Critical and non-critical fluctuations at RHIC BES

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EMMI Workshop on Critical Fluxtuations Near the QCD phase Boundary in Relativistic Nuclear Collisions

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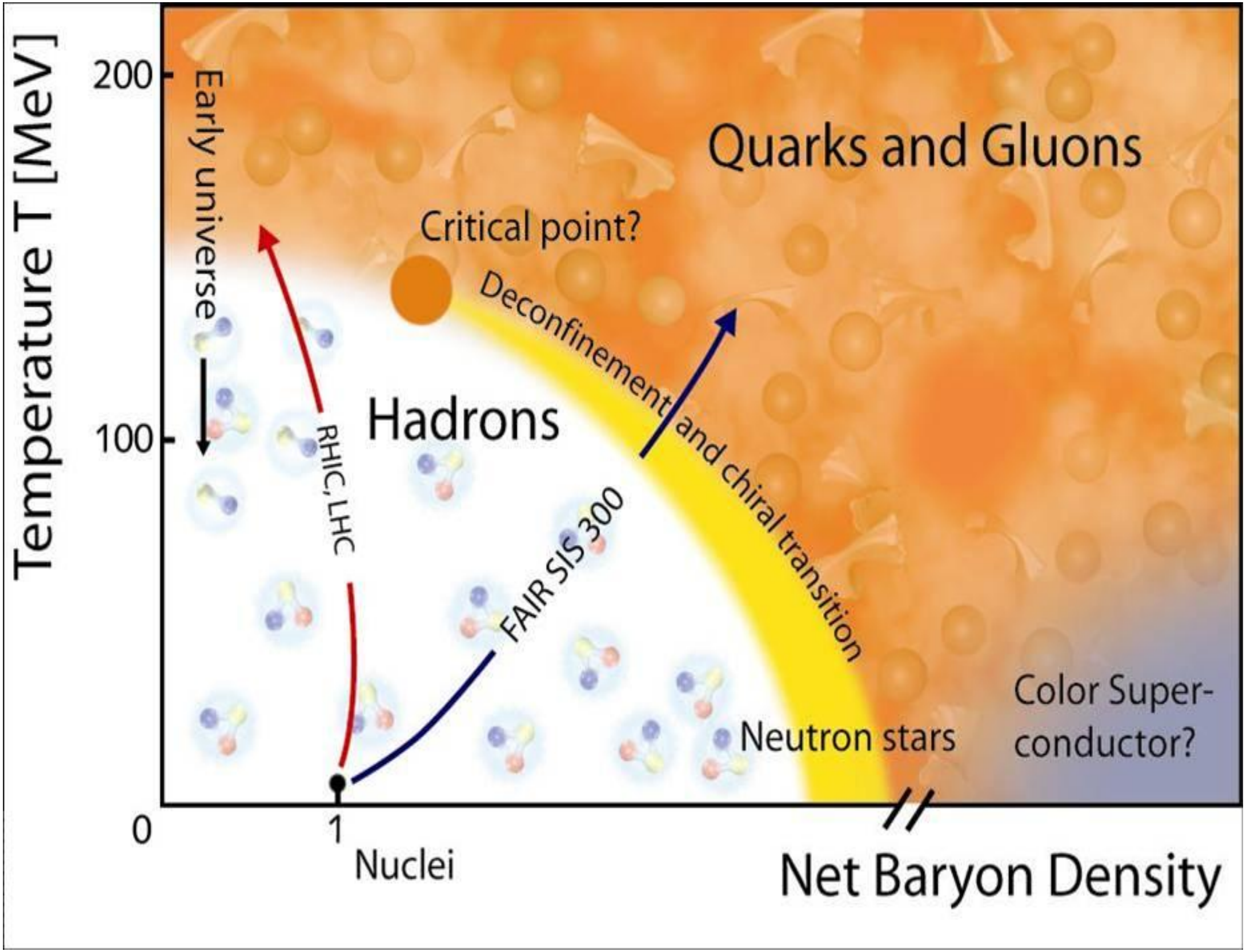
Temperature T [MeV]
200
100
0
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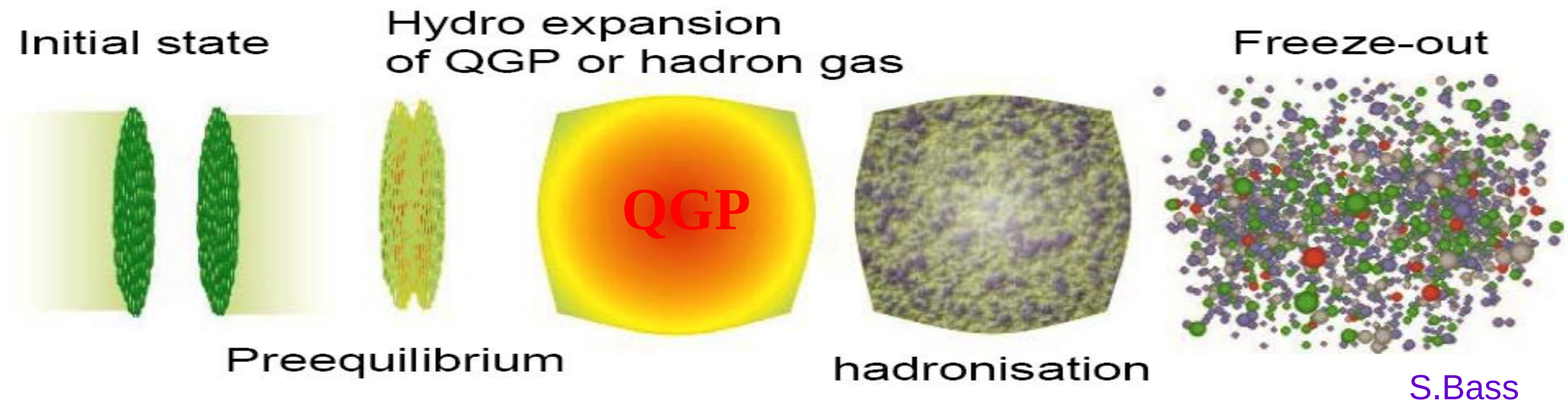
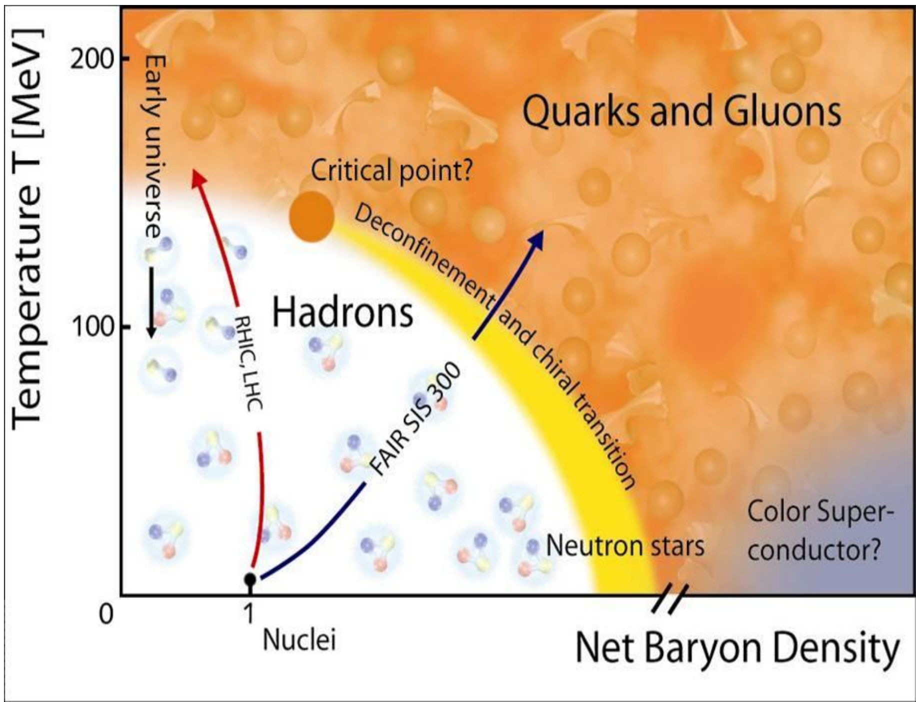
Nuclei
Net Baryon Density
Early universe
RHIC, LHC
FAIR SIS 300
Quarks and Gluons
Critical point?
Deconfinement and chiral transition
Hadrons
Neutron stars
Color Super-
conductor?



