Conserved charge fluctuations at vanishing and non-vanishing chemical potential

Frithjof Karsch
Brookhaven National Laboratory & Bielefeld University

BROOKHAVEN NATIONAL LABORATORY

Universität Bielefeld

- Taylor expansion of the QCD equation of state and estimators for the location of the critical point
- Conserved charge fluctuations at vanishing and non-vanishing values of the chemical potential
- Probing the structure of strongly interacting matter with conserved charge fluctuations and correlations

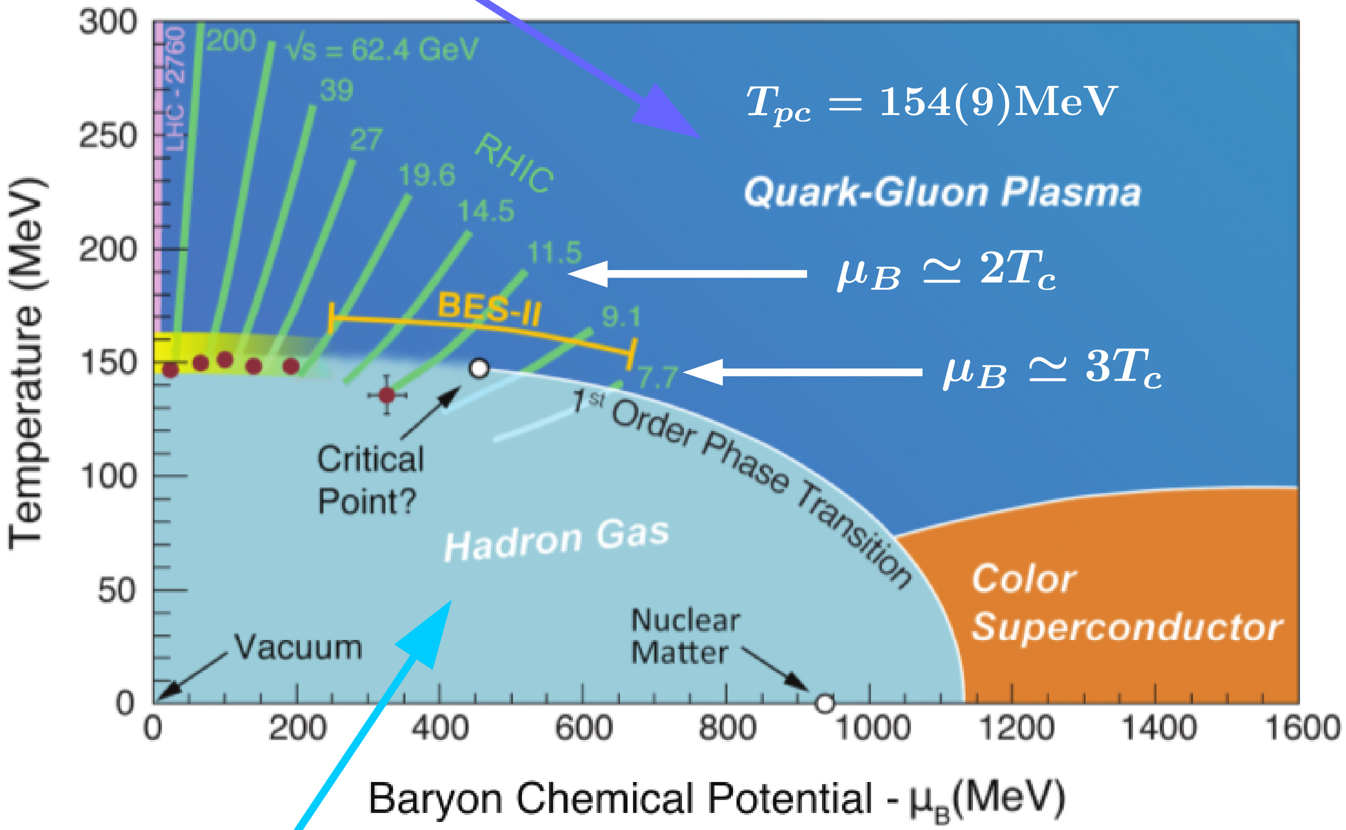
BEST COLLABORATION

CRC-211 TR

DFG Deutsche Forschungsgemeinschaft

Phases of strong-interaction matter

thermodynamics of quarks and gluons:
high-T – QCD perturbation theory



$T_{pc} = 154(9)\mathrm{MeV}$

$\mu_B \simeq 2T_c$

$\mu_B \simeq 3T_c$

thermodynamics of a hadron gas

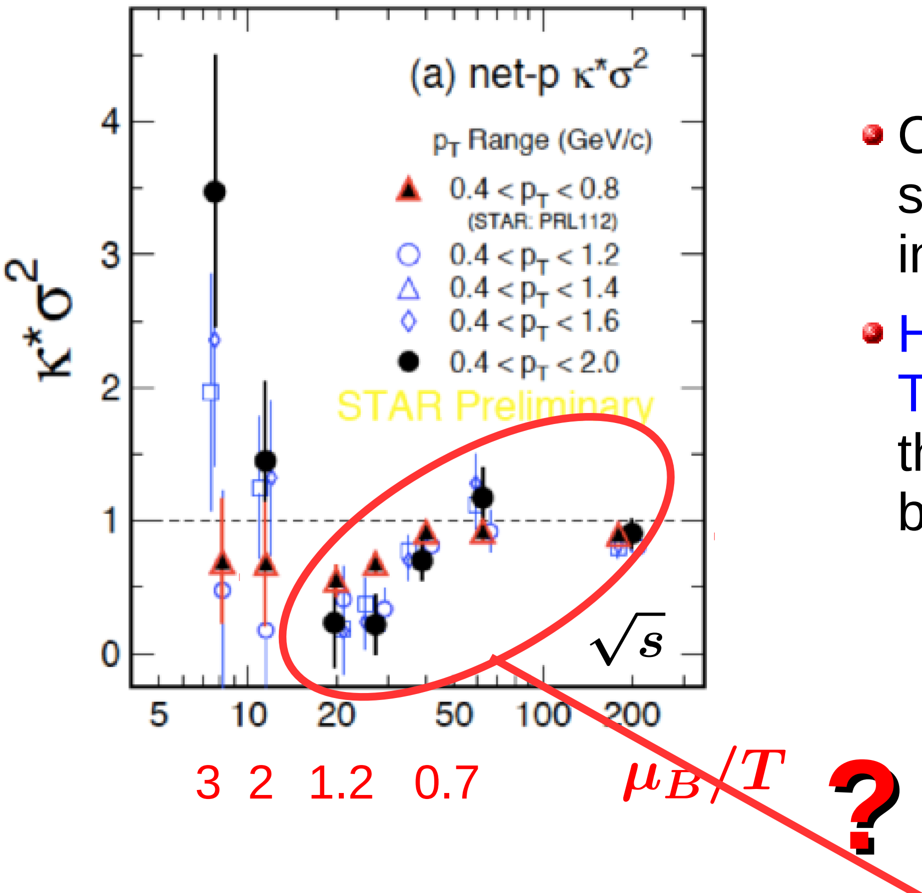
Where is the critical point?

most relevant for current experiments is to reach knowledge on the EoS for

$$0 \le \mu_B/T \le 3$$

$$(T \simeq T_c \simeq 155 \text{ MeV})$$

Exploring the QCD phase diagram



$$\kappa_X \sigma_X^2 = \frac{\chi_4^X}{\chi_2^X}$$

More moderate questions:

- Can we understand the systematics seen in cumulants of charge fluctuations in terms of QCD thermodynamics ?
- How far do we get with low order Taylor expansions of **QCD** in explaining the obvious deviations from HRG model behavior ?

- For $\sqrt{s} \geq 19.6\ \mathrm{GeV}$:
  Structure of net-proton cumulants can be understood in terms of **QCD thermodynamics in a next-to-leading order Taylor expansion**

A. Bazavov et al. (HotQCD), arXiv:1708.04897 to appear in Phys. Rev. D

Probing the properties of matter through the analysis of conserved charge fluctuations

Taylor expansion of the **QCD** pressure: $\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(T, V, \mu_B, \mu_Q, \mu_S)$

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

cumulants of net-charge fluctuations and correlations:

$$\chi_{ijk}^{BQS} = \left.\frac{\partial^{i+j+k} P/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k}\right|_{\mu_{B,Q,S}=0} \quad , \quad \hat{\mu}_X \equiv \frac{\mu_X}{T}$$

the pressure in hadron resonance gas (**HRG**) models:

$$\frac{p}{T^4} = \sum_{m \in meson} \ln Z_m^b(T, V, \mu) + \sum_{m \in baryon} \ln Z_m^f(T, V, \mu)$$

$$\sim \mathrm{e}^{-m_H/T} \mathrm{e}^{(B\mu_B + S\mu_S + Q\mu_Q)/T}$$

Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{i,j,k}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

the simplest case: $\mu_S = \mu_Q = 0$

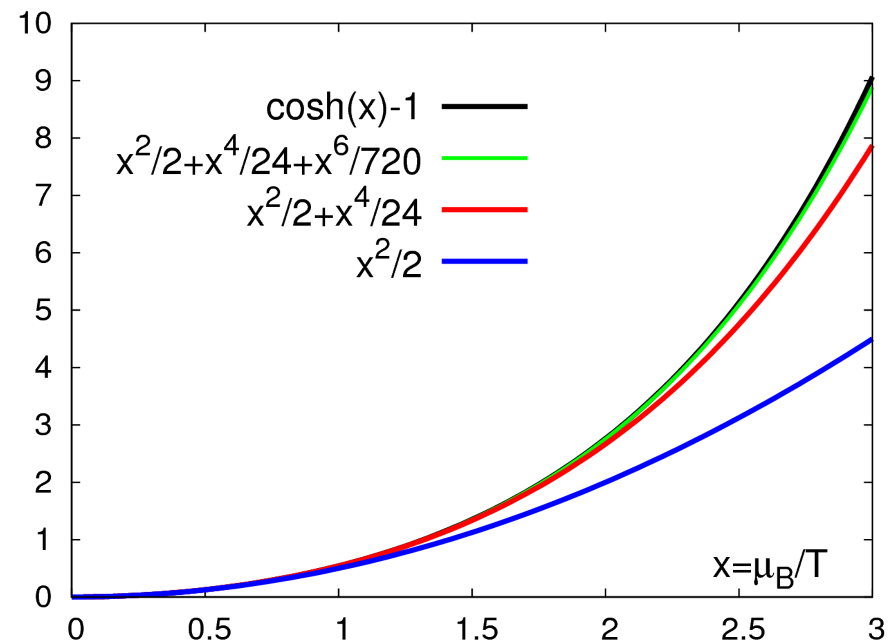
$$\frac{P(T,\mu_B)}{T^4} = \frac{P(T,0)}{T^4} + \frac{\chi_2^B(T)}{2}\left(\frac{\mu_B}{T}\right)^2 + \frac{\chi_4^B(T)}{24}\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}((\mu_B/T)^6)$$

An $\mathcal{O}((\mu_B/T)^4)$ expansion is exact in a QGP up to $\mathcal{O}(g^2)$

HRG vs. QCD:

$\mathcal{O}((\mu_B/T)^4)$: difference is less than 3% at $\mu_B/T = 2$

$\mathcal{O}((\mu_B/T)^6)$: difference is less than 2% at $\mu_B/T = 3$



Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{i,j,k}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

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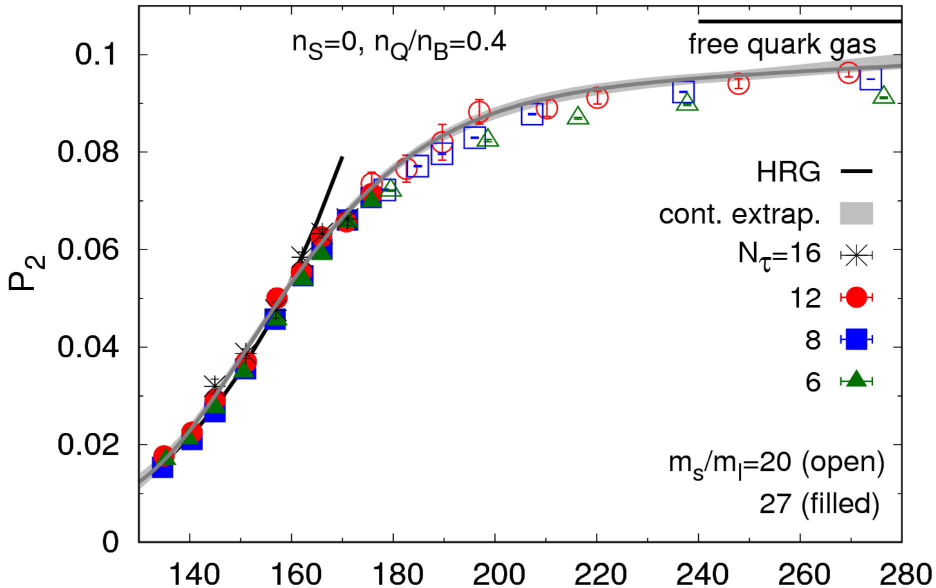
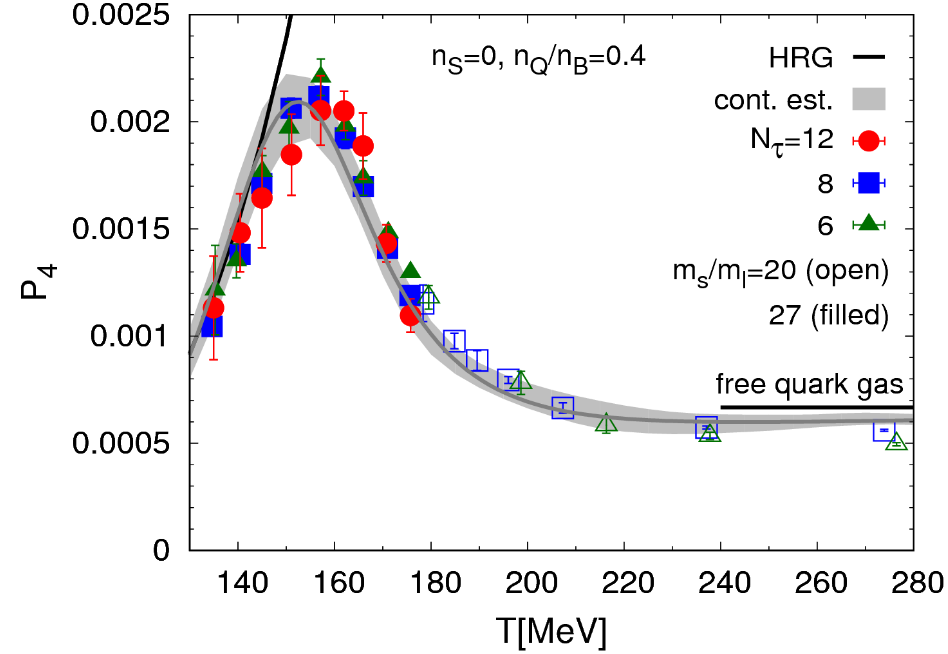
$$\frac{P(T,\mu_B)}{T^4} = \frac{P(T,0)}{T^4} + \frac{\chi_2^B(T)}{2}\left(\frac{\mu_B}{T}\right)^2 + \frac{\chi_4^B(T)}{24}\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}((\mu_B/T)^6)$$

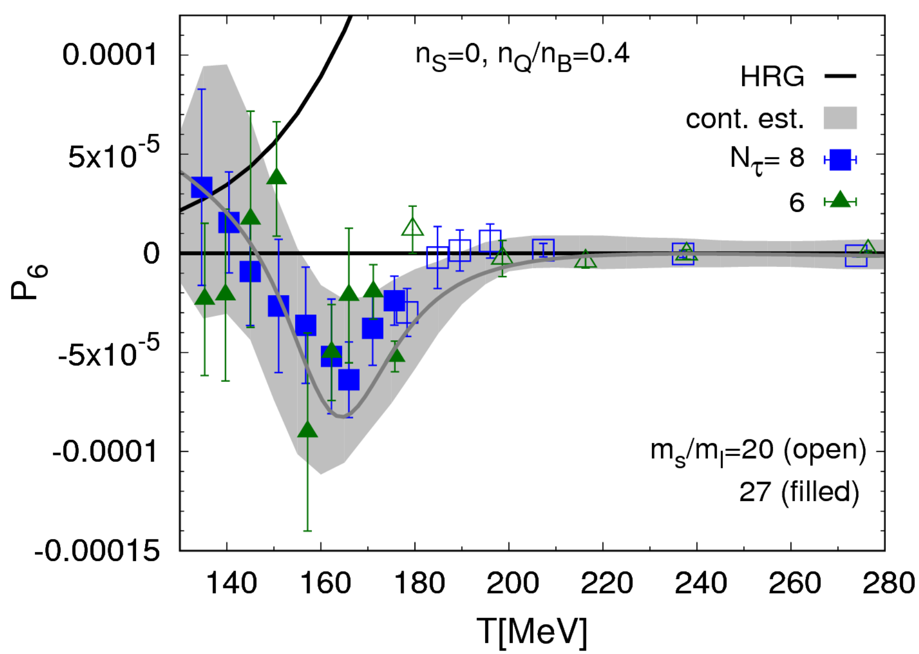
strangeness neutral: $n_S = 0\ ,\ n_Q/n_B = 0.4\ \Rightarrow\ \mu_{S,Q} \equiv \mu_{S,Q}(\mu_B)$

$$\frac{P(T,\mu_B)}{T^4} = P_0(T) + P_2(T)\left(\frac{\mu_B}{T}\right)^2 + P_4(T)\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}((\mu_B/T)^6)$$

Taylor expansion coefficients





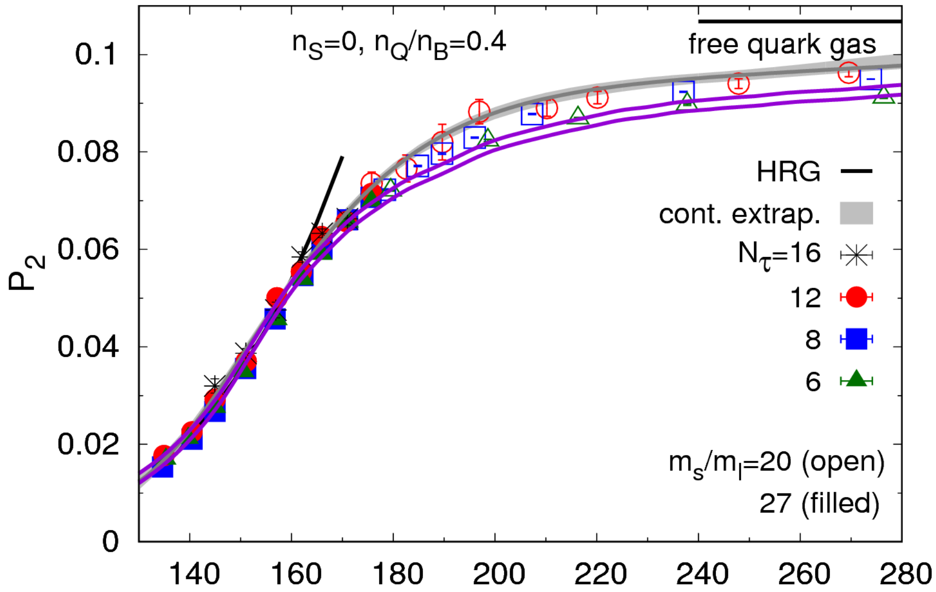


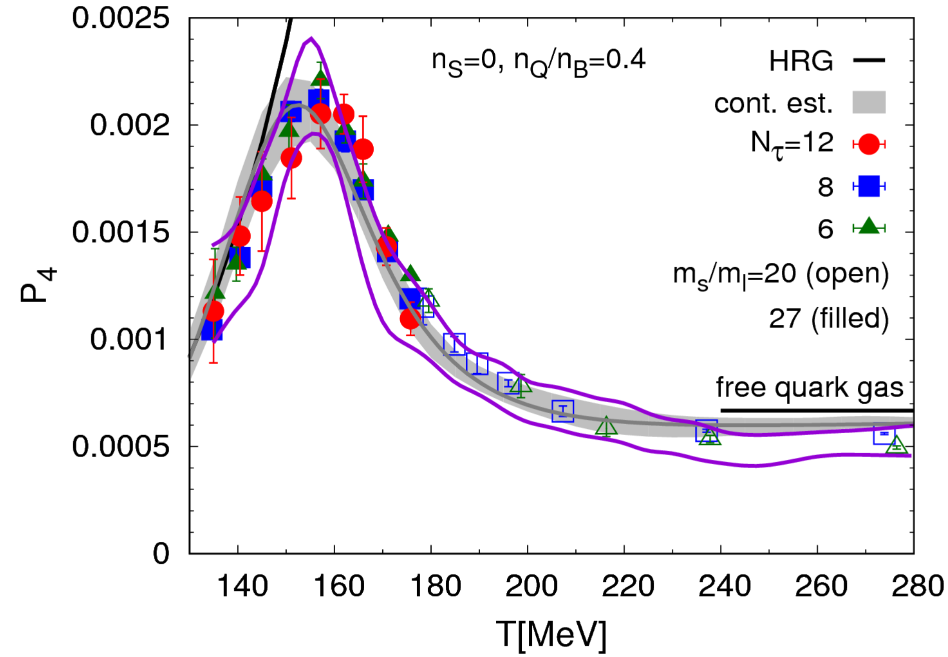
fits from: A. Bazavov et al., Phys. Rev. D 95 (2017) 054504

