Making sense of (anti)proton cumulants and correlation functions at RHIC

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Outline

STAR results

cumulants, correlation functions

volume fluctuation

baryon conservation

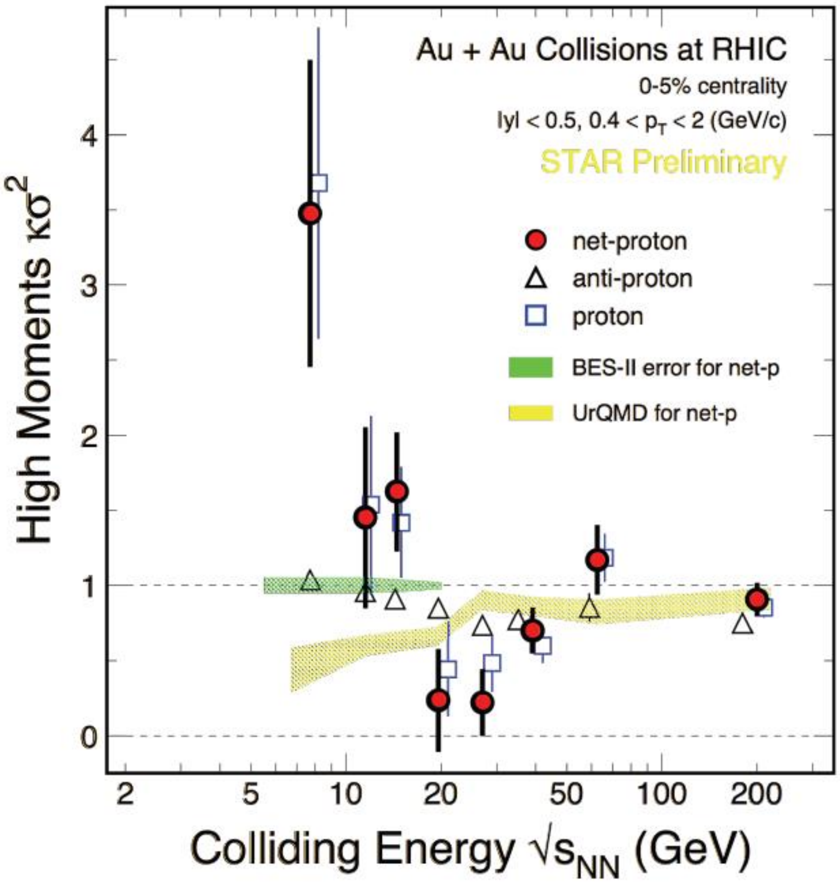
rapidity dependence

STAR data from *ordinary* multiplicity distributions

conclusions

Preliminary STAR data

X.Luo, N.Xu, 1701.02105



my notation

$$K_4/K_2$$

Is proton signal at 7.7 GeV large?

Is antiproton signal at 7.7 GeV small?

Is proton signal at 200 GeV small?

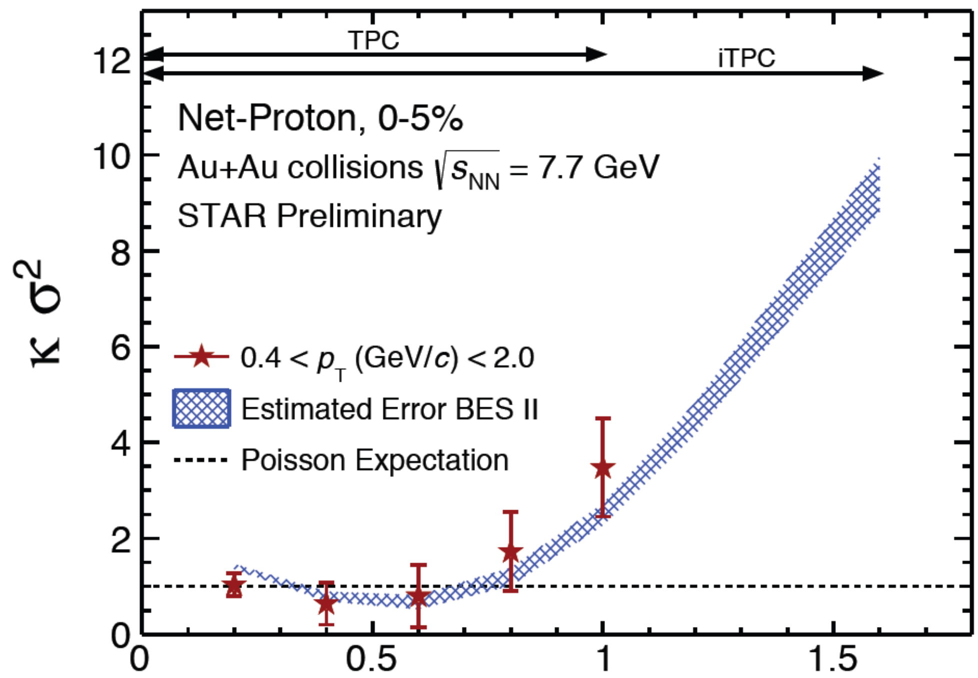
Can we and how to directly compare different energies?

STAR Preliminary at 7.7 GeV

my notation

K_4/K_2

X.Luo, N.Xu, 1701.02105



Δy (rapidity window size)

$$-(\Delta y)/2 < y < (\Delta y)/2$$

Is this dependence expected?

Is it somehow related to the QCD phase diagram?

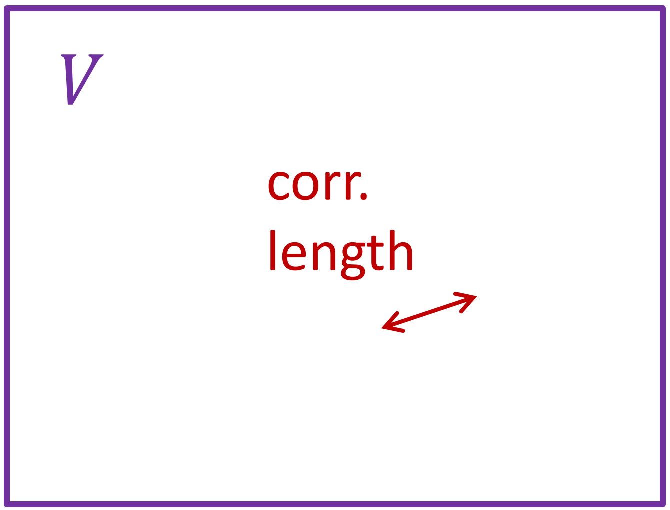
General remarks:

"Cumulant ratios do not depend on volume"

but depend on volume fluctuation

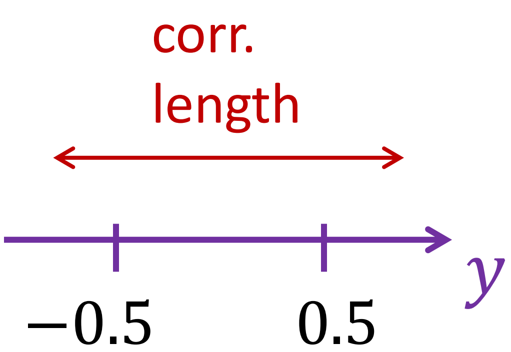
It is true if a correlation length is much smaller than the system size

real coordinate space



Here this condition is satisfied

momentum rapidity space



Correlation length is usually larger than one unit of rapidity.

Cumulant ratios are expected to depend on rapidity "volume"

Cumulants are not optimal

$K_2 = \langle (\delta N)^2 \rangle$ $\delta N = N - \langle N \rangle$ N – number of protons

we neglect anti-protons,
good at low energies

$K_3 = \langle (\delta N)^3 \rangle$

$K_4 = \langle (\delta N)^4 \rangle - 3\langle (\delta N)^2 \rangle^2$

$$K_n = \langle N \rangle + physics[2, \dots, n]$$

$physics$ = two-, three-, n-particle correlation functions

for Poisson distribution $K_n = \langle N \rangle$, $(physics = 0)$

We have

$$K_2 = \langle N \rangle + \boldsymbol{C_2}$$

$$K_3 = \langle N \rangle + 3\boldsymbol{C_2} + \boldsymbol{C_3}$$

$$K_4 = \langle N \rangle + 7\boldsymbol{C_2} + 6\boldsymbol{C_3} + \boldsymbol{C_4}$$

cumulants mix correlation functions of different orders

For example:

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2) + \boldsymbol{C_2}(y_1, y_2)$$

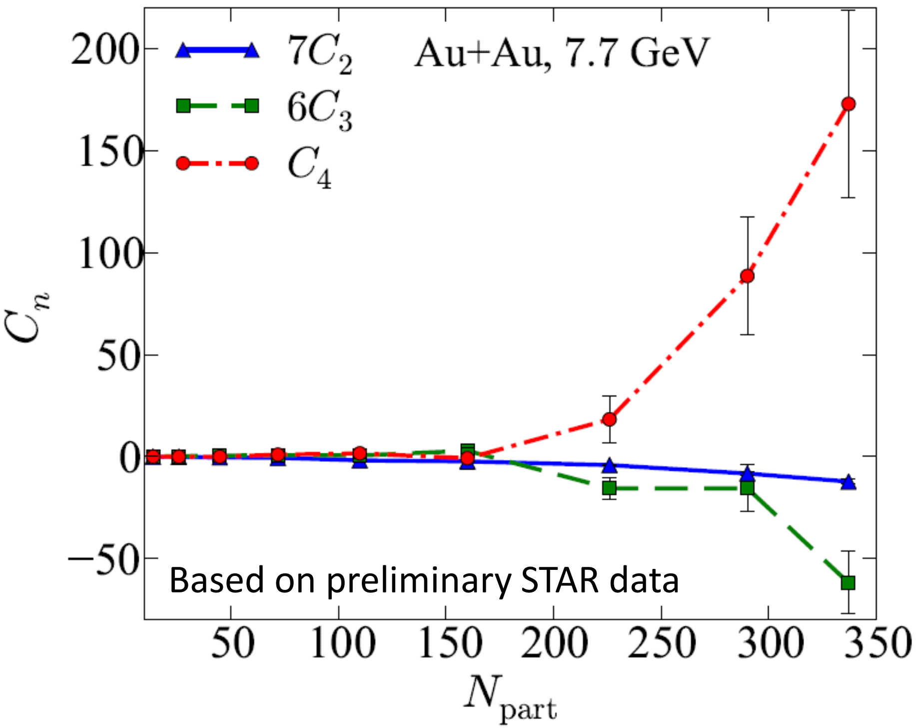
$$\boldsymbol{C_2} = \int \boldsymbol{C_2}(y_1, y_2)\, dy_1 dy_2$$

integrated correlation function

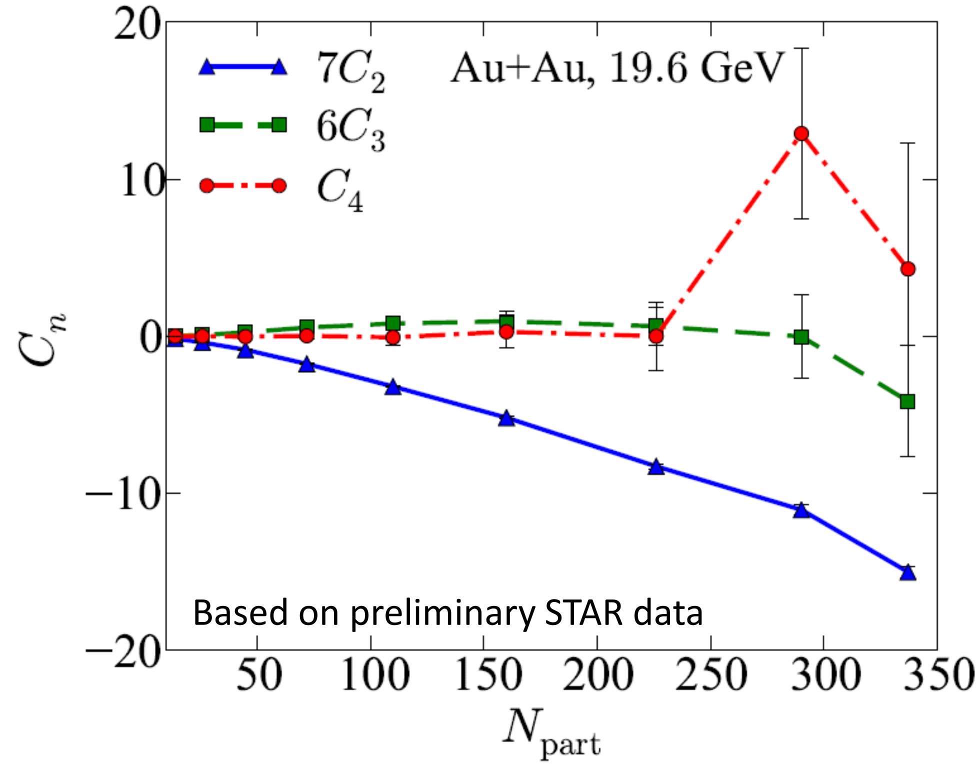
See, e.g.,
B. Ling, M. Stephanov, PRC 93 (2016) 034915
AB, V. Koch, N. Strodthoff , PRC 95 (2017) 054906

