Discrete Symmetries

- Parity
- Parity Violation
- Charge Conjugation
- CP Violation
- Time Reversal
- The CPT Theorem
- Lepton number

Parity

- A parity transformation, P, inverts every spatial coordinate: $P(t, \mathbf{x}) = (t, -\mathbf{x})$
  $P^2 = I$, and therefore the eigenvalues of P are ± 1.

- Ordinary vector $\mathbf{v}$. $P(\mathbf{v}) = -\mathbf{v}$.

- Scalar from $\mathbf{v}$: $s = \mathbf{v} \cdot \mathbf{v}$
  $P(s) = P(\mathbf{v} \cdot \mathbf{v}) = (-\mathbf{v}) \cdot (-\mathbf{v}) = \mathbf{v} \cdot \mathbf{v} = +s$

- Cross product of two vectors: $\mathbf{a} = \mathbf{v} \times \mathbf{w}$
  $P(\mathbf{a}) = P(\mathbf{v} \times \mathbf{w}) = (-\mathbf{v}) \times (-\mathbf{w}) = \mathbf{v} \times \mathbf{w} = +\mathbf{a}$

- Scalar from $\mathbf{a}$ and $\mathbf{v}$: $p = \mathbf{a} \cdot \mathbf{v}$
  $P(p) = P(\mathbf{a} \cdot \mathbf{v}) = (+\mathbf{a}) \cdot (-\mathbf{v}) = -\mathbf{a} \cdot \mathbf{v} = -p$

Scalar	$P(s) = +s$
Pseudoscalar	$P(p) = -p$
Vector	$P(\mathbf{v}) = -\mathbf{v}$
Pseudovector	$P(\mathbf{a}) = +\mathbf{a}$

Parity in Physical Systems

- Two-body systems have parity $p_A p_B (-1)^\ell$
  $P\phi(12) = p_1 p_2 (-1)^\ell \phi(12)$
- Intrinsically, particles and antiparticles have opposite parity
  Bound states like positronium $e^+ \, e^-$ and mesons $q\overline{q}$ have parity of $(-1)^{\ell+1}$.
- Photons have a parity of (-1), and this underlies the $\Delta\ell = \pm 1$ selection rule in atomic transitions.
- Note that parity is a *multiplicative* quantum number.
  This is true for all discrete symmetries.
  Continuous symmetries have *additive* quantum numbers.

Example: $u\overline{u}$ mesons

By conventions: u-quarks have spin $1/2$ and + parity and
$\overline{u}$-quarks have spin $1/2$ and - parity

Parity of a $u\overline{u}$ meson is $P = p_u p_{\overline{u}} (-1)^\ell$

The intrinsic spin (**S**) of the $u\overline{u}$ meson is 0 or 1
but may have any orbital angular momentum (**L**) value.

S	L	J^P	particle
0	0	0^-	π^0
1	0	1^-	ρ^0
0	1	1^+	$b_1(1235)$

See the PDG for a table of the quantum numbers of the low-mass mesons.

Parity in the Standard Model

In 1956, Yang and Lee realized that parity invariance had never been tested experimentally for weak interactions.

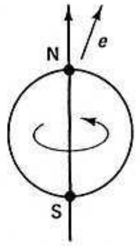


Figure 4.7 In the beta decay of cobalt 60, most electrons are emitted in the direction of the nuclear spin.

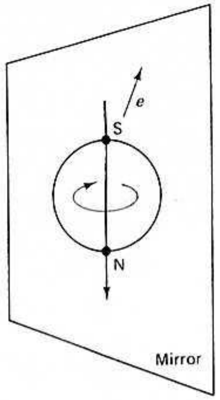


Figure 4.8 Mirror image of Figure 4.7: Most electrons are emitted *opposite* to nuclear spin.

Wu's experiment: recorded the direction of the emitted electron from a ^{60}Co β-decay when the nuclear spin was aligned up and down

The electron was emitted in the same direction independent of the spin.