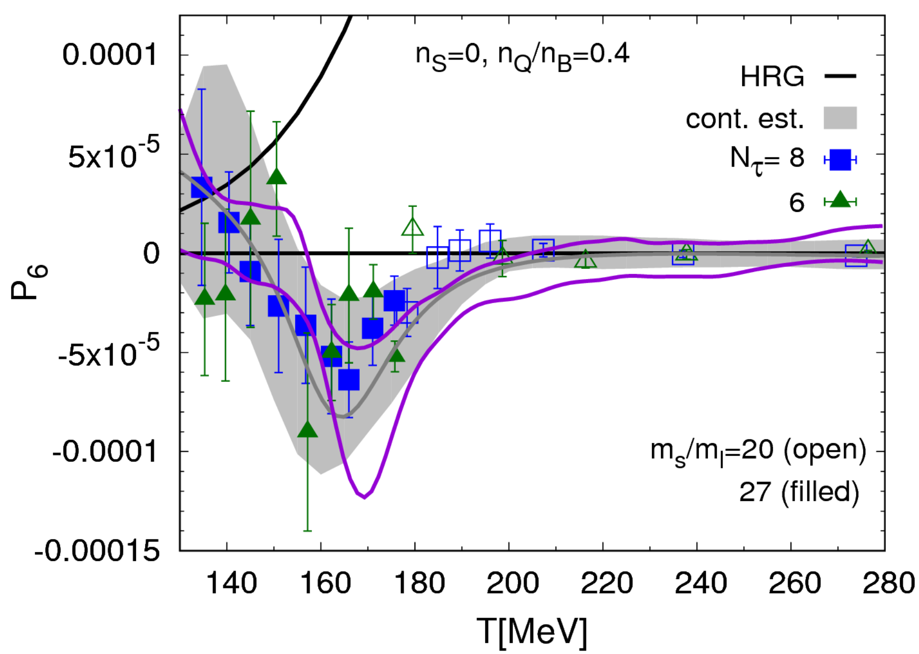
data are updated: HotQCD 2017

$$P_6 < 0 \text{ for } T \gtrsim 150 \text{ MeV}$$

Taylor expansion coefficients







fits from: A. Bazavov et al., Phys. Rev. D 95 (2017) 054504

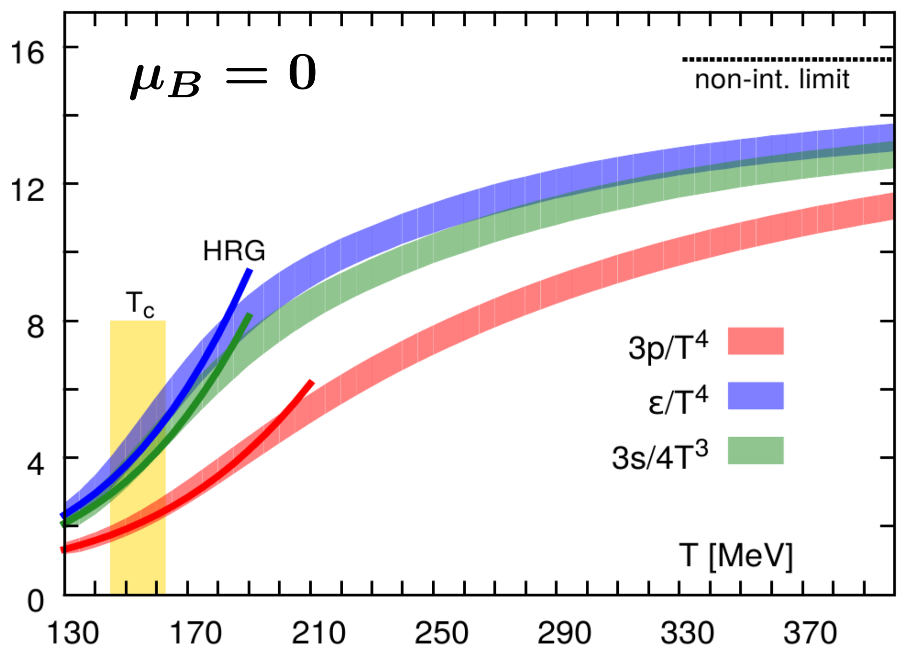
data are updated: hotQCD 2017

$$P_6 < 0 \text{ for } T \gtrsim 150 \text{ MeV}$$

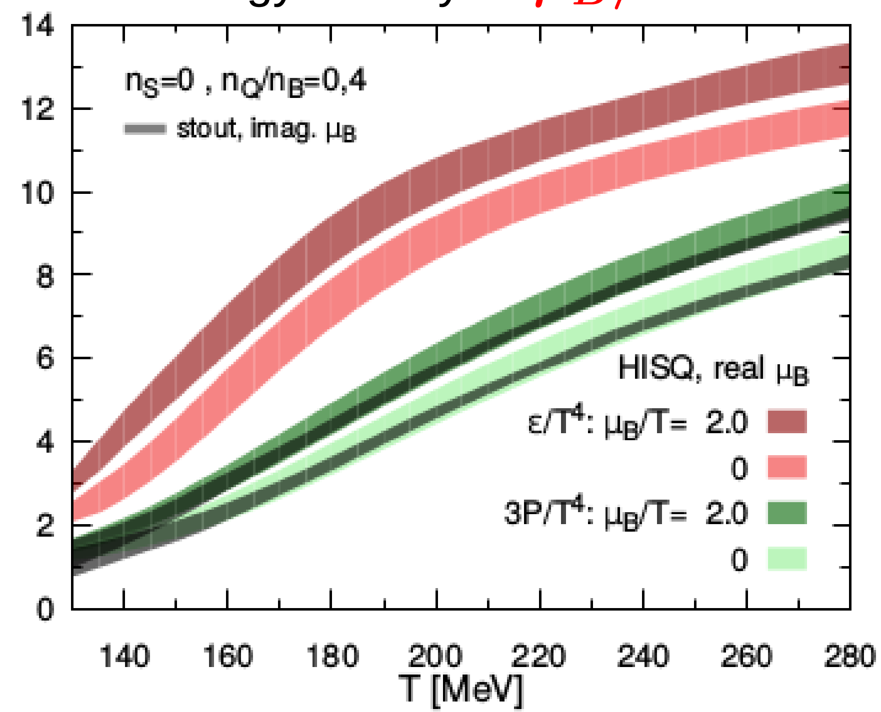
consistent with results obtained from analytic continuation:
J. Gunther et al., EPJ Web Conf. 137 (2017) 07008

Equation of state of (2+1)-flavor QCD: $\mu_B/T > 0$

$$\frac{\Delta(T,\mu_B)}{T^4} = \frac{P(T,\mu_B) - P(T,0)}{T^4} = \frac{\chi_2^B}{2}\left(\frac{\mu_B}{T}\right)^2 + \frac{\chi_4^B}{24}\left(\frac{\mu_B}{T}\right)^4 + \frac{\chi_6^B}{720}\left(\frac{\mu_B}{T}\right)^6 + \dots$$

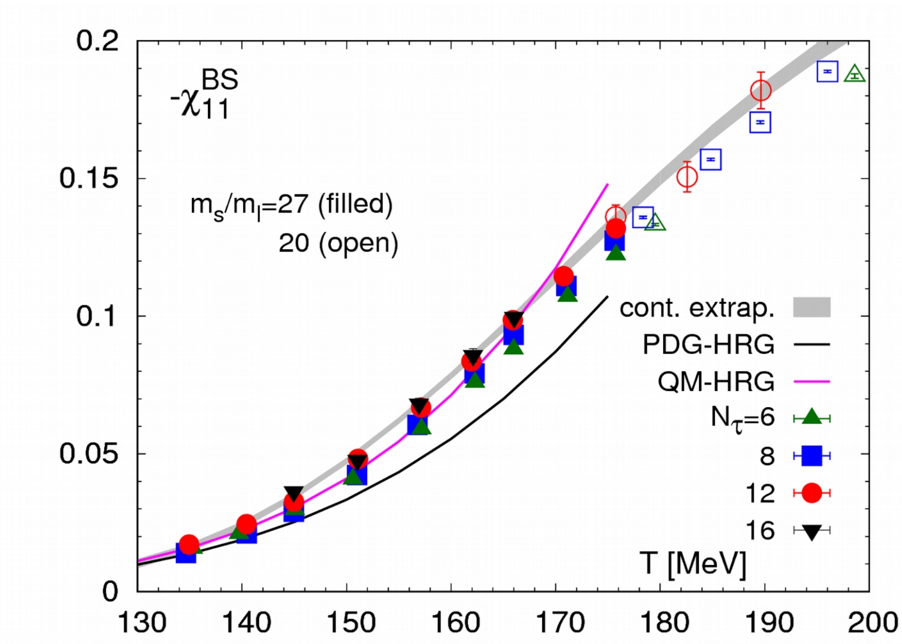


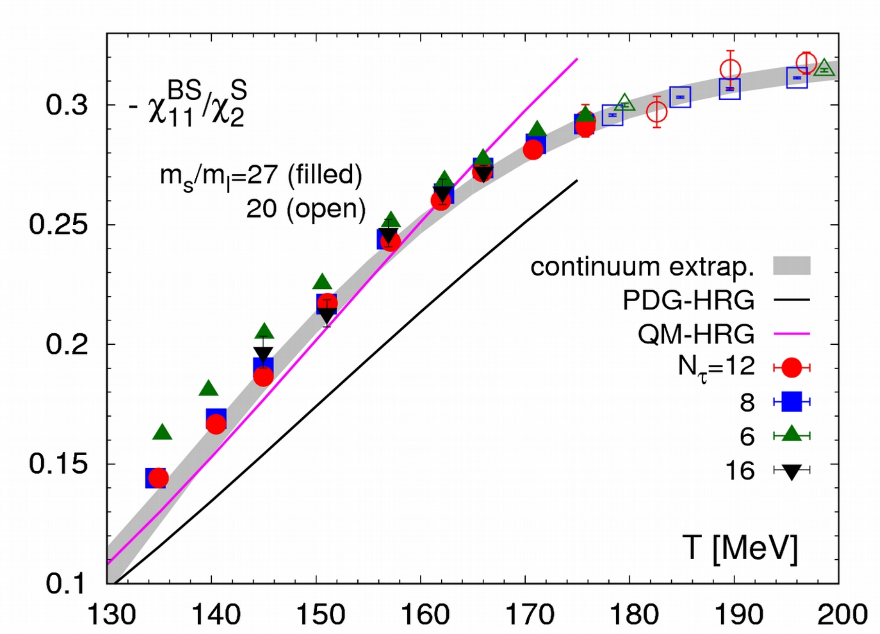
(10-50)% contribution to total energy density at $\mu_B/T = 2$

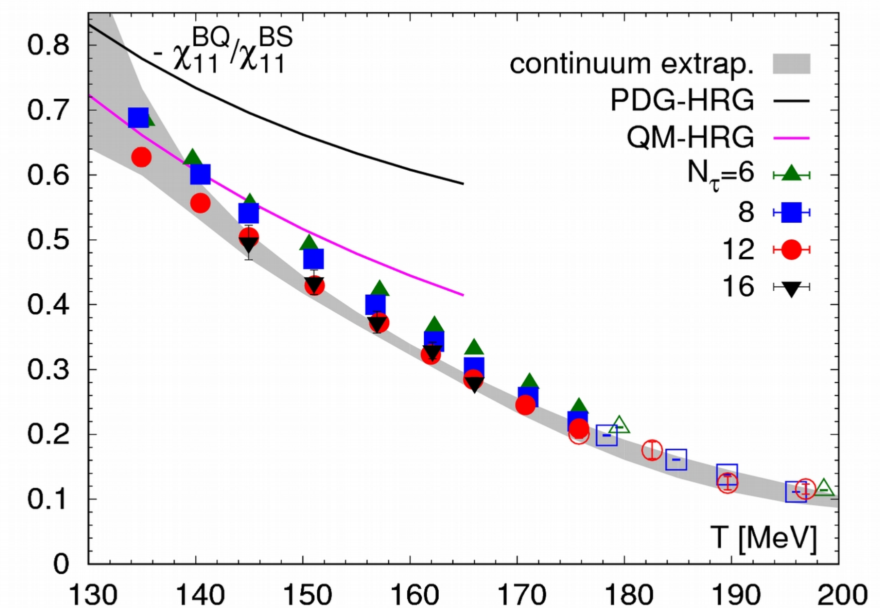


The EoS is well controlled for $\mu_B/T \leq 2$
or equivalently $\sqrt{s_{NN}} \geq 12$ GeV

Taylor expansion coefficients at $\mu_B = 0$
– observables at the LHC ? –





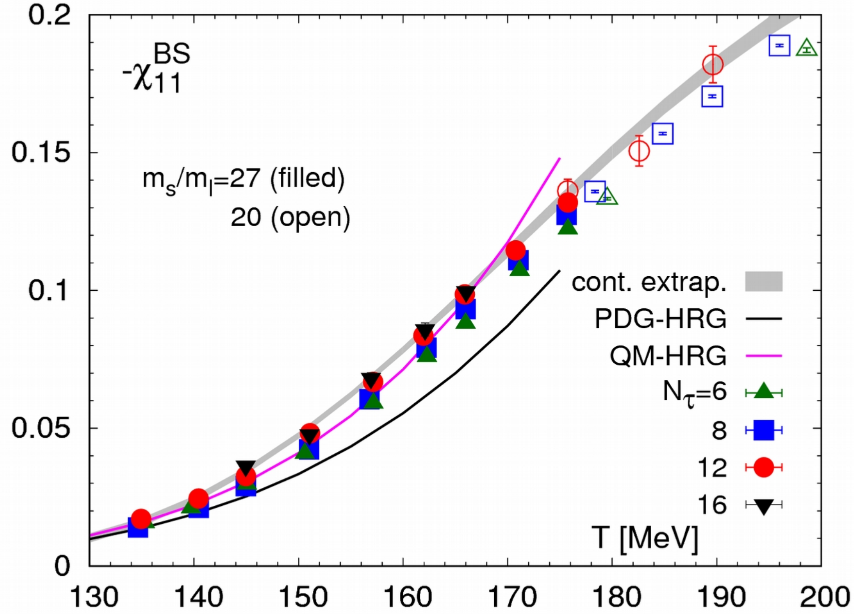


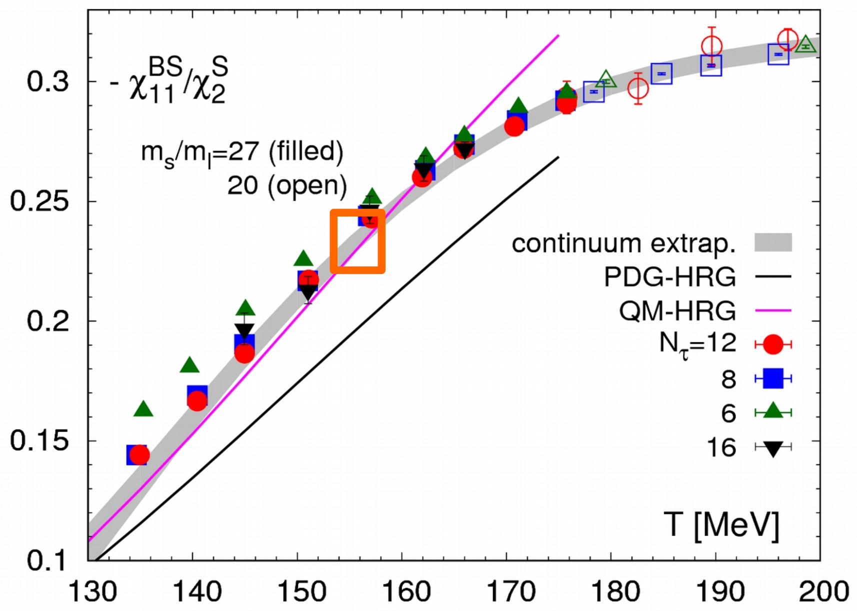
all fits and results are taken from:
lattice QCD calculation of 6th order Taylor expansion of the QCD equation of state, using the Highly Improved Staggered Quark (HISQ) action,

A. Bazavov et al. (Bielefeld-BNL-CCNU)
arXiv:1701.04325

data for $N_\tau = 12$ are updated: hotQCD 2017

Taylor expansion coefficients at $\mu_B = 0$
– observables at the LHC ? –

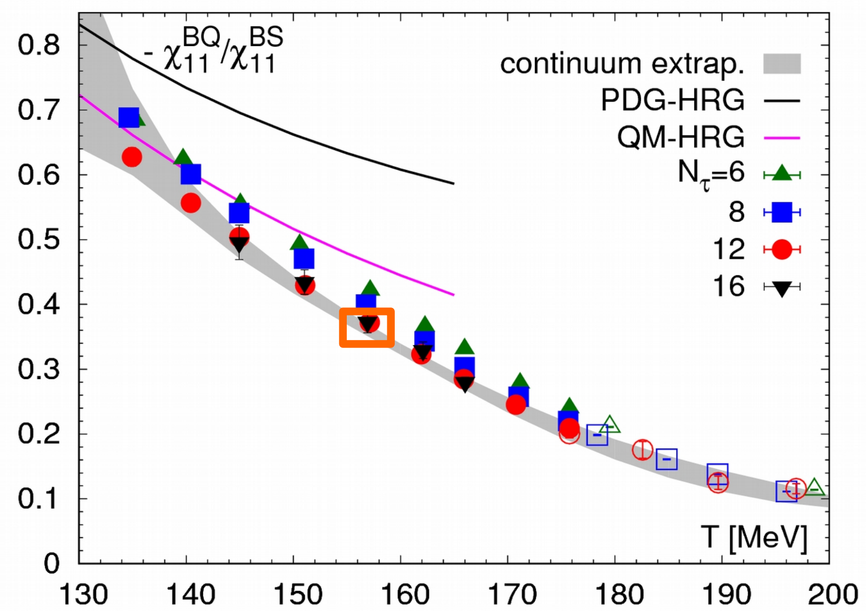




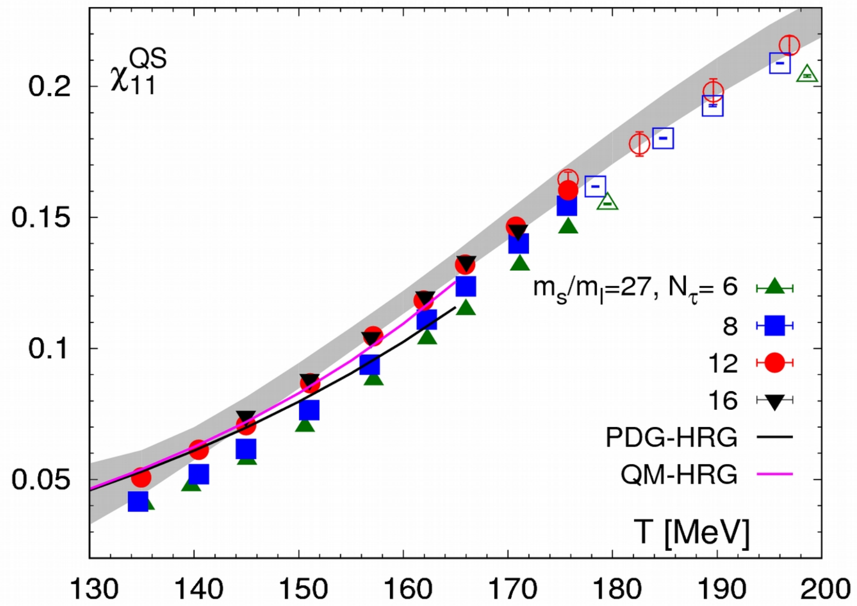
at ALICE freeze-out temperature
$T_{fo} = 156(3)\mathrm{MeV}$

