

FLUCTUATIONS FROM LATTICE QCD

Claudia Ratti

University of Houston (USA)

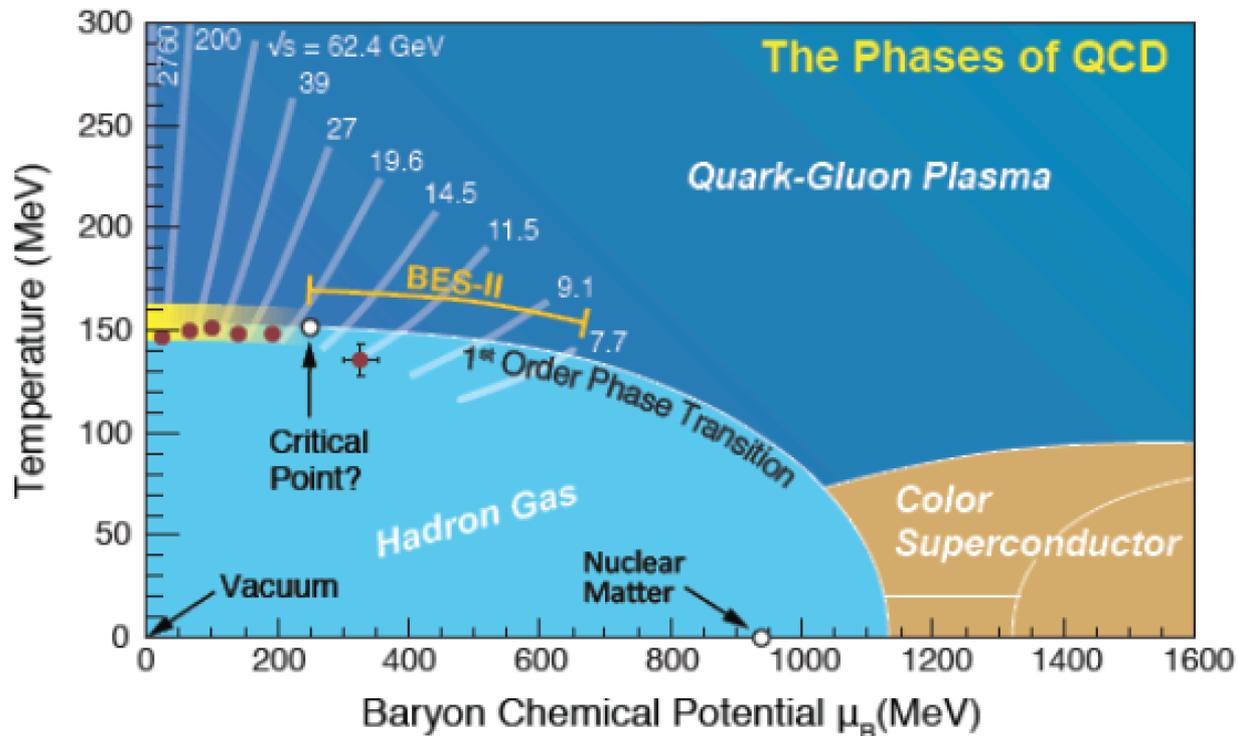
Collaborators: Paolo Alba, Rene Bellwied, Szabolcs Borsanyi, Renato Critelli, Zoltan Fodor, Jana Guenther, Sandor Katz, Stefan Krieg, Valentina Mantovani-Sarti, Jorge Noronha, Jaki Noronha-Hostler, Paolo Parotto, Attila Pasztor, Israel Portillo, Romulo Rougemont, Kalman Szabo



BEST
COLLABORATION

Ultimate goals

- Map the phase diagram of Quantum Chromodynamics
- Locate the critical point
- Find it in experiments



Sign problem

- The QCD path integral is computed by Monte Carlo algorithms which samples field configurations with a weight proportional to the exponential of the action

$$Z(\mu_B, T) = \text{Tr} \left(e^{-\frac{H_{\text{QCD}} - \mu_B N_B}{T}} \right) = \int \mathcal{D}U e^{-S_G[U]} \det M[U, \mu_B]$$

- $\det M[\mu_B]$ complex \rightarrow Monte Carlo simulations are not feasible
- We can rely on a few approximate methods, viable for small μ_B/T :
 - ▣ Taylor expansion of physical quantities around $\mu_B=0$ (Bielefeld-Swansea collaboration 2002; R. Gai, S. Gupta 2003)
 - ▣ Simulations at imaginary chemical potentials (plus analytic continuation) (Alford, Kapustin, Wilczek, 1999; de Forcrand, Philipsen, 2002; D'Elia, Lombardo 2003)

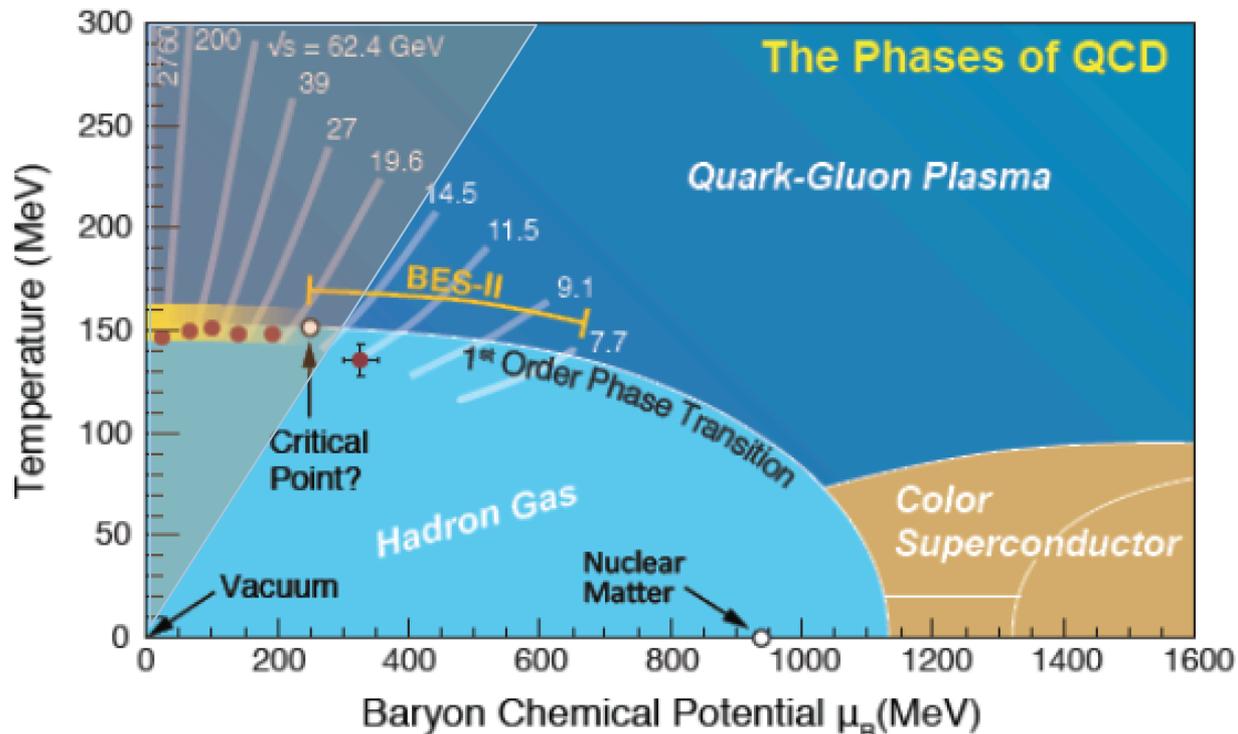
Equation of state as a Taylor expansion in μ_B

- Notation:

$$\hat{\mu}_B \equiv \mu_B/T \quad \hat{p} \equiv p/T^4 \quad \hat{n} \equiv n_B/T^3 \quad \hat{s} \equiv s/T^3$$

- Taylor expansion for the pressure:

$$\hat{p} = c_0(T) + c_2(T) \cdot \hat{\mu}_B^2 + c_4(T) \cdot \hat{\mu}_B^4 + c_6(T) \cdot \hat{\mu}_B^6 + \dots$$



Physics at imaginary μ

- At imaginary μ there is no sign problem
- The partition function is periodic in μ_I with period $2\pi T$

$$Z = \text{Tr} \left(e^{-\beta \hat{H} + i\beta \mu_I \hat{N}} \right)$$

- For more chemical potentials: μ_B , μ_Q , μ_S , several trajectories are possible \rightarrow useful for different physics
 - Here we use:

$$\langle n_S \rangle = 0$$

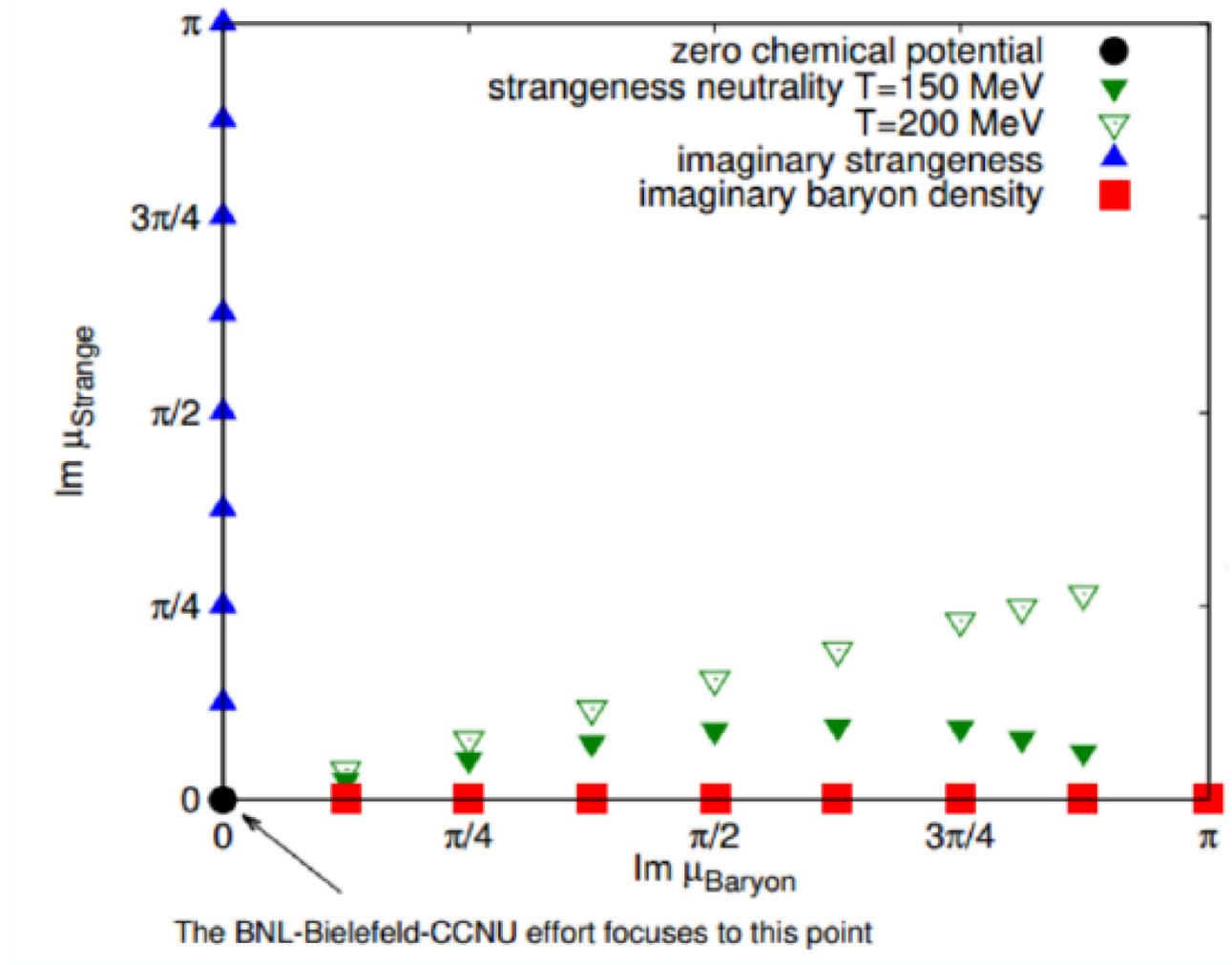
$$\langle n_Q \rangle = 0.4 \langle n_B \rangle$$

