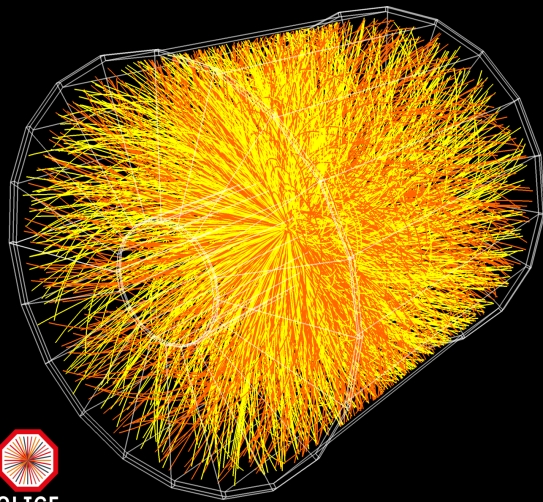


NET-PROTON FLUCTUATIONS AND NON-DYNAMICAL CONTRIBUTIONS *FROM SPS TO LHC*

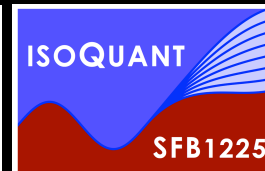
Anar Rustamov

Universität Heidelberg, GSI, BSU, NNRC

EMMI WORKSHOP



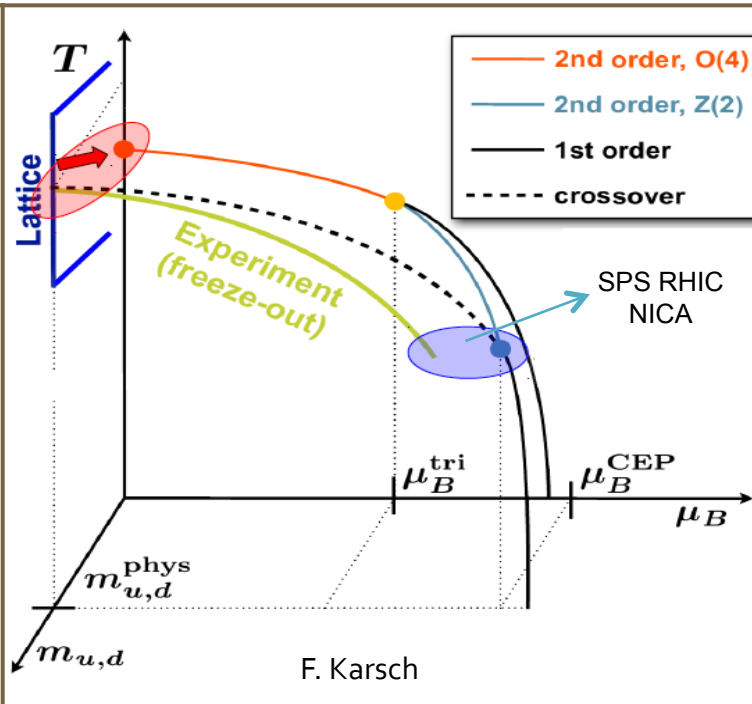
- ⊙ Motivations
- ⊙ News from SPS
- ⊙ Results From ALICE
- ⊙ Fluctuations in CE
- ⊙ Summary



The Ultimate Goal

- ⊙ To probe the structure of strongly interacting matter
 - ⊙ Locate phase boundaries
 - ⊙ Search for critical phenomena
 - ⊙ ...

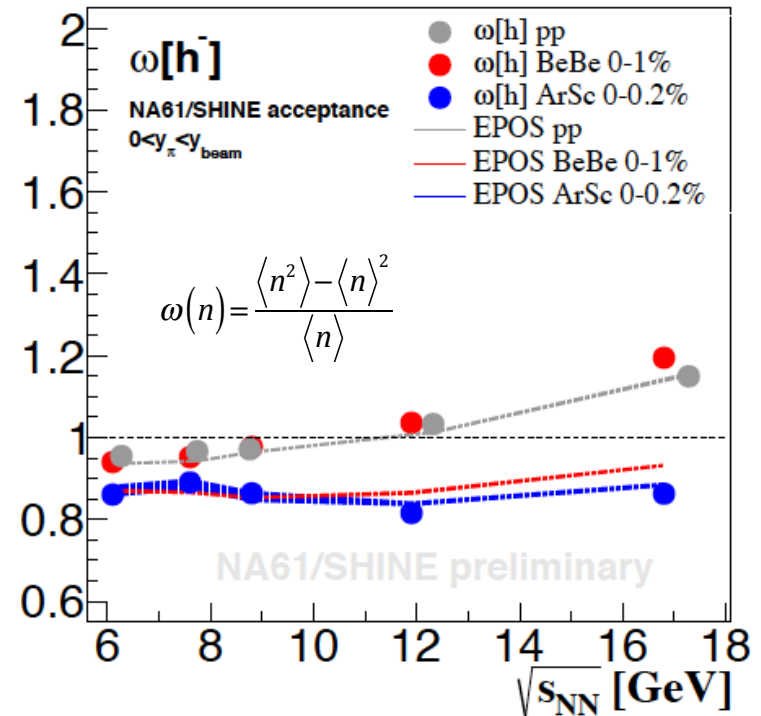
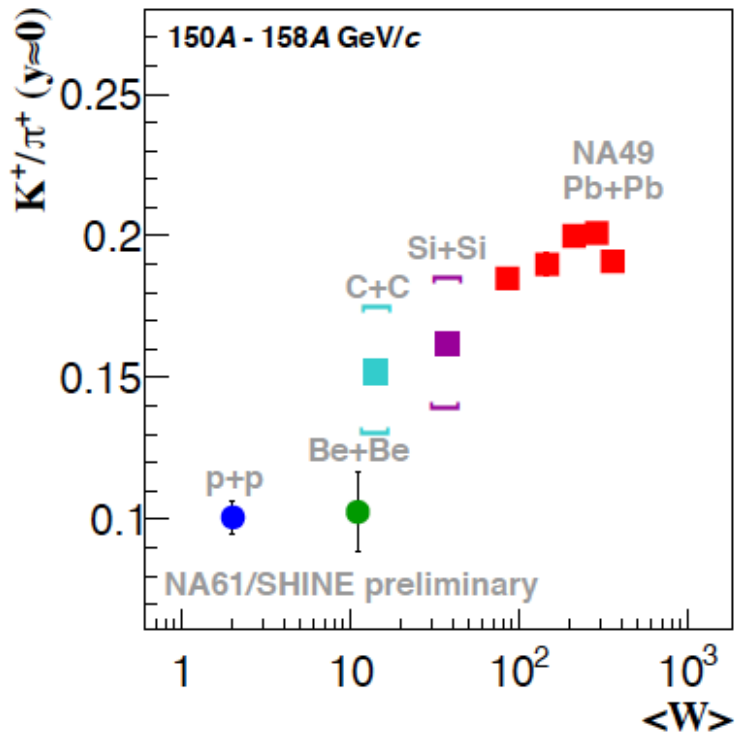
E-by-E fluctuations are predicted within Grand Canonical Ensemble



fingerprints of criticality for $m_{u,d} = 0$
survive at crossover with $m_{u,d} \neq 0$

[A. Bazavov et al., Phys.Rev. D85 \(2012\) 054503](#)

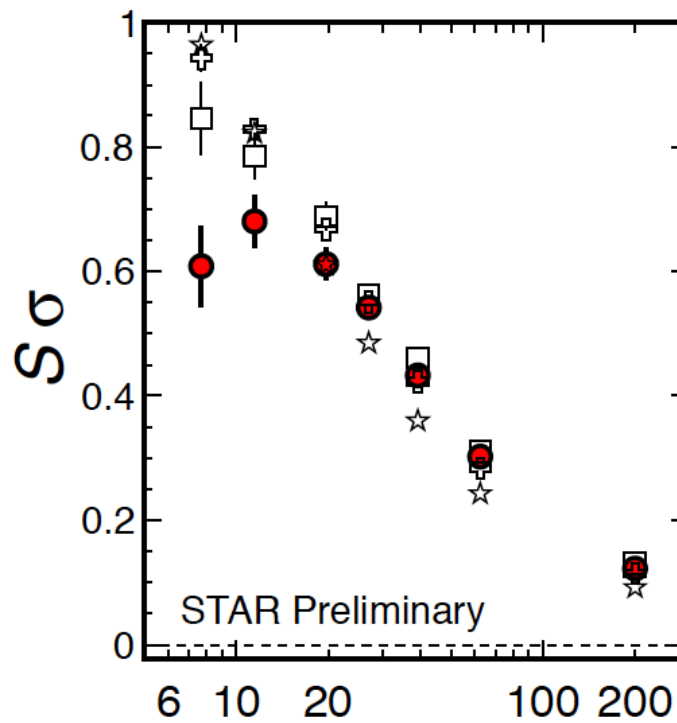
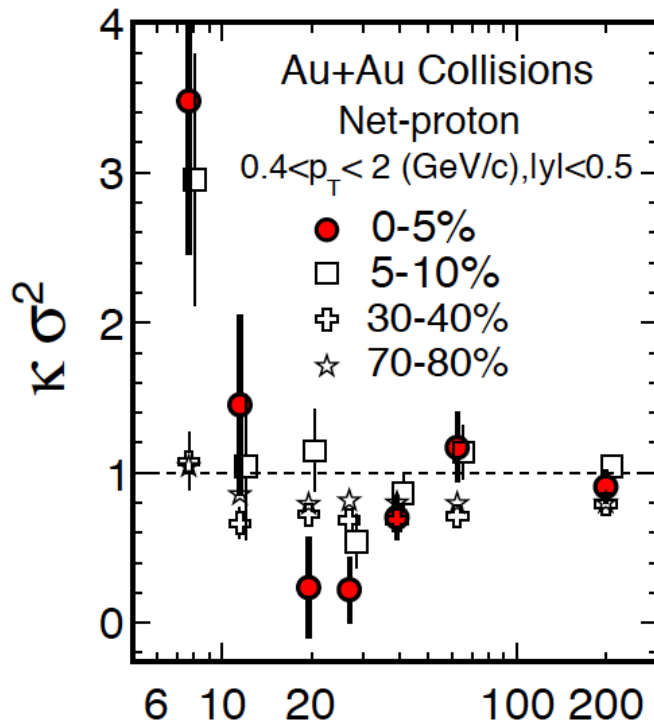
News from NA61/SHINE



- ⊙ Be+Be results are very close to p+p
- ⊙ Consistent behavior between first and second moments

- ⊙ Similar energy dependence for p+p and Be+Be
- ⊙ Lower values for Ar+Sc (conservation laws)
- ⊙ Deviation of Be+Be results from EPOS values

Results from STAR



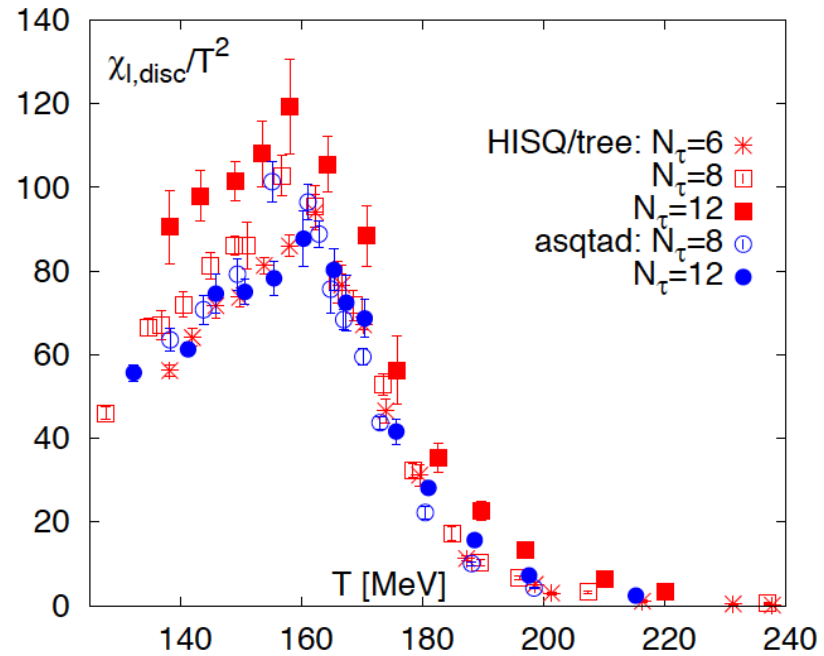
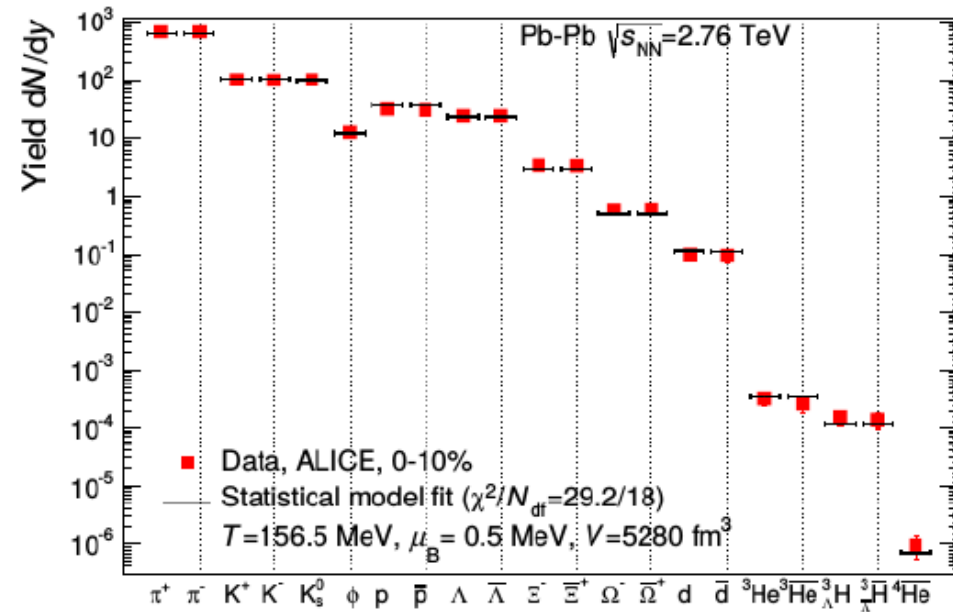
Colliding Energy $\sqrt{s_{NN}}$ (GeV)