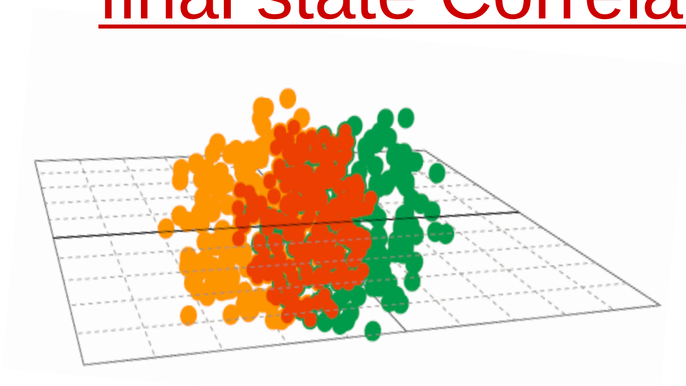
Correlations & fluctuations

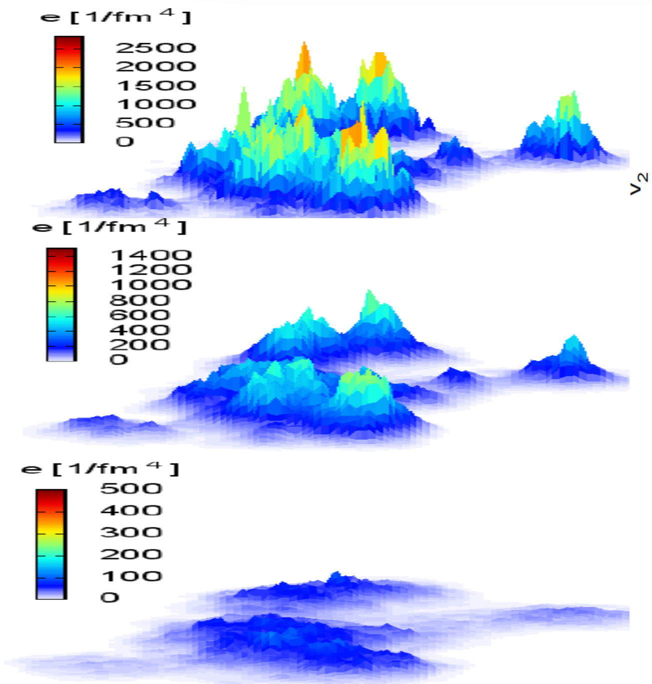
-Correlated fluctuations near the QCD critical point

-Non-critical (thermal) fluctuations

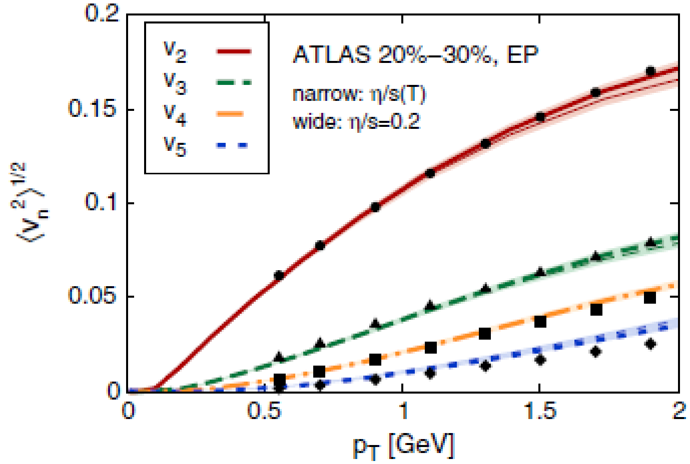


Initial state fluctuations & final state Correlations

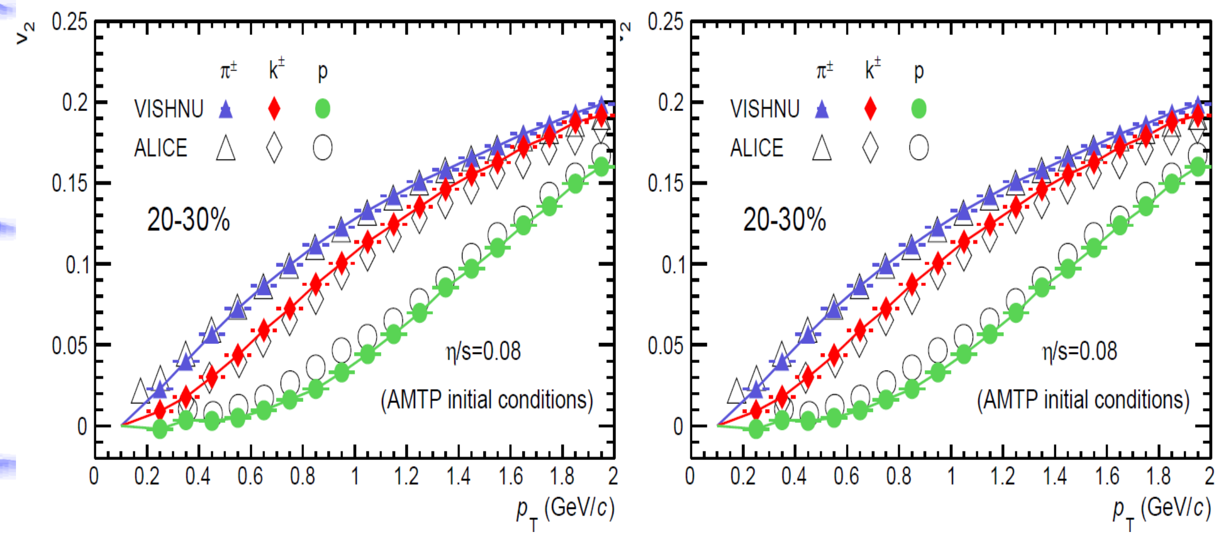




C. Gale, PRL2013



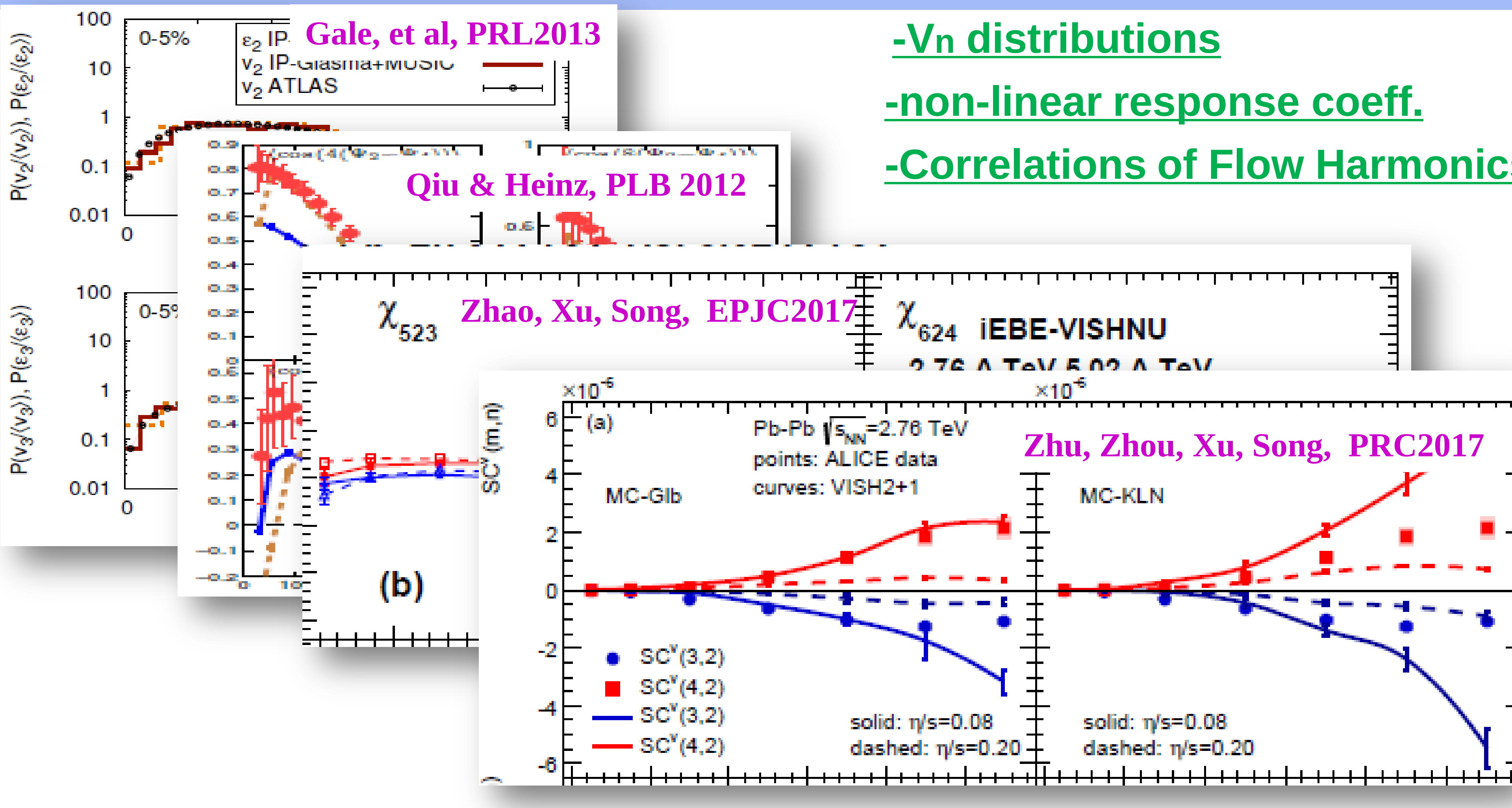
Xu, Li & Song, PRC 2016



$$E\frac{dN}{d^3p}=\frac{dN}{dyp_Tdp_Td\varphi}=\frac{1}{2\pi}\frac{dN}{dyp_Tdp_T}[1+2v_1(p_T,b)\cos(\varphi)+2v_2(p_T,b)\cos(2\varphi)+2v_3(p_T,b)\cos(3\varphi)......]$$