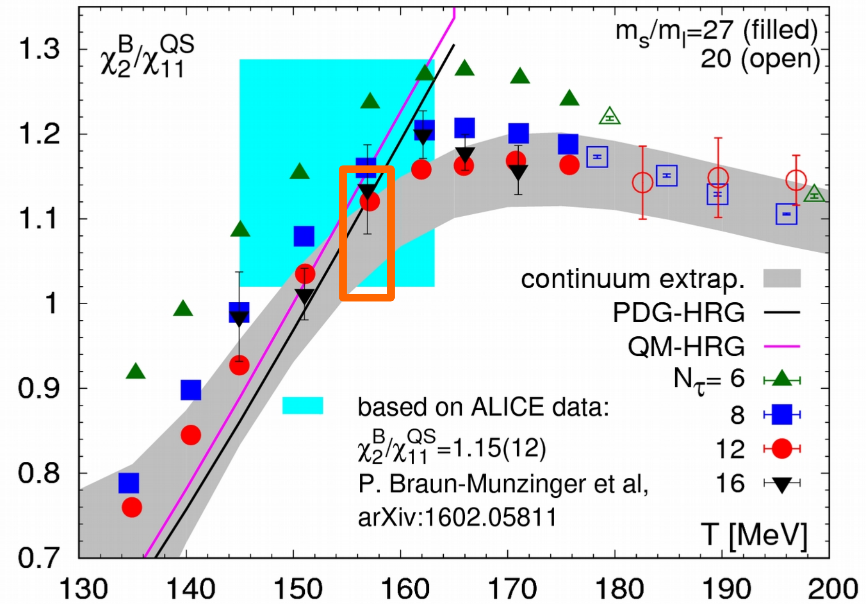
$$\chi_{11}^{BS}/\chi_2^S = -0.235(15)$$

$$\chi_{11}^{BQ}/\chi_{11}^{BS} = -0.37(3)$$



Taylor expansion coefficients at $\mu_B = 0$
– observables at the LHC ? –



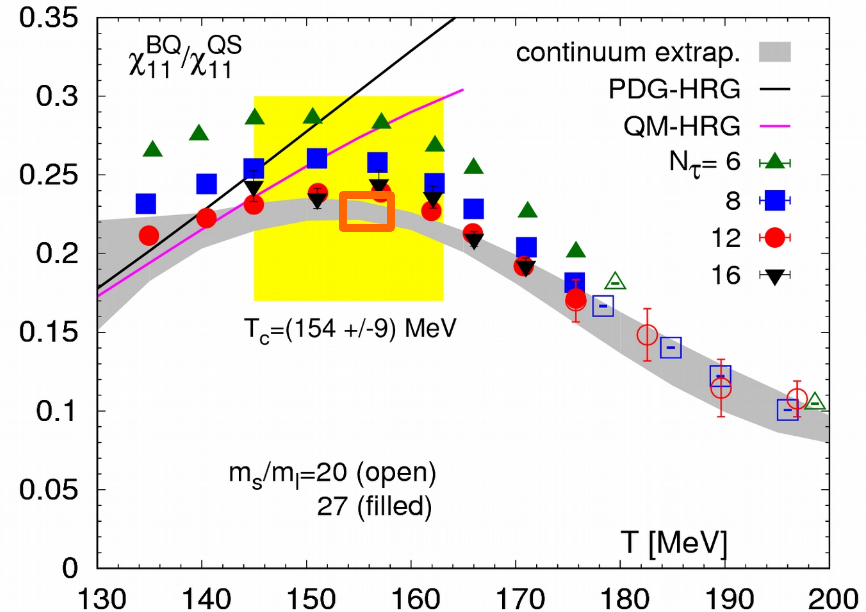


at ALICE freeze-out temperature

$$T_{fo} = 156(3)\mathrm{MeV}$$

$$\chi_2^B/\chi_{11}^{QS} = 1.10(8)$$

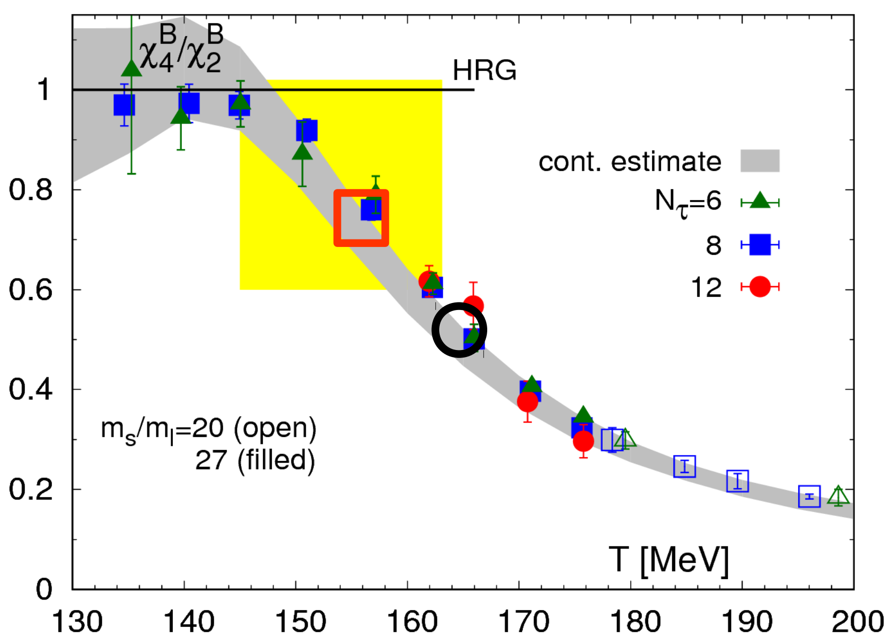
$$\chi_{11}^{BQ}/\chi_{11}^{QS} = 0.225(15)$$

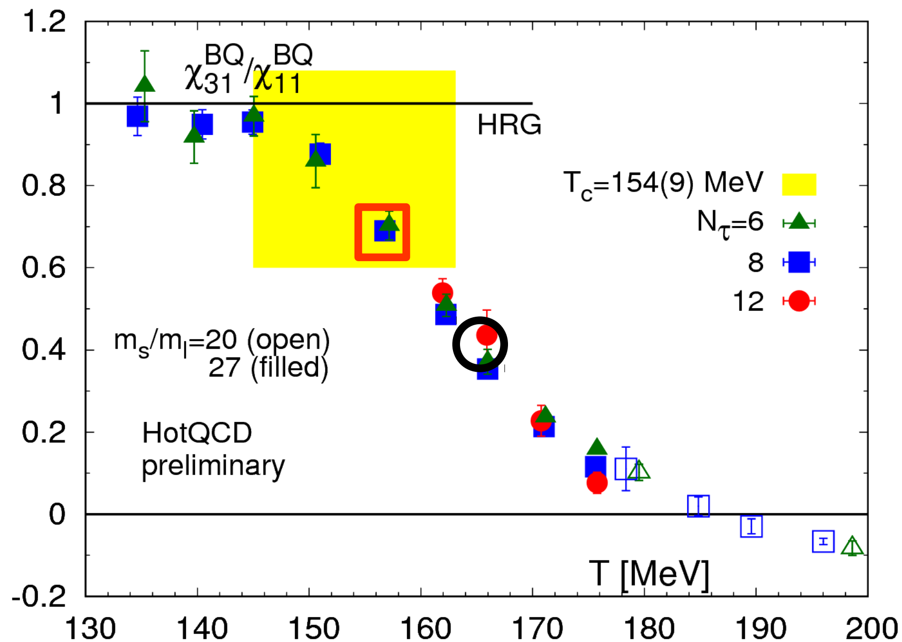


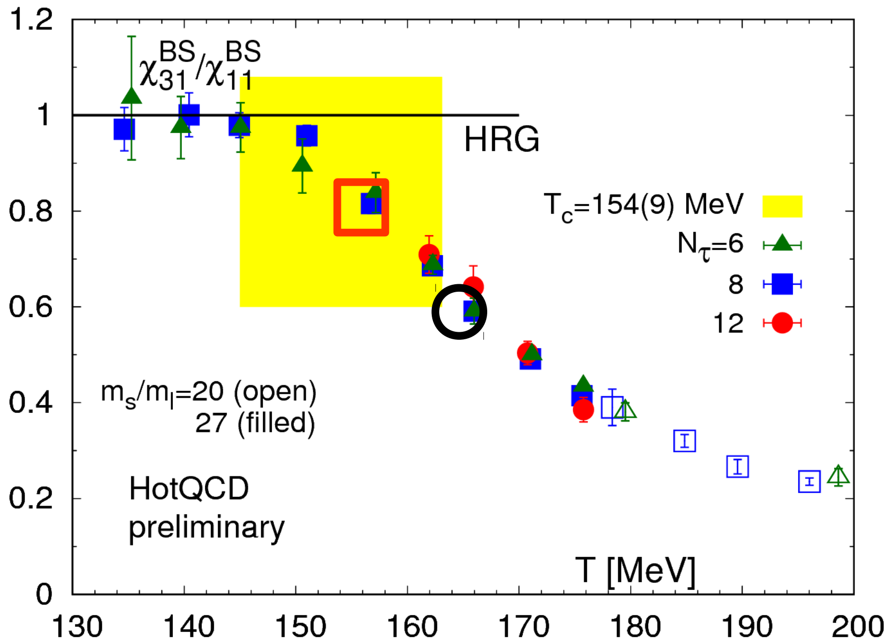
a QCD bound on ratios of charge correlations:

$$\chi_{11}^{BQ}/\chi_{11}^{QS} \leq 0.24$$

Ratios of 4th and 2nd order cumulants







– ratios of 4th and 2nd order cumulants differ from non-inter. HRG for T>145 MeV

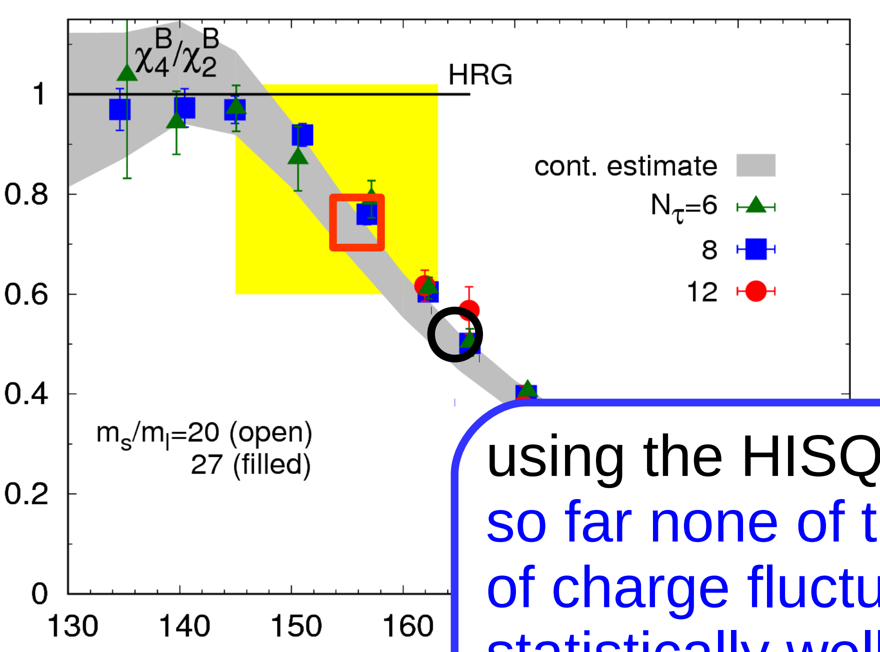
– they change by ~40% in the crossover region

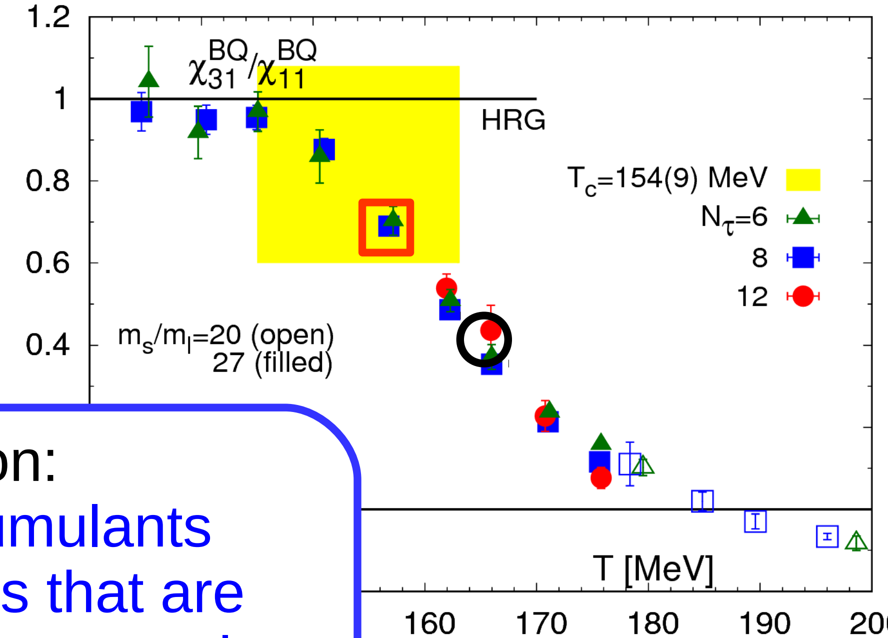
sensitive probes for freeze-out

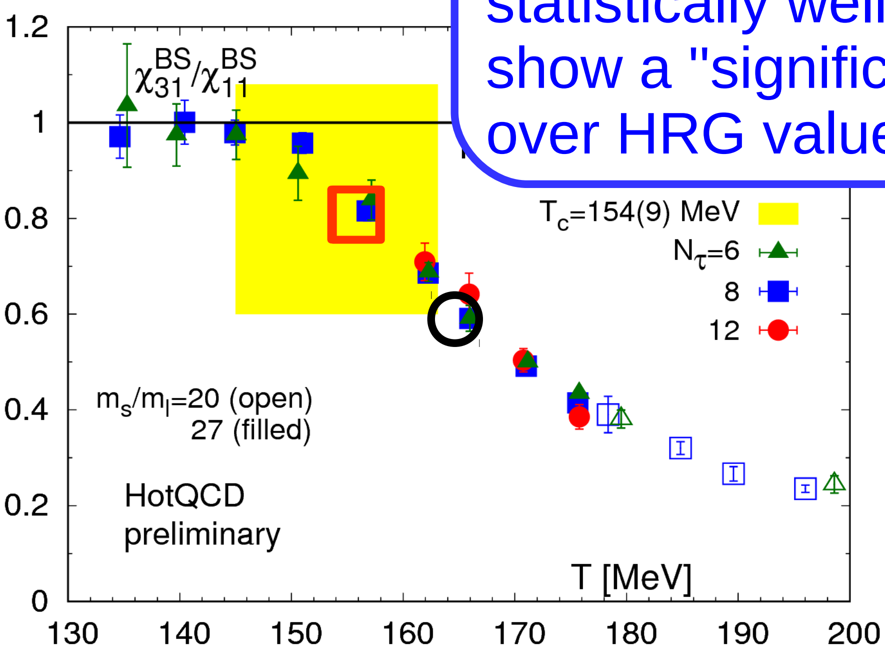
at ALICE freeze-out temperature $T_{fo} = 156(3)\mathrm{MeV}$

STAR at 200 GeV: $T_{fo} = 165(3)\mathrm{MeV}$

Ratios of 4th and 2nd order cumulants







using the HISQ action:
so far none of the cumulants of charge fluctuations that are statistically well under control show a "significant" enhancement over HRG values

d 2nd order cumulants differer
. HRG for T>145 MeV

– they change by ~40% in the crossover region

sensitive probes for freeze-out

at ALICE freeze-out temperature $T_{fo} = 156(3)\text{MeV}$

STAR at 200 GeV: $T_{fo} = 165(3)\text{MeV}$

Taylor expansion of the pressure and critical point

$$\frac{P}{T^4} = \sum_{n=0}^{\infty} \frac{1}{n!} \chi_n^B(T) \left(\frac{\mu_B}{T}\right)^n$$

for simplicity : $\mu_Q = \mu_S = 0$

estimator for the radius of convergence:

$$\left(\frac{\mu_B}{T}\right)^{\chi}_{crit,n} \equiv r_n^{\chi} = \sqrt{\left|\frac{n(n-1)\chi_n^B}{\chi_{n+2}^B}\right|}$$

– radius of convergence corresponds to a critical point only, iff

$$\chi_n > 0 \text{ for all } n \geq n_0$$

forces P/T^4 and $\chi_n^B(T,\mu_B)$ to be monotonically growing with μ_B/T

at T_{CP} : $\kappa_B \sigma_B^2 = \frac{\chi_4^B(T,\mu_B)}{\chi_2^B(T,\mu_B)} > 1$

if not:

– radius of convergence does not determine the critical point

– Taylor expansion can not be used close to the critical point

Taylor expansion of the pressure and critical point

$$\frac{P}{T^4} = \sum_{n=0}^{\infty} \frac{1}{n!} \chi_n^B(T) \left(\frac{\mu_B}{T}\right)^n$$

