

Critical Enhancement and Diffusion of Non-Gaussian Fluctuations

Masakiyo Kitazawa
(Osaka U.)

Asakawa, MK, Prog. Part. Nucl. Phys. 90, 299 (2016)

MK, Luo, Phys. Rev. D95 (2017)

Nonaka, MK, Esumi, Phys. Rev. C95 (2017)

Sakaida, Asakawa, Fujii, MK, Phys. Rev. C95, 064905 (2017)

Ohnishi, MK, Asakawa, Phys. Rev. C94, 044905 (2016)

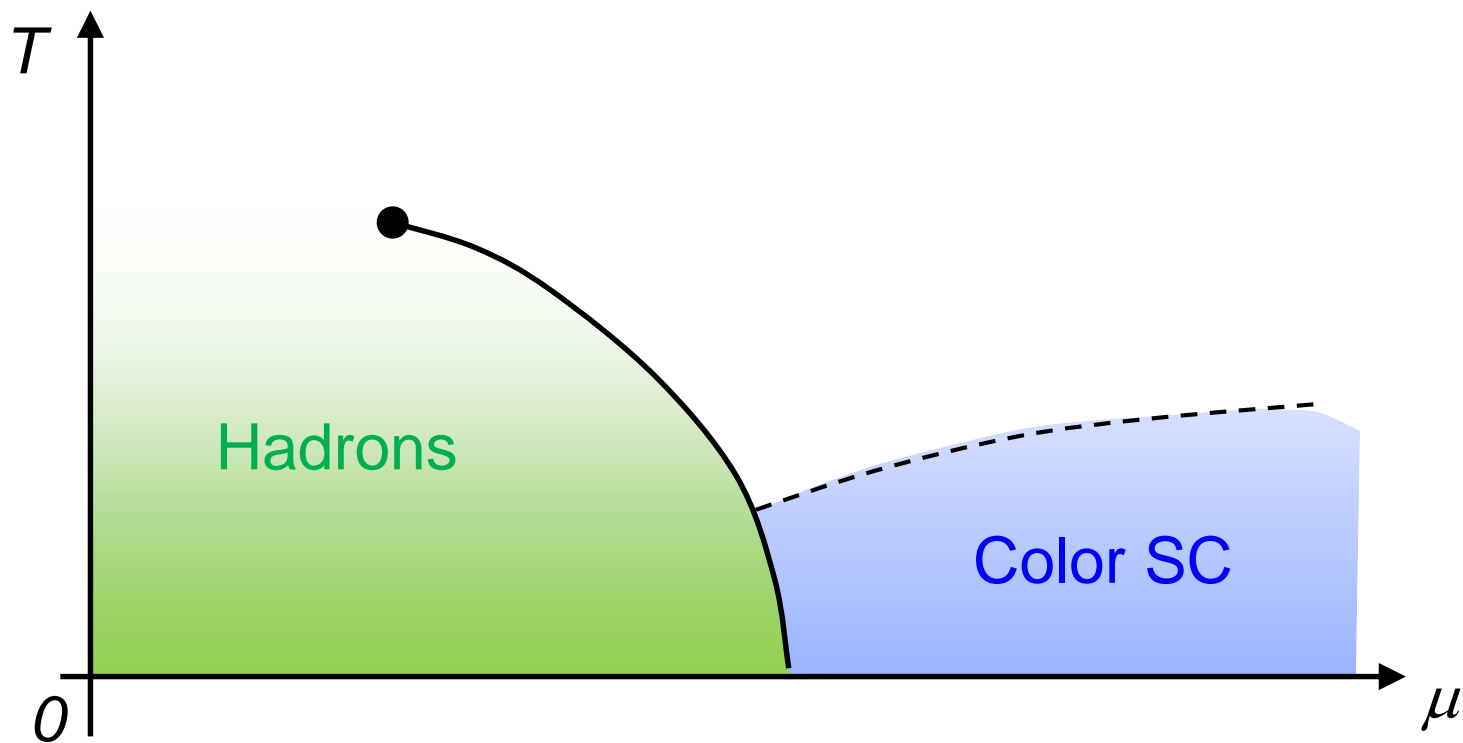
MK, Nucl. Phys. A942, 65 (2015)

Sakaida, Asakawa, MK, Phys. Rev. C90, 064901 (2014)

MK, Asakawa, Ono, Phys. Lett. B728, 386 (2014)

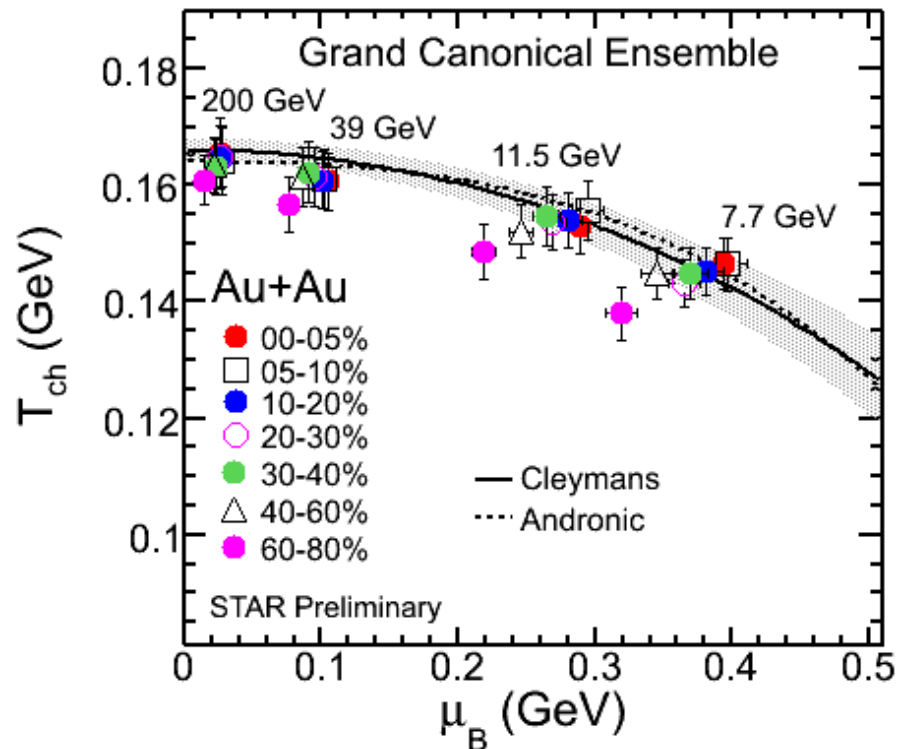
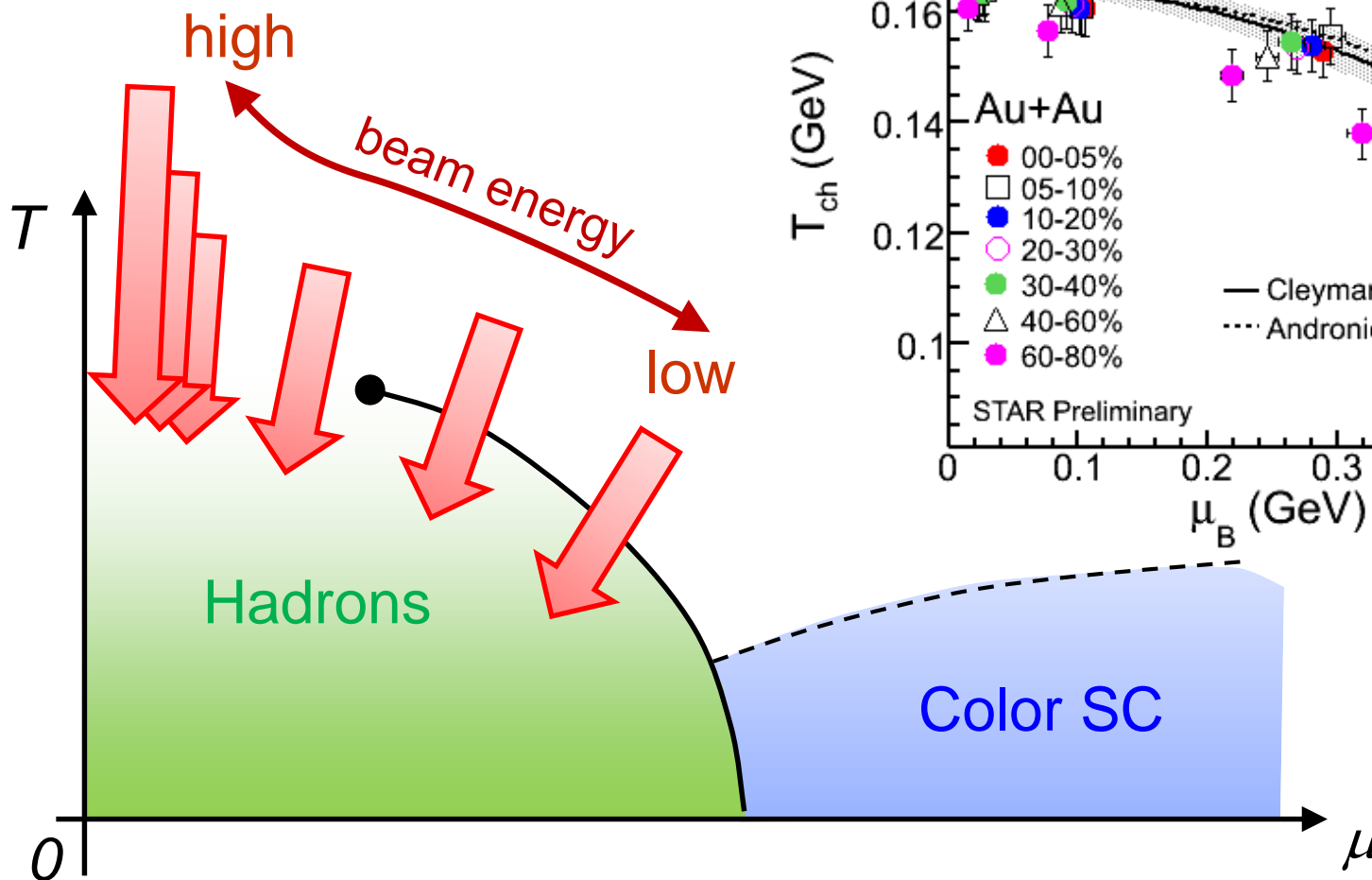
EMMI workshop on Critical Fluctuations, Oct. 10-13, 2017, CCNU, China

Beam-Energy Scan



Beam-Energy Scan

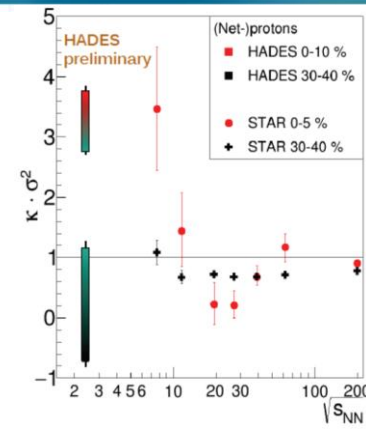
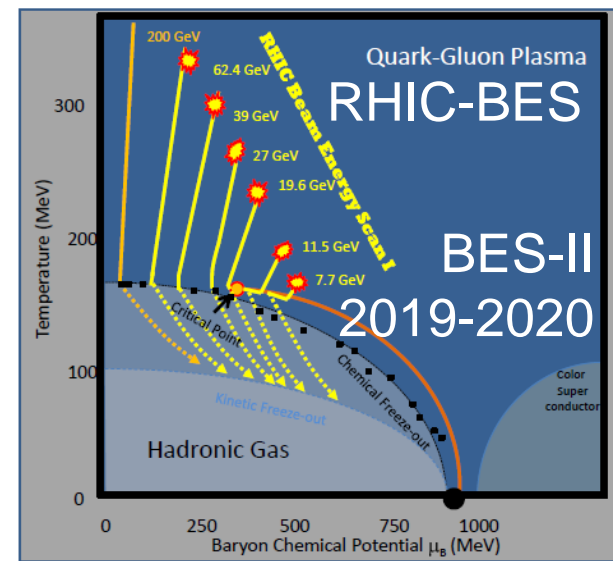
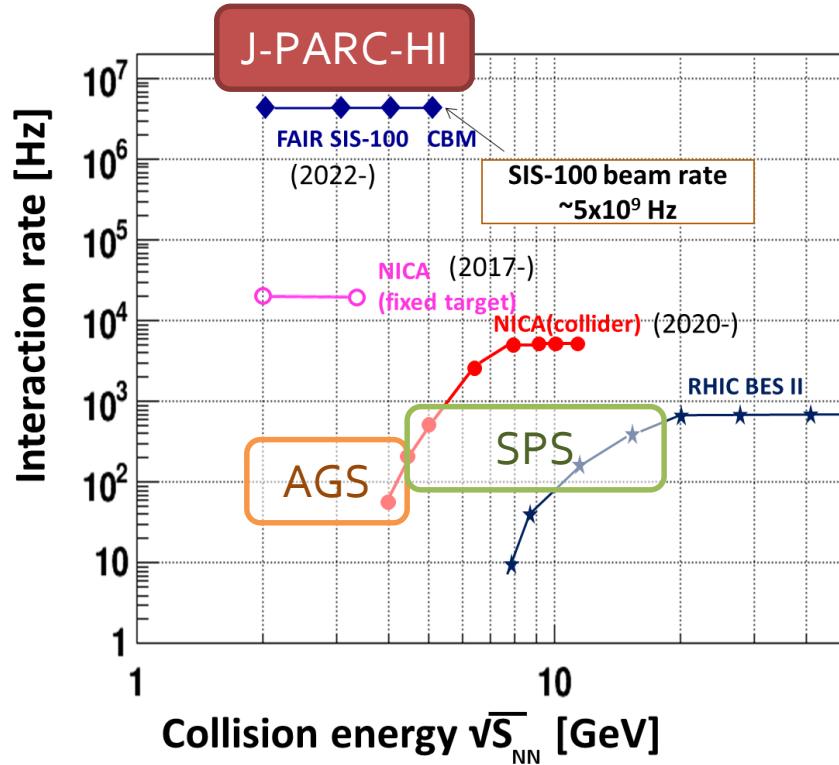
STAR 2012



STAR Preliminary

Beam-Energy Scan

Active experimental researches/plans for the beam-energy scan

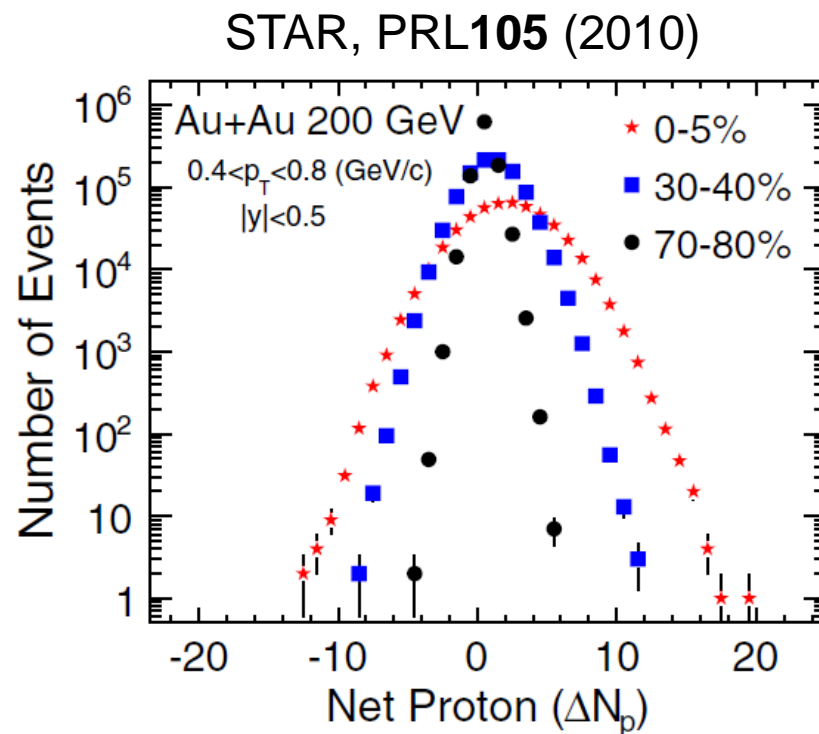
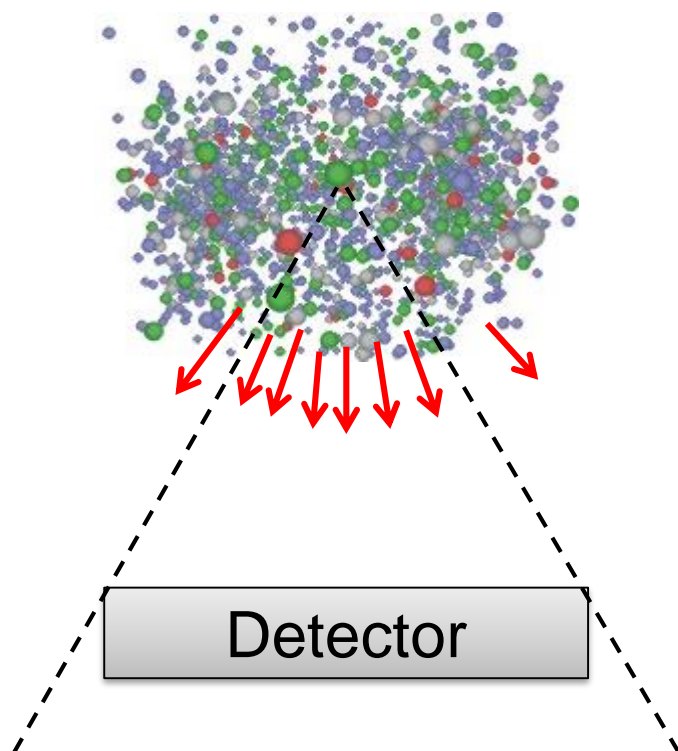


➔ Search for QCD phase structure / critical point

Event-by-Event Fluctuations

Review: Asakawa, MK, PPNP **90** (2016)

Fluctuations can be measured by e-by-e analysis in experiments.



