Fluctuation Observables and Equilibration in Heavy Ion Collisions

Krzysztof Redlich, University of Wroclaw & EMMI/GSI

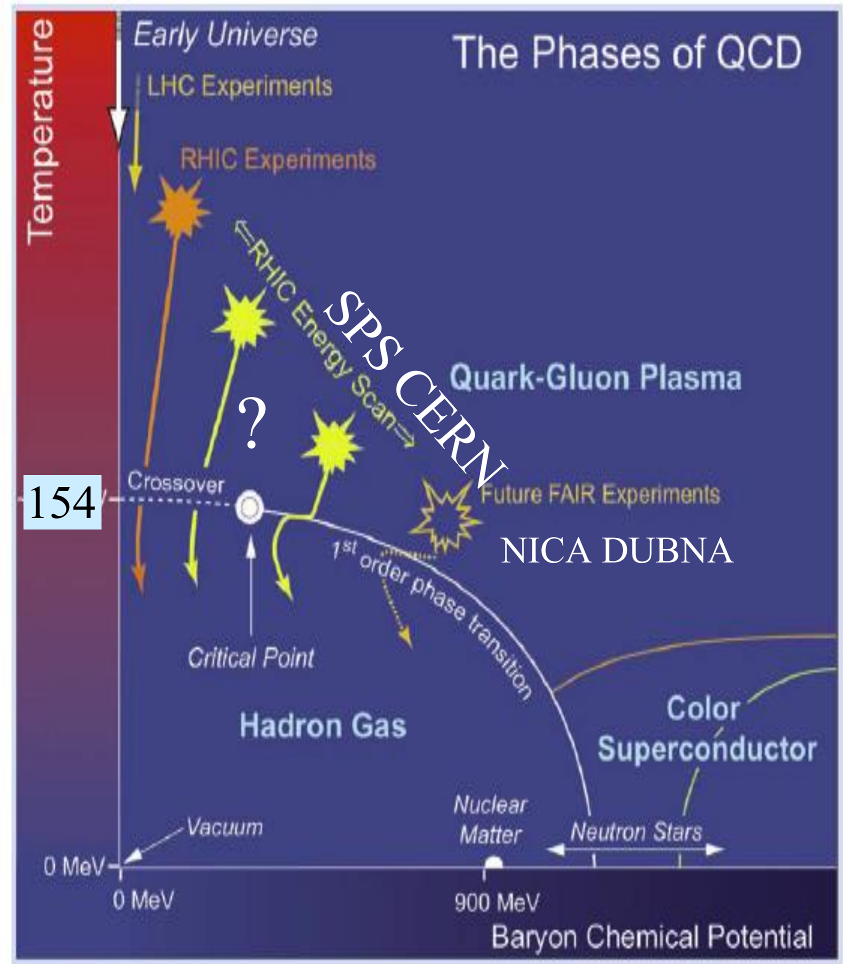
- Probing thermalization, composition and parameters of the collision fireball in HIC

  ⇨ linking LQCD results to HIC data of ALICE coll.

- Modelling QCD thermodynamic with the HRG statistical operator

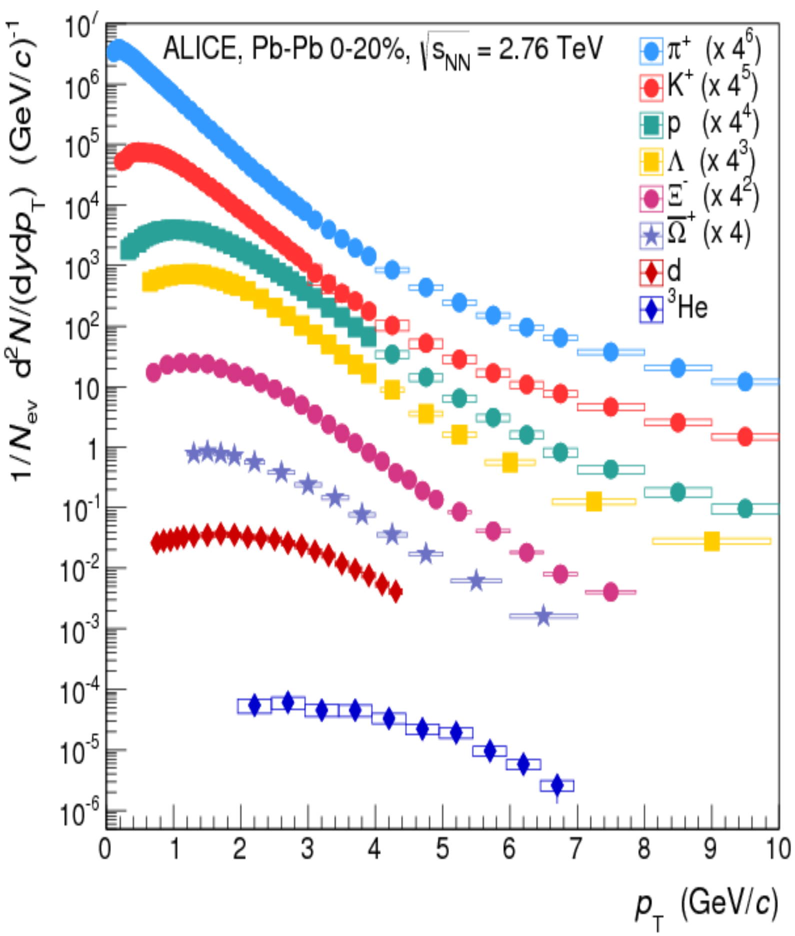
  ⇨ importance of dynamical widths and non-resonance interactions: the S- matrix approach

⇨ Higher order charge fluctuations as a probe of critical chiral dynamics: STAR data on net proton fluctuations



Can the thermal nature and composition of the collision fireball in HIC be verified with respect to the QCD partition function ?

HIC ⟺ **Lattice QCD**



- The strategy:
  - Compare directly measured fluctuations and correlations with LGT

    F. Karsch and K. R, Phys. Lett. B 695, 136 (2011)
    F. Karsch, Central Eur. J. Phys. 10, 1234 (2012)
    A. Bazavov et al., Phys. Rev. Lett. 109, 192302 (2012):

see talks: Frithjof Karsch, Claudia Ratti, Rene Bellwied

  - Construct the 2nd order fluctuations and correlations from measured yields and compare with LGT

P. Braun-Munzinger, A. Kalweit, J. Stachel, K.R. Phys. Lett. B 47, 292 (2015), Nucl.Phys. A956, 805 (2016)

Consider fluctuations and correlations of conserved charges to be compared with LQCD

Excellent probe of:

- QCD criticality

A. Asakawa at. al.

S. Ejiri et al.,…

M. Stephanov et al.,

K. Rajagopal et al.

B. Frimann et al.

- freezeout conditions in HIC

F. Karsch &

S. Mukherjee et al.,

C. Ratti et al.

P. Braun-Munzinger et al.

- They are quantified by susceptibilities:

If $P(T,\mu_B,\mu_Q,\mu_S)$ denotes pressure, then

$$\frac{\chi_N}{T^2} = \frac{\partial^2(P)}{\partial(\mu_N)^2} \qquad \frac{\chi_{NM}}{T^2} = \frac{\partial^2(P)}{\partial\mu_N\partial\mu_M}$$

$$N = N_q - N_{-q},\quad N,M = (B,S,Q),\quad \mu = \mu/T,\quad P = P/T^4$$

- Susceptibility is connected with variance

$$\frac{\chi_N}{T^2} = \frac{1}{VT^3}(<N^2> - <N>^2)$$

- If $\mathrm{P}(N)$ probability distribution of N then

$$<N^n> = \sum_N N^n \mathrm{P}(N)$$

Consider special case:

- Baryon and anti-antibaryon Poisson distributed, then for the net charge N
- $\mathrm{P}(N)$ is the Skellam distribution

$$\mathrm{P}(N) = \left(\frac{N_q}{N_{-q}} \right)^{N/2} I_N(2\sqrt{N_q N_{-q}}) \exp[-(N_q + N_{-q})]$$

$$< N_q > \equiv N_q \quad =>$$

Charge carrying by particles $q = \pm 1$

- The susceptibility

$$\frac{\chi_N}{T^2} = \frac{1}{VT^3}(< N_q > + < N_{-q} >)$$

Consider special case: particles carrying $q = \pm 1, \pm 2, \pm 3$

- The probability distribution

P. Braun-Munzinger,
B. Friman, F. Karsch,
V Skokov &K.R.
Phys .Rev. C84 (2011) 064911
Nucl. Phys. A880 (2012) 48)

$< S_{-q} >\equiv S_{-q}$

$q = \pm 1, \pm 2, \pm 3$

$$P(S) = \left(\frac{S_1}{S_{\bar{1}}}\right)^{\frac{S}{2}} \exp\left[\sum_{n=1}^{3}\left(S_n + S_{\bar{n}}\right)\right]$$

$$\sum_{i=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \left(\frac{S_3}{S_{\bar{3}}}\right)^{\frac{k}{2}} I_k\left(2\sqrt{S_3 S_{\bar{3}}}\right)\left(\frac{S_2}{S_{\bar{2}}}\right)^{\frac{i}{2}} I_i\left(2\sqrt{S_2 S_{\bar{2}}}\right)$$

$$\left(\frac{S_1}{S_{\bar{1}}}\right)^{-i-\frac{3k}{2}} I_{2i+3k-S}\left(2\sqrt{S_1 S_{\bar{1}}}\right)$$

Fluctuations

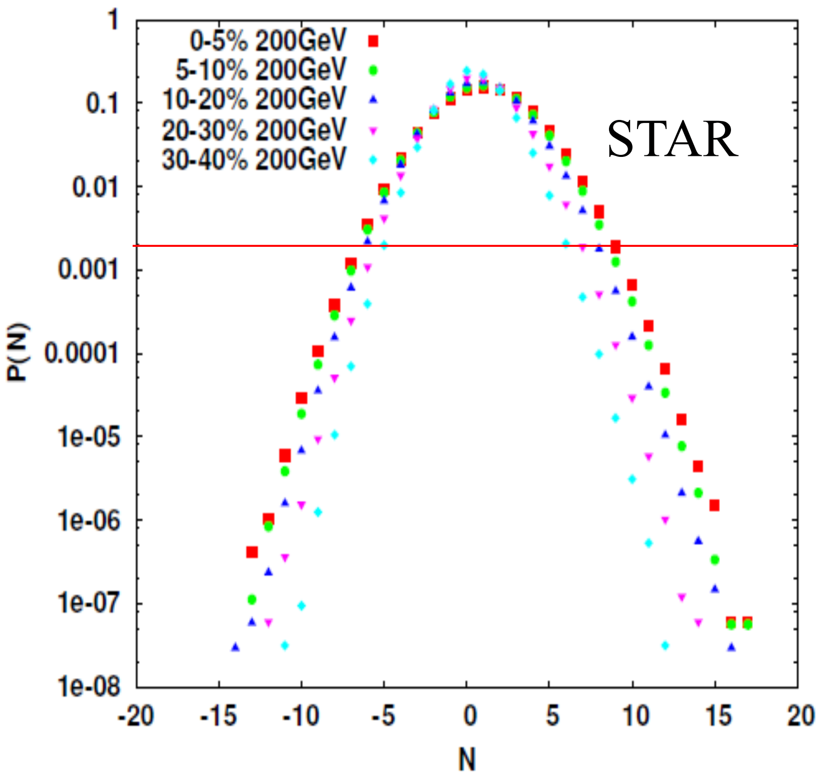
$$\frac{\chi_S}{T^2} = \frac{1}{VT^3}\sum_{n=1}^{|q|} n^2\left(\langle S_n\rangle + \langle S_{-n}\rangle\right)$$