Using preliminary STAR data we obtain C_n

central signal at 7.7 GeV is driven by large 4-particle correlations



central signal at 19.6 GeV is driven by 2-particle correlations



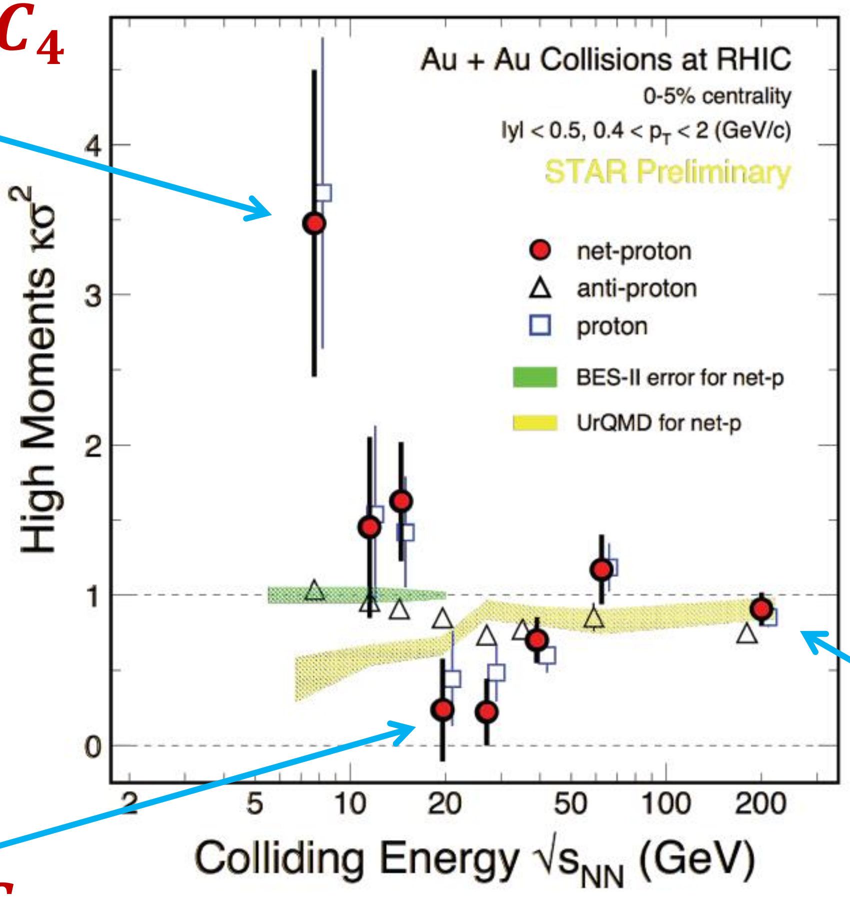
$$C_4(7.7) \sim 170$$

C_4 and $6C_3$ cancelation in most central coll.

AB, V. Koch, N. Strodthoff, PRC 95 (2017) 054906

STAR Preliminary
X.Luo, N.Xu, 1701.02105

here we see C_4



?

we need...

and here C_2

e.g., baryon conservation

… mixed integrated correlation functions

$$C_2^{(2,0)} = -F_{1,0}^2 + F_{2,0}$$

$$C_2^{(1,1)} = -F_{0,1}F_{1,0} + F_{1,1}$$

$$C_3^{(3,0)} = 2F_{1,0}^3 - 3F_{1,0}F_{2,0} + F_{3,0}$$

$$C_3^{(2,1)} = 2F_{0,1}F_{1,0}^2 - 2F_{1,0}F_{1,1} - F_{0,1}F_{2,0} + F_{2,1}$$

$$F_{i,k} \equiv \left\langle \frac{N!}{(N-i)!} \frac{\bar{N}!}{(\bar{N}-k)!} \right\rangle$$

Cumulants

$$K_2 = \langle N \rangle + \langle \bar{N} \rangle + C_2^{(2,0)} + C_2^{(0,2)} - 2C_2^{(1,1)}$$

$$K_3 = \langle N \rangle - \langle \bar{N} \rangle + 3C_2^{(2,0)} - 3C_2^{(0,2)} + C_3^{(3,0)} - C_3^{(0,3)} - 3C_3^{(2,1)} + 3C_3^{(1,2)}$$

For $C_4^{(i,k)}$ and K_4 see the appendix of

AB, V. Koch, N. Strodthoff , 1607.07375

First model (AMPT) calculations by

Yufu Lin, Lizhu Chen, Zhiming Li, 1707.04375

AB, V. Koch, V. Skokov, 1612.05128

Let's put the STAR numbers in perspective.

Suppose that we have clusters (distributed according to Poisson) decaying always to 4 protons

$$\boldsymbol{C_k} = \langle N_{\mathrm{cl}} \rangle \cdot 4!/(4-k)!$$

mean number of clusters

$$\boldsymbol{C_4} = \langle N_{\mathrm{cl}} \rangle \cdot 24$$

for 5-proton clusters:

$$\boldsymbol{C_k} = \langle N_{\mathrm{cl}} \rangle \cdot 5!/(5-k)!$$

$$\boldsymbol{C_4} = \langle N_{\mathrm{cl}} \rangle \cdot 120$$

and $\langle N_{\mathrm{cl}} \rangle \sim 1$

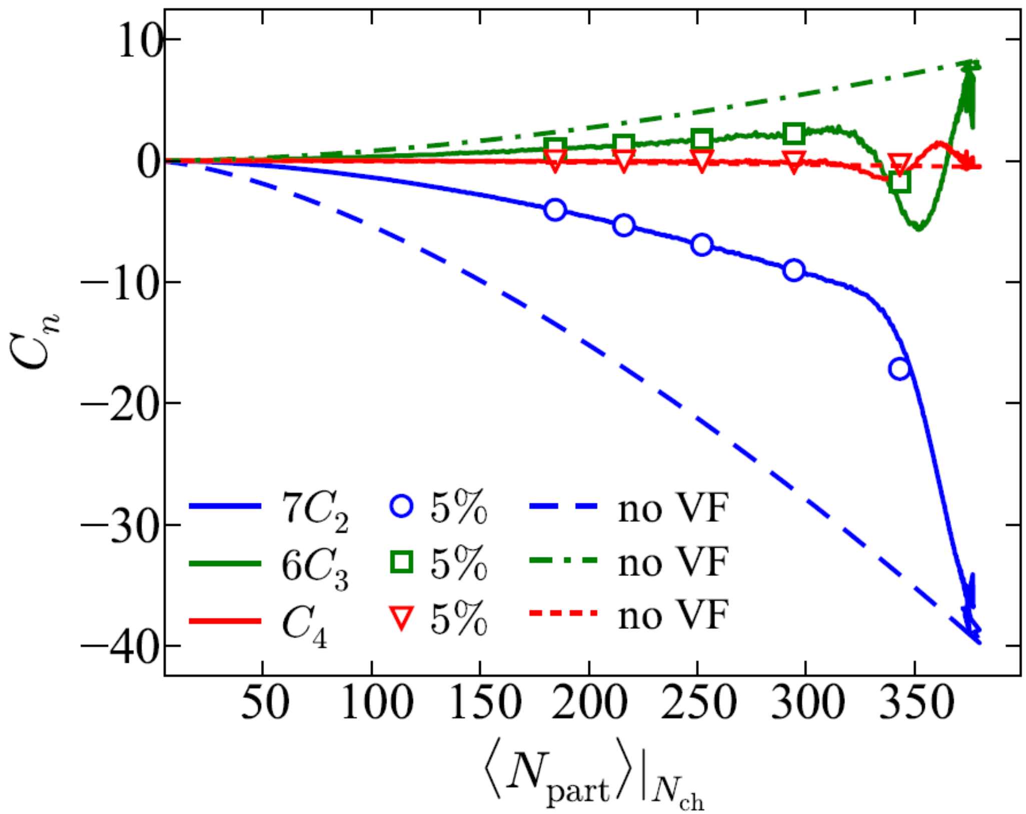
To obtain $\boldsymbol{C_4} \approx 170$ we need $\langle N_{\mathrm{cl}} \rangle \sim 7$, it means 28 protons.
STAR sees on average 40 protons in central collisions.

In this model $C_2 > 0$ and $C_3 > 0$ contrary to the STAR data

Volume fluctuation: minimal model at low energies

- independent baryon stopping (baryon conservation by construction)
- N_{part} fluctuations (volume fluctuation - VF)

Au+Au, $\sqrt{s} = 7.7$ GeV



STAR

$C_4 \sim 170$

$6C_3 \sim -60$

$7C_2 \sim -15$

we follow the STAR way (centrality etc.) as closely as possible

AB, V. Koch, V. Skokov, 1612.05128

See also, P. Braun-Munzinger, A. Rustamov, J. Stachel, 1612.00702

Volume fluctuation seems to be important for C_2.

When compared to the STAR data at 7.7 GeV, volume fluctuation is irrelevant for C_3 and C_4.

C_4 observed by STAR is larger by almost **three orders of magnitude** than the minimal model.

To explain C_4 we need a strong source of multi-proton correlations. **Proton clusters**?

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2) + \boldsymbol{C_2(y_1, y_2)}$$

correlation function

$$\rho_2(y_1, y_2) = \rho(y_1)\rho(y_2)[1 + \boldsymbol{c_2}(y_1, y_2)]$$

reduced correlation function

e.g., does not depend on binomial efficiency

integrated reduced correlation function "coupling"

$$\boldsymbol{c_2} = \frac{\int \rho(y_1)\rho(y_2)\boldsymbol{c_2}(y_1, y_2)dy_1 dy_2}{\int \rho(y_1)\rho(y_2)dy_1 dy_2} = \frac{\boldsymbol{C_2}}{\langle N\rangle^2}$$

and the second order cumulant

$$K_2 = \langle N\rangle + \underbrace{\langle N\rangle^2 \boldsymbol{c_2}}_{\boldsymbol{C_2}}$$

Finally we obtain

$$\boldsymbol{c_2} = \frac{\int \rho(y_1)\rho(y_2)\boldsymbol{c_2}(y_1, y_2)dy_1dy_2}{\int \rho(y_1)\rho(y_2)dy_1dy_2}$$

$$K_2 = \langle N \rangle + \langle N \rangle^2 \boldsymbol{c_2}$$

$$K_3 = \langle N \rangle + 3\langle N \rangle^2 \boldsymbol{c_2} + \langle N \rangle^3 \boldsymbol{c_3}$$

$$K_4 = \langle N \rangle + 7\langle N \rangle^2 \boldsymbol{c_2} + 6\langle N \rangle^3 \boldsymbol{c_3} + \langle N \rangle^4 \boldsymbol{c_4}$$

For $\boldsymbol{c_n}(y_1, \dots, y_n) = const$, K_n strongly depends on rapidity window size since $\langle N \rangle \sim \Delta y$

btw, K_n is strongly efficiency dependent through $\langle N \rangle$

At 7.7 GeV, $K_4/K_2 \sim \langle N \rangle^3 \sim (\Delta y)^3$

See the appendix of [AB, V. Koch, N. Strodthoff , 1607.07375] for net-proton K_n

When, e.g., $K_4/K_2 \approx 1$?