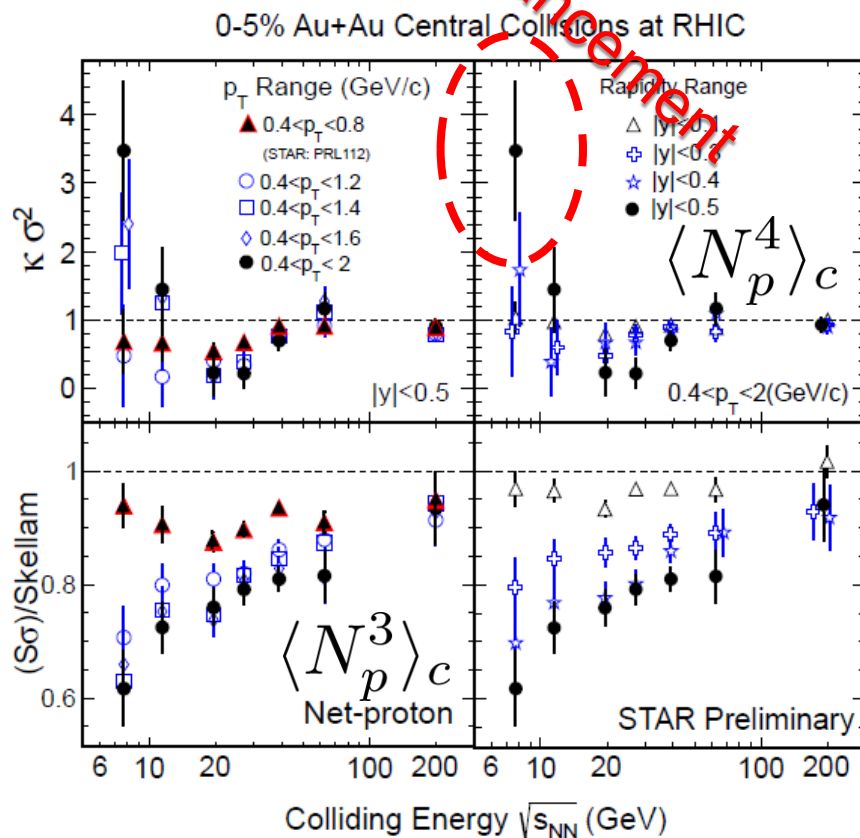
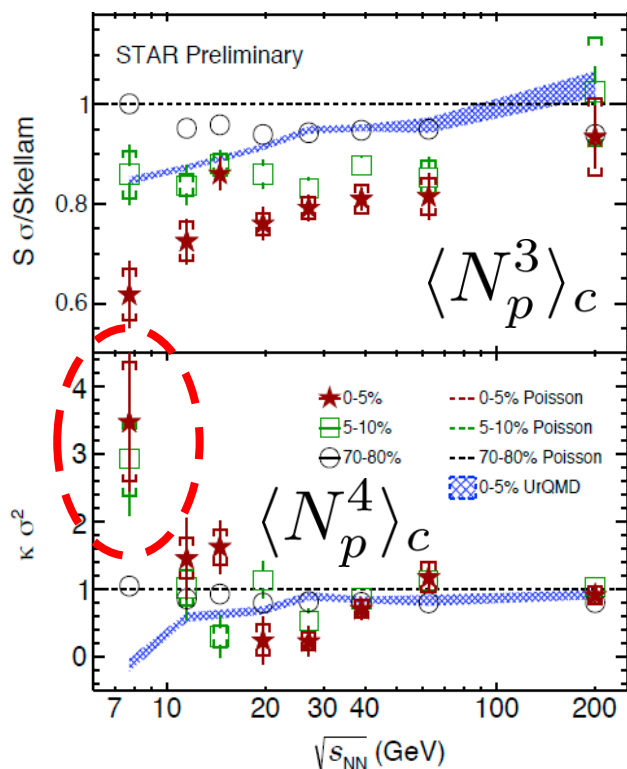
Cumulants

$$\langle \delta N_p^2 \rangle, \langle \delta N_p^3 \rangle, \langle \delta N_p^4 \rangle_c$$



Higher-Order Cumulants

STAR Collab.
2010~

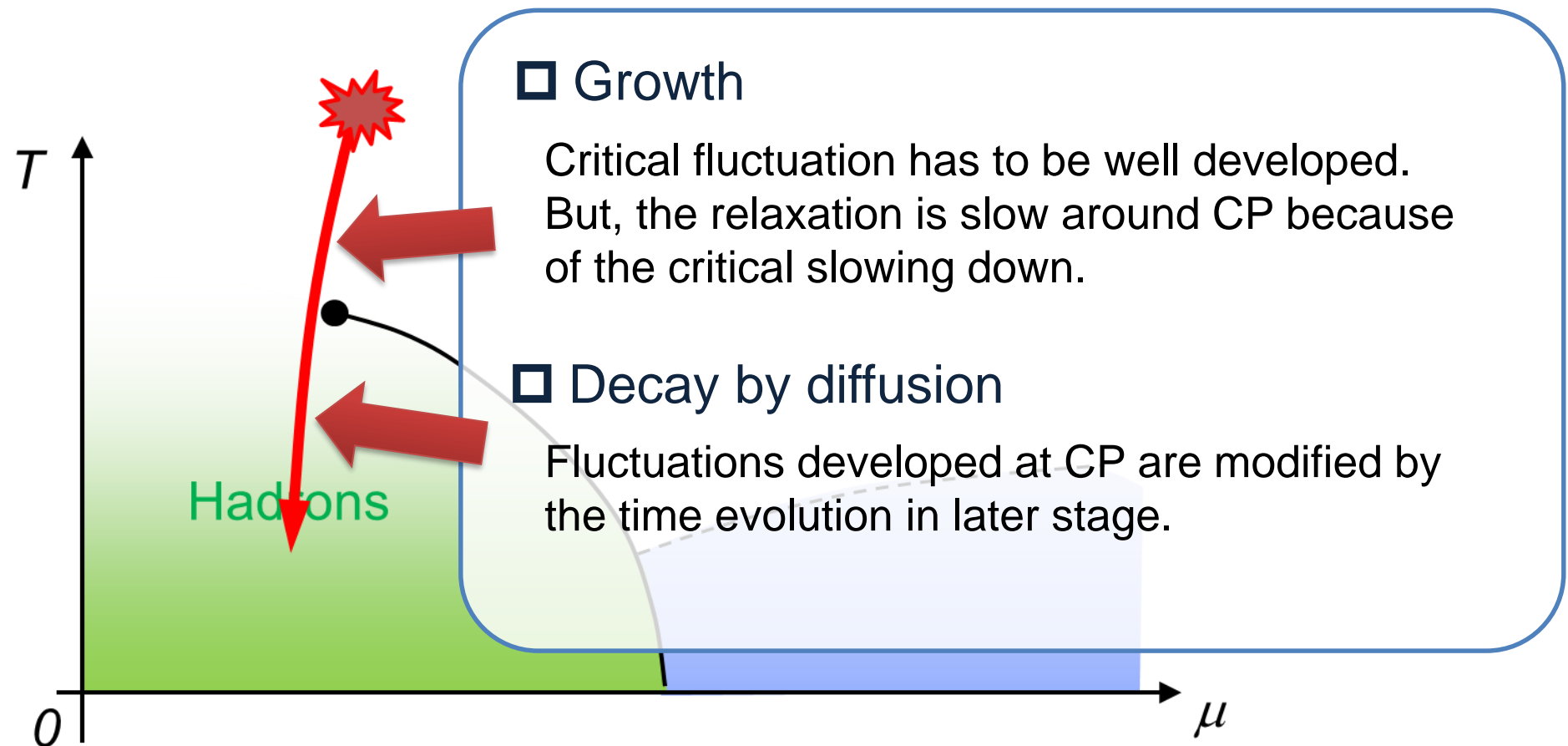


Non-zero non-Gaussian and non-Poissonian cumulants have been established!

Have we measured critical fluctuations?

Remark on Critical Fluctuation

Experiments cannot observe critical fluctuation in equilibrium directly.



Three Topics

1. Diffusion of fluctuations
2. Analysis of fluctuations with factorial cumulants
3. Critical enhancement and diffusion

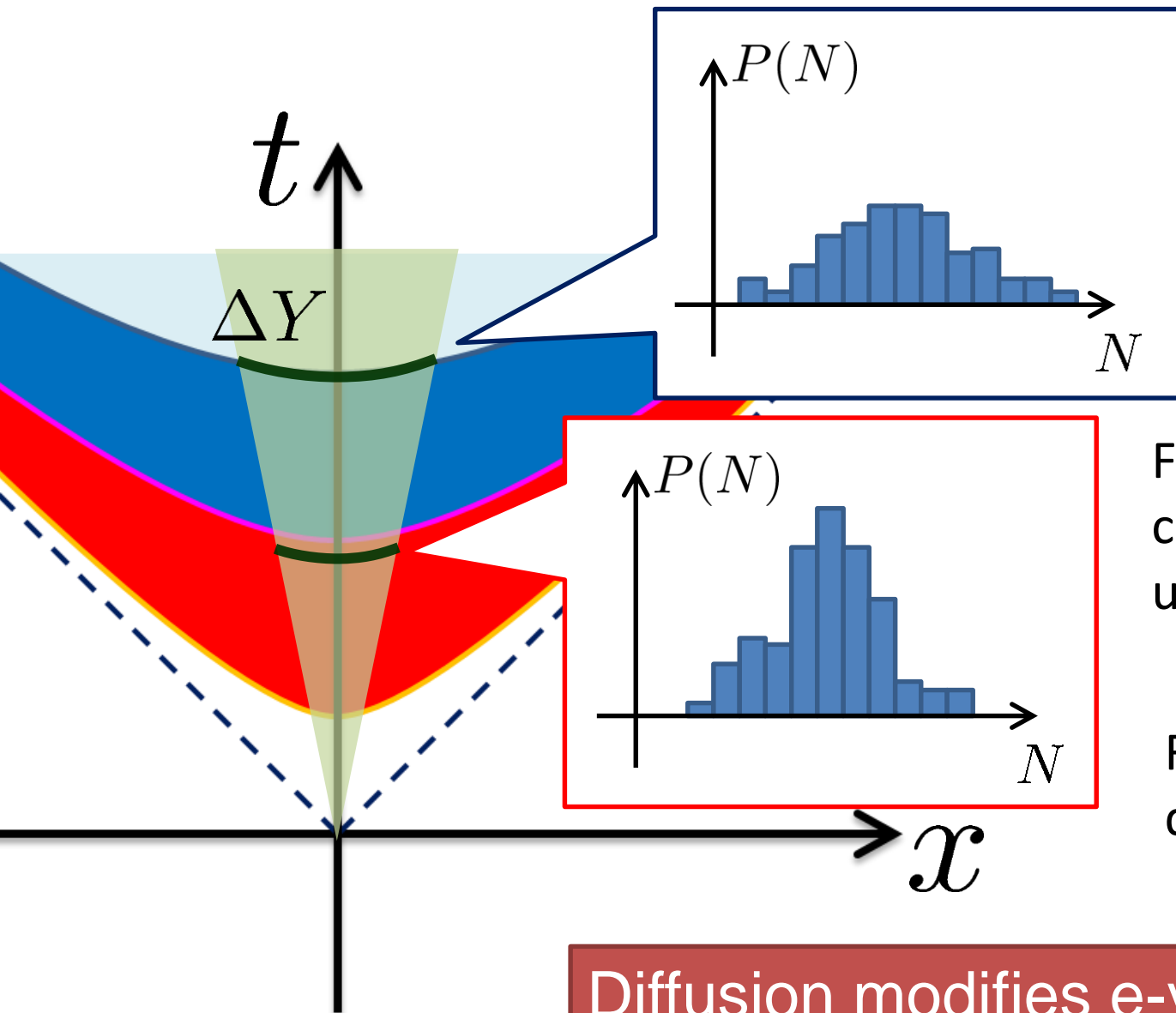
Diffusion of Fluctuations

Ohnishi, MK, Asakawa, Phys. Rev. C94, 044905 (2016)

MK, Nucl. Phys. A942, 65 (2015)

MK, Asakawa, Ono, Phys. Lett. B728, 386 (2014)

Diffusion of Fluctuations



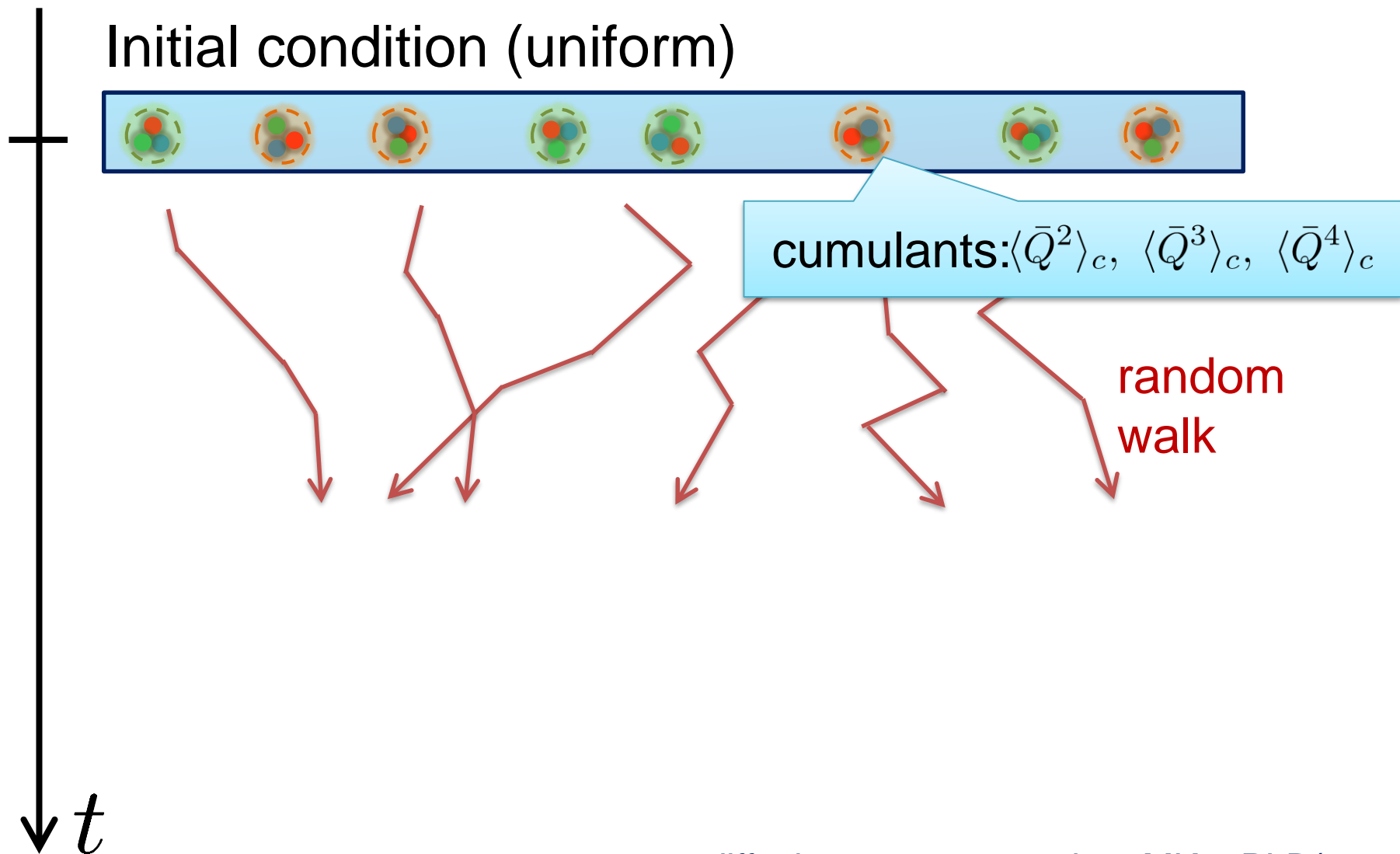
Fluctuations in ΔY continue to change until kinetic f.o.



