Electric fluctuations and conductivity at finite density under a magnetic field

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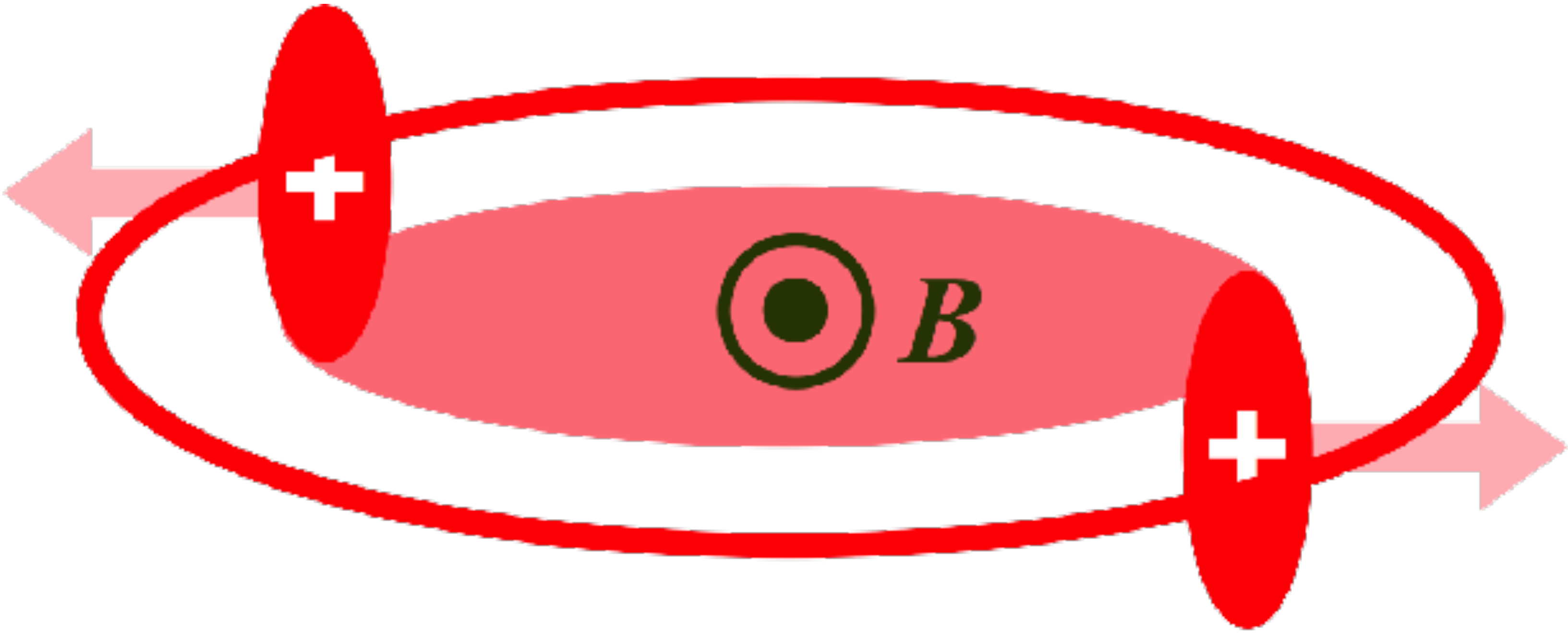
Fukushima-Hidaka, arXiv:1710.xxxxx

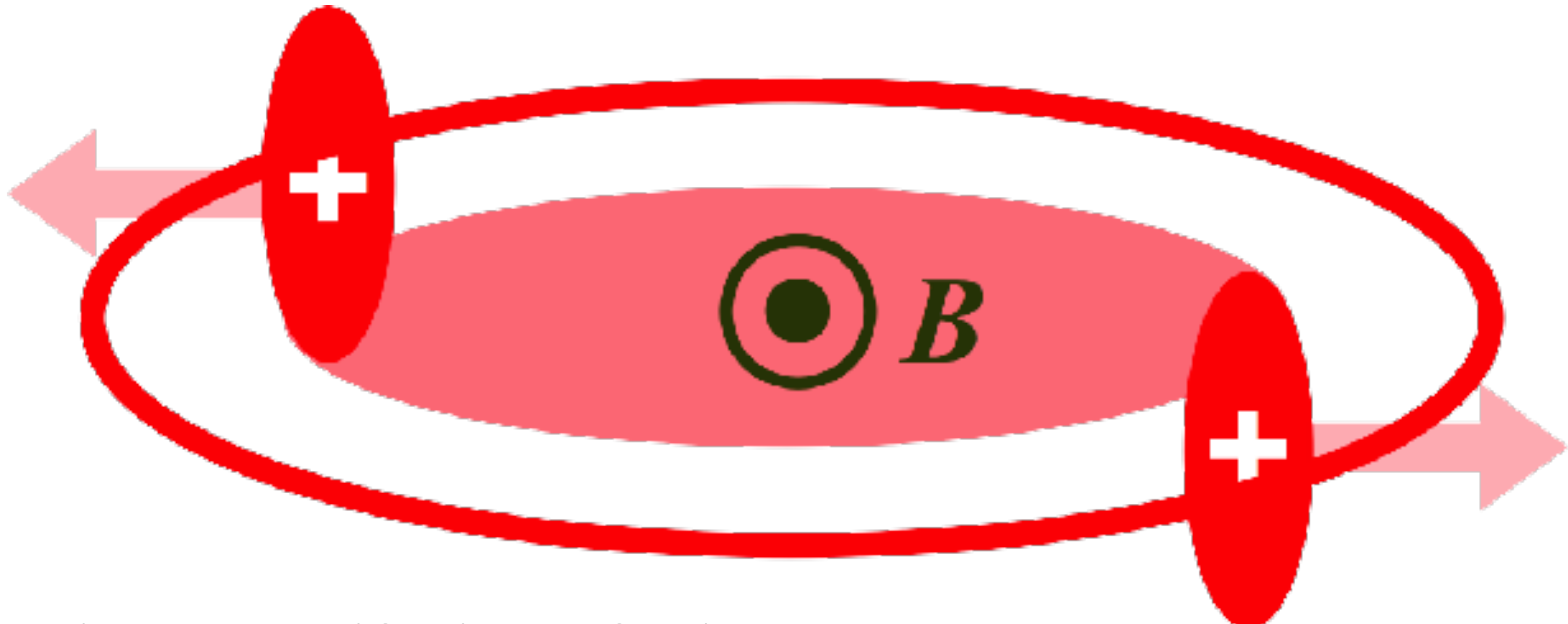
— Critical Fluctuations Near the QCD Phase Boundary in Relativistic Nuclear Collisions —