$\longrightarrow$ Parity is not conserved in the weak interations

Parity Violation in π Decay

- Consider the weak decay $\pi^+ \rightarrow \mu^+ + \nu_\mu$.
  Since the π is spin-0 and the μ and ν emerge back-to-back in the CM frame, the spins of the μ and ν must cancel.
- Experiments show that *every* μ^+ is left-handed, and therefore *every* ν_μ is also left-handed.
- Similarly, in π^- decay, both the μ^- and $\overline{\nu}_\mu$ *always* emerge right-handed.
- If parity were conserved by the weak interaction, we would expect left-handed pairs and right-handed pairs with equal probability (just as we observe with $\pi^0 \rightarrow 2\gamma$).
- Assuming that neutrinos are massless,
  ALL neutrinos are left-handed
  ALL antineutrinos are right-handed

Charge Conjugation I

- The charge conjugation operator, C, converts a particle to i ts antiparticle.
  $C\,|p\rangle = |\overline{p}\rangle$
- In particular, C reverses *every* internal quantum number
  (e.g. charge, baryon/lepton number, strangeness, etc.).
- $C^2 = I$ implies that the only allowed eigenvalues of C are ± 1.
- Unlike parity, very few particles are C eigenstates.
  Only particles that are their own antiparticles (π^0, η, γ) are C eigenstates.

  For example,
  $C\left|\pi^+\right\rangle = \left|\pi^-\right\rangle$
  $C\,|\gamma\rangle = -\,|\gamma\rangle$

Charge Conjugation II

- The photon has a $C = -1$
- $f\overline{f}$ bound states have $C = (-1)^{\ell+s}$
- Charge conjugation is respected by both the strong and electromagnetic interactions.
- Example: the π^0 $(\ell = s = 0 \;\Rightarrow\; C = +1)$ can decay into 2γ but not 3γ

$$C\left|n\gamma\right\rangle = (-1)^n \left|\gamma\right\rangle$$

$$C\left|\pi^0\right\rangle = \left|\pi^0\right\rangle$$

$\pi^0 \longrightarrow 2\gamma$ is allowed (and observed)
$\pi^0 \longrightarrow 2\gamma$ is not allowed (and not observed $< 3.1 \times 10^{-8}$)

G-Parity I

- C-symmetry is of limited use.
  $\Rightarrow$ Most particles are not C eigenstates
- The C operator converts π^+ to π^-.
- These two particles have isospin assignments $|\,1\;1\,\rangle$ and $|\,1\;\;-1\,\rangle$.
- A 180° isospin rotation gives $|\,1\;1\,\rangle = e^{i\pi I_2}\,|\,1\;\;-1\,\rangle$.
- The charged pions are eigenstates under the G-parity, which combines C with a 180° isospin rotation: $G = Ce^{i\pi I_2}$
- G-parity is mainly used to examine decays to pions (which have $G = -1$).
  $G\,|n\pi\rangle = (-1)^n\,|n\pi\rangle$

G-Parity II

G-Parity of a few mesons

Particle	J^P	I	G	Decay
$\rho(770)$	1^-	1	+1	2π
$\omega(783)$	1^-	0	-1	3π
$\phi(1020)$	1^-	0	-1	3π
$f(1270)$	2^+	0	+1	2π

For example, the $\rho(770)$ has $G = 1$ which means it should only decay to an even number of pions. Experimentally we find that

$$\begin{aligned} \rho \longrightarrow \quad & \pi\pi \quad && 100\% \\ & \pi\pi\pi && < 1.2 \times 10^{-4} \end{aligned}$$

CP **Symmetry**

The combination of C and P (and time reversal T) have special significance.

The violation of CP is the reason we live in a matter universe

CPT is required to be conserved in Quantum Field Theory (QFT)