Fluctuations are ΔY dependent

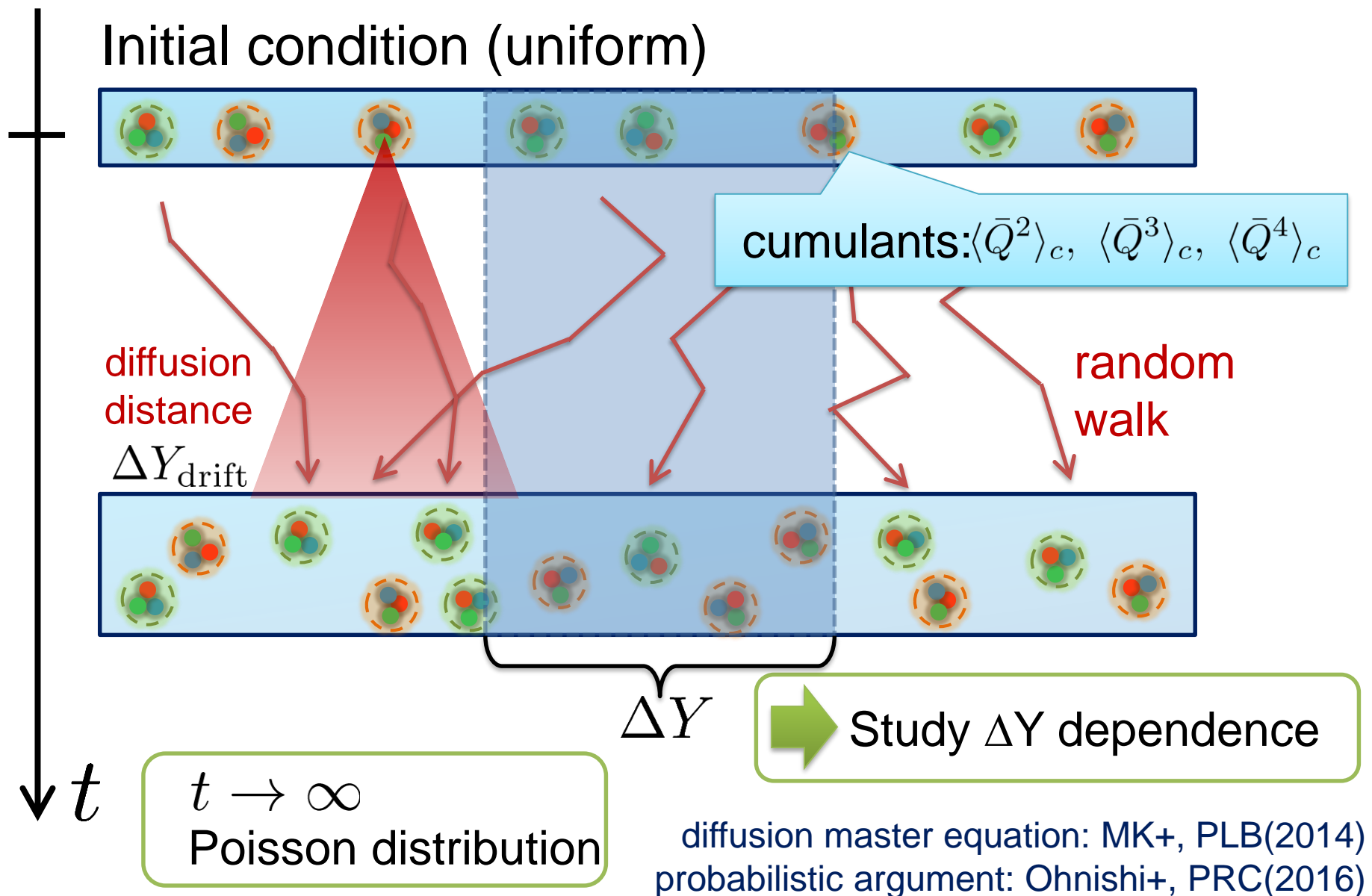
Diffusion modifies e-v-e fluctuations

(Non-Interacting) Brownian Particle Model



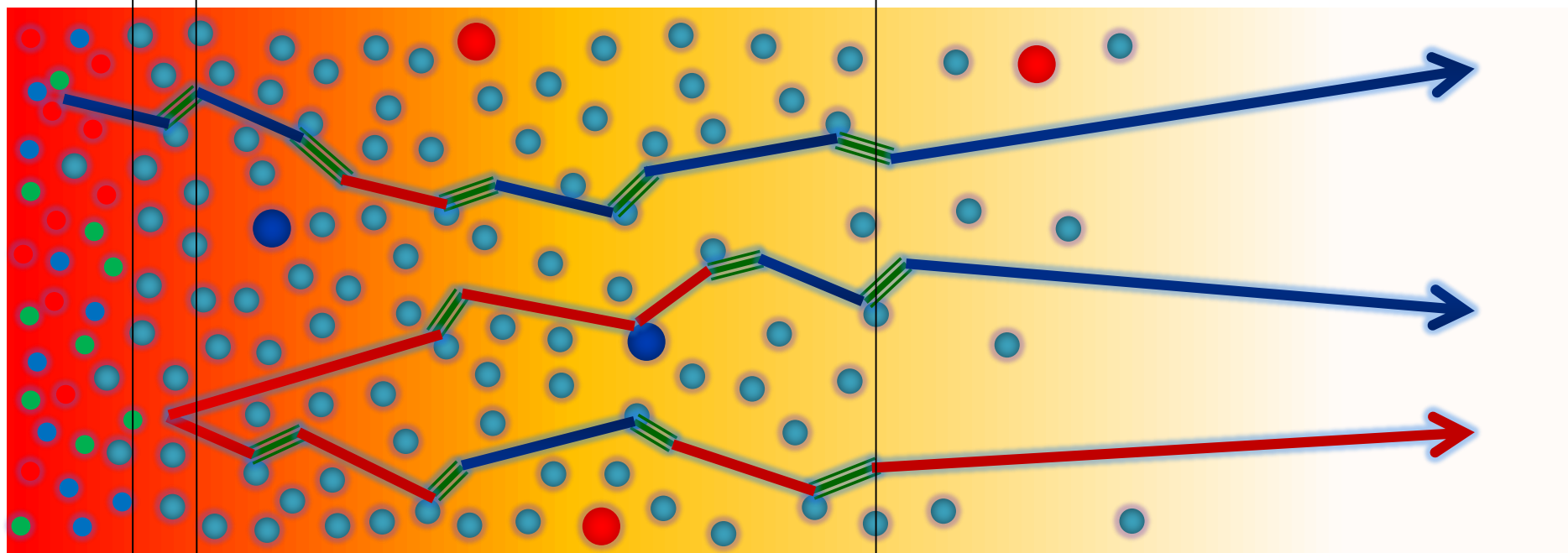
diffusion master equation: MK+, PLB(2014)
probabilistic argument: Ohnishi+, PRC(2016)

(Non-Interacting) Brownian Particle Model



Baryons in Hadronic Phase






time →



hadronize
chem. f.o.

10~20fm

kinetic f.o.

-  p, \bar{p}
-  n, \bar{n}
-  $\Delta(1232)$
-  mesons
-  baryons

Baryons behave like
Brownian pollens in water

Rapidity Window Dep.

4th-order cumulant

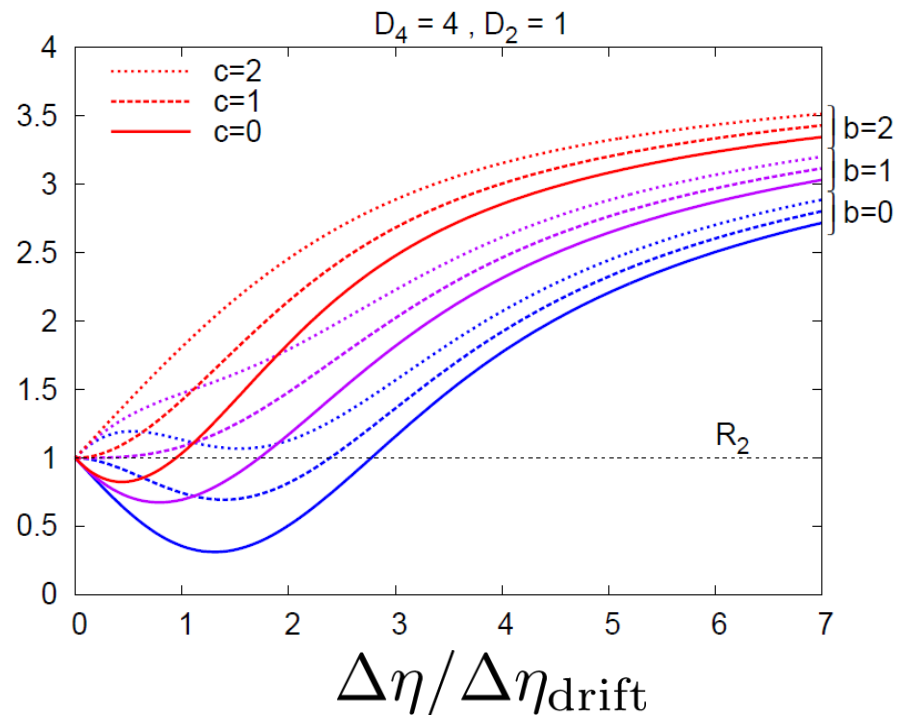
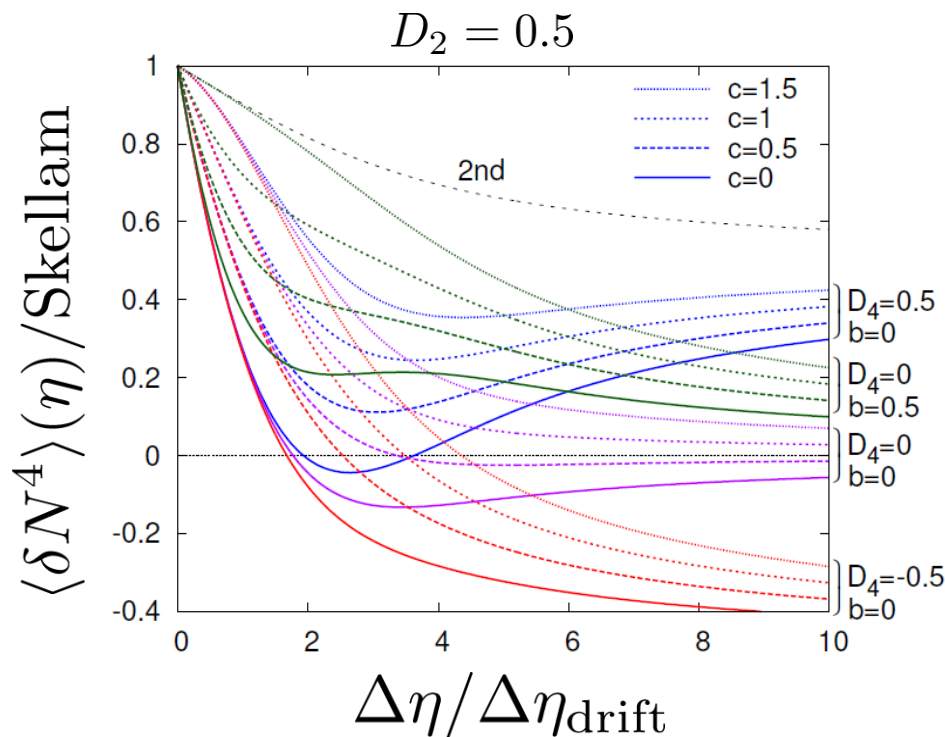
MK+, 2014

MK, 2015

Initial Conditions

$$D_4 = \frac{\langle Q_{(\text{net})}^4 \rangle_c}{\langle Q_{(\text{tot})} \rangle} \quad b = \frac{\langle Q_{(\text{net})}^2 Q_{(\text{tot})} \rangle_c}{\langle Q_{(\text{net})} \rangle}$$

$$D_2 = \frac{\langle Q_{(\text{net})}^2 \rangle_c}{\langle Q_{(\text{tot})} \rangle} \quad c = \frac{\langle Q_{(\text{tot})}^2 \rangle_c}{\langle Q_{(\text{tot})} \rangle}$$



- Different initial conditions give rise to different characteristic $\Delta\eta$ dependence.
- Non-monotonic behaviors can appear in $\Delta\eta$ dependence.

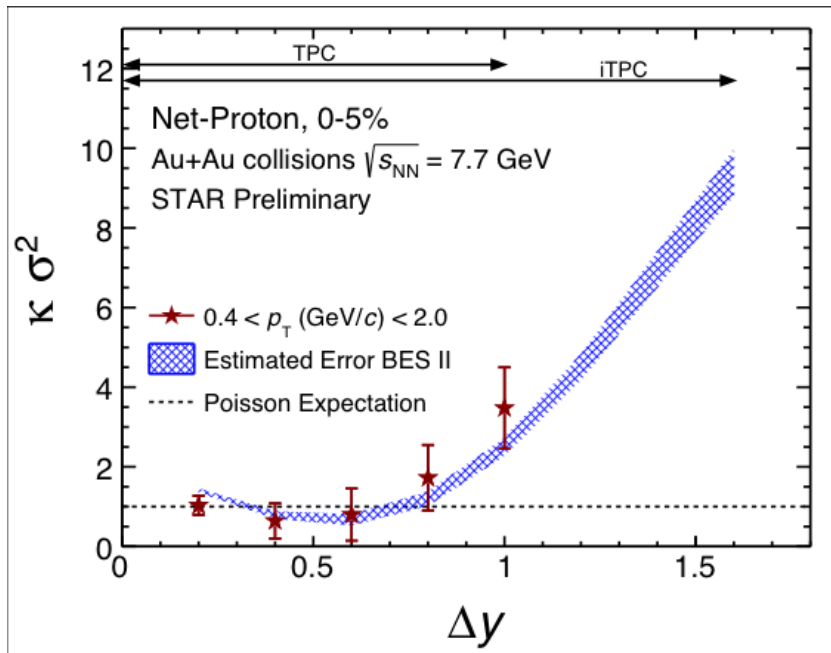
Finite volume effects: Sakaida+, PRC90 (2015)

Rapidity Window Dep.

4th-order cumulant

MK+, 2014
MK, 2015

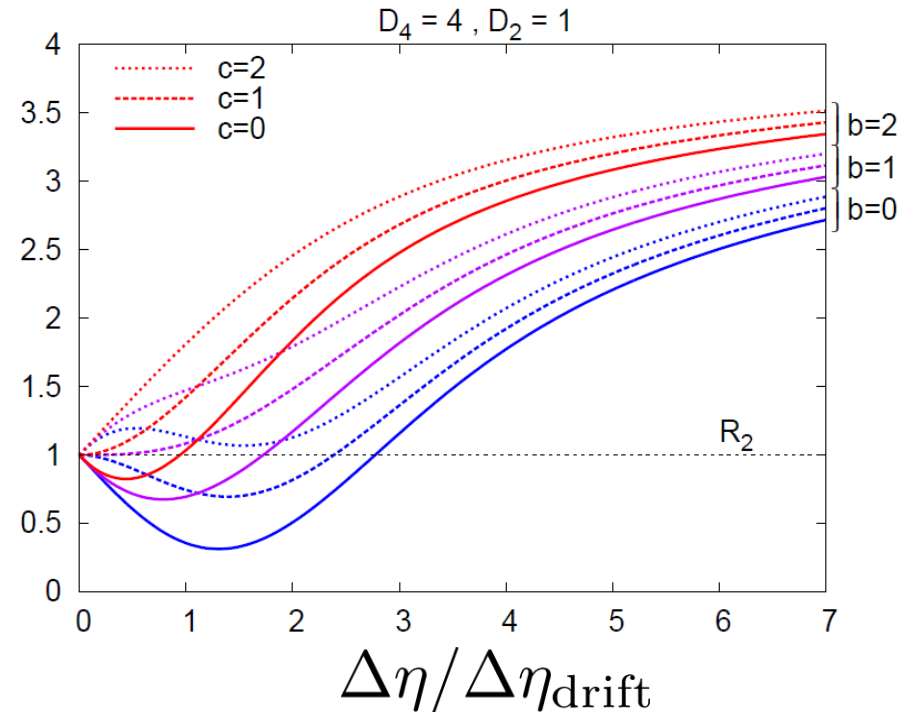
STAR Collab. (X. Luo, CPOD2014)



Initial Conditions

$$D_4 = \frac{\langle Q_{(\text{net})}^4 \rangle_c}{\langle Q_{(\text{tot})} \rangle} \quad b = \frac{\langle Q_{(\text{net})}^2 Q_{(\text{tot})} \rangle_c}{\langle Q_{(\text{net})} \rangle}$$

$$D_2 = \frac{\langle Q_{(\text{net})}^2 \rangle_c}{\langle Q_{(\text{tot})} \rangle} \quad c = \frac{\langle Q_{(\text{tot})}^2 \rangle_c}{\langle Q_{(\text{tot})} \rangle}$$



- Different initial conditions give rise to different characteristic $\Delta\eta$ dependence.
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Finite volume effects: Sakaida+, PRC90 (2015)

Analysis of Fluctuations with Factorial Cumulants

MK, Luo, Phys. Rev. C94, 044905 (2016)
Nonaka, MK, Esumi, Phys. Rev. C95, (2016)

Definition of Terminology

□ Cumulants

$$\langle N \rangle_c = \langle N \rangle, \quad \langle N^2 \rangle_c = \langle \delta N^2 \rangle, \quad \langle N^3 \rangle_c = \langle \delta N^3 \rangle,$$

$$\langle N^4 \rangle_c = \langle \delta N^4 \rangle - 3\langle \delta N^2 \rangle^2,$$

$$K(\theta) \equiv \ln \langle e^{N\theta} \rangle = \sum_m \frac{\theta^m}{m!} \langle N^m \rangle_c$$

□ Factorial Cumulants

$$\langle N \rangle_c = \langle N \rangle, \quad \langle N^2 \rangle_{fc} = \langle N(N-1) \rangle_c,$$