- Other choices are possible, e.g.:

$$\mu_S = 0$$

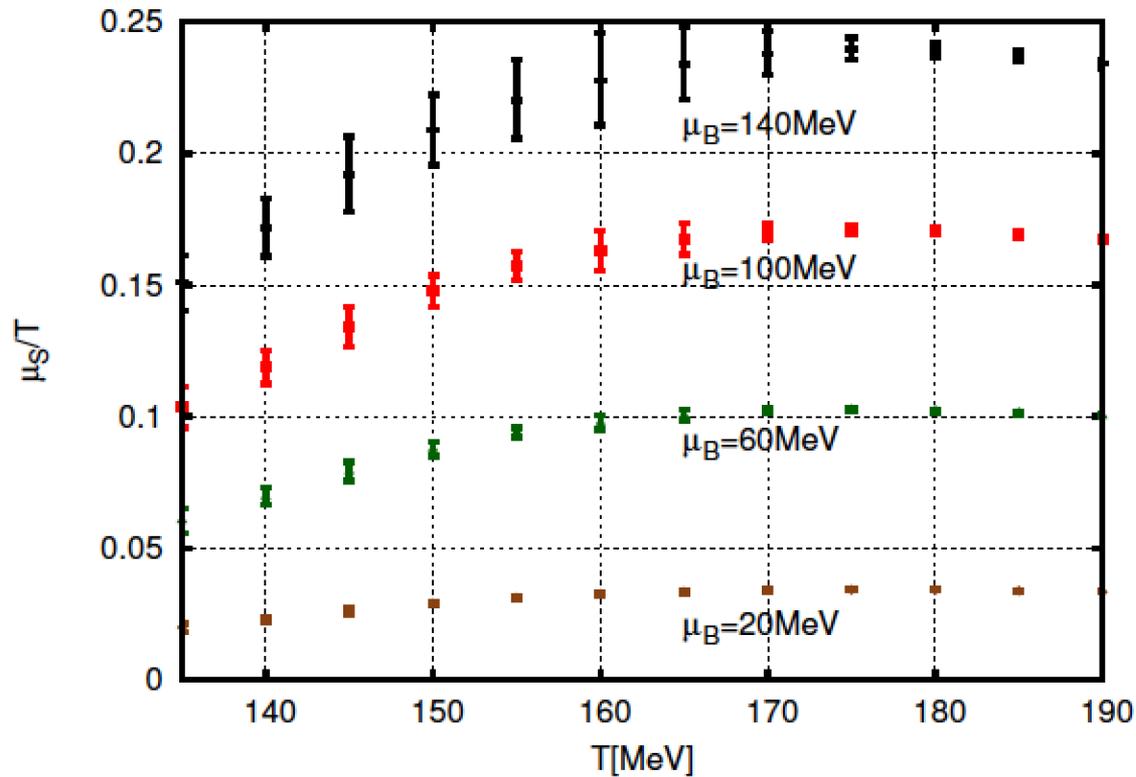
$$\mu_Q = 0$$

Simulation landscape



Strangeness neutrality

- We simulate at μ_B, μ_S pairs such that $\langle n_S \rangle = 0$
- This requires a non-trivial fine tuning



Thermodynamic identities

- For the pressure we measure:

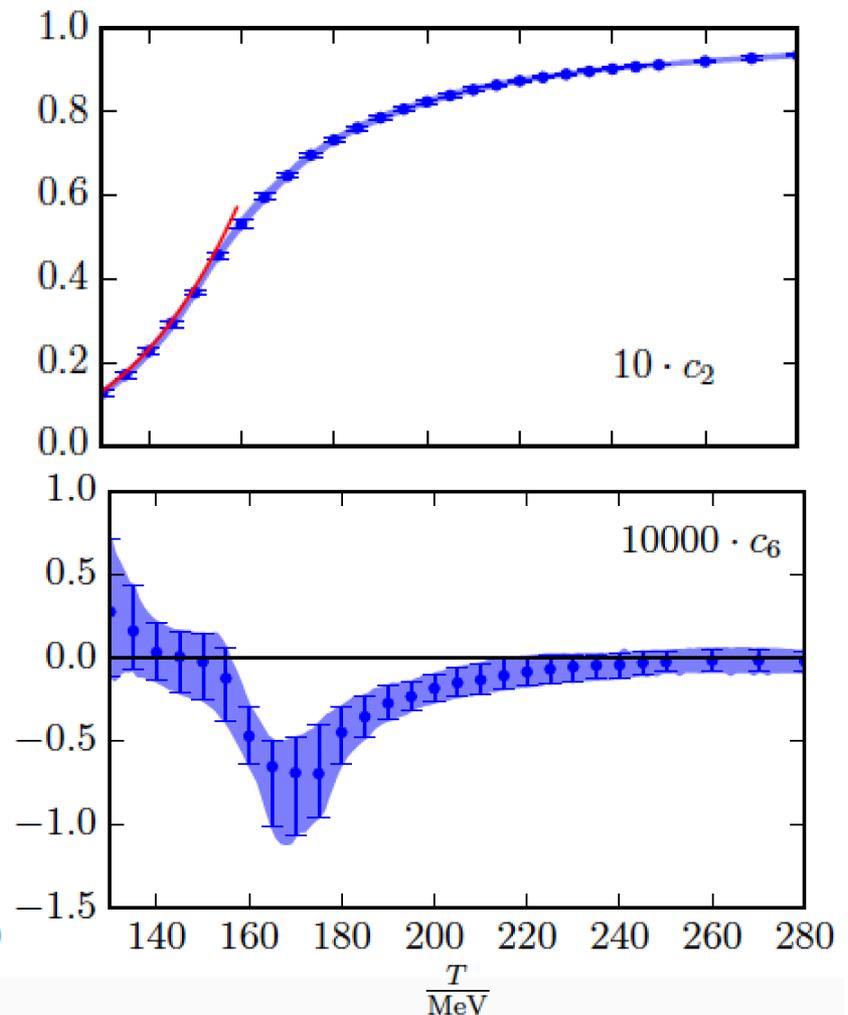
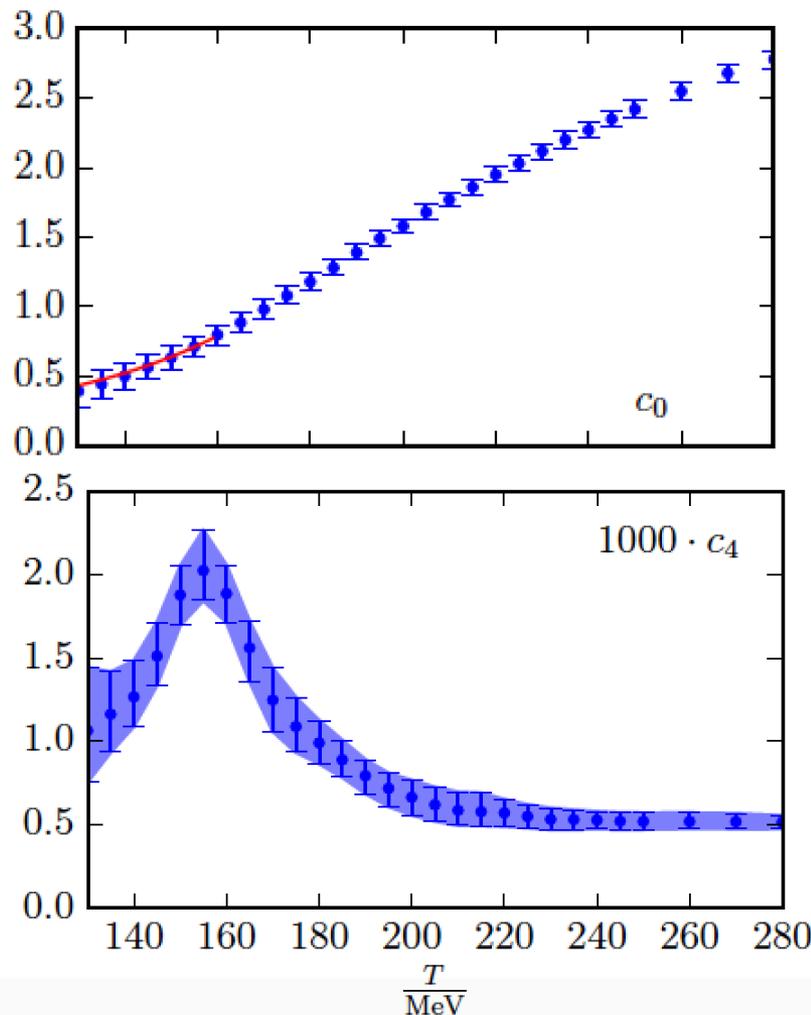
$$\frac{n}{\mu_B T^2} = \frac{T}{\mu_B} \frac{d(p/T^4)}{d(\mu_B/T)} \Big|_{\langle n_S \rangle=0, \langle n_Q \rangle=0.4\langle n_B \rangle, T=\text{const.}}$$
$$= n_B \left(1 + 0.4 \frac{d\mu_Q}{d\mu_B} \right) = 2c_2 + 4c_4 \left(\frac{\mu_B}{T} \right)^2 + 6c_6 \left(\frac{\mu_B}{T} \right)^4 + \dots$$

- For the entropy and energy:

$$s = [T^4 \partial / \partial T + 4T^3](p/T^4)$$

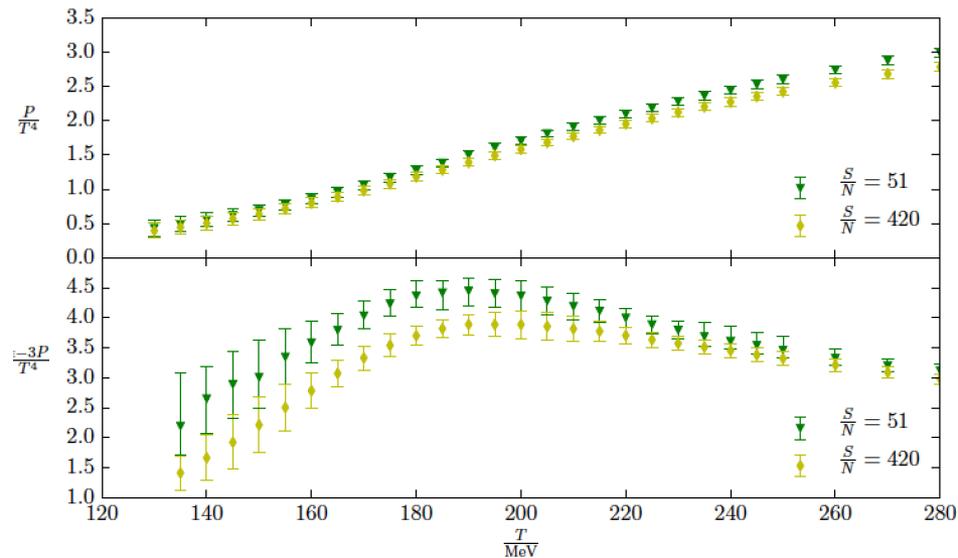
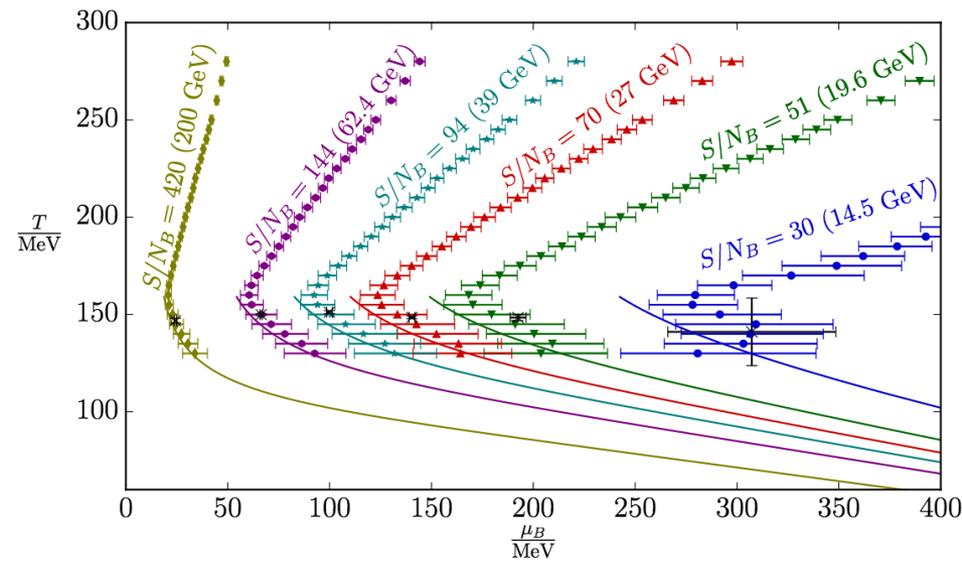
$$\hat{\epsilon} = \hat{s} - \hat{p} + \hat{\mu}_Q \hat{n}_Q + \hat{\mu}_B \hat{n}_B$$