Motivation



Interesting physics expected IF *B* lives long

Motivation



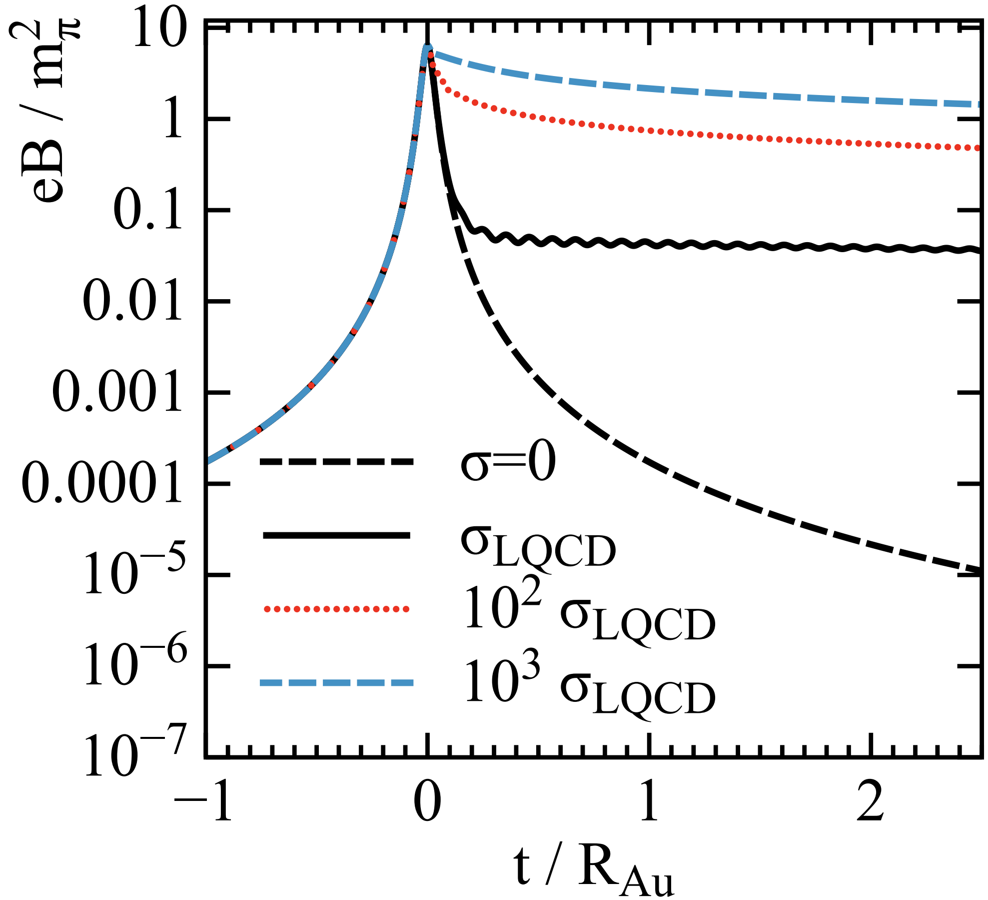
Naively, the life time of *B* is very short

$$eB_0 = (47.6\ \mathrm{MeV})^2\left(\frac{1\ \mathrm{fm}}{b}\right)^2 Z\sinh Y$$

$$t_0 = \frac{b}{2\sinh(Y)}$$

Motivation

McLerran-Skokov (2013)



Electric conductivity σ crucial for the fate of B

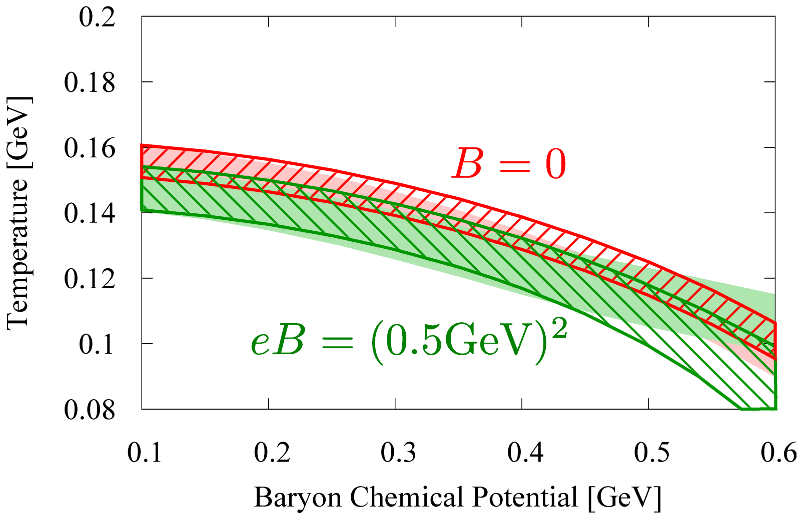
σ affected by strong B ?
Maybe larger at finite μ ?