Correlations

$$\frac{\chi_{NM}}{T^2} = \frac{1}{VT^3}\sum_{m=-q_M}^{q_M}\sum_{n=-q_N}^{q_N} nm\langle S_{n,m}\rangle$$

$\langle S_{n,m}\rangle$ is the mean number of particles carrying charge $N = n$ and $M = m$

Variance at 200 GeV AA central coll. at RHIC

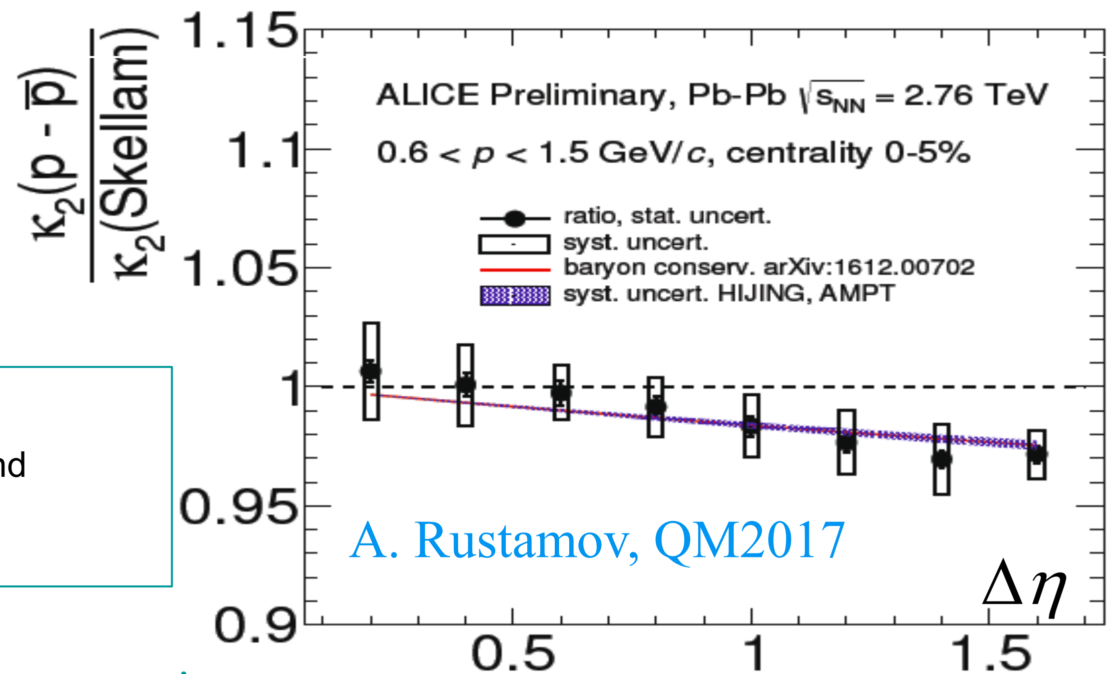


STAR Collaboration data in central coll. 200 GeV

- Consistent with Skellam distribution

$$\frac{<p>+<\bar{p}>}{\sigma^2} = 1.022 \pm 0.016 \qquad \frac{\chi_1}{\chi_3} = 1.076 \pm 0.035$$

- ALICE data consistent with Skellam $\Delta\eta < 1$



- Skellam distribution is a good approximation to calculate the 2nd order charge fluctuations in HIC

The influence of baryon number conservation: P. Braun-Munzinger, A. Rustamov, J. Stachel. Nucl Phys. A960 (2017) 114

Variance at 200 GeV AA central coll. at RHIC

K. Morita, B. Friman and K.R. Phys.Lett. B741 (2015) 178

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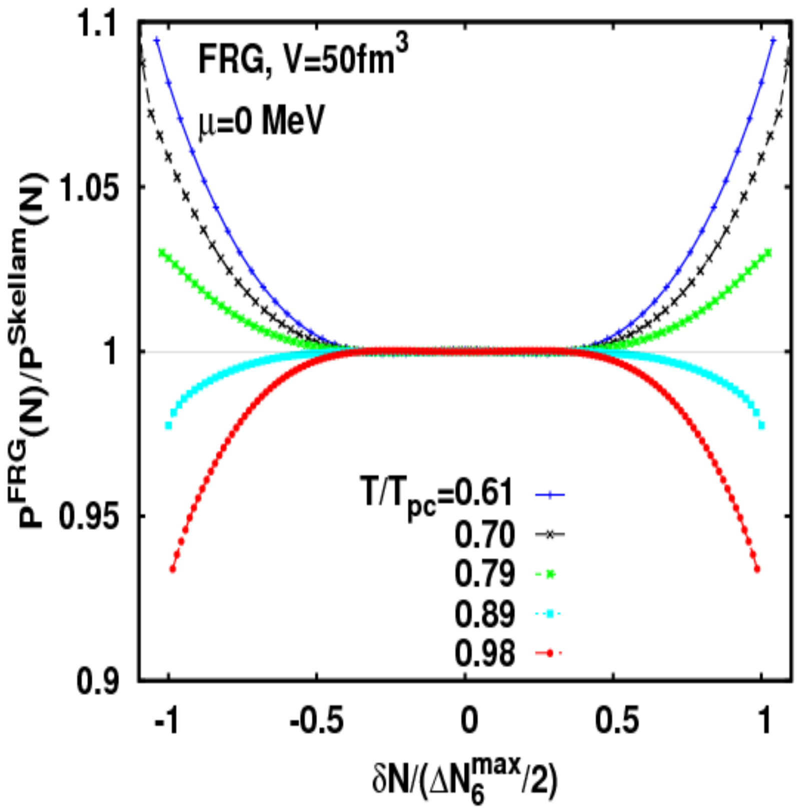

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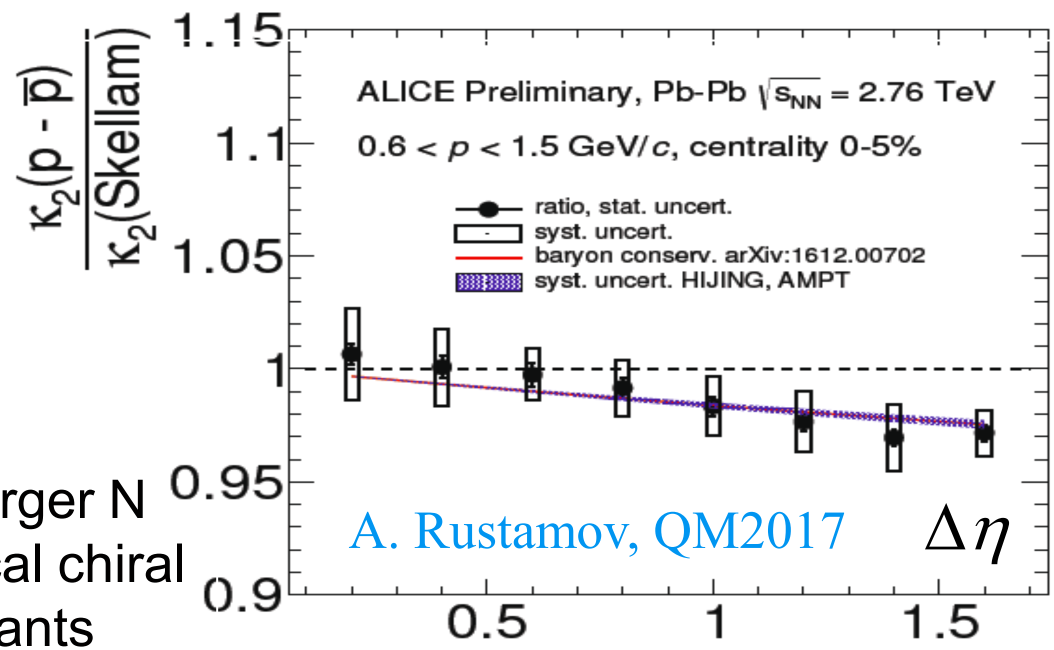

- Shrinking of Skellam distr. at larger N needed to capture the O(4) critical chiral properties of higher order cumulants

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K. Morita, B. Friman and K.R. Phys.Lett. B741 (2015) 178

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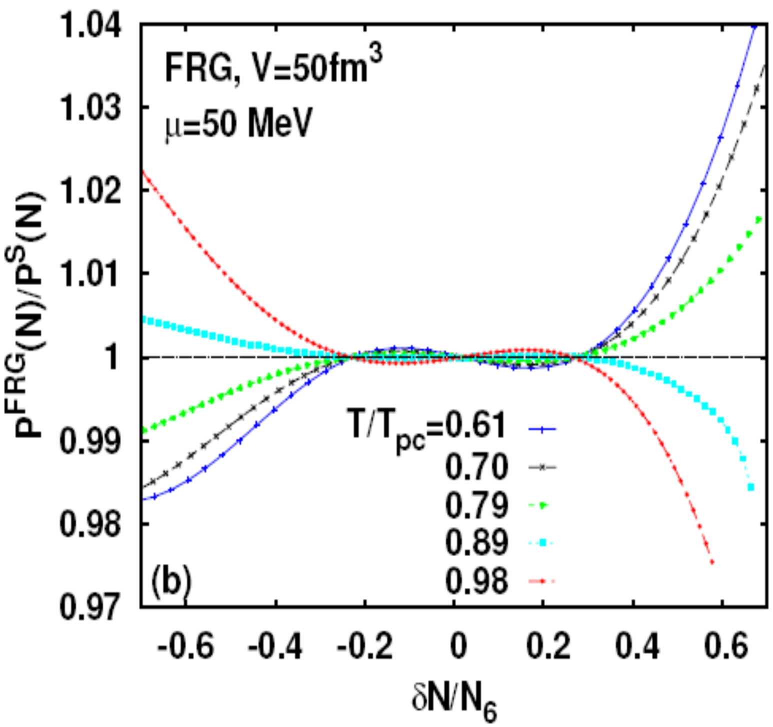