Taylor expansion of the pressure for $\langle n_s \rangle = 0$



Equation of state at $\mu_B > 0$

- Extract the isentropic trajectory that the system follows in the absence of dissipation
- Calculate the EoS along these constant S/N trajectories



Fluctuations of conserved charges

- Definition:

$$\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n} p / T^4}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}.$$

- Relationship between chemical potentials:

$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q;$$

$$\mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q;$$

$$\mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S.$$

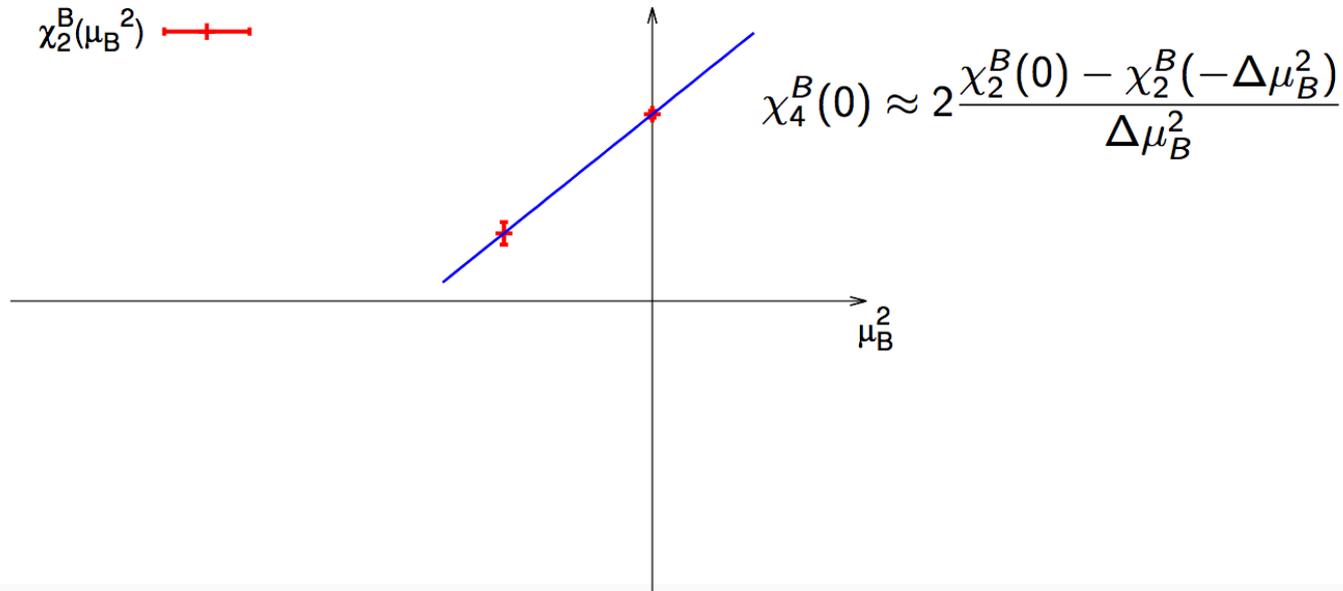
- They can be calculated on the lattice and compared to experiment

The imaginary μ_B approach

The standard method to calculate higher order fluctuations is to calculate the non-gaussianity of the baryon number distribution at $\mu_B = 0$.

$$\chi_4^B(0) = \frac{1}{VT^3} (\langle N_B^4 \rangle - 3\langle N_B^2 \rangle^2)$$

Alternatively one measures the variance $\chi_2^B(\mu_B^2)$ at non-zero chemical potentials.

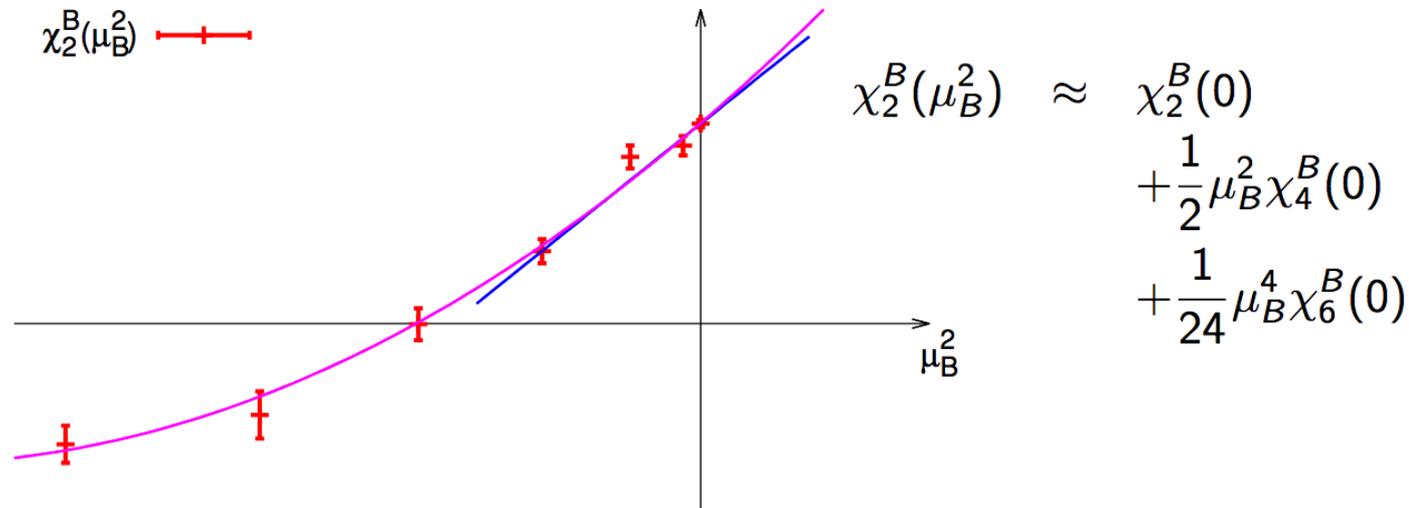


The imaginary μ_B approach

The standard method to calculate higher order fluctuations is to calculate the non-gaussianity of the baryon number distribution at $\mu_B = 0$.

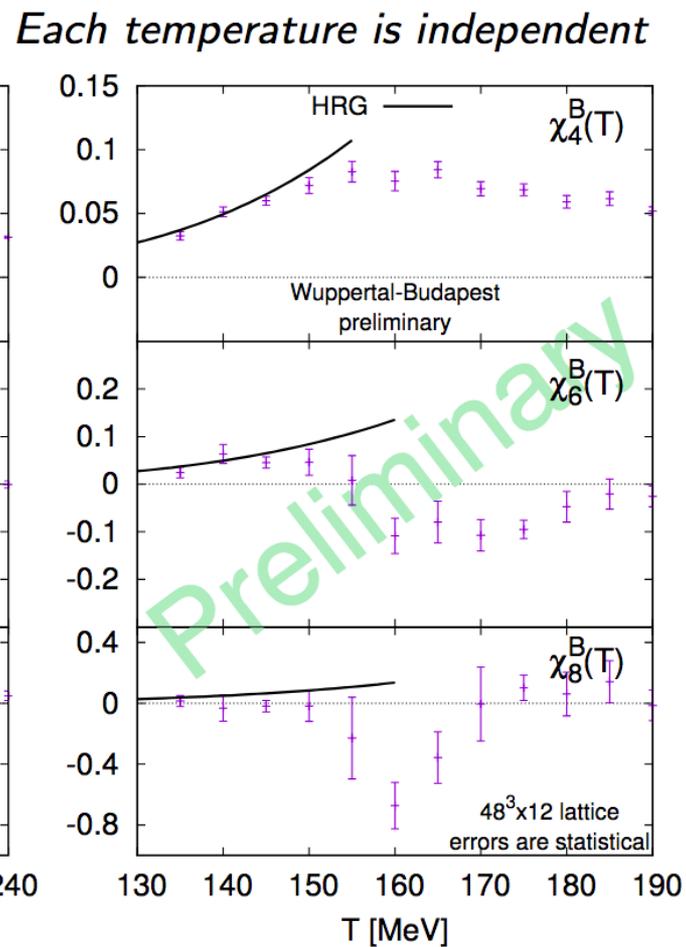
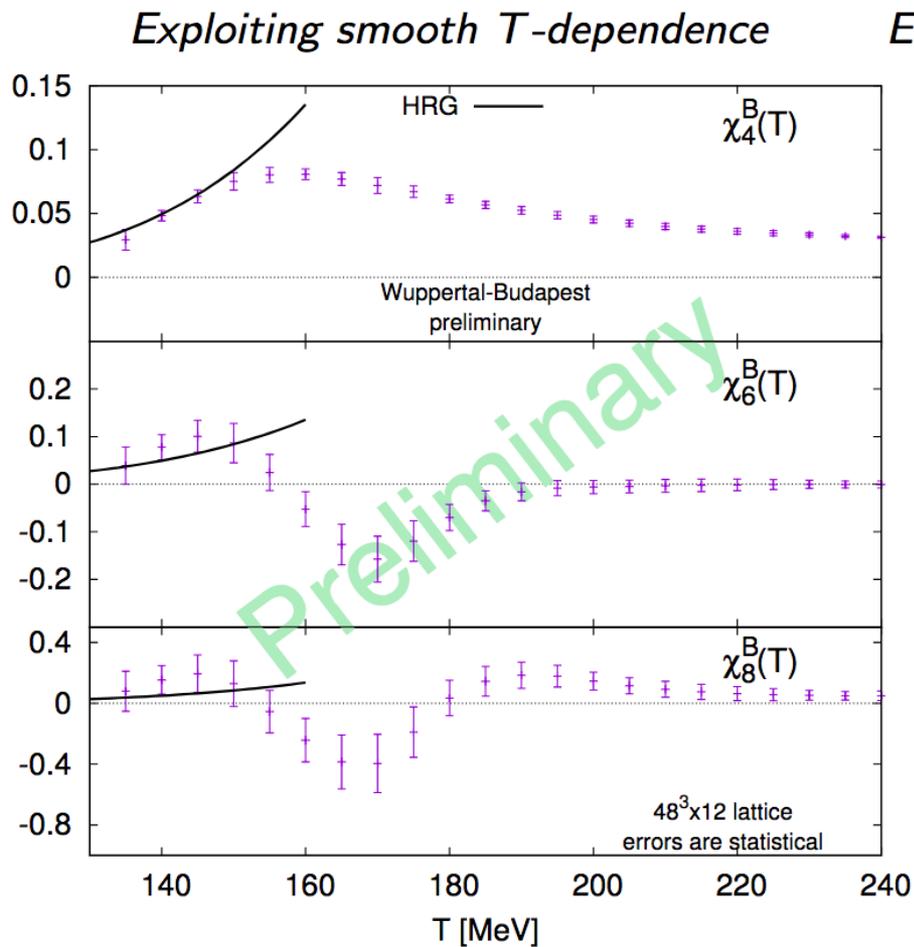
$$\chi_4^B(0) = \frac{1}{VT^3} (\langle N_B^4 \rangle - 3\langle N_B^2 \rangle^2)$$

Alternatively one measures the variance $\chi_2^B(\mu_B^2)$ at several non-zero chemical potentials.



