transformation modulo a simple rotation. Parity invariance requires all situations to be equally likely to occur, e.g. right-handed electrons and left-handed electrons should be equally likely, and so should right-handed anti-neutrinos and left-handed anti-neutrinos. Experimental evidence is quite different: situation A is found *very much less frequently* than situation B! All is consistent with the statement that "Parity is maximally broken" in weak interactions, i.e. situation A is completely suppressed in the limit $\theta \to 0$. Another way of viewing this is the statement that weak interactions involve only left-handed fermions (and right-handed anti-fermions) in the limit that the fermion (rest-)masses are small compared to their energies.

The form of the angular distribution that is predicted for the weak decay by the Standard Model is parity non-conserving and has the form:

$$I(\theta) \propto 1 - \frac{\mathbf{p}_e \cdot \mathbf{J}_{^{60}\mathrm{Co}}}{E_e \left|\mathbf{J}_{^{60}\mathrm{Co}}\right|} = 1 - \hat{\boldsymbol{\beta}}_e \cdot \hat{\mathbf{J}} \tag{I.66}$$

even parity term — odd parity term

where both terms have equal maximum magnitudes ($|\beta_e| \approx 1$ for high-energy electrons); $I(\theta)$ has no definite parity. Many more, and more sophisticated experiments in atomic, nuclear, and particle physics have since confirmed the violation of parity invariance by the weak interaction.

The θ–τ puzzle can now be explained by the finding that the weak decay of the Kaon (~495 MeV) does not conserve parity and can result in final states of either two or three pions.

3.5 The Charge Conjugation Operator

The transformation of "Charge Conjugation" is somewhat of a misnomer: the transformation **C** is defined as a full switch in sign of all discrete ***additive*** quantum numbers of particles: Baryon number, Lepton number, Isospin, Strangeness, Charm, etc., as well as Charge (but NOT the space-time properties like mass, momentum, spin; and NOT parity which is a multiplicative q.n.). In this way, **C** transforms a particle into its anti-particle: e.g. $\mathbf{C}|p\rangle = |\bar{p}\rangle$, $\mathbf{C}|\pi^-\rangle = |\pi^+\rangle$, $\mathbf{C}|e^-\rangle = |e^+\rangle$, etc. Particles and antiparticles have exactly the same masses.

Note that only the *neutral* objects (in the general sense of having zero charge, isospin, baryon-number, etc.), i.e. mesons ($q\bar{q}$ states) like π^0, ρ^0, φ, ω, J/ψ, or particle-antiparticle systems like e^+e^- or $n\bar{n}$ are actually eigenstates of **C**. For example, for charged objects **C** and **Q** do not commute:

$$\mathbf{CQ}|q\rangle = q\mathbf{C}|q\rangle = q|-q\rangle, \text{ and } \mathbf{QC}|q\rangle = \mathbf{Q}|-q\rangle = -q|-q\rangle, \quad \Rightarrow \quad [\mathbf{C},\mathbf{Q}] = 2\mathbf{CQ} \tag{I.67}$$

The eigenvalue of **C** for the neutral pion is easily derived using the fact that it decays into two photons. A photon is represented by the vector field **A**, which is generated by a circulating current of electrons. The **C** operation transforms the charge carriers into their antiparticles, and therefore, under **C** the current changes direction and so must **A**: $\mathbf{C}|\gamma\rangle = -|\gamma\rangle$. Therefore $\mathbf{C}|\pi^0\rangle = \mathbf{C}|\gamma\gamma\rangle = (-1)^2|\gamma\gamma\rangle = +|\pi^0\rangle$. In analogy with the neutral pion, one defines $\mathbf{C}|\pi^+\rangle = +|\pi^-\rangle$: the pion has $J^{PC}=0^{-+}$.