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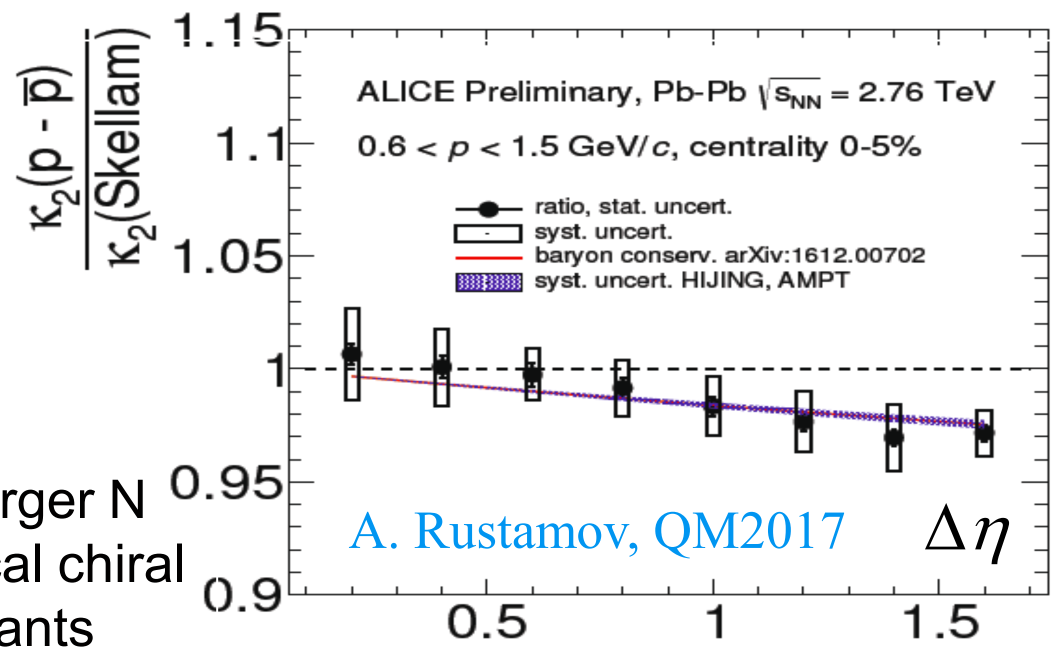

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Direct comparisons of Heavy ion data at LHC with LQCD

χ_{NM} with $N,M=\{B,Q,S\}$ are expressed by particle yields

$$\frac{\chi_B}{T^2}\approx\frac{1}{VT^3}(\langle p\rangle+\langle N\rangle+\langle\Lambda+\Sigma_0\rangle+\langle\Sigma^+\rangle+\langle\Sigma^-\rangle+\langle\Xi^-\rangle+\langle\Xi^0\rangle+\langle\Omega^-\rangle+\overline{par}\ )$$

LQCD From ALICE DATA

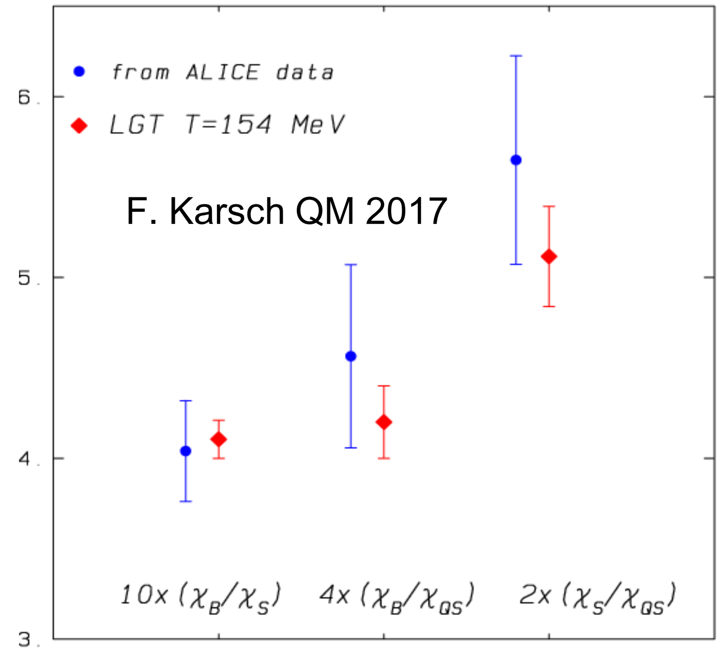
$$\frac{\chi_B}{T^2}=\frac{1}{VT^3}(203.7\pm11.4)$$

$$\frac{\chi_S}{T^2}=\frac{1}{VT^3}(504.2\pm16.8)$$

$$\frac{\chi_{QS}}{T^2}=\frac{1}{VT^3}(191.1\pm12)$$

- The Volume at $T\approx154\ MeV$

$$V_{T_c}=3800\pm500\ fm^3$$

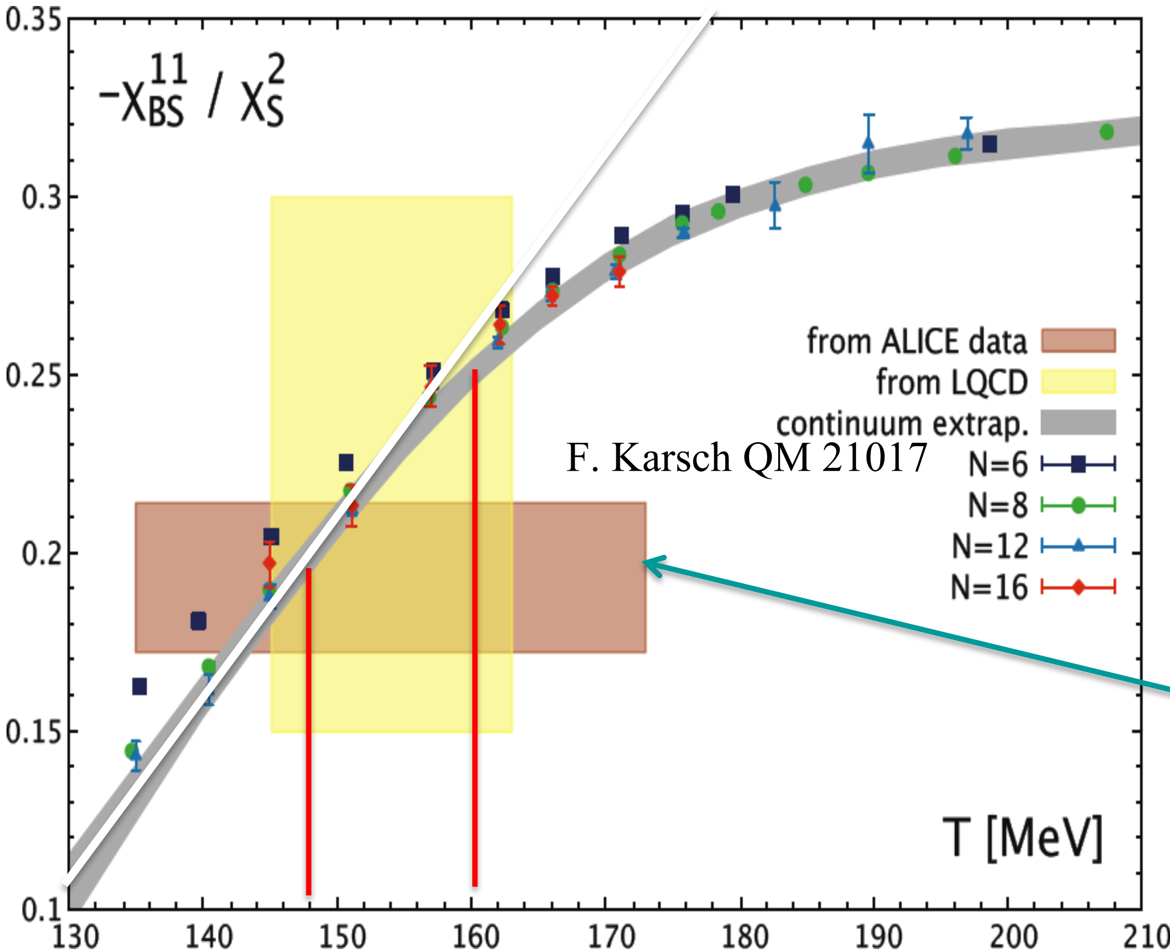


The cumulant ratios extracted from ALICE data are consistent with LQCD results at

$$0.148\le T_f<160\ MeV$$

Evidence for thermalization and saturation of yields/ 2nd order fluctuations at the phase boundary

Constraining chemical freezeout temperature at the LHC



At the LHC energy the fireball created in HIC is a QCD medium at the chiral cross over temperature.

$$C_{BS} = -\frac{<(\delta B)(\delta S)>}{<(\delta S)^2>} = -\frac{\chi_{BS}}{\chi_S}$$

- Excellent observable to fix the temperature

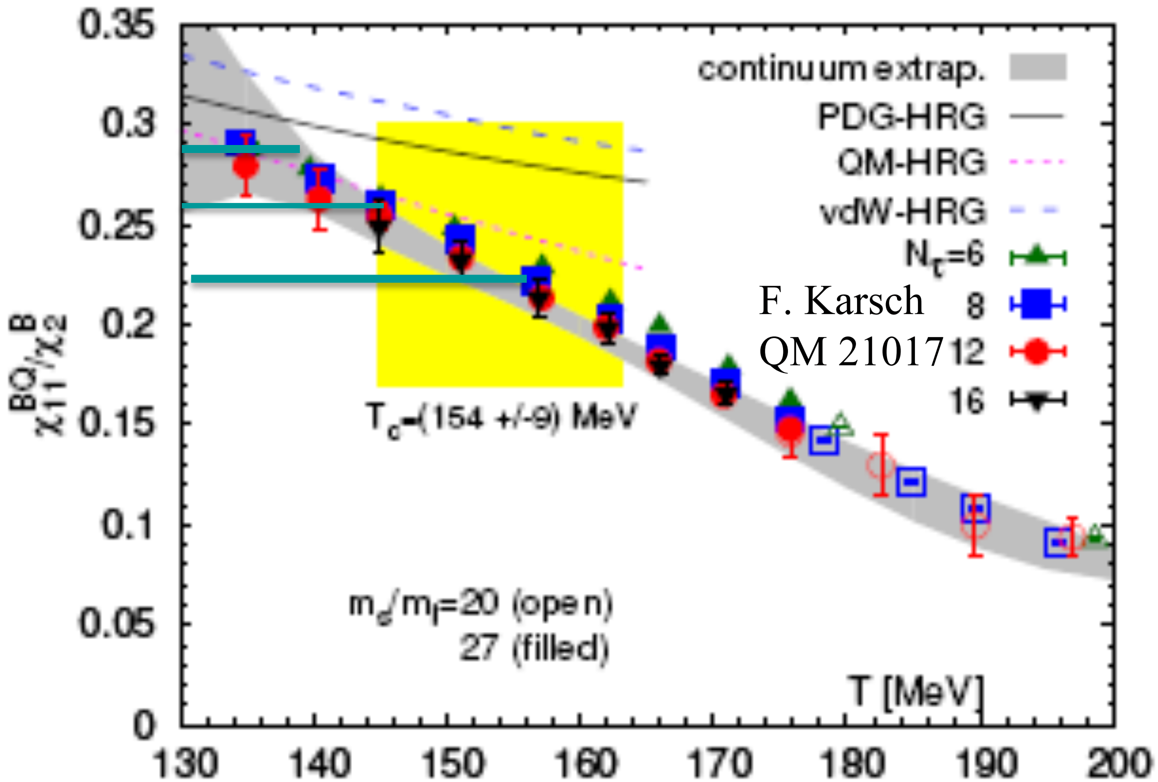
$$-\frac{\chi_{BS}}{T^2} \approx \frac{1}{VT^3}[2<\Lambda+\Sigma^0>+4<\Sigma^+>$$
$$+8<\Xi>+6<\Omega^->] = (97.4 \pm 5.8)/VT^3$$

However, this is the lower limit since e.g. $\Sigma^*(\geq 1660) \rightarrow N\bar{K}$ $\Lambda^*(\geq 1520) \rightarrow N\bar{K}$ are not included