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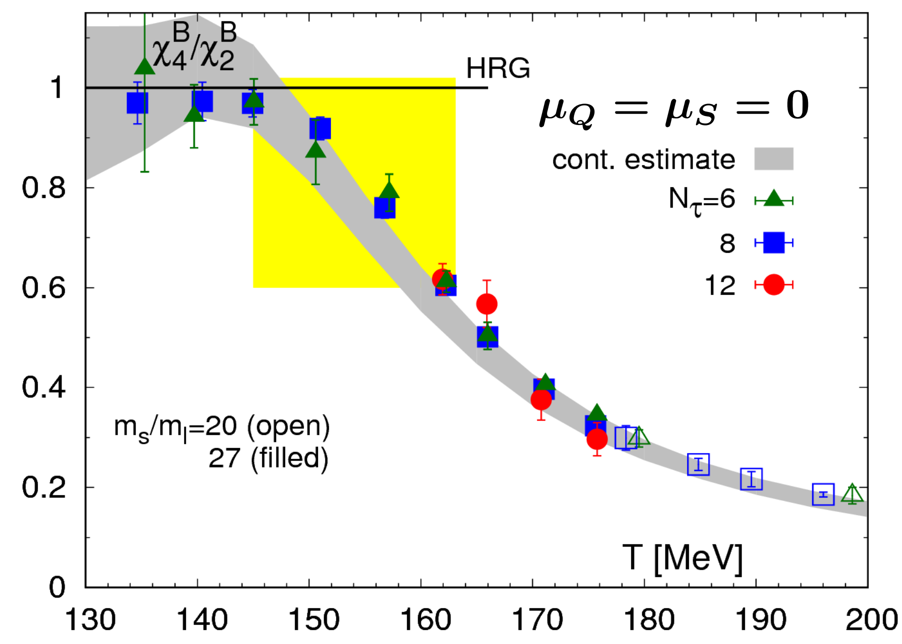
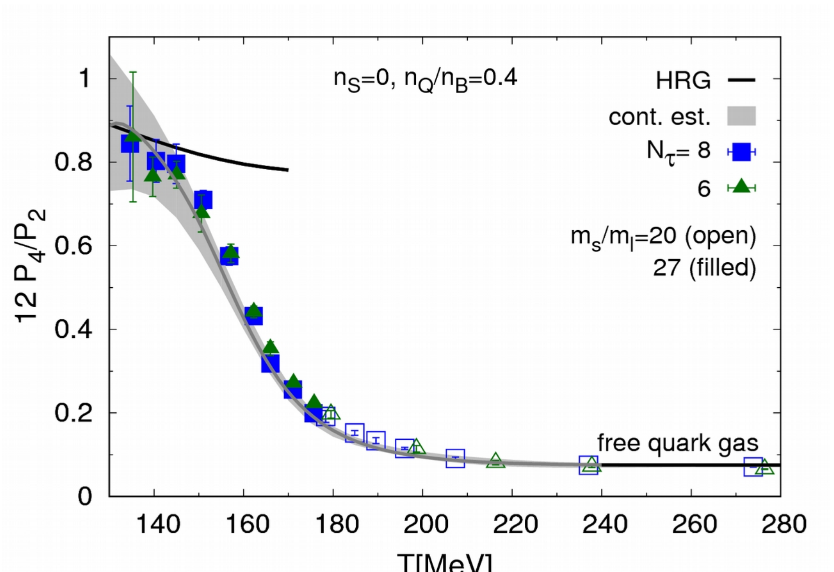
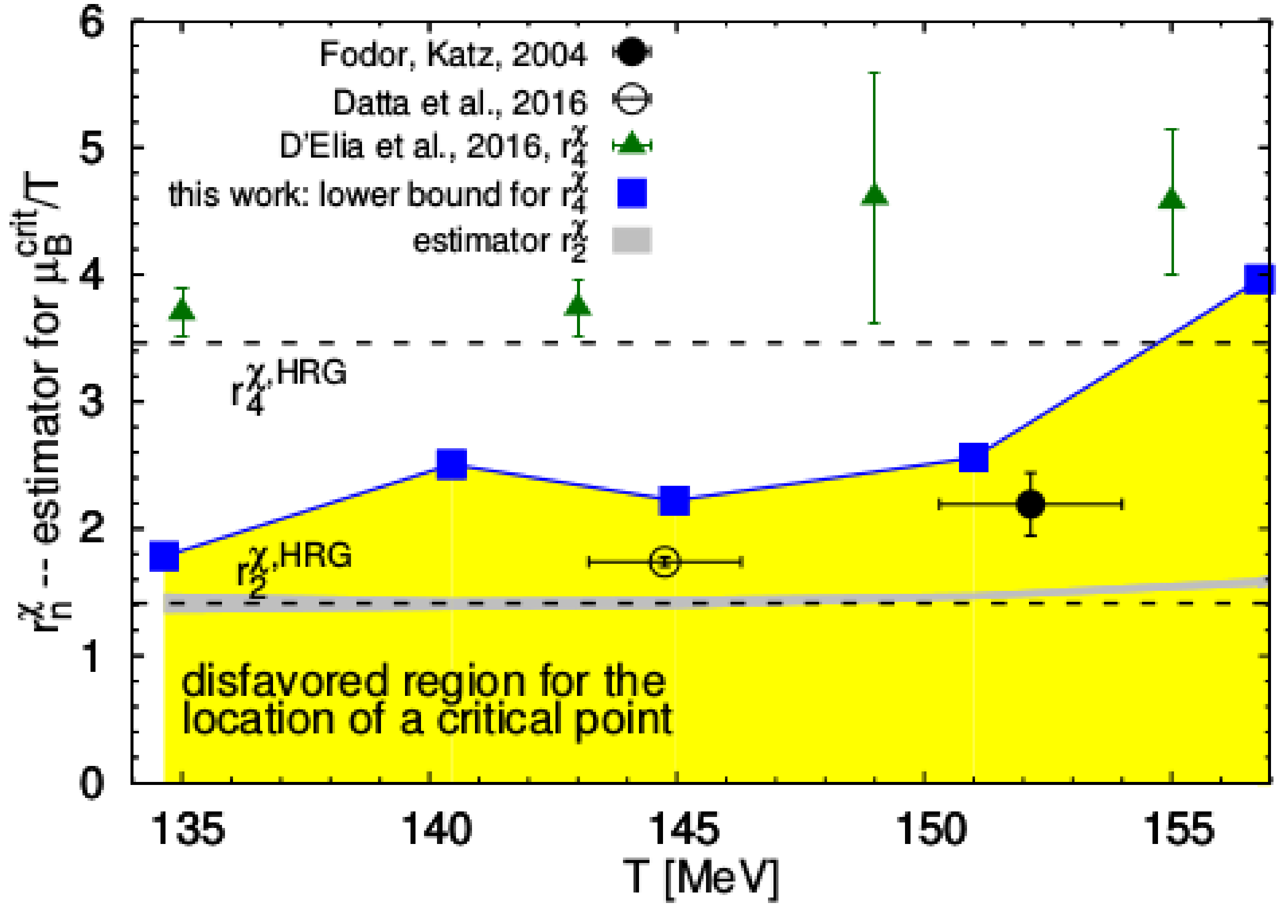
$$\left(\frac{\mu_B}{T}\right)^{\chi}_{crit,n} \equiv r_n^{\chi} = \sqrt{\left|\frac{n(n-1)\chi_n^B}{\chi_{n+2}^B}\right|}$$

$\frac{\chi_{n+2}^B}{\chi_n^B}$ needs to grow like n^2 in order to obtain a finite radius of convergence



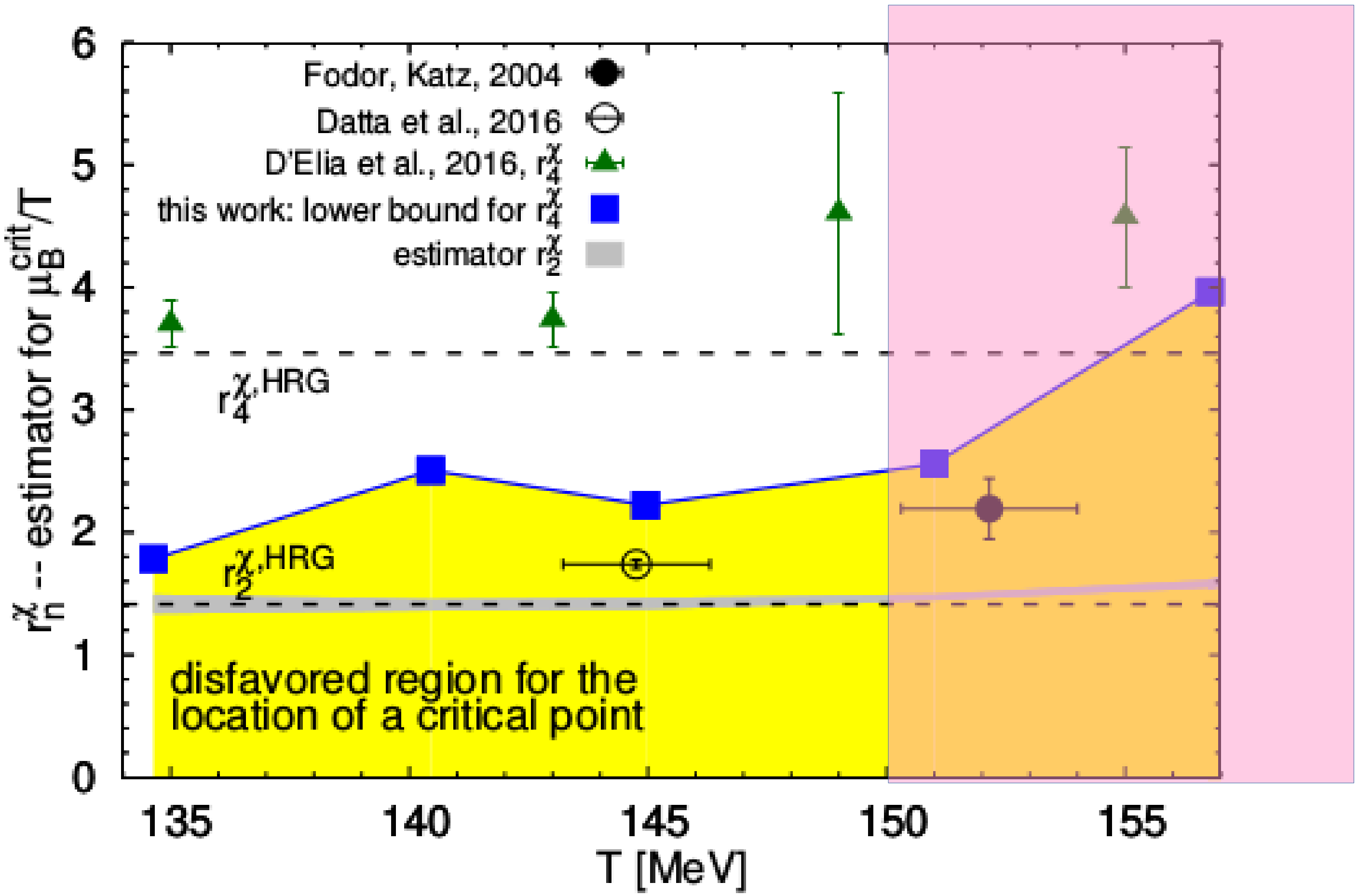


estimates/constraints on critical point location



based on ongoing calculations of 6th order Taylor expansion coefficients performed by the Bielefeld-BNL-CCNU collaboration
A. Bazavov et al., arXiv:1701.04325

estimates/constraints on critical point location



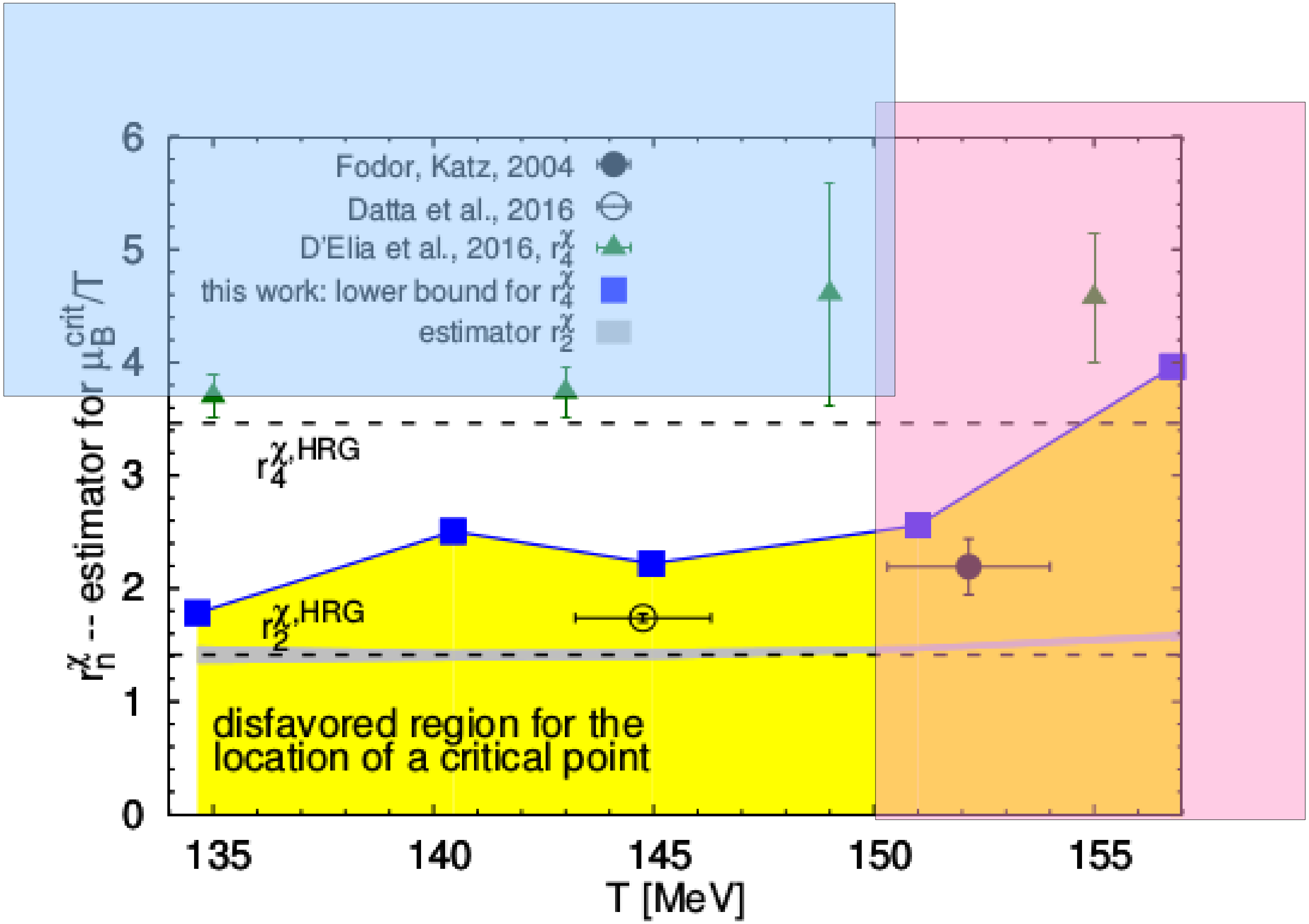
strongly disfavored as

$$\chi_6^B < 0$$

based on ongoing calculations of 6th order Taylor expansion coefficients performed by the Bielefeld-BNL-CCNU collaboration
A. Bazavov et al., arXiv:1701.04325

estimates/constraints on critical point location

not accessible in BES@RHIC collider mode



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Lines of constant physics in the QCD phase diagram

– freeze-out (hadronization) expected to happen at "approximately" identical physical conditions, i.e. **constant energy density or entropy density**....

– consider lines of constant observable "f" : $f(T,\mu_B) = \sum_{k=0}^{\infty} f_{2k}(T)(\mu_B/T)^{2k}$

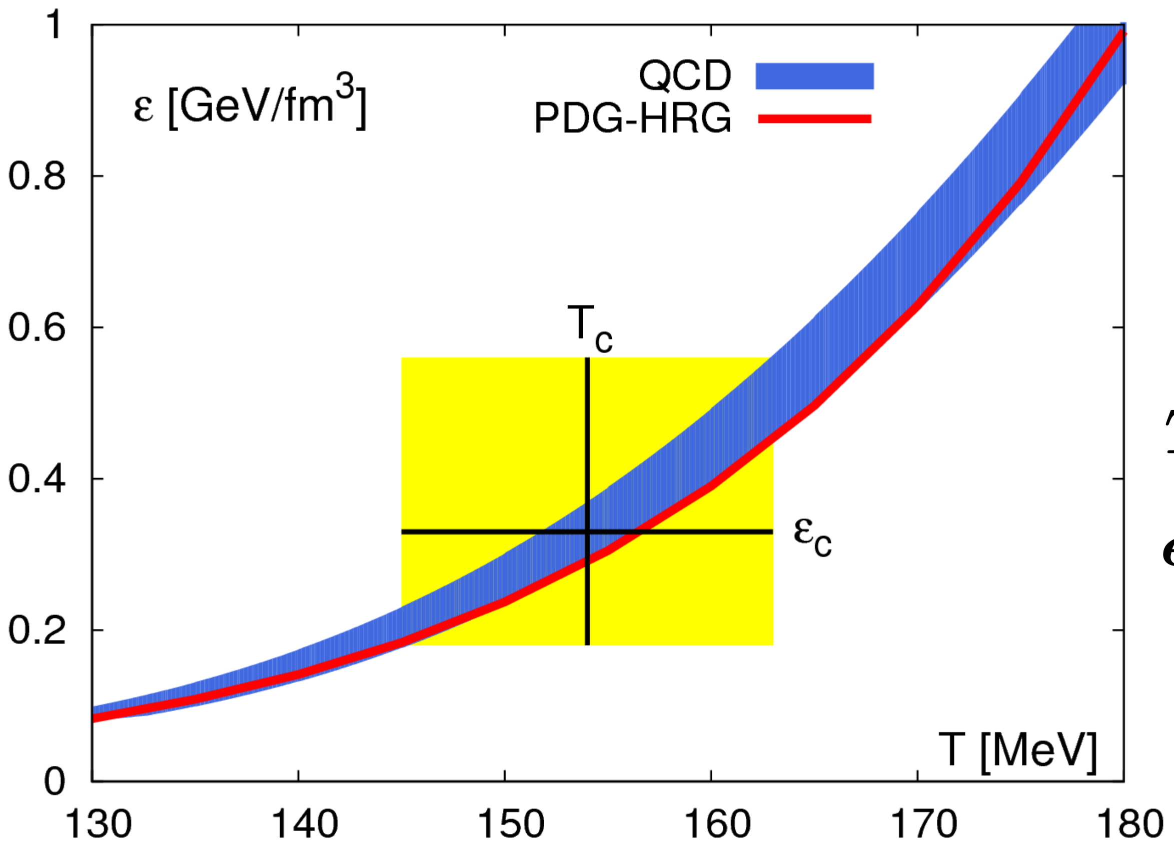
$$\Rightarrow \quad T_f(\mu_B) = T_0 \left(1 - \kappa_2^f \left(\frac{\mu_B}{T_0}\right)^2 - \kappa_4^f \left(\frac{\mu_B}{T_0}\right)^4 \right)$$

$$\kappa_2^f = \frac{T_0}{2} \frac{\left.\frac{\partial^2 f(T,\mu_B)}{\partial \mu_B^2}\right|_{(T_0,0)}}{\left.\frac{\partial f(T,\mu_B)}{\partial T}\right|_{(T_0,0)}}$$

$$\kappa_4^f = \frac{\frac{1}{2} T_0^2 \left.\frac{\partial^2 f(T,\mu_B)}{\partial T^2}\right|_{(T_0,0)} (\kappa_2^f)^2 - \frac{1}{2}\, T_0^3 \frac{\partial}{\partial T} \left.\frac{\partial^2 f(T,\mu_B)}{\partial \mu_B^2}\right|_{(T_0,0)} \kappa_2^f + \frac{1}{4!} T_0^4 \left.\frac{\partial^4 f(T,\mu_B)}{\partial \mu_B^4}\right|_{(T_0,0)}}{T_0 \left.\frac{\partial f(T,\mu_B)}{\partial T}\right|_{(T_0,0)}}$$

Crossover transition parameters

PDG: Particle Data Group hadron spectrum



$$\mu_B/T = 0$$

$$T_c = (154 \pm 9)\ \mathrm{MeV}$$

$$\epsilon_c = (0.34 \pm 0.16)\mathrm{GeV/fm^3}$$

compare with:

$$\epsilon^{\text{nucl. mat.}} \simeq 150\ \mathrm{MeV/fm^3}$$

$$\epsilon^{\text{nucleon}} \simeq 450\ \mathrm{MeV/fm^3}$$

A. Bazavov et al. (hotQCD) ,
Phys. Rev. D90 (2014) 094503

Lines of constant physics

– freeze-out (hadronization) expected to happen at "approximately" identical physical conditions, I.e constant energy density or entropy density....