AB, V. Koch, 1707.02640

$$7\langle N\rangle^2 \boldsymbol{c_2} \ll \langle N\rangle$$

$$6\langle N\rangle^3 \boldsymbol{c_3} \ll \langle N\rangle$$

$$\langle N\rangle^4 \boldsymbol{c_4} \ll \langle N\rangle$$

STAR at 7 GeV:

$\boldsymbol{c_2} \sim -1 \cdot 10^{-3}$

$\boldsymbol{c_3} \sim -2 \cdot 10^{-4}$

$\boldsymbol{c_4} \sim +7 \cdot 10^{-5}$

$\langle N\rangle \approx 40$ protons

two obvious options:

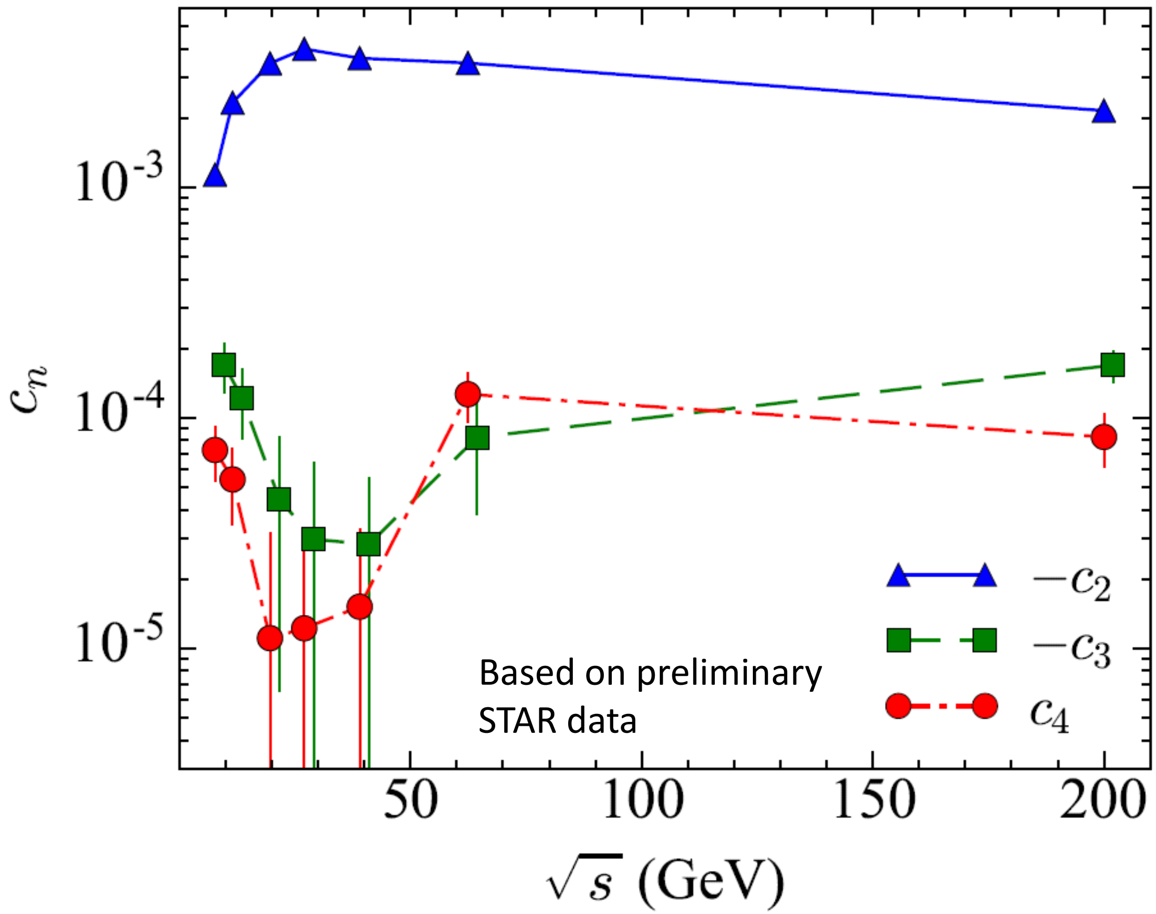
$\boldsymbol{c_n} \approx 0$, that is we are close to Poisson distribution

$\boldsymbol{c_n}$ is "large" but $\langle N\rangle$ is "small" $\longrightarrow$ anti-protons at 7.7 GeV ?

So for small $\langle N\rangle$ (rare particles, efficiency, acceptance) $K_4/K_2 \approx 1$

Couplings' point of view and **global baryon conservation**

AB, VK, preliminary



Global baryon conservation

$$-c_2 = 1/B \approx 2 \cdot 10^{-3}$$

$$-c_3 = -2/B^2 \approx -10^{-5}$$

$$c_4 = -6/B^3 \approx \underline{-10^{-7}}$$

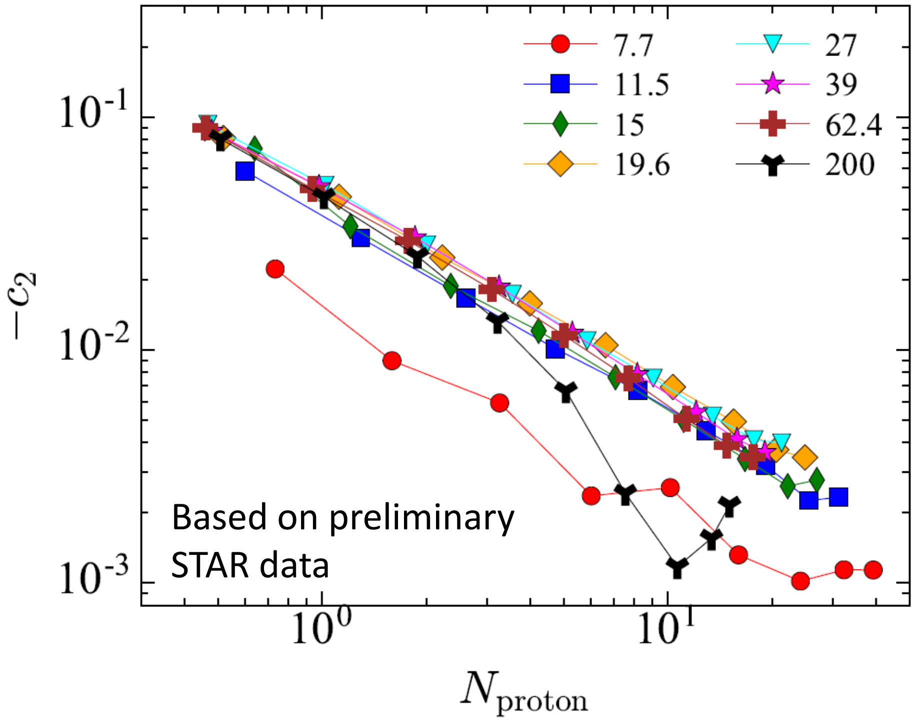
$$B \approx 400$$

c_n – integrated reduced correlation function (coupling)

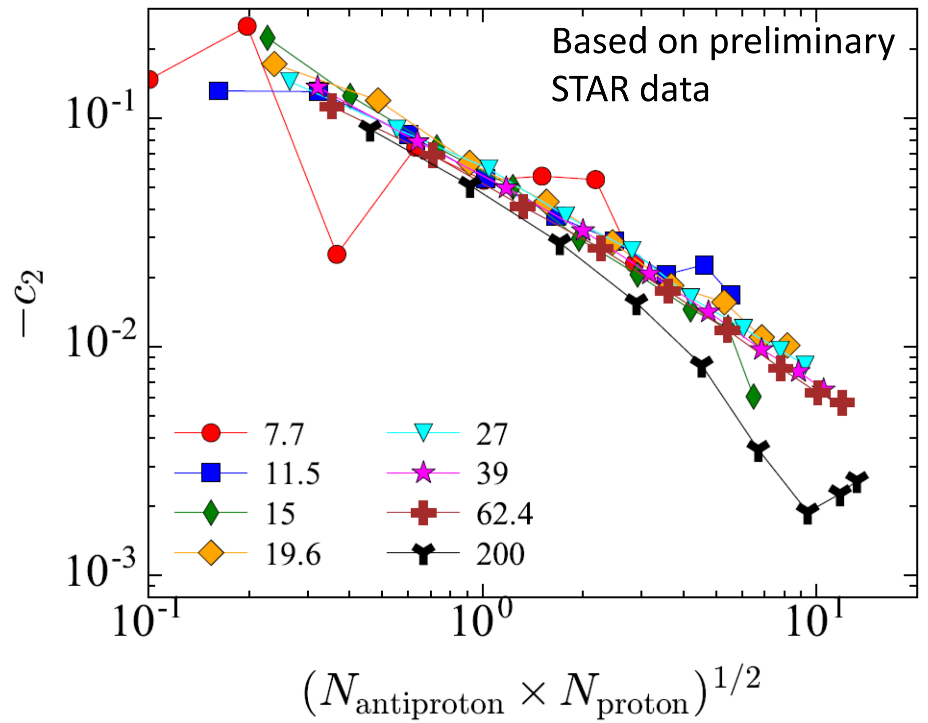
At 7.7 GeV baryon conservation is important for c_2 but irrelevant for c_3 and c_4 when compared to the STAR data

c_2 has some interesting features

protons



antiprotons



Is it consistent with baryon conservation?

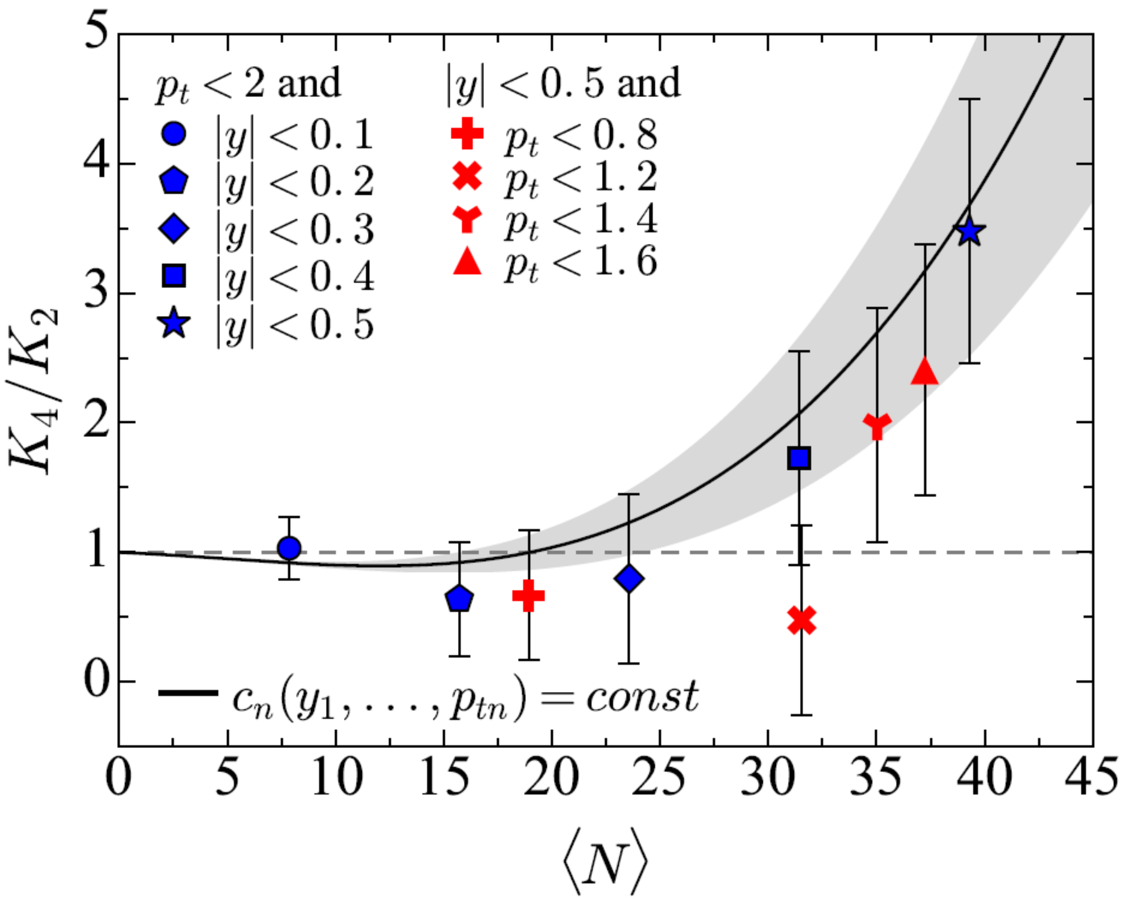
Central 200 GeV ?