- Data on χ_B/χ_S and χ_B/χ_{QS} χ_{BQ}/χ_B consistent with LQCD results for

$$148 \leq T_f < 160\ MeV$$

Constraining the upper value of the chemical freeze-out temperature at the LHC



- Considering the ratio

$$\frac{<(\delta B)(\delta Q)>}{<(\delta B)^2>} = \frac{\chi_{BQ}}{\chi_B} = 0.26 \pm 0.03$$

one gets $T < 156\ MeV$

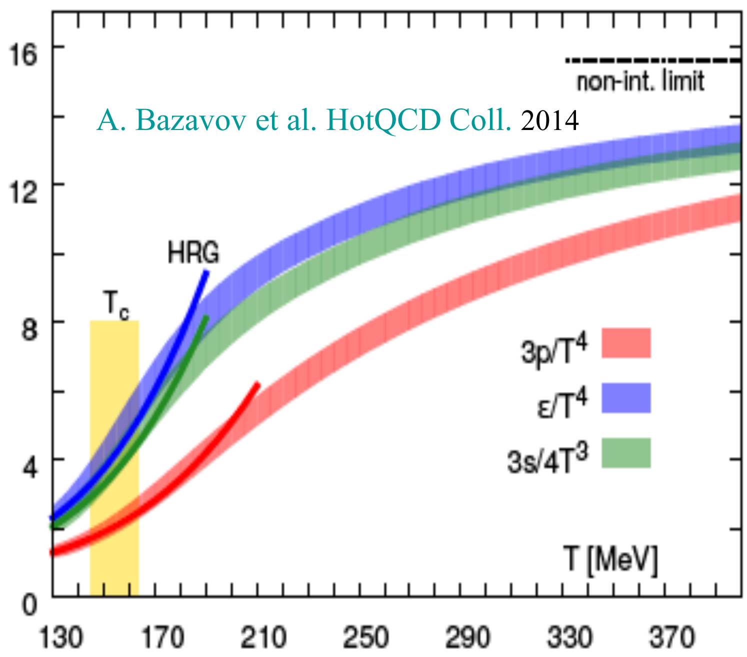
- From the comparison of 2nd order fluctuations and correlations observables constructed from ALICE data and LQCD, one gets agreement at

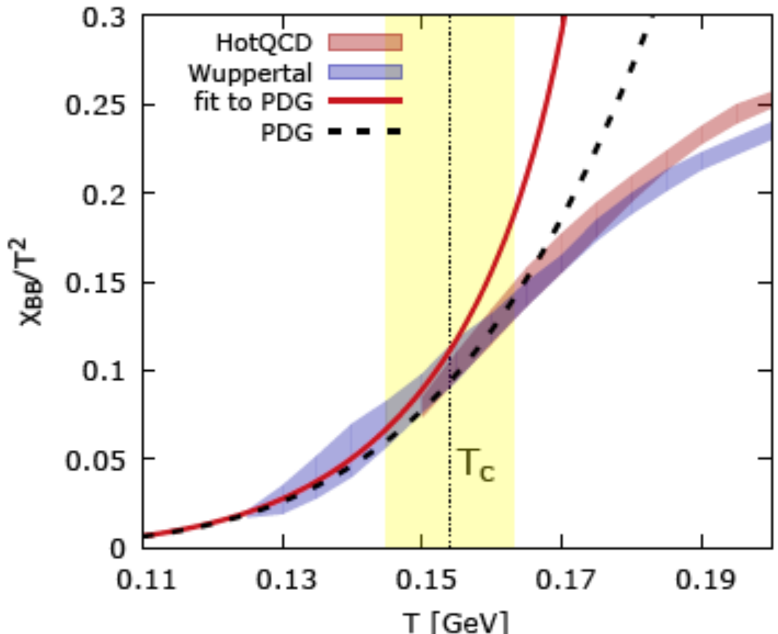
$$148 \le T_f < 156\ MeV$$

Particle yields data at the LHC consistent with LQCD at the phase boundary

Good description of the QCD Equation of States by Hadron Resonance Gas

The observed Skellam destribution of 2nd order fluctuations supports Modelling statistical operator of QCD in the hadronic phase as the Hadron Resonance Gas (HRG): mixture of ideal gases of all known stable hadrons and resonances





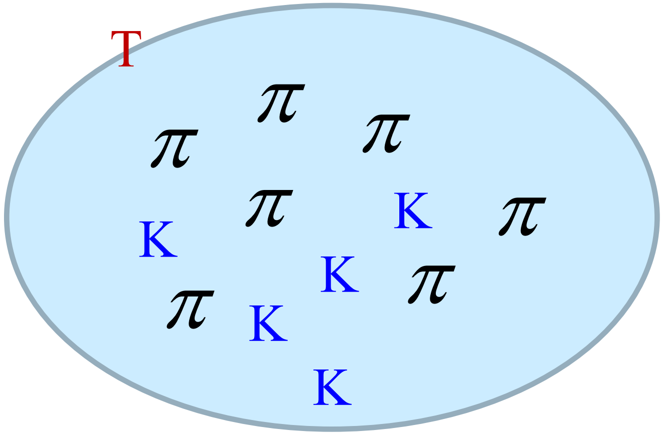
P.M. Lo, M Marczenko et al. Eur. Phys.J. A52 (2016)

- Hadron Gas thermodynamic potential provides an excellent approximation of the QCD equation of states in confined phase
- as well as, good description of the net-baryon number fluctuations which can be improved by adding baryonic resonances expected in the Hagedorn mass spectrum

HRG in the S-MATRIX APPROACH

R. Dashen, S. K. Ma and H. J. Bernstein,
Phys. Rev. 187, 345 (1969)

W. Weinhold, & B. Friman
Phys. Lett. B 433, 236 (1998).



- Consider interacting pions and kaons gas in thermal equilibrium at temperature T
- Due to *K*π scattering resonances are formed

  I =1/2, s -wave : κ(800), K0*(1430) [*JP* = 0+]

  I =1/2, p -wave : *K**(892), *K**(1410), *K**(1680) [*JP* =1−]
- In the S-matrix approach the thermodynamic pressure in the low density approximation

$$P(T) \approx P_{\pi}^{id} + P_{K}^{id} + P_{\pi K}^{\text{int}}$$

Thermodynamic pressure of an ideal gas:

$$P = P^{id} / T^4 = -\int \frac{d^3 p}{(2\pi)^3} \left\{ \ln\left[1 - e^{-\sqrt{p^2+M^2}-\mu}\right] + \ln\left[1 - e^{-\sqrt{p^2+M^2}+\mu}\right] \right\}$$

S-MATRIX APPROACH: INTERACTIG PART

The leading order corrections , determined by the two-body scattering phase shift, which is equivalent to the second virial coefficient

$$P_{\text{int}} = \int_{m_{th}}^{\infty} \frac{dM}{2\pi} B(M) P_T(M)$$

$$B(M) = 2 \frac{d}{dM} \delta(M)$$

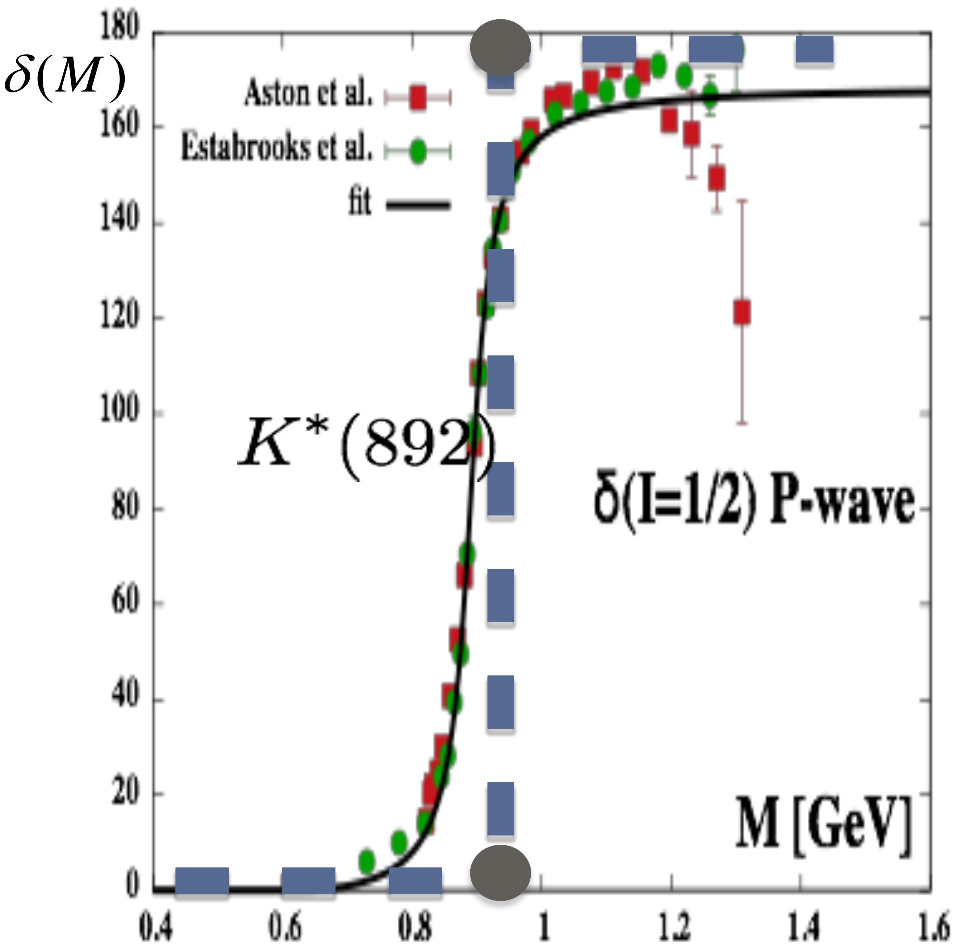
$$\int_{m_{th}}^{\infty} \frac{dM}{2\pi} B(M) = 1$$

Effective weight function — *Scattering phase shift* — *Normalization*

Pressure of an ideal gas of resonaces with an invariant mass M

$$P_T(M) = -2 \int \frac{d^3 p}{(2\pi)^3} \left\{ \ln\left[1 - e^{-\sqrt{p^2+M^2} - \mu} \right] + \ln\left[1 - e^{-\sqrt{p^2+M^2} + \mu} \right] \right\}$$