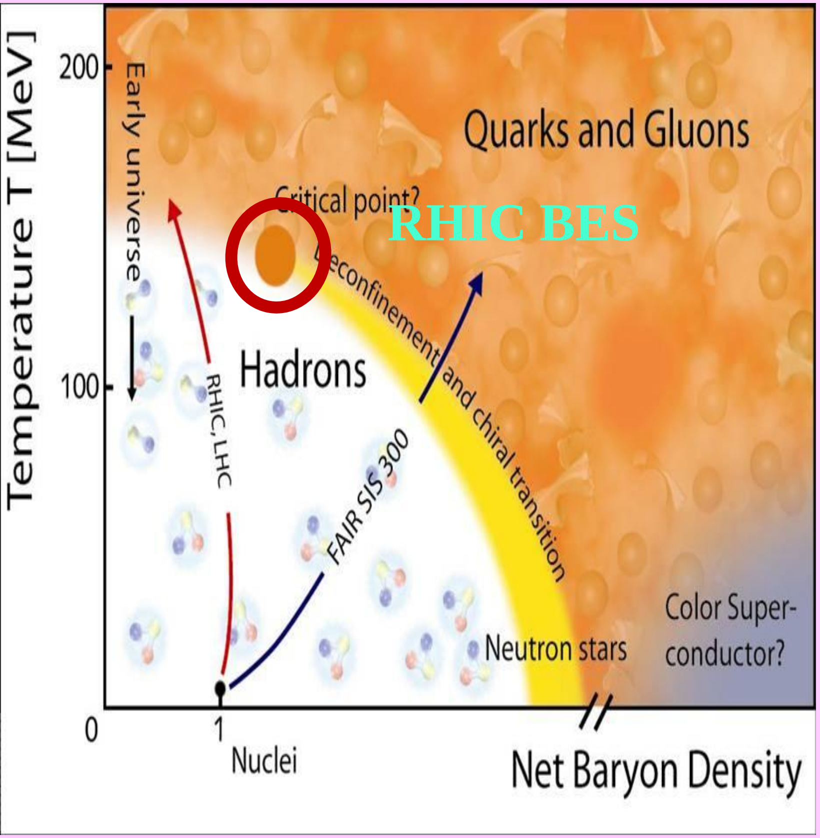
More observables for initial state fluctuations

- **-V_n distributions**
- **-non-linear response coeff.**
- **-Correlations of Flow Harmonics**



-Various flow data reflect the information of initial state fluctuations, some of them provide strong constraint for initial conditions

Correlated fluctuations near the QCD critical point



<u>Initial State Fluctuations</u>

-QGP fireball evolutions smear-out the initial fluctuations

-uncorrelated (in general)

<u>Fluctuations near the critical point</u>

-dramatically increase near Tc

-Strongly correlated

-Static vs dynamical critical fluct.

Static (equilibrium) critical fluctuations

Theoretical predictions on critical fluctuations

$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\}, \qquad \Omega = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \cdots\right]$$

$$\langle\sigma_0^2\rangle = \frac{T}{V}\xi^2 \qquad \langle\sigma_0^3\rangle = \frac{2\lambda_3 T}{V}\xi^6; \qquad \langle\sigma_0^4\rangle_c = \frac{6T}{V}[2(\lambda_3\xi)^2 - \lambda_4]\xi^8.$$

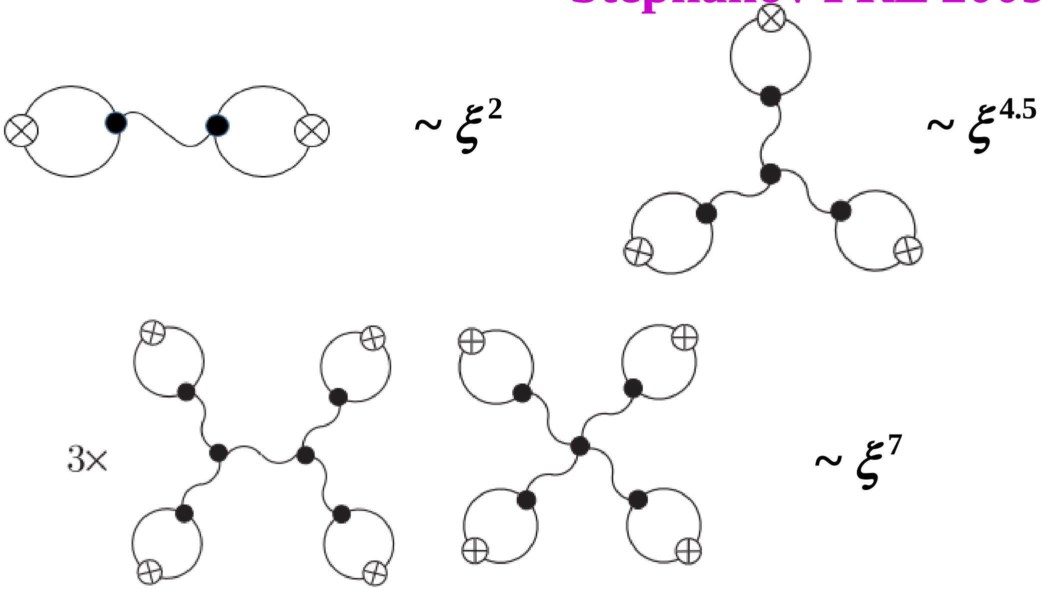
Stephanov PRL 2009

Critical Fluctuations of particles :

$$\langle(\delta N)^2\rangle \sim \xi^2$$

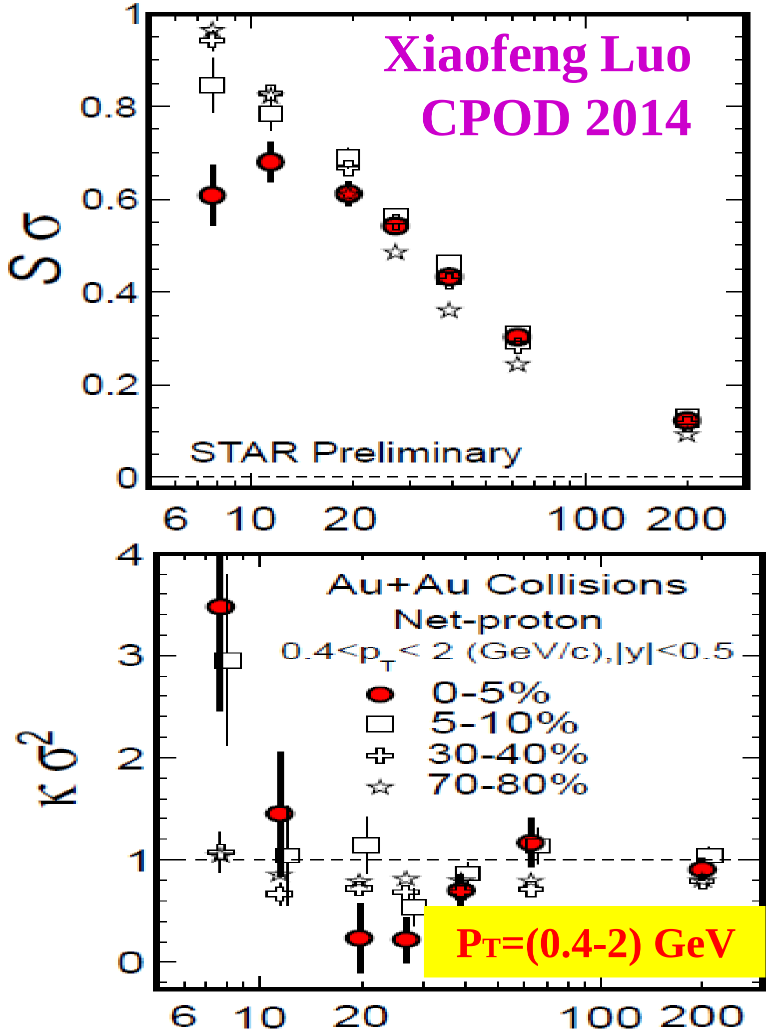
$$\langle(\delta N)^3\rangle \sim \xi^{4.5}$$