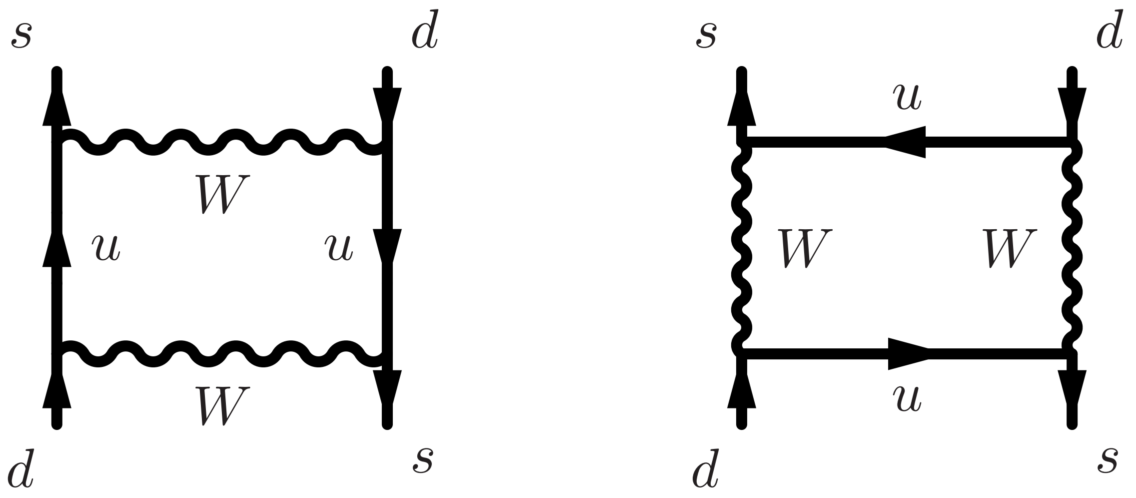
Look at a pion decay example:

- In the pion decay $\pi^+ \rightarrow \mu^+(R) + \nu_\mu(L)$, the ν_μ is always left-handed (LH)
- Under C, this becomes $\pi^- \rightarrow \mu^-(R) + \overline{\nu}_\mu(L)$, but the $\overline{\nu}_\mu$ is still LH
  ⇒ which *does not* occur in nature.
- With C and P, though, we get a RH antineutrino. $\pi^- \rightarrow \mu^-(L) + \overline{\nu}_\mu(R)$
  ⇒ whiich is allowed in nature

CP Violation in the Kaon Sector I

Consider the neutral kaons K^0 ($d\bar{s}$) and $\bar{K}^0$ ($s\bar{d}$)

These particles can mix via a second-order weak interaction:



This section will focus on the CP aspects. The details on the time dependence of K^0 $\overline{K}^0$ oscillations can be found in most textbooks.

CP Violation in the Kaon Sector II

- Both K^0 and $\bar{K}^0$ are pseudoscalar mesons, therefore $P = -1$. and the K^0 and $\bar{K}^0$ are a particle-antiparticle pair.
  As a result, under CP, we have

$$CP\,|K^0\rangle = -\,|\bar{K}^0\rangle \qquad CP\,|\bar{K}^0\rangle = -\,|K^0\rangle$$

- Defining

$$\begin{aligned} |K_1\rangle &= \left(|K^0\rangle - |\bar{K}^0\rangle\right)/\sqrt{2} \\ |K_2\rangle &= \left(|K^0\rangle + |\bar{K}^0\rangle\right)/\sqrt{2} \end{aligned}$$

  we have

$$\begin{aligned} CP\,|K_1\rangle &= +\,|K_1\rangle \\ CP\,|K_2\rangle &= -\,|K_2\rangle \end{aligned}$$

- If CP is conserved, then
  $|K_1\rangle$ can only decay to 2π $(CP = +1)$
  $|K_2\rangle$ can only decay to 3π $(CP = -1)$.

CP Violation in the Kaon Sector III

- K^0 and the $\overline{K}^0$ are mass eigenstates and are each others antiparticles
- K_1 and the K_2 are CP eigenstates
  $|K^0\rangle = \frac{1}{\sqrt{2}}(|K_1\rangle + |K_2\rangle)$ $|\overline{K}^0\rangle = \frac{1}{\sqrt{2}}(|K_1\rangle - |K_2\rangle)$
- K^0_S and the K^0_L are the observed states and are nearly identical to the CP eigenstates (K^0_S and K^0_L are not antiparticles)
  $|K^0_L\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}(|K_2\rangle + \epsilon\,|K_1\rangle)$ $|K^0_S\rangle = \frac{1}{\sqrt{1+|\epsilon|^2}}(|K_2\rangle - \epsilon\,|K_1\rangle)$
- Experimentally we observe

$$K_S \rightarrow \pi\pi \qquad \tau = 0.9 \times 10^{-10} s$$

$$K_L \rightarrow \pi\pi\pi \qquad \tau = 0.5 \times 10^{-7} s$$

However, we find about 1 $K^0_L \rightarrow \pi^+\pi^-$ in 440 K^0_L decays

Other Tests of CP Violation

- There are other CP-violating observables that have been measured in the kaon sector. For example, there is an asymmetry between the branching ratios of K_L to $\pi^+ + e^- + \bar{\nu}_e$ versus $\pi^- + e^+ + \nu_e$
- Within the last few years, the BaBar and Belle experiments have measured CP violation in the B-meson sector.
- CP violation should also be observable in the D-meson (charm) sector, though this will be a small effect that will be very difficult to measure.
- CP violation observed in the K and B mesons is not enough to explain the domination of matter in the universe
- With the observation that neutrino has mass, it is expected that we will observe CP violation in the neutrino sector

Why study CP violation?