Experimental phase shift in the P-wave channel



B. Friman, P. M. Lo, M. Marczenko, K. Redlich and C. Sasaki, Phys. Rev. D 92, no. 7, 074003 (2015)

For narrow resonance

$$B(M) = 2\frac{d}{dM}\delta(M)$$

very well described by the Breit-Wigner form

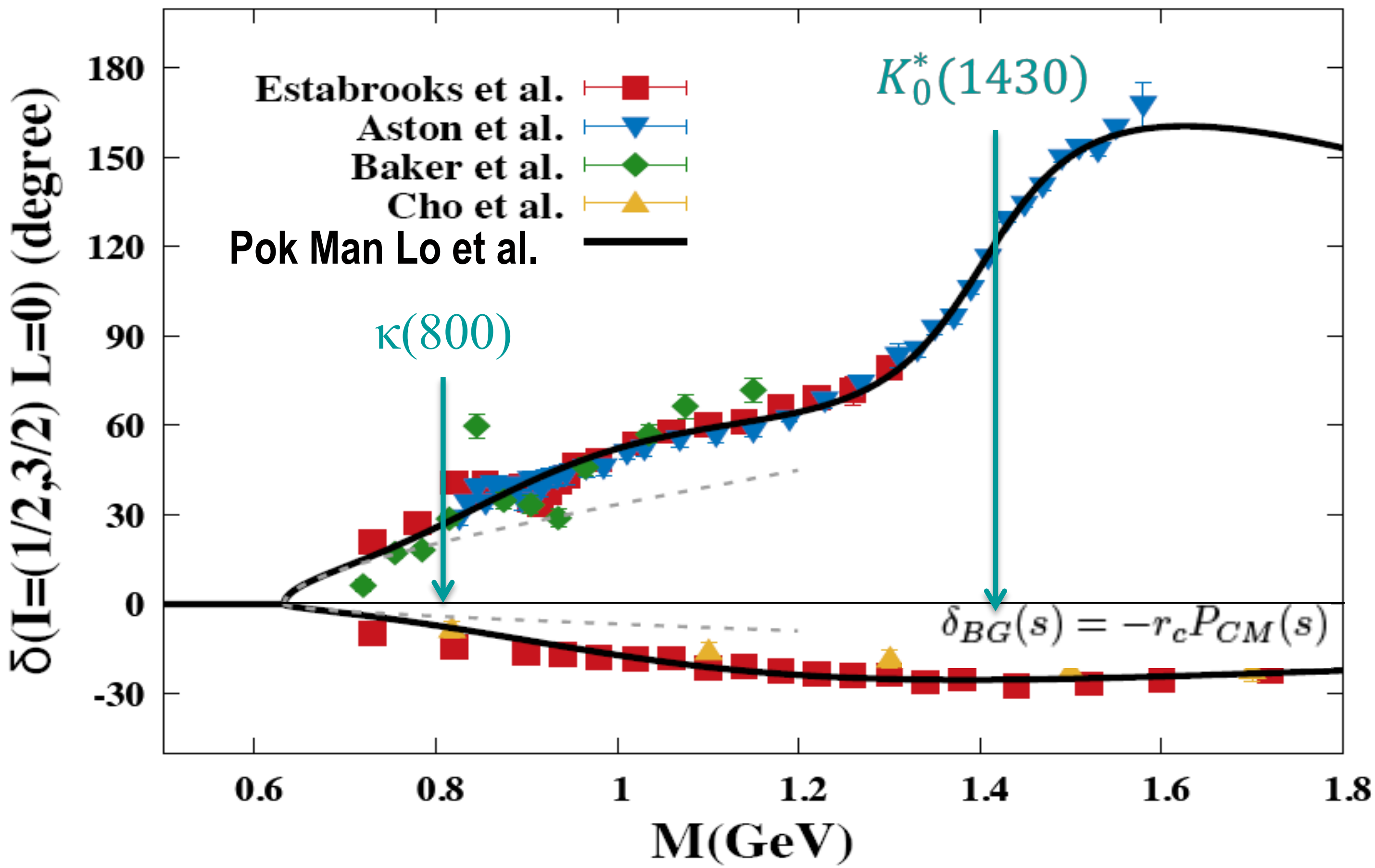
$$B(M) \approx M \frac{2M\gamma_{\rm BW}}{(M^2 - M_0^2)^2 + M^2\gamma_{\rm BW}^2}$$

for $\gamma_{BW} \to 0$

$$B(M) = \delta(M^2 - M_0^2)$$ **and**

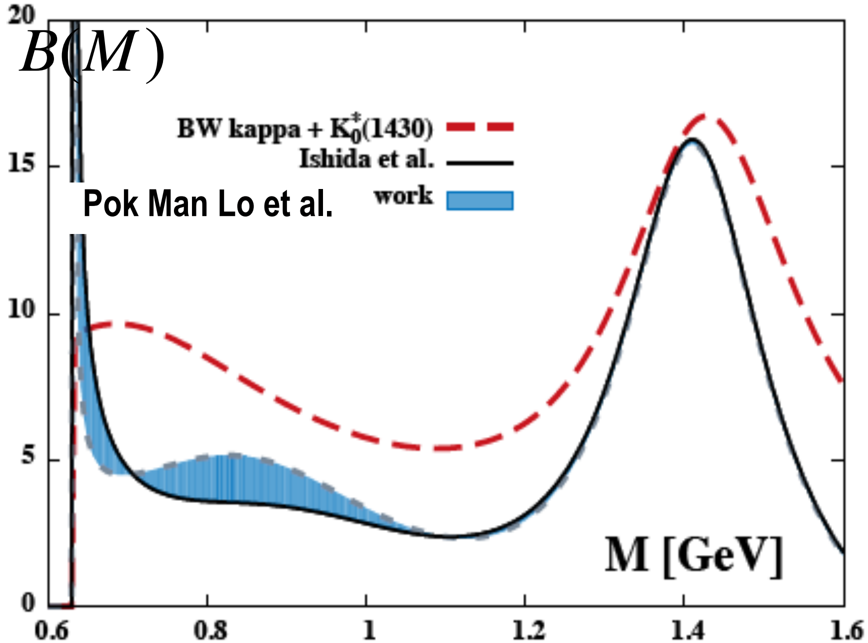
$$P_{\pi K}^{\rm int}(T) \approx P_{K^*}^{id}(T)$$

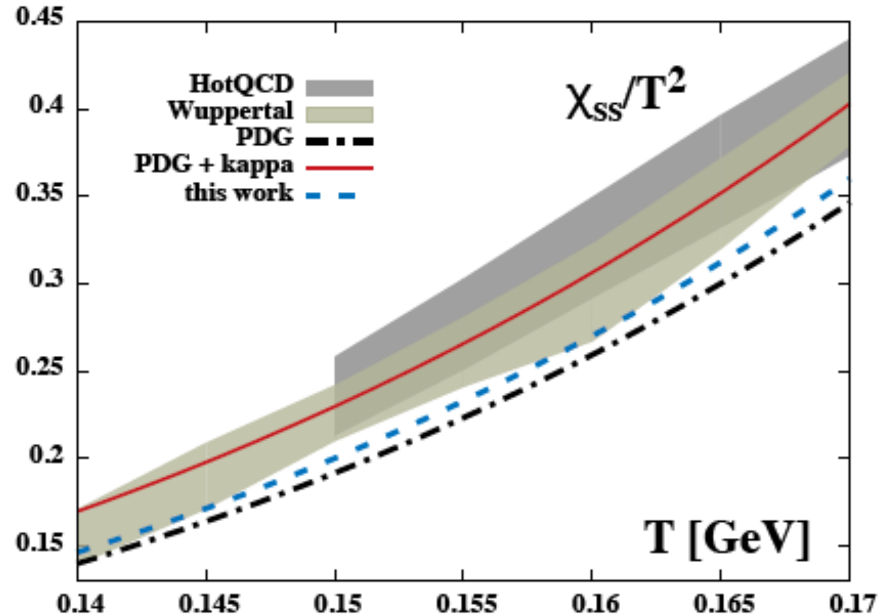
Non-resonance contribution- negative phase shift in S-wave channel



$$\delta_{All}^{L=0} = \delta_\kappa + \delta_{K_0^*(1430)} + \delta_{BG} \Rightarrow B(M) = 2\frac{d}{dM}\delta_{All}^{L=0}(M) \neq \sum_{i=1}^{2}\delta(M^2 - M_i^2)$$

S-matrix approach to strangeness fluctuations





In the S-matrix approach essential reduction of the contribution of S-wave resonances relative to the BW approach in the HRG

B. Friman, P. M. Lo, M. Marczenko, K. Redlich and C. Sasaki, Phys. Rev. D 92, no. 7, 074003 (2015)

Similar arguments also apply to sigma meson

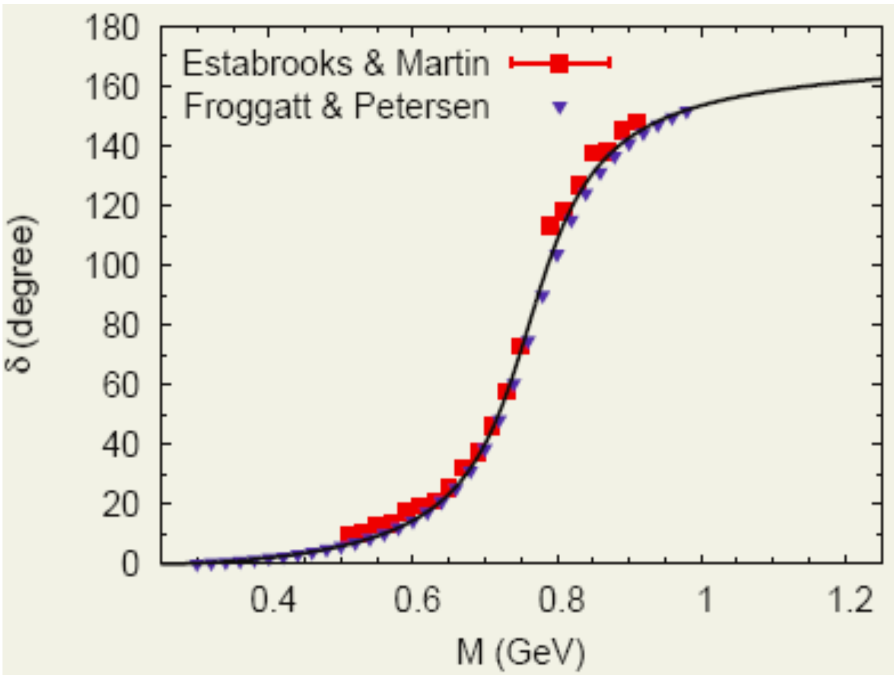
V. Begun and W. Florkowski Phys.Rev. C91 (2015) 054909

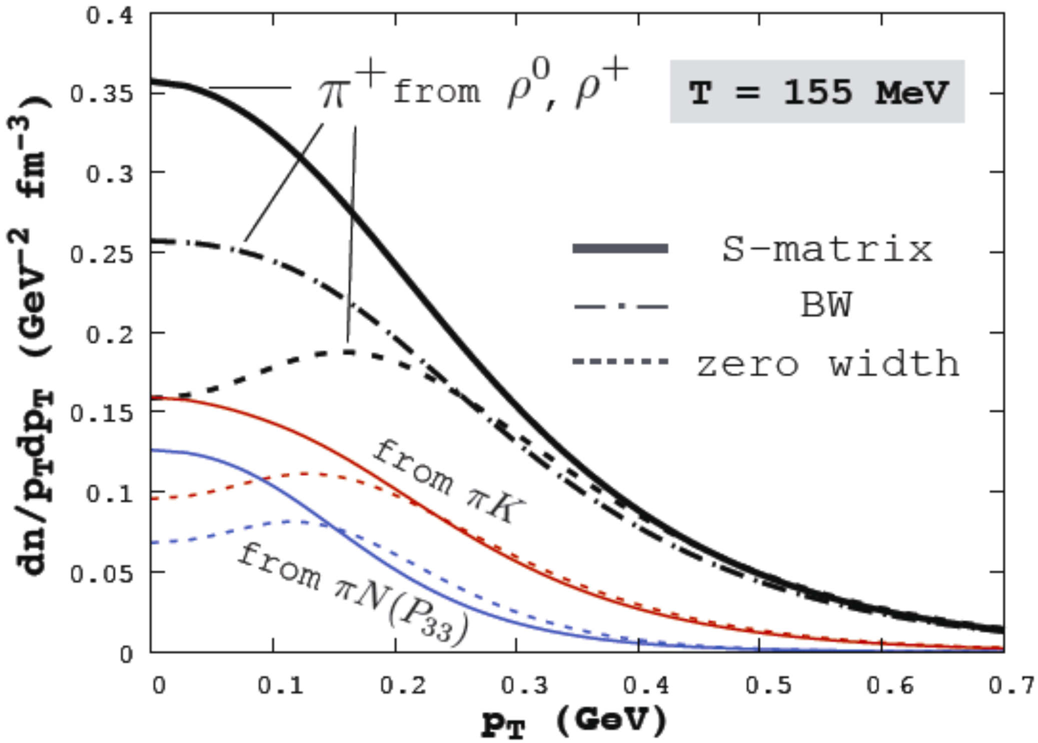
To quantify χ_{SS}, χ_{BS} in the hadronic phase within HRG one needs additional resonances beyond that known in the PDG (F. Karsch at al.)