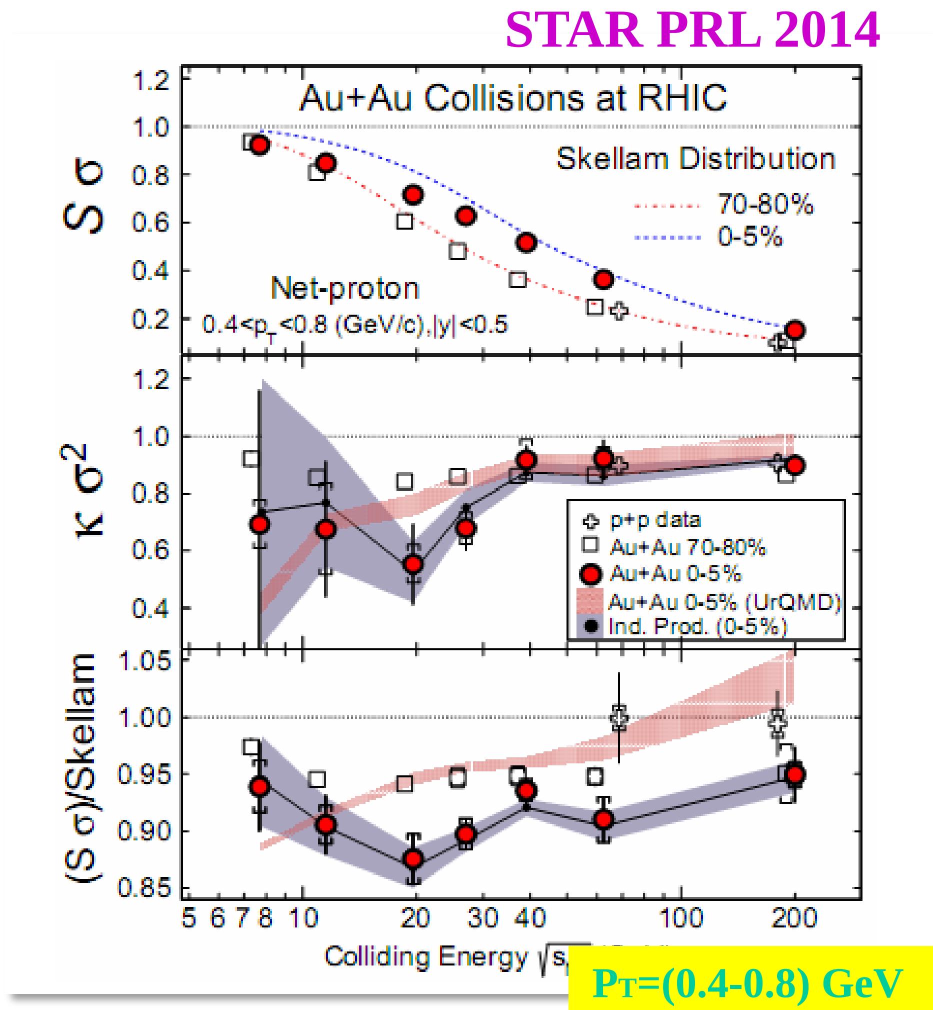
$$\langle(\delta N)^4\rangle \sim \xi^7$$



Higher cummulants (ratios) of net protons are sensitive observables to probe the QCD critical point

STAR BES: Cumulant ratios

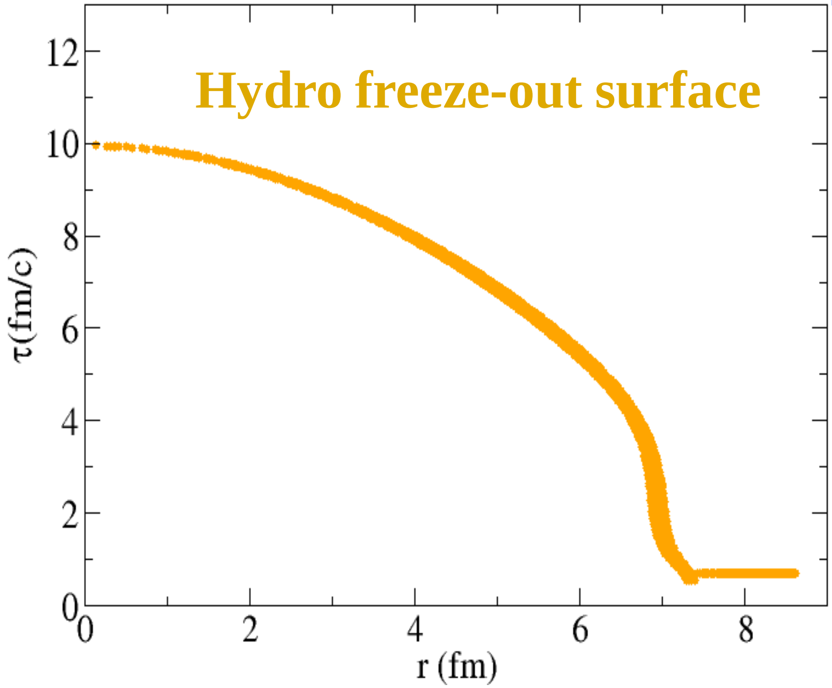




-Non-monotonic behavior, large deviation from the Poisson baseline

How to systematically describe the collision energy, centrality and acceptance cut dependence ?

Freeze-out Scheme near the Critical Points



Jiang, Li & Song, PRC 2016

Particle emissions in traditional hydro

$$E\frac{dN}{d^3p}=\int_\Sigma \frac{p_\mu d\sigma^\mu}{2\pi^3} f(x,p)$$

Particle emissions near T_{cr}

$$M \longrightarrow g\sigma(x)$$

$$f(x,p)=f_0(x,p)[1-g\sigma(x)/(\gamma T)]$$
$$= f_0+\delta f$$

$$\langle \delta f_1 \delta f_2\rangle_\sigma = f_{01}f_{02}f_{03}\left(\frac{g^2}{\gamma_1\gamma_2}\frac{1}{T^3}\right)\langle\sigma_1\sigma_2\rangle_c\,,$$

$$\langle \delta f_1 \delta f_2 \delta f_3\rangle_\sigma = f_{01}f_{02}f_{03}\left(-\frac{g^3}{\gamma_1\gamma_2\gamma_3}\frac{1}{T^3}\right)\langle\sigma_1\sigma_2\sigma_3\rangle_c\,,$$

$$\langle \delta f_1 \delta f_2 \delta f_3 \delta f_4\rangle_\sigma = f_{01}f_{02}f_{03}f_{04}\left(\frac{g^4}{\gamma_1\gamma_2\gamma_3\gamma_4}\frac{1}{T^4}\right)\langle\sigma_1\sigma_2\sigma_3\sigma_4\rangle_c\,.$$

CORRELATED particle emissions along the freeze-out surface

$$\left\langle (\delta N)^2 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^2 \left(\prod_{i=1,2} \left(\frac{1}{E_i} \int d^3p_i \int_{\Sigma_i} p_{i\mu} d\sigma_i^\mu d\eta_i \right) \right) \frac{f_{01} f_{02}}{\gamma_1 \gamma_2} \frac{g^2}{T^2} \langle \sigma_1 \sigma_2 \rangle_c \,,$$

$$\left\langle (\delta N)^3 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^3 \left(\prod_{i=1,2,3} \left(\frac{1}{E_i} \int d^3p_i \int_{\Sigma_i} p_{i\mu} d\sigma_i^\mu d\eta_i \right) \right) \frac{f_{01} f_{02} f_{03}}{\gamma_1 \gamma_2 \gamma_3} \left(-\frac{g^3}{T^3} \langle \sigma_1 \sigma_2 \sigma_3 \rangle_c \right) ,$$

$$\left\langle (\delta N)^4 \right\rangle_c = \left(\frac{g_i}{(2\pi)^3}\right)^4 \left(\prod_{i=1,2,3,4} \left(\frac{1}{E_i} \int d^3p_i \int_{\Sigma_i} p_{i\mu} d\sigma_i^\mu d\eta_i \right) \right) \frac{f_{01} f_{02} f_{03} f_{04}}{\gamma_1 \gamma_2 \gamma_3 \gamma_4} \frac{g^4}{T^4} \langle \sigma_1 \sigma_2 \sigma_3 \sigma_4 \rangle_c$$

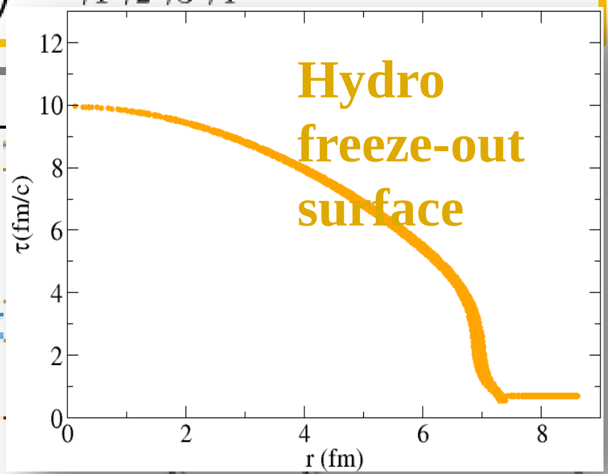
$$P[\sigma] \sim \exp\{-\Omega[\sigma]/T\}, \qquad \Omega[\sigma] = \int d^3x \left[\frac{1}{2} (\nabla\sigma)^2 + \ldots \right.$$

$$\langle \sigma_1 \sigma_2 \rangle_c = T D(x_1 - x_2),$$

$$\langle \sigma_1 \sigma_2 \sigma_3 \rangle_c = -2T^2 \lambda_3 \int d^3z D(x_1 - z) D(x_2 - z) D(x_3 - z \ldots$$

$$\langle \sigma_1 \sigma_2 \sigma_3 \sigma_4 \rangle_c = -6T^3 \lambda_4 \int d^3z D(x_1 - z) D(x_2 - z) D(x_3 \ldots$$

$$+12T^3 \lambda_3^2 \int d^3u \int d^3v D(x_1 - u) D(x_2 - u) D(x_3 - v) D(x_4 - v) D(u - v).$$