- Sakarov pointed out that it is possible to start from a matter-antimatter symmetric universe and end up in one that is asymmetric

  This requires that there be some process (or processes) that violates the CP symmetry.

- The SM does not predict CP violation (it can accomodate no CPV or CPV). However, the SM provides only one source of CP violation (CKM phase angle) which is only possible if there are more than 3 quark generations.

- The currently observed SM CPV (using K and B mesons) is too small to explain the matter-dominated universe.

- There is another possible source of CPV within the SM in the neutrino sector analoguous to the CKM matrix.

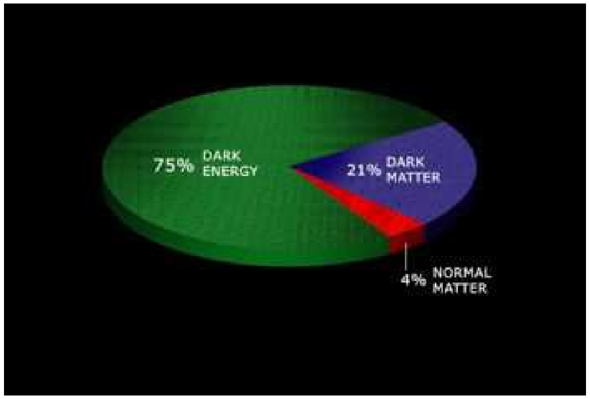
- Beyond the SM: CP violation is ubiquitous in theories of New Physics
  $\Rightarrow$ SUSY can provide enough CP violation to be observable at low energies.

Composition of the universe

The universe is more complicated and it is observed that only a small fraction (4%) of the universe is made of known matter.

Observations also show that there is more matter (dark matter) from unknown sources (21%).

In addition, there is another component (dark energy) which we know even less about (75%).



CP Violation in τ decays

The goal is to search for direct CP violation in the decay $\tau^- \to h^- K_S^0 \left(\geq 0\pi^0\right) \nu_\tau$

There is no charge asymmetry between $\tau^- \to \pi^- K^0 \nu_\tau$ and $\tau^+ \to \pi^+ \overline{K}^0 \overline{\nu}_\tau$

Experimentally we observe the K_S^0 and K_L^0 mesons, which are linear combinations of K^0 and $\overline{K}^0$ states.

Earlier experiments have shown that the K_S^0 and K_L^0 are not exact CP eigenstates. As a result the charge asymmetry is

$$A_Q = \frac{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \overline{\nu}_\tau\right) - \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}{\Gamma\left(\tau^+ \to \pi^+ K_S^0 \overline{\nu}_\tau\right) + \Gamma\left(\tau^- \to \pi^- K_S^0 \nu_\tau\right)}$$

Theory predicts $A_Q = (3.3 \pm 0.1) \times 10^{-3}$

Time Reversal Symmetry

- Time reversal symmetry, as you might guess, reverses the time component:

$$T(t, \mathbf{x}) = T(-t, \mathbf{x})$$

- Although we expect the weak interaction to violate T, direct T violation has not been definitively observed yet.
- Experimentally, one tries to measure the rate of a reaction in both directions $A + B \rightarrow C + D$ but this is not so easy

The CPT Theorem

- The combination CPT is *always* conserved in any local quantum field theory. CPT violation in essentially synonymous with a violation of Lorentz invariance
- CPT symmetry mandates that particles and antiparticles must have certain identical properties, such as the same mass, lifetime, charge, and magnetic moment
- See the PDG summary tables: http://pdg.lbl.gov/2011/tables/rpp2011-conservation-laws.pdf