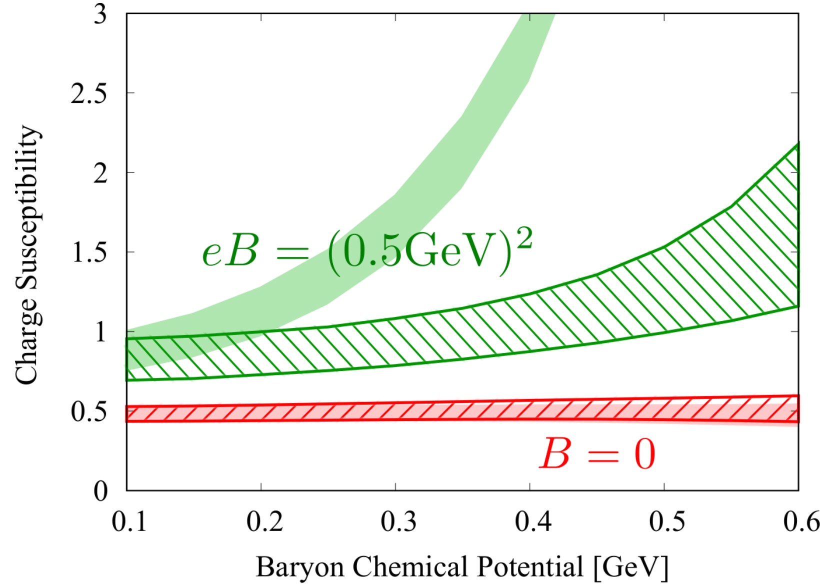
However, lowest Landau level approximation is NOT good due to phase space restriction

(Hattori-Satow-Li-Yee 2016)

Motivation

Fukushima-Hidaka (2016)





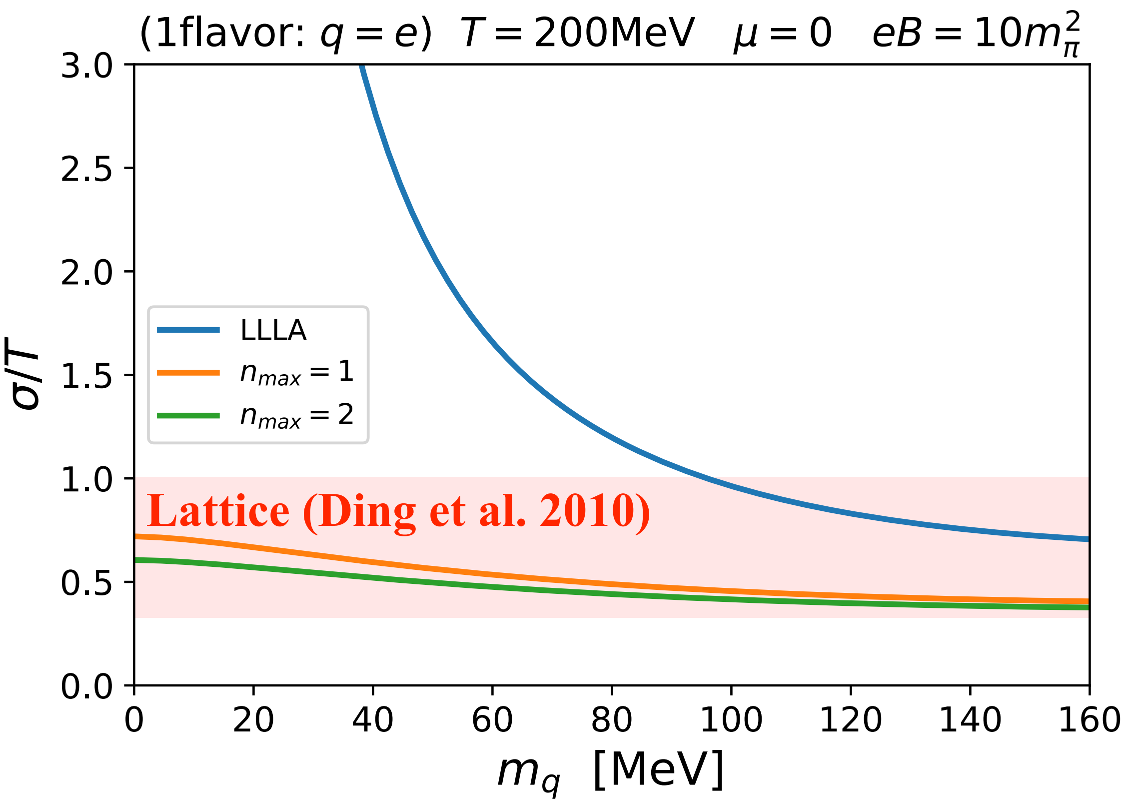
HRG model with *B*
* Inverse magnetic catalysis
* Enhanced charge fluctuations

Density → More protons
B → Lighter protons

Natural to anticipate enhanced electric conductivity ?