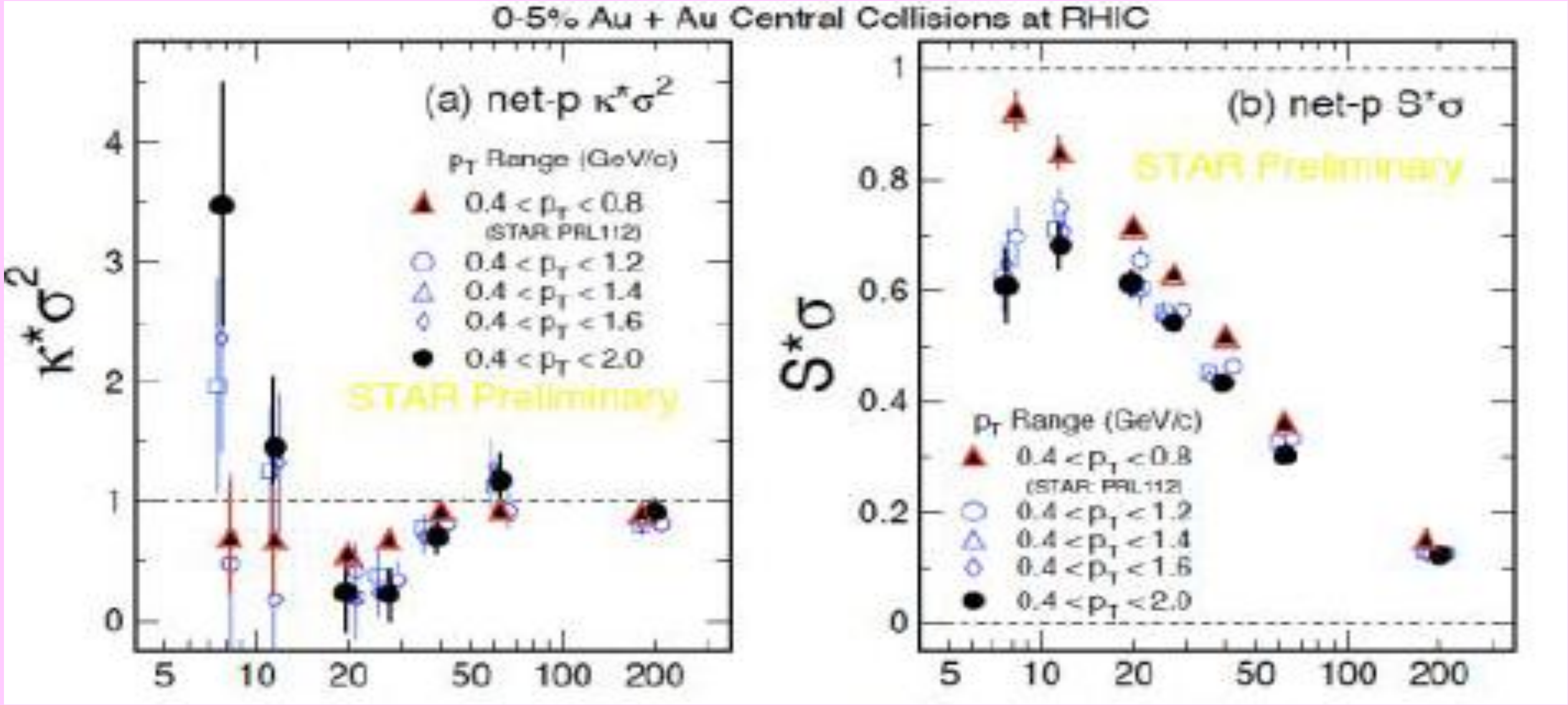


-- Static critical fluctuations along the freeze-out surface

Note: for static & infinite medium, the results in Stephanov PRL09 can be reproduced

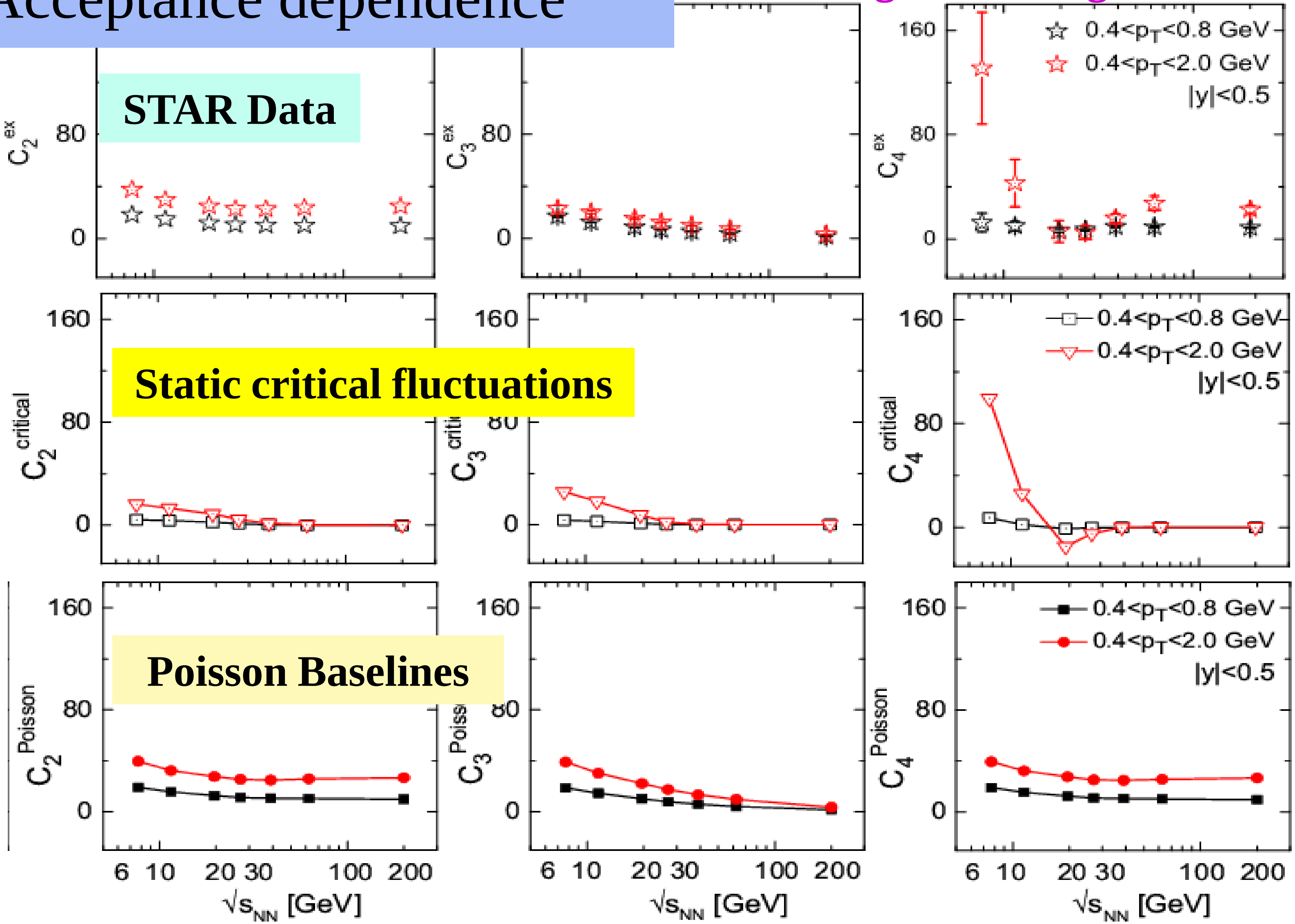
Static (equil.) critical fluctuations
-comparison with the exp.data