AB, V.Koch, in progress

Constant correlation

$$c_2 = \frac{\int \rho(y_1)\rho(y_2) c_2(y_1, y_2) dy_1 dy_2}{\int \rho(y_1)\rho(y_2) dy_1 dy_2}$$

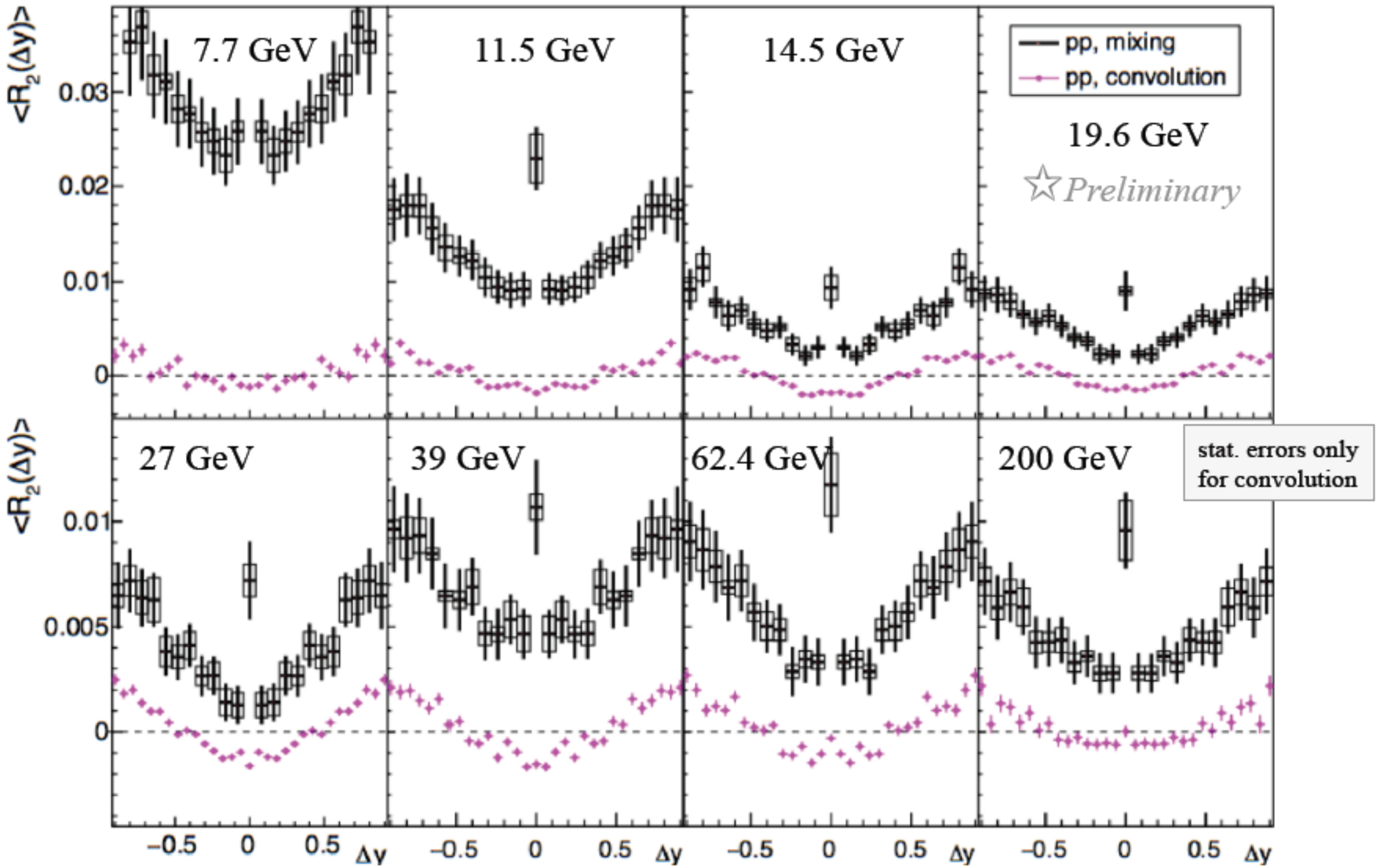
$$c_n(y_1, p_{t1}, \dots, y_n, p_{tn}) = c_n^0 = const \quad \rightarrow \quad c_n = c_n^0$$



$\sqrt{s} = 7.7$ GeV

AB, V. Koch, 1707.02640

R2(Δy) for LS protons vs. $\sqrt{s_{NN}}$, 0-5% central, convolution & mixing



Better control of finite multiplicity effects from convolution
LS proton anticorrelation for Δy~0. Weak beam energy dependence.

AB, V. Koch, 1707.02640

Repulsive vs attractive rapidity correlations

$$c_2(y_1, y_2) = c_2^0 + \gamma_2 \left(y_1 - y_2\right)^2$$

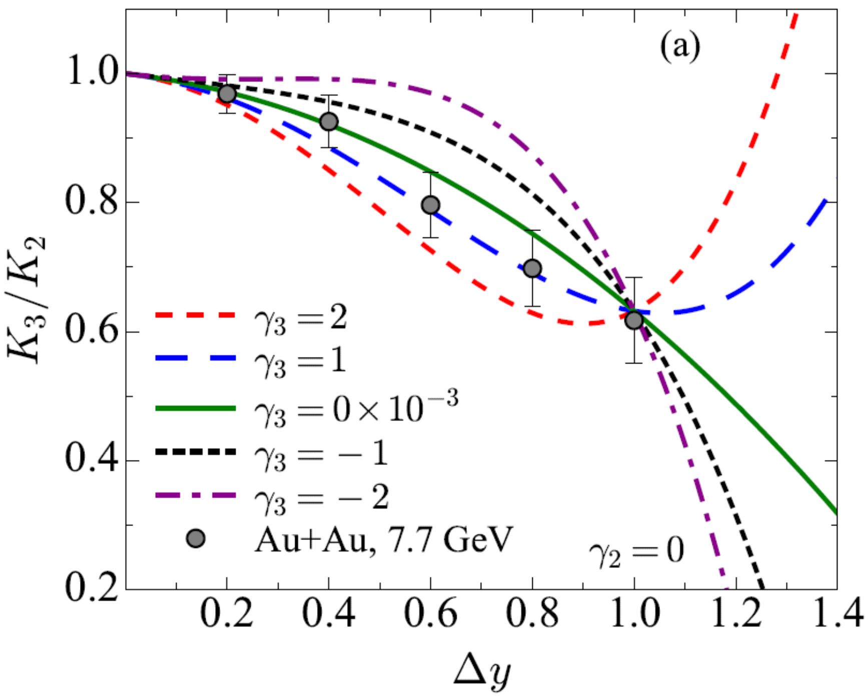
$$c_3(y_1, y_2, y_3) = c_3^0 + \gamma_3 \frac{1}{3} \left[\left(y_1 - y_2\right)^2 + \left(y_1 - y_3\right)^2 + \left(y_2 - y_3\right)^2\right]$$

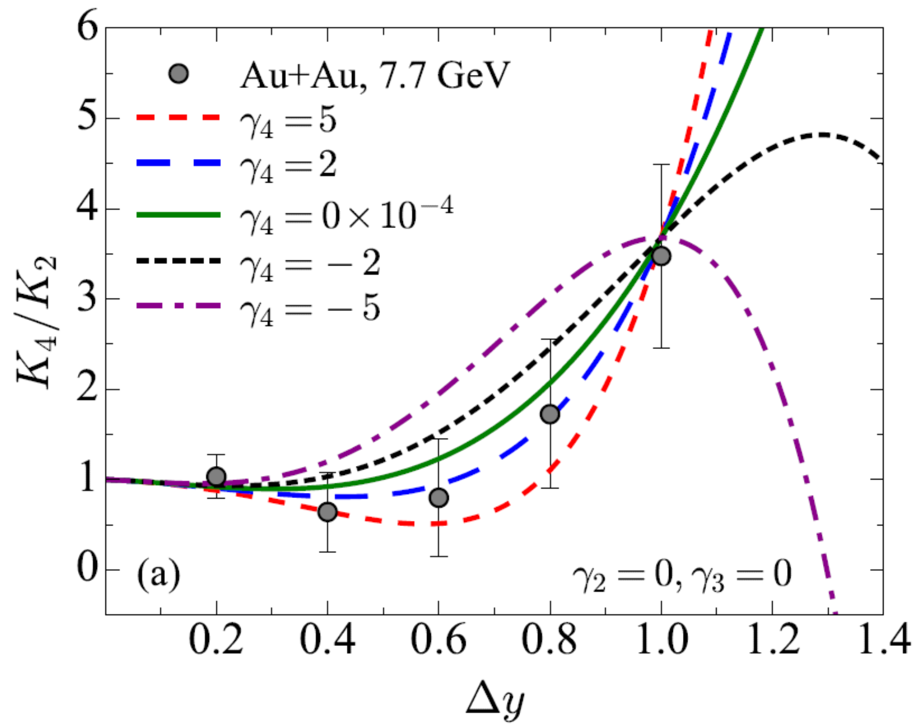
$$c_4(y_1, y_2, y_3, y_4) = c_4^0 + \gamma_4 \frac{1}{6} \Big[\left(y_1 - y_2\right)^2 + \left(y_1 - y_3\right)^2 + \left(y_1 - y_4\right)^2 + \left(y_2 - y_3\right)^2 + \left(y_2 - y_4\right)^2 + \left(y_3 - y_4\right)^2\Big]$$

$\gamma_n > 0$ - rapidity “repulsion”

$\gamma_n < 0$ - rapidity “attraction”

It seems that rapidity repulsion ($\gamma_{3,4} > 0$) is favored



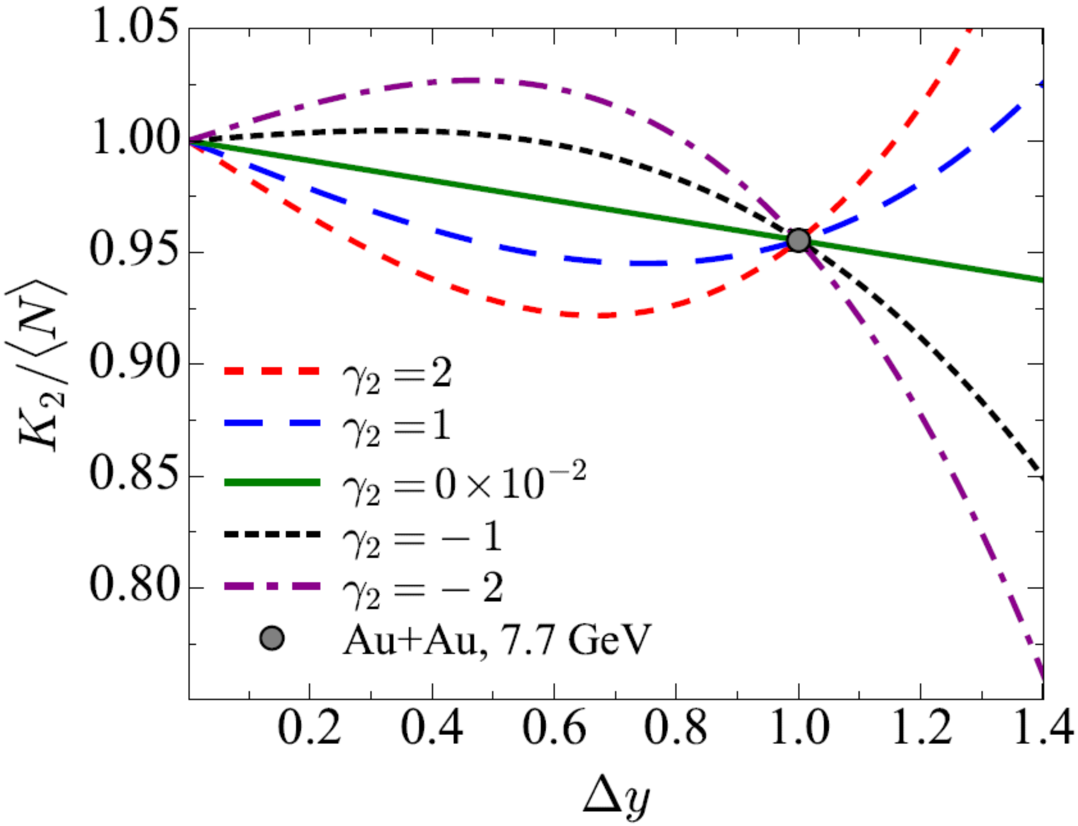


K_3/K_2 above $\Delta y > 1$ could even start growing

$\gamma_{3,4} < 0$ (attraction) seems to be excluded

Presence of proton clusters would naively result in $\gamma_{2,3,4} < 0$ ...