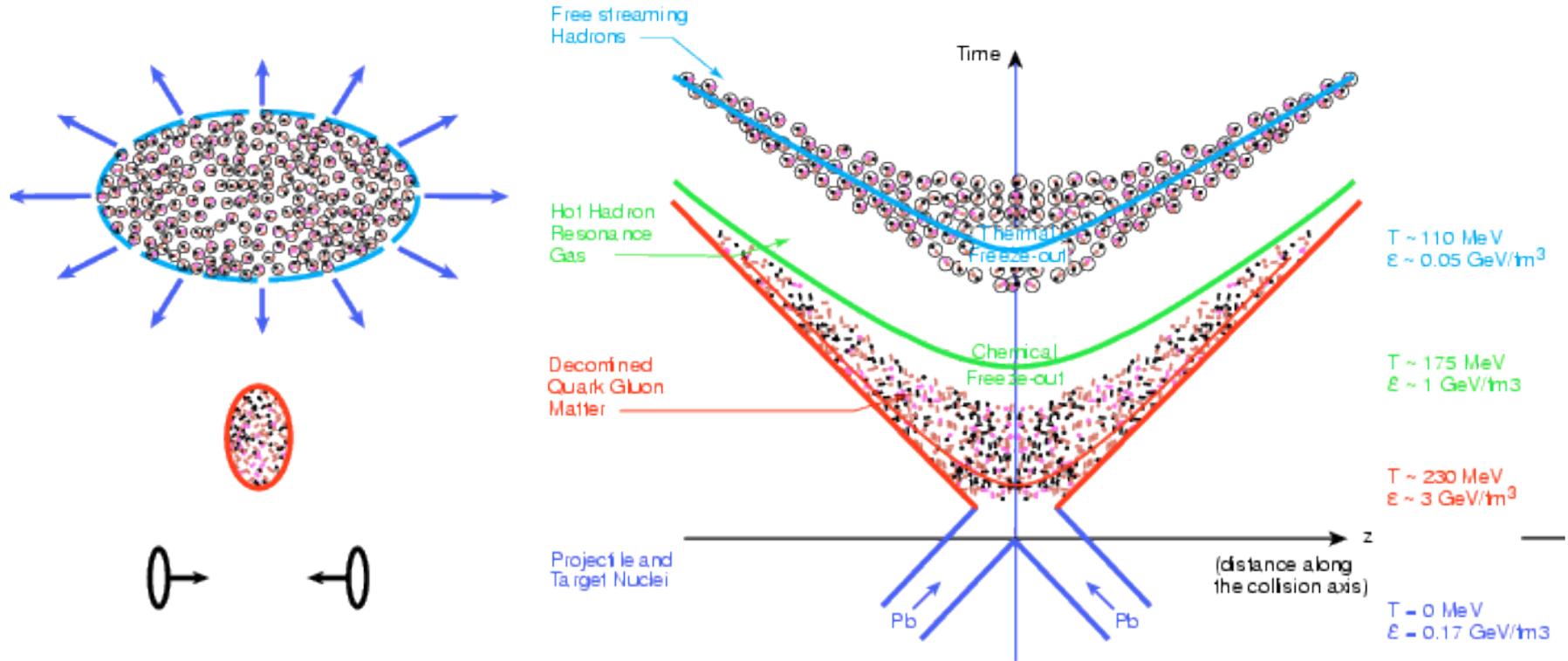
See also: [Wuppertal-Budapest 1607.02493, D'Elia et al 1611.08285]

Fluctuations at $\mu_s = \mu_Q = 0$



Systematic errors are not shown

Evolution of a Heavy Ion Collision



- **Chemical freeze-out:** inelastic reactions cease: the chemical composition of the system is fixed (particle yields and fluctuations)
- **Kinetic freeze-out:** elastic reactions cease: spectra and correlations are frozen (free streaming of hadrons)
- Hadrons reach the detector

Connection to experiment

- **Fluctuations** of conserved charges are the **cumulants** of their event-by-event distribution

$$\text{mean : } M = \chi_1$$

$$\text{variance : } \sigma^2 = \chi_2$$

$$\text{skewness : } S = \chi_3/\chi_2^{3/2}$$

$$\text{kurtosis : } \kappa = \chi_4/\chi_2^2$$

$$S\sigma = \chi_3/\chi_2$$

$$\kappa\sigma^2 = \chi_4/\chi_2$$

$$M/\sigma^2 = \chi_1/\chi_2$$

$$S\sigma^3/M = \chi_3/\chi_1$$

F. Karsch: Centr. Eur. J. Phys. (2012)

- The chemical potentials are not independent: fixed to match the experimental conditions:

$$\langle n_S \rangle = 0$$

$$\langle n_Q \rangle = 0.4 \langle n_B \rangle$$

Things to keep in mind

- Effects due to volume variation because of finite centrality bin width
 - ▣ Experimentally corrected by centrality-bin-width correction method
V. Skokov et al., PRC (2013)
- Finite reconstruction efficiency
 - ▣ Experimentally corrected based on binomial distribution A.Bzdak,V.Koch, PRC (2012)
- Spallation protons
 - ▣ Experimentally removed with proper cuts in p_T
- Canonical vs Grand Canonical ensemble
 - ▣ Experimental cuts in the kinematics and acceptance V. Koch, S. Jeon, PRL (2000)
- Proton multiplicity distributions vs baryon number fluctuations
 - ▣ Recipes for treating proton fluctuations
M. Asakawa and M. Kitazawa, PRC(2012), M. Nahrgang et al., 1402.1238
- Final-state interactions in the hadronic phase
 - ▣ Consistency between different charges = fundamental test
J.Steinheimer et al., PRL (2013)

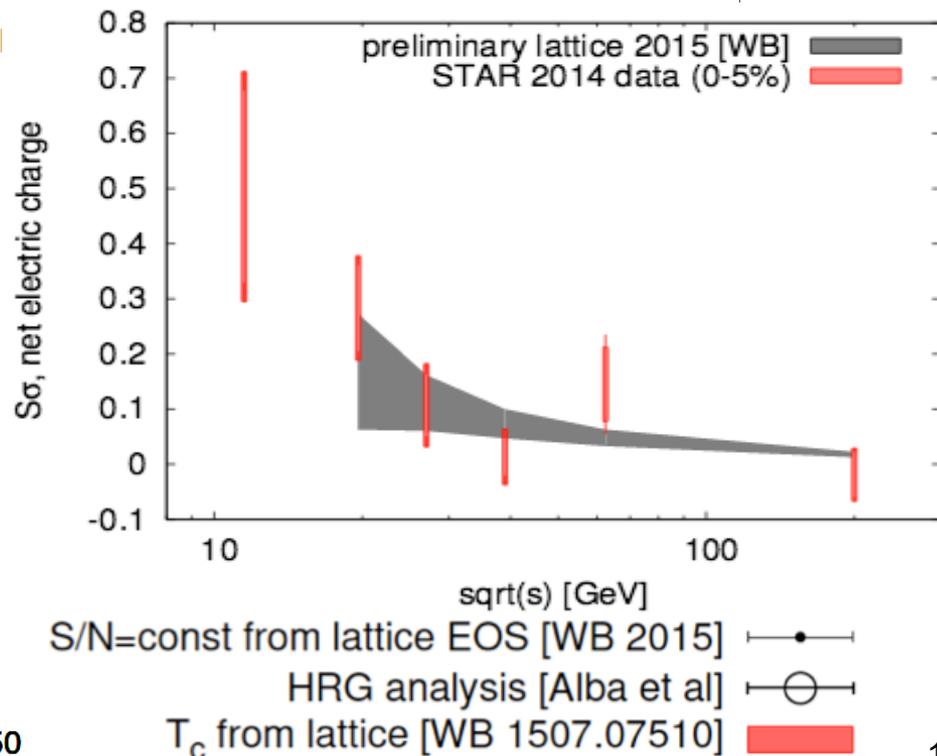
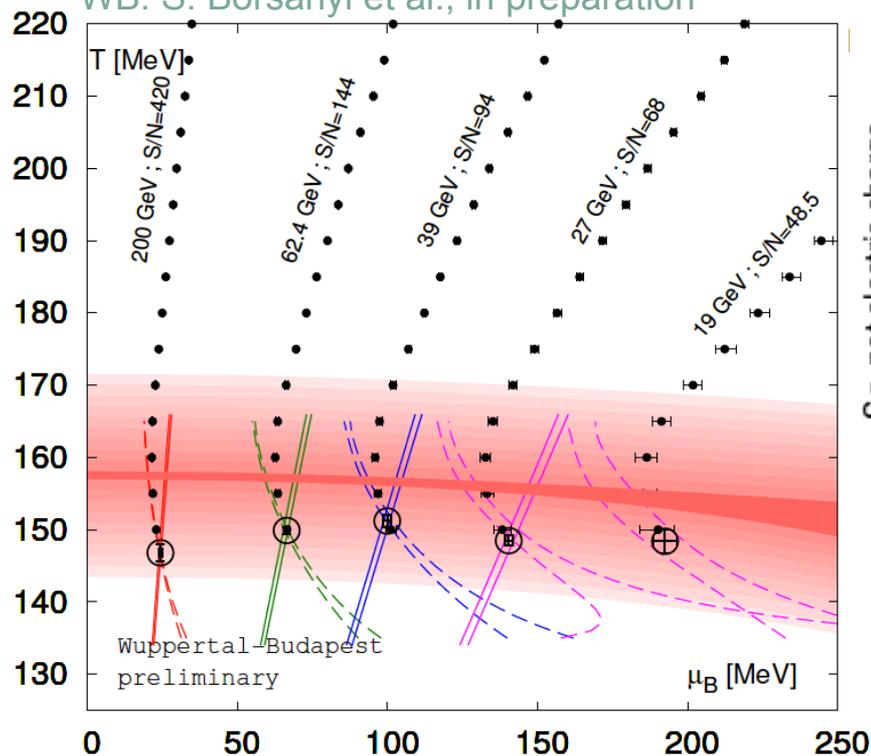
Freeze-out line from first principles

- Use T - and μ_B -dependence of R_{12}^Q and R_{12}^B for a combined fit:

$$R_{12}^Q(T, \mu_B) = \frac{\chi_1^Q(T, \mu_B)}{\chi_2^Q(T, \mu_B)} = \frac{\chi_{11}^{QB}(T, 0) + \chi_2^Q(T, 0)q_1(T) + \chi_{11}^{QS}(T, 0)s_1(T)}{\chi_2^Q(T, 0)} \frac{\mu_B}{T} + \mathcal{O}(\mu_B^3).$$

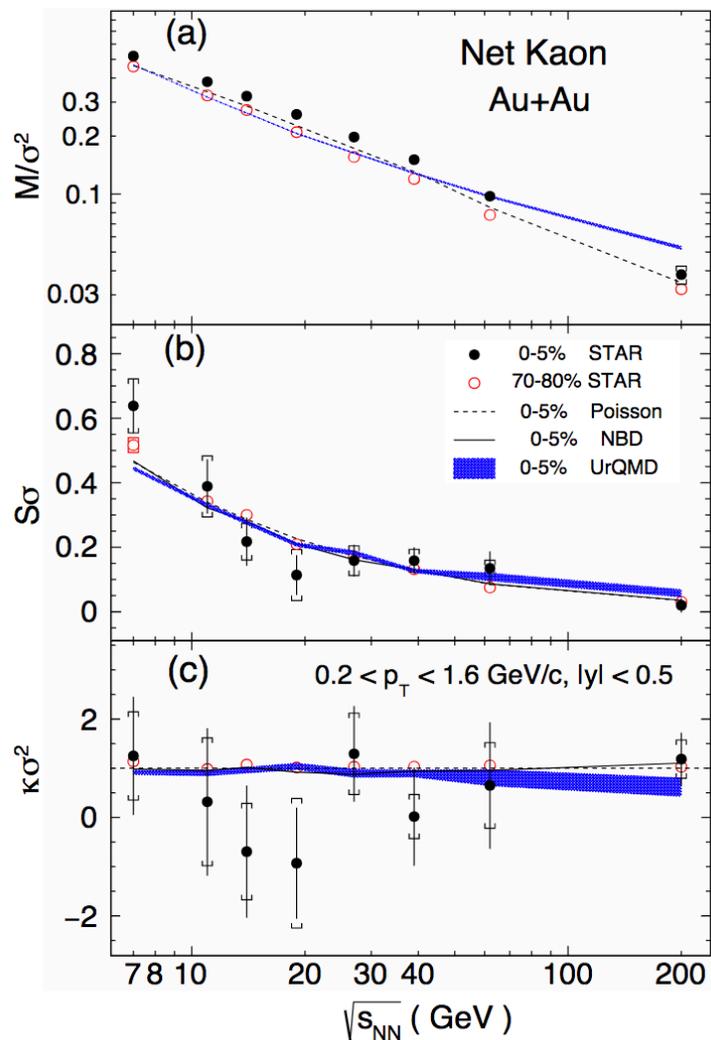
$$R_{12}^B(T, \mu_B) = \frac{\chi_1^B(T, \mu_B)}{\chi_2^B(T, \mu_B)} = \frac{\chi_2^B(T, 0) + \chi_{11}^{BQ}(T, 0)q_1(T) + \chi_{11}^{BS}(T, 0)s_1(T)}{\chi_2^B(T, 0)} \frac{\mu_B}{T} + \mathcal{O}(\mu_B^3)$$