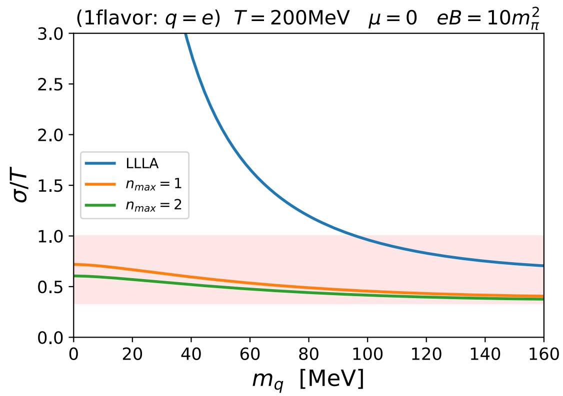
Conclusion First

Single flavor (with unit charge; C_{em}=1)



We assume: $T \gtrsim \sqrt{qB} \gg gT$ (thermal screening neglected)

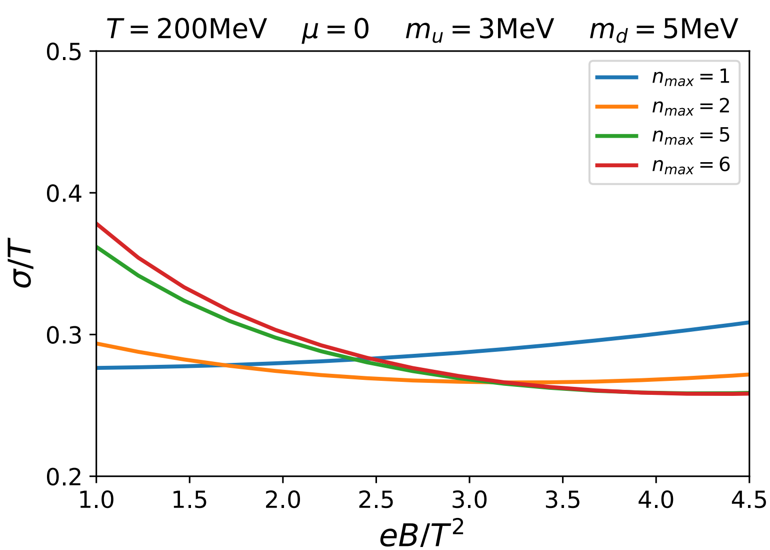
Conclusion First

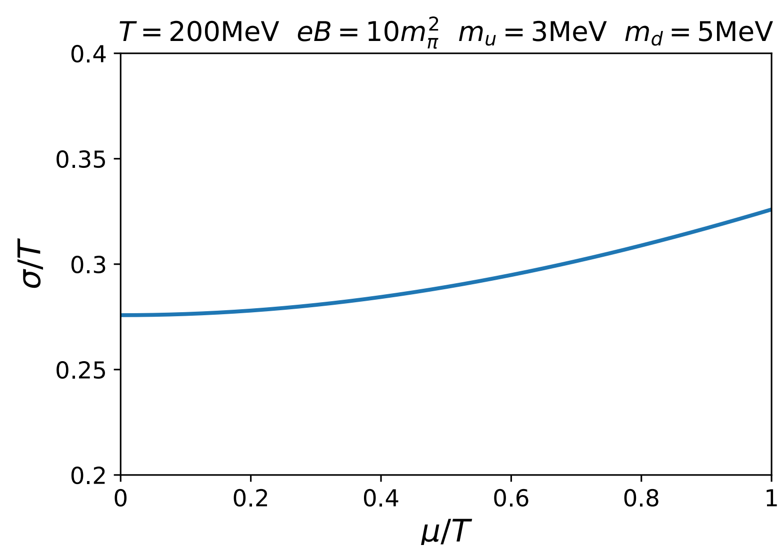


LLLA is not good (massless quarks cannot scatter in (1+1) D), but **Next-to-LLLA (NLLLA) is very good** (convergence is pretty fast)

Results are surprisingly close to (quenched) lattice estimates

***B* and *μ* dependence mild ? → Yes !?**





Calculations

Kubo Formula

$$\sigma^{ij} = \lim_{k_0 \to 0} \lim_{\boldsymbol{k} \to \boldsymbol{0}} \frac{1}{2\mathrm{i}k_0} \left[\Pi_R^{ij}(k) - \Pi_A^{ij}(k)\right]$$

$$\Pi_R^{\mu\nu}(k) := \mathrm{i} \int \mathrm{d}^4 x \, \mathrm{e}^{\mathrm{i}k\cdot x} \, \theta(t) \langle [j^\mu(x), j^\nu(0)] \rangle \,,$$

$$\Pi_A^{\mu\nu}(k) := -\mathrm{i} \int \mathrm{d}^4 x \, \mathrm{e}^{\mathrm{i}k\cdot x} \, \theta(-t) \langle [j^\mu(x), j^\nu(0)] \rangle$$

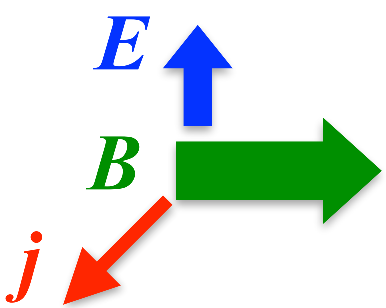
$$j^i_{\mathrm{em}} - n_0 T^{0i} / (\mathcal{E} + \mathcal{P}_i)$$