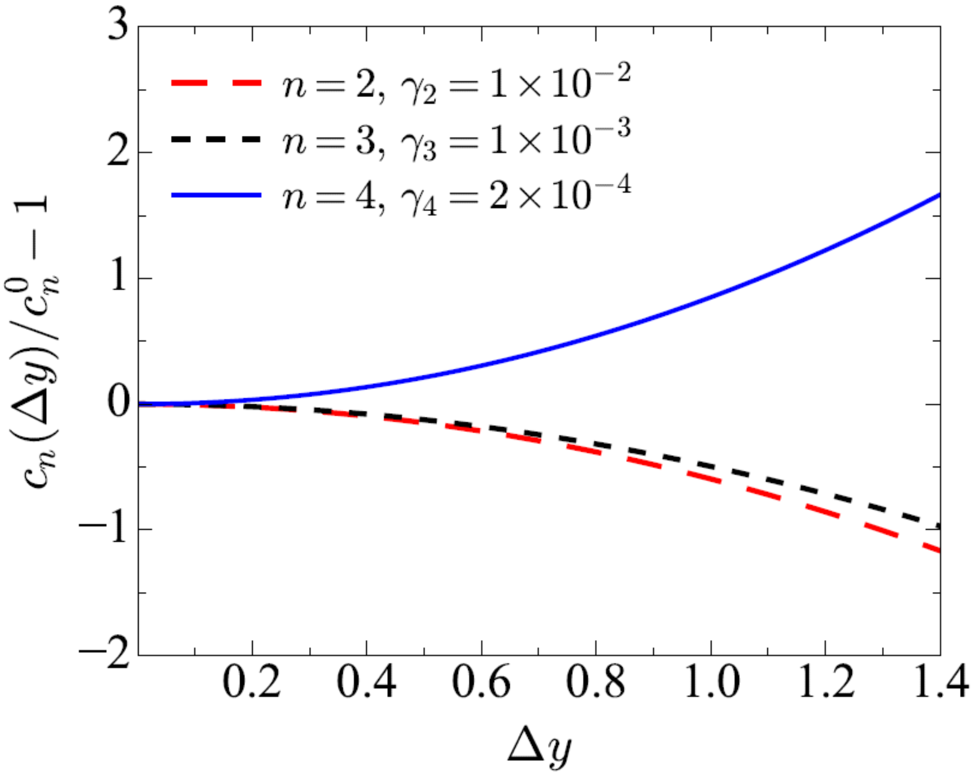
γ_2 is well visible in $K_2/\langle N \rangle$



We should study the integrated reduced correlation function

$$c_n(\Delta y) = \frac{C_n}{\langle N \rangle^n} = c_n^0 + \gamma_n \frac{1}{6}(\Delta y)^2$$

$$\boldsymbol{c_2} = \frac{\int \rho(y_1)\rho(y_2)\boldsymbol{c_2}(y_1, y_2)dy_1 dy_2}{\int \rho(y_1)\rho(y_2)dy_1 dy_2}$$



Can we describe the STAR data at 7.7 GeV with *ordinary* multiplicity distributions?

Model with **two event classes**

$$P(N) = (1-\alpha)P_{(a)}(N) + \alpha P_{(b)}(N)$$

Poisson, binomial, etc..

Poisson, binomial, etc.

That is, with probability $1-\alpha$ we have $P_{(a)}(N)$ and with probability α we have $P_{(b)}(N)$

AB, V. Koch, J. Steinheimer, in progress

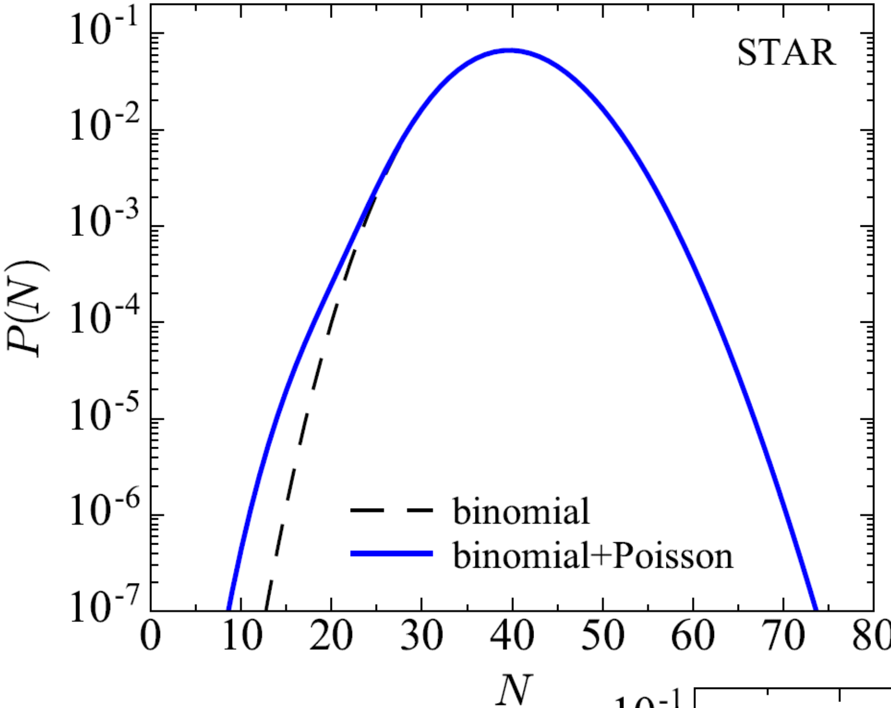
$$C_2 \approx C_2^{(a)} + \alpha N_\Delta^2$$
$$C_3 \approx C_3^{(a)} - \alpha N_\Delta^3$$
$$C_4 \approx C_4^{(a)} + \alpha N_\Delta^4 \qquad N_\Delta = \langle N \rangle - \langle N_{(b)} \rangle$$

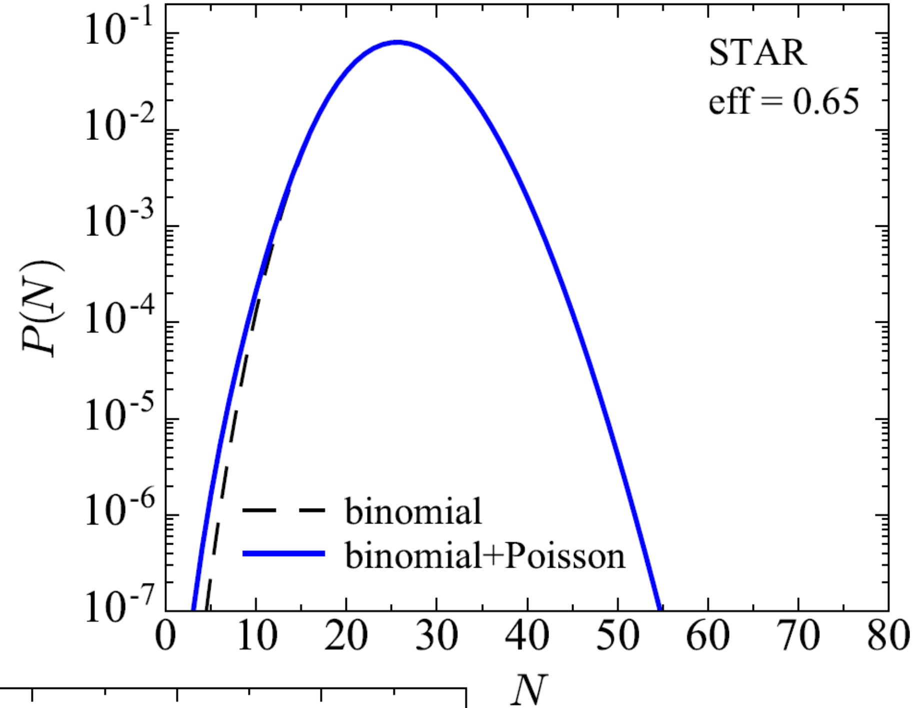
If $N_\Delta > 0$ we obtain $C_3 < 0$ and $C_4 > 0$

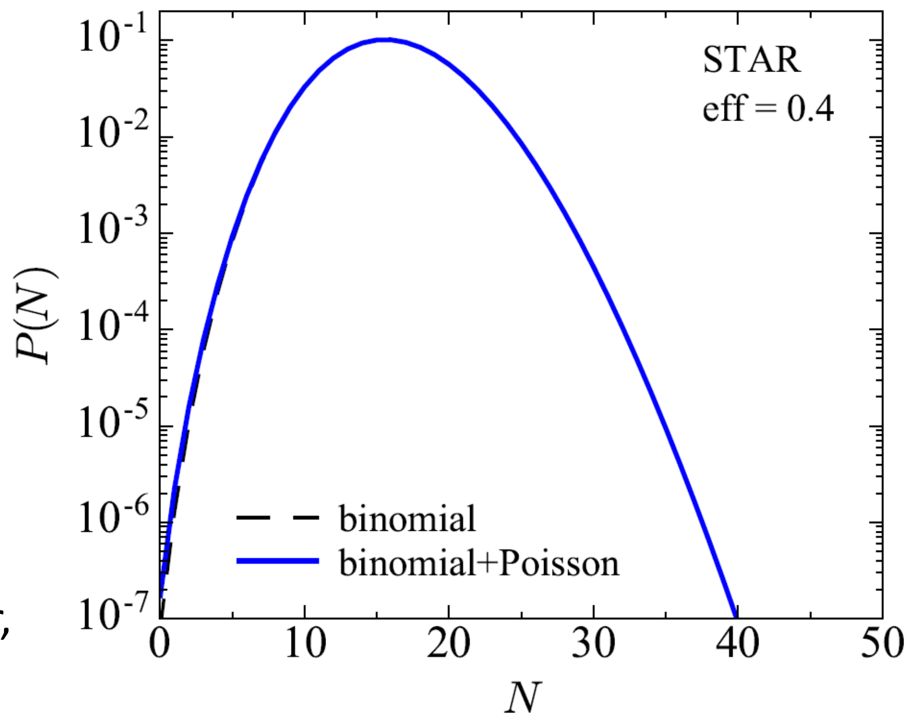
We can describe the data with $\alpha \approx 0.005$, $N_\Delta \approx 13$

$$\langle N_{(a)} \rangle \approx 40,\ \langle N_{(b)} \rangle \approx 27$$

Now we can plot $P(N)$ …







eff = efficiency

AB, V. Koch, J. Steinheimer, in progress

Conclusions

Four-proton correlation function at 7.7 GeV is surprisingly large.
We need a strong source of multi-proton correlation.
Proton clusters?

Volume fluctuation and baryon conservation seem to be irrelevant for C_3 and C_4. C_2 is likely dominated by background.

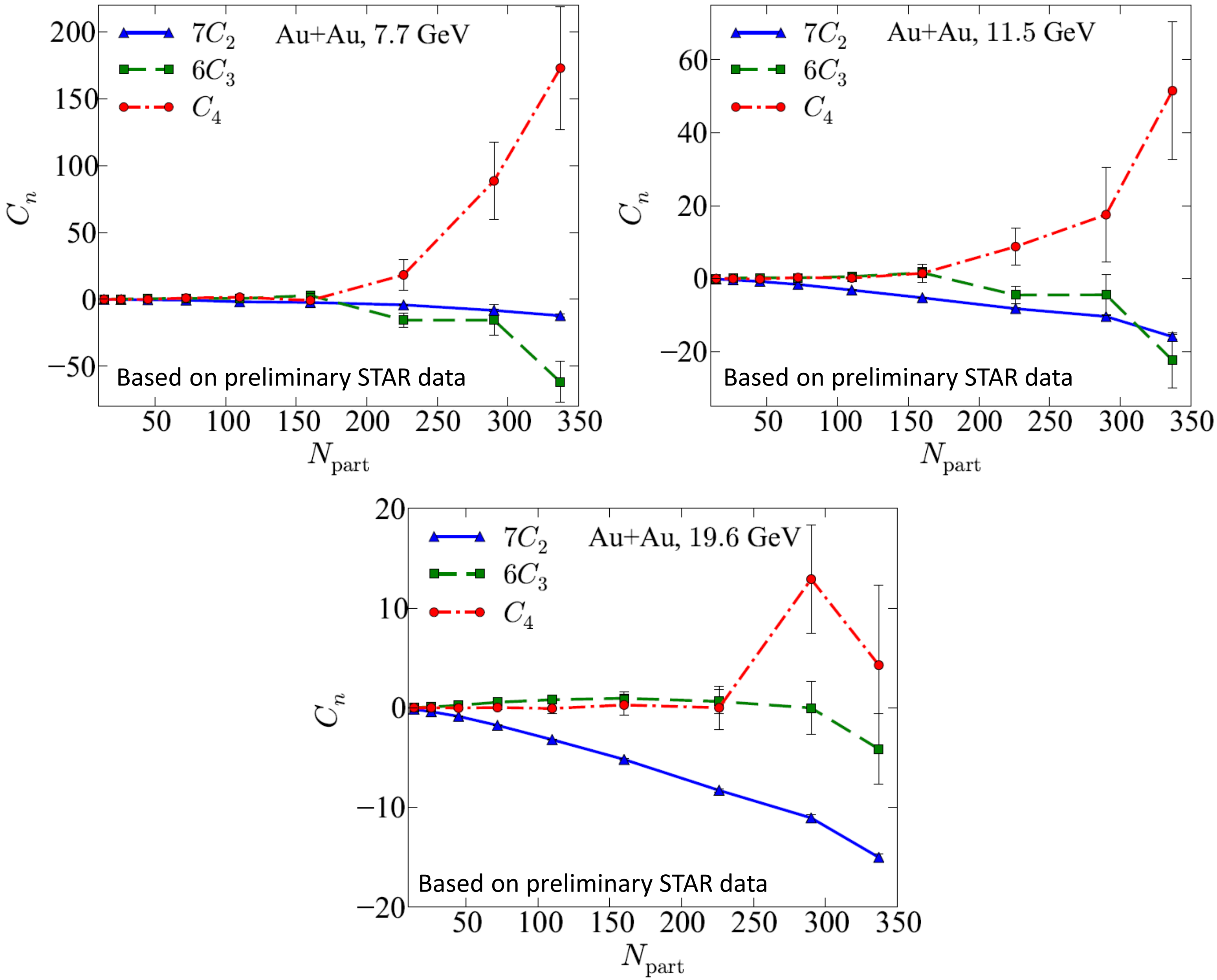
The STAR data at 7.7 GeV is consistent with constant correlation functions. A small multi-proton rapidity repulsion is slightly favored.

