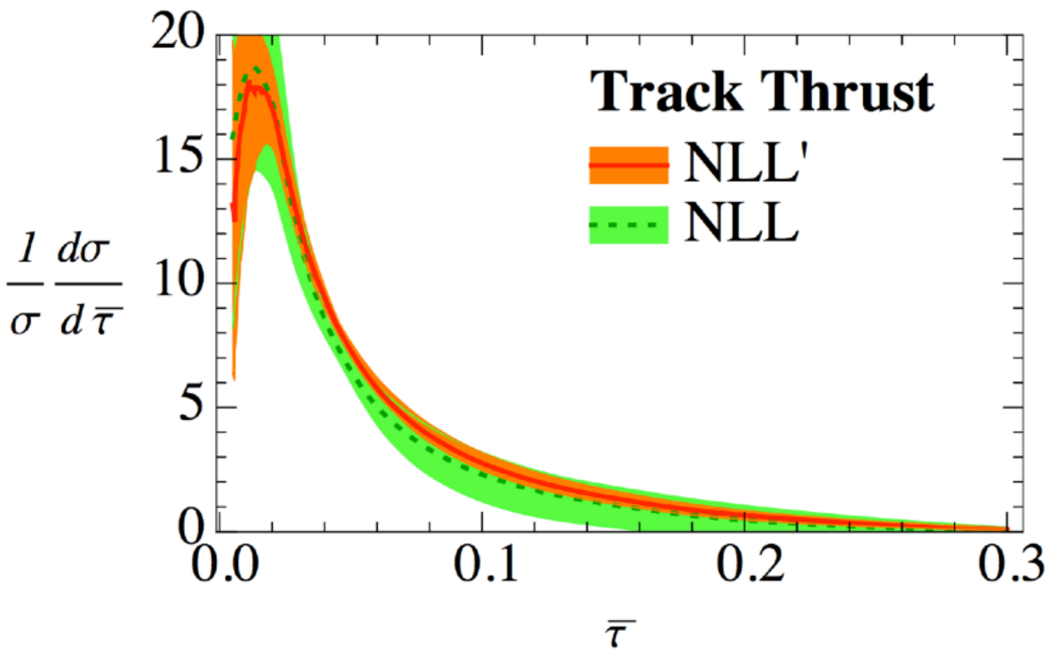
- For quark and gluon track functions from PYTHIA

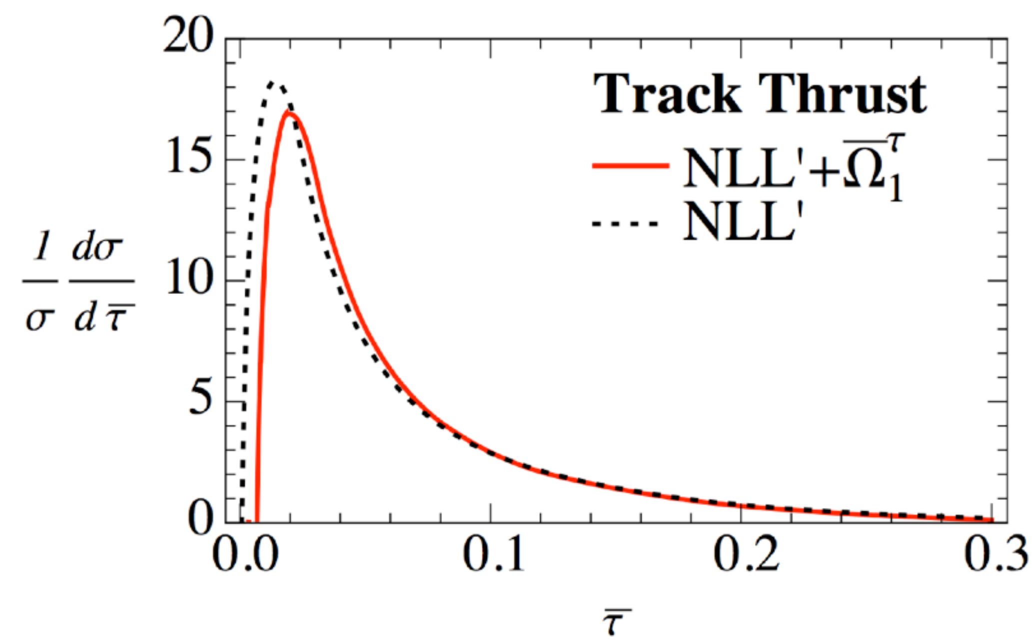
$$\boxed{\Delta = \frac{2\alpha_s C_F}{\pi}(g_1^L - q^L) \approx 0}$$

$$q^L(\mu) \equiv \int \mathrm{d}x_A\, \mathrm{d}x_B\, T_q(x_A,\mu) \times T_q(x_B,\mu)\, \ln\Big(\frac{x_A + x_B}{2}\Big)$$