Examples (fractional differences):

$$M(K^0) - M(\overline{K}^0) < 8 \times 10^{-19}$$

$$M(e^+) - M(e^-) < 8 \times 10^{-9}$$

$$\tau(\mu^+) - \tau(\mu^-) < 2 \times 10^{-5}$$

$$g(e^+) - g(e^-) < 10^{-12}$$

Lepton Number

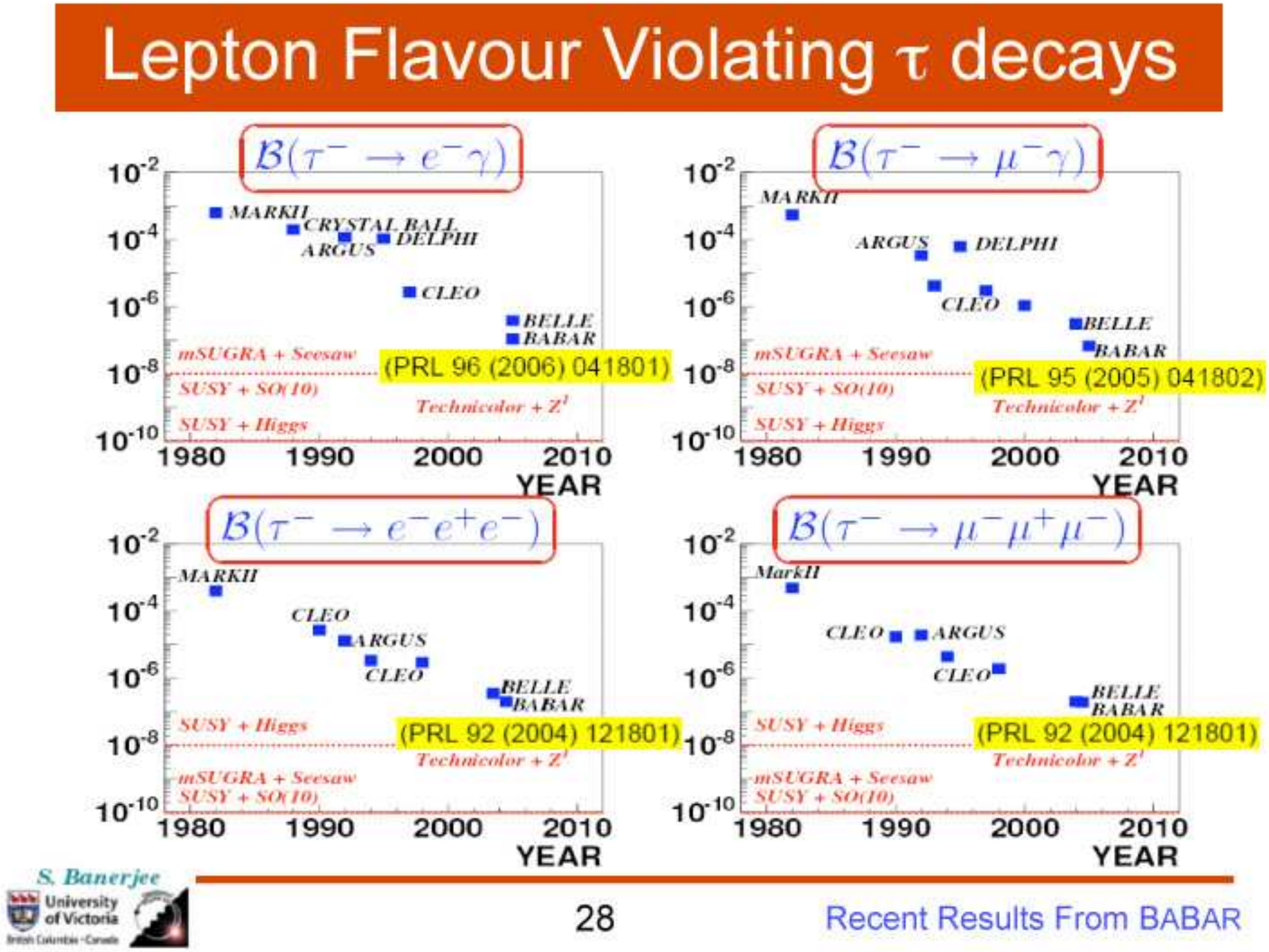
- There are 3 lepton numbers: L_e, L_μ and L_τ
  $L_e = +1$ for e^- and ν_e
  $L_e = -1$ for e^+ and $\overline{\nu}_e$
- Conserved in the EM and Weak interactions $\gamma \rightarrow e^+e^-$ and $\pi^+ \rightarrow \mu^+\overline{\nu}_\mu$ are allowed whereas $\mu^+ \rightarrow e^+\gamma$ is forbidden
- Neutrino oscillations imply that lepton number is violated (at a very small level)
- See the PDG summary tables:
  http://pdg.lbl.gov/2011/tables/rpp2011-conservation-laws.pdf