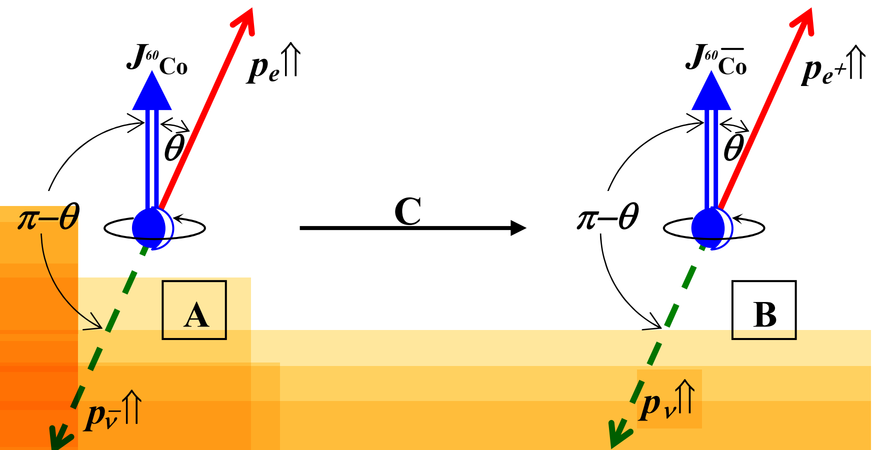
One can turn this fact around, and search for $\pi^0 \to \gamma\gamma\gamma$ decays whose existence would signal the violation of Charge Conjugation symmetry in the electromagnetic interaction; experimentally one finds: $\Gamma(\pi^0 \to \gamma\gamma\gamma)/\Gamma(\pi^0 \to \gamma\gamma) < 3.8\times10^{-7}$ (90%CL).

For charged particles, an extended operator, the so-called **G**-parity operator, has been defined, which combines an I_3 flip (e.g. a θ=180° rotation around the 2-axis in isospin space: $e^{i\pi I_2} = e^{\frac{1}{2}i\pi\sigma_2} = \mathbf{1}\cos{}^{\pi}/_2 + i\sigma_2\sin{}^{\pi}/_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$) of the meson with **C** (see also (I.47)):

$$\mathbf{G}\,|\,\pi^-\rangle = \mathbf{G}\,|-d\bar{u}\rangle = \mathbf{C}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}|\,(d)(-\bar{u})\rangle = \mathbf{C}\Big(|\,(u)(\bar{d})\rangle\Big) = |\,\bar{u}d\,\rangle = -|-\bar{u}d\,\rangle = -|\,\pi^-\rangle\,, \tag{I.68}$$

so that also charged (but *not* strange, nor charmed, etc.) mesons are <u>eigenstates</u> of this new operator. In general: **G**=**C**$(-1)^I$. The **G**-operator is useful in predicting whether an even or an odd number of pions is produced in a strong interaction: e.g. it codifies the fact that the $\rho^\pm\to\pi^\pm\pi^0$ but not: $\rho^\pm\to\pi^\pm\pi^0\pi^0$; the G-parity of the ρ-meson (C_ρ=–1) is G_ρ=(–)(–1)^1 = +1; thus the ρ will only decay in *even* numbers of pions. In contrast the I=0 omega-meson ω (783 MeV, C_ω=–1) has G_ω=(–)(–1)^0= –1, and the ω must therefore decay in *odd* numbers of pions.

3.5.1 The CP Operation



The Charge-Conjugation, as well as Parity quantum numbers are conserved in strong and electromagnetic interactions (with the caveat about only the neutral states being eigenstates of **C**; which can be amended by using **G**, see below, instead of **C** for charged objects).

It is clear that in addition to **P**, also **C** is violated in the weak interaction: applying the **C** operation to the ^{60}Co decay: because situation A is NOT observed (i.e. the electrons are emitted preferentially in the direction <u>opposite</u> to the ^{60}Co spin), the weak interaction does NOT couple with left-handed *anti*neutrinos, but with *right*-handed antineutrinos. Similarly, in β^+ decays it is found that only left-handed neutrinos participate in the weak interaction, as are present in situation B: situations A and B have very different probabilities, contrary to the expectation of **C** invariance.