Higher-order and leading NP corrections

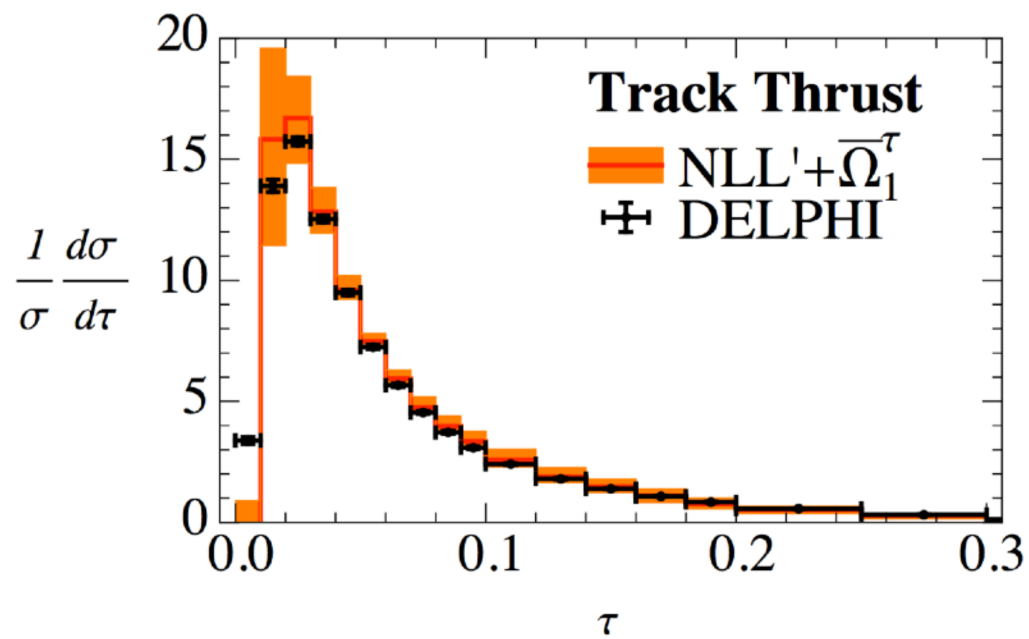


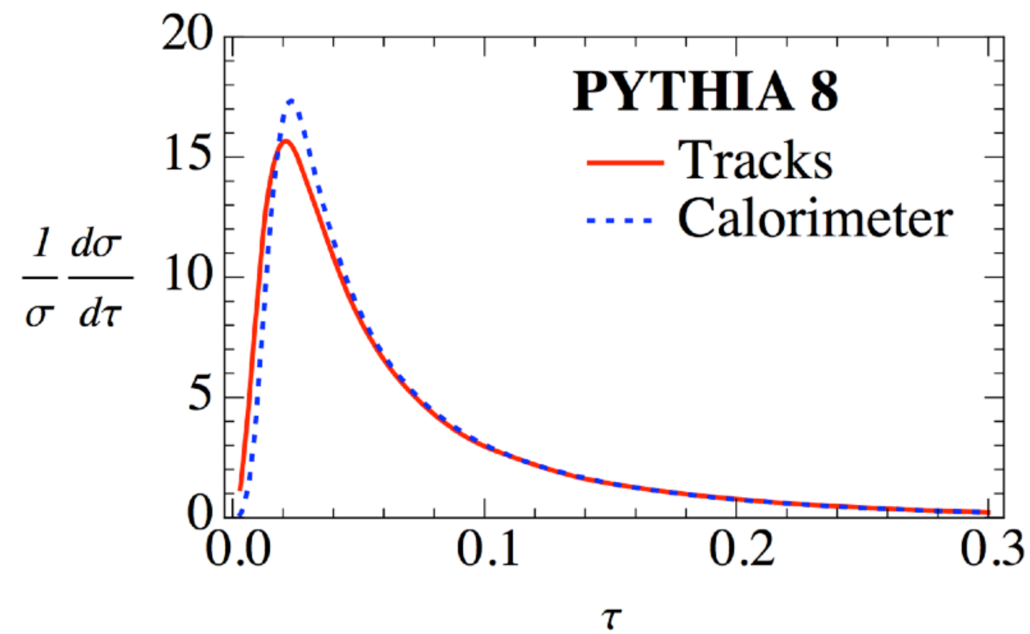


- At NLL′, fixed-order matching contribution at one order higher compared to NLL
- Leading non-perturbative correction :

$$\bar{S}(\bar{k},\mu) \simeq \bar{S}^{\text{part}}(\bar{k} - \bar{\Omega}_1^\tau, \mu) \qquad \text{with} \qquad \bar{\Omega}_1^\tau \simeq \langle x \rangle\, \Omega_1^\tau = 0.3\ \text{GeV}$$

Results for thrust distribution





- Our result agrees well with DELPHI measurements
- Good agreement with PYTHIA in the tail region, differences in the peak region due to non-perturbative corrections

Conclusions and Outlook

- GFFs for soft-safe and collinear-unsafe jet observables defined on subsets of hadrons. GFF RGE is self-similar for fractal jet observables, which recursively probe collective properties of hadrons produced in jet fragmentation. Framework to perform novel systematically improvable QCD calculations with theory uncertainties, both at fixed order and with logarithmic resummation.

- Extraction of GFFs, LO RGE vs parton showers, applications to quark/gluon discrimination: associative vs non-associative fractal observables

- Future work: higher-order evolution and extraction of GFFs, multivariate analyses

- Thrust distribution, charged vs charged + neutral: remarkable similarity related to properties of track functions.
  Same for other dimensionless ratio observables? Reduced hadronization smearing