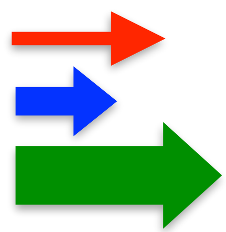
Hydro zero-mode subtracted if the charge density non-zero

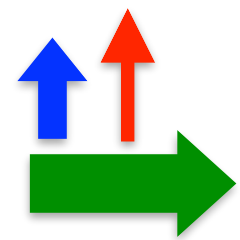
Calculations

Tensor Decomposition

$$\sigma^{ij} = \sigma_H \, \epsilon^{ijk} \hat{B}^k + \sigma_{\parallel} \hat{B}^i \hat{B}^j + \sigma_{\perp} (\delta^{ij} - \hat{B}^i \hat{B}^j)$$







$$\sigma_H = \frac{n_0}{B}$$

Hall conductivity

?

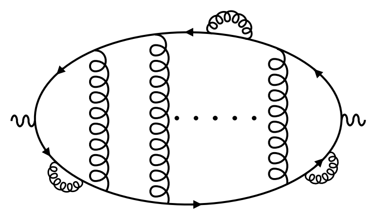
$$\frac{\sigma_\perp}{T} \sim \frac{g^2 T^2}{|qB|}$$

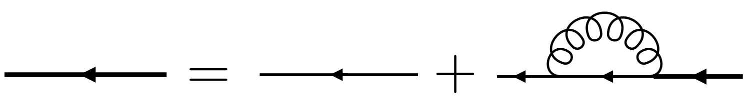
Suppressed

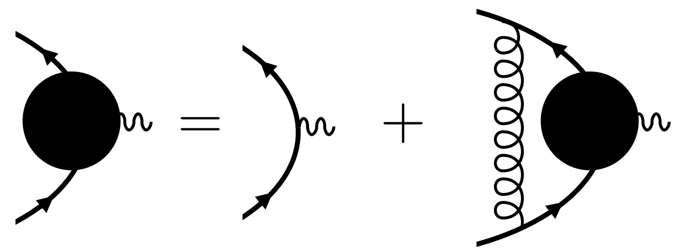
$$T \gtrsim \sqrt{qB} \gg gT$$

Calculations

Pinch singularities must be avoided by resummation

Higher order diagrams  **generated by**





leading to

$$2P_p^{\mu}\left(\partial_\mu + q_f F_{\nu\mu}\partial_{p_\nu}\right) f_p = -C[f]$$

$$2\bar{P}_{p'}^{\mu}\left(\partial_\mu - q_f F_{\nu\mu}\partial_{p'_\nu}\right) \bar{f}_{p'} = -\bar{C}[f]$$

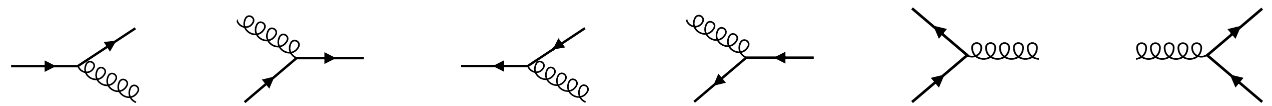
$$2k^\mu \partial_\mu g_k = -\tilde{C}[f]$$

$$2P_p^{\mu} := \bar{u}(p)\gamma^\mu u(p)$$

$$2\bar{P}_{p'}^{\mu} := \bar{v}(p')\gamma^\mu v(p')$$

Calculations

Strategy



(a1) $q \to qg$ (a2) $qg \to q$ (b1) $\bar{q} \to \bar{q}g$ (b2) $\bar{q}g \to \bar{q}$ (c1) $q\bar{q} \to g$ (c2) $g \to q\bar{q}$

Solve the Boltzmann equations for given collision terms

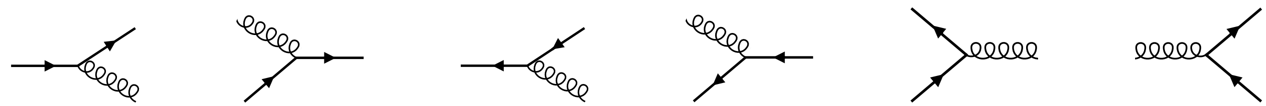
Expand distribution f around thermal equilibrium $f_{\rm eq}$

Linear deviation δf proportional to external E

$$\delta f_p = \beta f_{\rm eq}(p)[1 - f_{\rm eq}(p)]\, E_z\, \chi_p \,,$$
$$\delta \bar{f}_{p'} = \beta \bar{f}_{\rm eq}(p')[1 - \bar{f}_{\rm eq}(p')]\, E_z\, \bar{\chi}_{p'} \,,$$
$$\delta g_k = \beta g_{\rm eq}(k)[1 + g_{\rm eq}(k)]\, E_z\, \tilde{\chi}_k \,.$$

Calculations

Strategy



(a1) $q \to qg$ (a2) $qg \to q$ (b1) $\bar{q} \to \bar{q}g$ (b2) $\bar{q}g \to \bar{q}$ (c1) $q\bar{q} \to g$ (c2) $g \to q\bar{q}$