S-matrix approach: Pion spectra

$\pi\pi$ **scattering, P-wave, i.e.** ρ **resonance contribution**

P. Huovinen, P.M. Lo, M. Marczenko, K. Morita, K. Redlich and C. Sasaki, Phys. Lett. B 769, 509 (2017)



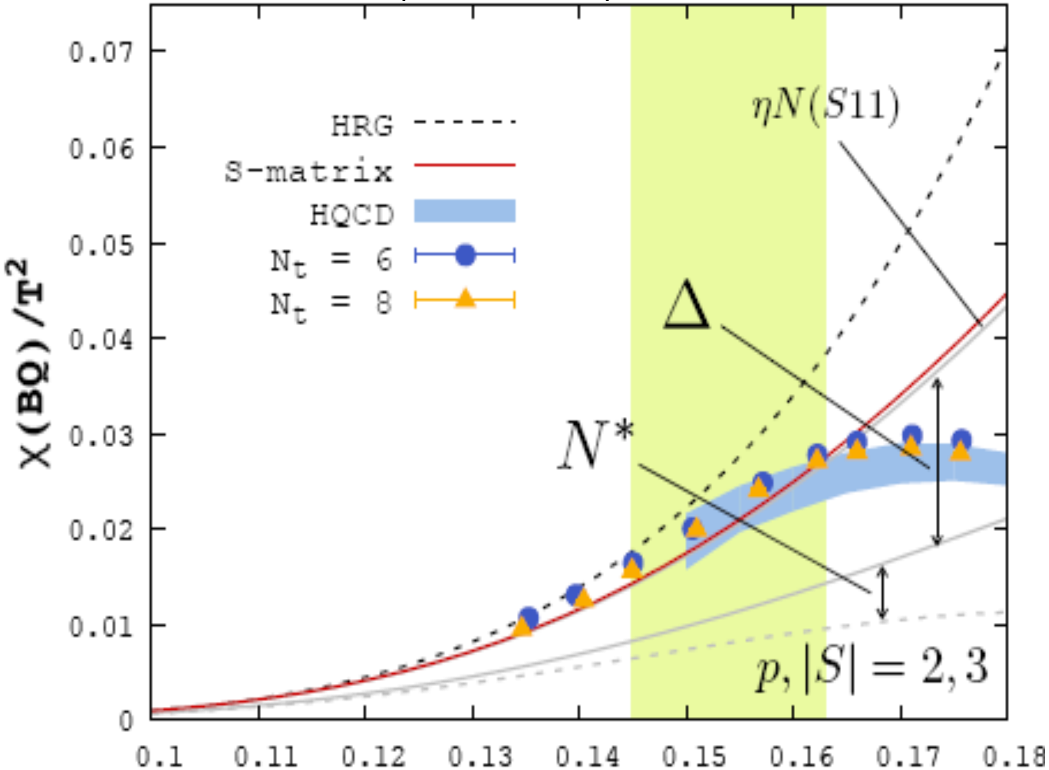


$$\frac{dN_\pi^{de}}{dy\, p_T\, dp_T\, d\phi} = V E_\pi \int dM_\rho \frac{1}{2\pi} \mathcal{B}(M_\rho) \times \frac{M_\rho}{2\, p_\pi p_{\mathrm{CM}}} \int_{E_\rho^-}^{E_\rho^+} dE_\rho\, E_\rho \frac{d_\rho}{(2\pi)^3} f_\rho(E(M_\rho), T),$$

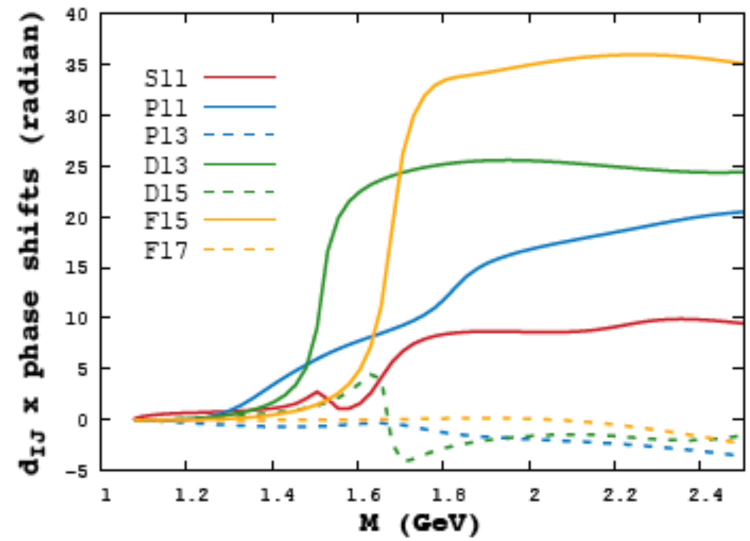
Large increase of soft pions obtained in the S-matrix approach

Probing non-strange baryon sector

Pok Man Lo, B. Friman, C. Sasaki & K.R.



$$\chi_{BQ} = (\chi_{BB} - |\chi_{BS}|)/2$$



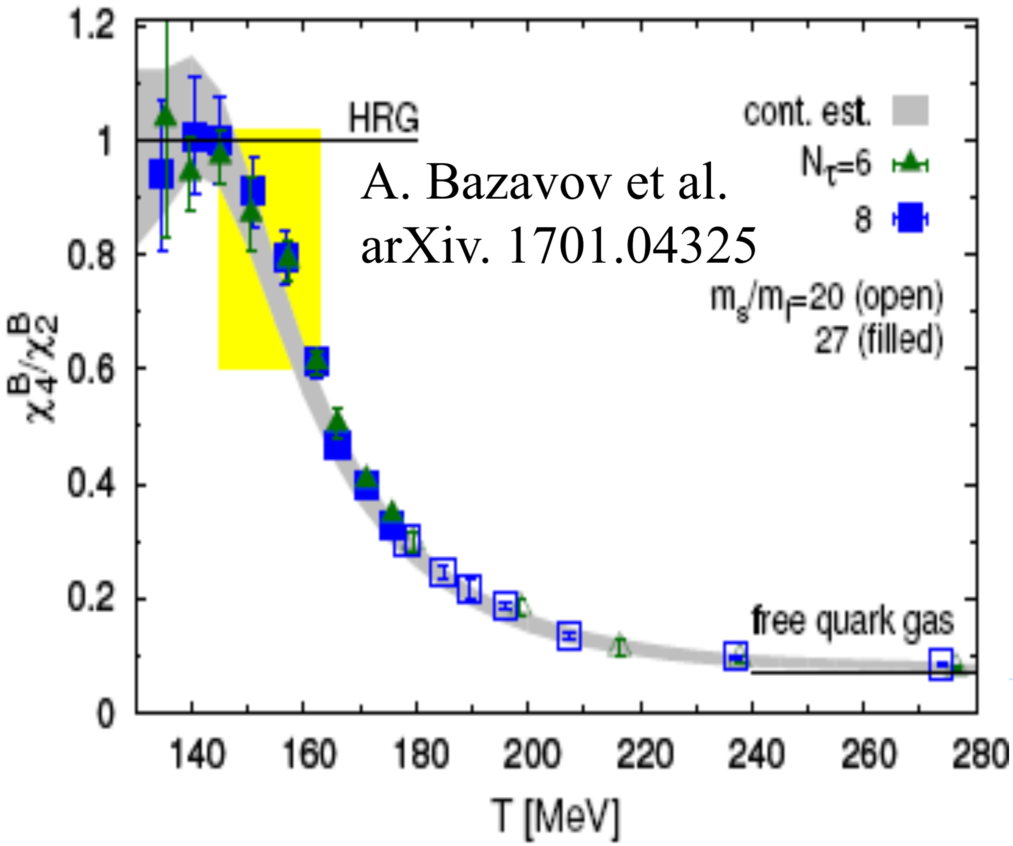
$$\frac{dN_\pi^{de}}{dy\, p_T\, dp_T\, d\phi} = V E_\pi \int dM_\rho \, \frac{1}{2\pi} \mathcal{B}(M_\rho)$$

$$\times \frac{M_\rho}{2\, p_\pi p_{\rm CM}} \int_{E_\rho^-}^{E_\rho^+} dE_\rho \, E_\rho \frac{d_\rho}{(2\pi)^3} f_\rho(E(M_\rho), T),$$

- Considering contributions of all N^*, Δ^* resonances to χ_{BQ} with correctly implemented dynamical widths within S-matrix approach imply the reduction of the HRG contribution towards the LQCD in the chiral crossover

Deviations of Fluctuations of net charges

due to deconfinement and partial chiral symmetry restoration in QCD



$$\chi_n^B = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n}$$

- HRG factorization of pressure:

$$P^B(T,\mu_q) = F(T)\cosh(B\mu_B/T)$$

- Kurtosis measures the squared of the baryon number carried by leading particles in a medium

S. Ejiri, F. Karsch & K.R. (06)

$\frac{1}{9}$

$$\kappa\,\sigma^2 = \frac{\chi_4^B}{\chi_2^B} \approx B^2 = \begin{pmatrix} 1 & T < T_{PC} \\ \frac{1}{9} & T > T_{PC} \end{pmatrix}$$

Modelling fluctuations in the O(4)/Z(2) universality class

Gabor Almasi, Bengt Friman & K.R., Phys. Rev. D96 (2017) no.1, 014027

$$\mathcal{L} = \bar{q}\left(i\gamma^\mu D_\mu - g(\sigma + i\gamma_5 \vec{\tau}\vec{\pi}) - g_\omega \gamma^\mu \omega_\mu\right) q + \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 - U_m(\sigma, \vec{\pi})$$
$$- \mathcal{U}(\Phi, \bar{\Phi}; T) - \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

Effective potential is obtained by solving *the exact flow equation* (Wetterich eq.) with the approximations resulting in the O(4)/Z(2) critical exponents

$$\partial_k \Omega_k(\sigma) = \frac{V k^4}{12\pi^2}\left[\sum_{i=\pi,\sigma} \frac{d_i}{E_{i,k}}[1 + 2n_B(E_{i,k})] - \frac{2\nu_q}{E_{q,k}}[1 - n_F(E^+_{q,k}) - n_F(E^-_{q,k})]\right]$$