The signal at higher energies can be actually interesting.

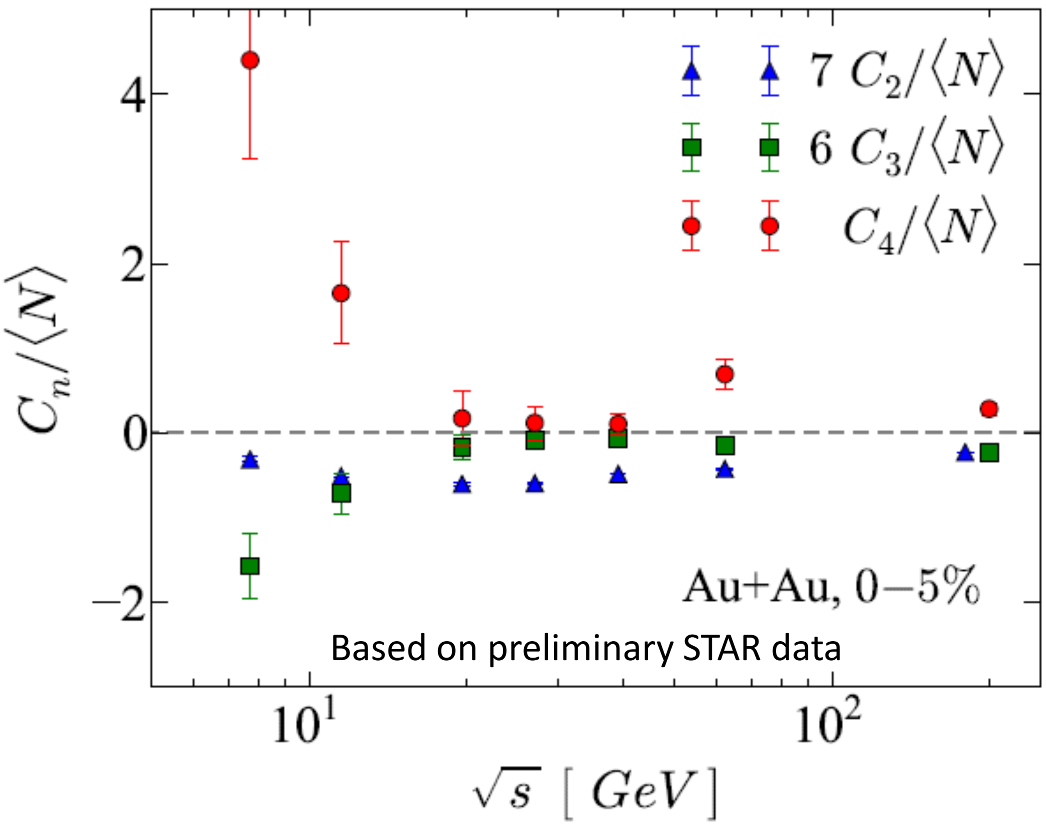
STAR data from *ordinary* multiplicity distributions, $\alpha \approx 0.005$

Backup

Comparison of 7.7, 11.5 and 19.6 GeV



AB, V. Koch, N. Strodthoff , 1607.07375



C_4 at 62 GeV !

Mixed correlation functions and cumulants

$$C_2^{(2,0)} = -F_{1,0}^2 + F_{2,0}$$
$$C_2^{(1,1)} = -F_{0,1}F_{1,0} + F_{1,1}$$
$$C_3^{(3,0)} = 2F_{1,0}^3 - 3F_{1,0}F_{2,0} + F_{3,0}$$
$$C_3^{(2,1)} = 2F_{0,1}F_{1,0}^2 - 2F_{1,0}F_{1,1} - F_{0,1}F_{2,0} + F_{2,1}$$
$$C_4^{(4,0)} = -6F_{1,0}^4 + 12F_{1,0}^2F_{2,0} - 3F_{2,0}^2 - 4F_{1,0}F_{3,0} + F_{4,0}$$
$$C_4^{(3,1)} = -6F_{0,1}F_{1,0}^3 + 6F_{1,0}^2F_{1,1} + 6F_{0,1}F_{1,0}F_{2,0} - 3F_{1,1}F_{2,0} - 3F_{1,0}F_{2,1} - F_{0,1}F_{3,0} + F_{3,1}$$
$$C_4^{(2,2)} = (-6F_{0,1}^2 + 2F_{0,2})F_{1,0}^2 + 8F_{0,1}F_{1,0}F_{1,1} - 2F_{1,1}^2 - 2F_{1,0}F_{1,2} + (2F_{0,1}^2 - F_{0,2})F_{2,0} - 2F_{0,1}F_{2,1} + F_{2,2}$$

$$K_2 = \langle N\rangle + \langle\bar{N}\rangle + C_2^{(2,0)} + C_2^{(0,2)} - 2C_2^{(1,1)}$$
$$K_3 = \langle N\rangle - \langle\bar{N}\rangle + 3C_2^{(2,0)} - 3C_2^{(0,2)} + C_3^{(3,0)} - C_3^{(0,3)} - 3C_3^{(2,1)} + 3C_3^{(1,2)}$$
$$K_4 = \langle N\rangle + \langle\bar{N}\rangle + 7C_2^{(2,0)} + 7C_2^{(0,2)} - 2C_2^{(1,1)} + 6C_3^{(3,0)} + 6C_3^{(0,3)} - 6C_3^{(2,1)} - 6C_3^{(1,2)} + C_4^{(4,0)} + C_4^{(0,4)} - 4C_4^{(3,1)} - 4C_4^{(1,3)} + 6C_4^{(2,2)}$$

$$c_{n+m}^{(n,m)} = \frac{C_{n+m}^{(n,m)}}{\langle N\rangle^n \langle\bar{N}\rangle^m}$$

AB, V. Koch, N. Strodthoff , 1607.07375

Full acceptance

$$N_{(b)}$$

$$\boxed{N_{(a)}}$$

$$N_{(a)} + N_{(b)} = B = const.$$

baryon conservation

$$K_{2,(a)} = K_{2,(b)} \qquad K_{3,(a)} = -K_{3,(b)}$$

$$K_{4,(a)} = K_{4,(b)} \qquad K_{5,(a)} = -K_{5,(b)}$$

$$\frac{K_4}{K_2} \to 1, \quad \frac{K_3}{K_2} \to -1$$

for full acceptance

$$c_2 = \frac{\int \rho(y_1)\rho(y_2)c_2(y_1, y_2)dy_1dy_2}{\int \rho(y_1)\rho(y_2)dy_1dy_2}$$

$$K_2 = \langle N \rangle + \langle N \rangle^2 c_2$$

$$K_4 = \langle N \rangle + 7\langle N \rangle^2 c_2 + 6\langle N \rangle^3 c_3 + \langle N \rangle^4 c_4$$

Rapidity dependence:

long-range correlation

$$c_n(y_1, \dots, y_n) = c_n^0$$

$$c_n = c_n^0$$

short-range correlation

$$c_2(y_1, y_2) = c_2^0 \delta(y_1 - y_2)$$

$$c_2 \sim 1/(\Delta y)$$

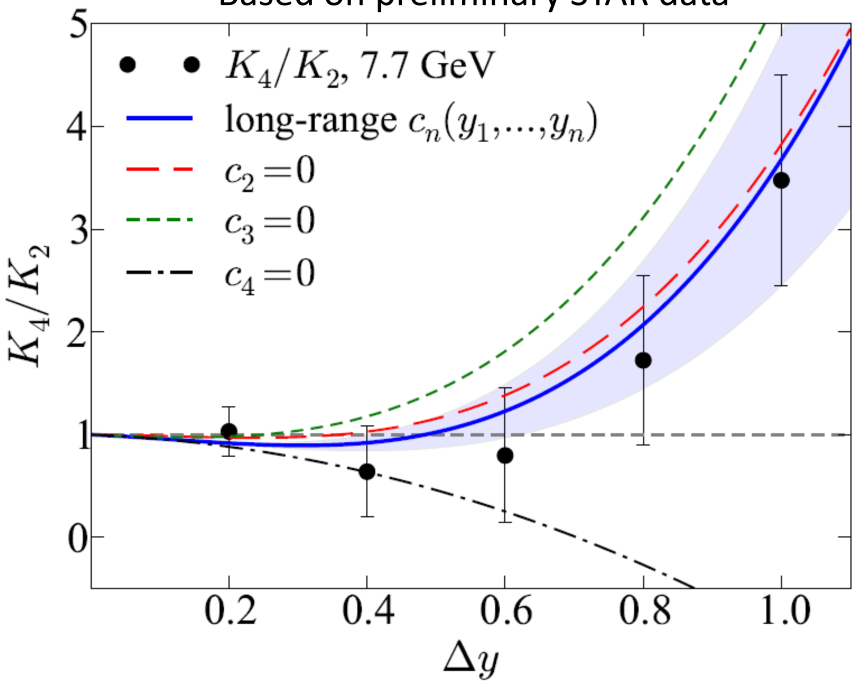
$$\boxed{K_n \sim \Delta y}$$

$$K_2 = \langle N \rangle + c_2^0 \langle N \rangle^2, \qquad \langle N \rangle \sim \Delta y$$

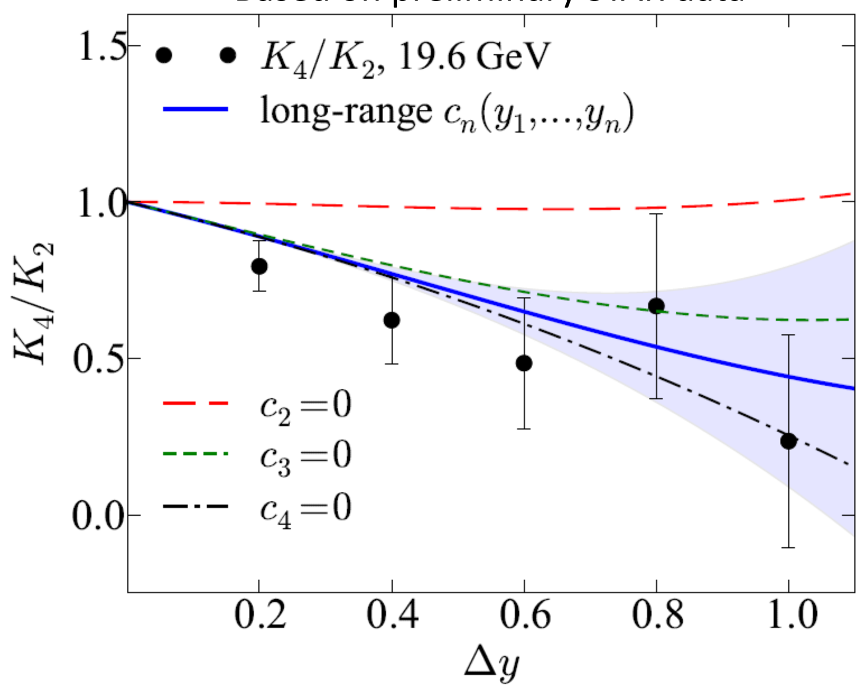
$$\boxed{K_4 = \langle N \rangle + 7c_2^0 \langle N \rangle^2 + 6c_3^0 \langle N \rangle^3 + c_4^0 \langle N \rangle^4}$$