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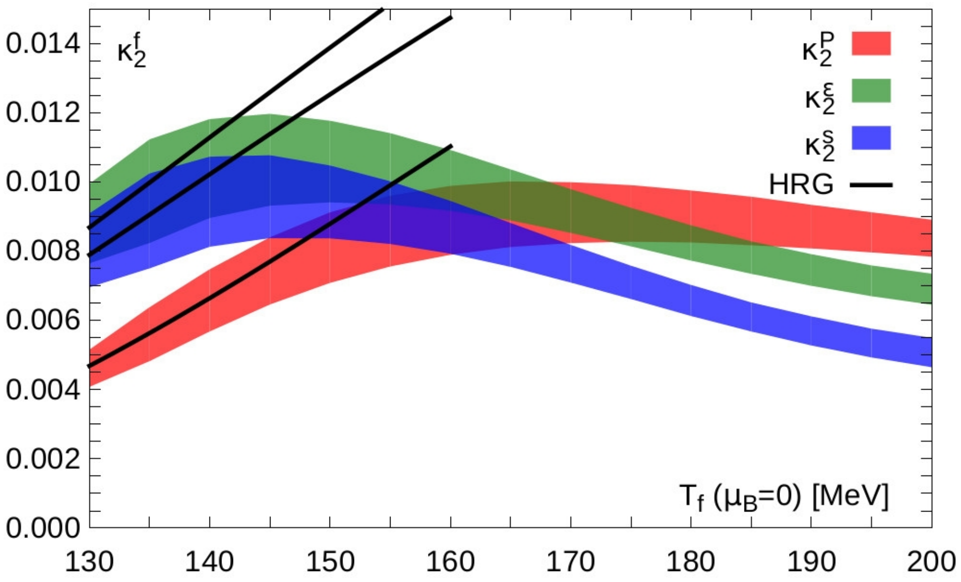
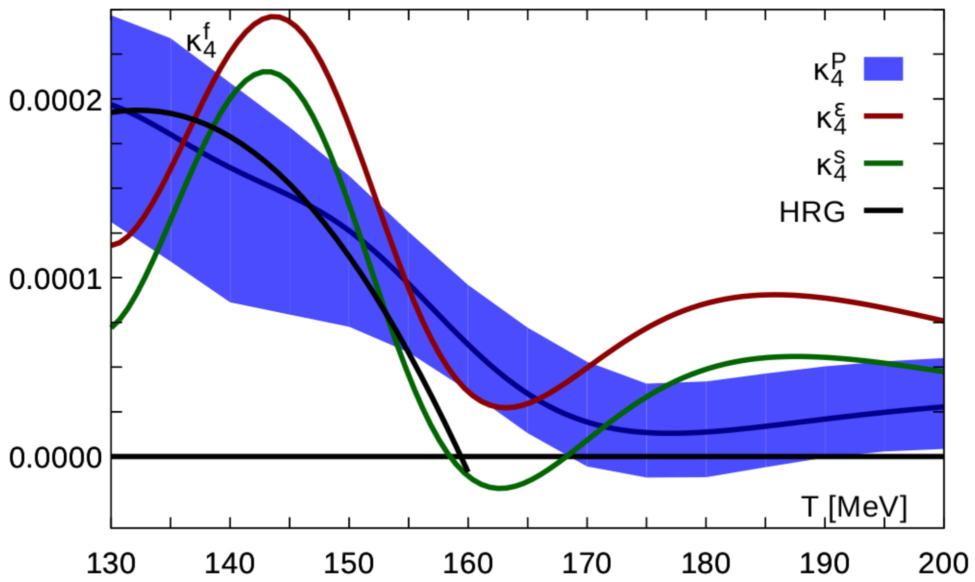




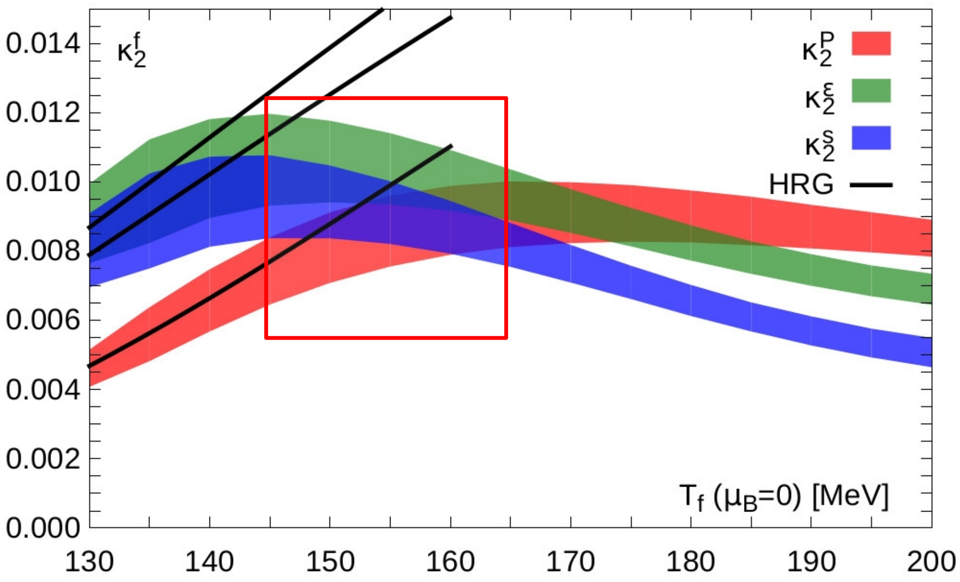
contributes less than 10% for $\mu_B/T < 2$

Lines of constant physics

– freeze-out (hadronization) expected to happen at "approximately" identical physical conditions, i.e. constant energy density or entropy density....

– consider lines of constant observable "f" : $f(T,\mu_B) = \sum_{k=0}^{\infty} f_{2k}(T)(\mu_B/T)^{2k}$

$$\Rightarrow \quad T_f(\mu_B) = T_0\left(1 - \kappa_2^f\left(\frac{\mu_B}{T_0}\right)^2 - \kappa_4^f\left(\frac{\mu_B}{T_0}\right)^4\right)$$



in the crossover region

$$T_c = (154 \pm 9)\ \mathrm{MeV}$$

all curvature coefficients are of similar magnitude

$$0.0064 \le \kappa_2^P \le 0.0101$$
$$0.0087 \le \kappa_2^\epsilon \le 0.012$$
$$0.0074 \le \kappa_2^s \le 0.011$$

and are similar to recent estimates for the curvature of the pseudo-critical line

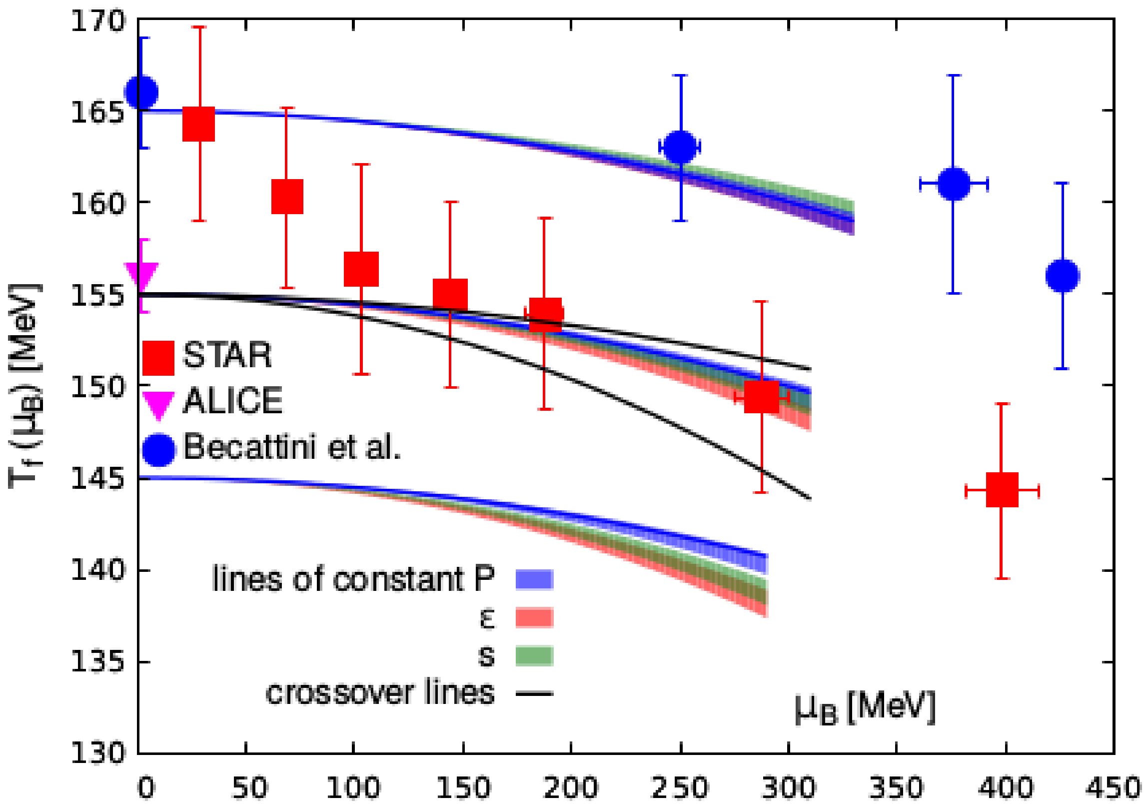
Chiral transition, hadronization and freeze-out

$\mu_B = 0:$ – pseudo-critical temperature $T_c = 154(9)\mathrm{MeV}$

– hadronization temperatures $T_h = 164(3)\ \mathrm{MeV}$

– freeze-out temperatures: $T_{fo} = 156(3)\ \mathrm{MeV}$

$T_{fo} = [164(5) - 168(4)]\ \mathrm{MeV}$



HOWEVER

physics is quite different at lower and upper end of the current error bar on Tc

probed with net-charge correlations&fluctuations

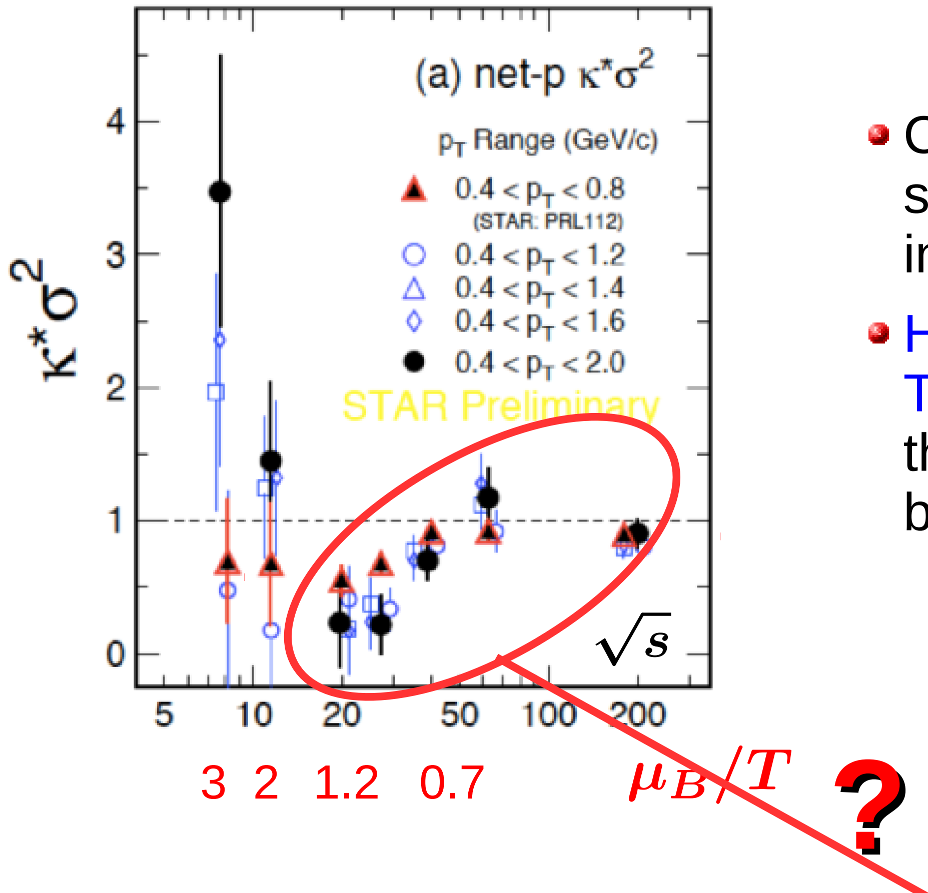
lines of constant physics from a 6th order Taylor expansion of the QCD equation of state

A. Bazavov et al. (Bielefeld-BNL-CCNU) arXiv:1701.04325

crossover transition lines:

G. Endrodi et al., arXiv:1102.1356, O. Kaczmarek et al., arXiv:1011.31.30
C. Bonati et al., arXiv:1507.03571, P. Cea et al., arXiv:1403.0821

Exploring the QCD phase diagram



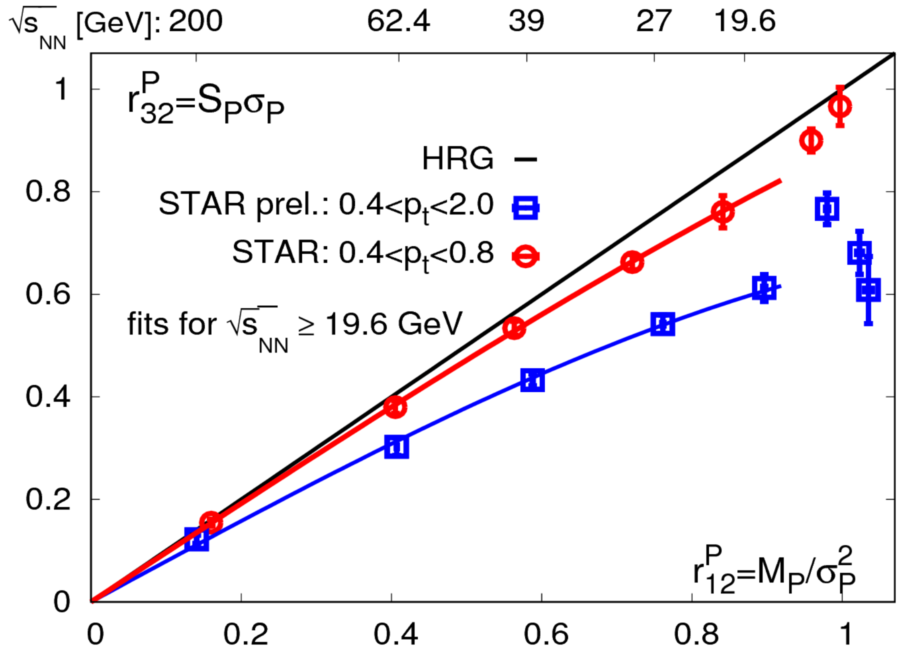
$$\kappa_X \sigma_X^2 = \frac{\chi_4^X}{\chi_2^X}$$

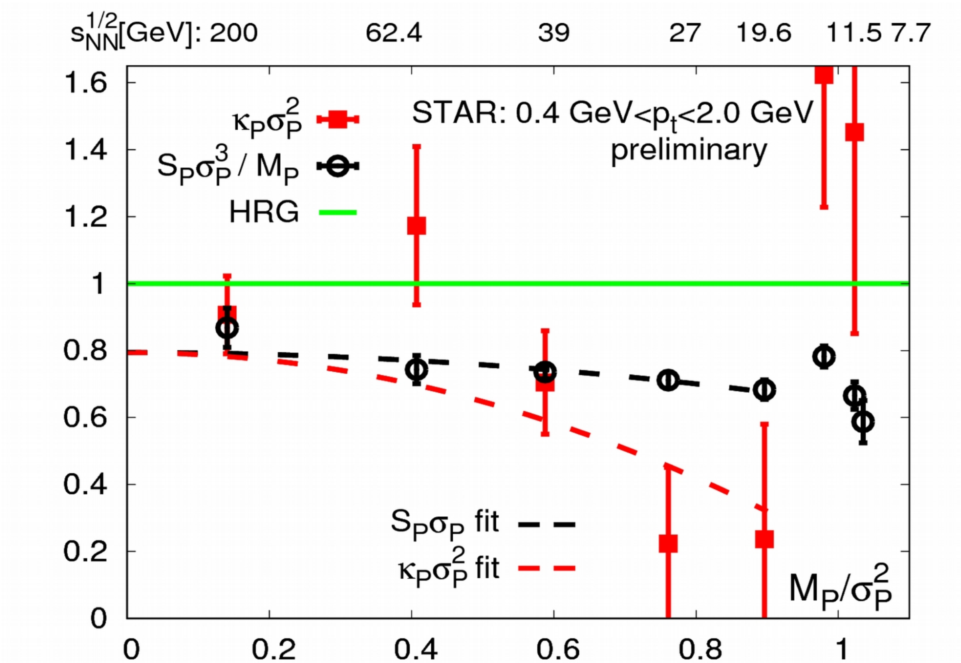
More moderate questions:

- Can we understand the systematics seen in cumulants of charge fluctuations in terms of QCD thermodynamics ?
- How far do we get with low order Taylor expansions of **QCD** in explaining the obvious deviations from HRG model behavior ?

- For $\sqrt{s} \geq 19.6$ GeV : Structure of net-proton cumulants can be understood in terms of **QCD thermodynamics in a next-to-leading order Taylor expansion**

A. Bazavov et al. (HotQCD), arXiv:1708.04897





STAR data and corresponding Taylor expansions of cumulant ratios evaluated in lattice QCD

$$\frac{M_B}{\sigma_B^2} = \frac{\mu_B}{T} \frac{1 + \frac{1}{6}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}{1 + \frac{1}{2}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2} \qquad (*)$$

$$S_B\sigma_B = \frac{\mu_B}{T}\frac{\chi_4^B}{\chi_2^B} \frac{1 + \frac{1}{6}\frac{\chi_6^B}{\chi_4^B}\left(\frac{\mu_B}{T}\right)^2}{1 + \frac{1}{2}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}$$

$$\kappa_B\sigma_B^2 = \frac{\chi_4^B}{\chi_2^B} \frac{1 + \frac{1}{2}\frac{\chi_6^B}{\chi_4^B}\left(\frac{\mu_B}{T}\right)^2}{1 + \frac{1}{2}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}$$

– use $(*)$ to eliminate μ_B/T

– evaluate on lines of constant energy density

Conserved charge fluctuations and freeze-out
mean, variance, skewness and kurtosis

in a NLO Taylor expansion $r_{31}^B \equiv S_B \sigma_B^3 / M_B$ and $r_{42}^B \equiv \kappa_B \sigma_B^2$ } **are closely related**

F. Karsch et al.,
arXiv:1512.06987,
arXiv:1708.04897

$$r_{31}^B = r_{31}^{B,0} + r_{31}^{B,2} \left(\frac{\mu_B}{T}\right)^2$$

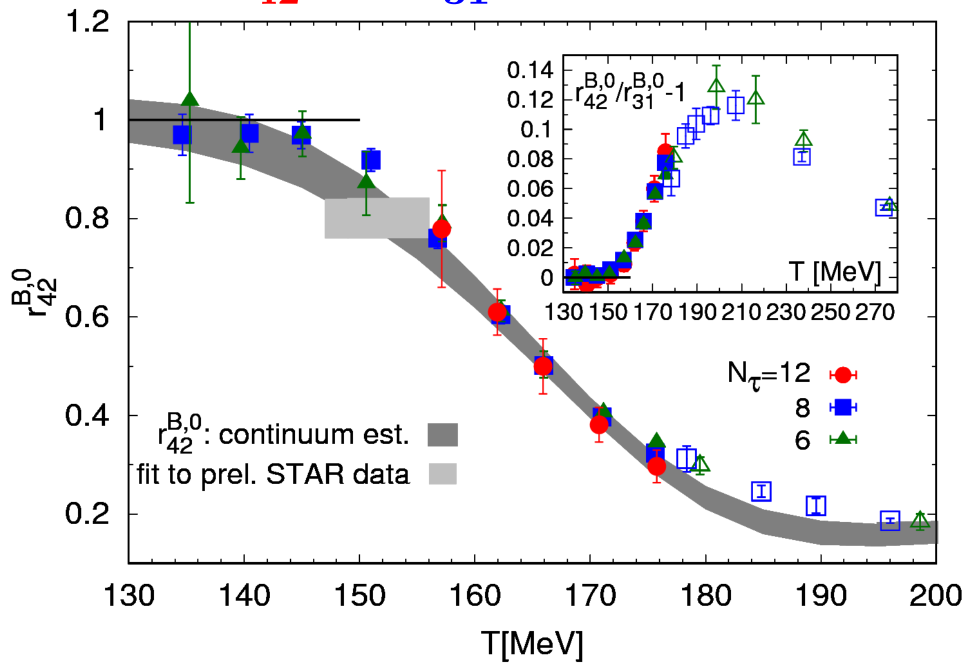
$$r_{42}^B = r_{42}^{B,0} + r_{42}^{B,2} \left(\frac{\mu_B}{T}\right)^2$$

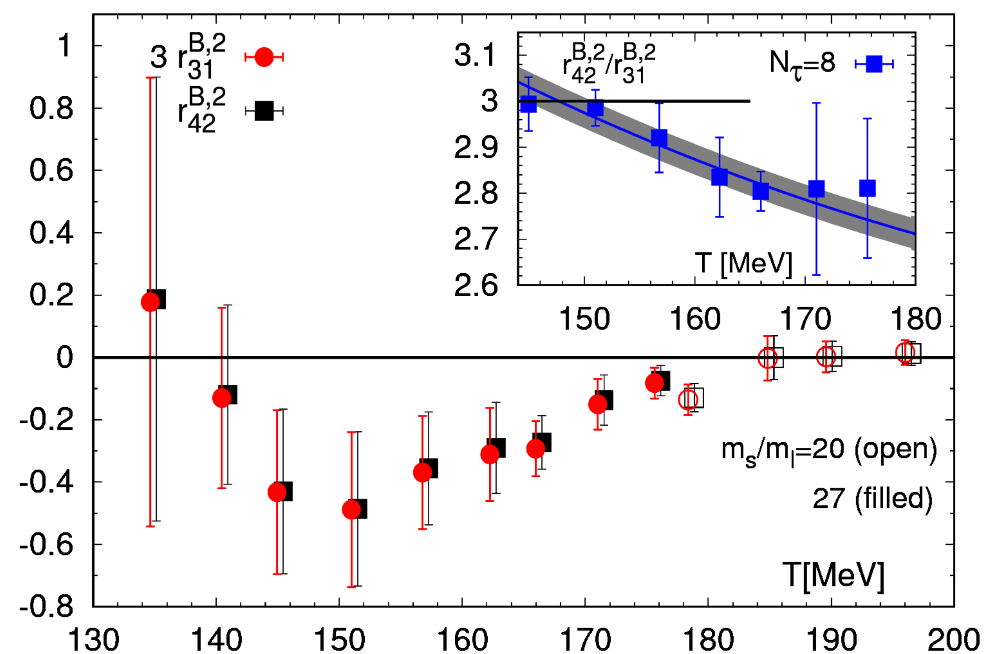
$\Rightarrow$ $r_{42}^{B,0} \simeq r_{31}^{B,0}$