Full propagators with $k < q < \Lambda$

$$E_{\pi,k} = \sqrt{k^2 + \Omega'_k}$$
$$E_{\sigma,k} = \sqrt{k^2 + \Omega'_k + 2\rho\Omega''_k}$$
$$E_{q,k} = \sqrt{k^2 + 2g^2\rho}$$

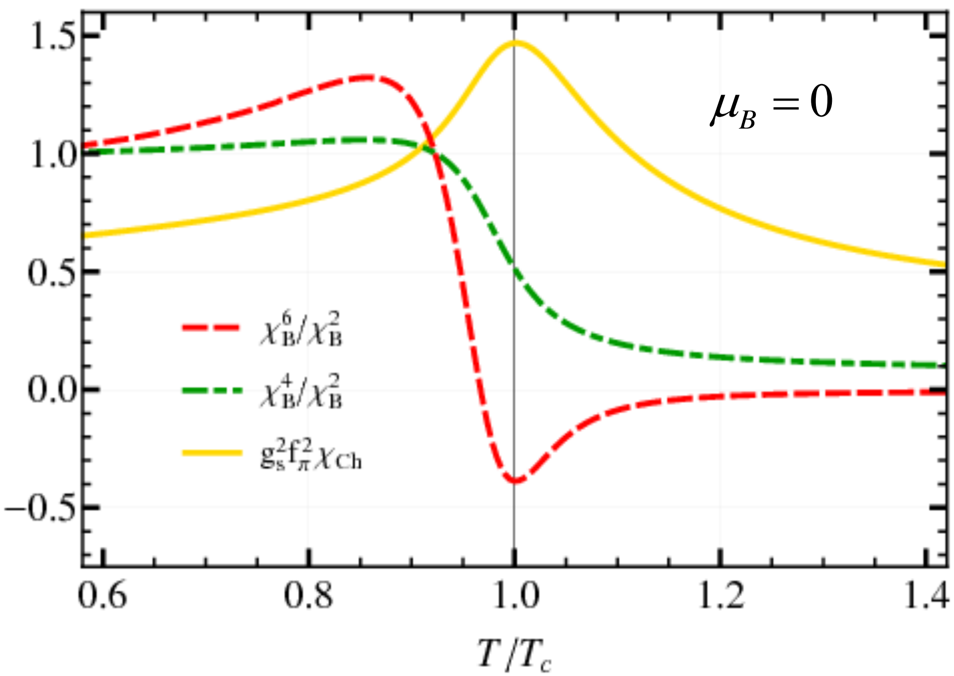
$\Gamma_\Lambda = S_{\text{classical}}$

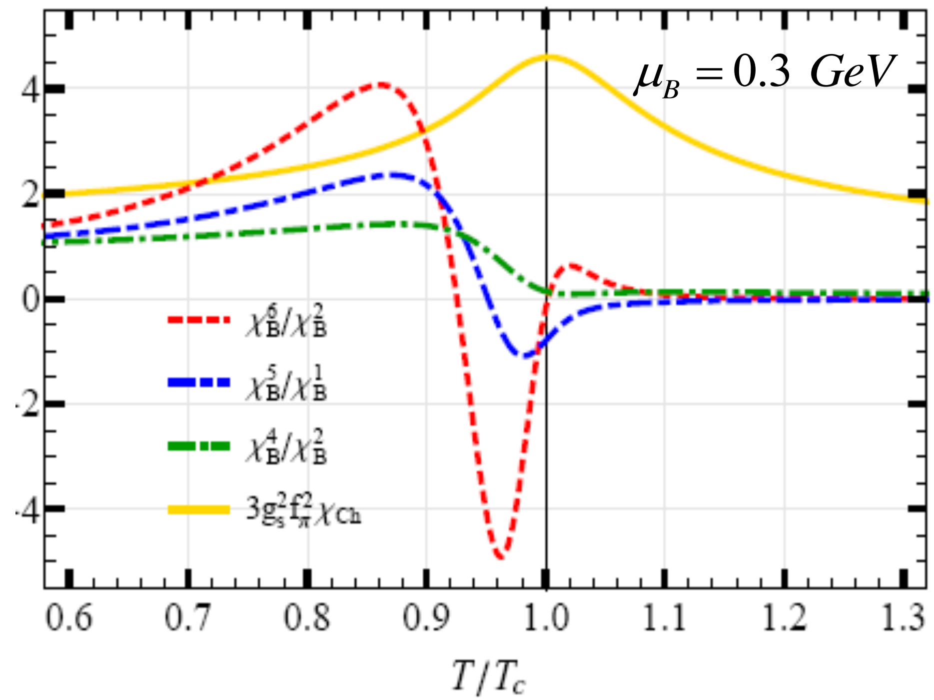
⇨ Integrating from $k=\Lambda$ to $k=0$ gives full quantum effective potential

$$\Omega'_k \equiv \frac{\partial \Omega_k}{\partial(\sigma^2/2)}$$

Higher order cumulants in effective chiral model within FRG approach, belongs to the O(4)/Z(2) universality class

B. Friman, V. Skokov &K.R. Phys. Rev. C83 (2011) 054904
G. Almasi, B. Friman &K.R. Phys. Rev. D96 (2017) no.1, 014027

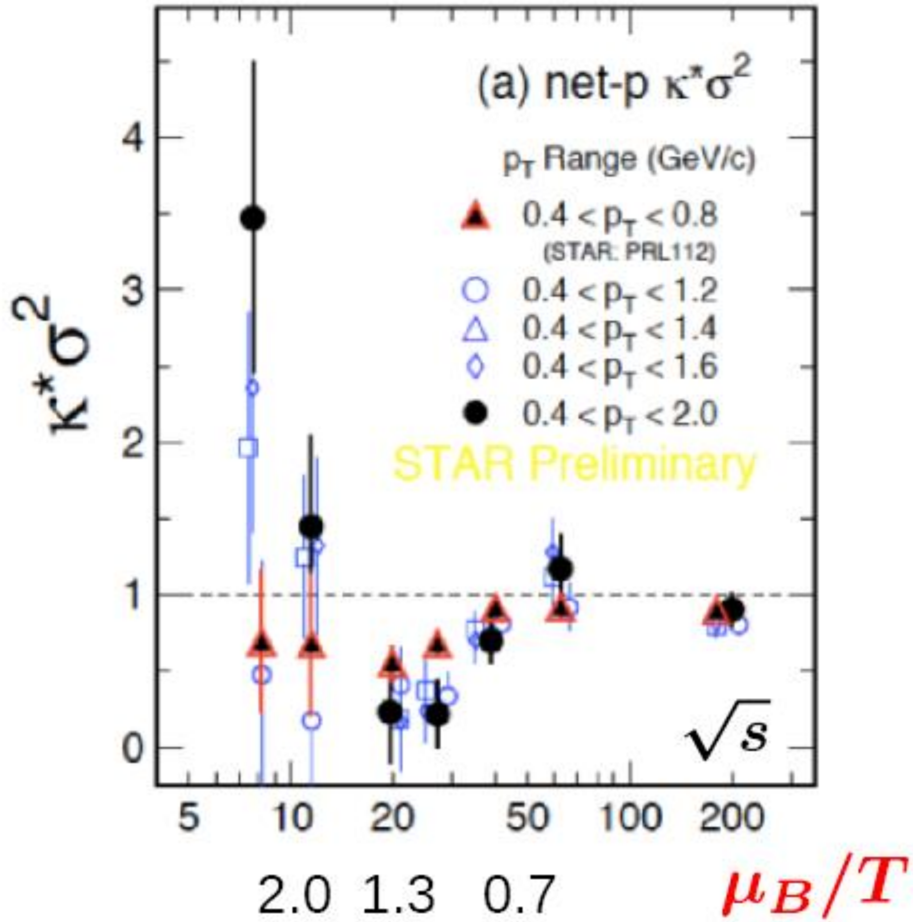




Deviations of cumulant ratios from Skellam distribution are increasing with the order of the cumulants and can be used to identify the chiral QCD phase boundary in HIC

STAR "BES" and recent results on net-proton fluctuations

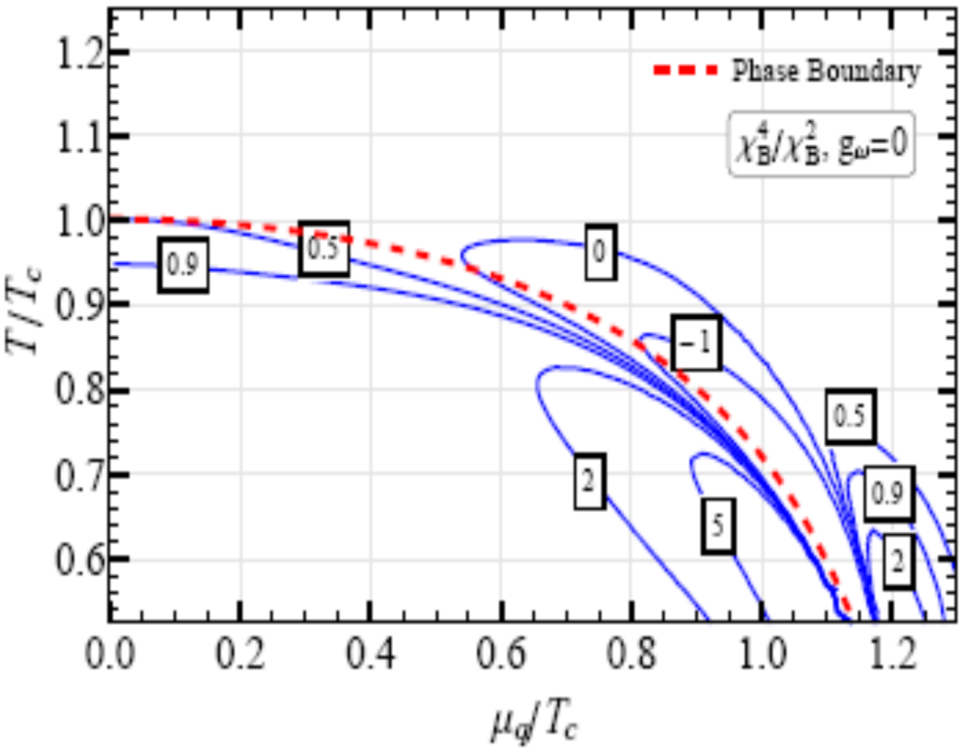
X. Luo et al. (2015), STAR Coll. Preliminary



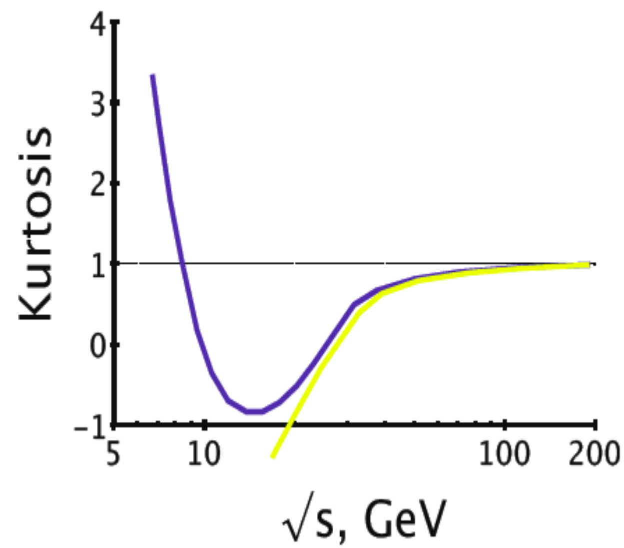
See also talk of Lijun Ruan

- With increasing acceptance of the transverse momentum, large increase of net-proton fluctuations at $\sqrt{s} < 20$ GeV beyond that of a non-critical reference of a HRG
- Is the above an Indication of the CEP?
- At $\sqrt{s} > 20$ GeV data consistent with LQCD results near the chiral crossover