- ⊙ Close to unity for peripheral collisions
- ⊙ Below 39 GeV hints for a non-monotonic behavior
- ⊙ **More statistics and precise control of systematics are needed to explore this region**

Drop at 7.7 GeV for central events

X. Luo, PoS CPOD2014, 019 (2015)
STAR: PRL 112, 032302 (2014)

Criticality at crossover



$$\langle N_i \rangle = V \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp\left[\frac{(E_i - \mu_i)}{T}\right] \pm 1}$$

$$\mu_i = \mu_B B_i + \mu_s S_i + \mu_I I_i$$

freeze-out at the phase boundary

$$T_c^{lattice} = 154 \pm 9 \text{ MeV}, \quad T_{fo}^{ALICE} = 156 \pm 3 \text{ MeV}$$

ALICE, PLB 726 (2013) 610

J. Stachel, A. Andronic, P. Braun-Munzinger and K. Redlich

J. Phys. Conf. Ser. 509 (2014) 012019

A. Bazavov et al., Phys.Rev. D85 (2012) 054503

y axis: 9 orders of magnitude; works in the energy range spanning by 3 orders of magnitude

Bridge from experiment to theory

for a thermal system in a fixed volume V
within the Grand Canonical Ensemble

$$\hat{\chi}_2^B = \frac{\langle \Delta N_B^2 \rangle - \langle \Delta N_B \rangle^2}{VT^3} = \frac{\kappa_2(\Delta N_B)}{VT^3}$$

$$\hat{\chi}_n^{N=B,S,Q} = \frac{\partial^n P/T^4}{\partial (\mu_N/T)^n} \quad \frac{P}{T^4} = \frac{1}{VT^3} \ln Z(V, T, \mu_{B,Q,S})$$

- In experiments**

- Volume (participants) fluctuates from E -to- E
- Global conservation laws are important

$$\frac{\kappa_4(\Delta N_B)}{\kappa_2(\Delta N_B)} \equiv \gamma_2 \sigma^2 \neq \frac{\hat{\chi}_4^B}{\hat{\chi}_2^B} \quad \frac{\kappa_3(\Delta N_B)}{\kappa_2(\Delta N_B)} \equiv \gamma_1 \sigma \neq \frac{\hat{\chi}_3^B}{\hat{\chi}_2^B}$$

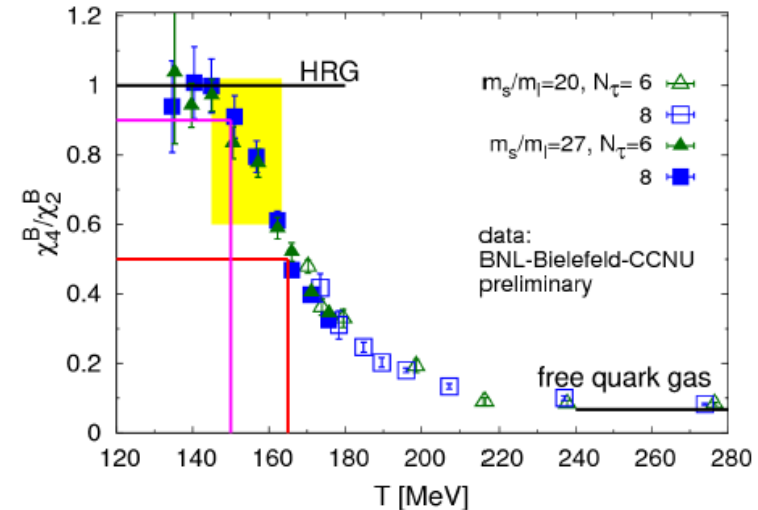
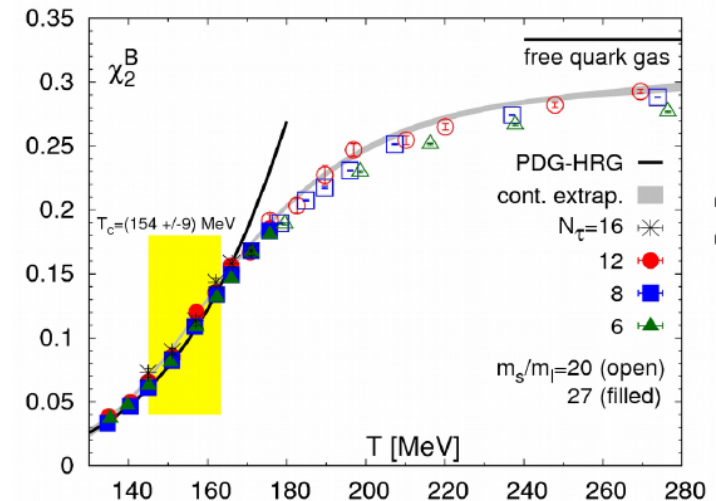
V. Skokov, B. Friman, and K. Redlich, Phys.Rev. C88 (2013) 034911

P. Braun-Munzinger, A. Rustamov, J. Stachel, arXiv:1612.00702, NPA 960 (2017) 114

At $s^{1/2} > 10$ GeV net-proton is a reasonable proxy for the net-baryon

M. Kitazawa, and M. Asakawa, Phys. Rev. C86 (2012) 024904

A. Rustamov, EMMI workshop on fluctuations, China, Wuhan, 10-13 October, 2017



smaller than in HRG for $T > 150$ MeV

F. Karsch, QM17, arXiv:1706.01620

O. Kaczmarek, QM17, arXiv:1705.10682

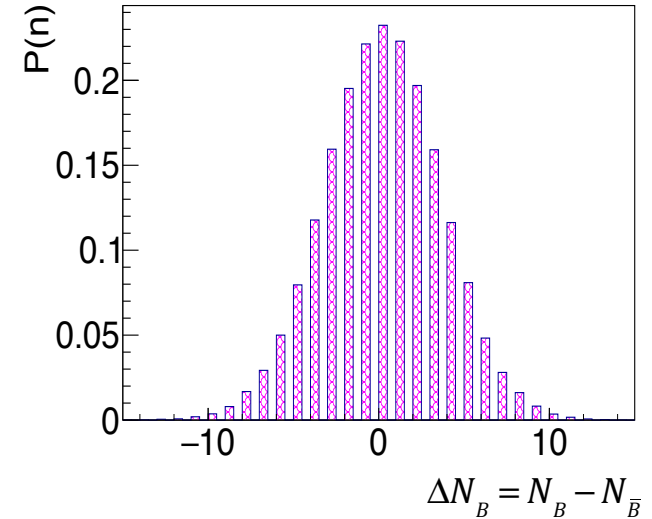
Net-cumulants, definitions

$$\kappa_1(X) = \langle X \rangle$$

$$\kappa_2(X) = \langle (X - \langle X \rangle)^2 \rangle$$

$$\kappa_3(X) = \langle (X - \langle X \rangle)^3 \rangle$$

$$\kappa_4(X) = \langle (X - \langle X \rangle)^4 \rangle - 3\kappa_2^2(X)$$



e.g., second cumulant of net-baryons

$$\kappa_2(N_B - N_{\bar{B}}) = \langle (N_B - N_{\bar{B}})^2 \rangle - \langle N_B - N_{\bar{B}} \rangle^2$$

$$\kappa_2(N_B - N_{\bar{B}}) = \kappa_2(N_B) + \kappa_2(N_{\bar{B}}) - 2(\langle N_B N_{\bar{B}} \rangle - \langle N_B \rangle \langle N_{\bar{B}} \rangle)$$

Correlation term may arise from:

1. Resonance contributions
2. Global conservation laws

Poisson limit:

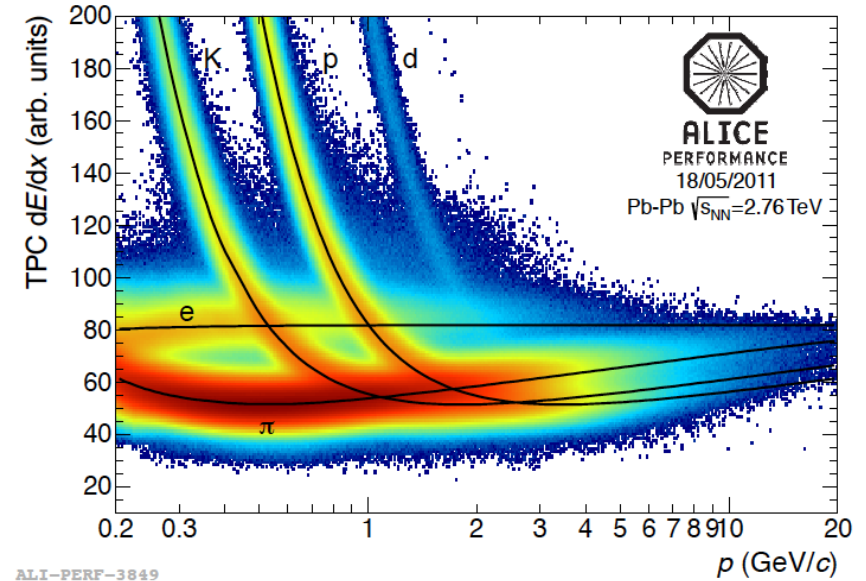
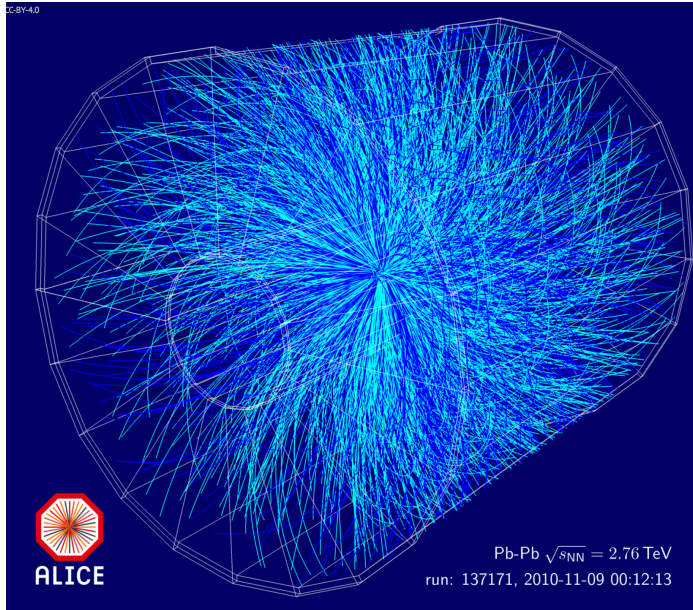
$$\kappa_2(N_B) = \langle N_B \rangle$$

$$\kappa_2(N_{\bar{B}}) = \langle N_{\bar{B}} \rangle$$

$$\kappa_2(N_{\bar{B}} - N_B) \xrightarrow{\langle N_B N_{\bar{B}} \rangle = \langle N_B \rangle \langle N_{\bar{B}} \rangle} \langle N_B \rangle + \langle N_{\bar{B}} \rangle$$

Skellam

ALICE Pb-Pb data at $\sqrt{s_{NN}} = 2.76\text{TeV}$



Acceptance selection:

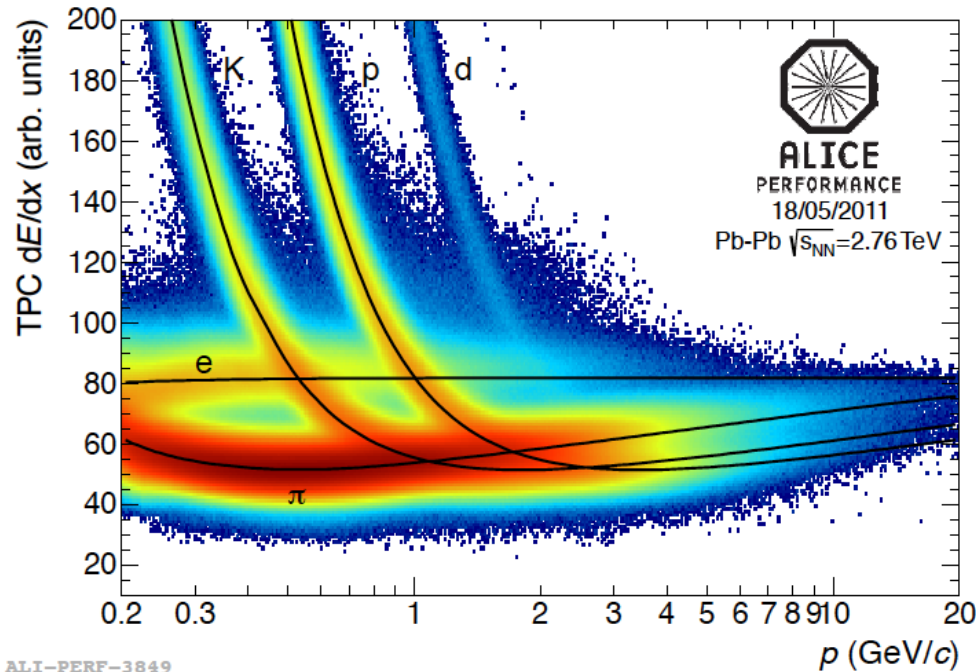
$$0.6 < p < 1.5 \text{ GeV}/c, \quad |\eta| < 0.8$$