Express a current associated with *E*

Equilibrium has no current
Current proportional to δf and thus E
Coefficient → Electric conductivity

$$j_z = \sigma_{\parallel} E_z = \int_p 2P_p^3 q_f \left(\delta f_p - \delta \bar{f}_p\right)$$

$$\sigma_{\parallel} = \beta N_{\mathrm{c}} \sum_f \frac{q_f |q_f B|}{2\pi} \sum_{n=0}^{\infty} \alpha_n \int \frac{\mathrm{d}p_z}{2\pi} \frac{p_z}{\varepsilon_{fn}} \left\{ f_{\mathrm{eq}}(p)[1 - f_{\mathrm{eq}}(p)]\chi_p - \bar{f}_{\mathrm{eq}}(p)[1 - \bar{f}_{\mathrm{eq}}(p)]\bar{\chi}_p \right\}$$

Calculations

Linearized Boltzmann equations

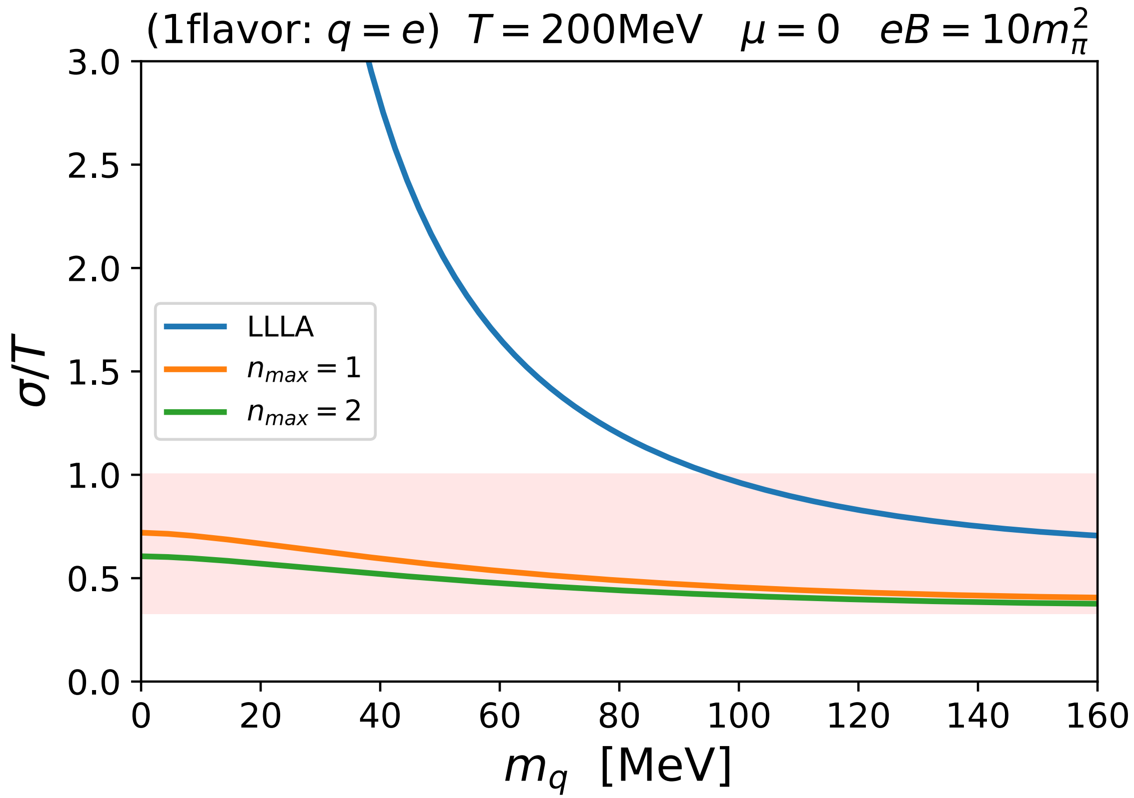
Symbolically $\mathcal{S} = \mathcal{J}^z - \mathcal{T}^{0z}/(\mathcal{E}+\mathcal{P}_z) = \boxed{\mathcal{L}\chi}$

$$\mathcal{J}^\mu := q_f \begin{pmatrix} p^\mu/\varepsilon_{fn} \\ -p'^\mu/\varepsilon_{fn'} \\ 0 \end{pmatrix}, \qquad \mathcal{T}^{0\mu} := \begin{pmatrix} p^\mu \\ p'^\mu \\ k^\mu \end{pmatrix}$$

Fixed from the collision terms
A huge matrix in Landau level space

This matrix has zero eigenvalue if hydro modes are not subtracted. In this approximation quark-gluon and flavor mixing occur only through the hydro mode subtraction.