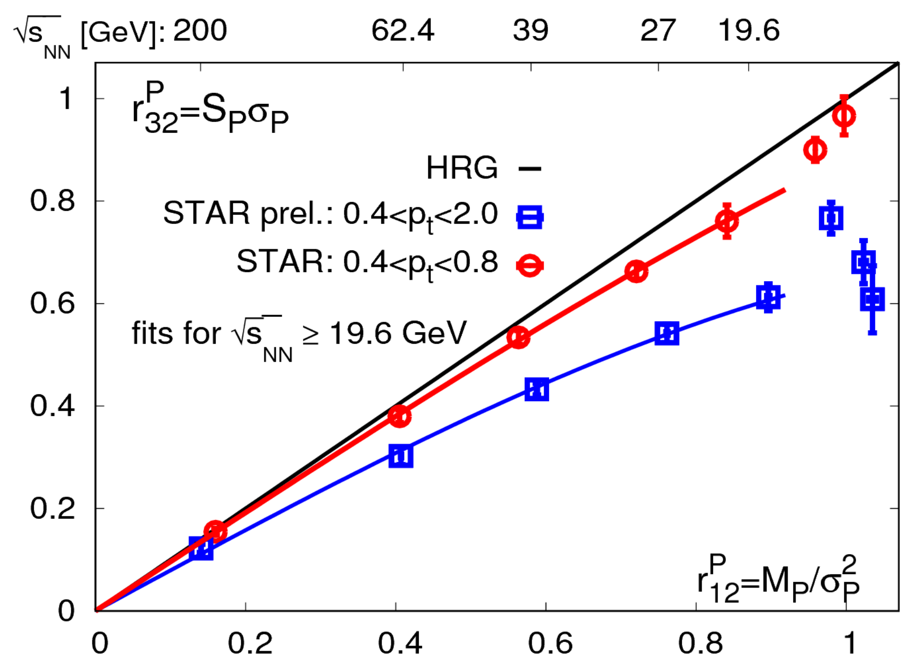
$\mu_S = \mu_Q = 0$:

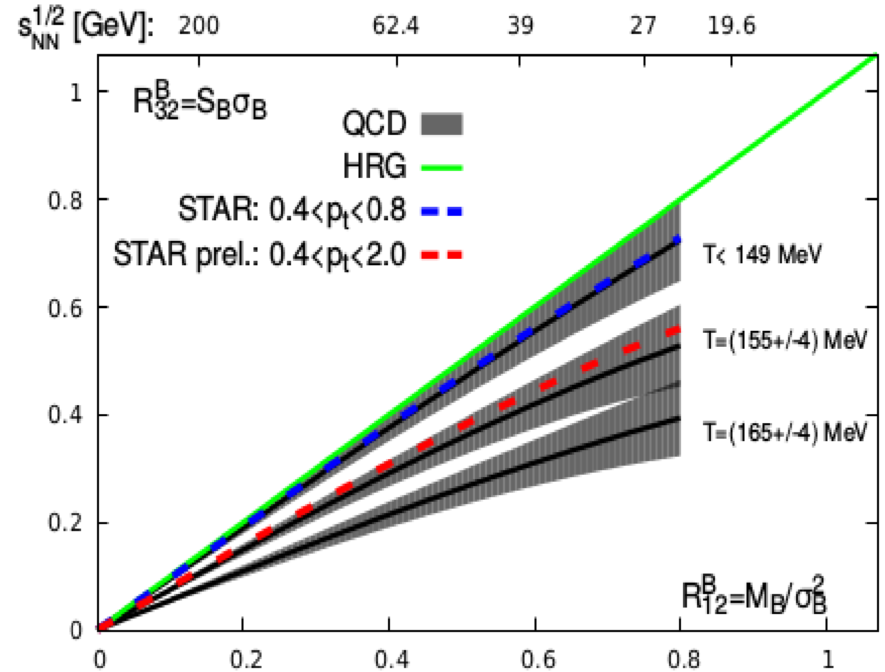
$$r_{42}^{B,2} = 3 r_{31}^{B,2} = \frac{1}{2}\left(\frac{\chi_6^B}{\chi_2^B} - \left(\frac{\chi_4^B}{\chi_2^B}\right)^2\right)$$

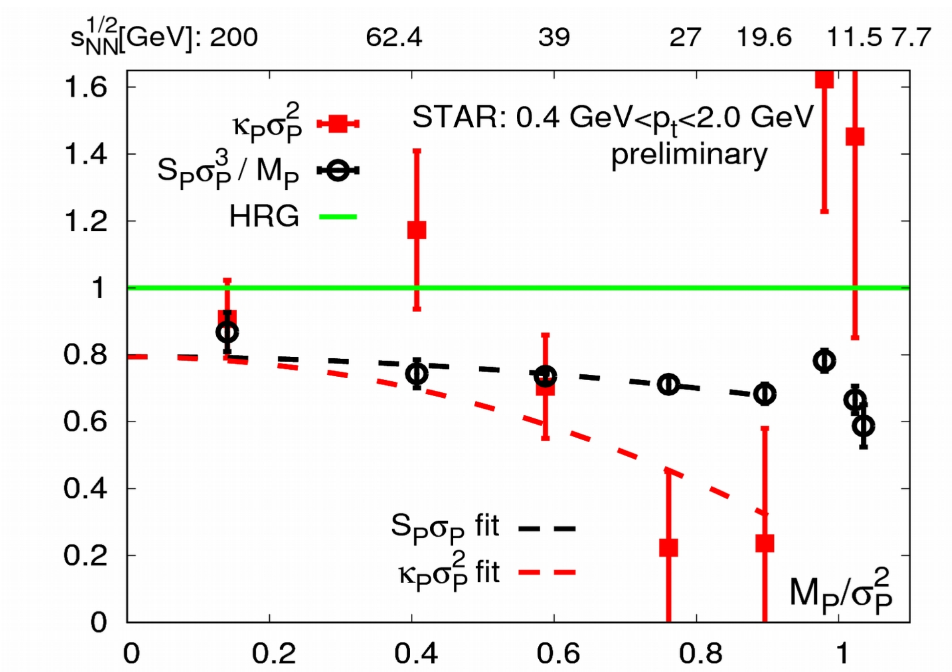
$\mu_S \neq \mu_Q \neq 0$:

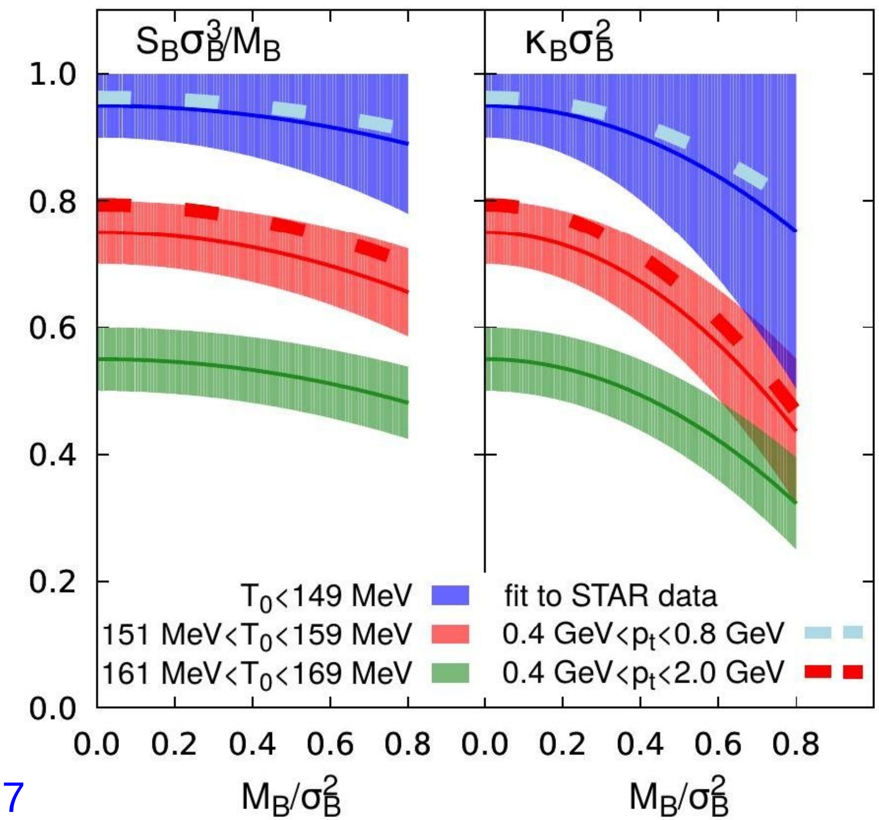












F. Karsch et al.,
arXiv:1708.04897

Cumulant ratios of net-strangeness number fluctuations at non-zero baryon number density

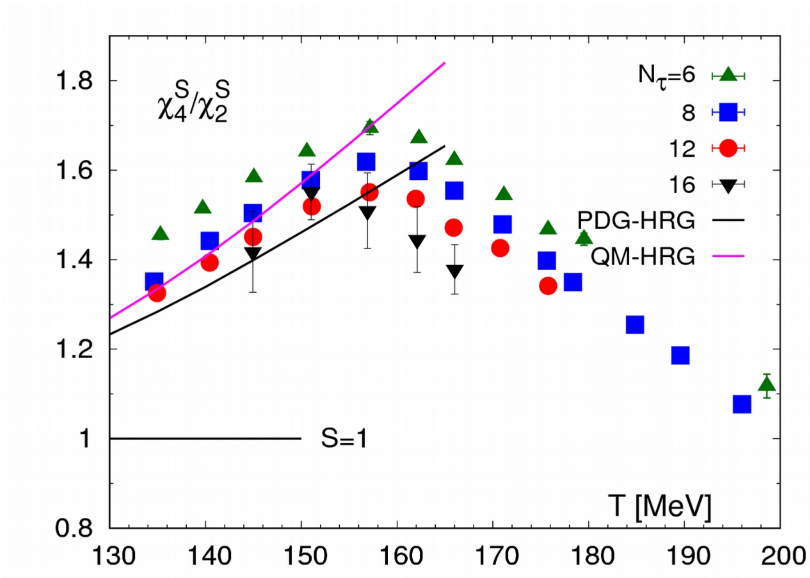
A high impact project on Cori-II

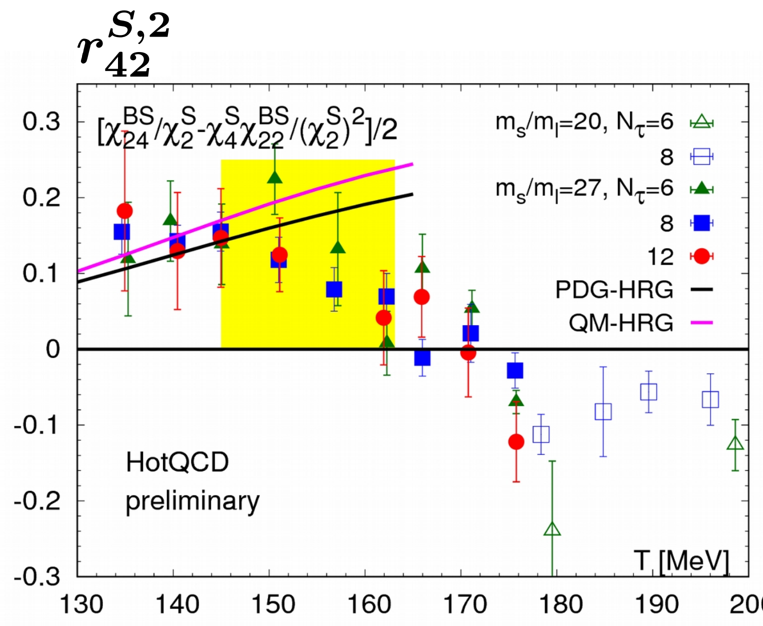
9000 KNL nodes for two full days in September 2017

$$\kappa_S \sigma_S^2 = \frac{\chi_4^S(T,\mu_B)}{\chi_2^S(T,\mu_B)}$$

$$= \left(\frac{\chi_4^S}{\chi_2^S}\right)_{\mu_B=0} + r_{42}^{S,2}\left(\frac{\mu_B}{T}\right)^2$$

→ $\frac{\chi_4^S}{\chi_2^S}$ slightly rises with μ_B for $T \leq 155\ \mathrm{MeV}$



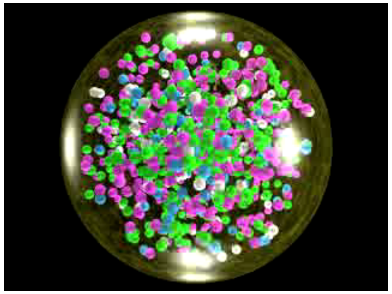


Explore the **structure of matter** close to the QCD transition temperature using **fluctuations of conserved charges**

baryon number, strangeness, electric charge

High T: ideal gas

ideal quark (fermi) gas, m=0

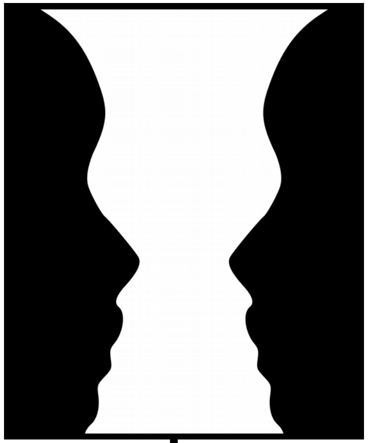


fractional charges

baryon number: B= +/- 1/3

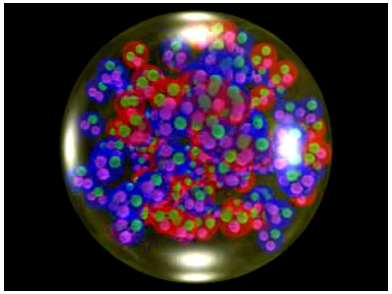
electric charge: Q= +/- 1/3, +/- 2/3

strangeness: S= 0, +/- 1



Low T: HRG

hadron resonance gas



integer charges

baryon number: B= +/-1

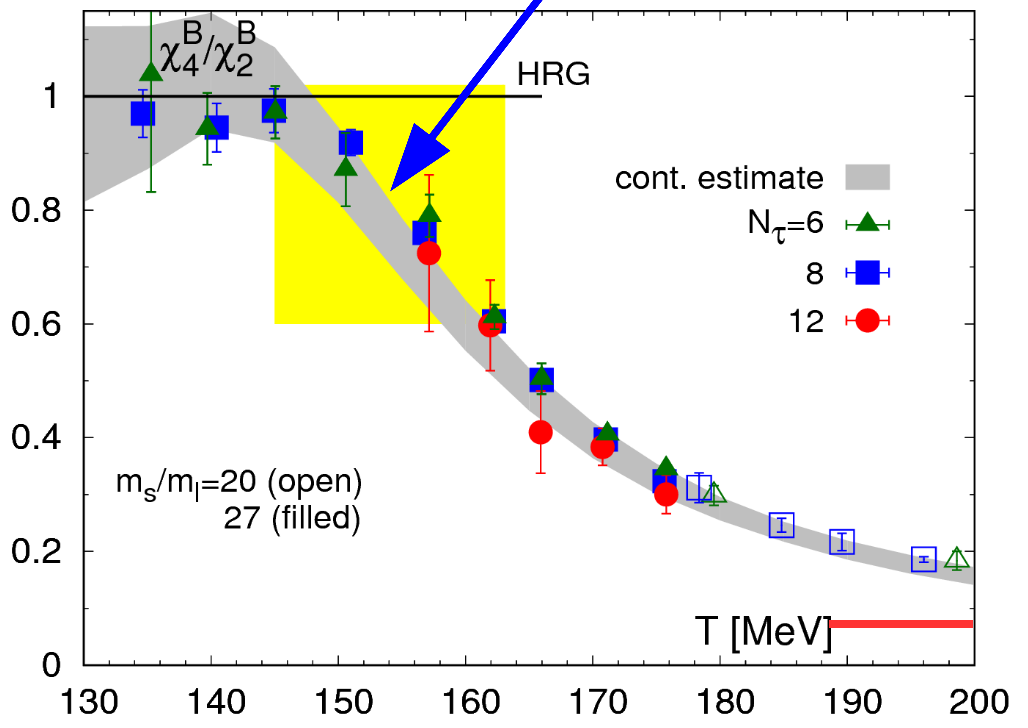
electric charge: Q= 0 =+/- 1, +/- 2

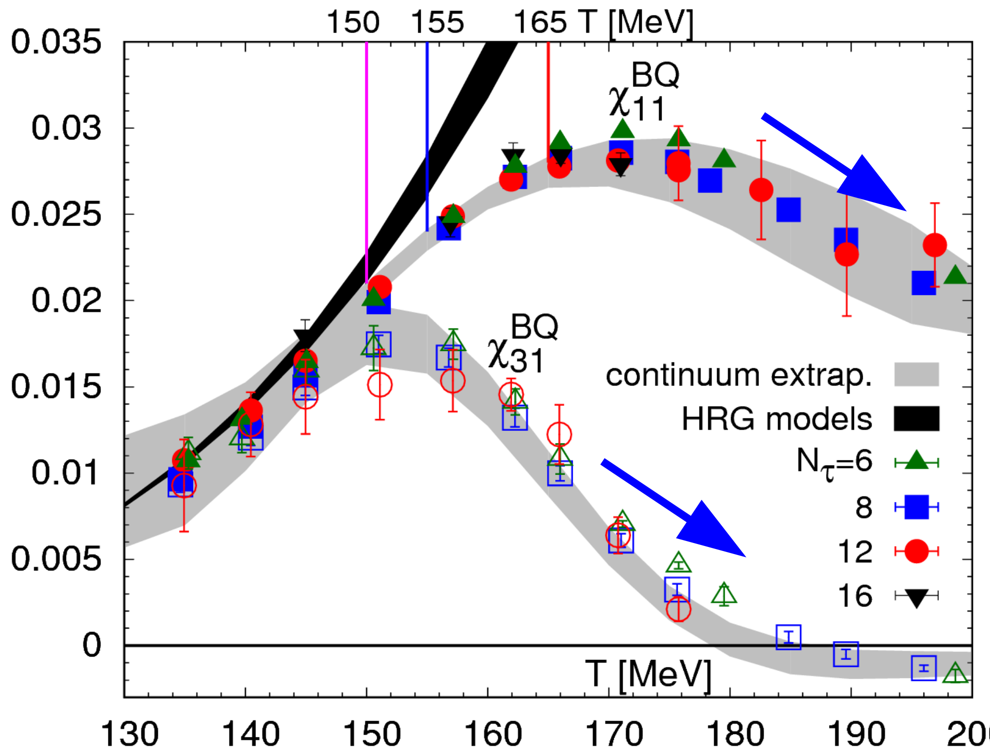
strangeness: S= 0, +/- 1, +/- 2, +/- 3

baryon number – electric charge correlations
2nd and 4th order cumulants

$$\kappa_B \sigma_B^2 \equiv \chi_4^B / \chi_2^B < 1$$

$$\chi_{11}^{BQ} = \frac{\partial^2 P/T^4}{\partial \hat{\mu}_B \partial \hat{\mu}_Q} = \langle B \cdot Q \rangle - \langle B \rangle \langle Q \rangle$$