Centrality selection:

charged particle multiplicities in
 $2.8 < \eta < 5.1$ and $-3.7 < \eta < -1.7$

Analysis technique: Identity Method

Analysis Strategy, PID

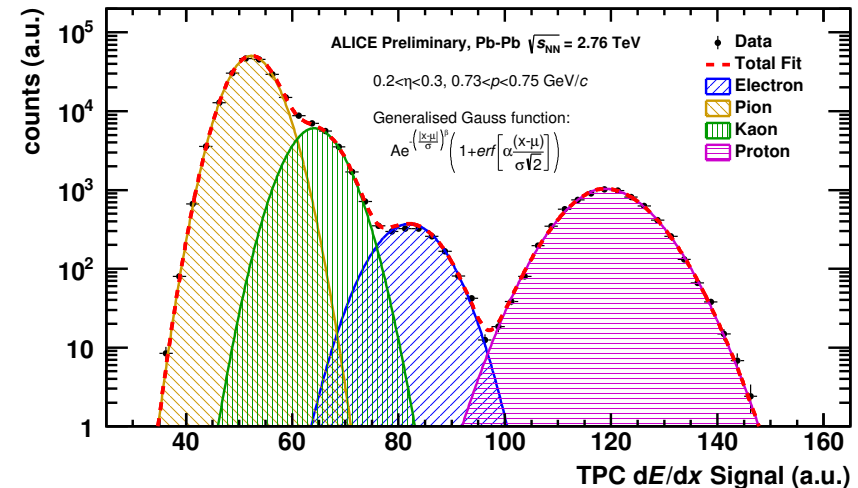


Analysis technique: Identity Method

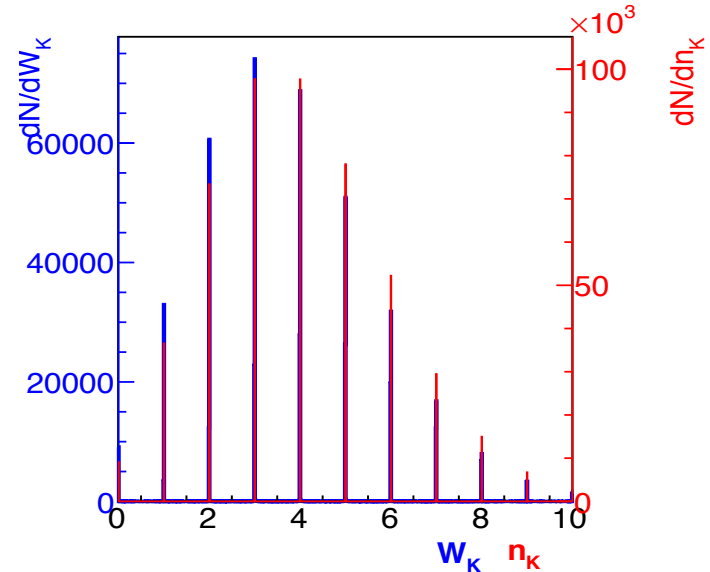
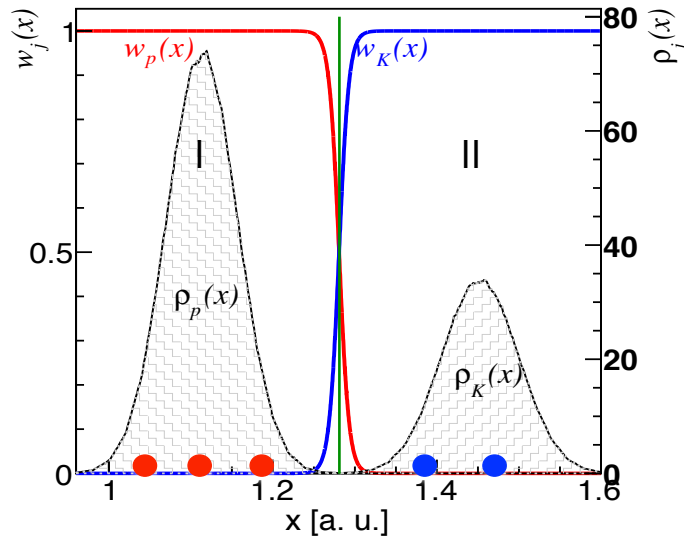
Tracking and cut efficiencies for protons:

- 75-80 % for $p_T > 0.5$ GeV/c
- no centrality dependence

ALICE: Phys.Rev. C88 (2013) 044910



Analysis method



single event example : 3 protons, 2 kaons

traditional approach

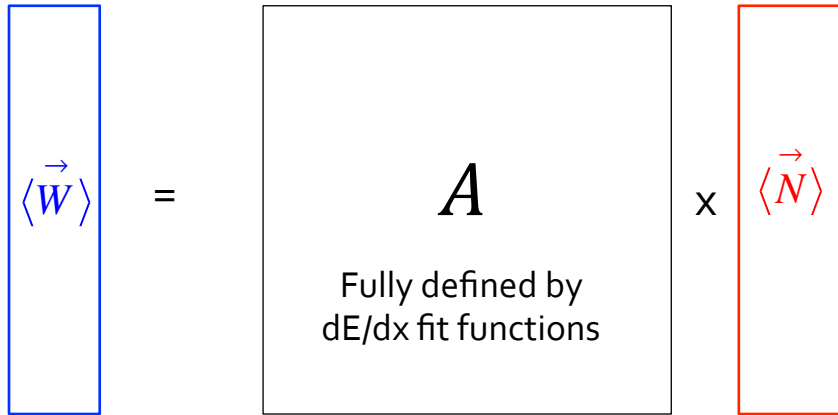
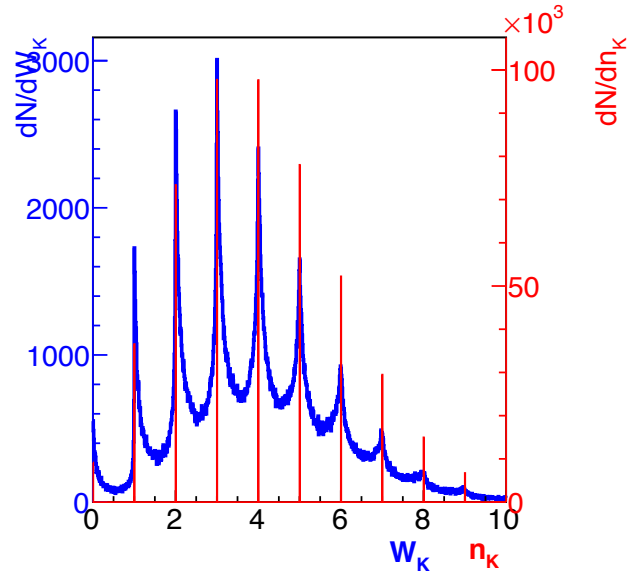
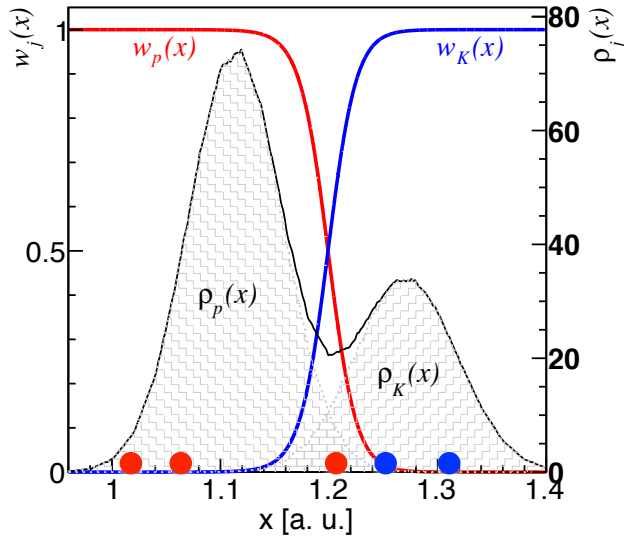
measurement in region I
count as proton
measurement in region II
count as kaon

Identity method approach

$$w_p(x_i) = \frac{\rho_p(x_i)}{\rho_p(x_i) + \rho_K(x_i)} \quad W_p = \sum_{i=1}^5 w_p(x_i)$$

$$w_K(x_i) = \frac{\rho_K(x_i)}{\rho_p(x_i) + \rho_K(x_i)} \quad W_K = \sum_{i=1}^5 w_K(x_i)$$

Analysis method



Identity method, basic idea:

$$\langle \vec{N} \rangle = A^{-1} \langle \vec{W} \rangle$$

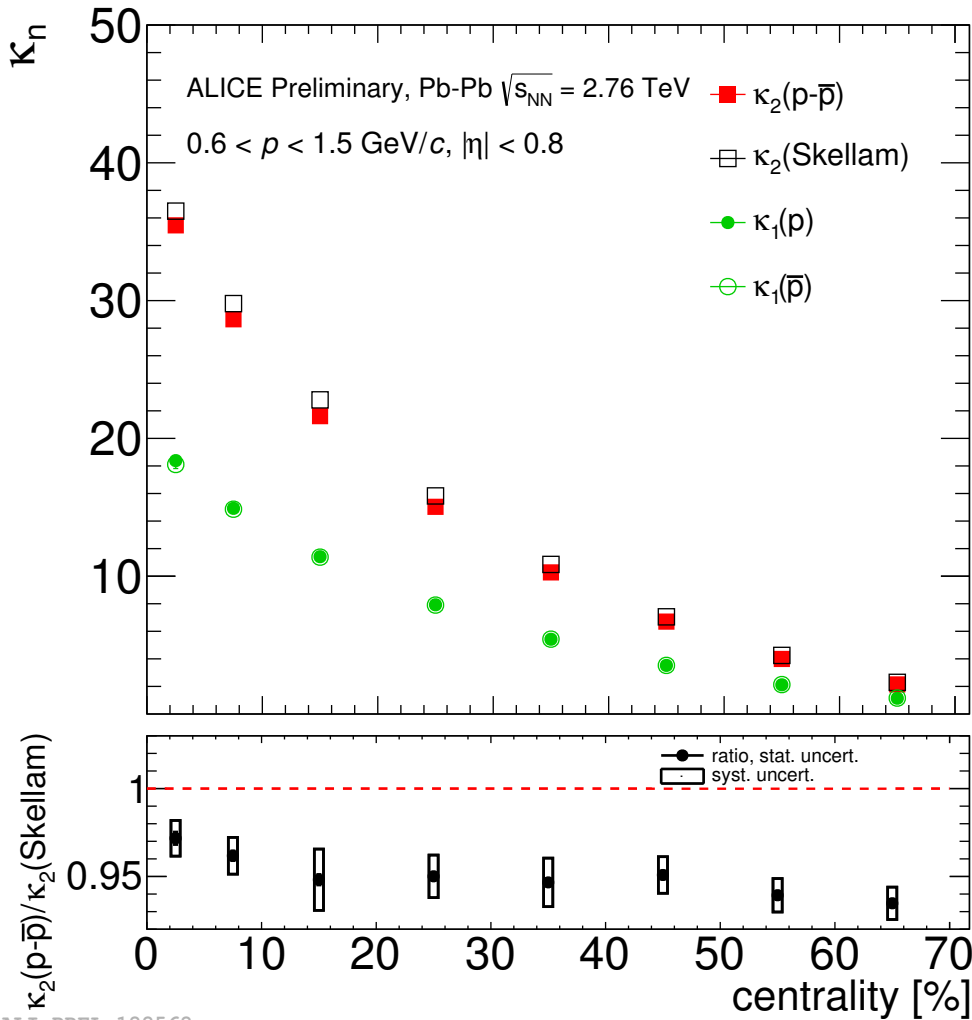
M. Gazdzicki et al., PRC 83, 054907 (2011)

M. I. Gorenstein, PRC 84, 024902 (2011)

A. Rustamov, M. I. Gorenstein, PRC 86, 044906 (2012)

Used in NA49, NA61/SHINE, ALICE

Net-protons



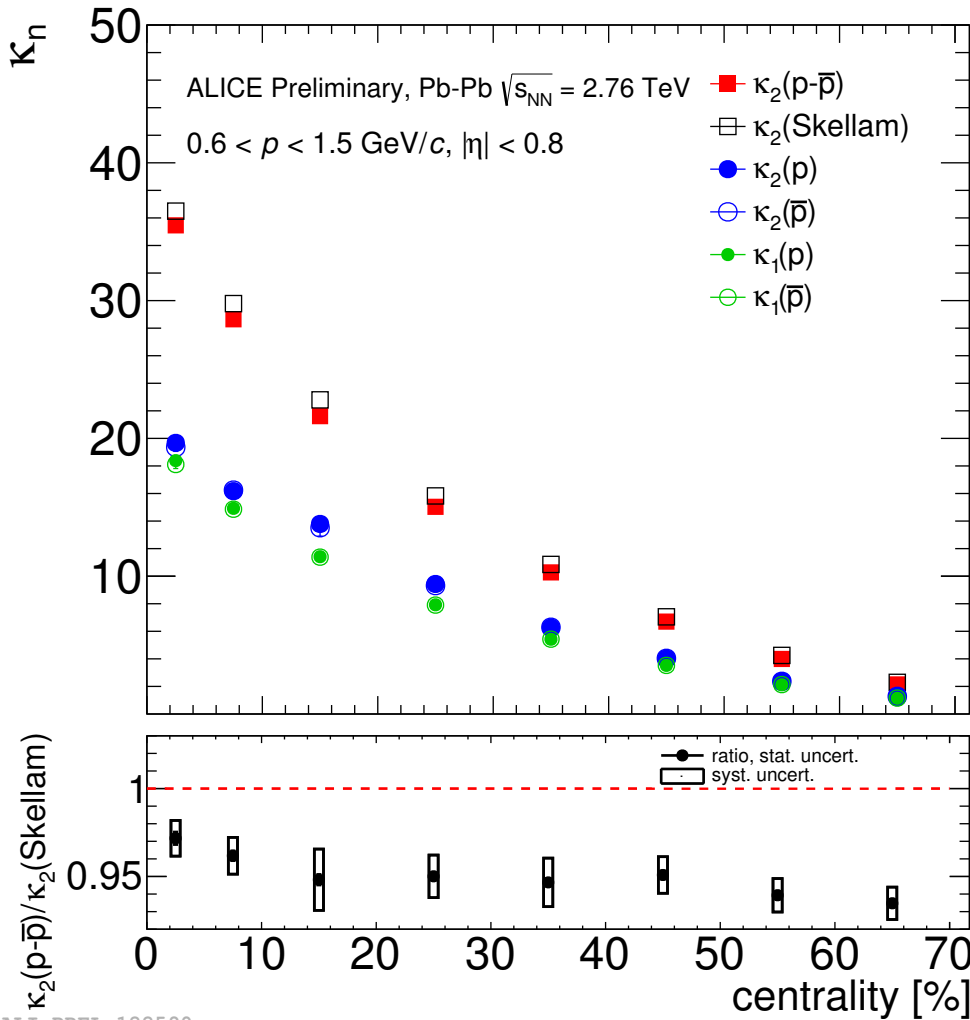
$$\kappa_2(p-\bar{p}) = \kappa_2(p) + \kappa_2(\bar{p}) - \underbrace{2(\langle p\bar{p} \rangle - \langle p \rangle \langle \bar{p} \rangle)}_{\text{correlation term}}$$

$$\kappa_2(\text{Skellam}) = \kappa_1(p) + \kappa_1(\bar{p})$$

$$\bullet + \circ = \square \neq \blacksquare$$

- correlation term?
- non Poisson (anti)protons?