WB: S. Borsanyi et al., in preparation

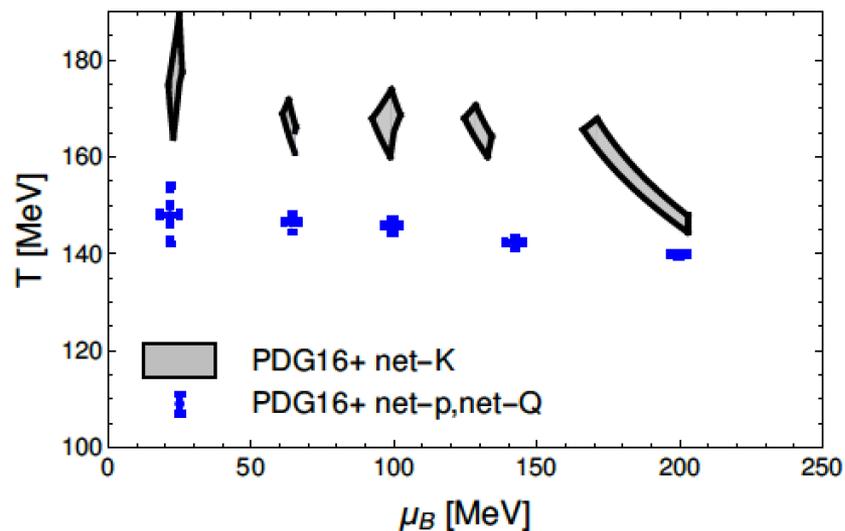


Freeze-out of kaons in the HRG model

STAR Collaboration (2017)



- Calculate χ_1/χ_2 for kaons in the HRG model, including resonance decays and acceptance cuts
- Calculate it along the isentropes



C. R. et al., in preparation

A different approach at large densities

- Use AdS/CFT correspondence
- Fix the parameters to reproduce everything we know from the lattice
- Calculate observables at finite density
- Fluctuations of conserved charges: they are sensitive to the critical point

Holographic model

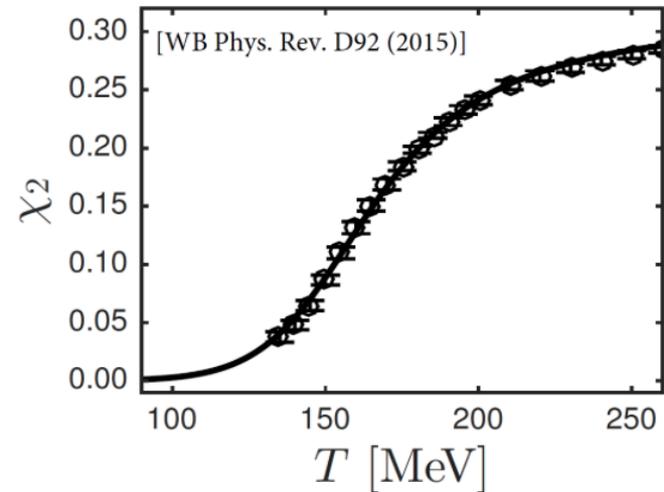
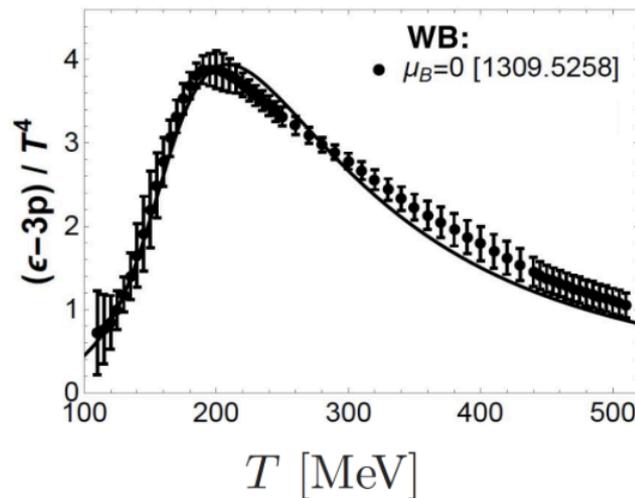
O DeWolfe, S S Gubser, and C Rosen, Phys. Rev. D 83, (2011)
 R Rougemont, A Ficnar, S Finazzo and J Noronha, JHEP (2016) 102

Non-conformal holographic gravity
 dual in 5 dimensions



Black Hole
 Solution

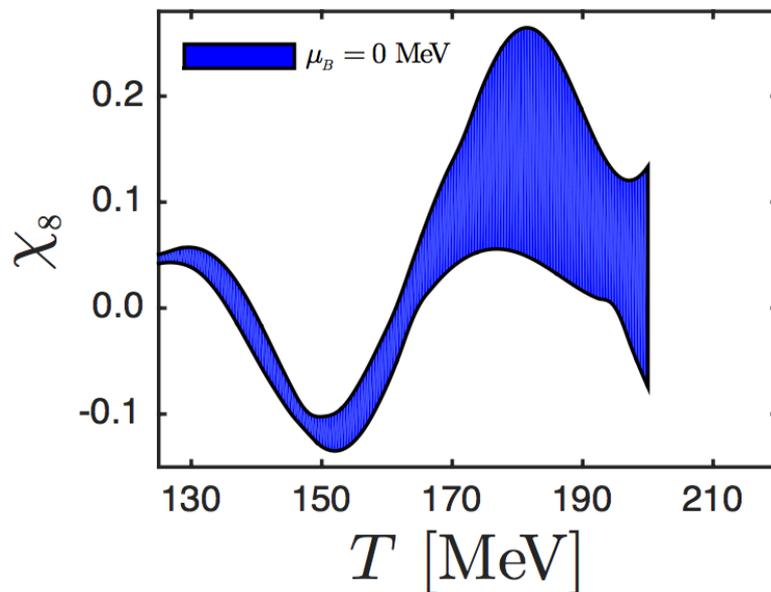
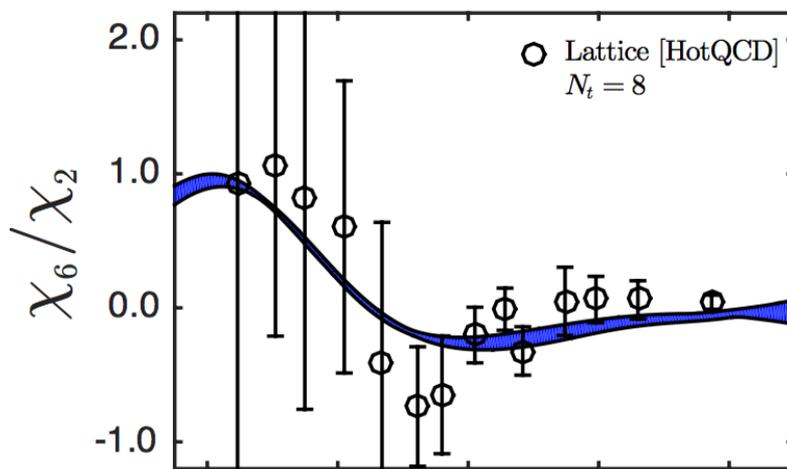
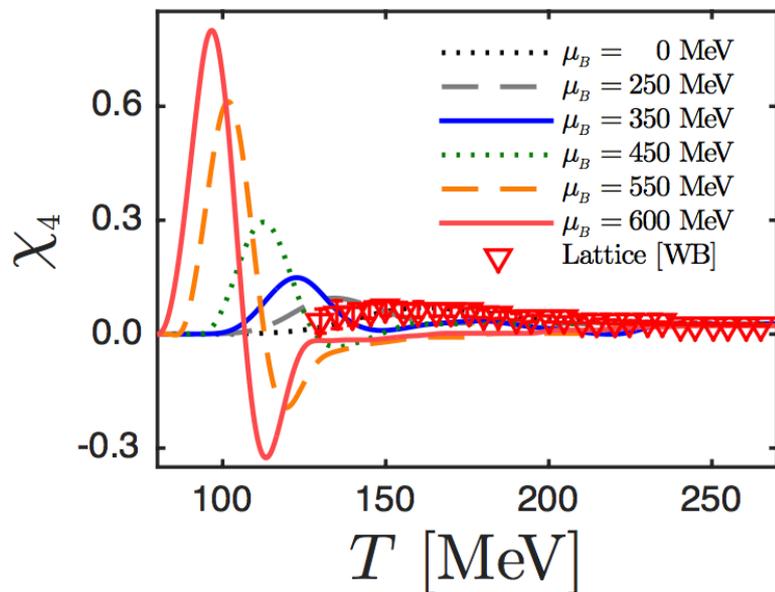
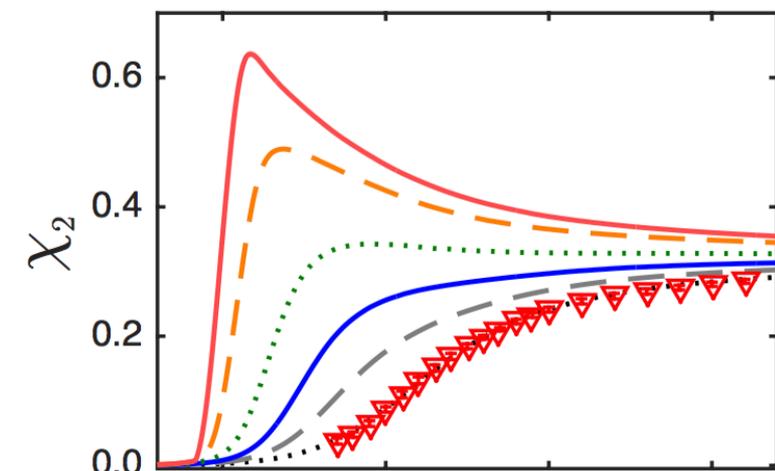
$$\mathcal{S} = \frac{1}{16\pi G_5} \int dx^5 \sqrt{-g} \left[\mathcal{R} - \frac{1}{2} (\partial_M \phi)^2 - \underbrace{V(\phi)}_{\text{nonconformal}} - \frac{1}{4} \underbrace{f(\phi) F_{MN}^2}_{\mu_B \neq 0} \right]$$



- Input parameters are fixed by lattice QCD results at $\mu_B = 0$
- Finite T and $\mu_B \rightarrow$ Predictions

Black Hole Susceptibilities

R. Critelli, C. R. et al., (2017)



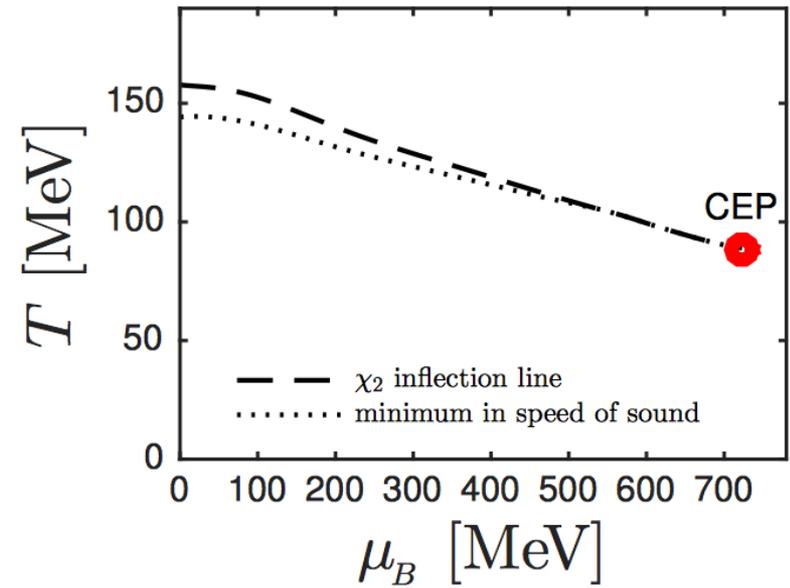
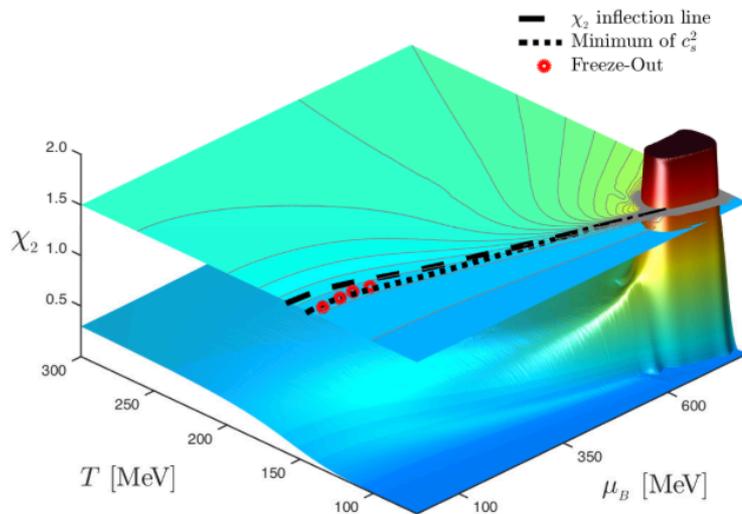
Black hole critical point