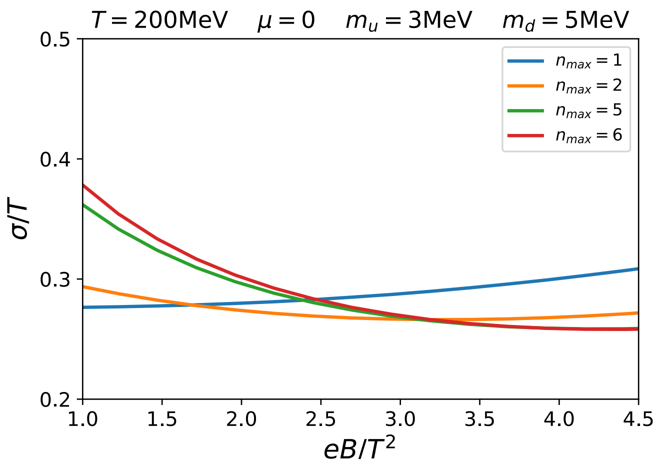
Results again



LLLA recovers Hattori-Satow-Li-Yee analytically

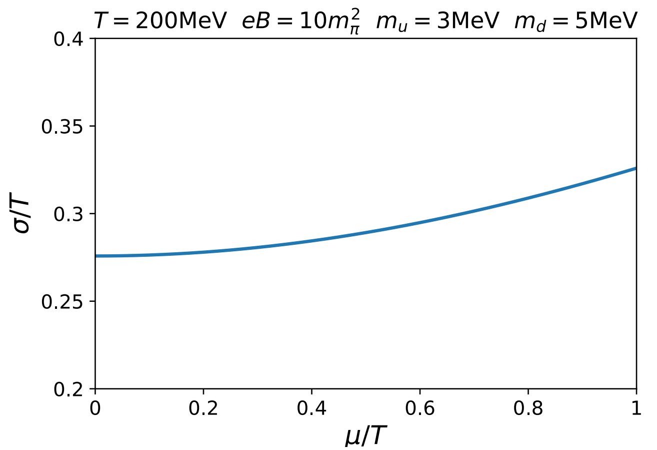
Large *B* is insufficient to justify LLLA

Results again



No order of magnitude enhancement

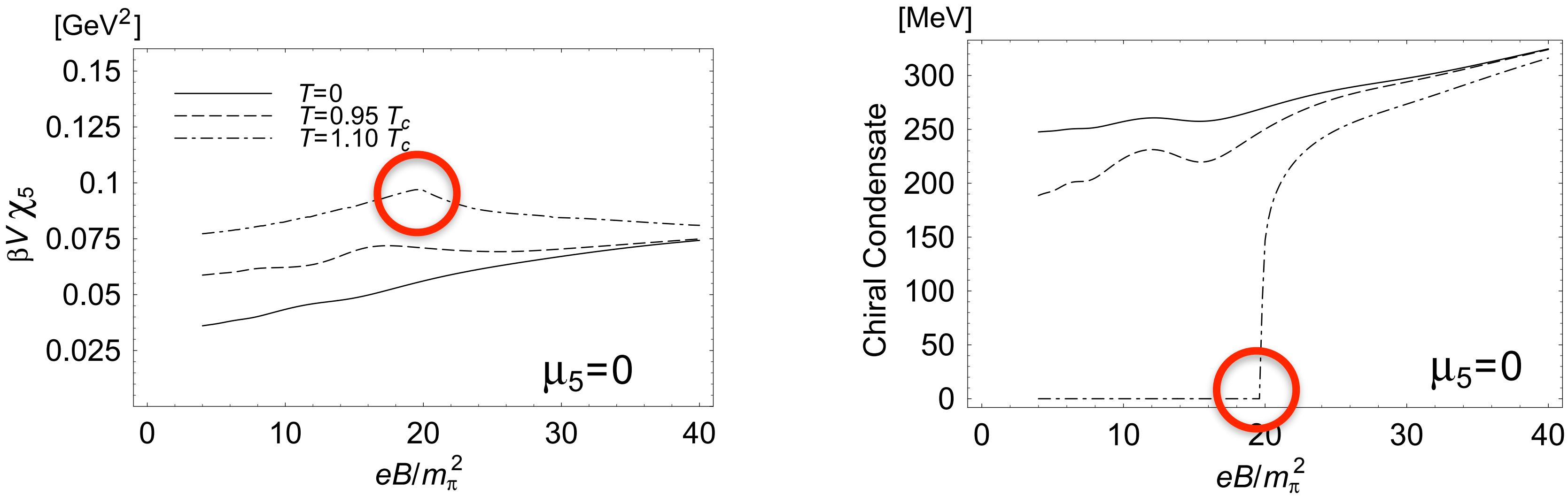
Unfortunately (and surprisingly)
the life time of B not much changed
by strong B and finite density
(Anyway, this is LONGITUDINAL)



This may be interesting because
ONLY QCD-Critical-Point may
significantly change σ.
(σ diverges at QCP)

CME near QCP ?

Fukushima-Ruggieri-Gatto (2010)