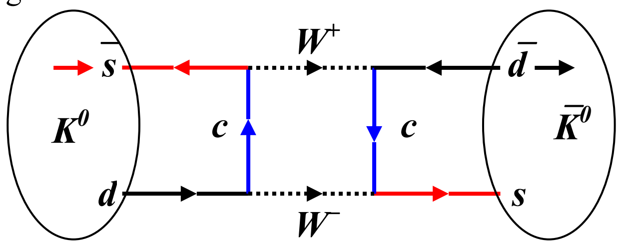
After the bad shock of parity violation in the weak interaction, it was hypothesized that at least the combined operation **CP** might be a symmetry of all interactions, including the weak interaction. That means that $I(\theta)$ and $\bar{I}(\pi-\theta)$ would be equal, where $\bar{I}$ is the distribution of *positrons* in the decay of *anti*-^{60}Co. However, it was soon found that, on a small scale, weak interactions also broke this symmetry. Evidence for that violation was first found in the neutral Kaon system.

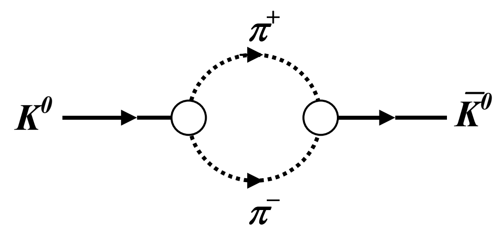
3.5.2 Neutral Kaons

Kaons are produced in large quantities in strong interactions. The prime example is the production of the K^0 in $\pi^- + p \to K^0(S{=}+1,B{=}0) + \Lambda^0(S{=}-1,B{=}1)$. As is easy to see, this strong interaction conserves baryon number and strangeness. In fact, the $\bar{K}^0$ is *not* so easy to produce: as the K^0, because the additive quantum number strangeness changes sign under **C**, the $\bar{K}^0$ has S=+1, and it cannot be produced together with a Λ^0. (In fact the easiest way to produce a $\bar{K}^0$ in a πp interaction is in: $\pi^+ + p \to \bar{K}^0(S{=}-1,B{=}0) + K^+(S{=}+1,B{=}0) + p$ which requires more energy, and has an inherently smaller production rate.) Both the K^0 and the $\bar{K}^0$ decay frequently into two pions ($\pi^+\pi^-$ and $\pi^0\pi^0$) with a lifetime typical for the weak interaction. Note that the states $\pi^+\pi^-$ and $\pi^0\pi^0$ are actually eigenstates of **CP**, but not the K^0 and $\bar{K}^0$: **CP**$|K^0\rangle$ =

$-|\bar{K}^0\rangle$, and $\mathbf{CP}|\bar{K}^0\rangle = -|K^0\rangle$. The minus sign comes from the parity operation: there is arbitrariness in the parity assignment of the Kaon; normally one defines the parity of the Kaon to be odd, identical to the parity of the pion (they are members of the same spin-0 multiplet), but apart from some sign changes, the discussion below stays valid if the parity assignment is chosen to be even.

Weak interactions do not conserve strangeness S and strangeness-violating transitions between K^0 and $\bar{K}^0$ will, and do, occur. The transition can be seen to occur via the common 2π intermediate states, or via the underlying quark diagrams that explicitly involve the weak intermediate vector bosons, e.g.:





Because the connection (via the W's or the pions) is awfully weak, we may treat the process using perturbation theory: we start with the interaction Hamiltonian $\mathbf{H}_0$ ($=\mathbf{H}_{\text{strong}}+\mathbf{H}_{\text{electromagnetic}}$) which conserves strangeness, and therefore cannot couple the K^0 and $\bar{K}^0$ states. The eigenstates of $\mathbf{H}_0$ are $|K^0\rangle$ and $|\bar{K}^0\rangle$. Once we "switch on" the weak interaction, transitions between the two Kaon states become possible, and the total Hamiltonian $\mathbf{H}$ contains the term $\mathbf{H}_{\text{weak}}$: $\mathbf{H}=\mathbf{H}_{\text{strong}}+\mathbf{H}_{\text{electromagnetic}}+\mathbf{H}_{\text{weak}}=\mathbf{H}_0+\mathbf{H}_w$.