Examples (fractional differences):

$$\Gamma(\mu^- \rightarrow e^-\gamma) < 10^{-11}$$

$$\Gamma(Z^0 \rightarrow e^-\mu^+) < 10^{-6}$$

$$\Gamma(\tau^- \rightarrow e^-e^-e^+) < 10^{-8}$$



Lepton Flavour Violating τ decays

(PRL 96 (2006) 041801)

(PRL 95 (2005) 041802)

(PRL 92 (2004) 121801)

(PRL 92 (2004) 121801)

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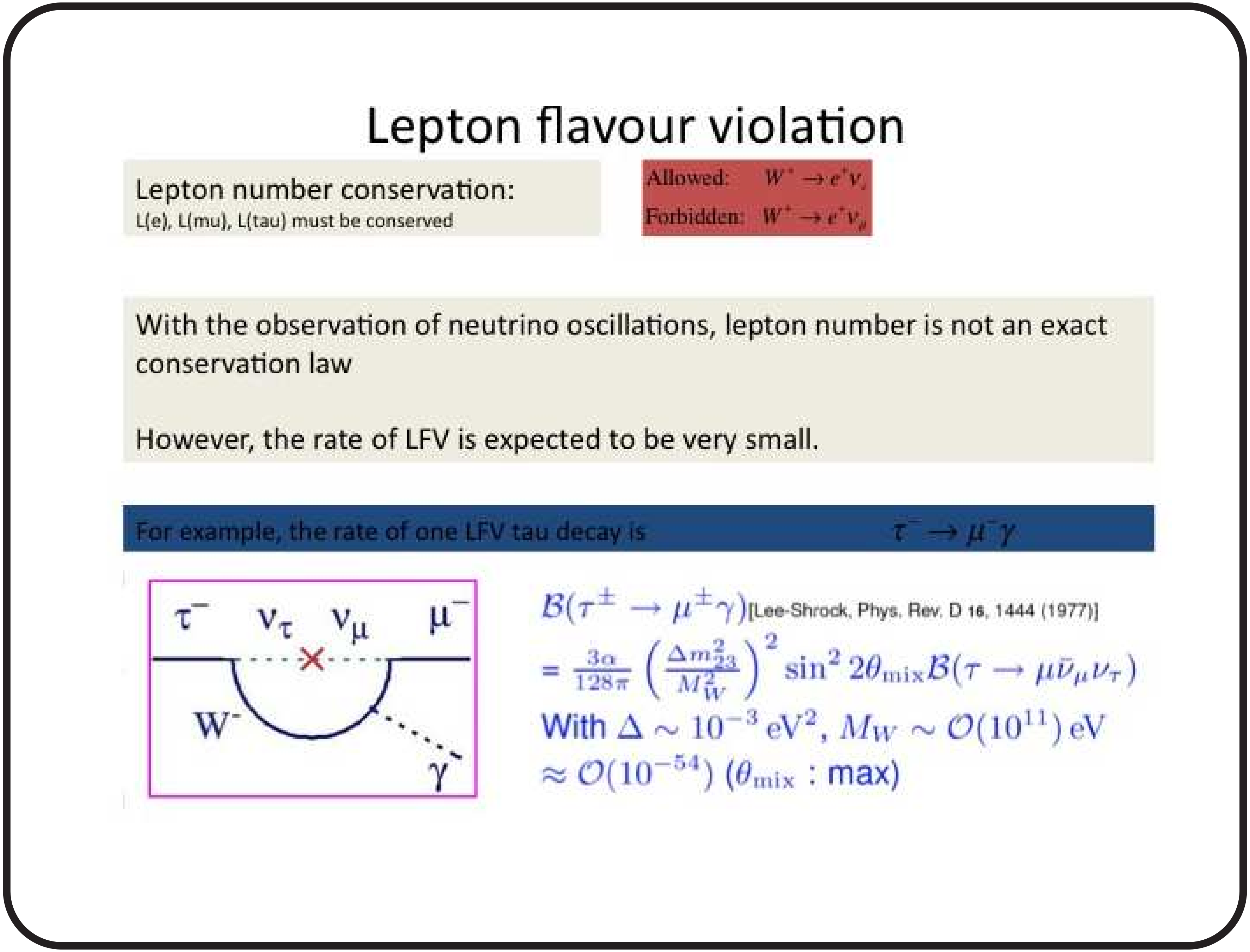
Recent Results From BABAR