Net-protons, protons, antiprotons



$$\kappa_2(p-\bar{p}) = \kappa_2(p) + \kappa_2(\bar{p}) - \underbrace{2(\langle p\bar{p} \rangle - \langle p \rangle \langle \bar{p} \rangle)}_{\text{correlation term}}$$

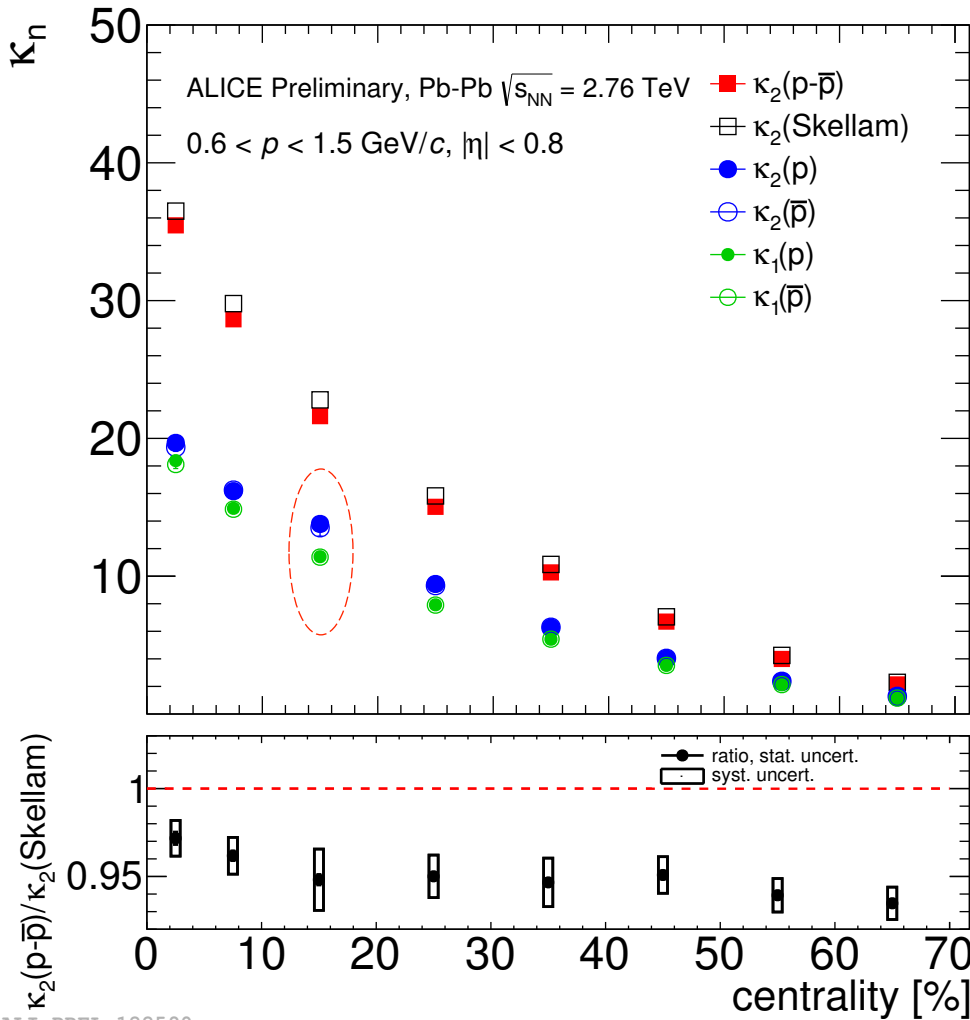
$$\kappa_2(\text{Skellam}) = \kappa_1(p) + \kappa_1(\bar{p})$$

$$\bullet + \circ = \square \neq \blacksquare$$

- correlation term?
- non Poisson (anti)protons?

$$\bullet, \circ \neq \bullet, \circ$$

Net-protons, protons, antiprotons



ALI-PREL-122590

$$\kappa_2(p-\bar{p}) = \kappa_2(p) + \kappa_2(\bar{p}) - \underbrace{2(\langle p\bar{p} \rangle - \langle p \rangle \langle \bar{p} \rangle)}_{\text{correlation term}}$$

$$\kappa_2(\text{Skellam}) = \kappa_1(p) + \kappa_1(\bar{p})$$

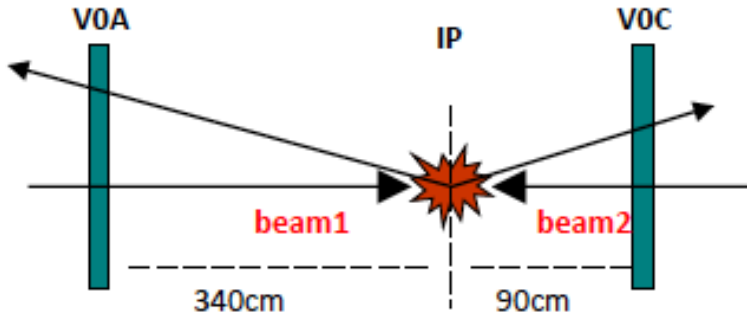
$$\bullet + \circ = \square \neq \blacksquare$$

- correlation term?
- non Poisson (anti)protons?

$$\bullet, \circ \neq \bullet, \circ$$

- more evident in the third centrality class
- participant fluctuations?

Centrality determination matters

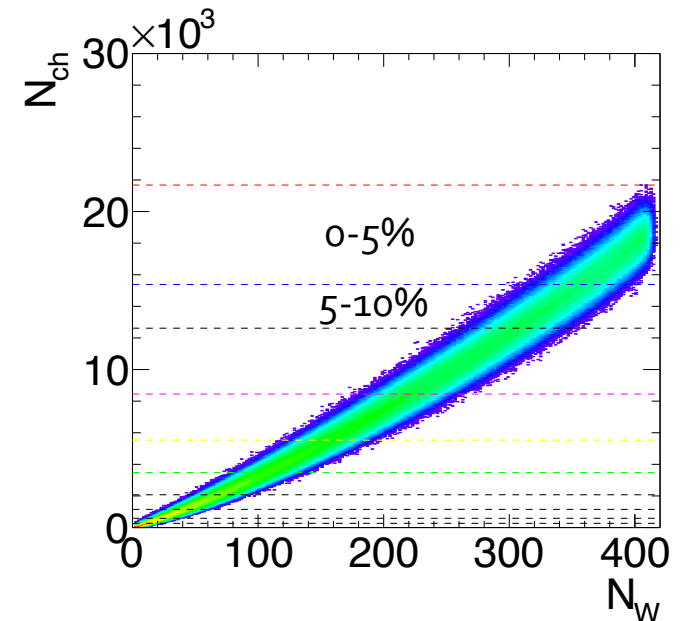
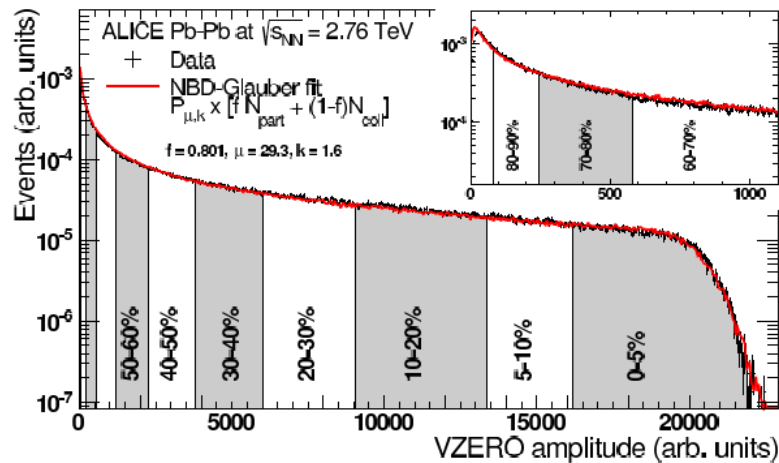


$$2.8 < \eta < 5.1 \quad \text{and} \quad -3.7 < \eta < -1.7$$

ALICE Phys.Rev. C88 (2013) no.4, 044909

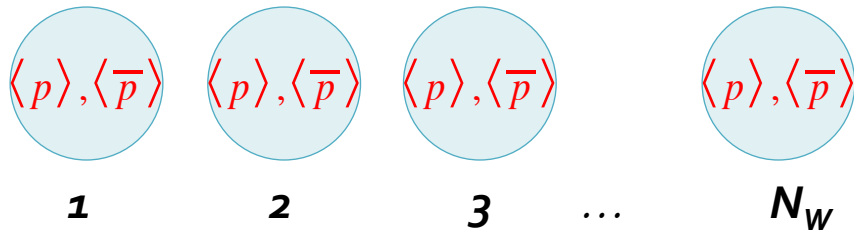
$$P_{\mu,k}(n) = \frac{\Gamma(n+k)}{\Gamma(n+1)\Gamma(k)} \frac{\left(\frac{\mu}{k}\right)^n}{\left(\frac{\mu}{k} + 1\right)^{n+k}}$$

$$N = fN_W + (1-f)N_{coll}$$

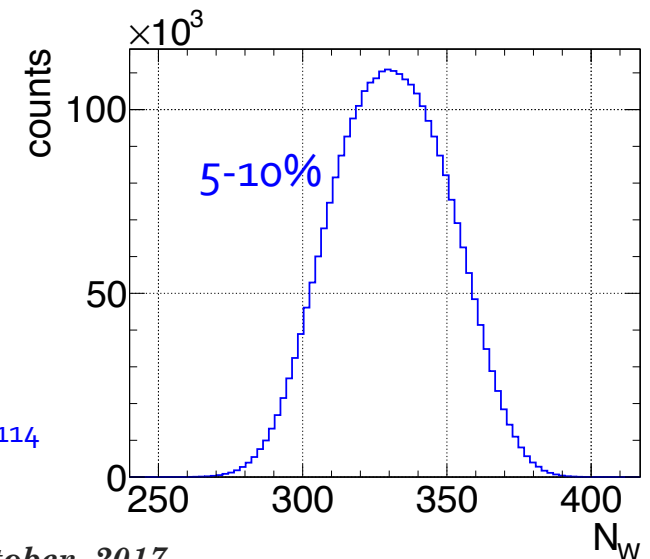
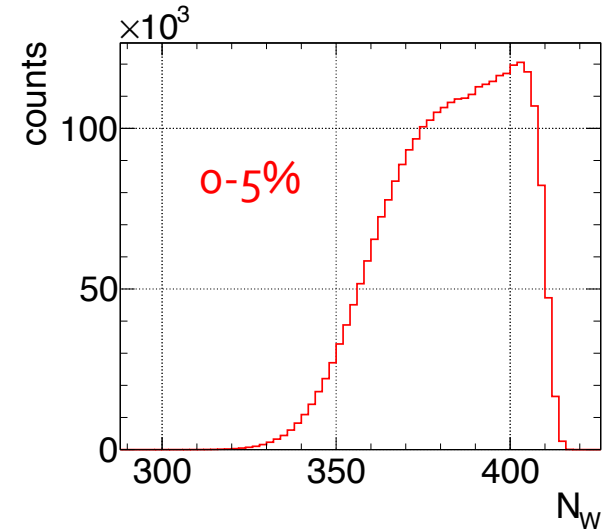


P. Braun-Munzinger, A. Rustamov, J. Stachel, arXiv:1612.00702, NPA in print

Participant (volume) fluctuations

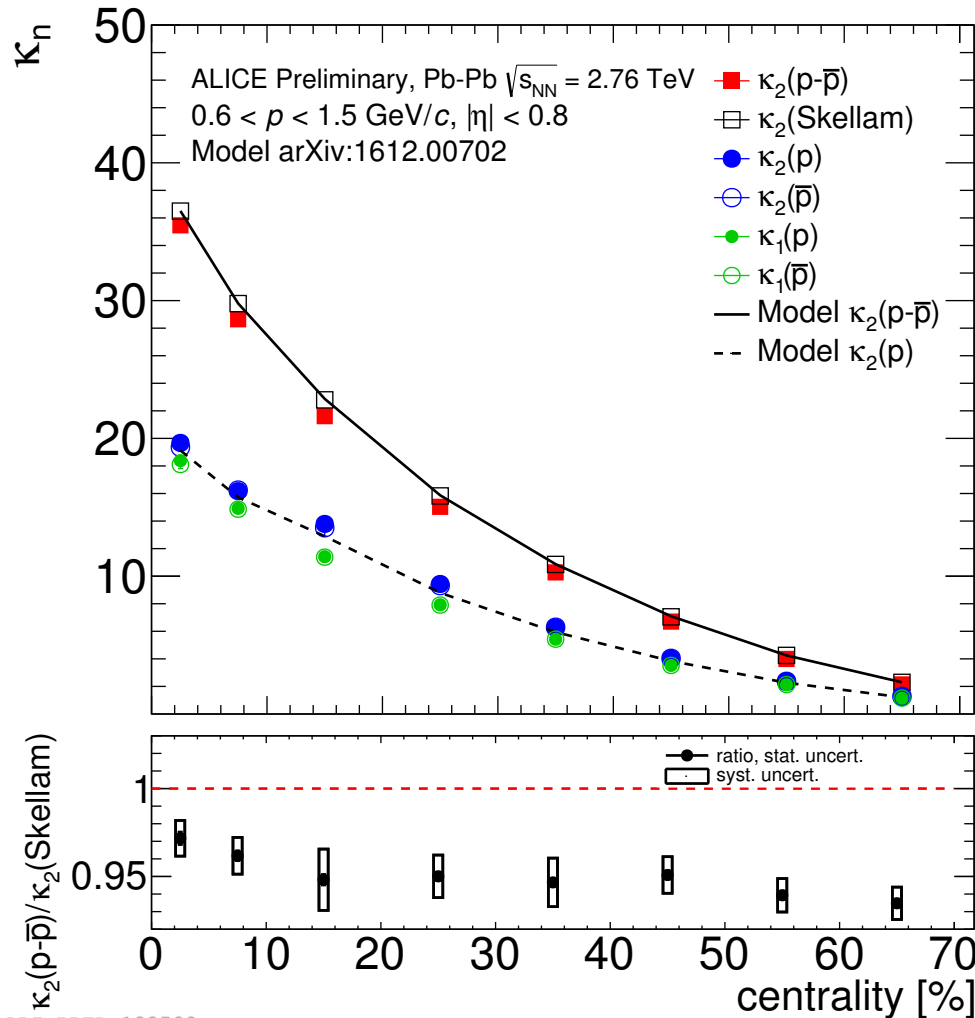


- ⊙ N_W fluctuates with MC Glauber initial conditions
- ⊙ Particles are produced from each source (GCE)
- ⊙ Inputs:
 - ⊙ Mean proton multiplicities $\langle p \rangle, \langle \bar{p} \rangle$
 - ⊙ Centrality selection like in experimental data