If **CP** is a good symmetry, it follows that $[\mathbf{H},\mathbf{CP}]=0$, and we should be able to construct common eigenstates of **H** and **CP**; indeed:

$$\left.\begin{aligned}|K_1^0\rangle &\equiv \tfrac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right)\\ |K_2^0\rangle &\equiv \tfrac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)\end{aligned}\right\} \text{ or } \left\{\begin{aligned}|K^0\rangle &= \tfrac{1}{\sqrt{2}}\left(|K_1^0\rangle + |K_2^0\rangle\right)\\ |\bar{K}^0\rangle &= \tfrac{1}{\sqrt{2}}\left(-|K_1^0\rangle + |K_2^0\rangle\right)\end{aligned}\right\} \text{ then: } \left\{\begin{aligned}\mathbf{CP}|K_1^0\rangle &= +|K_1^0\rangle\\ \mathbf{CP}|K_2^0\rangle &= -|K_2^0\rangle\end{aligned}\right., \tag{I.69}$$

proving that K_1 and K_2 are proper **CP** eigenstates. They are also eigenstates of **H**; because they are constructed from eigenstates of $\mathbf{H}_0$, and, further because for $\mathbf{H}_w$:

$$\langle K^0|\,\mathbf{H}_w\,|K^0\rangle = -\langle K^0|\,\mathbf{H}_w\mathbf{CP}\,|\bar{K}^0\rangle \stackrel{\text{if }[\mathbf{H}_w,\mathbf{CP}]=0}{=\!=\!=} -\langle K^0|\,\mathbf{CPH}_w\,|\bar{K}^0\rangle = -\langle K^0|\,(\overleftarrow{\mathbf{CP}})^\dagger\,\mathbf{H}_w\,|\bar{K}^0\rangle = \langle \bar{K}^0|\,\mathbf{H}_w\,|\bar{K}^0\rangle\,. \tag{I.70}$$

$\mathbf{H}_w$ does not connect nor mix the K_1 and K_2 states:

$$\langle K_2^0|\,\mathbf{H}_w\,|K_1^0\rangle = \langle K_2^0|\,\mathbf{H}_w\mathbf{CP}\,|K_1^0\rangle \stackrel{\text{if }[\mathbf{H}_w,\mathbf{CP}]=0}{=\!=\!=} \langle K_2^0|\,\mathbf{CPH}_w\,|K_1^0\rangle = \langle K_2^0|\,(\mathbf{CP})^\dagger\,\mathbf{H}_w\,|K_1^0\rangle = -\langle K_2^0|\,\mathbf{H}_w\,|K_1^0\rangle = 0\,. \tag{I.71}$$

and similar for $\mathbf{H}_0$, which commutes with CP. Thus, the K_1 and K_2 states must be eigenstates of the total **H.**

We may define the expectation values of $\mathbf{H}_w$ in K^0 and $\bar{K}^0$:

$$E' \equiv \langle K^0|\,\mathbf{H}_w\,|K^0\rangle = \langle \bar{K}^0|\,\mathbf{H}_w\,|\bar{K}^0\rangle, \quad \text{and} \quad \Delta E' \equiv \langle \bar{K}^0|\,\mathbf{H}_w\,|K^0\rangle \stackrel{\text{if }[\mathbf{H}_w,\mathbf{CP}]=0}{=\!=\!=} \langle K^0|\,\mathbf{H}_w\,|\bar{K}^0\rangle\,, \tag{I.72}$$

from which:

$$\langle K_1^0|\,\mathbf{H}\,|K_1^0\rangle = \langle K_1^0|\,\mathbf{H}_0|K_1^0\rangle + \langle K_1^0|\,\mathbf{H}_w|K_1^0\rangle = E_0 + \frac{1}{2}\left(\langle K^0|\mathbf{H}_w|K^0\rangle - \langle \overline{K}^0|\mathbf{H}_w|K^0\rangle - \langle K^0|\mathbf{H}_w|\overline{K}^0\rangle + \langle \overline{K}^0|\mathbf{H}_w|\overline{K}^0\rangle\right) = E_0 + E' - \Delta E' \quad . \qquad \text{(I.73)}$$

and similarly: $\langle K_2^0|\,\mathbf{H}\,|K_2^0\rangle = E_0 + E' + \Delta E'$

where E_0 is the eigenvalue of $\mathbf{H}_0$. In the Kaon rest system we have $E_0 = m_K - i\Gamma_K/2$, where m_K, Γ_K are the mass, respectively the width of the neutral Kaon, which differ for the K_1 and K_2: the K_1 and K_2 have different energies and (in their rest frames) different masses; they are different states: each has definite CP (+1 and −1 respectively); they decay very differently (e.g. $K_1 \rightarrow 2\pi + Q \approx 220$ MeV, $K_2 \rightarrow 3\pi + Q \approx 80$ MeV) and with quite different rates because of the difference in phase space: $\tau(K_1) = 8.958\times10^{-11}$ s, and $\tau(K_2) = 5.11\times10^{-8}$ s. The true 'particles' are *not* the K^0 and $\overline{K}^0$, but the K_1 and K_2. To quote Gell-Mann and Pais (*Phys. Rev.* **97** (1955) 1387.):

> "To sum up, our picture of the K^0 implies that it is a particle mixture exhibiting two distinct lifetimes, that each lifetime is associated with a different set of decay modes, and that not more than half of all K^0's can undergo the familiar decay into two pions." ...
> "Since we should properly reserve the word 'particle' for an object with a unique lifetime, it is the K_1 and the K_2 quanta that are the true 'particles.' The K^0 and the $\overline{K}^0$ must, strictly speaking, be considered 'particle mixtures'."

Gell-Mann and Pais predicted the observation that some meters after a target the short-lived K_1 would have died out, and purely $K_2 \rightarrow 3\pi$ decays should remain. Several other beautiful and purely quantum mechanical phenomena occur in the Kaon system: 'Strangeness oscillations' and 'Regeneration.' However, it is also clear that the K_1 nor the K_2 has a well defined strangeness nor a well defined quark content, so it remains very much a matter of taste whether one calls the set (K_1, K_2) more elementary than the set (K^0, $\overline{K}^0$)!

$\mathbf{K^0 \leftrightarrow \overline{K}^0}$ Oscillations

Starting with a pure K^0 state at the (strong interaction) production site, the K^0, a superposition of the weak eigenstates K_1 and K_2, will oscillate back-and-forth between a $\overline{K}^0$ and a K^0 as time passes.
Physically this is analog to the system of two weakly coupled pendula (e.g. two pendula hanging off a crossbar that is somewhat flexible): setting one pendulum in motion, the other pendulum starts slowly to pick up power, and while the first pendulum deceases in amplitude, the swing of the next one is picking up. Energy oscillates back-and-forth between the two, with a frequency dependent on the coupling strength. The added complication in the Kaon system is, of course, that the Kaon decays, so the oscillation is superimposed on the decay rate of the particle.
Mathematically the oscillation depends on the coupling strength between the two states, i.e. on $\Delta E'$. Starting with $\psi(t{=}0) = |K^0\rangle = (^1/_{\sqrt{2}})(K_1 + K_2)$, we find at time $t > 0$:

$$i\frac{\partial\psi(t)}{\partial t} = \left(\mathbf{H}_0 + \mathbf{H}_w\right)\psi(t) \qquad \text{(I.74)}$$