Modelling critical fluctuations



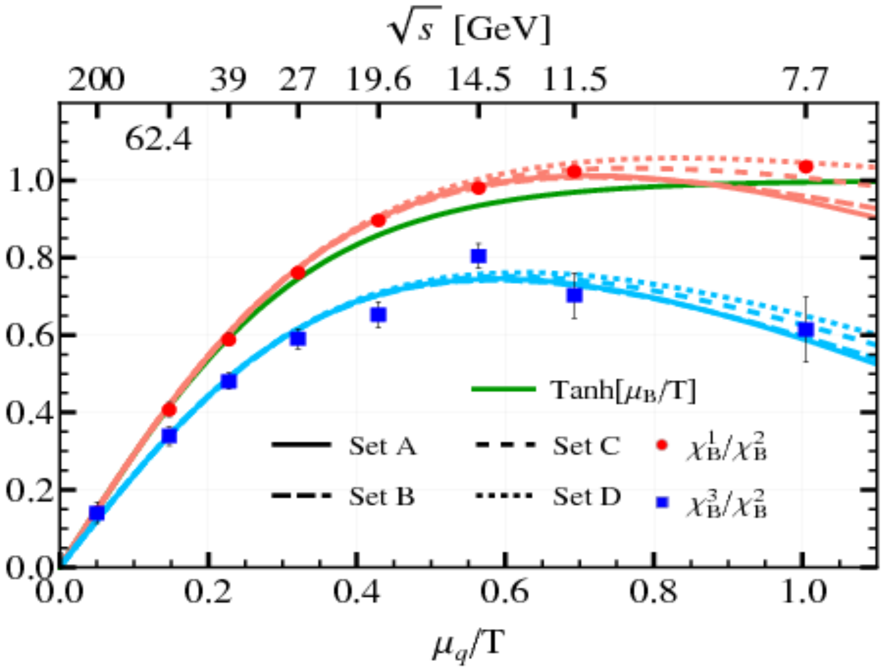
- It is possible to find the freeze-out line such that kurtosis exhibits the energy dependence as seen in data.

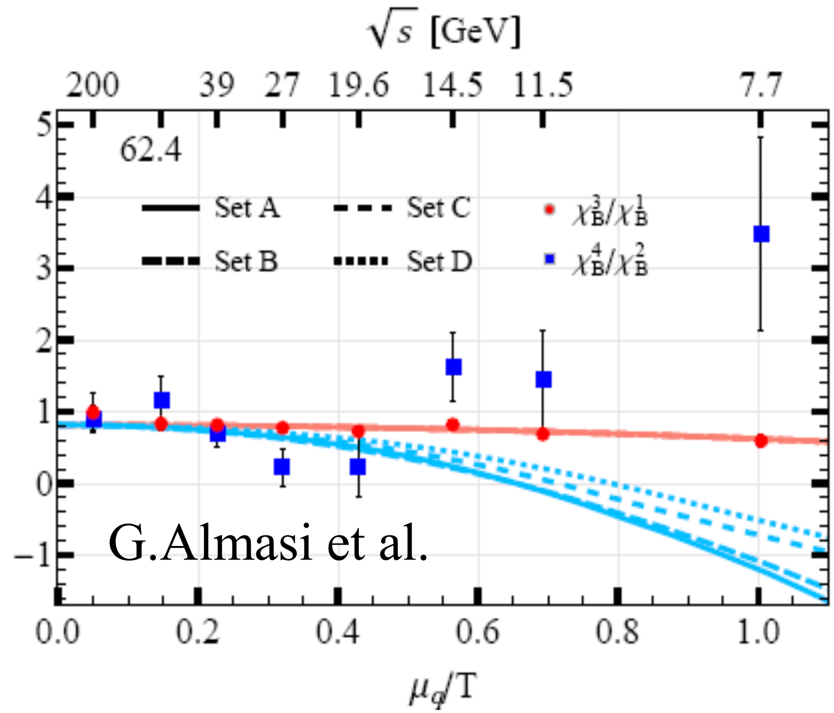


- However, are other cumulants consistent?

Self - consistent freeze-out and STAR data

- Freeze-out line in $(T,\mu)-$ plain determined by fitting χ_B^1/χ_B^2 to data
- Ratio $\chi_B^1/\chi_B^2 \approx \tanh(\mu/T)$ => further evidence of equilibrium and thermalisation at $7\ GeV \le \sqrt{s} < 5\ TeV$
- Ratio $\chi_B^1/\chi_B^2 \neq \chi_B^3/\chi_B^2$ expected due to critical chiral dynamics
- Enhancement of χ_B^4/χ_B^2 at $\sqrt{s} < 20\ GeV$ not reproduced





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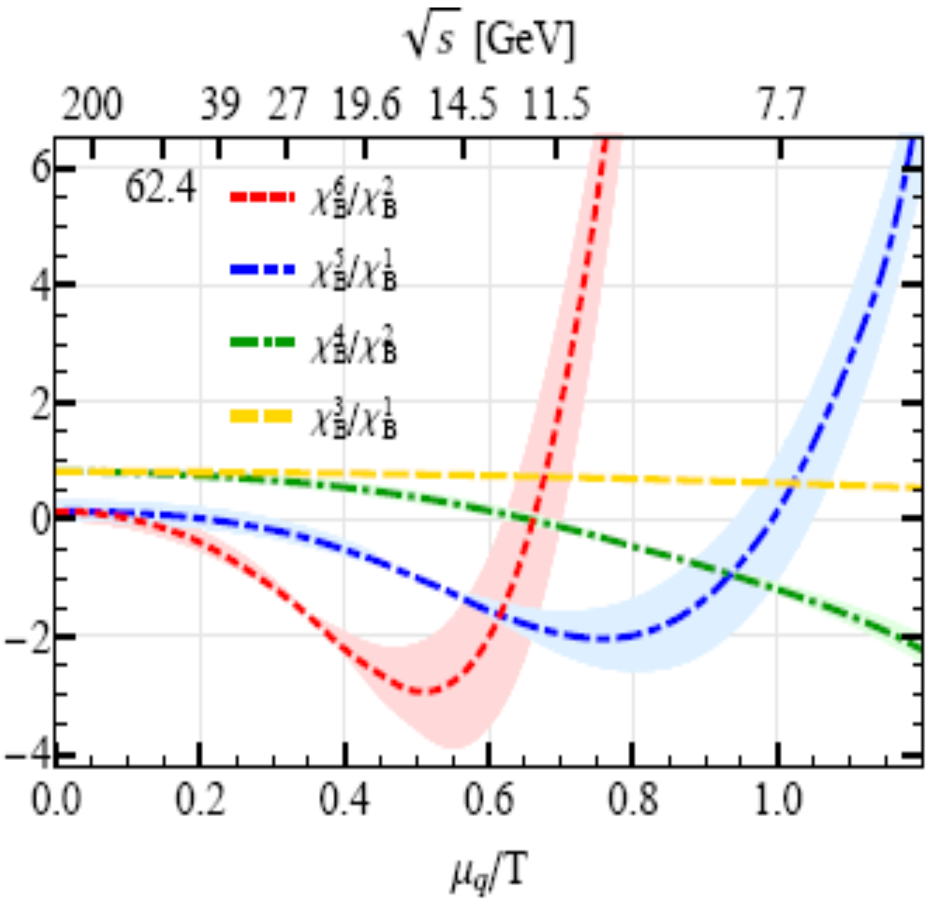
Higher order cumulants - energy dependence



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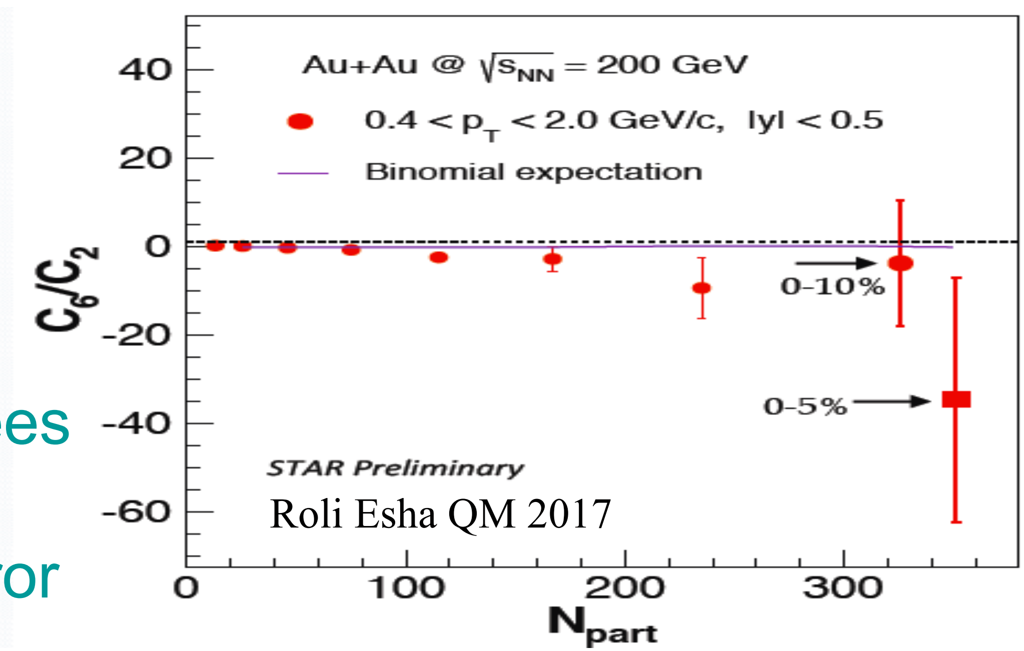
- Strong non-monotonic variation of higher order cumulants at lower $\sqrt{s}$
- Equality of different ratios excellent probes of equilibrium evolution in HIC



- At freeze-out fixed by χ_B^3/χ_B^1, the ratio $\chi_B^6/\chi_B^2 \approx 0$ which agrees with preliminary STAR data, albeit within still very large error

Conclusions:

The medium created in HIC at the LHC is of thermal origin and follows properties expected in LQCD near the phase boundary at

$$148 \leq T < 160 \ MeV$$

- The Hadron Resonance Gas is confirmed to be a very good approximation of QCD thermodynamics and provides also quantitative description of particle yields in HIC (see also talks by Peter Braun-Munzinger and Johanna Stachel)

  However to properly quantify fluctuation observables in the hadronic phase within HRG model one needs to include resonances beyond that known in the PDG, the dynamical widths of broad resonances and non-resonance interactions e.g. by using the phase shift data within S-matrix approach

- Systematics of the net-proton number fluctuations at $\sqrt{s} \geq 20$ GeV measured by STAR Coll. in HIC at RHIC is qualitatively consistent with the expectation, that they are influenced by the critical chiral dynamics and deconfinement