P. Braun-Munzinger, A. Rustamov, J. Stachel, arXiv:1612.00702, NPA 960 (2017) 114

Volume fluctuations



ALI-PREL-122598

Analysis of higher cumulants is ongoing!

$$\kappa_2(n_B - n_{\bar{B}}) = \kappa_2(n_B) + \kappa_2(n_{\bar{B}}) - 2(\langle n_B n_{\bar{B}} \rangle - \langle n_B \rangle \langle n_{\bar{B}} \rangle)$$

Input to the Model

$$\kappa_1(p), \kappa_1(\bar{p})$$

centrality selection procedure

Predictions

$$\text{—} \quad \kappa_2(p-\bar{p})$$

$$\text{- - -} \quad \kappa_2(p)$$

participants

vanishes at LHC

$$\kappa_2(N_B - N_{\bar{B}}) = \langle N_W \rangle \kappa_2(n_B - n_{\bar{B}}) + \langle n_B - n_{\bar{B}} \rangle^2 \kappa_2(N_W)$$

from single participant

Second cumulants of net-particles at LHC are not affected by participant fluctuations
easy control of systematics

Acceptance dependence

Contribution from global baryon number conservation

$$\frac{\kappa_2(p-\bar{p})}{\kappa_2(\text{Skellam})} = 1 - \alpha \quad \alpha = \frac{\langle p \rangle^{\text{measured}}}{\langle B \rangle^{4\pi}}$$

P. Braun-Munzinger, A. Rustamov, J. Stachel,
arXiv:1612.00702, NPA 960 (2017) 114

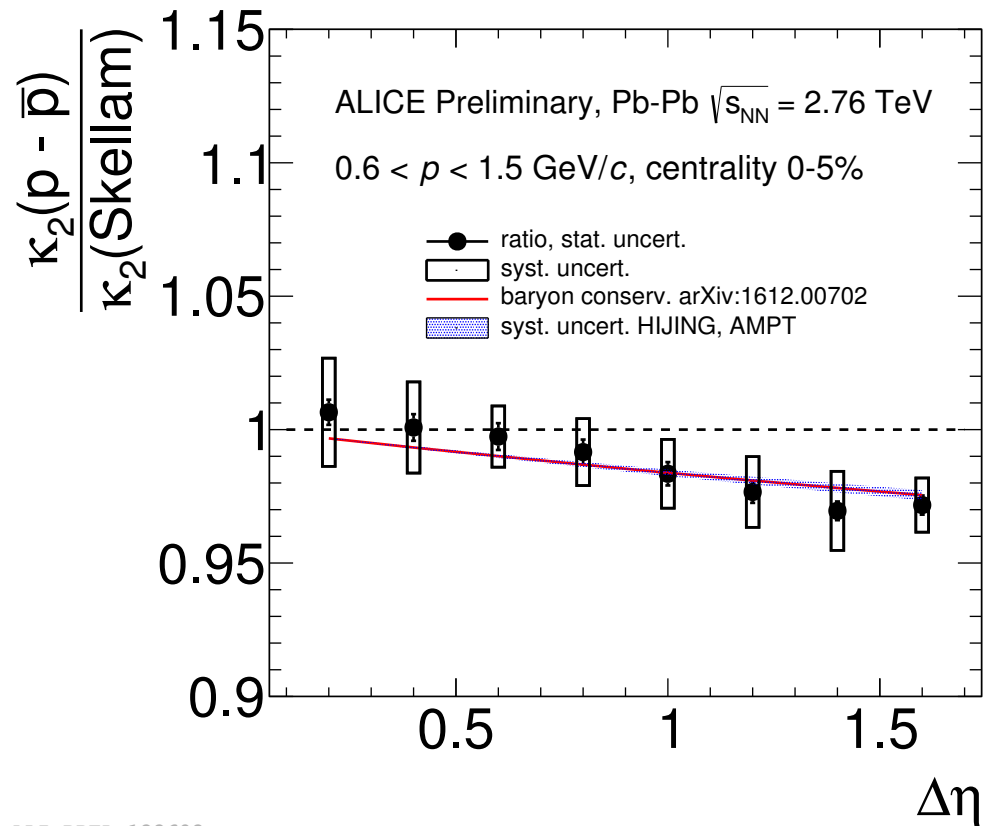
Inputs for $\langle B \rangle^{\text{acc}}$ from:

Phys. Lett. B 747, 292 (2015)
P. Braun-Munzinger, A. Kalweit, K. Redlich, J. Stachel

extrapolation from $\langle B \rangle^{\text{acc}}$ to $\langle B \rangle^{4\pi}$

using HIJING and AMPT models

A. Rustamov, QM2017, arXiv:1704.05329

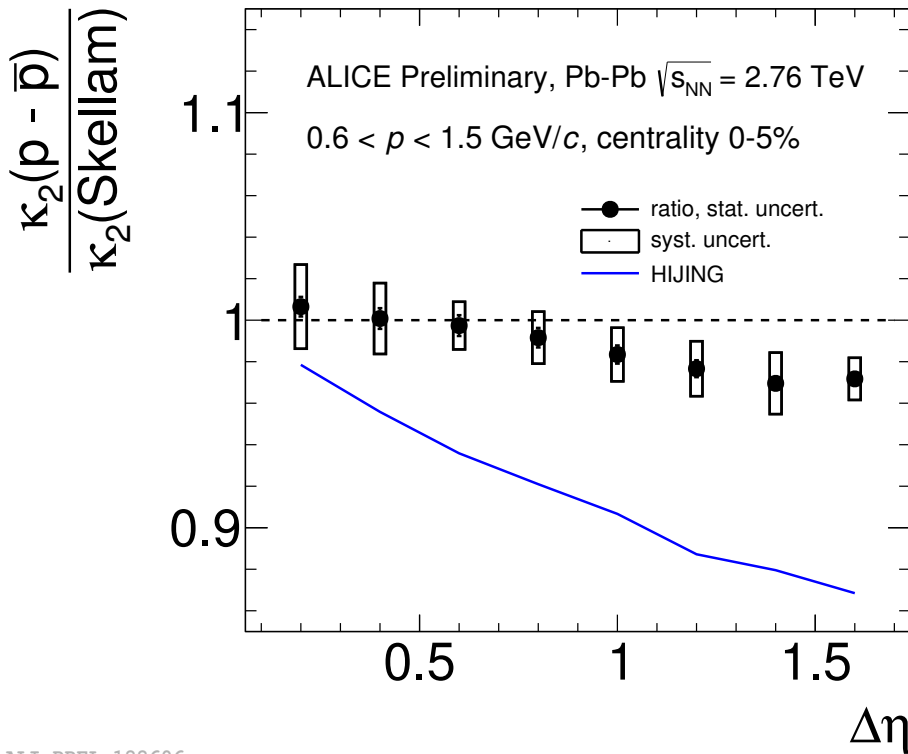


ALI-PREL-122602

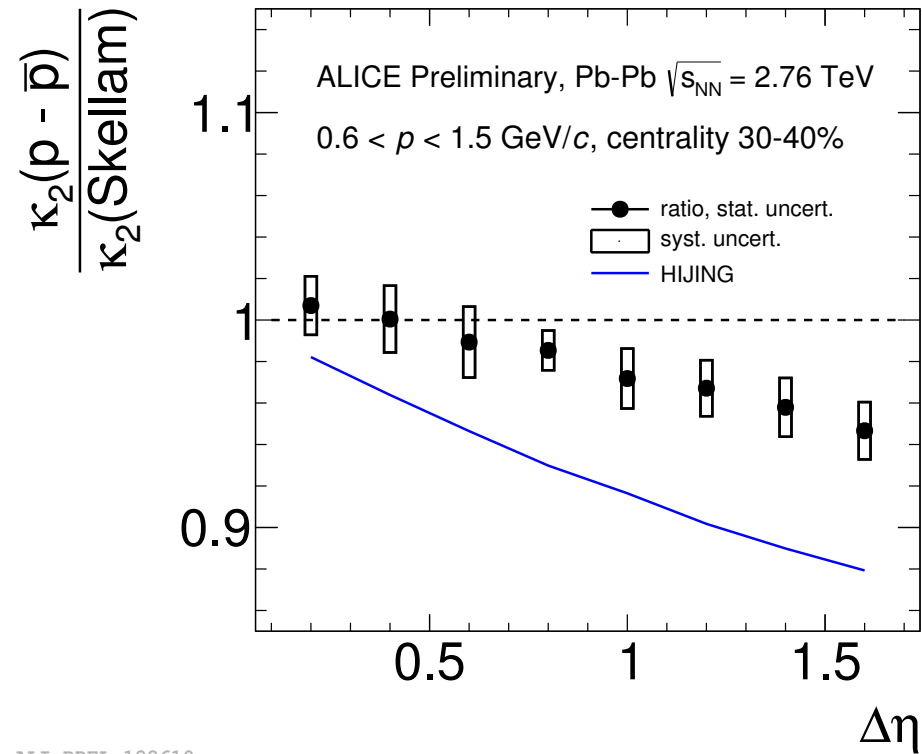
The deviation from Skellam is due to the global baryon number conservation.

Analysis of higher cumulants is ongoing!

Acceptance and centrality dependence



ALI-PREL-122606

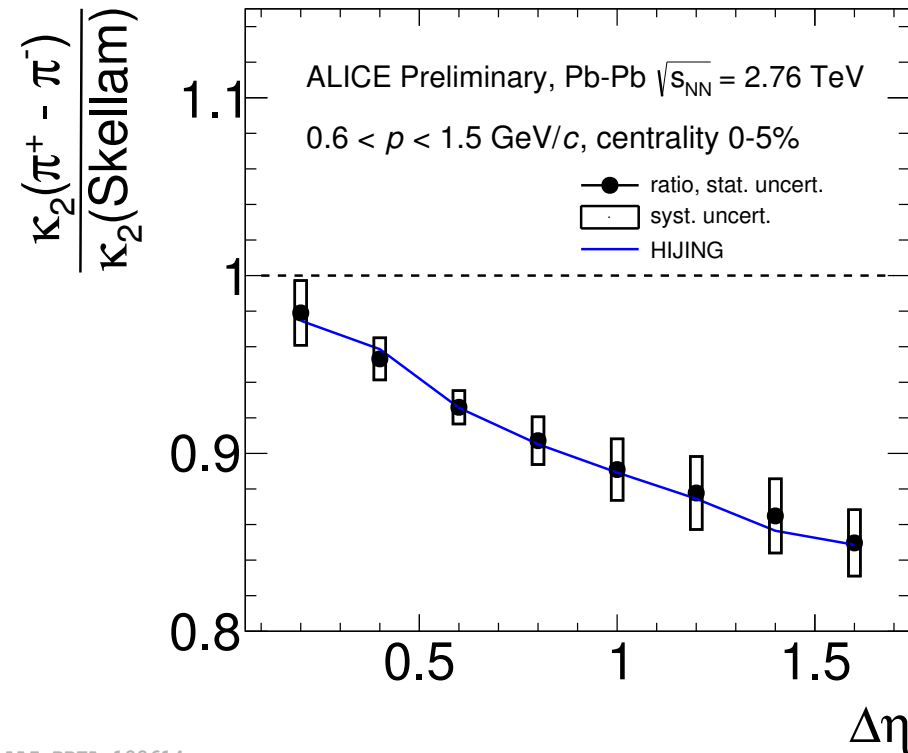


ALI-PREL-122610

Not properly described in most string fragmentation based event generators

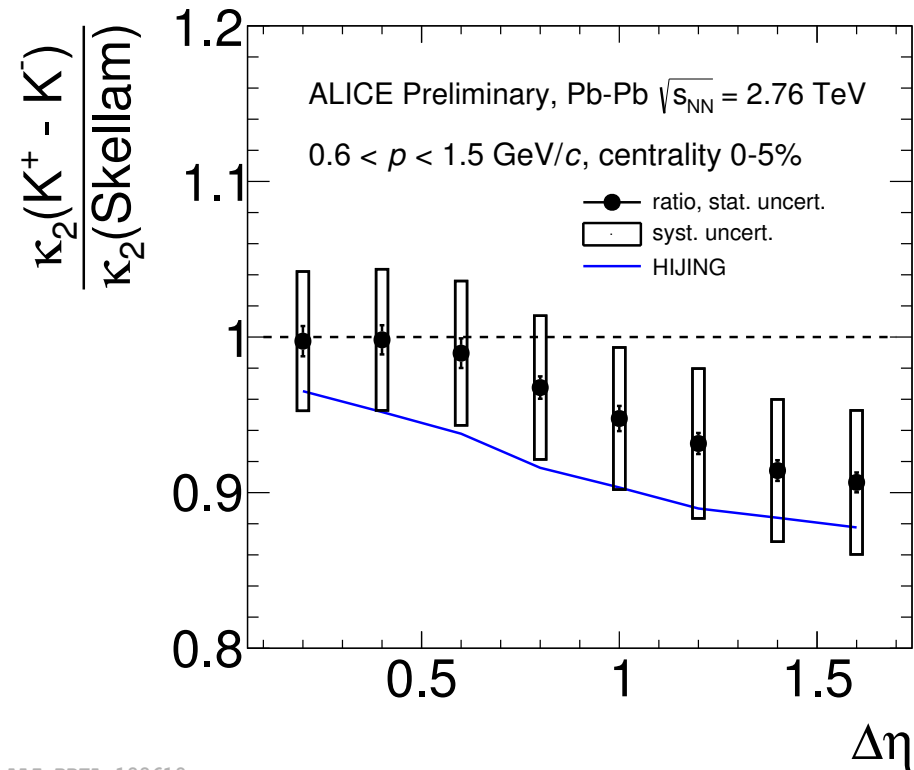
Net-pions and Net-kaons

net-pions



perfect agreement with HIJING

net-kaons



reasonable agreement with HIJING

resonance pion and kaon production is likely to explain the measured trend

Warning: Skellam is not a proper baseline for net-pions and net-kaons

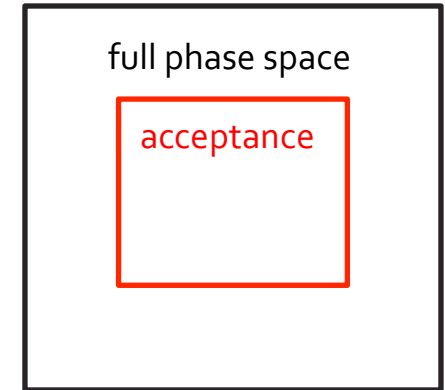
Effects of conservation laws on higher moments