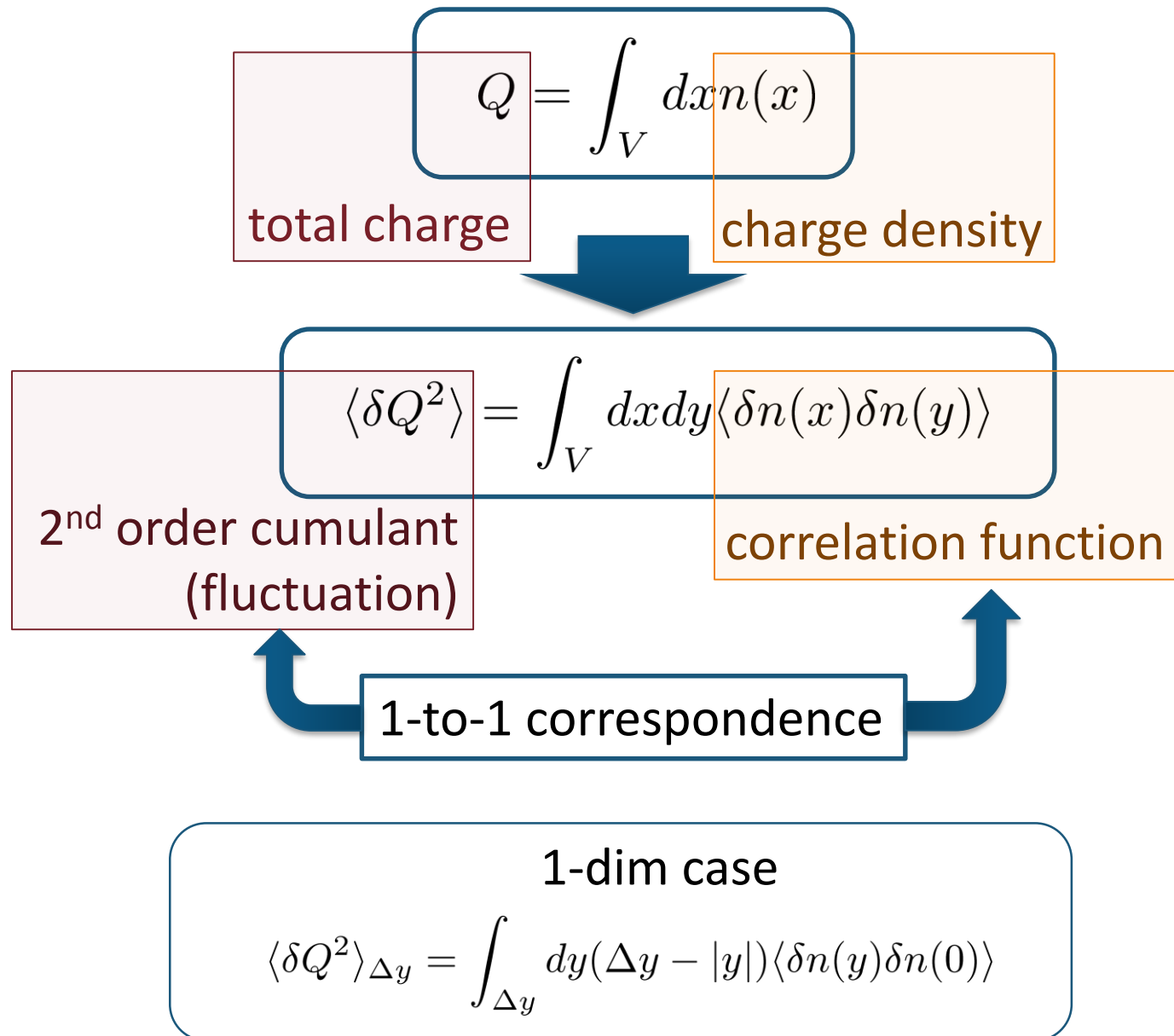
$$\langle N^3 \rangle_{fc} = \langle N(N-1)(N-2) \rangle_c,$$

$$K_f(s) \equiv \ln \langle (1+s)^N \rangle = \sum_m \frac{s^m}{m!} \langle N^m \rangle_{fc}$$

□ Correlation Functions

$$\langle n(x)n(y) \rangle_c, \quad \langle n(x)n(y)n(z) \rangle_c,$$

Cumulants and Correlation Function



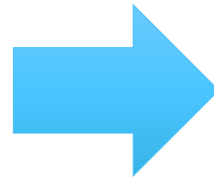
= Cumulants after removing “self-correlation”

Correlation function

$$C(x, y) = \langle \delta n(x) \delta n(y) \rangle$$



$$C(x, y) = C^*(x, y) + \langle n(x) \rangle \delta(x - y)$$



Cumulant

$$\langle Q^2 \rangle_c = \int_{\Delta} dx dy C(x, y)$$



Factorial cumulant

$$\langle Q^2 \rangle_{fc} = \int_{\Delta} dx dy C^*(x, y)$$

Caveats:

- ❑ This interpretation is valid only for classical particle systems.
- ❑ When quasi-particles are lost, f-cumulants are not well-defined.
- ❑ Values of f-cumulants are modified by inelastic scatterings.



Cumulants of conserved charges have desirable properties.

Usage of F-Cumulants

Suggestion

F-cumulants play useful roles in revealing underlying physics described (approximately) by the binomial model.

independent probabilistic events

Binomial Model

$$P_{\text{obs}}(n) = \sum_N B_p(n; N) P(N)$$

“observed”
distribution

“original”
distribution



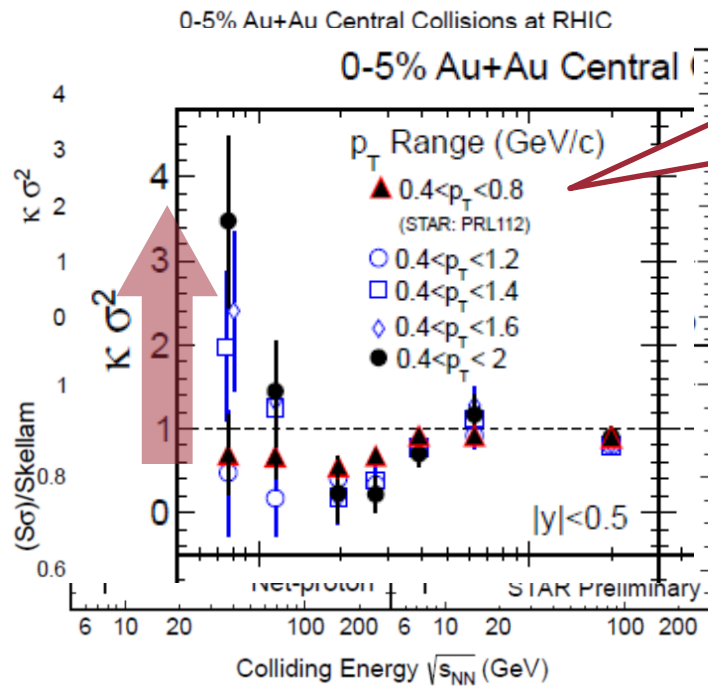
F-cumulants

$$\langle n^m \rangle_{\text{fc}} = \langle p^m N^m \rangle_{\text{fc}}$$

Ex: efficiency correction
Nonaka, MK, Esumi, 2017

p_T -cut Dependence

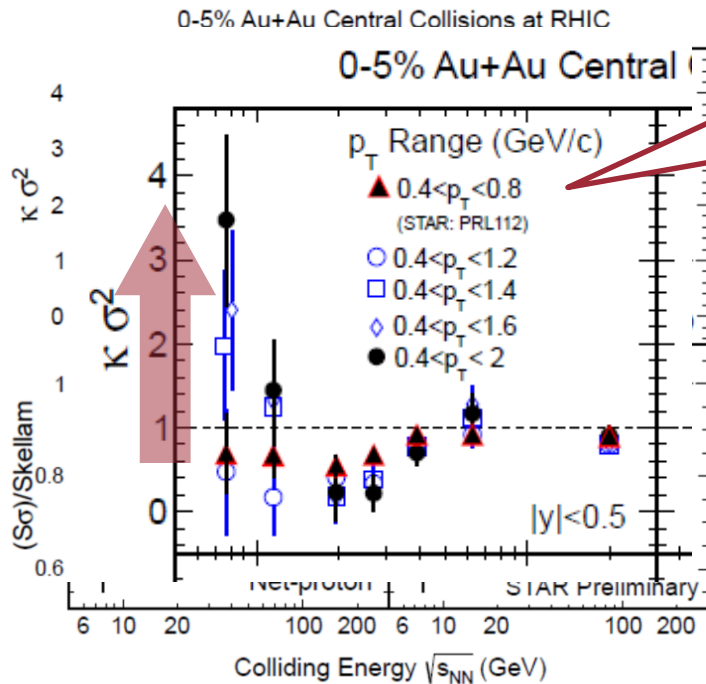
MK, Luo, PRC, 2017



Enhancement of cumulant
with increasing p_T acceptance

p_T -cut Dependence

MK, Luo, PRC, 2017

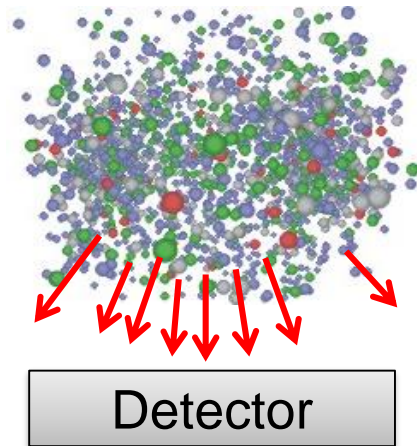


Enhancement of cumulant with increasing p_T acceptance

Assumption:
independent and random p_T distribution in the final state

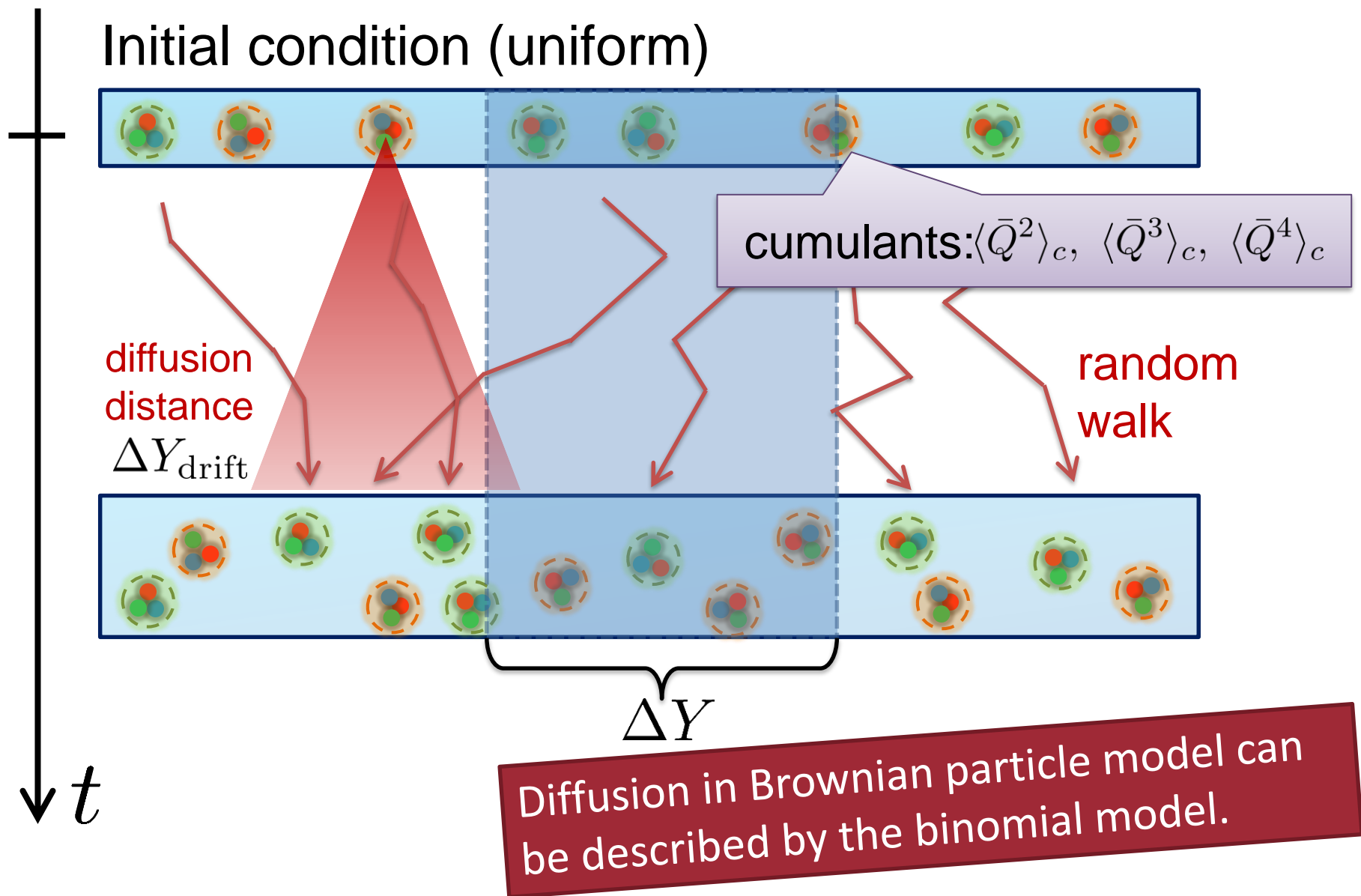
Power-law behavior of f-cum.

$$\frac{\langle n^m \bar{n}^{\bar{m}} \rangle_{fc, p_T}}{\langle n \rangle_{p_T}^m \langle \bar{n} \rangle_{p_T}^{\bar{m}}} = \frac{\langle N^m \bar{N}^{\bar{m}} \rangle_{fc}}{\langle N \rangle^m \langle \bar{N} \rangle^{\bar{m}}}$$



- ✓ Study particle emission mechanism
- ✓ Construction of cumulants

Non-Interacting Brownian Particle System



$\Delta\eta$ Dependences of F-Cumulants

MK, Luo, PRC, 2017

$$\langle n_p^m n_{\bar{p}}^{\bar{m}} \rangle_{fc} = \kappa_{m\bar{m}} \Delta y F_{m+\bar{m}}(\Delta y/d)$$

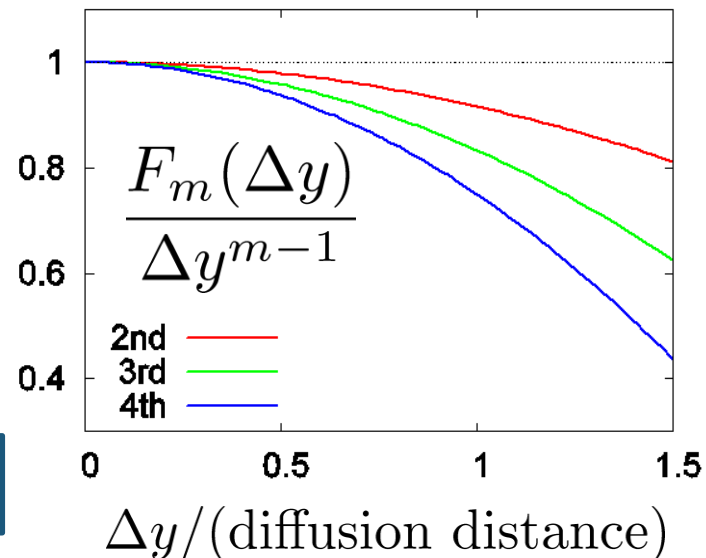
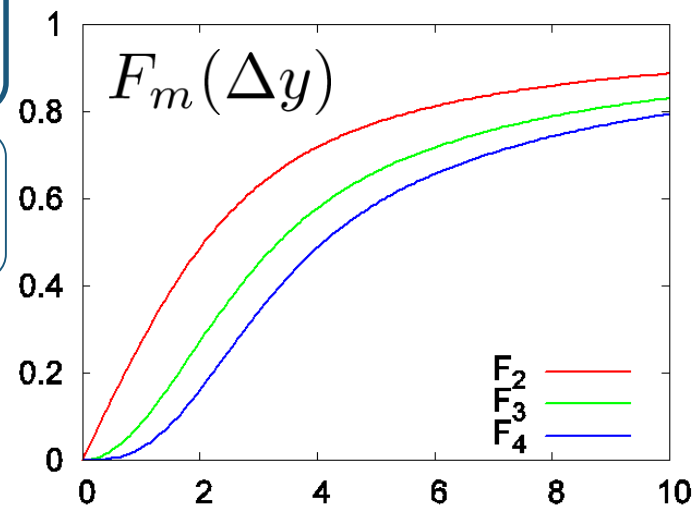
f-cumulant at
initial cond.

diffusion
distance

- All f-cumulants at the same order have the same Δy dependence up to normalization.
- f-cumulants and diffusion distance can be inferred.