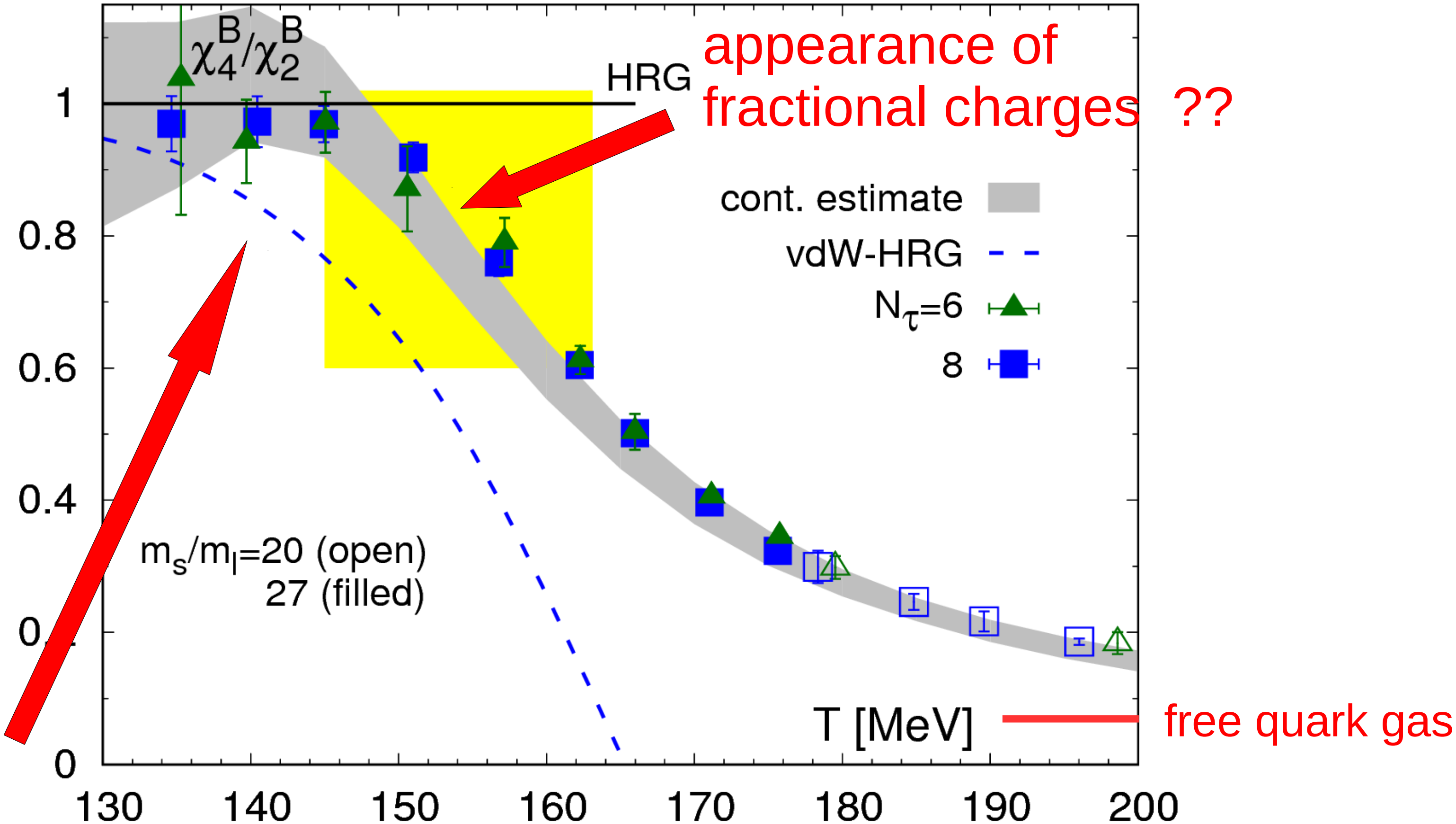
free quark gas

free quark gas: $\frac{1}{3}\left(\frac{2}{3} - \frac{1}{3} - \frac{1}{3}\right) = 0$

– change in composition of the thermal medium is detected through conserved charge correlations
=> this gets reflected in cumulant ratios of, e.g. net-baryon number fluctuations => no longer Skellam for $T \gtrsim 150\mathrm{MeV}$

Net baryon-number fluctuations

ratio of 4th and 2nd order cumulants:



validity range of HRG model (somewhat) extended in excluded volume models

V. Vovchenko et al., arXiv:1707.09215

BNL-Bielefeld-CCNU:
Phys. Rev. Lett. 111, 082301 (2013)
Phys. Lett. B737, 210 (2014)

Net quark-number fluctuations

C.R.Allton et al, PRD71,054508 (2005)

perturbation theory: $$\frac{\chi_{ff}(T,\mu)}{T^2} = \frac{\partial^2\Omega(T,\mu)}{\partial(\mu_f/T)^2} \quad , \quad \frac{\chi_{fk}(T,\mu)}{T^2} = \frac{\partial^2\Omega(T,\mu)}{\partial(\mu_f/T)\partial(\mu_k/T)}$$

$$\frac{p}{T^4}(T,\mu) = \Omega^{(0)}(T,\mu) + g^2\,\Omega^{(2)}(T,\mu) + g^3\,\Omega^{(3)}(T,\mu) + \mathcal{O}(g^4)$$

$$\frac{p_{SB}}{T^4} = \Omega^{(0)}(T,\mu) = \frac{8\pi^2}{45} + \sum_{f=u,d,..}\left[\frac{7\pi^2}{60} + \frac{1}{2}\left(\frac{\mu_f}{T}\right)^2 + \frac{1}{4\pi^2}\left(\frac{\mu_f}{T}\right)^4\right] \quad ,$$

$$\Omega^{(2)}(T,\mu) = -\left(\frac{1}{6} + \frac{5n_f}{72} + \frac{1}{4\pi^2} + \frac{1}{8\pi^4}\sum_{f=u,d,..}\left(\frac{\mu_f}{T}\right)^4\right) \quad ,$$

$$\Omega^{(3)}(T,\mu) = \frac{1}{6\pi}\left(\frac{m_E}{gT}\right)^3 = \frac{1}{6\pi}\left(1 + \frac{n_f}{6} + \frac{1}{2\pi^2}\sum_{f=u,d,...}\left(\frac{\mu_f}{T}\right)^2\right)^{3/2}$$

$$\frac{\chi_{ff}(T,\mu)}{T^2} = 1 + \frac{3}{\pi^2}\left(\frac{\mu_f}{T}\right)^2 + \mathcal{O}(g^2) \qquad \chi_4^f - \chi_4^{f,ideal} \sim \mathcal{O}(g^2)$$

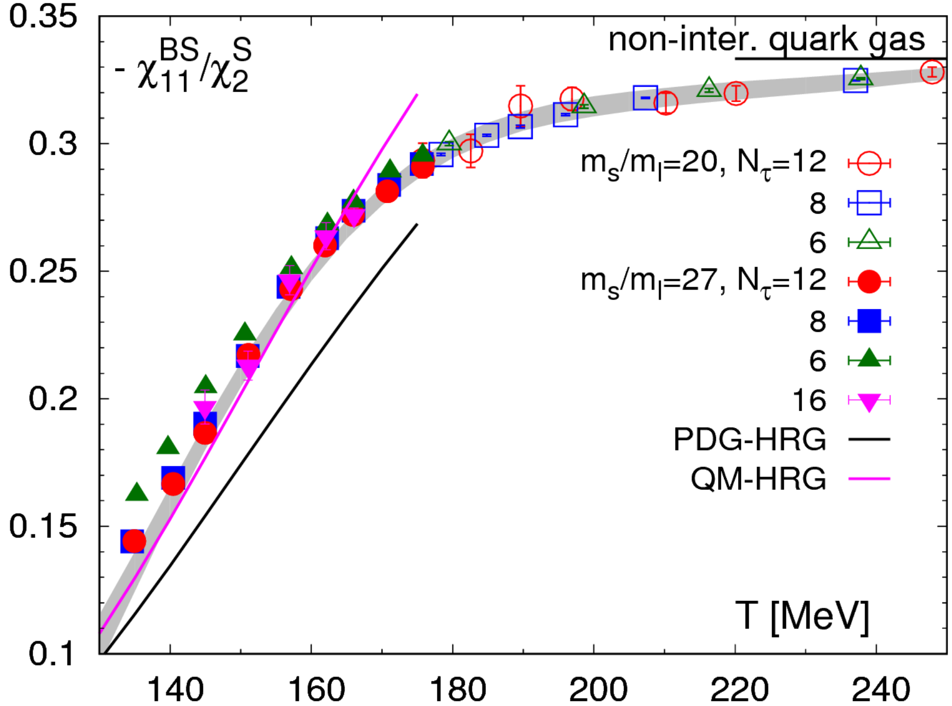
$$\frac{\chi_{fk}(T,\mu)}{T^2} = \frac{g^3}{2\pi^5}\left(1 + \frac{n_f}{6} + \frac{1}{2\pi^2}\sum_{f=u,d,...}\left(\frac{\mu_f}{T}\right)^2\right)^{-1/2}\frac{\mu_f}{T}\frac{\mu_k}{T} \qquad \chi_{22}^{fk} \sim \mathcal{O}(g^3)$$

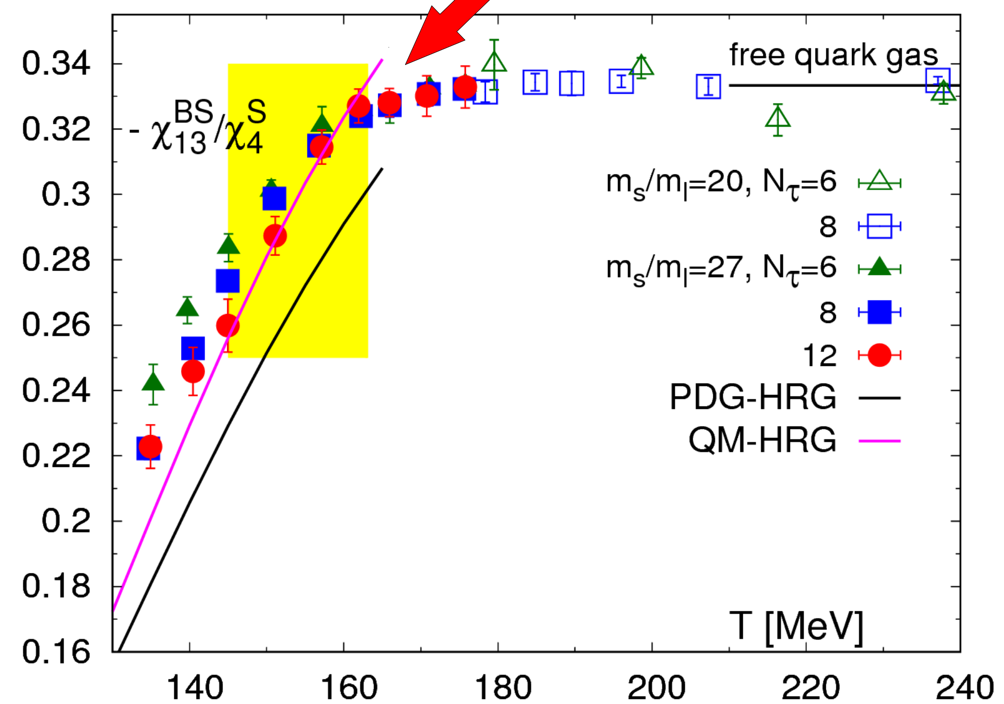
$$\frac{\chi_{fk}(T,0)}{T^2} \simeq -\frac{5}{144\pi^6}\,g^6\ln 1/g \qquad \chi_{13}^{fk} \sim \mathcal{O}(g^6\ln g)$$

Ratio of baryon number – strangeness correlation and net strangeness fluctuations

2nd & 4th order cumulants

rapid approach to free quark gas limit





conserved charge ⟺ quark number fluctuations:

$$\chi_{11}^{BS} = -\frac{1}{3}\chi_{11}^{us} - \frac{1}{3}\chi_{11}^{ds} - \frac{1}{3}\chi_2^{s}$$

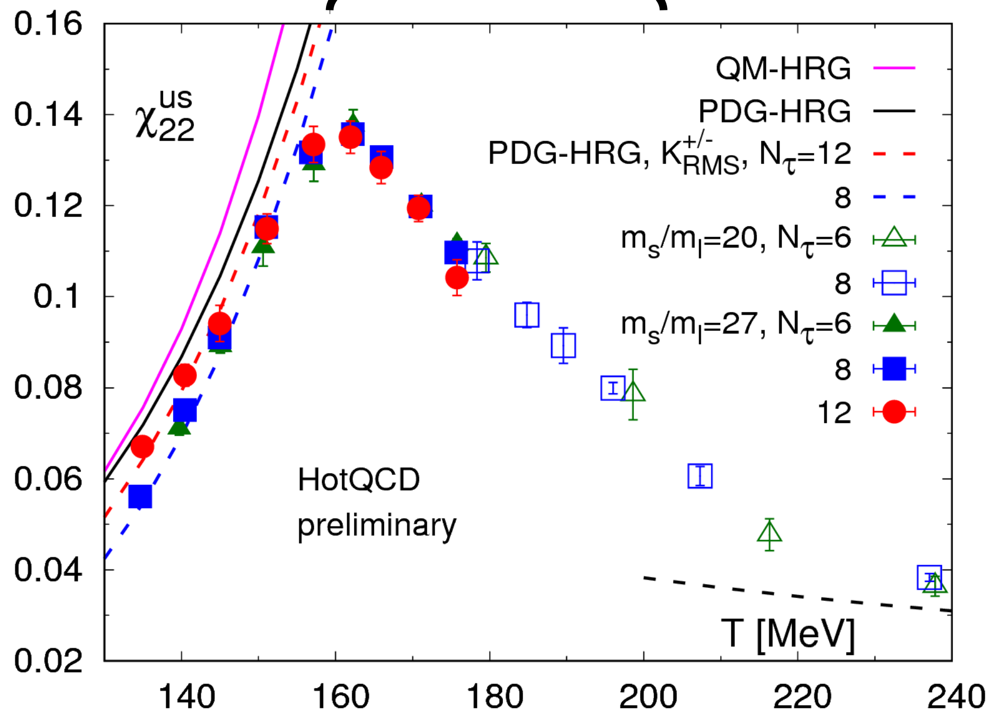
$$\chi_{13}^{BS} = -\frac{1}{3}\chi_{13}^{us} - \frac{1}{3}\chi_{13}^{ds} - \frac{1}{3}\chi_4^{s}$$

$$-\frac{\chi_{11}^{BS}}{\chi_2^S} = \frac{1}{3} + \frac{2}{3}\frac{\chi_{11}^{us}}{\chi_2^s}$$

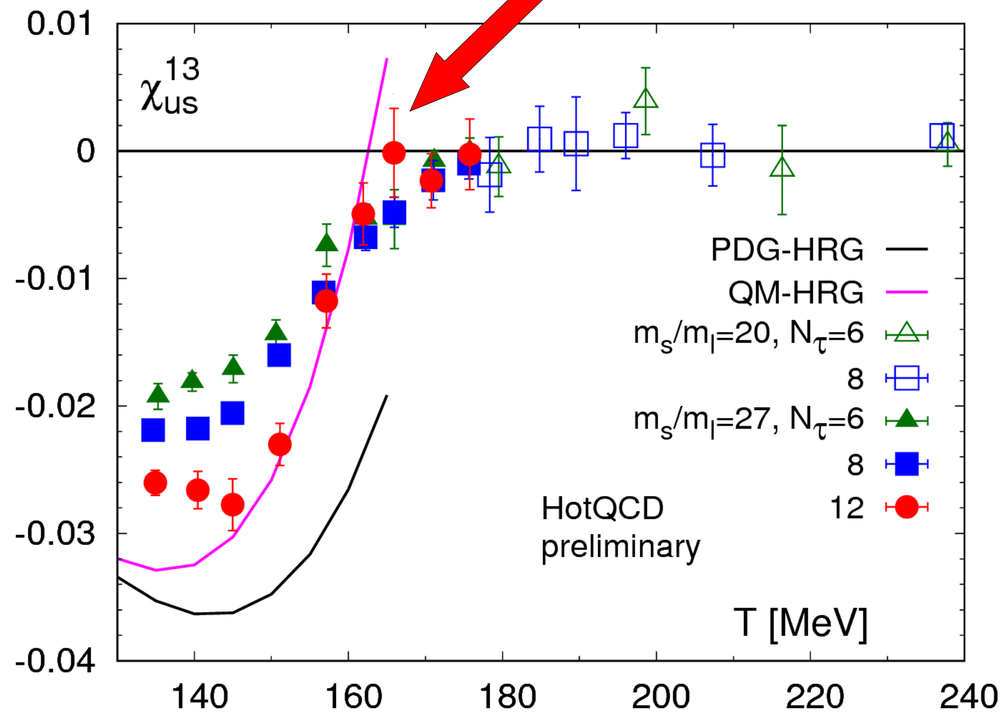
$$-\frac{\chi_{13}^{BS}}{\chi_4^S} = \frac{1}{3} + \frac{2}{3}\frac{\chi_{13}^{us}}{\chi_4^s}$$

Net quark-number fluctuations

strongly interacting
QGP



rapid approach to
free quark gas limit



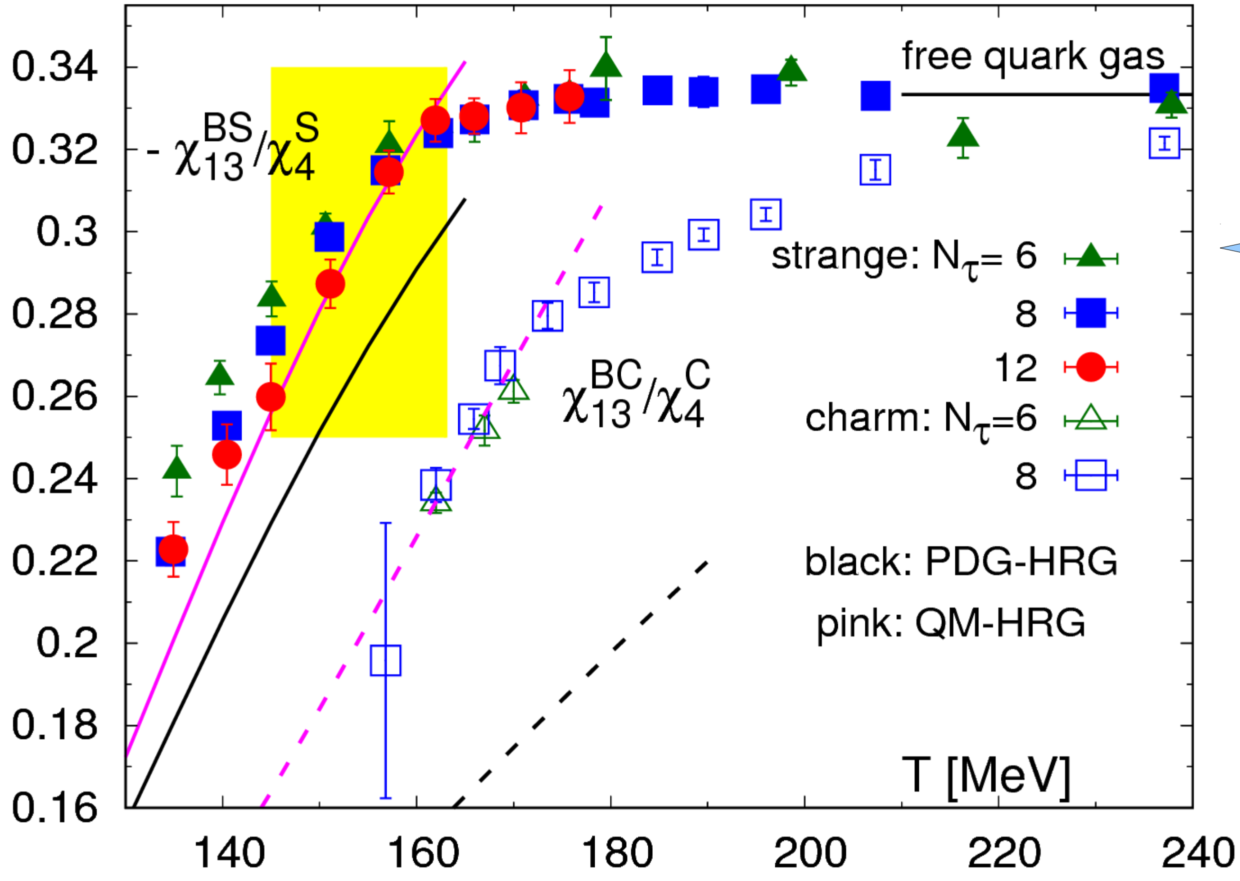
$\mathcal{O}(g^3)$

(Debye mass)