Approach reported before

1. Poisson distributions for both baryons and anti-baryons in 4π
2. Folding into acceptance with binomial distribution
3. Counting net-baryons inside and outside acceptance

$P_1(n_1)$ – probability to measure n_1 net-baryons inside acceptance

$P_2(n_2)$ – probability to measure n_2 net-baryons outside acceptance



Summing over net-baryons outside acceptance by imposing conservation in the full phase space

$$P_B(n_1) = \sum_{n_2} P_1(n_1) P_2(n_2) \delta(n_1 + n_2 - B)$$

Input to the model:

number of baryons and anti-baryons in 4π

experimental acceptance

Our approach: MC implementation of canonical ensemble

Two baryon species with the baryon numbers +1 and -1 in the ideal Boltzmann gas

$$Z_{GCE}(V, T, \mu) = \sum_{N_B=0}^{\infty} \sum_{N_{\bar{B}}=0}^{\infty} \frac{(\lambda_B z)^{N_B}}{N_B!} \frac{(\lambda_{\bar{B}} z)^{N_{\bar{B}}}}{N_{\bar{B}}!} = e^{2z \cosh\left(\frac{\mu}{T}\right)}, \quad \lambda_{B, \bar{B}} = e^{\pm \frac{\mu}{T}}$$

$$Z_{GC}(V, T, B) = \sum_{N_B=0}^{\infty} \sum_{N_{\bar{B}}=0}^{\infty} \frac{(\lambda_B z)^{N_B}}{N_B!} \frac{(\lambda_{\bar{B}} z)^{N_{\bar{B}}}}{N_{\bar{B}}!} \delta(N_B - N_{\bar{B}} - B) = I_B(2z) \Big|_{\lambda_B = \lambda_{\bar{B}} = 1}$$

$$\langle N_{B, \bar{B}} \rangle_{GCE} = \lambda_{B, \bar{B}} \frac{\partial \ln Z_{GCE}}{\partial \lambda_{B, \bar{B}}} = e^{\pm \frac{\mu}{T}} z, \quad z = \sqrt{\langle N_B \rangle_{GCE} \langle N_{\bar{B}} \rangle_{GCE}}$$

$$\langle N_{B, \bar{B}} \rangle_{CE} = \sqrt{\langle N_B \rangle_{GCE} \langle N_{\bar{B}} \rangle_{GCE}} \frac{I_{B \mp 1} \left(2 \sqrt{\langle N_B \rangle_{GCE} \langle N_{\bar{B}} \rangle_{GCE}} \right)}{I_B \left(2 \sqrt{\langle N_B \rangle_{GCE} \langle N_{\bar{B}} \rangle_{GCE}} \right)}$$

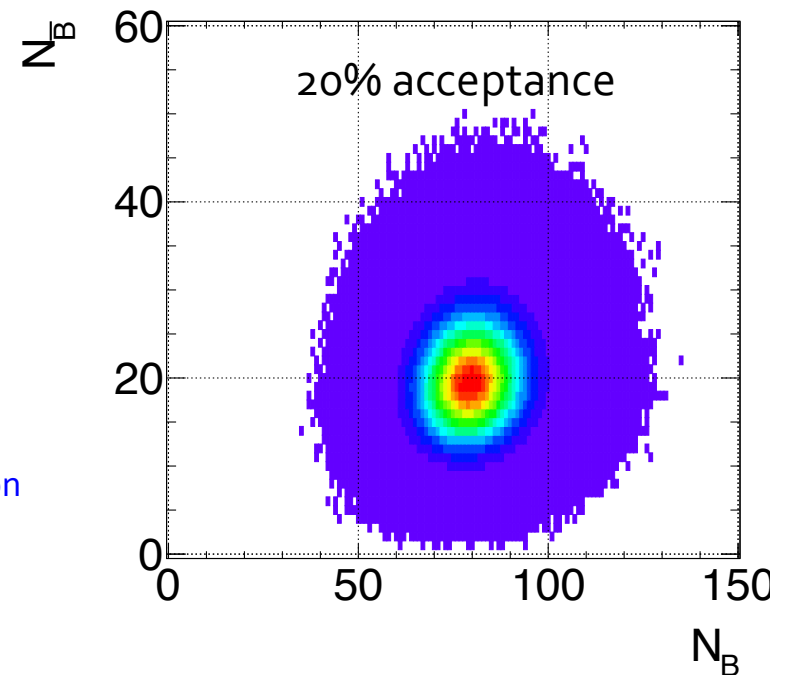
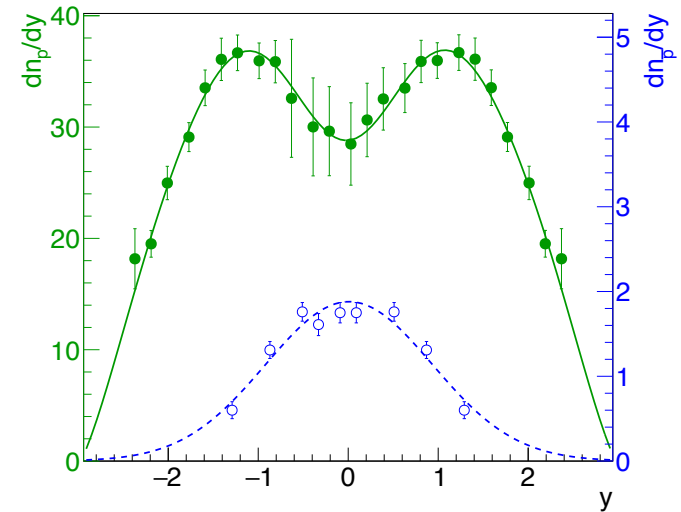
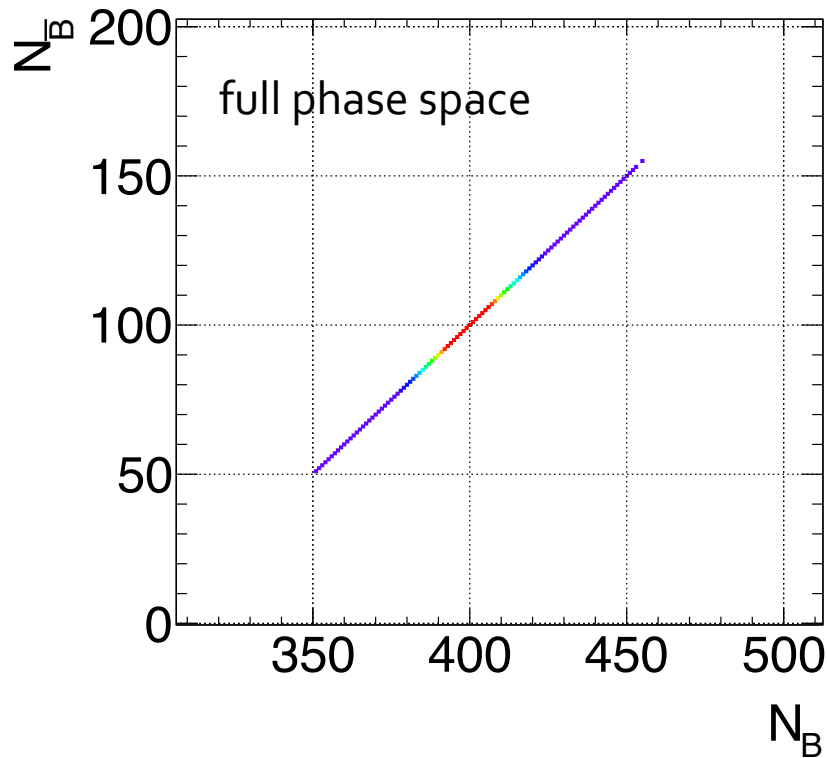
V.V. Begun, M. I. Gorenstein, O. S. Zozulya, PRC 72 (2005) 014902

M. I. Gorenstein, W. Greiner, A. R., PLB 731 (2014) 302-306

P. Braun-Munzinger, B. Friman, F. Karsch, K. Redlich, V. Skokov, NPA 880 (2012)

Technical details

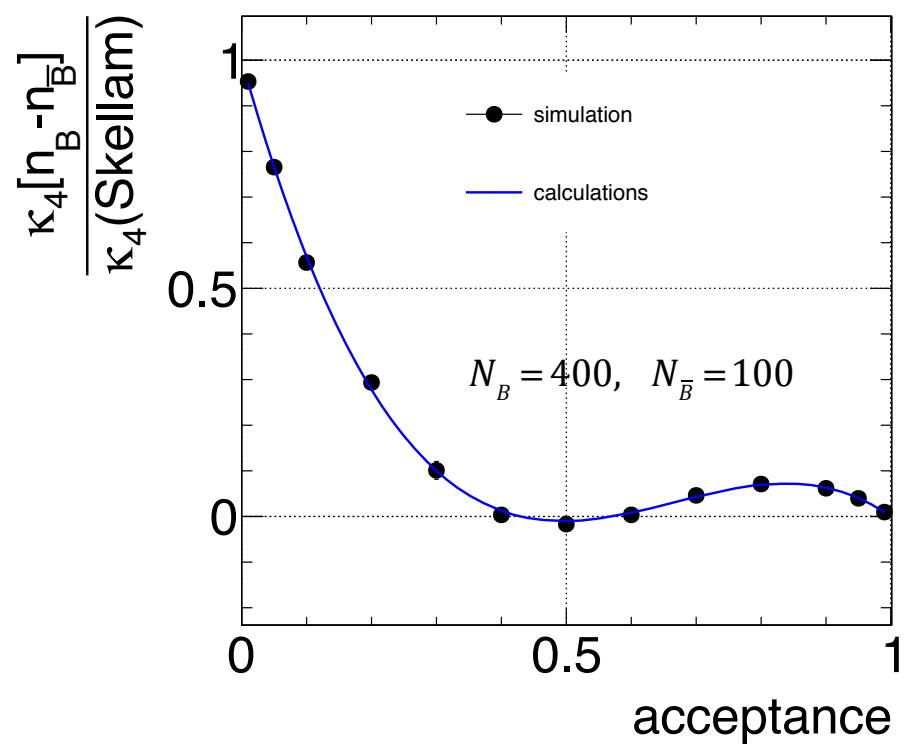
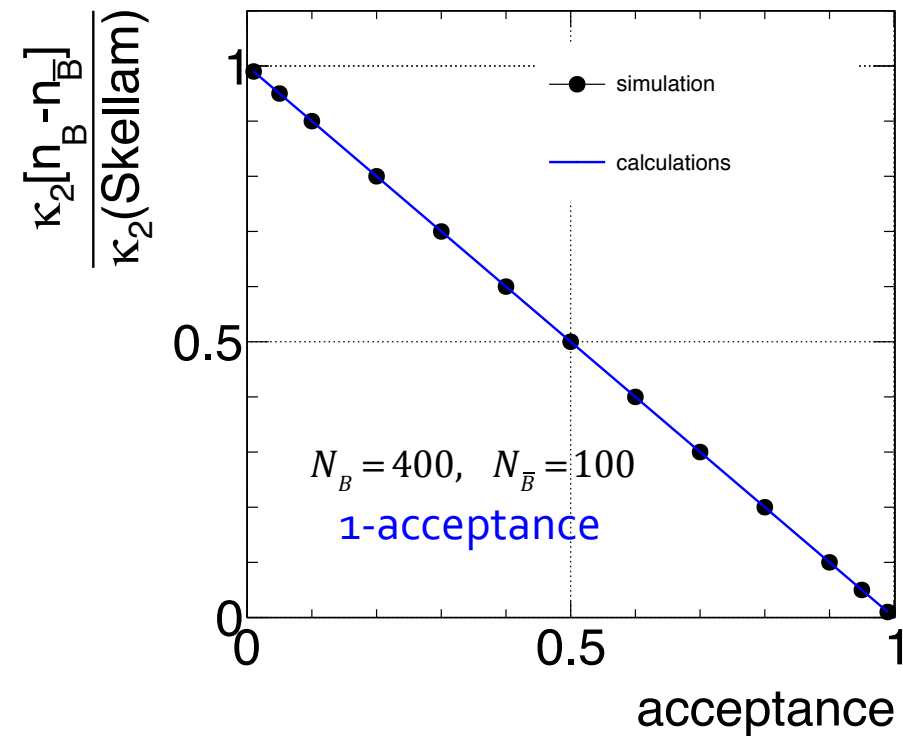
$$N_B = 400, \quad N_{\bar{B}} = 100$$