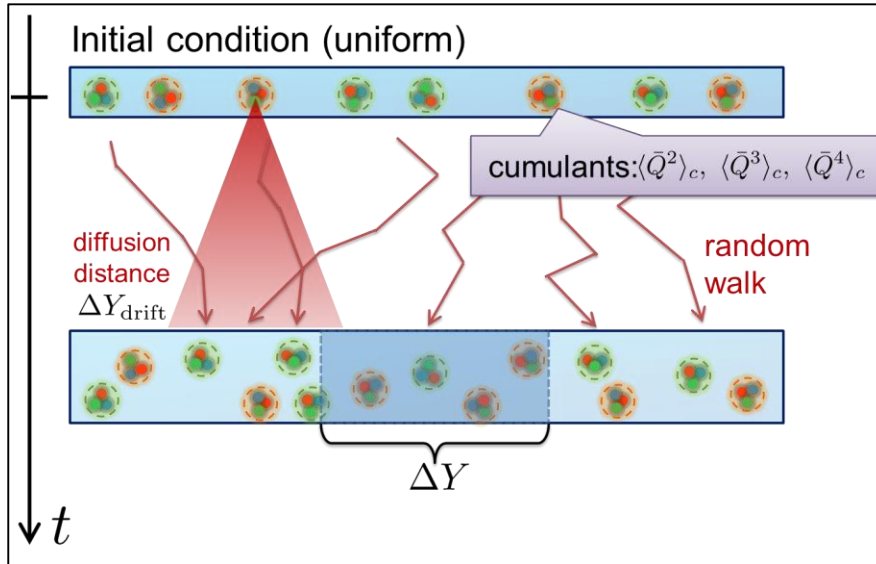


reconstruct cumulants of conserved charges



Translating Languages

Brownian particle model

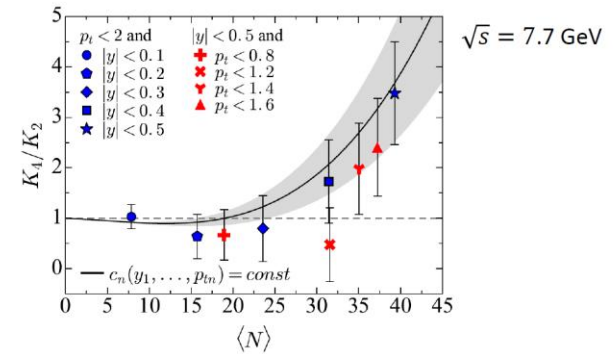


From Bzdak's talk

Constant correlation

$$c_2 = \frac{\int \rho(y_1) \rho(y_2) c_2(y_1, y_2) dy_1 dy_2}{\int \rho(y_1) \rho(y_2) dy_1 dy_2}$$

$$c_n(y_1, p_{t1}, \dots, y_n, p_{tn}) = c_n^0 = \text{const} \rightarrow c_n = c_n^0$$



AB, V. Koch, 1707.02640

19

$$\langle n^m \bar{n}^{\bar{m}} \rangle_{fc} = \kappa_{m\bar{m}} \Delta y F_{m+\bar{m}}(\Delta y/d)$$

$$c_{m\bar{m}}^0 = \frac{1}{2} \frac{\partial^2}{\partial \Delta y^2} \langle n^m \bar{n}^{\bar{m}} \rangle_{fc} \Big|_{\Delta y \rightarrow 0} = \kappa_{m\bar{m}} \frac{\partial}{\partial \Delta y} F_{m+\bar{m}}(\Delta y/d) \Big|_{\Delta y \rightarrow 0}$$

$$= \frac{\kappa_{m\bar{m}}}{d} \frac{1}{\sqrt{(m+\bar{m})(2\pi)^{m+\bar{m}-1}}}$$

$\kappa_{m\bar{m}}$: F cumulants at initial condition

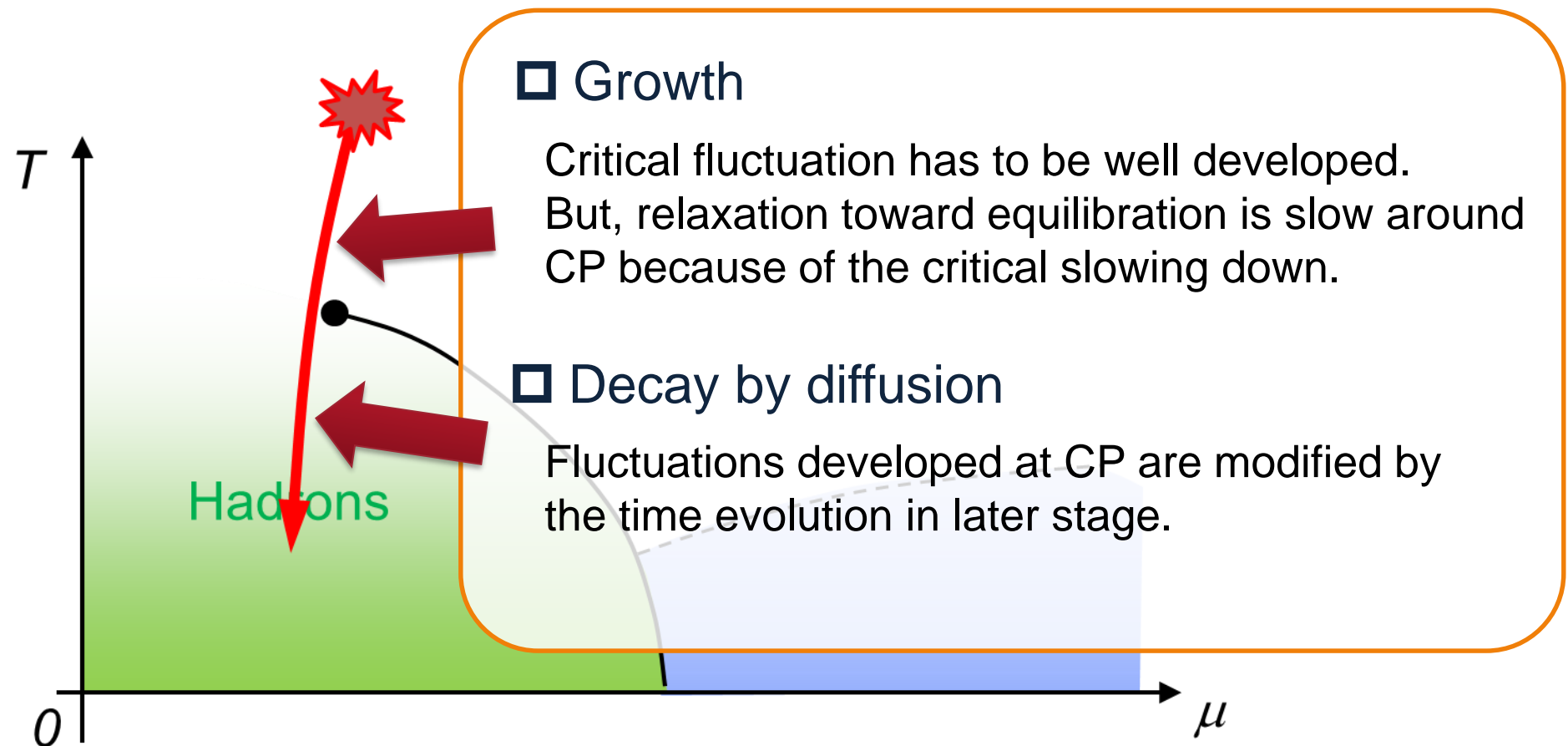
d : diffusion distance

Critical Enhancement and Diffusion

Sakaida, Asakawa, Fujii, MK, Phys. Rev. C95, 064905 (2017)

Remarks on Critical Fluctuation 1

Experiments cannot observe critical fluctuation in equilibrium directly.



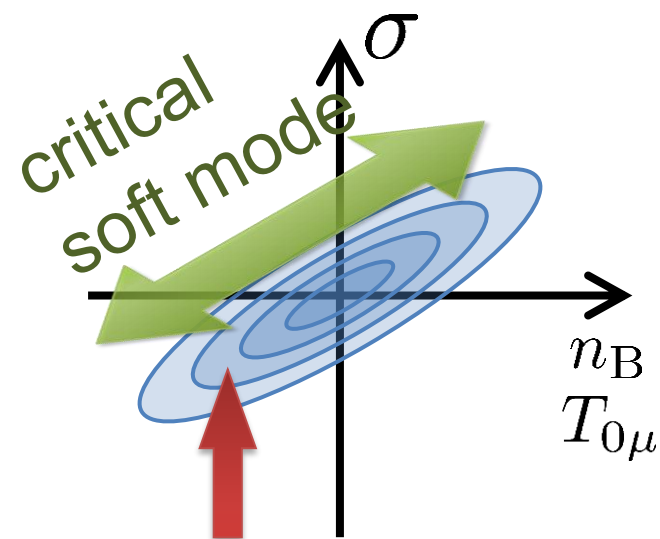
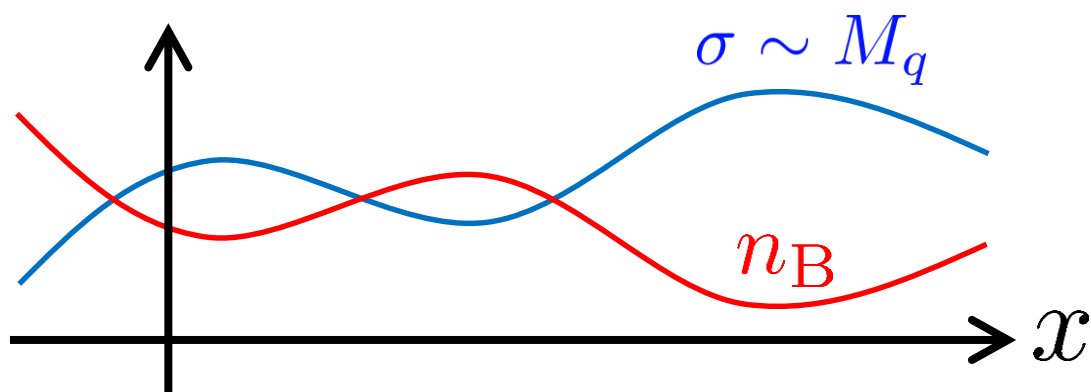
Remarks on Critical Fluctuation 2

Critical fluctuation is a conserved mode!

Fujii 2003; Fujii, Ohtani, 2004; Son, Stephanov, 2004

Fluctuations of σ and n_B are coupled around the CP!

$$\delta\sigma \simeq \delta n_B$$



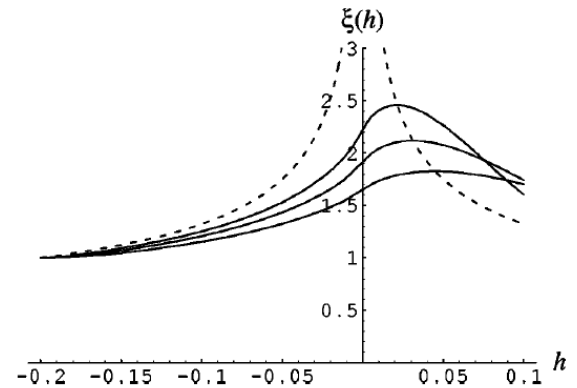
σ : fast damping

$$F(\sigma, n) = A\sigma^2 + B\sigma n + Cn^2 + \dots$$

Dynamical Evolution of Critical Fluctuations

□ Evolution of correlation length

Berdnikov, Rajagopal (2000)
Asakawa, Nonaka (2002)

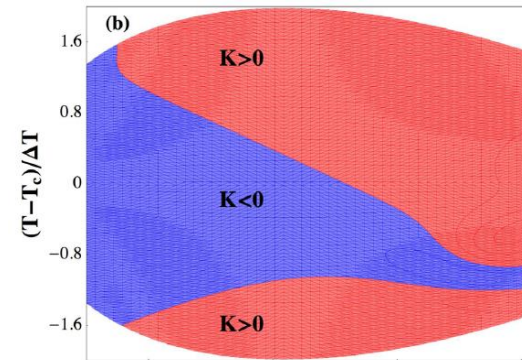


□ Higher orders (spatially uniform “ σ ” mode)

Mukherjee, Venugopalan, Yin (2015)

□ Dynamical evolution in chiral fluid model

Nahrgang, Herold, ... (2014~)



□ Correlation functions

Kapusta, Torres-Rincon (2012)

Aim of This Study

- ❑ Describe **conserved nature** of critical fluctuation.
- ❑ We want to study **experimental observables**.
 - ❑ focus on a **conserved charge (baryon number)**
 - ❑ study evolution of **conserved-charge** fluctuation
- ❑ Concentrate on **2nd order** fluctuation. (not higher)
- ❑ We study
 - ❑ **rapidity window dependence** of the cumulant
 - ❑ 2-particle **correlation function**

Our Main Conclusion

Non-monotonicity in
cumulants or correlation func.

=

Signal of
QCD-CP

Stochastic Diffusion Equation (SDE)

□ Diffusion equation

$$\partial_{\tau} n = D \partial_{\eta}^2 n$$

- Describe a relaxation of a conserved density n toward uniform state **without fluctuation**

□ Stochastic diffusion equation

$$\partial_{\tau} n = D \partial_{\eta}^2 n + \partial_{\eta} \xi(\eta, \tau)$$

$$\langle \xi(\eta_1) \xi(\eta_2) \rangle \sim \chi \delta(\eta_1 - \eta_2)$$

- Describe a relaxation toward **fluctuating** uniform state
- χ : susceptibility (fluctuation in equil.)

Soft Mode of QCD Critical Point

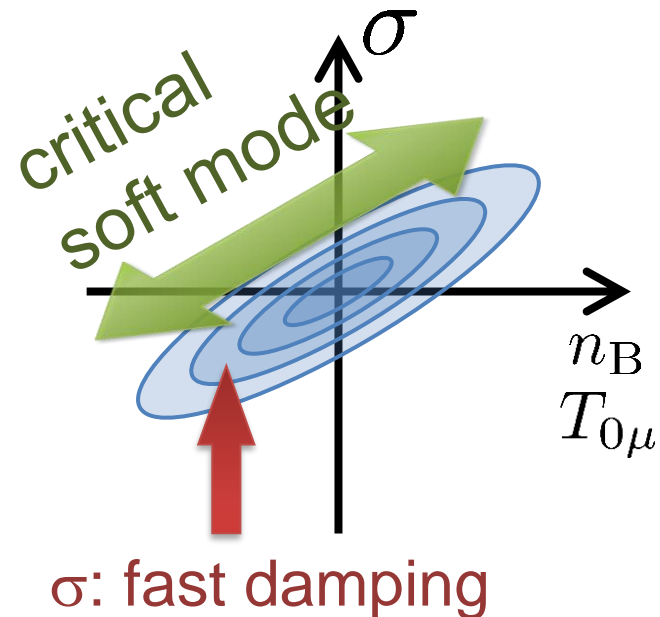
Fujii 2003; Fujii, Ohtani, 2004; Son, Stephanov, 2004

□ Effective potential

$$F(\sigma, n) = A\sigma^2 + B\sigma n + Cn^2 + \dots$$

□ Time dependent Ginzburg-Landau

$$\begin{pmatrix} \dot{\sigma} \\ \dot{n} \end{pmatrix} = \begin{pmatrix} \Gamma_{\sigma\sigma} & \Gamma_{\sigma n} \\ \Gamma_{n\sigma} & \Gamma_{nn} \end{pmatrix} \begin{pmatrix} \sigma \\ n \end{pmatrix} \sim k^2$$



For slow and long wavelength,

$$\text{SDE} \quad \partial_\tau n = D(\tau) \partial_\eta^2 n + \partial_\eta \xi$$

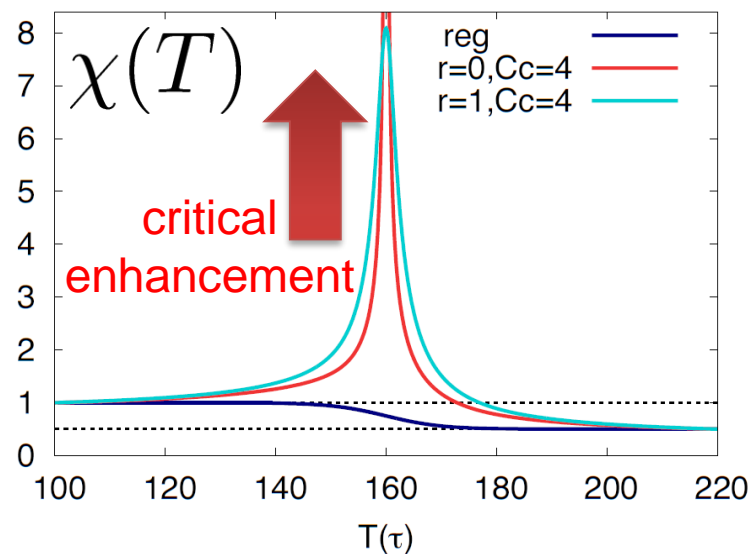
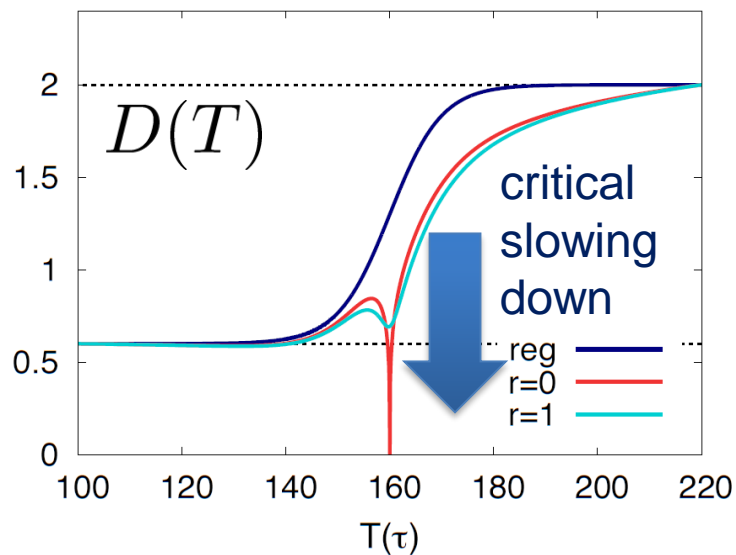
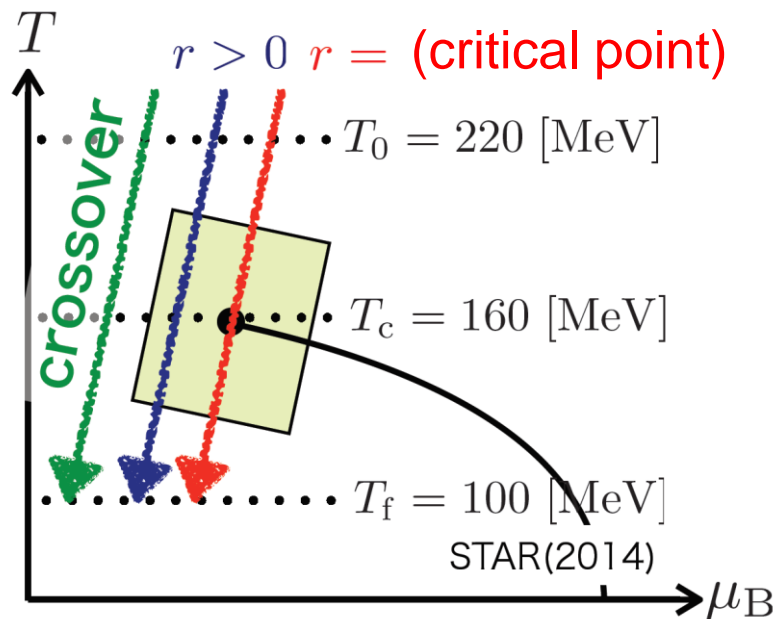
singularities in $D(\tau)$ and $\chi(\tau)$