-Acceptance dependence
-Cumulants & cummulant ratios



Acceptance dependence

Jiang, Li & Song, PRC2016

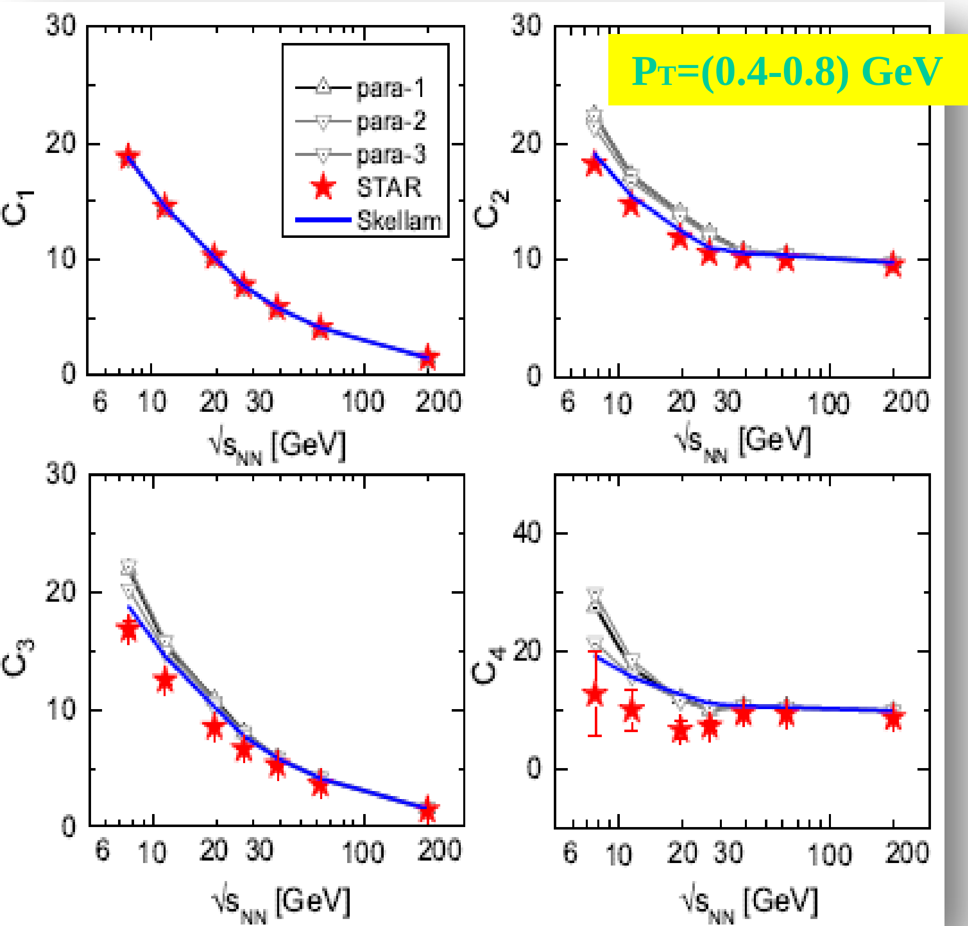


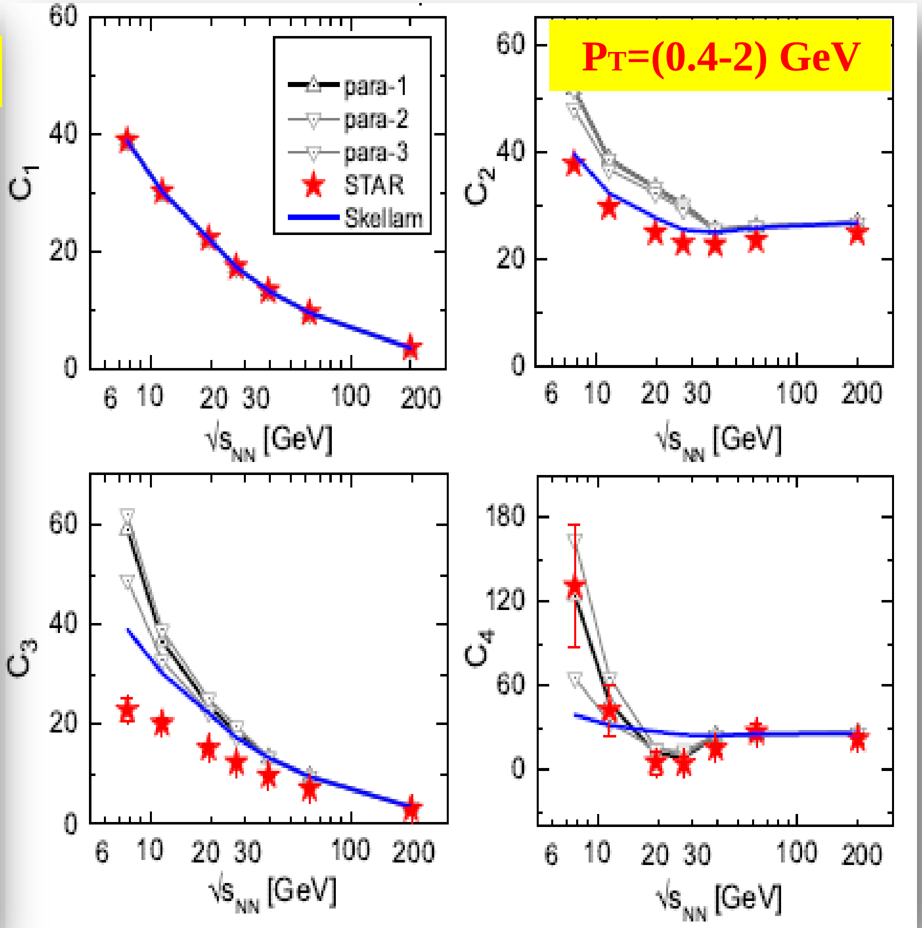
-Static critical fluctuations can qualitatively explain the acceptance dependence of the STAR data

Cumulants of net protons

Net Protons 0-5%

Jiang, Li & Song, PRC2016





Static critical fluctuations give positive contribution to C_2 , C_3; well above the poisson baselines, can NOT explain/describe the C_2 , C_3 data

Dynamical (non-equilibrium) Critical Fluctuations

-Static (equilibrium) universality class

-QCD matter, 3-D Ising model, gas-liquid

-Dynamical universality class

-whether or not the order parameter is conserved
-other conserved quantities in the system

-Model A: non-conserved order parameter
-Model B: conserved order parameter;
-Model H: conserved energy and baryons density,
non-conserved order parameter

… … … ….

-Other dynamical model: chiral hydrodynamics, hydro+

Effects from dynamical evolutions

-Model A

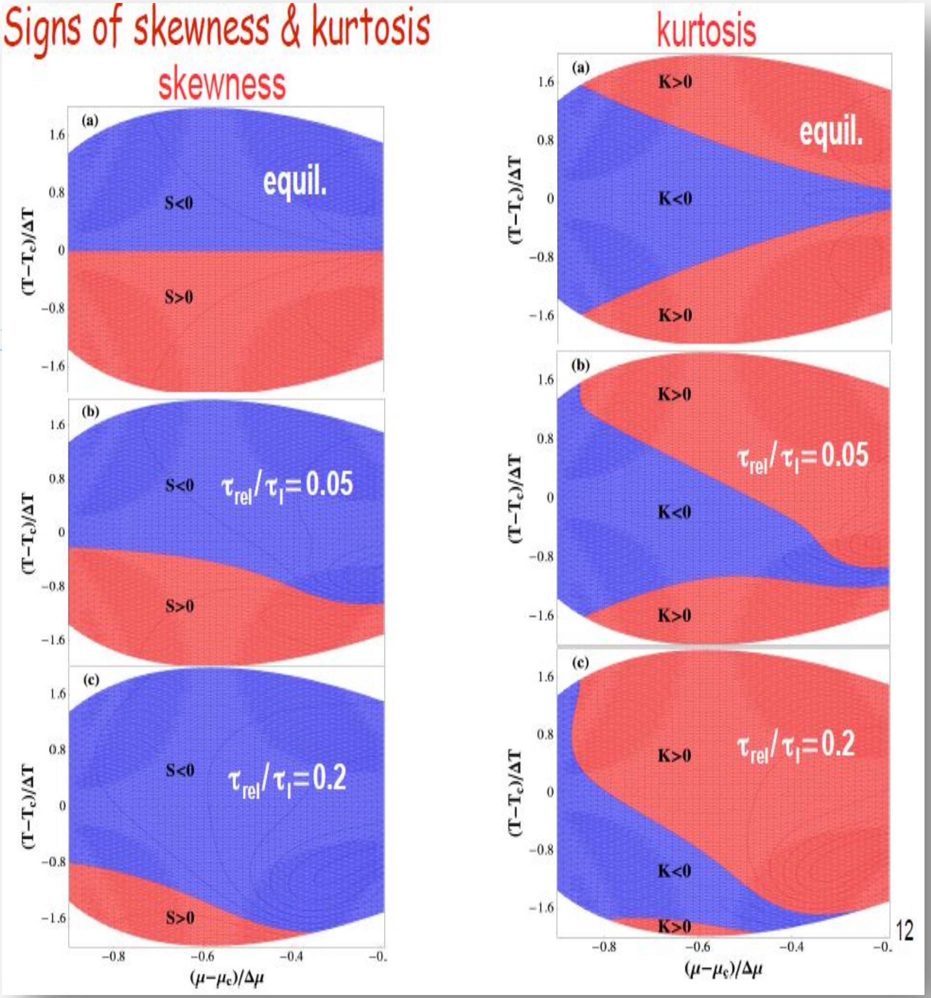
$$\partial_\tau P(\sigma;\tau) = \frac{1}{m_\sigma^2 \tau_{eff}} \left[\partial_\sigma \left[\partial_\sigma \Omega_0(\sigma) + V_4^{-1} \partial_\sigma\right] P(\sigma;\tau)\right]$$

near-equilibrium limit: $\delta\,\kappa_n = \kappa_n - \kappa_n^{eq}$

$$\partial_\tau \kappa_2 = -2\,\tau_{eff}^{-1}\, a_2\,\delta\,\kappa_2$$

$$\partial_\tau \kappa_3 = -3\,\tau_{eff}^{-1}\left[a_2\,\delta\,\kappa_2 + a_3\,\delta\,\kappa_3\right]$$

$$\partial_\tau \kappa_4 = -4\,\tau_{eff}^{-1}\left[a_2\,\delta\,\kappa_2 + a_3\,\delta\,\kappa_3 + a_4\,\delta\,\kappa\right.$$