⊙ Fluctuations of net-baryons appear only inside finite acceptance

P. Braun-Munzinger, A. Rustamov, J. Stachel, in preparation

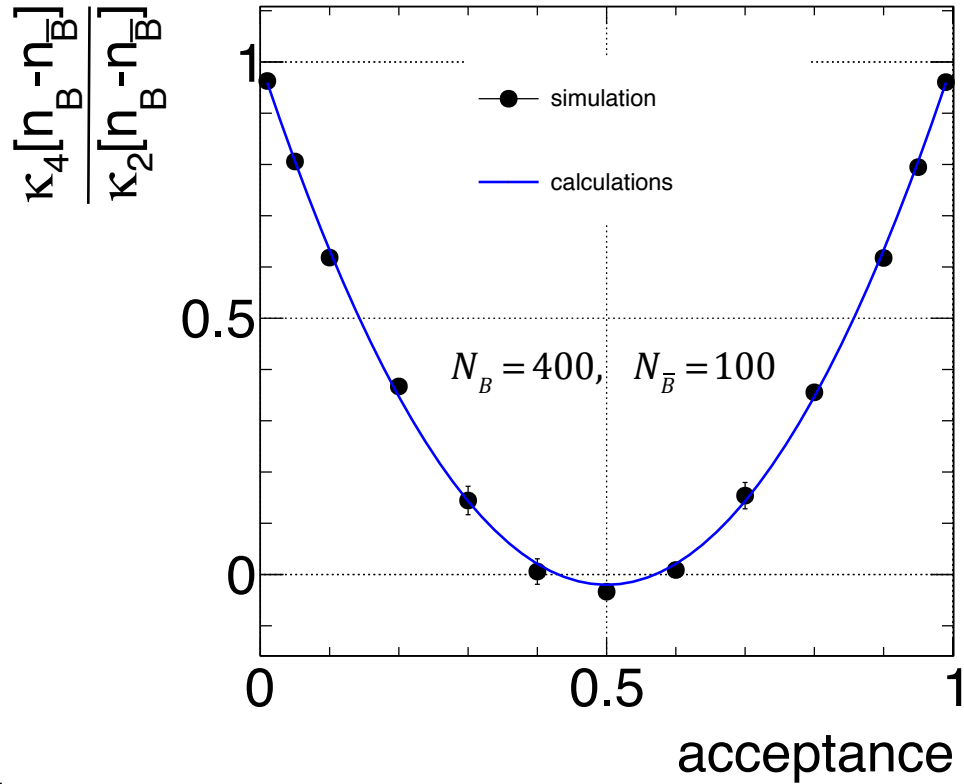
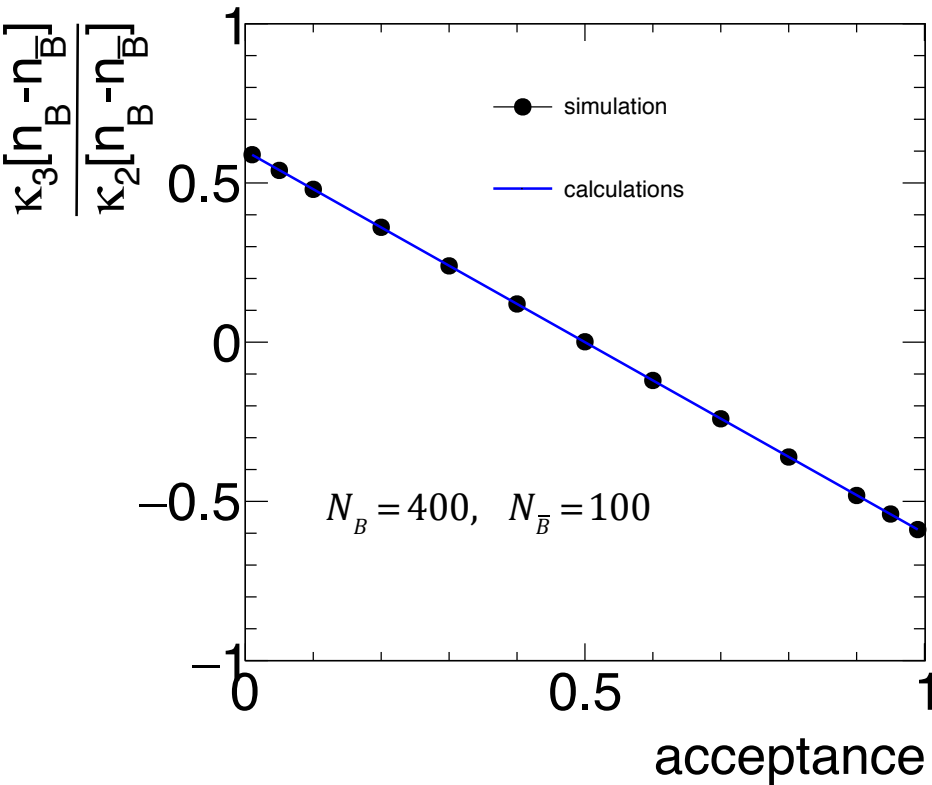
Importance of acceptance



- ⊙ Approach to independent Poisson (Skellam) for a small acceptance
- ⊙ Approach to zero for full acceptance
- ⊙ Acceptance is more crucial for the 4th cumulant

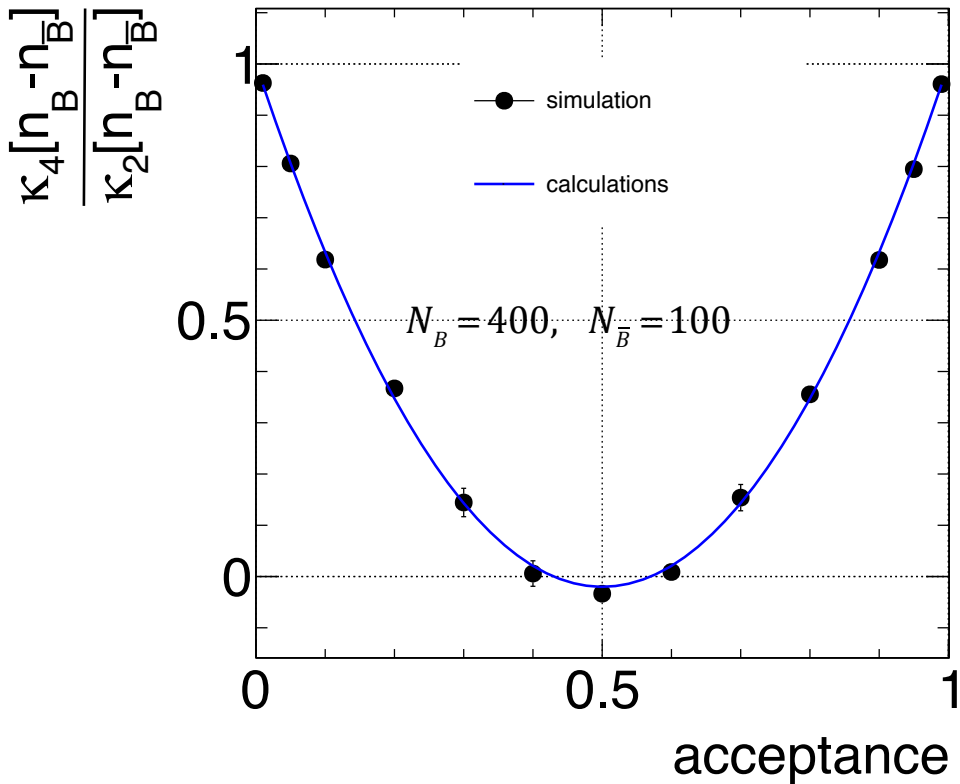
Third and fourth cumulants

$$\frac{\kappa_3}{\kappa_2} = \frac{\langle n_B - n_{\bar{B}} \rangle_{CE}}{\langle n_B + n_{\bar{B}} \rangle_{CE}} (1 - 2\alpha) \xrightarrow{\langle n_{\bar{B}} \rightarrow 0} (1 - 2\alpha)$$

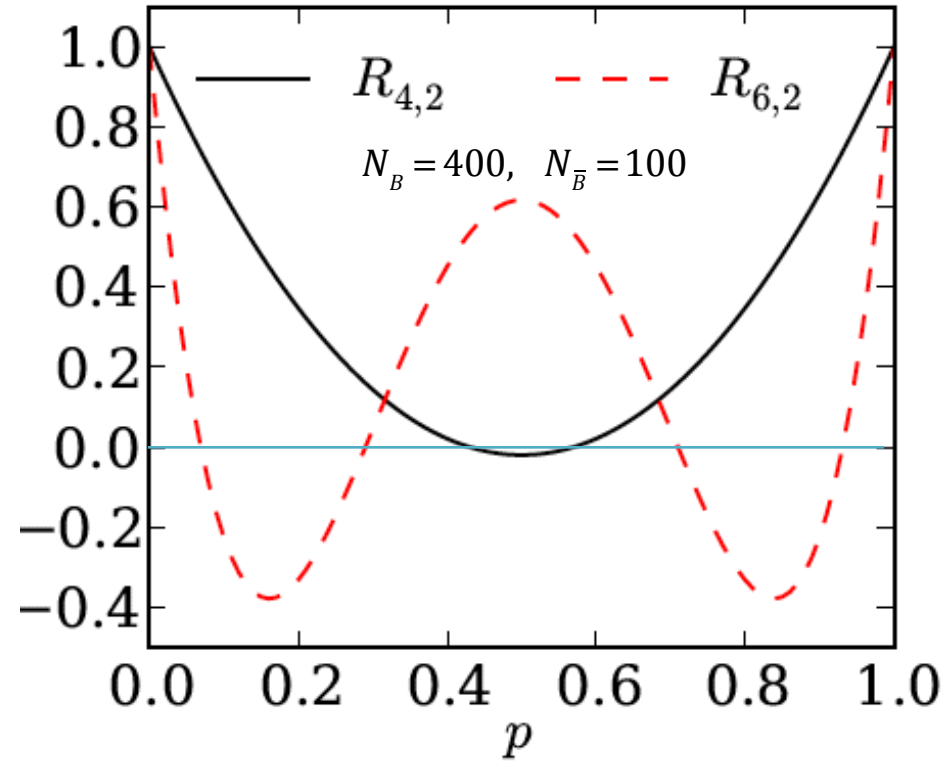


$$\frac{\kappa_4}{\kappa_2} = 1 - 6\alpha(1 - \alpha) \left(1 - \frac{2}{\langle n_B + n_{\bar{B}} \rangle_{CE}} \left[\langle n_B \rangle_{GCE} \langle n_{\bar{B}} \rangle_{GCE} - \langle n_B \rangle_{CE} \langle n_{\bar{B}} \rangle_{CE} \right] \right)$$

Comparisons between the two



P. Braun-Munzinger, A. Rustamov, J. Stachel, in preparation

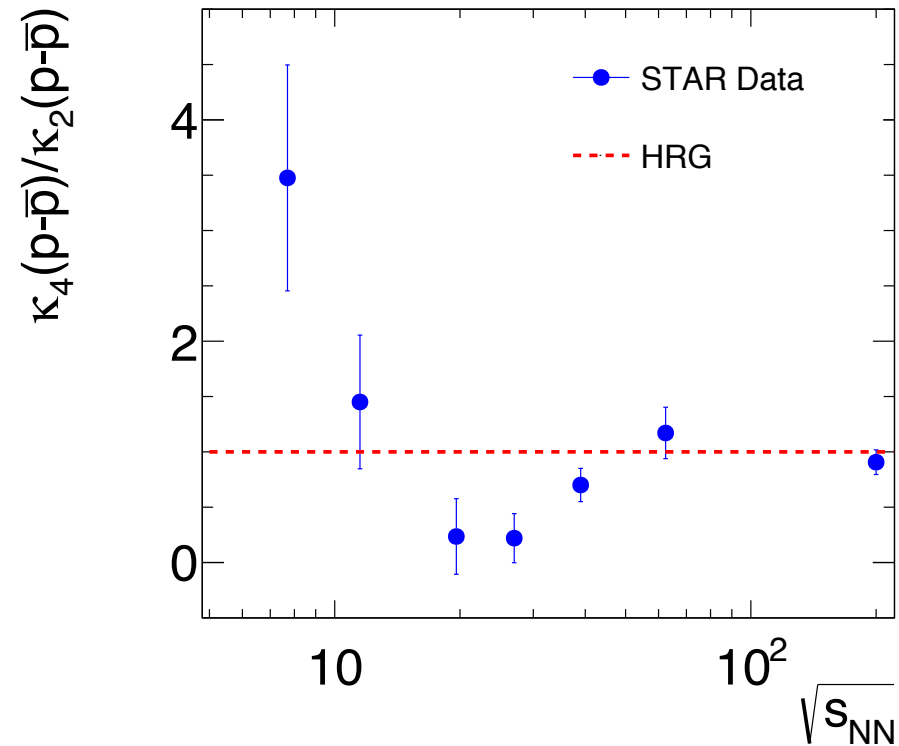
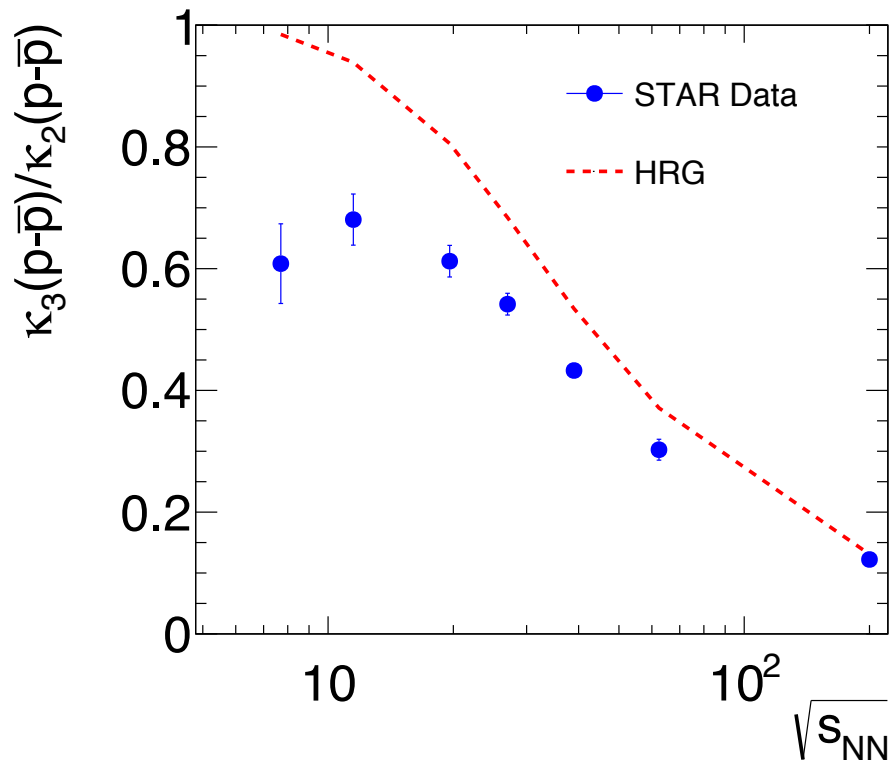


A. Bzdak, V. Koch, V. Skokov, PRC87 (2013) 014901

Same results in both approaches

Results from STAR, deviations from HRG

At low energies : opposite deviations from HRG baseline for 3rd and 4th cumulants!