Developing $\psi(t)$ in terms of the two t=0 eigenfunctions of $\mathbf{H}_0$ we write: $\psi(t) = a(t)|K^0\rangle + b(t)|\overline{K}^0\rangle$, with the normalization and boundary conditions: $a^2+b^2=1$; $a(t{=}0)=1$. Filling this general solution into (I.74), we obtain a set of coupled differential equations for $a(t)$ and $b(t)$:

$$\left.\begin{array}{l} i\dfrac{\partial a(t)}{\partial t} = (E_0 + E')a(t) + \Delta E' b(t) \\ i\dfrac{\partial b(t)}{\partial t} = (E_0 + E')b(t) + \Delta E' a(t) \end{array}\right\} \Rightarrow \begin{array}{l} a(t) = \exp\{-i(E_0 + E')t\}\cos(\Delta E' t) \\ b(t) = -i\exp\{-i(E_0 + E')t\}\sin(\Delta E' t) \end{array} \tag{I.75}$$

and: $$\psi(t) = \mathrm{e}^{-i(E_0+E')t}\left(\cos(\Delta E' t)\,|\mathrm{K}^0\rangle - i\sin(\Delta E' t)\,|\overline{\mathrm{K}}^0\rangle\right)$$

Clearly, the probability of finding a $\overline{\mathrm{K}}^0$ sometime after production oscillates: Prob($|\overline{\mathrm{K}}^0\rangle,t$) = $\langle\,\overline{\mathrm{K}}^0|\,\psi(t)\rangle = \sin^2(\Delta E' t)$; it oscillates between 0 and 1, with average value ½ (in the absence of decays)!

The appearance of $\overline{\mathrm{K}}^0$ after production can be verified experimentally by detecting strong interactions in a secondary target from Kaons produced in a primary target up-stream: after being produced as K^0, the beam is inspected at various distances by inter-posing a secondary target and collecting the interactions. One observes final states of the type $\pi^+ + \Lambda^0$, which can *only* be produced in strong, strangeness conserving, $\mathrm{K}^0 p$ interactions, see Figure 3.[12]

If it were possible to create a mono-energetic K^0 beam, the oscillations would be simply observable as function of distance – note that the time t is in the rest system of the Kaon. However, by measuring the interaction products of a particular final state fully as function of distance, one can derive the initial Kaon energy, and then correct to the proper time t. Indeed, a beautiful confirmation of the oscillation is found.

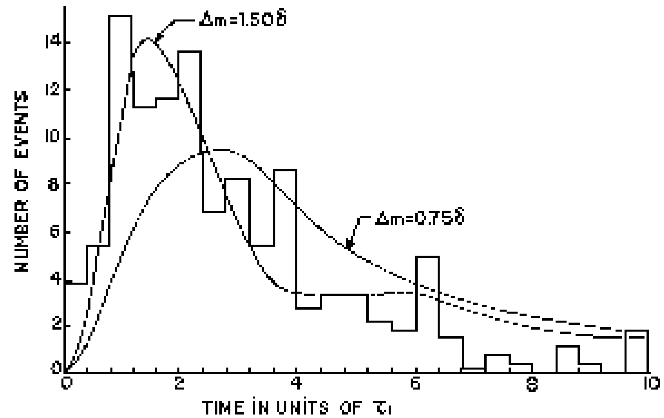


Figure 3. Number of $\overline{\mathrm{K}}^0$ interactions as function of time (units of proper lifetime τ_1). The curves are for two values of Δm (natural units). Note that the number of events goes to zero for small times.

Regeneration

Instead of considering the K^0 and $\overline{\mathrm{K}}^0$ states, one can also consider the produced Kaon as a superposition of K_1 and K_2. Because only the K_1 state (CP-even) can decay into 2π, it has a much shorter lifetime than the K_2 state (CP-odd), which must go to at least 3π. Therefore, after some tens of centimeters away from the production target, the K_1 component of the neutral Kaon beam has died out completely, and a **pure K_2 beam remains** (in vacuum), as evidenced by the observation of only 3π decays in the vacuum for many meters after the production target.