S. Mukherjee, R. Venugopalan, Y. Yin, PRC92 (2015)

sign of non-Gaussian cumulants can be different from equilibrium one

Dynamical critical fluctuations of the sigma field

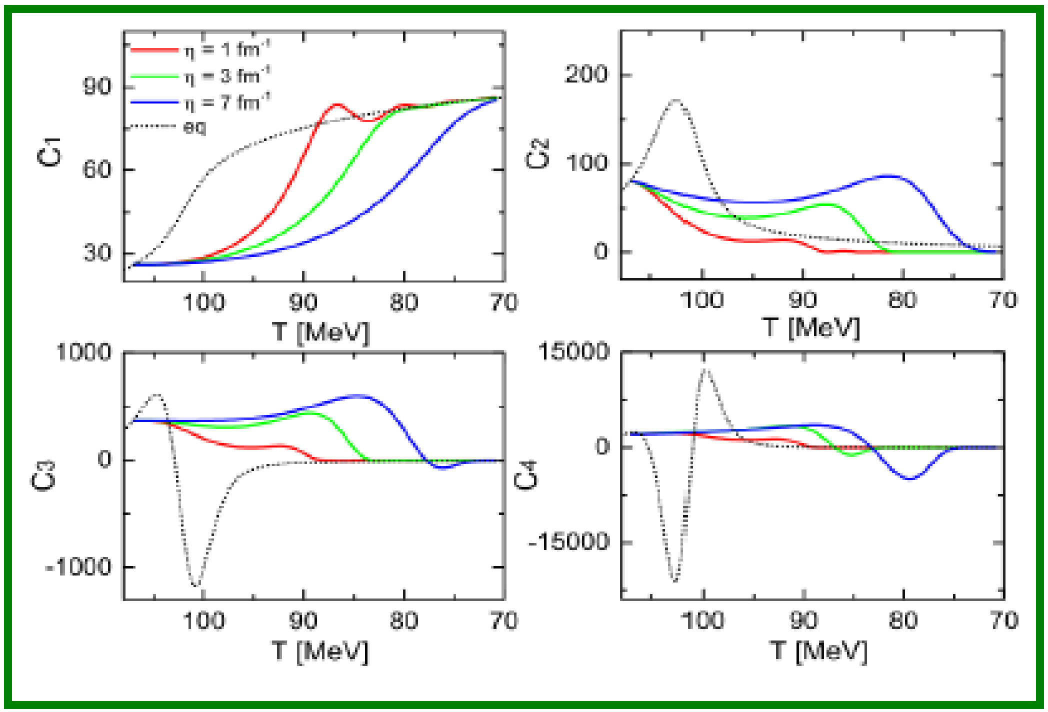
Langevin dynamics: $\partial^{\mu}\partial_{\mu}\sigma(t,x)+\eta\partial_t\sigma(t,x)+V'_{eff}(\sigma)=\xi(t,x)$ **-Model A**

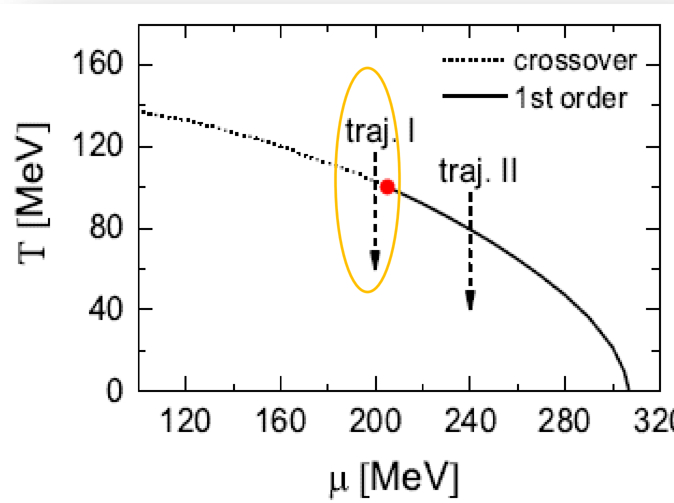
with effective potential from linear sigma model with constituent quarks

$$V_{eff}(\sigma)=U(\sigma)+\Omega_{\bar{q}q}(T,\sigma)=\frac{\lambda^2}{4}\left(\sigma^2-\nu^2\right)^2-h_q\sigma-U_0-2d_qT\int\frac{d^3p}{(2\pi)^3}\ln\left(1+\exp\left(-\frac{E}{T}\right)\right)$$

Jiang, Wu, Song, NPA 2017, paper in preparation

On the crossover side





-The signs of C_3 & C_4 are different from the equil. ones due to memory effects

-in the near future: maping with 3D Ising model; extend to model B; dynamical universal behavior

Dynamical critical fluctuations of the sigma field

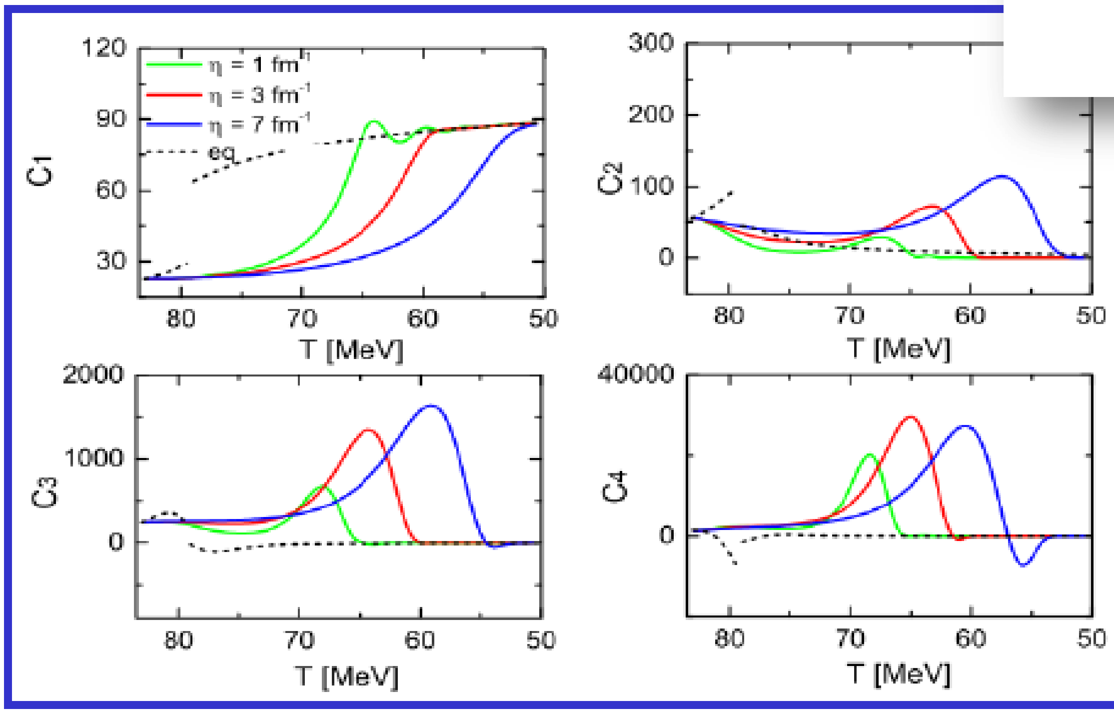
Langevin dynamics: $\partial^\mu\partial_\mu\sigma(t,x)+\eta\partial_t\sigma(t,x)$ ... odel A