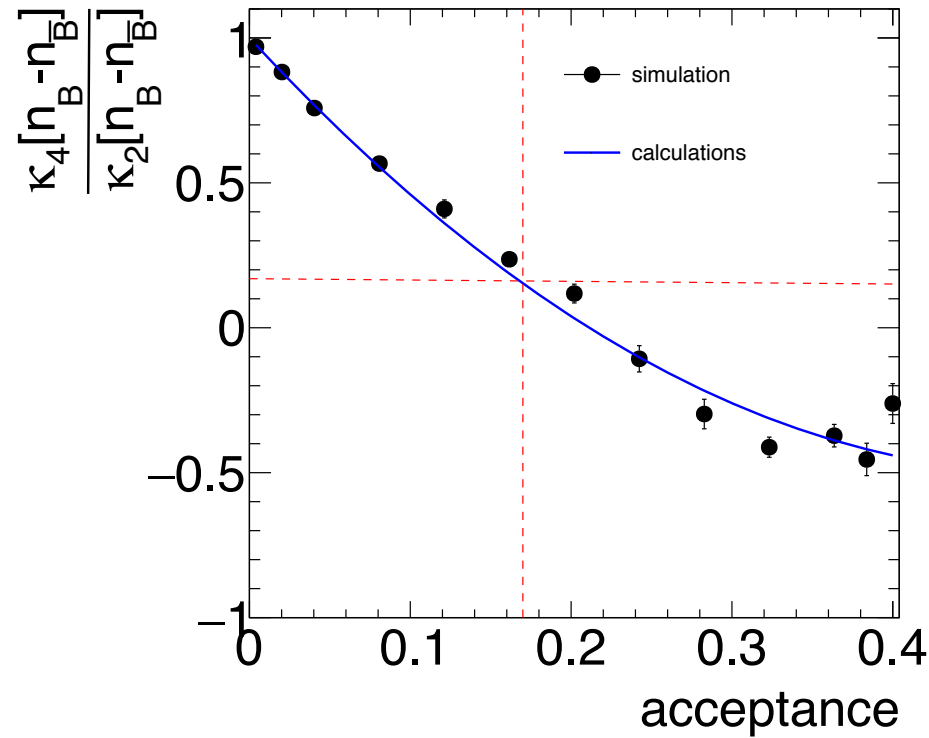
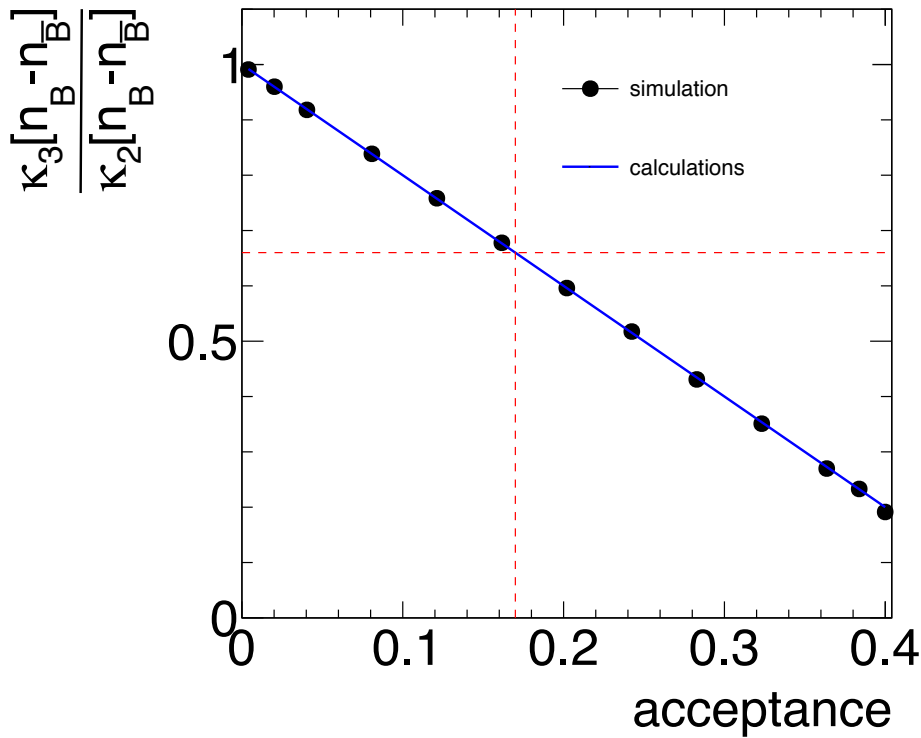
$$\frac{\kappa_3}{\kappa_2^2} = \frac{\langle n_B - n_{\bar{B}} \rangle_{CE}}{\langle n_B + n_{\bar{B}} \rangle_{CE}} (1 - 2\alpha), \quad \alpha_{\sqrt{s}=7.7\text{GeV}} = 0.19 \pm 0.03, \quad \alpha_{\sqrt{s}=19.6\text{GeV}} = 0.12 \pm 0.016$$

Based on data from X. Luo, PoS CPOD2014, 019 (2015)

40 GeV lab momentum, $\sqrt{s} \approx 8.8$ GeV



Input to the model (in 4π): $\langle N_B \rangle = 351$, $\langle N_{\bar{B}} \rangle = 0$, experimentally measured rapidity dist.

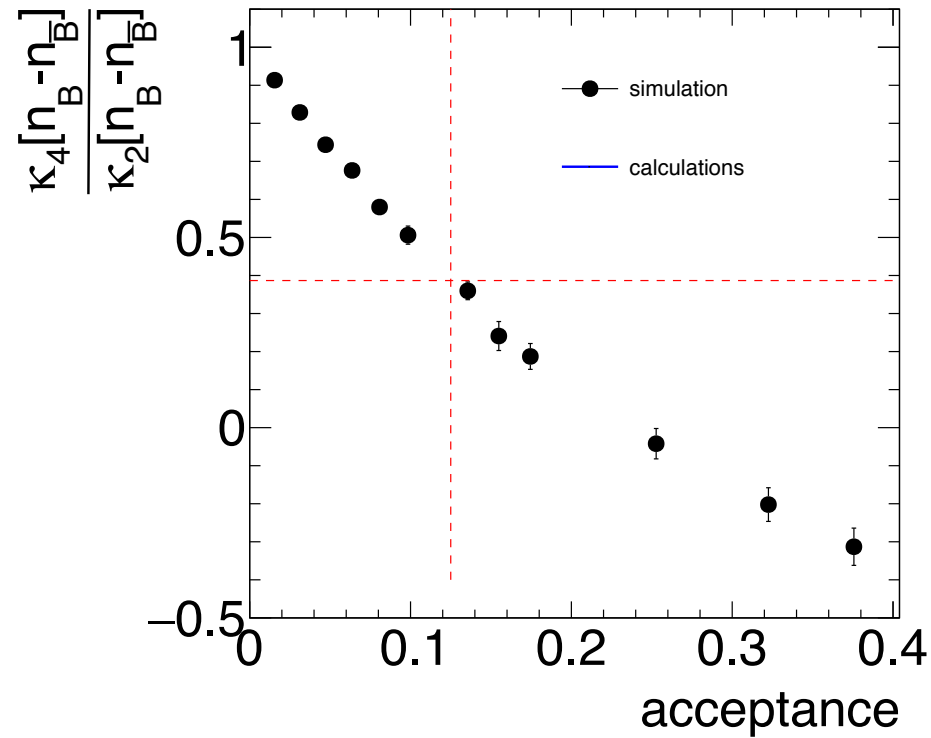
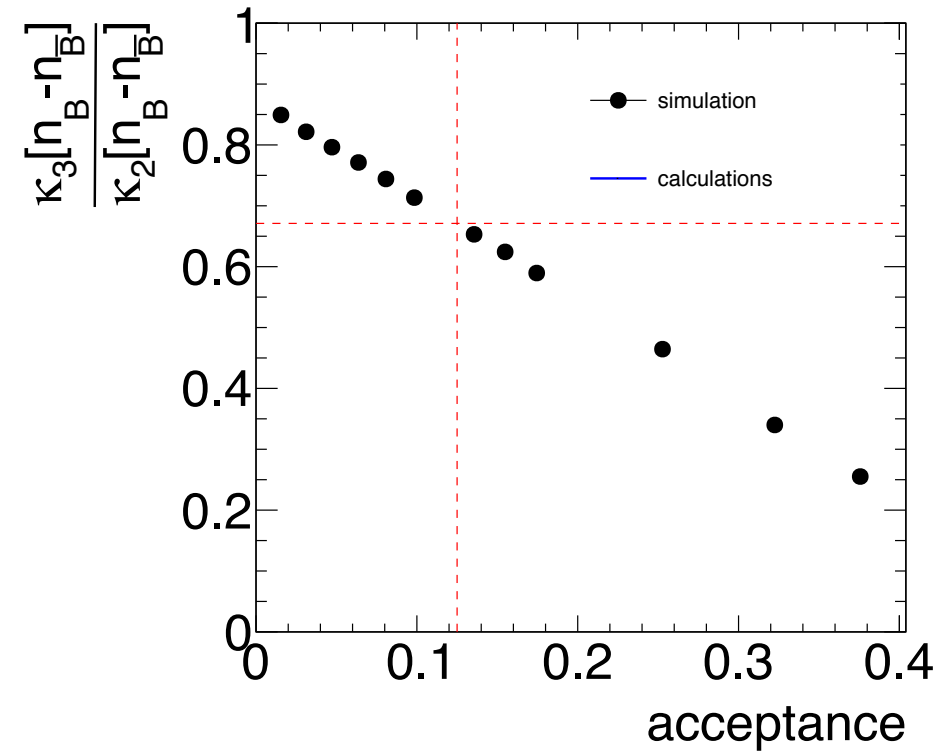


$\alpha_{\sqrt{s}=8.8\text{GeV}} = 0.17$ estimated from energy dependence

158 GeV lab momentum, $\sqrt{s} \approx 17.3$ GeV

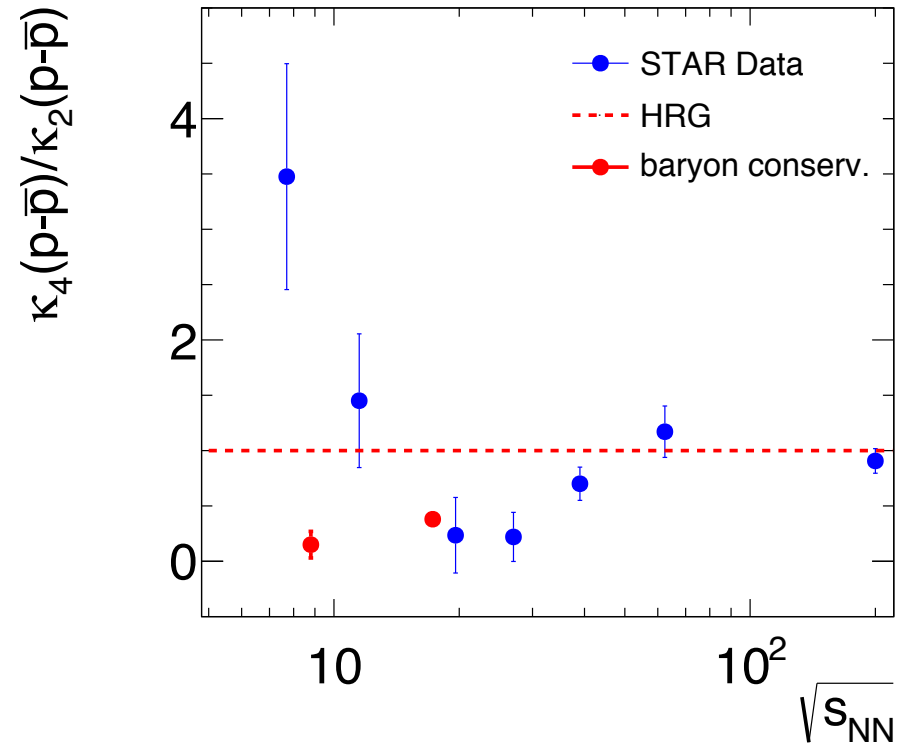
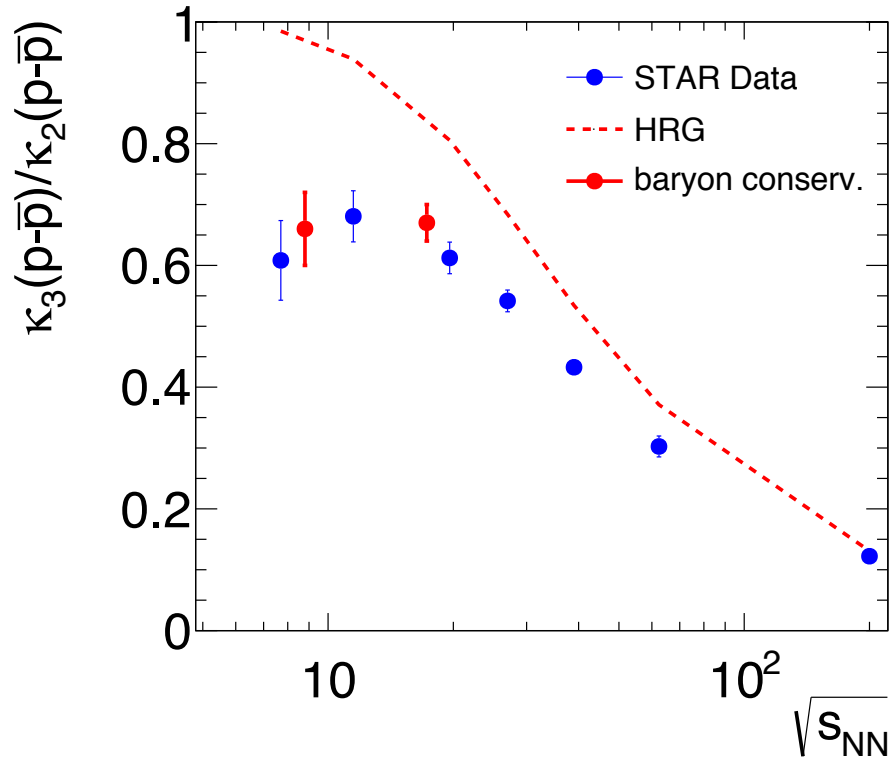


Input to the model (in 4π): $\langle N_B \rangle = 373$, $\langle N_{\bar{B}} \rangle = 20$, experimentally measured rapidity dist.



$\alpha_{\sqrt{s}=17.3\text{GeV}} = 0.125$ estimated from energy dependence

Our predictions



At 7.7 GeV k_3/k_2 and k_4/k_2 cannot be simultaneously explained

Summary

- ⊙ NA61/SHINE data show a significant jump in K^+/π^+ ratio from light to heavy systems
- ⊙ No anomalies are observed in the energy dependence of fluctuations for p+p, Be+Be and Ar+Sc.
- ⊙ The measured second cumulants of net-protons at ALICE are, after accounting for baryon number conservation, in agreement with the corresponding second cumulants of the Skellam distribution.
 - ⊙ LQCD predicts a Skellam behavior for κ_2 of net-baryons at 150 MeV.
- ⊙ Net-proton measurements from STAR hint for a non-monotonic behavior for energies below 39 GeV. More statistics and control of systematics are needed.
- ⊙ At 7.7 GeV the energy dependence of k_3/k_2 and k_4/k_2 of net-protons from STAR cannot be simultaneously described by baryon-number conservation
- ⊙ At 19.6 GeV both k_3/k_2 and k_4/k_2 are in agreement with the baryon number conservation

The analysis of higher cumulants are ongoing in ALICE, which is extremely important for understanding the nature of transition at vanishing μ_B