When one interposes a secondary target ("matter") into this pure K_2 beam, the K^0 and $\overline{\mathrm{K}}^0$ components interact with different strengths (see the section above); the $\overline{\mathrm{K}}^0$ ($S=-1$) component is much more likely to undergo strong interactions than the K^0 component:

[12] Camerini et al., Phys. Rev. Lett. (197x) ~365. see also: Gewenniger et al.;

$$\overline{\mathrm{K}}^0(S{=}-1)+\left(p/n\right)\rightarrow\pi^{+/0}+\Lambda^0(S{=}-1)$$

$$\mathrm{K}^0(S{=}+1)+\left(p/n\right)\not\rightarrow\pi^{+/0}+\Lambda^0(S{=}-1), \tag{I.76}$$

$$\left(\text{e.g. only:}\quad \mathrm{K}^0(S{=}+1)+\left(p/n\right)(B{=}+1)\rightarrow\overline{\Lambda}^0(S{=}+1,B{=}-1)+n(B{=}+1)+\left(p/n\right)(B{=}+1)\right)$$

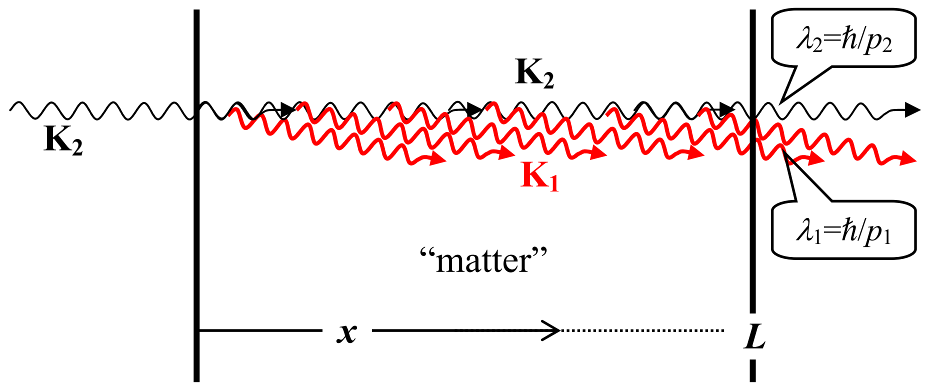
Formally, after "regeneration" (of the K^0 component) in the interposed matter, the total amplitude can be written as:

$$\psi_{reg}=\tfrac{1}{\sqrt{2}}\left(a\,|\mathrm{K}^0\rangle+\overline{a}\,|\overline{\mathrm{K}}^0\rangle\right)=\tfrac{1}{2}(a-\overline{a})\,|\mathrm{K}_1^0\rangle+\tfrac{1}{2}(a+\overline{a})\,|\mathrm{K}_2^0\rangle\,, \tag{I.77}$$

where the coefficients a and $\overline{a}$ are now *different* from one another after passing through matter, and a K_1 component reappears, as evidenced by a multitude of 2π decays in the first few centimeters after the secondary target!

$\mathrm{K}_1 - \mathrm{K}_2$ Mass Difference

Coherent regeneration is used to determine the $\mathrm{K}_1 - \mathrm{K}_2$ mass difference. Coherence exists when only the regenerated K_1 component in the *forward* direction (θ=0) is considered. Again, for simplicity, assume that the incoming K_2 beam is monochromatic (=mono-energetic), and is represented by a traveling wave $\exp(ip_2x)$. The regenerated K_1 component in the forward direction is then also mono-energetic, but, because the K_1 has a slightly different mass, will have a slightly different momentum p_1, and is represented by a traveling wave $\exp(ip_1x)$. The K_1 component is generated all along the path inside the material, and waves generated at x=0 and at $x=L/2$ may interfere destructively when the phase difference $\Delta\varphi = 2\pi\,(^L/_2)(1/\lambda_2 - 1/\lambda_1) = 2\pi\,L/(2\hbar)\,(p_2 - p_1) = (2n{-}1)\pi$: waves from the first half of the material will pair-wise extinguish waves generated in the second half. In general, interference will take place depending on $\exp[iL(p_2 - p_1)] \approx \exp[iL(m_1 - m_2)m/p]$, where $m\equiv(m_1+m_2)/2$ and $p\equiv(p_1+p_2)/2$, and where we used $dp/dm = -m/p$. These strength variations in the number of regenerated K_1 can be measured as function of average Kaon momentum p (controlled by the energy of the production π^- beam) and material thickness L, and lead to a very sensitive determination of $m_2 - m_1 = 3.48\times10^{-6}$ eV. This is to be compared to the measurement of a possible difference in K^0 and $\overline{\mathrm{K}}^0$ masses (a test of the validity of invariance under the combined operation of **TCP**): $m_{\mathrm{K}^0} - m_{\overline{\mathrm{K}}^0} < 5\times10^{-10}$ eV.