Rapidity dependence consistent with long-range correlations

Based on preliminary STAR data

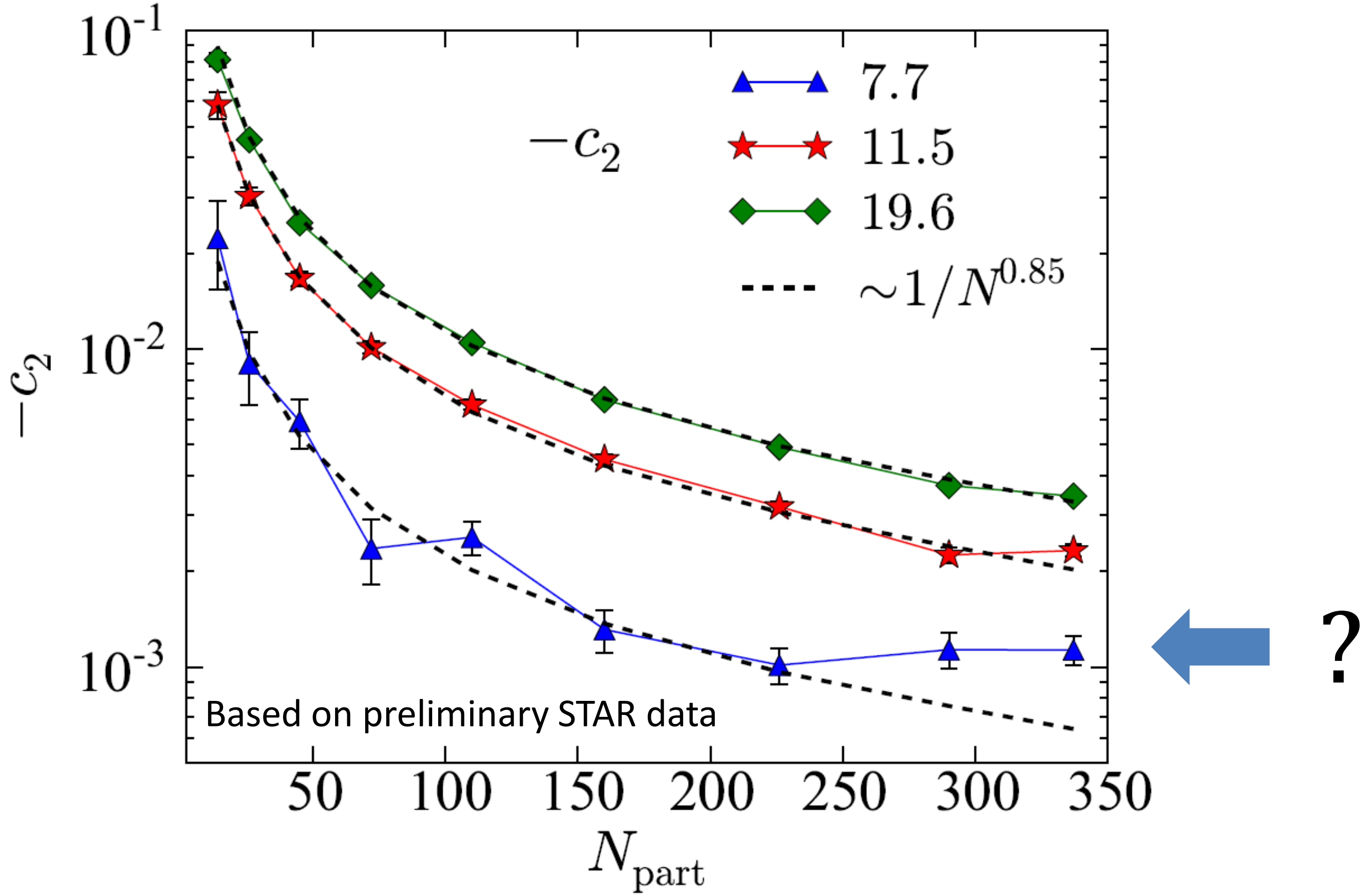


Based on preliminary STAR data



AB, V. Koch, N. Strodthoff , 1607.07375

results for c_2

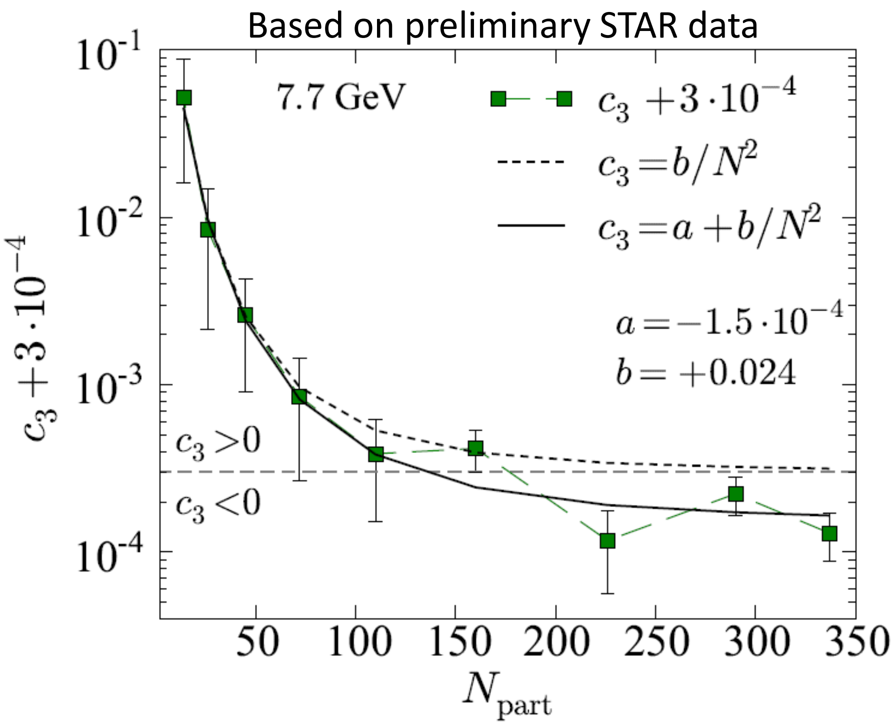


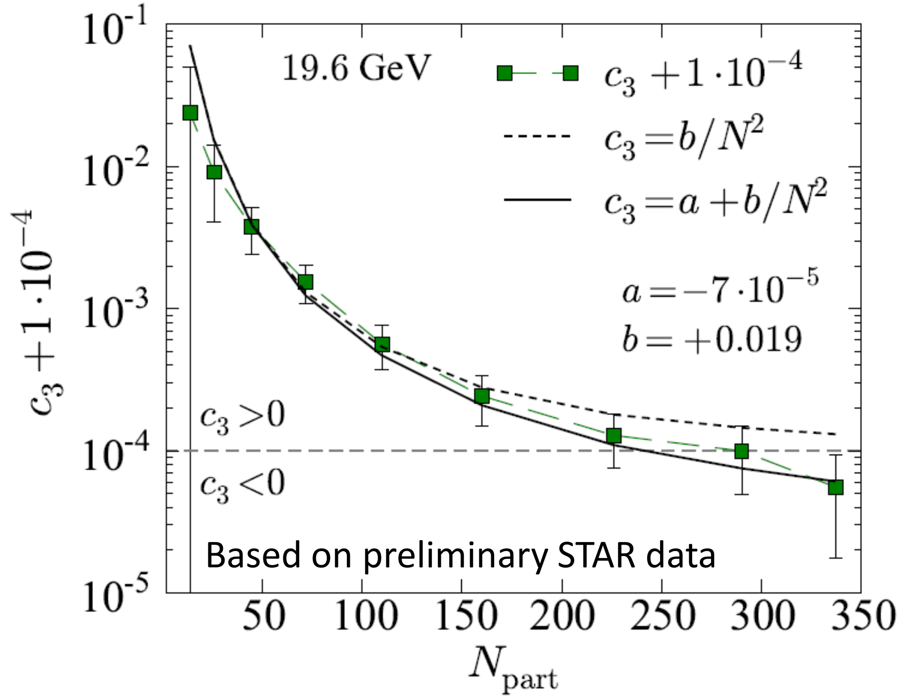
central 7 GeV points are somehow special

AB, V. Koch, N. Strodthoff , 1607.07375

Using preliminary STAR data we obtain c_3

AB, V. Koch, N. Strodthoff, 1607.07375

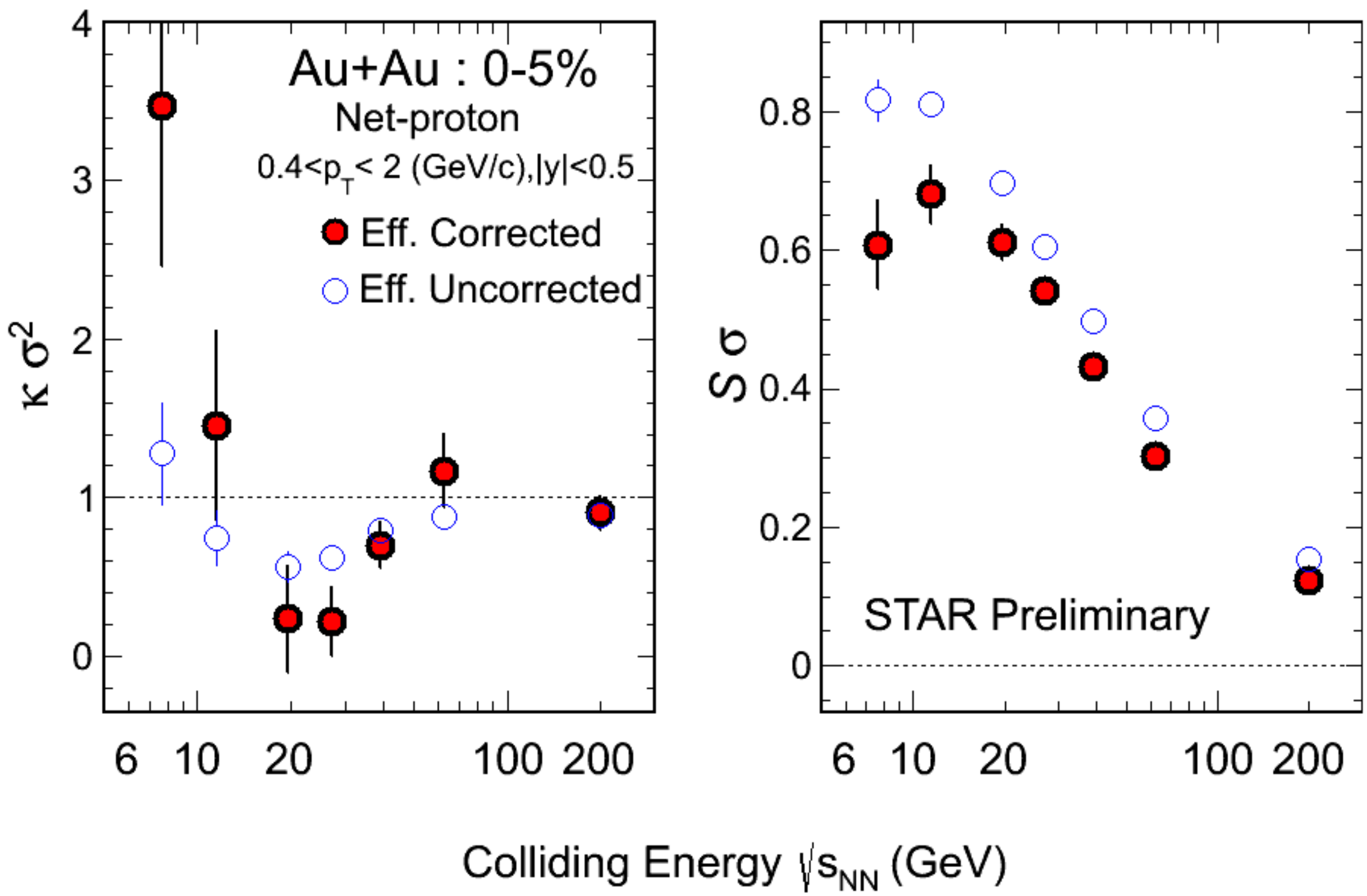




At 7.7 GeV we see $1/N^2$ for small N_{part} then c_3 **changes sign** and stays roughly constant…

Similar story for c_4

Preliminary STAR data



K_4/K_2 my notation K_3/K_2

Genuine three-proton correlation

$$\rho_3(y_1, y_2, y_3) = \rho(y_1)\rho(y_2)\rho(y_3)[1 + \boldsymbol{c_2}(y_1, y_2) + \cdots + \boldsymbol{c_3}(y_1, y_2, y_3)]$$

$$F_3 = \langle N(N-1)(N-2)\rangle = \langle N\rangle^3 + 3\langle N\rangle^3 \boldsymbol{c_2} + \langle N\rangle^3 \boldsymbol{c_3}$$