Parametrizing $D(\tau)$ and $\chi(\tau)$

□ Critical behavior

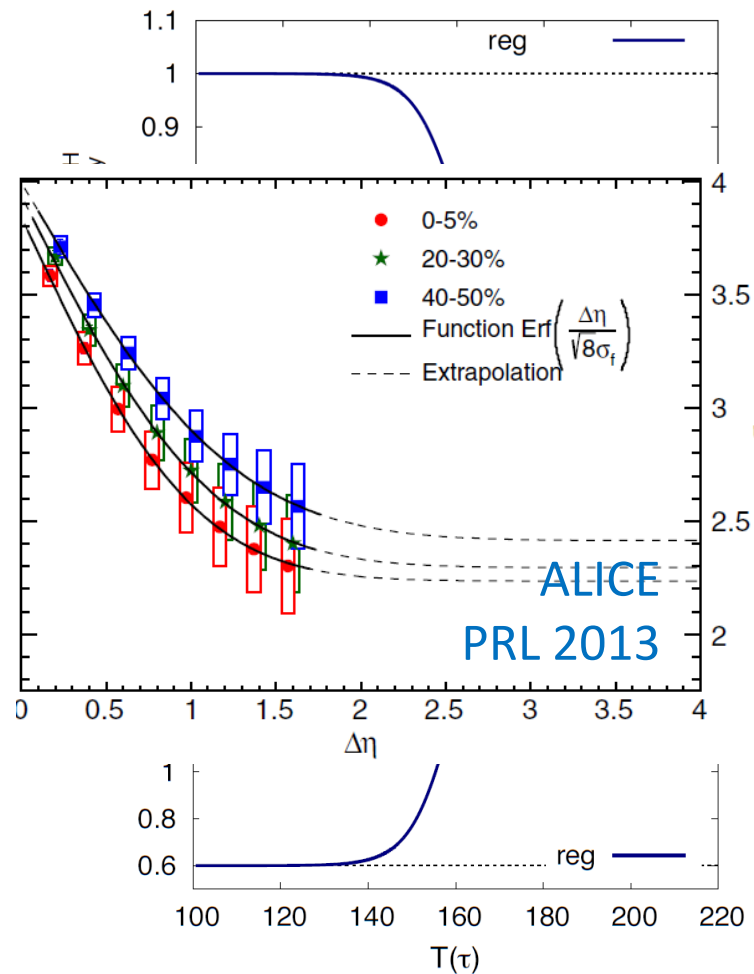
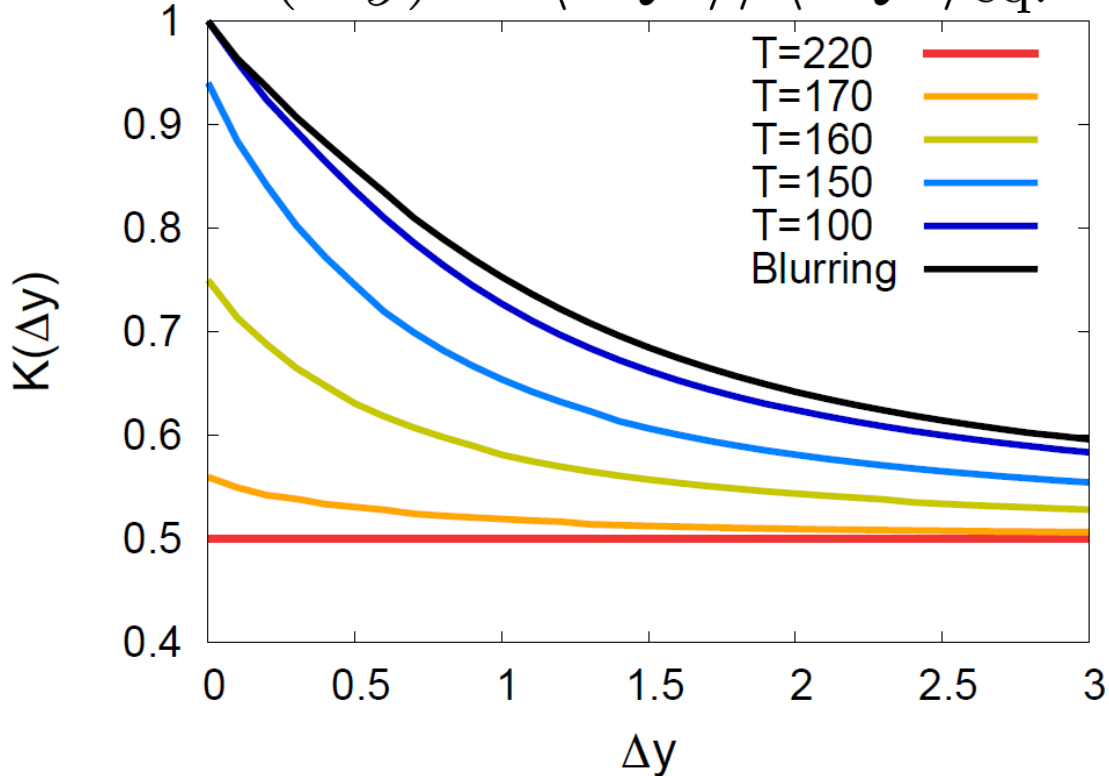
- 3D Ising (r, H)
- model H

□ Temperature dep.

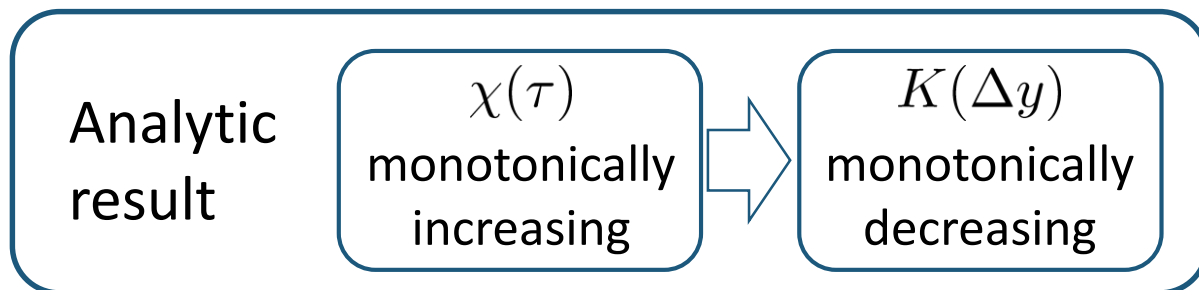


Crossover / Cumulant

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\text{eq.}}$$

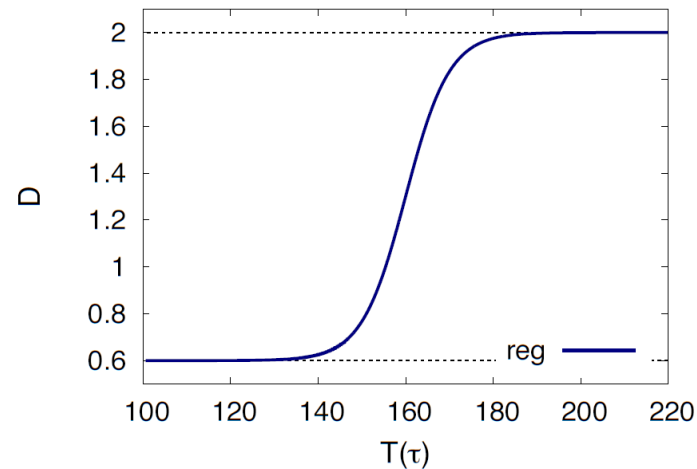
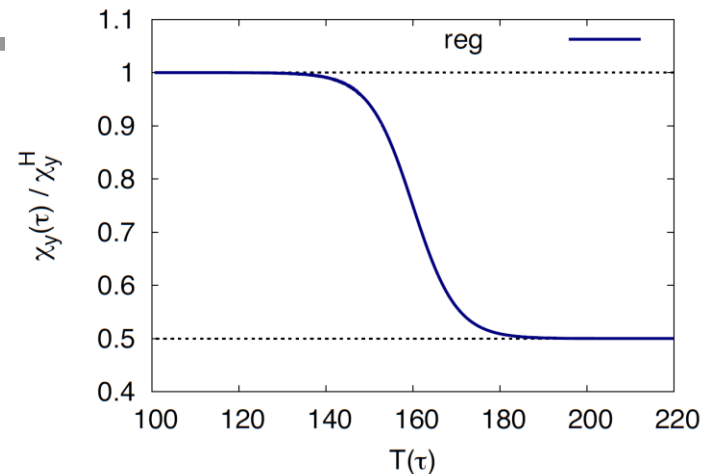
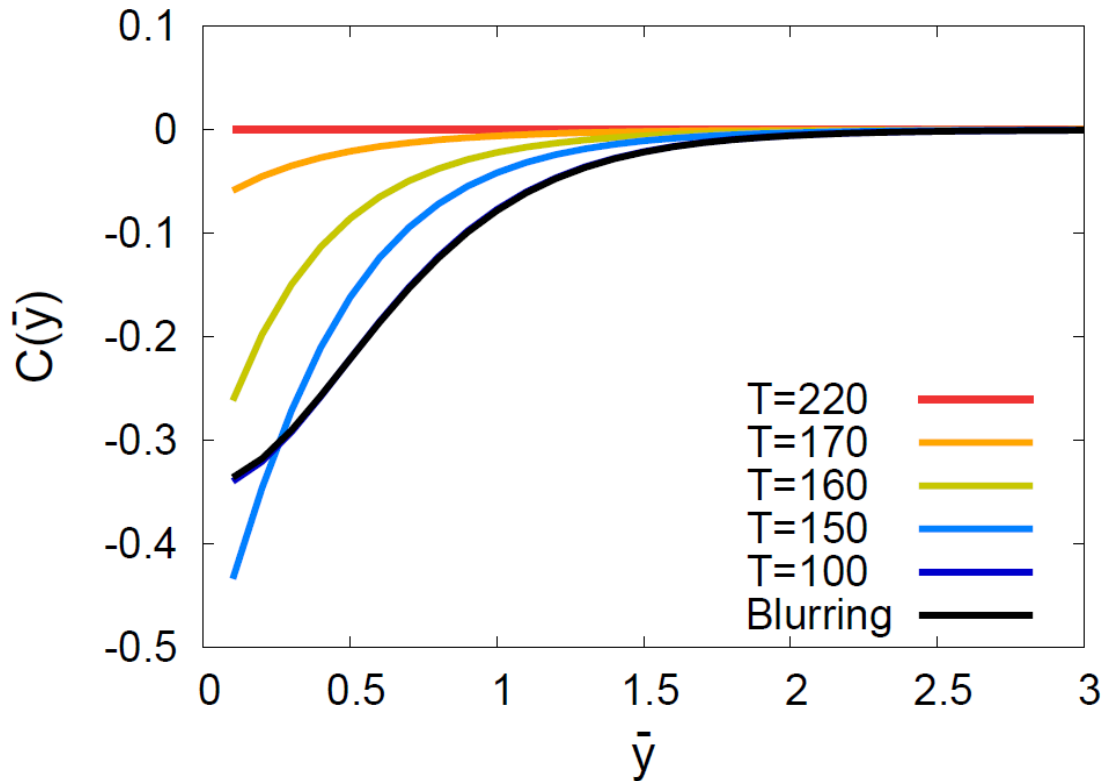


□ monotonically decreasing

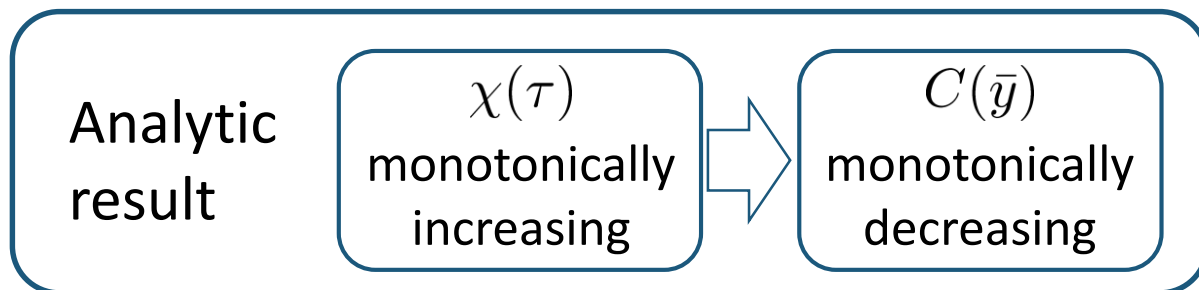


Crossover / Correlation Func.