$\mathcal{O}(g^6 \ln g)$

Correlation of net-baryon number with net strangeness and net charm

4^{th} order cumulants



evidence for experimentally not yet observed strange and charmed baryons?

$$-\frac{\chi_{13}^{BS}}{\chi_4^S} = \frac{1}{3} + \frac{2}{3}\frac{\chi_{13}^{us}}{\chi_4^s}$$

$$\frac{\chi_{13}^{BC}}{\chi_4^C} = \frac{1}{3} + \frac{2}{3}\frac{\chi_{13}^{uc}}{\chi_4^c} + \frac{1}{3}\frac{\chi_{13}^{sc}}{\chi_4^c}$$

PDG-HRG: uses experimentally known hadron spectrum listed by the Particle Data Group
QM-HRG: uses additional hadrons predicted to exist in Quark Model calculations

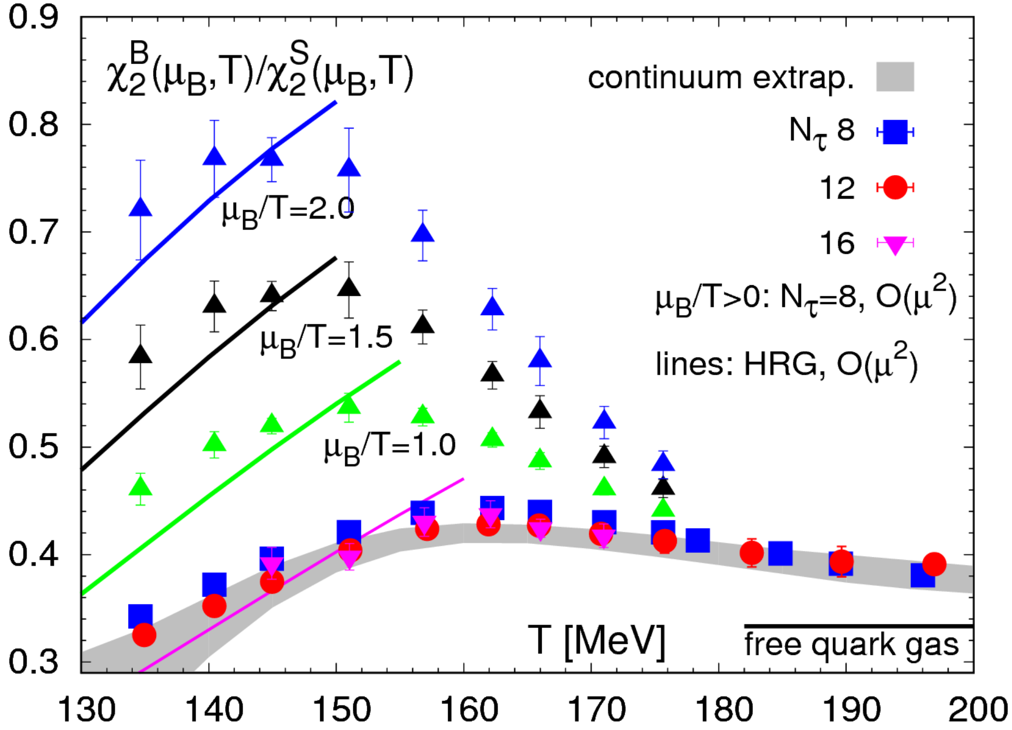
HRG vs. QCD
baryon number vs strangeness fluctuations

$$\mu_B/T > 0$$

for simplicity: $\mu_Q = \mu_S = 0$

$$\chi_2^B(T,\mu_B) = \chi_2^B + \frac{1}{2}\chi_4^B \left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}(\mu_B^4)$$

$$\chi_2^S(T,\mu_B) = \chi_2^S + \frac{1}{2}\chi_{22}^{BS} \left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}(\mu_B^4)$$



– agreement between HRG and QCD starts to deteriorate for T>150 MeV, and even earlier for $\mu_B/T > 0$

HRG: $\chi_4^B = \chi_2^B$

$\chi_{22}^{BS} << \chi_2^S$

– increase of B-fluctuation with baryon chemical potential smaller in QCD than HRG

$\Rightarrow$ deviations from HRG become larger with increasing baryon chemical potential

Conclusions

– 6th order Taylor expansions allow to control basic bulk thermodynamic observables in (2+1)-flavor QCD with physical quark mass up to $\mu_B/T \simeq 2$, which covers beam energies in heavy ion collisions down to $\sqrt{s_{NN}} \simeq 12\ \mathrm{GeV}$

– in this range of net baryon number densities, $0 \leq n_B \leq 0.06/\mathrm{fm}^3$ no evidence for "critical fluctuations", i.e. the presence of a critical point have been observed

– conserved charge fluctuations are quite well described by (non-interacting) HRG model calculations below $T \simeq 145\ \mathrm{MeV}$, if supplemented by additional strange degrees of freedom

– for $T > 160\ \mathrm{MeV}$ net quark number correlations (in different flavor channels) provide evidence for "liberated" quark degrees of freedom

Probing the properties of matter through the analysis of conserved charge fluctuations

Taylor expansion of the **QCD** pressure: $\frac{P}{T^4} = \frac{1}{VT^3} \ln Z(T, V, \mu_B, \mu_Q, \mu_S)$

$$\frac{P}{T^4} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS}(T) \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k$$

generalized susceptibilities: $\chi_{ijk}^{BQS} = \left. \frac{\partial^{i+j+k} p/T^4}{\partial \hat{\mu}_B^i \partial \hat{\mu}_Q^j \partial \hat{\mu}_S^k} \right|_{\mu=0}$, $\hat{\mu}_X \equiv \frac{\mu_X}{T}$

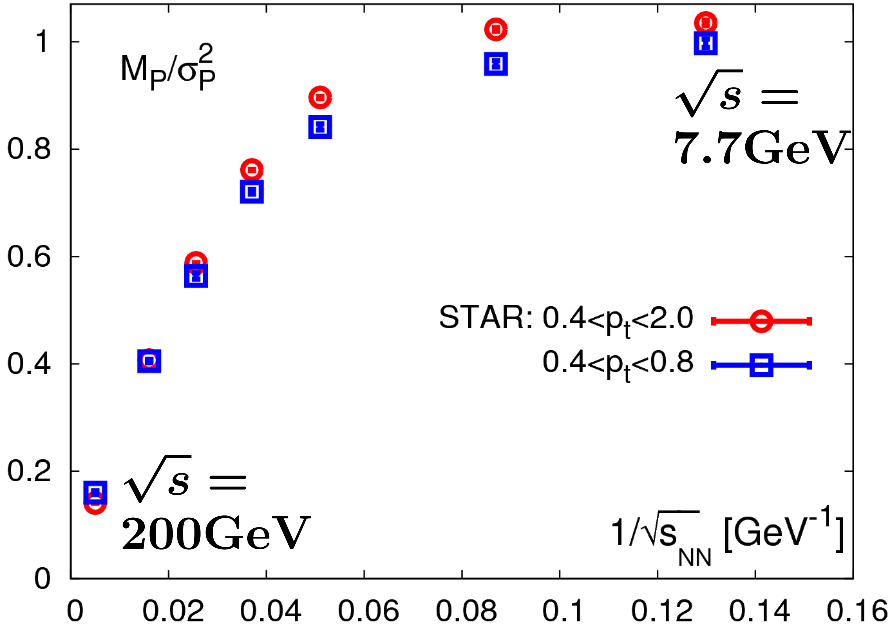
conserved charge fluctuations: $\chi_n^X(T, \mu_B, \ldots) = \frac{\partial^n P/T^4}{\partial \hat{\mu}_X^n}$ $X = B,\ Q,\ S$

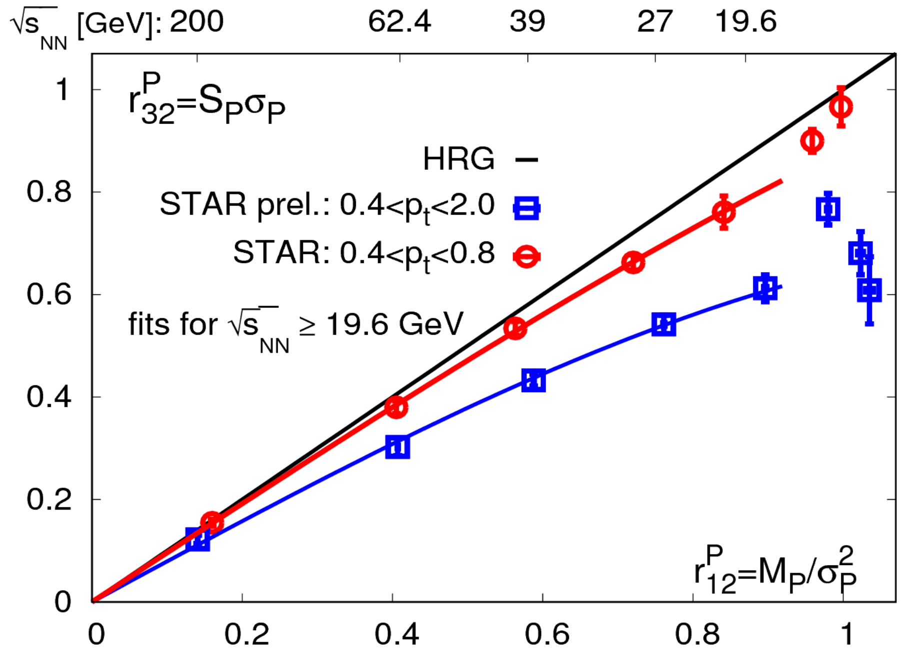
cumulant ratios:

$$\frac{M_X}{\sigma_X^2} = \frac{\chi_1^X(T,\mu)}{\chi_2^X(T,\mu)} \ , \quad S_X \sigma_X = \frac{\chi_3^X(T,\mu)}{\chi_2^X(T,\mu)} \ , \quad \kappa_X \sigma_X^2 = \frac{\chi_4^X(T,\mu)}{\chi_2^X(T,\mu)}$$

$\mu \equiv (\mu_B, \mu_Q, \mu_S)$

STAR data on cumulant ratios of net-proton number fluctuations





$$r_{12}^P \equiv \frac{M_P}{\sigma_P^2}$$ is monotonic functions of $\sqrt{s_{NN}}$

$$r_{32}^P \equiv S_P \sigma_P < r_{12}^P$$

→ and hence also of μ_B

(this is not trivial;
It will not hold close to a critical point !)

→ replace μ_B/T in favor of r_{12}^B , e.g.

$$\frac{\mu_B}{T} = m_1^B r_{12}^B + m_3^B \left(r_{12}^B\right)^3 + \mathcal{O}\left(\left(r_{12}^B\right)^5\right)$$, e.g. $m_1^B = \frac{1}{r_{12}^{B,1}}$

Conserved charge fluctuations and freeze-out
mean and variance

for simplicity: $\mu_S = \mu_Q = 0$

ratio of cumulants on "a line" in the (T, μ_B) plane (NLO Taylor expansion)

$$\frac{M_B}{\sigma_B^2} = \frac{\mu_B}{T} \frac{1 + \frac{1}{6}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}{1 + \frac{1}{2}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}$$

$$S_B\sigma_B = \frac{\mu_B}{T} \frac{\chi_4^B}{\chi_2^B} \frac{1 + \frac{1}{6}\frac{\chi_6^B}{\chi_4^B}\left(\frac{\mu_B}{T}\right)^2}{1 + \frac{1}{2}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}$$

$$\frac{\mu_B}{T} = m_1^B r_{12}^B + m_3^B \left(r_{12}^B\right)^3$$

Conserved charge fluctuations and freeze-out
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$$S_B\sigma_B = \frac{\mu_B}{T}\frac{\chi_4^B}{\chi_2^B} \frac{1 + \frac{1}{6}\frac{\chi_6^B}{\chi_4^B}\left(\frac{\mu_B}{T}\right)^2}{1 + \frac{1}{2}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}$$

$$\frac{\mu_B}{T} = m_1^B r_{12}^B + m_3^B \left(r_{12}^B\right)^3$$

$\mu_S = \mu_Q = 0$

$$\Rightarrow \quad S_B\sigma_B \equiv r_{32} = \frac{\chi_4^B}{\chi_2^B} r_{12}^B + \mathcal{O}((r_{12}^B)^3)$$

Conserved charge fluctuations and freeze-out
mean and variance

for simplicity: $\mu_S = \mu_Q = 0$

freeze-out line

$$T_f(\mu_B) = T_{f,0}\left(1 - \kappa_2^f \left(\frac{\mu_B}{T}\right)^2\right)$$

ratio of cumulants on "a line" in the (T, μ_B) plane (NLO Taylor expansion)

$$\frac{M_B}{\sigma_B^2} = \frac{\mu_B}{T} \frac{1 + \frac{1}{6}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}{1 + \frac{1}{2}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}$$

$$S_B\sigma_B = \frac{\mu_B}{T}\frac{\chi_4^B}{\chi_2^B} \frac{1 + \frac{1}{6}\frac{\chi_6^B}{\chi_4^B}\left(\frac{\mu_B}{T}\right)^2}{1 + \frac{1}{2}\frac{\chi_4^B}{\chi_2^B}\left(\frac{\mu_B}{T}\right)^2}$$

$$\frac{\mu_B}{T} = m_1^B r_{12}^B + m_3^B \left(r_{12}^B\right)^3$$

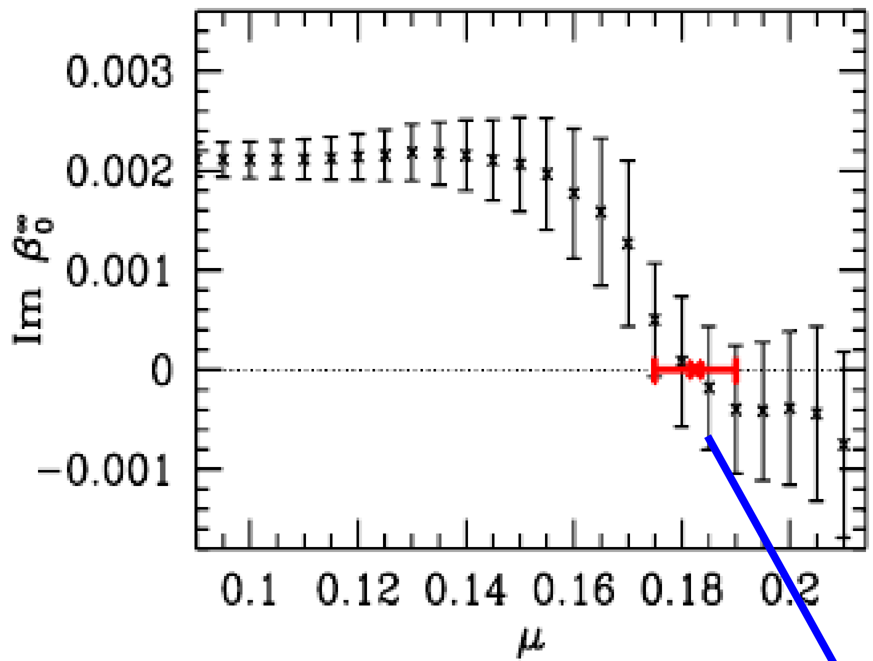
$$\left(\frac{\chi_4^B}{\chi_2^B}\right)_{T_{f,0}} - \kappa_2^f T_{f,0}\left(\frac{\chi_4^B}{\chi_2^B}\right)' \left(\frac{\mu_B}{T}\right)^2$$

all $\chi_n \equiv \chi_n(T)$ eventually need to be expanded in T

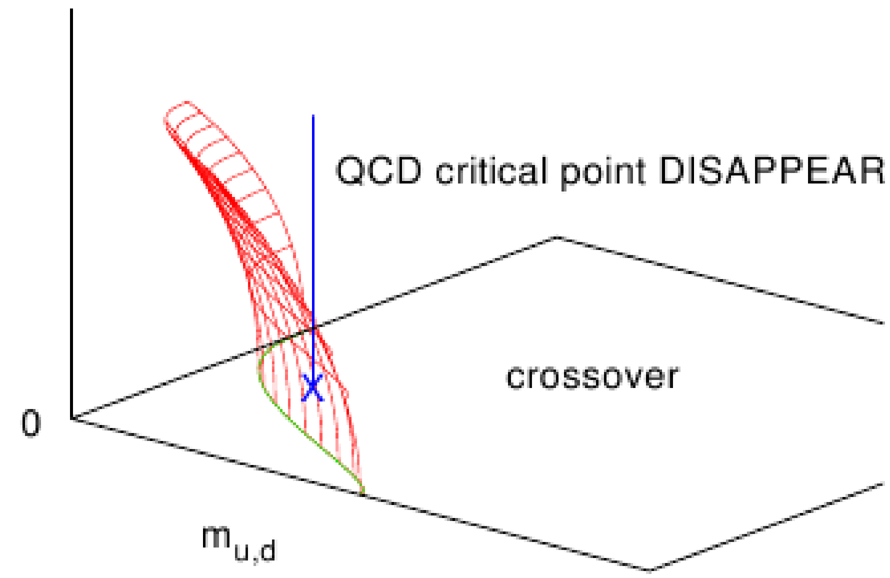
$\mu_S = \mu_Q = 0$

$$\Rightarrow \quad S_B\sigma_B \equiv r_{32} = \frac{\chi_4^B}{\chi_2^B} r_{12}^B + \mathcal{O}((r_{12}^B)^3)$$

LGT attempts to find the critical point



Z. Fodor, S. Katz. 2001, 2004



P. deForcrand, O. Philipsen, 2002

these calculations were possible because
(I) the lattices were coarse,
(II) the discretization schemes were crude (standars staggered)

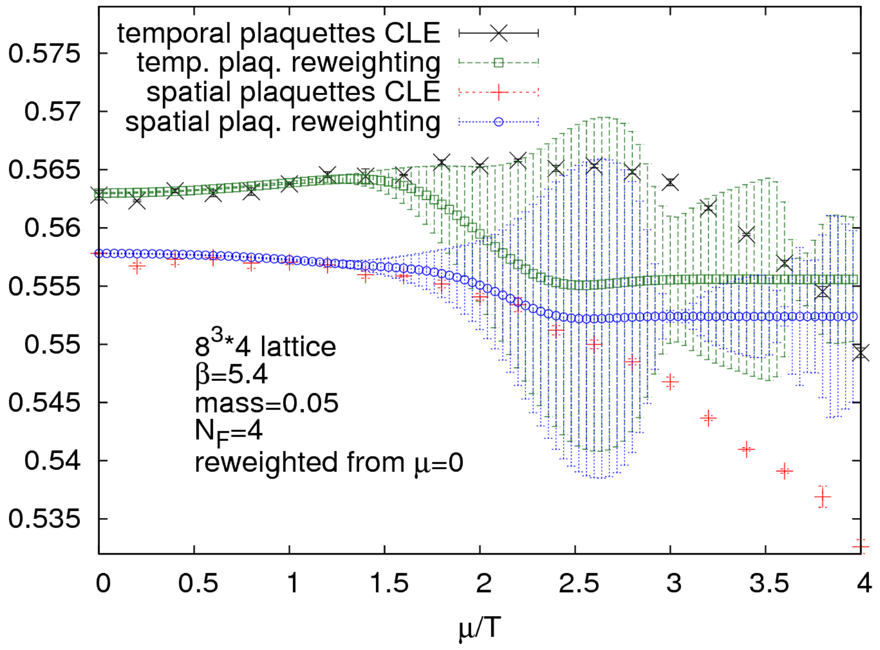
critical point or breakdown of the reweighting approach (loosing the overlap) ?

S. Ejiri, PRD69, 094506 (2004)

since 15 years no progress along this line

Complex Langevin vs. Reweighting

– the silent death of the Fodor/Katz critical point ? –



Z. Fodor, S. Katz. D Sexty, C. Torok,
Phys. Rev. D 92 (2015) 094516

from Conclusion:

...reweighting from zero μ breaks down
because of the overlap and sign problems
around

$$\frac{\mu}{T} = 1 - 1.5$$

i.e. $$\frac{\mu_B}{T} = 3 - 4.5$$

this should be compared to the
first Fodor/Katz critical point estimate
on lattices with comparable parameters:

$$\frac{\mu_B^{crit}}{T} = 4.5(3)$$

Z. Fodor, S. Katz. JHEP 0203 (2002) 014

(calculations with physical quark masses
eventually lead to a twice smaller estimate
for the critical chemical potential)