R. Critelli, C. R. et al., (2017)

The black hole model contains a critical end point at

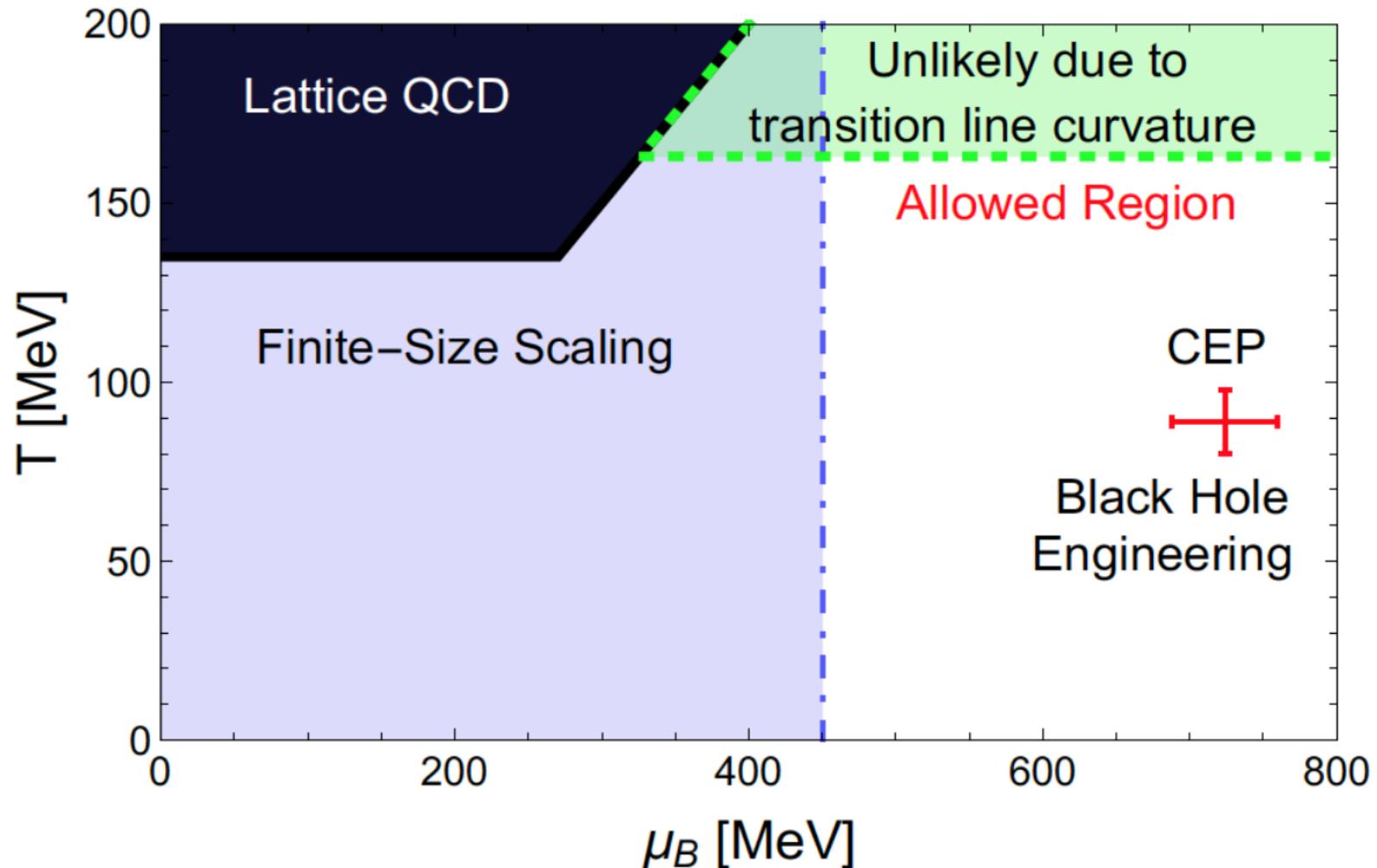
■ $\mu_B = 723 \pm 36 \text{ MeV}$

■ $T = 89 \pm 11 \text{ MeV}$



Black hole critical point

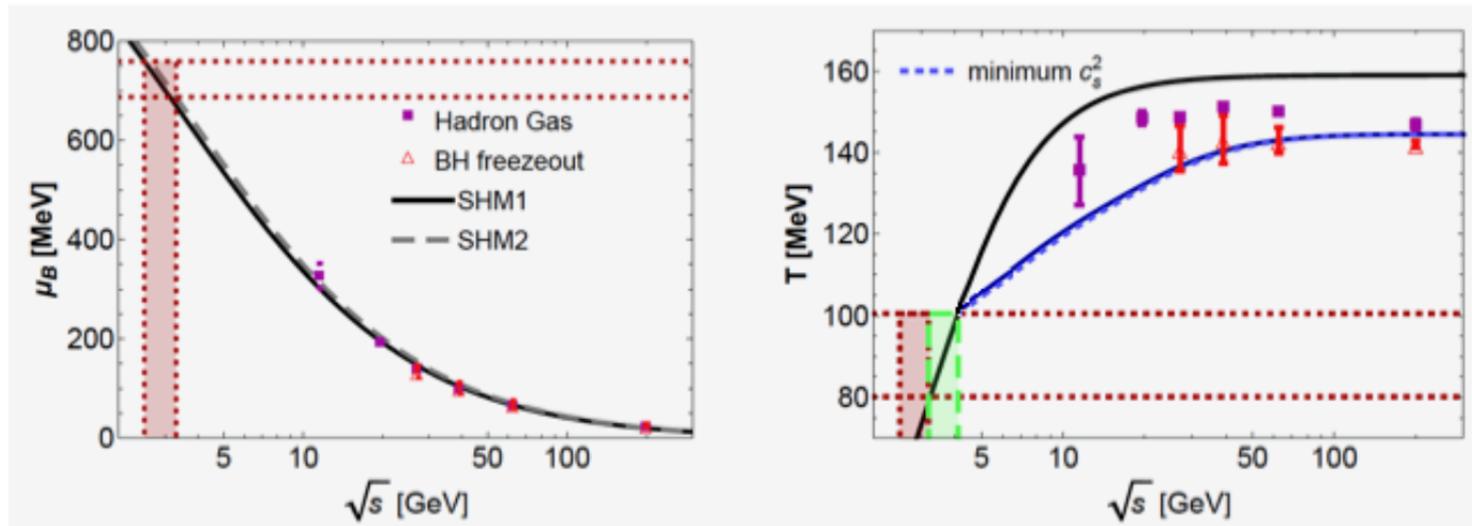
R. Critelli, C. R. et al., (2017)



Collision energy estimate

We estimate a collision energy needed to hit the CEP

- $\sqrt{s} = 2.5 - 4.1$ GeV



- The collision energy is reachable by the next generation of colliders

[BH] R. Critelli, C. R. et al., (2017)

[HRG] Paolo Alba et al. Phys. Lett. B738 (2014),

[SHM1] A. Andronic et al. Phys. Lett. B673 (2009).

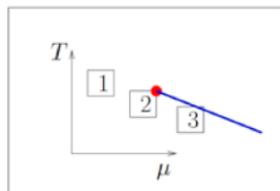
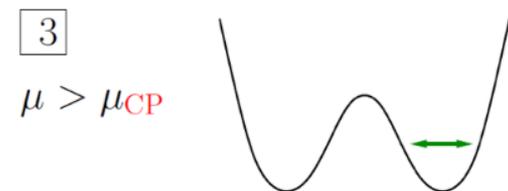
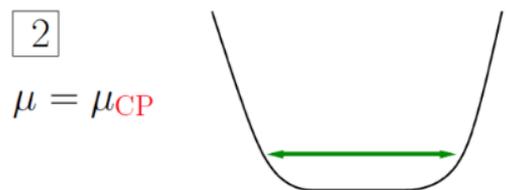
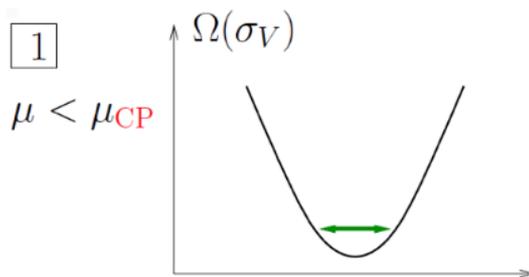
[SHM2] J. Cleymans et al. Phys. Rev. C73 (2006).

Conclusions

- Unprecedented precision in lattice QCD data allows a direct comparison to experiment for the first time
- QCD thermodynamics at $\mu_B=0$ can be simulated with high accuracy
- Extensions to finite density are under control up to $O(\mu_B^6)$
- Comparison with experiment allows to determine properties of strongly interacting matter from first principles
- Black Hole model engineered to reproduce lattice QCD results predicts a critical point which can be reached by the next generation colliders

Fluctuations at the critical point

M. Stephanov, PRL (2009).



The probability distribution for the order parameter

$$P[\sigma] \sim \exp \{-\Omega[\sigma]/T\}$$

$$\Omega = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \dots \right]$$

The **correlation length** ($\xi = m_\sigma^{-1}$)

$$\xi \sim |T - T_c|^{-\nu} \text{ where } \nu > 0$$

$$\chi_2 = VT\xi^2$$

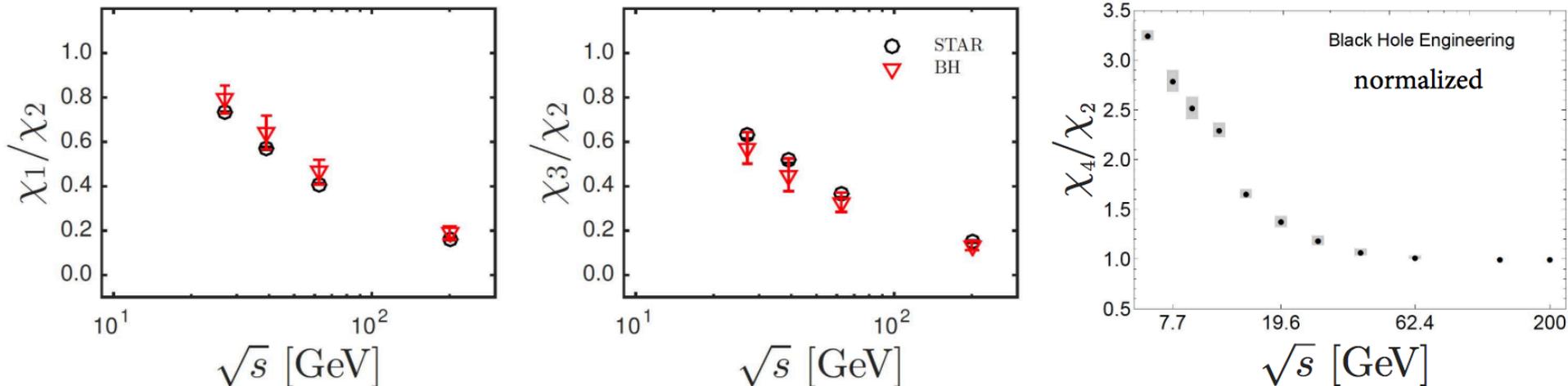
$$\chi_3 = 2VT^{3/2}\hat{\lambda}_3\xi^{9/2}$$

$$\chi_4 = 6VT^2[2\hat{\lambda}_3^2 - \hat{\lambda}_4]\xi^7$$

Connection to experiment

R. Critelli, C. R. et al., (2017)

- We want to estimate the collision energy we need to find the critical point in experiments
- We compare the baryonic BH susceptibilities ratios with the net-proton moments measured at STAR
- Freeze-out parameters are extracted by fitting the experimental values for χ_1/χ_2 and χ_3/χ_2



[STAR] Phys. Rev. Lett. **112** (2014)

Testing the Taylor expansion

R. Critelli, C. R. et al., forthcoming

Taylor expansion of observables in terms of susceptibilities

$$\chi_n = \chi_n^B(T, \mu_B = 0)$$

- Pressure

$$\frac{p(T, \mu_B) - p(T, \mu_B = 0)}{T^4} = \sum_{n=1}^{\infty} \frac{\chi_{2n}}{(2n)!} \left(\frac{\mu_B}{T}\right)^{2n}$$