$$C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\text{hadron}}$$

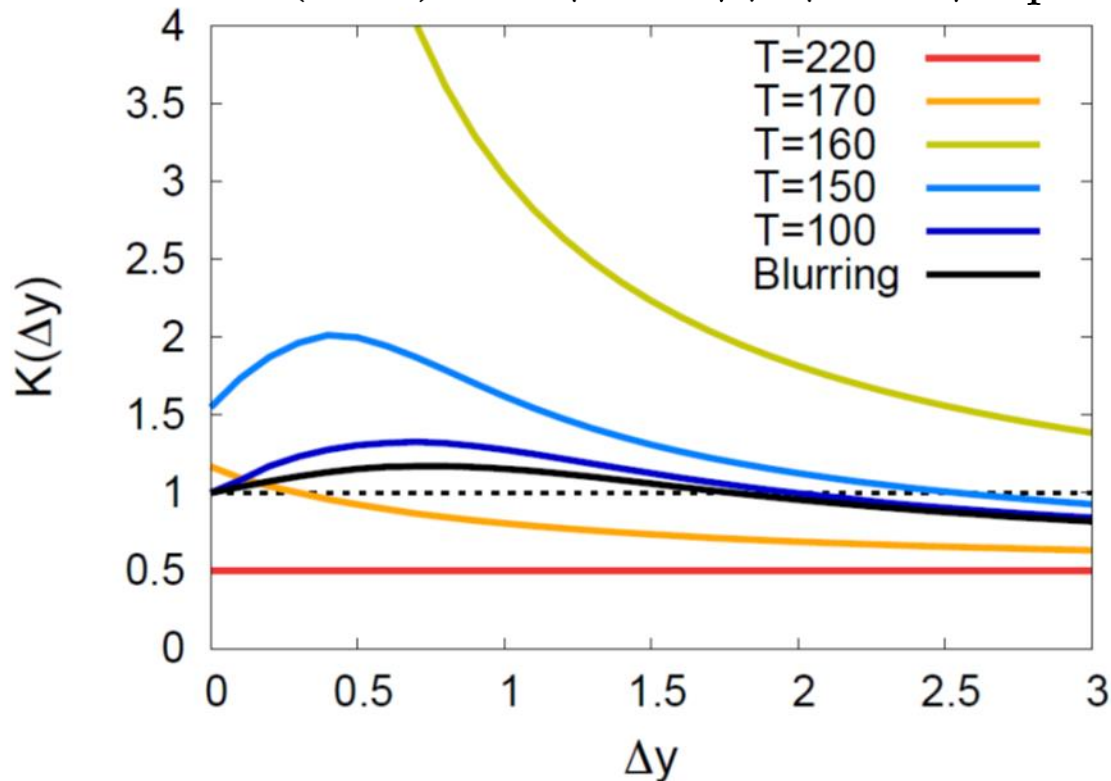


□ monotonically decreasing

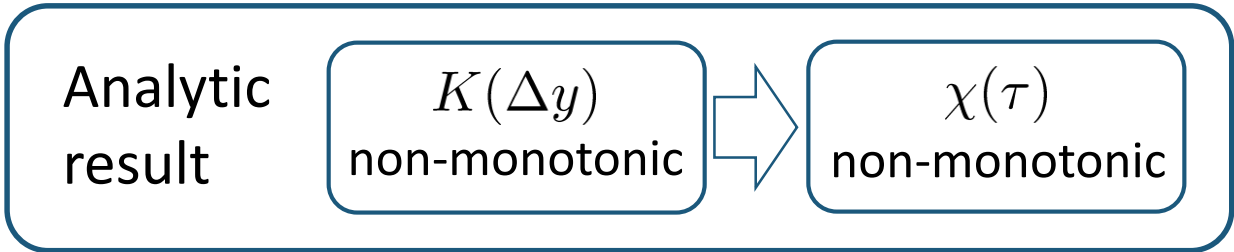
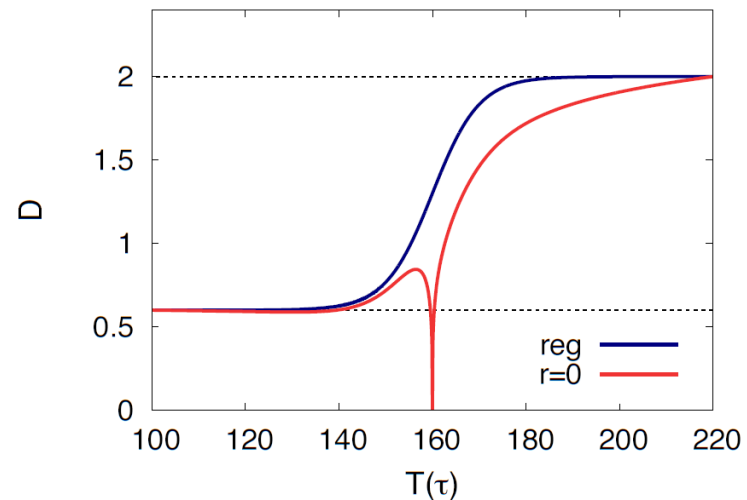
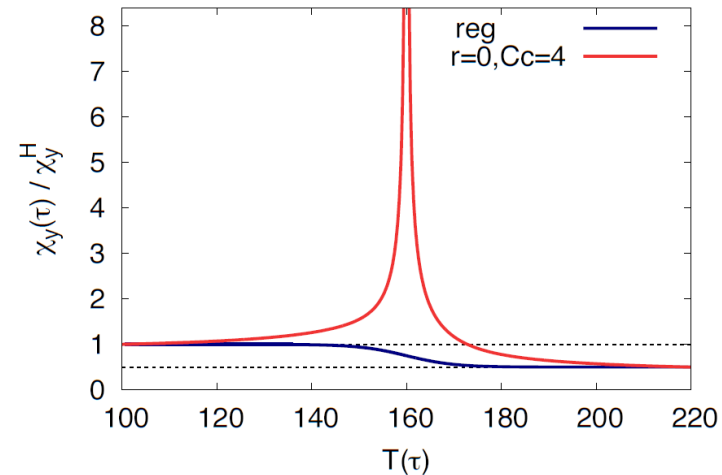


Critical Point / Cumulant

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\text{eq.}}$$

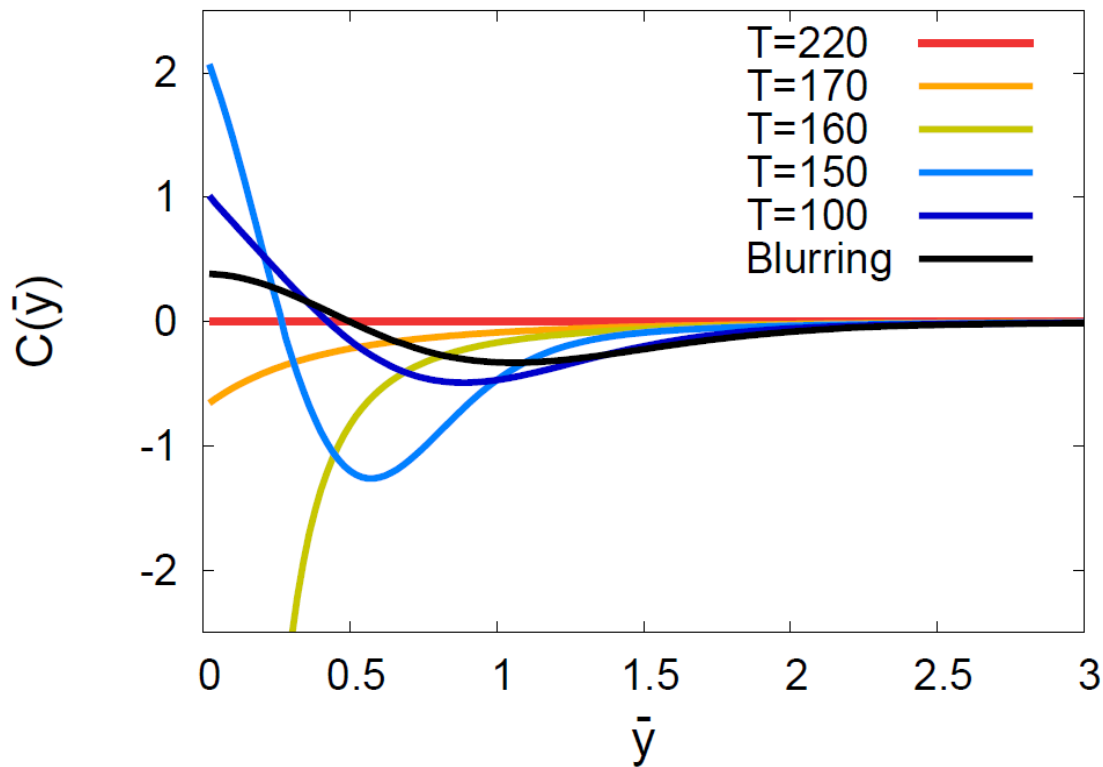


□ non-monotonic Δy dep.

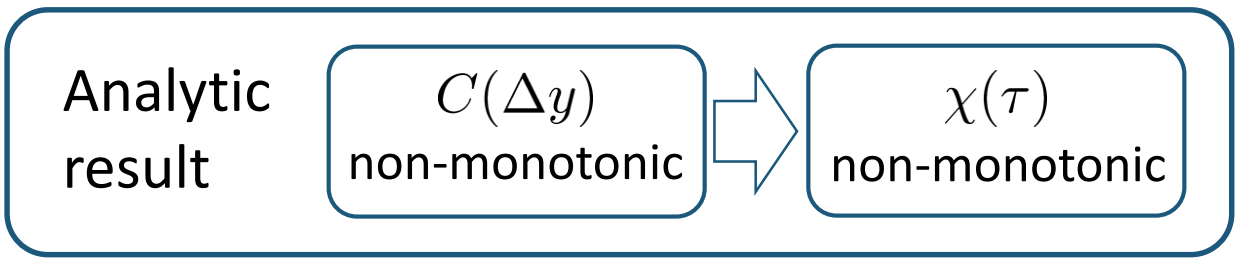
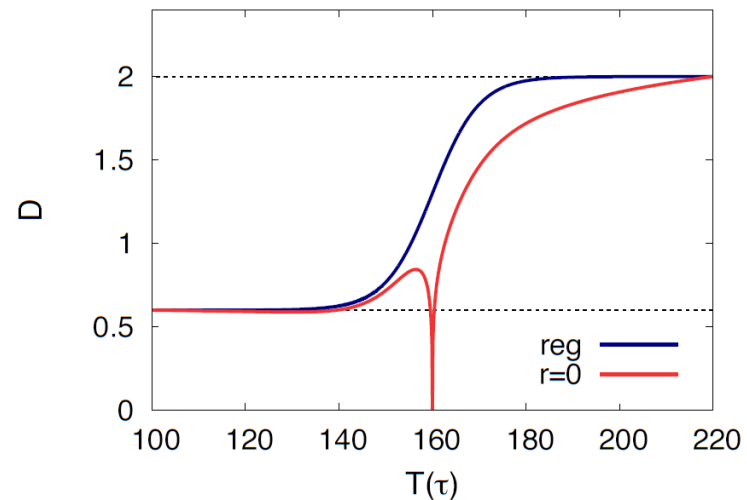
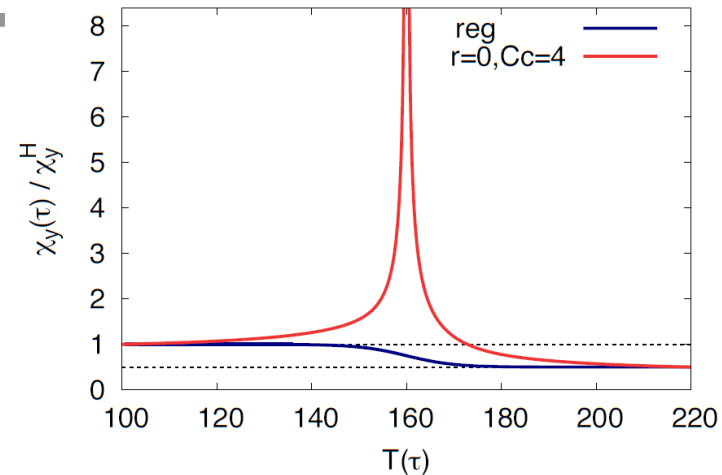


Criticap Point / Correlation Func.

$$C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\text{hadron}}$$

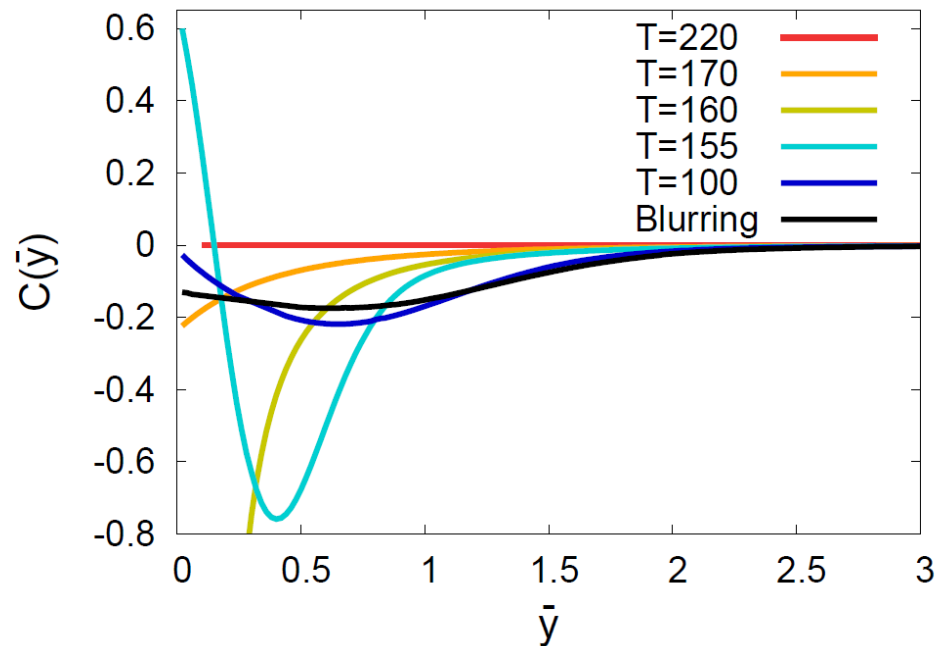
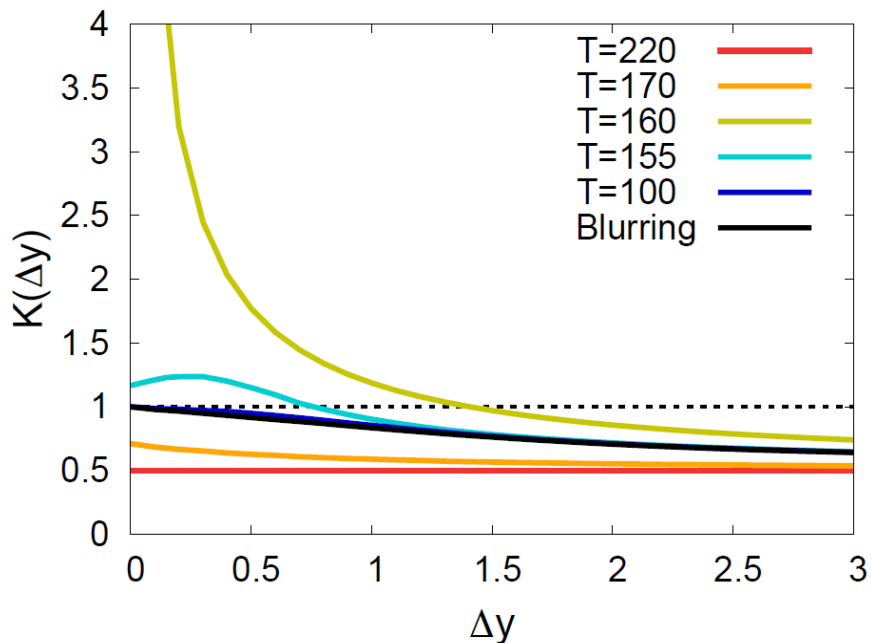


□ non-monotonic Δy dep.



Weaker Critical Enhancement

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\text{eq.}} \quad C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\text{hadron}}$$



❑ Non-monotonicity in $K(\Delta y)$ disappears.

❑ But $C(y)$ is still non-monotonic.

Analytic
result

$K(\Delta y), C(\bar{y})$
monotonic

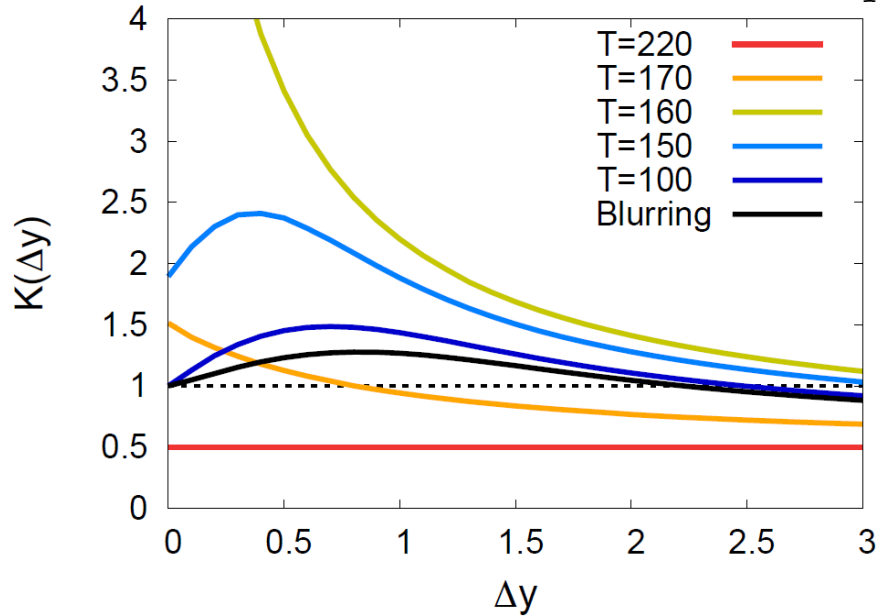


no information on
 $\chi(\tau)$

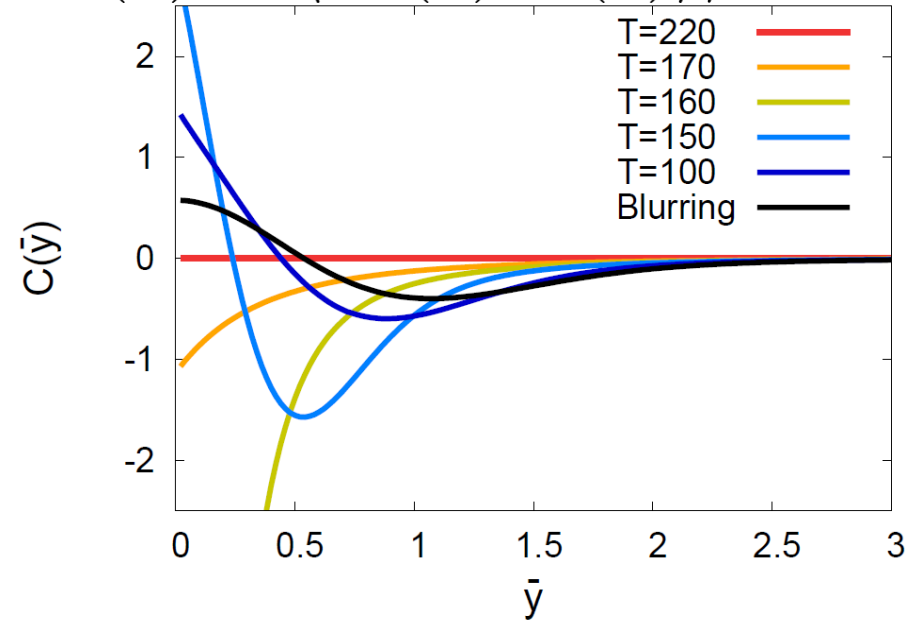
❑ $C(y)$ is better to see non-monotonicity.