CP Violation in Weak Interactions

In 1973 a Brookhaven experiment found that even the combination of **C** and **P** was not a perfect symmetry of the weak interaction: far from a production target, where – if **CP** would be a perfect symmetry – only the CP-odd state K_2 should be present (in vacuum), a few 2π (CP-even) were found!

$$@57\text{ m: } \frac{\Gamma(\mathrm{K}_\mathrm{L}^0 \to \pi\pi)}{\Gamma(\mathrm{K}_\mathrm{L}^0 \to \text{all})} = \frac{45}{22{,}700} \approx 2\times 10^{-3},$$

where we used the notation of K_L to indicate the long-lived Kaon, and to distinguish it from the true CP-eigenstate K_2; K_L would be identical to K_2 if CP were conserved in the weak interaction. Similarly, one uses K_S for the short-lived variant ($\approx K_1$). The "wrong" decay rate is small, about 1 in 500 of all decays, but it means that **CP** is not a symmetry of the weak interaction either, and that one can actually distinguish particles from antiparticles by the weak interaction. The K_L and K_S are not pure CP-eigenstates, and are mixtures of the CP-eigenstates K_1 and K_2:

$$\mathrm{K}_\mathrm{L}^0 = \frac{1}{\sqrt{1+\varepsilon^2}}\left(\mathrm{K}_2^0 + \varepsilon \mathrm{K}_1^0\right), \quad \varepsilon \approx 2.3\times 10^{-3}, \quad \text{and: } \frac{\Gamma(K_L^0 \to \pi^+ + e^- + \overline{\nu}_e)}{\Gamma(K_L^0 \to \pi^- + e^+ + \nu_e)} \neq 1 \qquad \text{(I.78)}$$

The finding of **CP** violation implies that **T** – Time Reversal symmetry – must also be violated in the weak interaction, because **TCP** invariance is a feature of (almost) all possible quantum theories and very hard to get around.

3.6 Time Reversal Symmetry

Time reversal symmetry is a somewhat funny symmetry, because no conservation law is related to it. Instead of a unitary transformation it is a anti-unitary transformation. This can be seen already in the simple Schrodinger equation: a replacement of *t* by –*t*, changes not only the wavefunction but also the sign of the time derivative (which is to first power, unlike the second-order spatial derivative!), and thus changes the *form* of the equation! The proper **T** transformation is therefore:

$$\psi(\mathbf{r},t) \xrightarrow{\mathbf{T}} \psi^*(\mathbf{r},-t); \quad \text{i.e. Not an eigenvalue equation!}$$

$$i\frac{\partial \psi(\mathbf{r},t)}{\partial t} = \mathbf{H}\psi(\mathbf{r},t) \xrightarrow{\mathbf{T}} i\frac{\partial \psi^*(\mathbf{r},-t)}{-\partial t} = \mathbf{H}\psi^*(\mathbf{r},-t) = \left(i\frac{\partial \psi(\mathbf{r},-t)}{\partial t}\right)^* = \left(\mathbf{H}\psi(\mathbf{r},-t)\right)^* \qquad \text{(I.79)}$$

We will not further discuss **T**.