Lepton flavour violation

Lepton number conservation:
L(e), L(mu), L(tau) must be conserved

Allowed: $W^+ \to e^+ \nu_e$

Forbidden: $W^+ \to e^+ \nu_\mu$

With the observation of neutrino oscillations, lepton number is not an exact conservation law

However, the rate of LFV is expected to be very small.

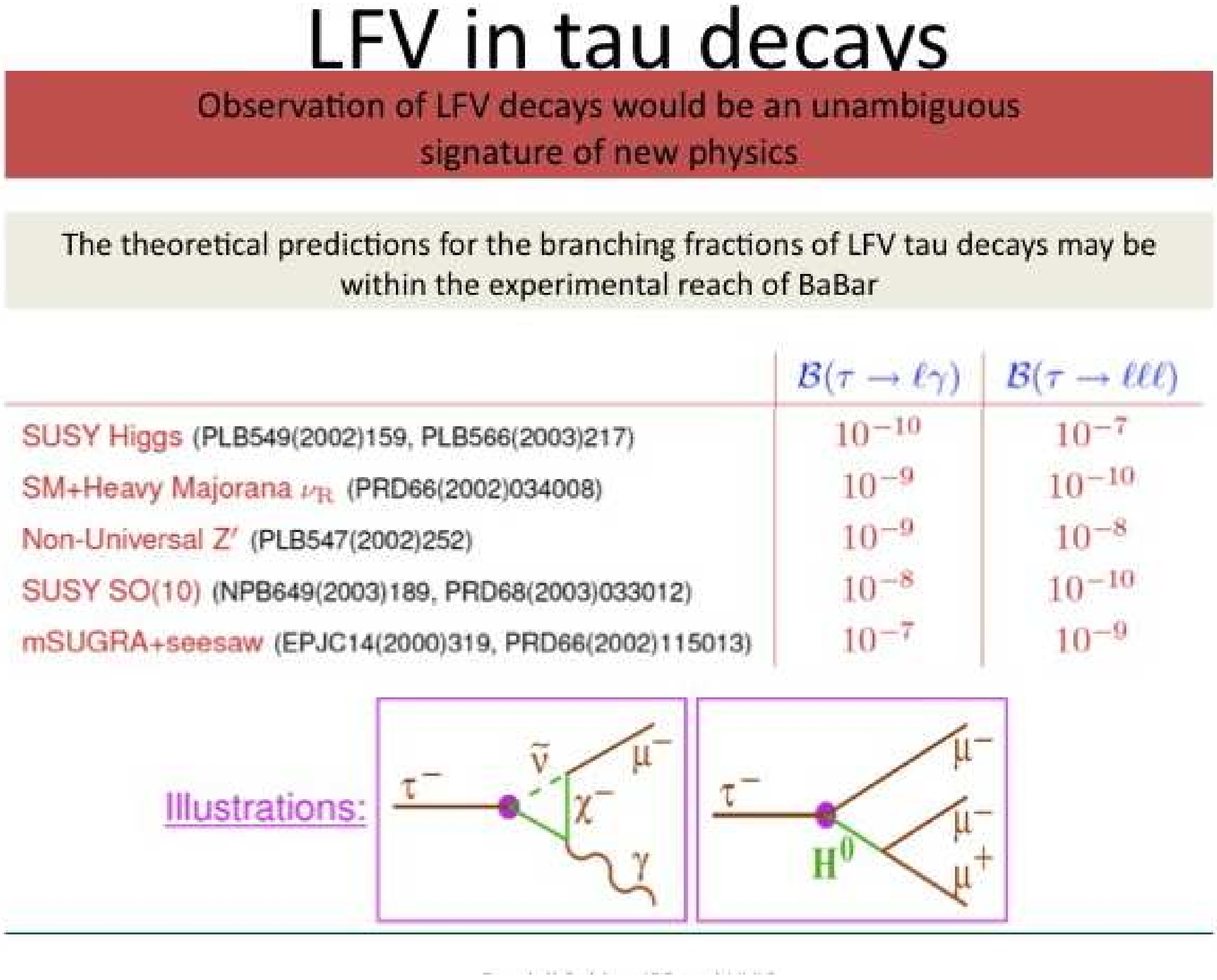
For example, the rate of one LFV tau decay is $\tau^- \to \mu^- \gamma$