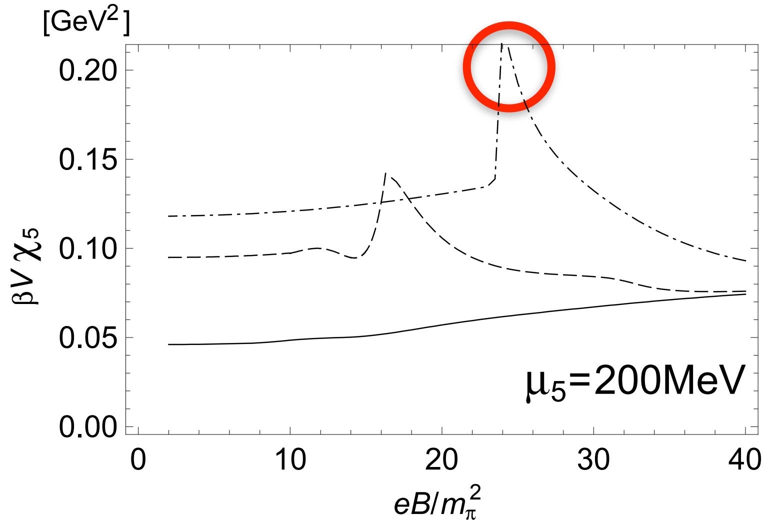


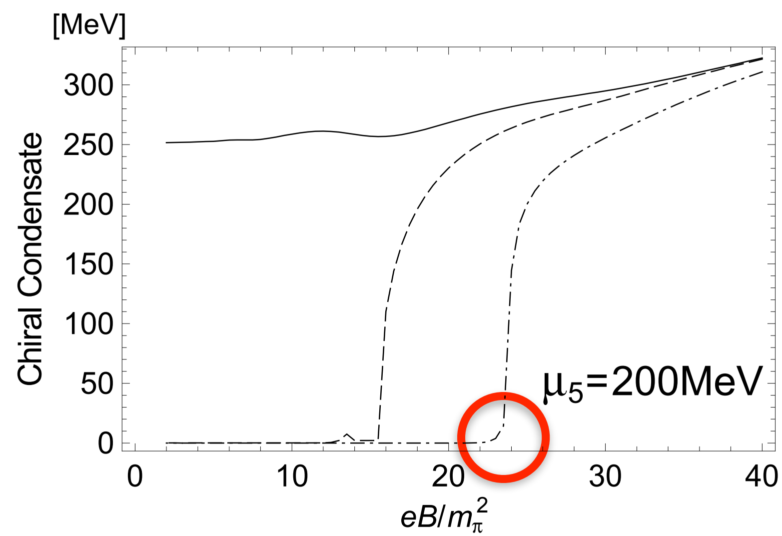
$$\chi_5 = \langle n_5^2 \rangle - \langle n_5 \rangle^2 = -\frac{1}{\beta V}\frac{\partial^2 \Omega}{\partial \mu_5^2}$$

Chiral charge fluctuations enhanced at phase transition

CME near QCP ?

Fukushima-Ruggieri-Gatto (2010)



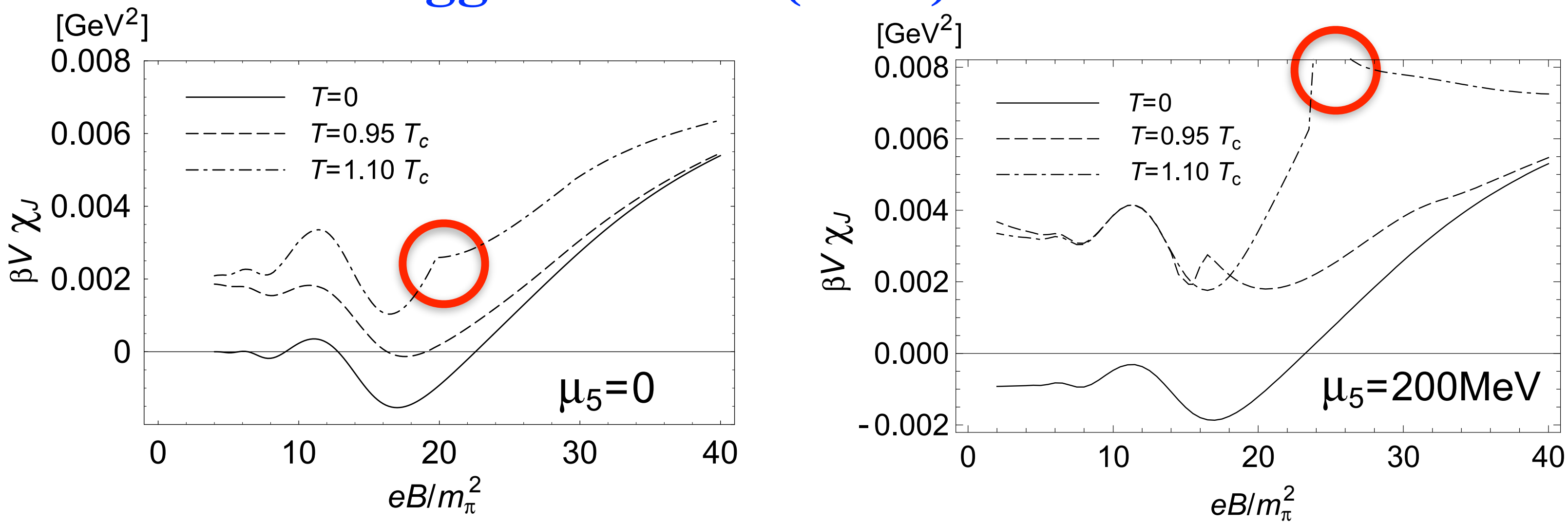


Chiral charge fluctuations enhanced at phase transition

At finite chiral chemical potential there is direct mixing between the chiral charge and the scalar (σ meson).

CME near QCP ?

Fukushima-Ruggieri-Gatto (2010)



$$\chi_J = \langle j_3^2 \rangle - \langle j_3 \rangle^2 = -\frac{1}{\beta V} \frac{\partial^2 \Omega}{\partial A_3^2}\bigg|_{A_3=0}$$

Current fluctuations enhanced at phase transition

Implications

QCP would significantly change σ.

If μ_5 is non-zero, the chiral phase transition would significantly change chirality and current fluctuations.

σ diverges more around QCP if μ_5 is non-zero.

An interesting question:

CME signals are *P*-odd fluctuations (which are *P*-even).
They should be affected by QCP — how?
γ-correlator near QCP should deserve more investigations.

I do not have an answer yet…

Conclusions

- **We have done the electric conductivity calculation at strong magnetic field and at finite density.**

- **We found weak dependence on the quark mass, the magnetic field, and the chemical potential (after the hydro zero-mode subtraction).**

- **Though our calculations assume a strong magnetic field, our results are surprisingly close to the (quenched) lattice QCD results.**