Away from the CP

$$K(\Delta y) = \langle \delta Q^2 \rangle / \langle \delta Q^2 \rangle_{\text{eq.}}$$



$$C(\bar{y}) = \langle \delta n(\bar{y}) \delta n(0) \rangle / \chi_{\text{hadron}}$$



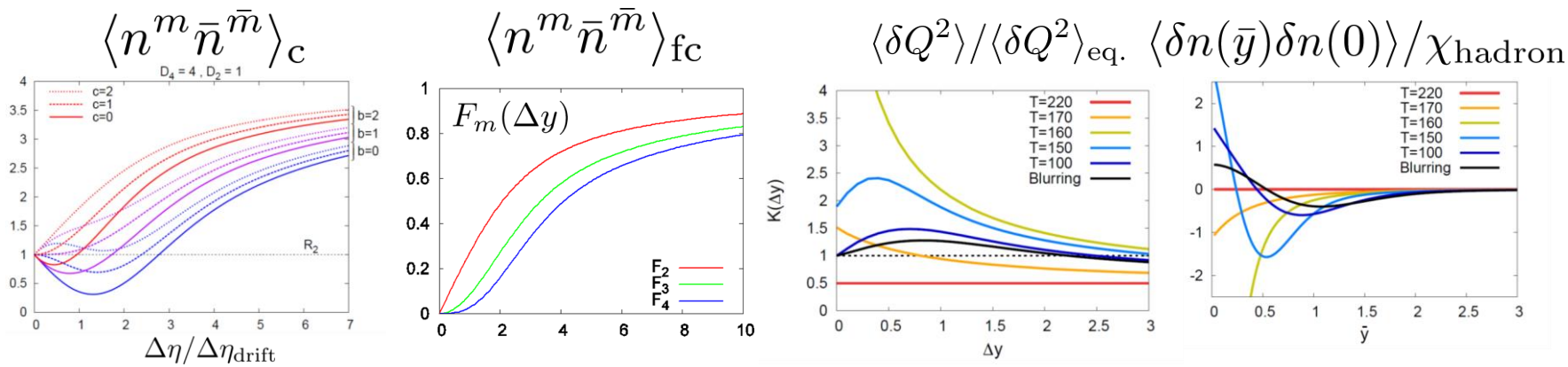
□ Signal of the critical enhancement can be clearer on a path away from the CP.

Away from the CP \rightarrow Weaker critical slowing down

Summary

- ❑ Non-equilibrium dynamics is crucial for a proper understanding of experimentally-observed fluctuations.
- ❑ $\Delta\eta$ dependences of (factorial) cumulants and correlation functions are useful information to understand fluctuations.
- ❑ A non-monotonic behavior of **2nd-order** cumulant or correlation function is the signal of the critical enhancement!

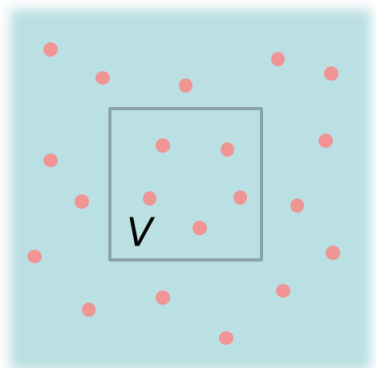
Suggestion to experimentalists



- cumulants / factorial cumulants / correlation funcs.
- $\Delta\eta$ dependence / p_T dependence / **Baryon number** / etc.

Fluctuations: Theory vs Experiment

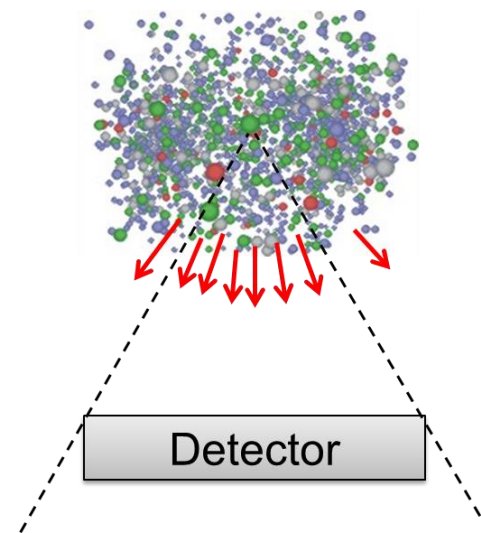
Theoretical analyses
based on statistical mechanics



lattice, critical point,
effective models, ...

Fluctuation in
a spatial volume

Experiments

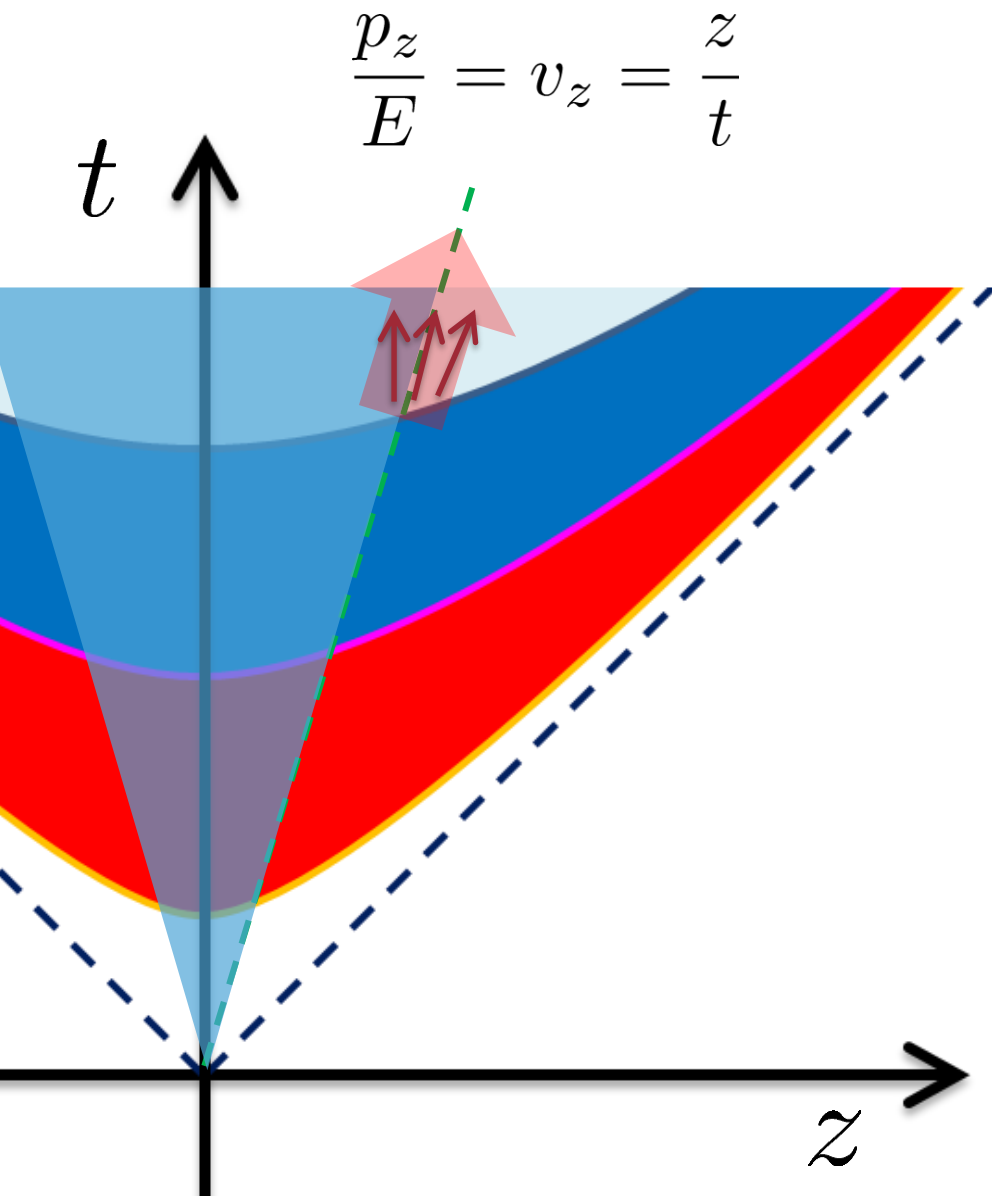


Fluctuations in
a momentum space

discrepancy in phase spaces

Thermal Blurring

Ohnishi, MK, Asakawa,
PRC94, 044905 (2016)



Under Bjorken picture,

coordinate-space rapidity Y

||

momentum-space rapidity y
of **medium**

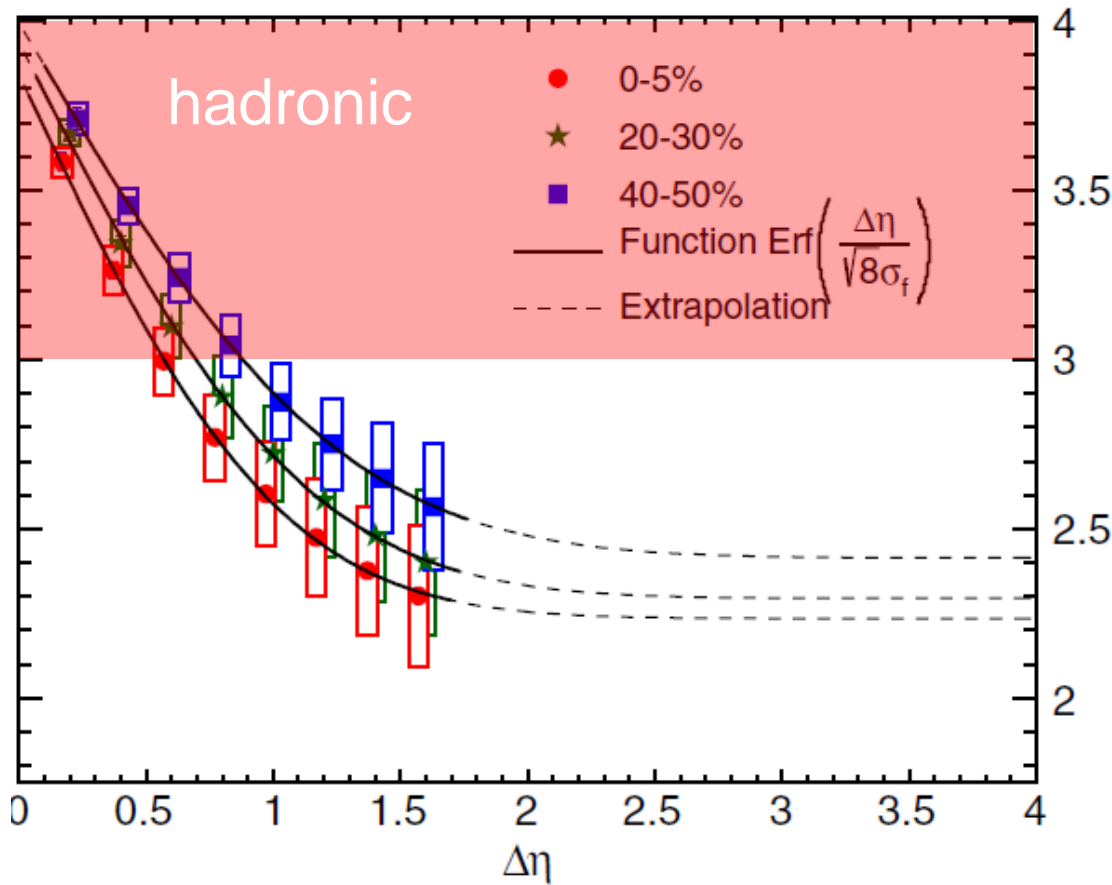
|}

momentum-space rapidity y
of **individual particles**

$$\Delta y \simeq \Delta Y$$

$\Delta\eta$ Dependence @ ALICE

ALICE
PRL 2013



$\Delta\eta$

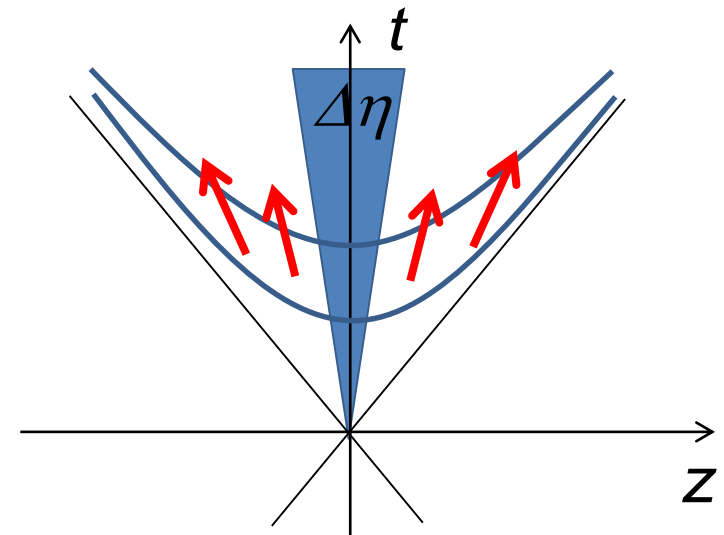
↑

rapidity window

D-measure

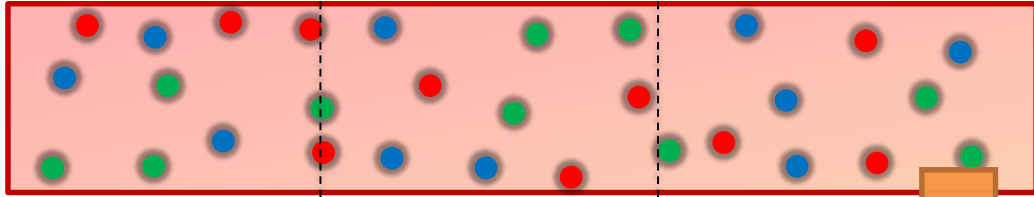
$$D = 4 \frac{\langle \delta N_Q^2 \rangle}{\langle N_Q^+ + N_Q^- \rangle}$$

- $D \sim 3-4$ Hadronic
- $D \sim 1-1.5$ Quark

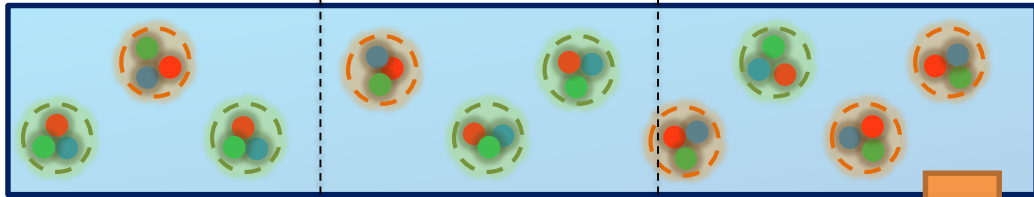


Time Evolution of Fluctuations

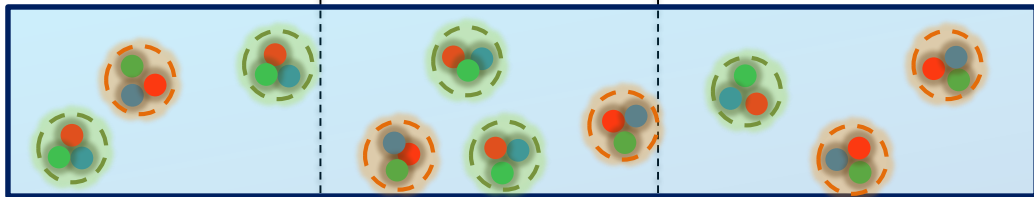
Quark-Gluon Plasma



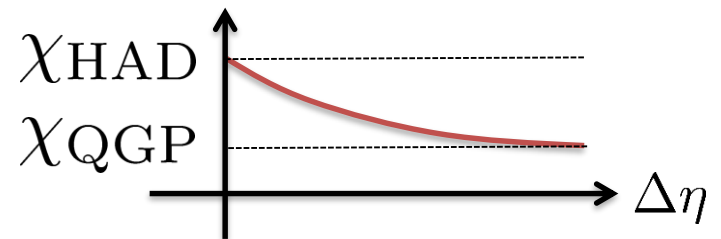
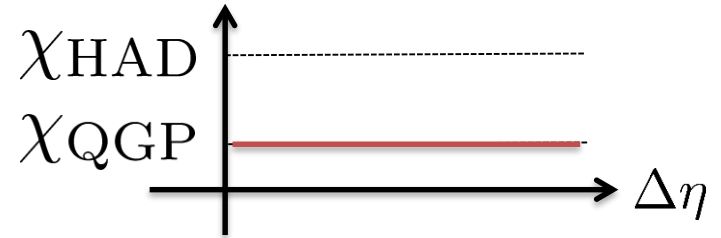
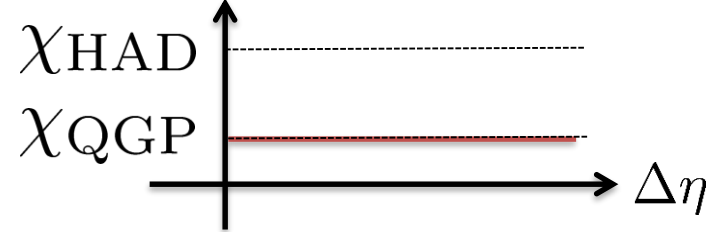
Hadronization



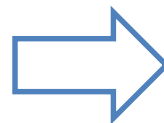
Freezeout



$$\frac{\langle \Delta N^2 \rangle}{\Delta \eta}$$



Variation of a conserved charge is achieved only through diffusion.



The larger $\Delta \eta$, the slower diffusion