$\mathcal{B}(\tau^\pm \to \mu^\pm \gamma)$ [Lee-Shrock, Phys. Rev. D **16**, 1444 (1977)]

$$= \frac{3\alpha}{128\pi}\left(\frac{\Delta m^2_{23}}{M^2_W}\right)^2 \sin^2 2\theta_{\rm mix} \mathcal{B}(\tau \to \mu \bar{\nu}_\mu \nu_\tau)$$

With $\Delta \sim 10^{-3}\,\mathrm{eV}^2$, $M_W \sim \mathcal{O}(10^{11})\,\mathrm{eV}$

$\approx \mathcal{O}(10^{-54})$ ($\theta_{\rm mix}$: max)



LFV in tau decays

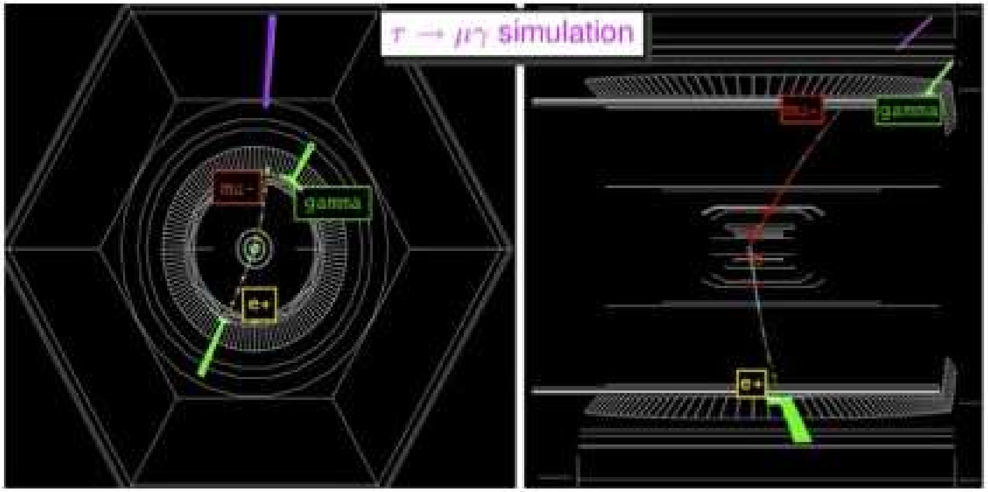
Observation of LFV decays would be an unambiguous signature of new physics

The theoretical predictions for the branching fractions of LFV tau decays may be within the experimental reach of BaBar

	$\mathcal{B}(\tau \to \ell\gamma)$	$\mathcal{B}(\tau \to \ell\ell\ell)$
SUSY Higgs (PLB549(2002)159, PLB566(2003)217)	10^{-10}	10^{-7}
SM+Heavy Majorana ν_R (PRD66(2002)034008)	10^{-9}	10^{-10}
Non-Universal Z' (PLB547(2002)252)	10^{-9}	10^{-8}
SUSY SO(10) (NPB649(2003)189, PRD68(2003)033012)	10^{-8}	10^{-10}
mSUGRA+seesaw (EPJC14(2000)319, PRD66(2002)115013)	10^{-7}	10^{-9}

Illustrations:

LFV Results



Very clean signature for (lepton + photon) decay

Challenge is understanding contribution from background

90% confidence level lower limits:

$B(\tau \to e\gamma) < 1.1 \times 10^{-7}$

$B(\tau \to \mu\gamma) < 6.8 \times 10^{-8}$

Results do not include the full data sample

Starting to probe the predictions of the new physics models
(1E-8 to 1E-10 level)