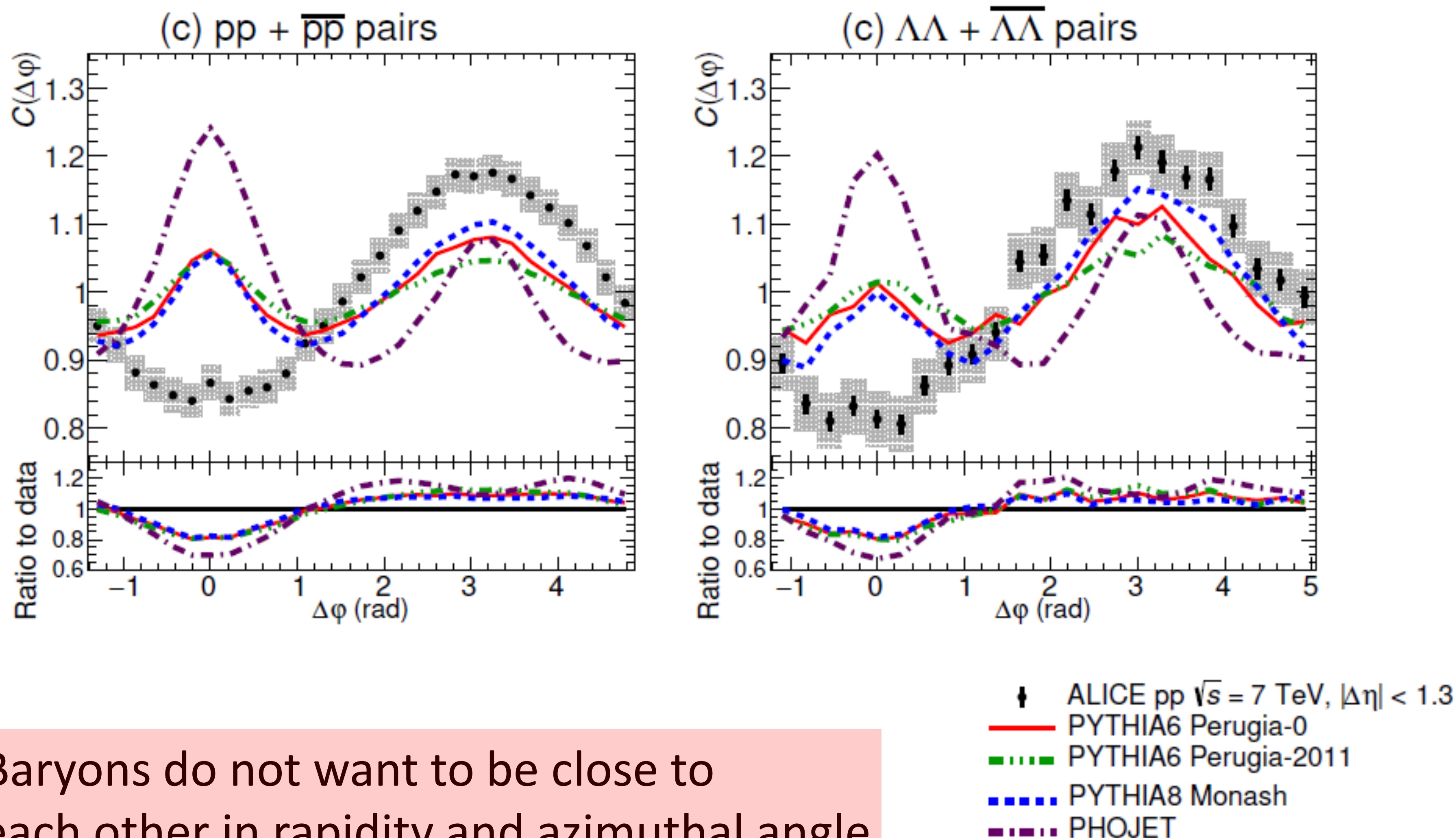
$$\boldsymbol{c_3} = \frac{\int \rho(y_1)\rho(y_2)\rho(y_3)\boldsymbol{c_3}(y_1, y_2, y_3)dy_1 dy_2 dy_3}{\int \rho(y_1)\rho(y_2)\rho(y_3)dy_1 dy_2 dy_3}$$

and the third order cumulant

$$K_3 = \langle N\rangle + \underbrace{3\langle N\rangle^2 \boldsymbol{c_2}}_{3\boldsymbol{C_2}} + \underbrace{\langle N\rangle^3 \boldsymbol{c_3}}_{\boldsymbol{C_3}}$$

ALICE

ALICE, 1612.08975

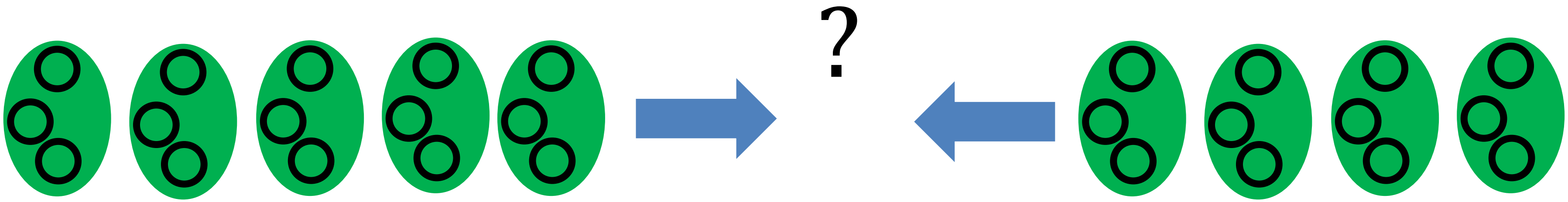


Baryons do not want to be close to each other in rapidity and azimuthal angle

First seen by TPC/Two Gamma Collaboration in e+e- annihilation at 29-GeV, PRL 57, 3140 (1986).

How to properly model baryon stopping?

Do baryons stop independently or maybe we have some multi-baryon correlations?



Important questions if we want to understand proton cumulants as measured by STAR and HADES.

STAR and HADES see rather exciting multi-proton correlations at lower energies.

Proton stopping – is it obvious that we produce high baryon density?

At low energy protons are not produced. They are transferred from incoming nucleus.

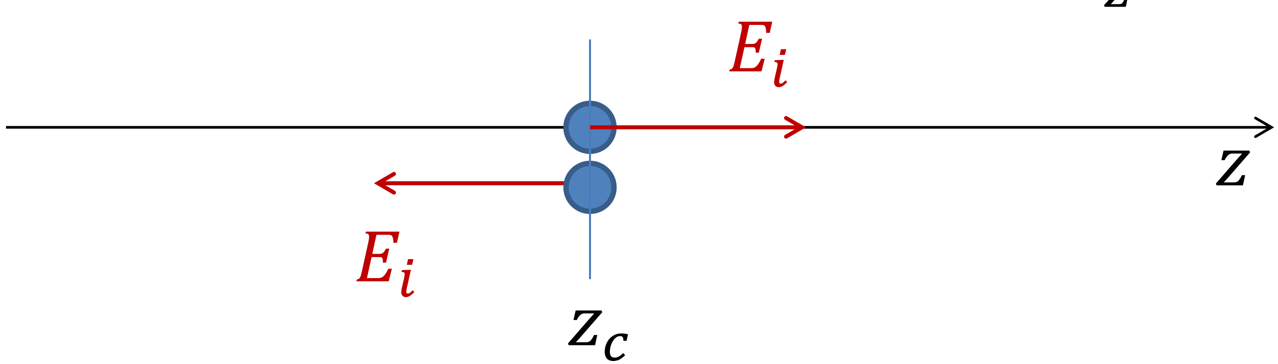
There is no infinite deceleration. It take some time and length to slow down or stop a proton. Thus we expect (?) the stopped protons to be away from $z = 0$.

$$E_z = E_i - \sigma(z - z_c)$$

E_i – initial energy; z_c – collision point; E_z – energy at a point z

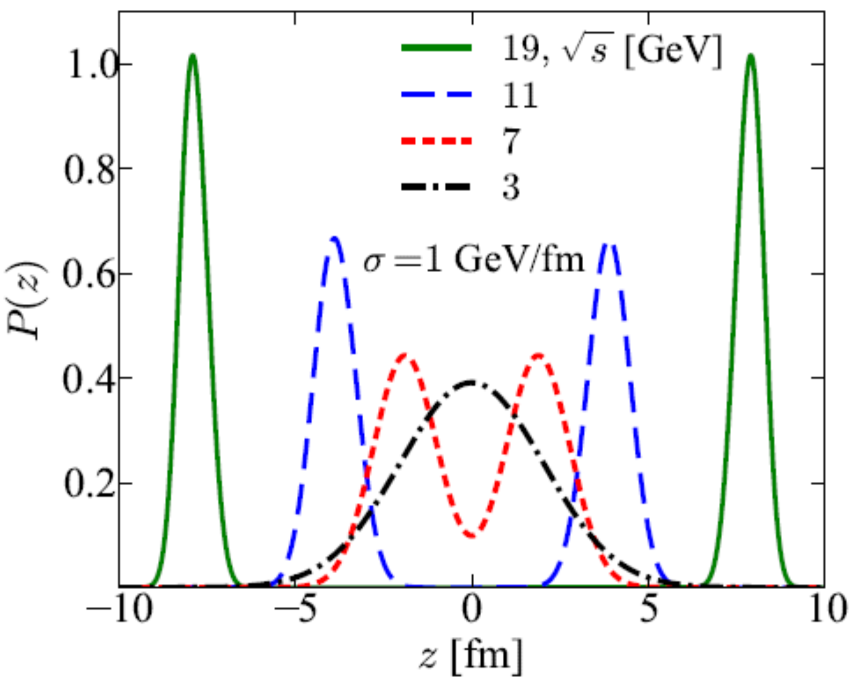
σ – energy loss per unit length

$$E_z \rightarrow M_t \cosh(y)$$

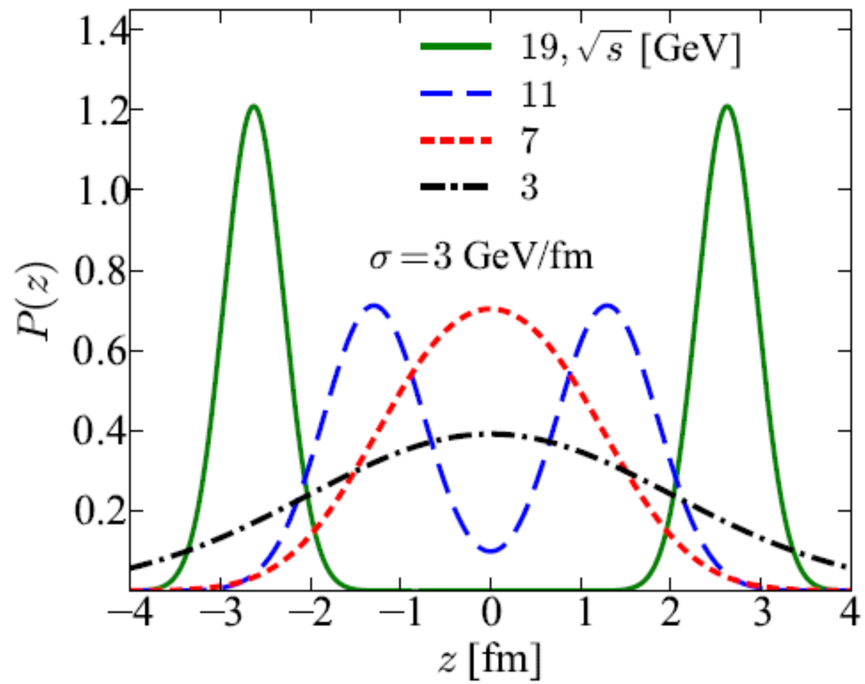


Now we need to average over the collision points z_c in A+A, and over the measured rapidity bin.

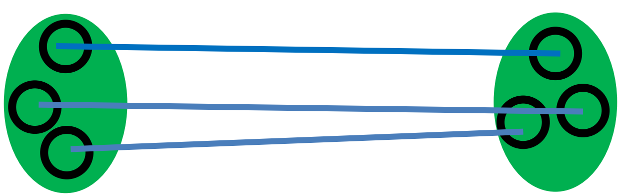
Here $|y| < 1$, $R_A = 6.5$ fm, $P_t = 1$ GeV. $M_t^2 = M^2 + P_t^2$



wounded nucleon model



wounded quark model



A.Bialas, AB, V.Koch, 1608.07041