- Baryonic density

$$\frac{\rho_B(T, \mu_B)}{T^3} = \sum_{n=1}^{\infty} \frac{\chi_{2n}}{(2n-1)!} \left(\frac{\mu_B}{T}\right)^{2n-1}$$

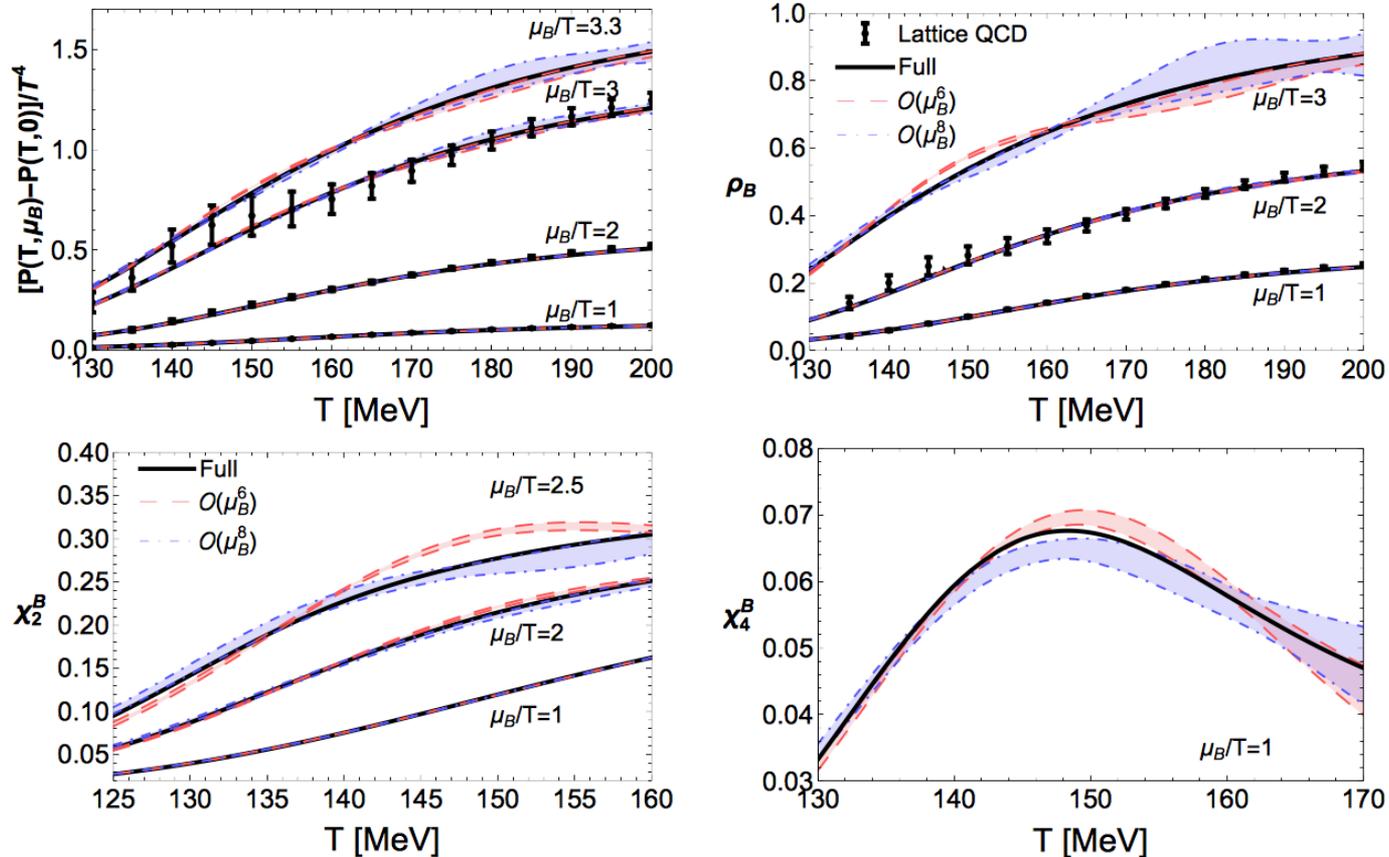
- Susceptibilities χ_2 and χ_4

$$\chi_2(T, \mu_B) = \sum_{n=0}^{\infty} \frac{\chi_{2n+2}}{(2n)!} \left(\frac{\mu_B}{T}\right)^{2n} \quad \chi_4(T, \mu_B) = \sum_{n=0}^{\infty} \frac{\chi_{2n+4}}{(2n)!} \left(\frac{\mu_B}{T}\right)^{2n}$$

Testing the Taylor expansion

R. Critelli, C. R. et al., forthcoming

Reconstruction of thermodynamic quantities at different values of μ_B/T via Taylor series from calculations at $\mu_B = 0$.



Lattice details

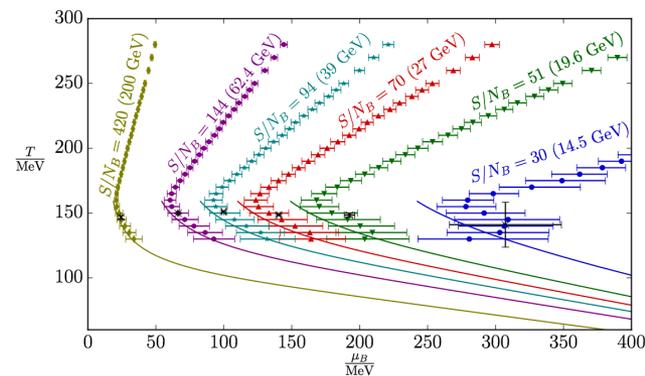
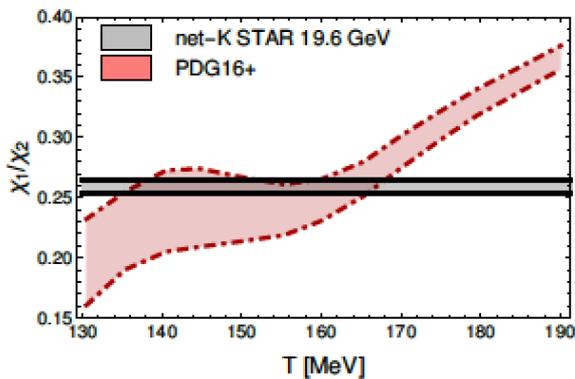
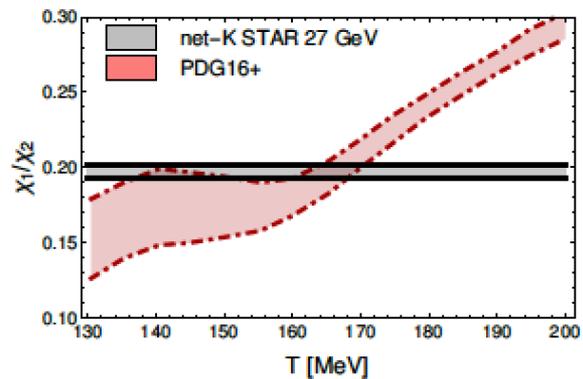
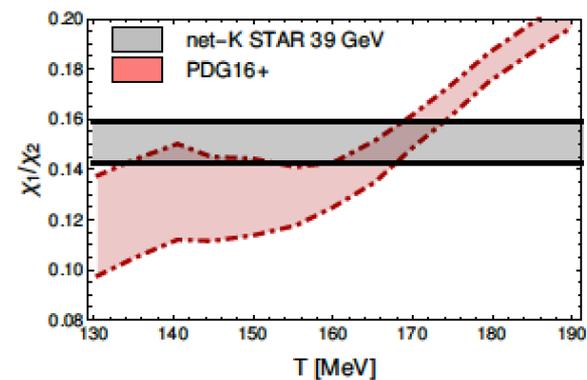
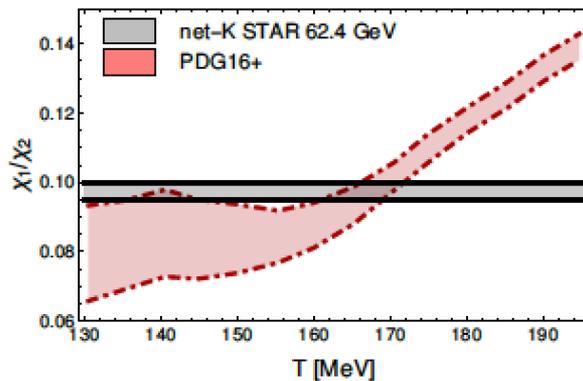
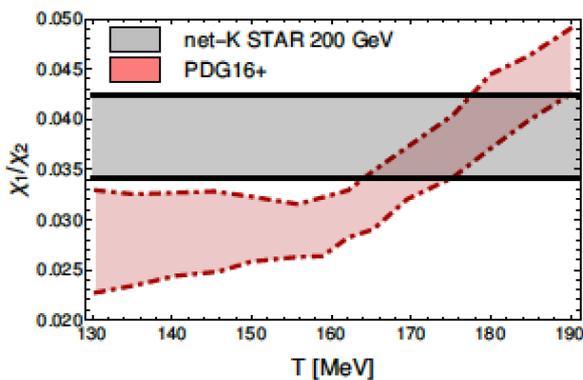
□ The 4stout staggered action

- 2+1+1 dynamical flavors
- 4 levels of stout smearing in the fermionic action
- The masses are set by bracketing both the pion and the kaon masses within a few percent, keeping $m_c/m_s=11.85$
- The scale is set in two ways: f_π and w_0 (with Wilson flow). The scale setting procedure is one of the source of the systematic error in all of the plots

□ Ensembles

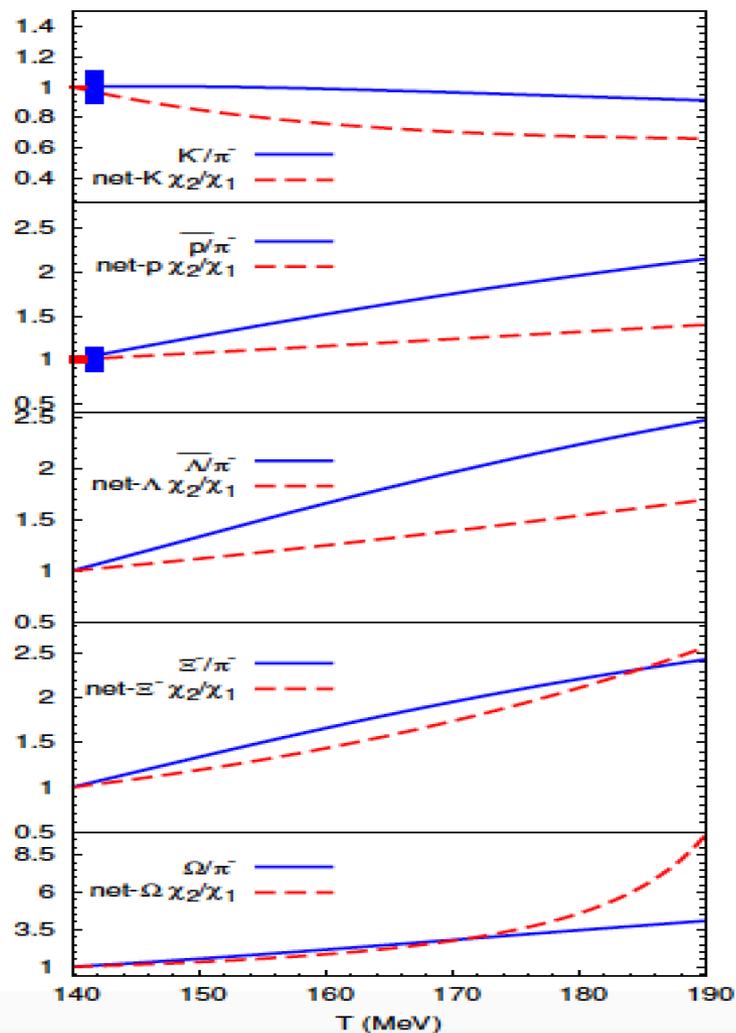
- Continuum limit from $N_t=10, 12, 16$
- For imaginary μ we have $\mu_B=iT\pi j/8$, with $j=3, 4, 5, 6, 6.5, 7$

Kaon freeze-out fit



C. R. et al., in preparation

Sensitivity of kaon fluctuations to freeze-out parameters

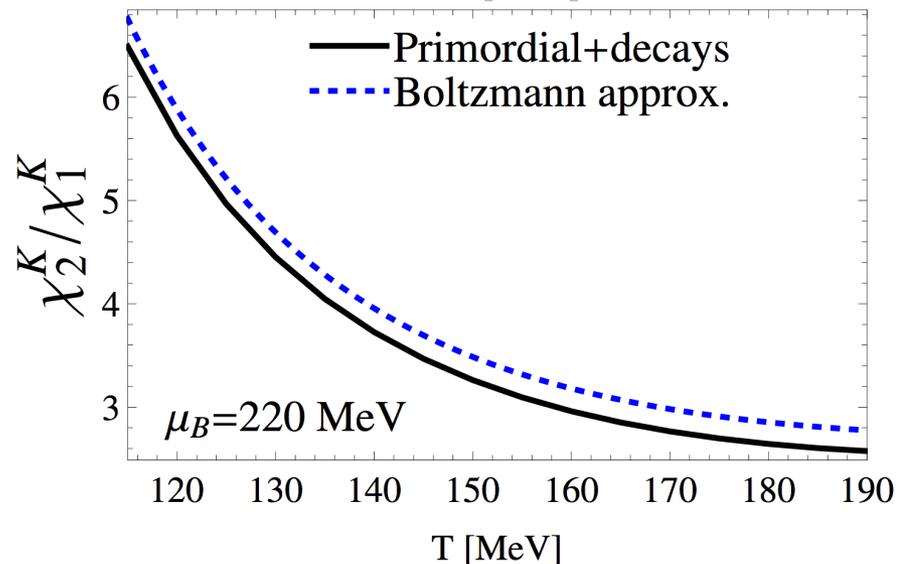
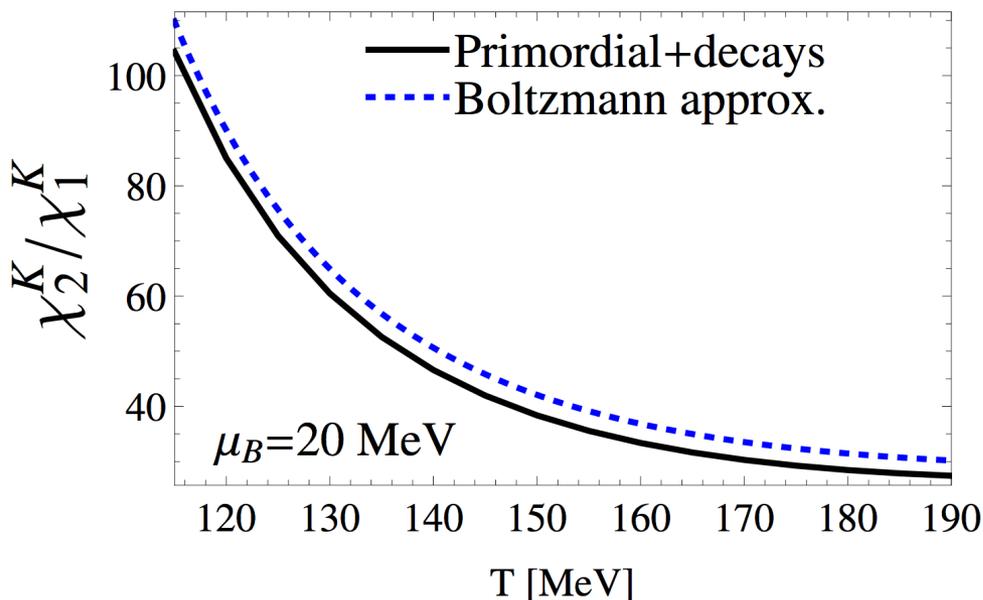


Kaon fluctuations are more sensitive to the freeze-out parameters, compared to yields

P. Alba et al., PRC (2015)

Kaon fluctuations on the lattice

J. Noronha-Hostler, C.R. et al., 1607.02527



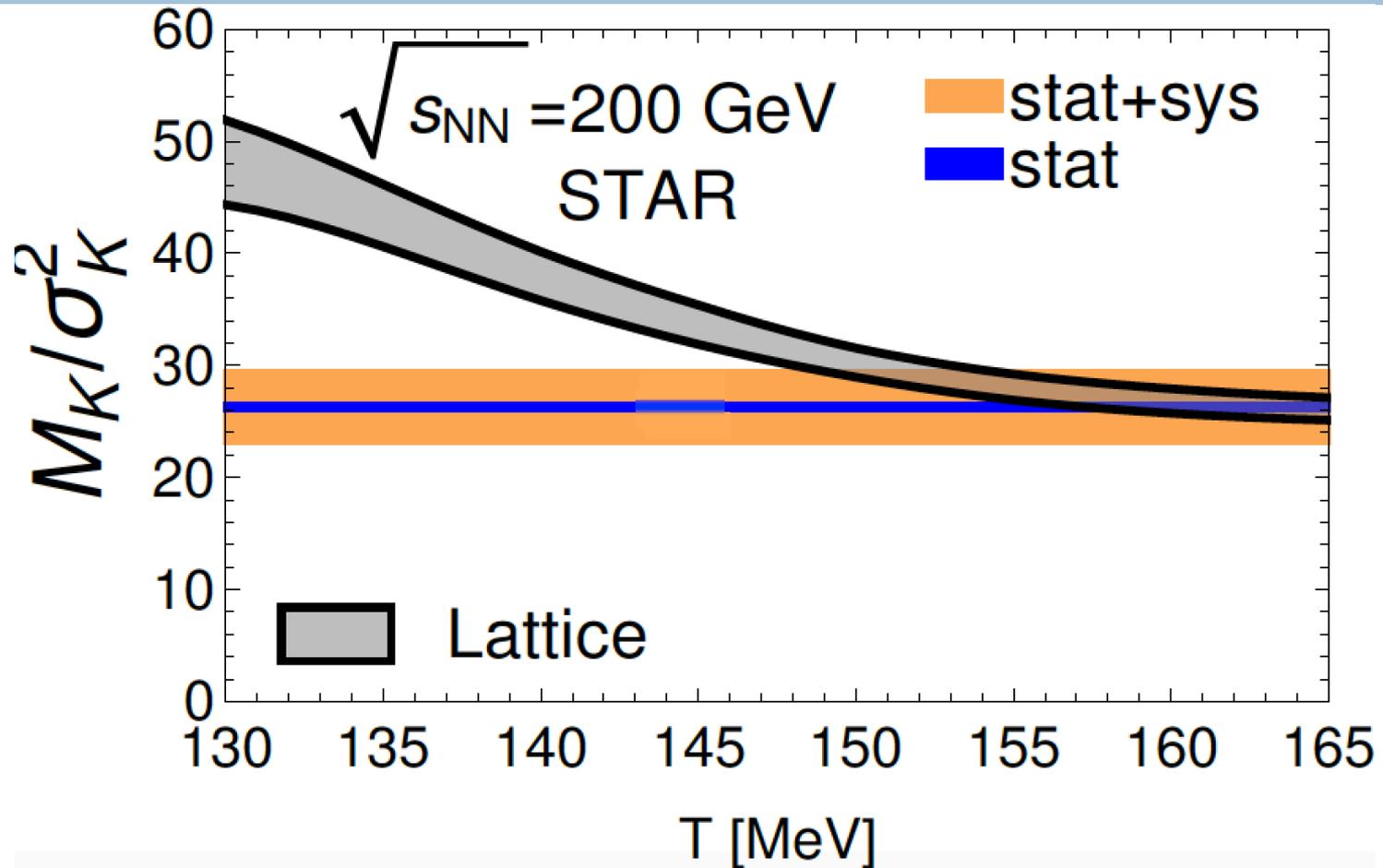
- Boltzmann approximation works well for lower order kaon fluctuations

$$\frac{\chi_2^K}{\chi_1^K} = \frac{\cosh(\hat{\mu}_S + \hat{\mu}_Q)}{\sinh(\hat{\mu}_S + \hat{\mu}_Q)}$$

- χ_2^K / χ_1^K from primordial kaons + decays is very close to the one in the Boltzmann approximation

Kaon fluctuations on the lattice

J. Noronha-Hostler, C.R. et al., 1607.02527

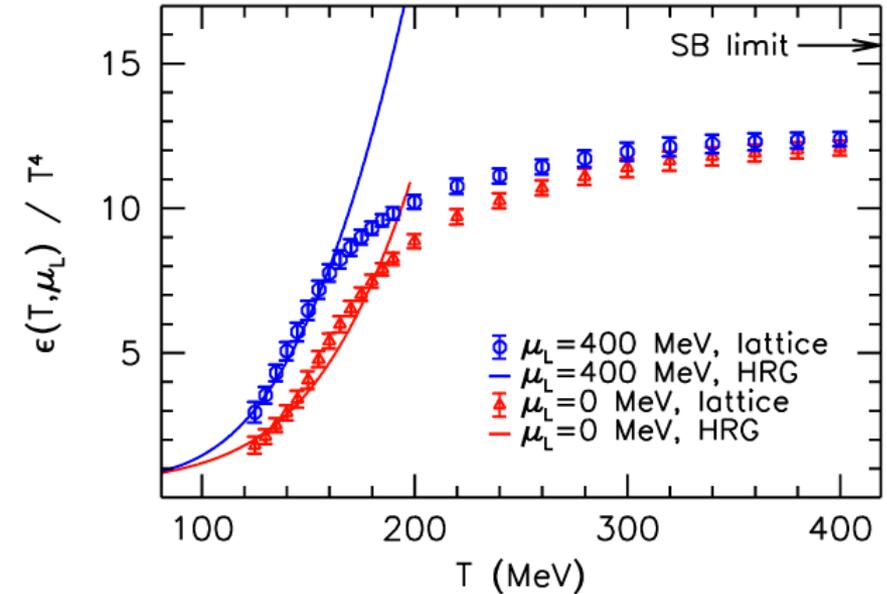
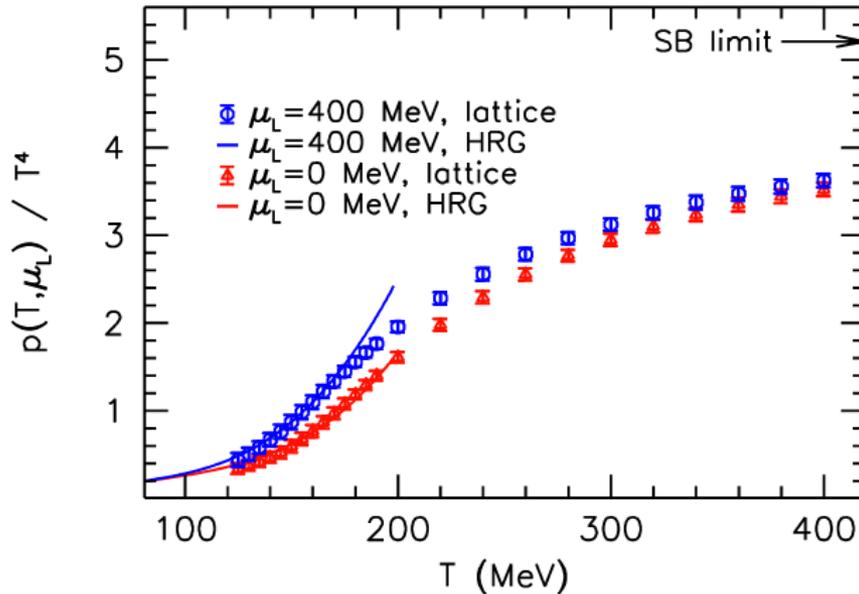


- Experimental uncertainty does not allow a precise determination of T_f^K
- It looks like $T_f^K > 150$ MeV

Equation of state at $\mu_B > 0$

- Expand the pressure in powers of μ_B (or $\mu_L = 3/2(\mu_u + \mu_d)$)

$$\frac{p(T, \{\mu_i\})}{T^4} = \frac{p(T, \{0\})}{T^4} + \frac{1}{2} \sum_{i,j} \frac{\mu_i \mu_j}{T^2} \chi_2^{ij} \quad \text{with} \quad \chi_2^{ij} \equiv \frac{T}{V} \frac{1}{T^2} \frac{\partial^2 \log \mathcal{Z}}{\partial \mu_i \partial \mu_j} \Big|_{\mu_i = \mu_j = 0}$$



S. Borsanyi et al., JHEP (2012)

- Continuum extrapolated results at the physical mass

Analytical continuation – illustration of systematics

Analytical continuation on $N_t = 12$ raw data