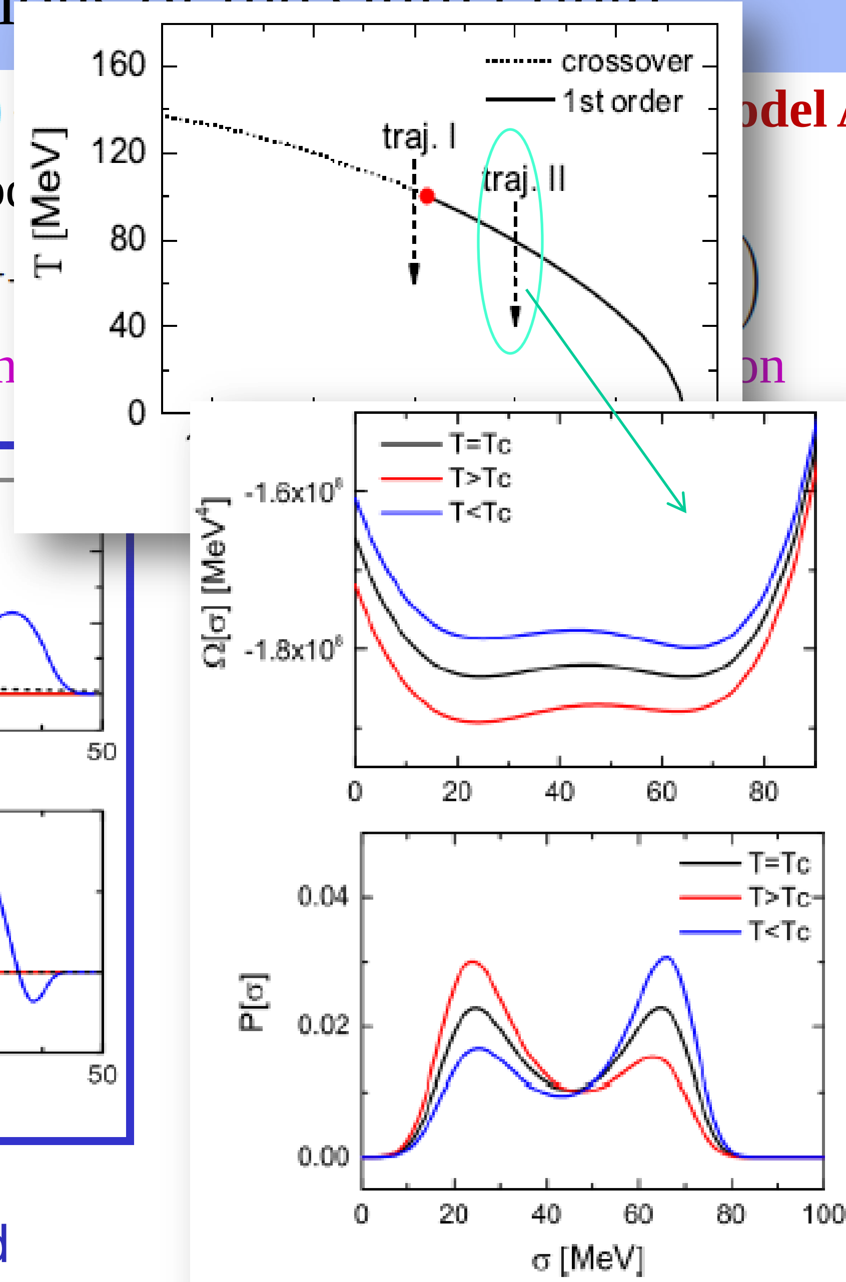
with effective potential from linear sigma mod...

$$V_{eff}(\sigma)=U(\sigma)+\Omega_{\bar{q}q}(T,\sigma)=\frac{\lambda^2}{4}\left(\sigma^2-\nu^2\right)^2-h_q\sigma \ldots$$

Jiang, Wu, Son... ...on

On the 1st order side





-Super cooling & bubble formations:

C_3 & C_4 are largely enhanced compared with the equil. ones

Non-Critical (Thermal) Fluctuations

- **Detection and analysis technology**

The efficiency corrections and acceptance of the detector

Bzdak A, Holzmann R, Koch V. arXiv preprint arXiv:1603.09057, 2016…

Bin width effect and centrality dependence

McDonald D, STAR Collaboration. Nuclear Physics A, 2013, 904: 907c-910c…

Auto-correlation effects(ACE)

Luo X, Xu J, Mohanty B, et al. JPG, 2013, 40(10): 105104…

Acceptance dependence of fluctuation

Ling B, Stephanov M A. arXiv preprint arXiv:1512.09125, 2015; Bzdak A, Koch V. Phys. Rev. C, 2012, 86(4): 044904;
Masayuki Asakawa and Masakiyo Kitazawa. arXiV:1512.0038…

- **physical effect**

Conservations law for charges and baryons

Bzdak A, Koch V, Skokov V. PRC, 2013, 87(1): 014901…

Volume fluctuations

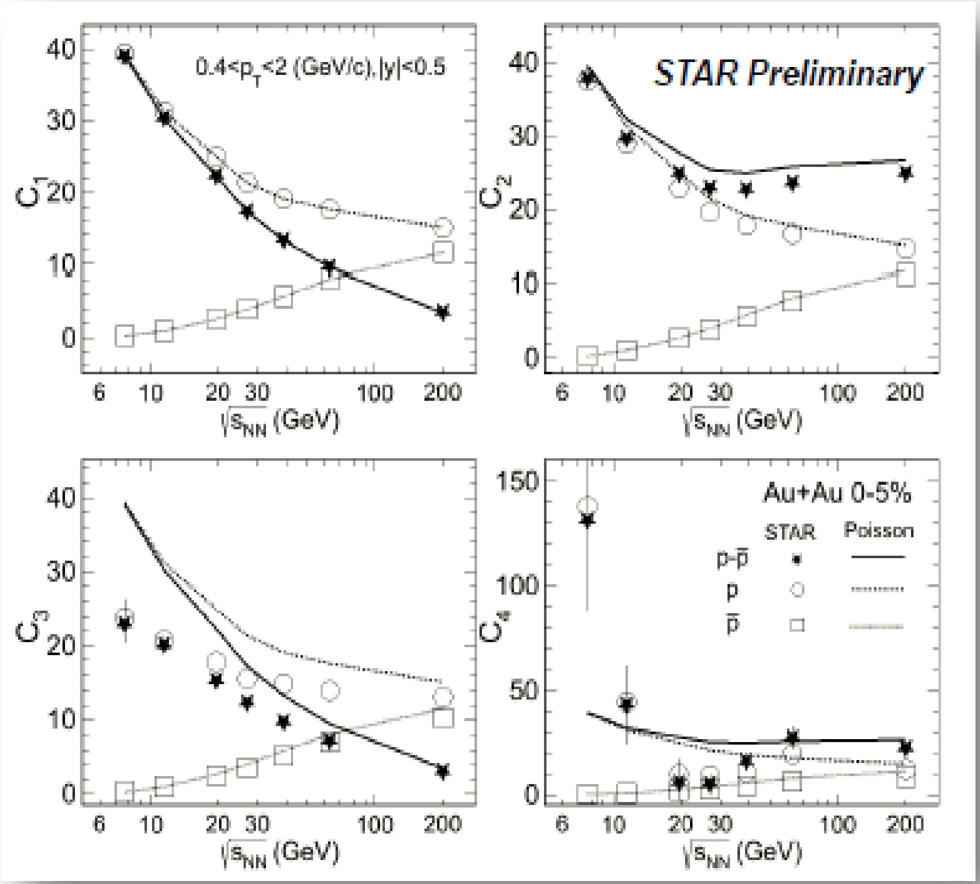
Xu H..arXiv:1602.07089, 2016; Xu H. arXiv:1602.06378, 2016; S. Jeon, hep-ph/0304012; M. I. Gorenstein, Phys.Rev. C 78, 041902;
V. Skokov, Phys.Rev. C 88, 034911…

Hadronic evolution & rescattering

X.Luo,J. Xu, B. Mohanty,and N. Xu, J.P.G 40,105104(2013); Xu, Ji; Yu, Shili; Liu, Feng; Luo, Xiaofeng arXiv:1606.03900 …

Resonance decay

Garg P, Mishra D K, et al. Phys. Lett. B, 2013, 726(4): 691-696; Andronic A, Braun-Munzinger P, Stachel J. Nucl. Phys. A, 2006, 772(3): 167-199; Andronic A, Braun-Munzinger P. Phys. Lett. B, 2009, 673(2): 142-145;
Cleymans J, Kämpfer B, Kaneta M, et al.. Phys. Rev. C, 2005, 71(5): 054901…



-In experiment, Poisson fluctuations are generally served as the thermal fluctuation baselines, especially for the multiplicity fluct. of (anti)protons

-Where does the Poisson baselines come from?

-How various factors influence / destroy Poisson distributions

(volume fluctuations, hadronic evolution, resonance decays … …)

Hadron Resonance Gas Model

-Grand canonical ensemble(GCE)

$$\ln Z(T,\mu_B,\mu_Q,\mu_S) = \sum_{i\in \text{mesons}} \ln Z_i^{+}(T,\mu_Q,\mu_S) + \sum_{i\in \text{baryons}} \ln Z_i^{-}(T,\mu_B,\mu_Q,\mu_S)$$

-With Boltzmann approximation

$$\ln Z_i^{\pm}(T,V,\vec{\mu}) = \frac{VTg_i m_i^2}{2\pi^2} K_2(m_i/T)\exp(\vec{\mu}/T)$$

-The susceptibilities

$$\chi_q^{(n)}(T,\mu_B,\mu_S,\mu_Q) = \left.\frac{\partial^n (P/T^4)}{\partial(\mu_q/T)^n}\right|_T \qquad P/T^4 = \lim_{V\to\infty}\frac{1}{VT^3}\ln Z(T,V,\vec{\mu})$$

$$\Rightarrow \chi_q^{(n)}(T,\mu_B,\mu_S,\mu_Q) = \chi_q^{(1)}(T,\mu_B,\mu_S,\mu_Q)$$

$$\Rightarrow C_{n,q} = C_{1,q} = \text{Mean}_q$$

Poisson Baselines!

Garg P, Mishra D K, Netrakanti P K, et al. Phys. Lett. B, 2013, 726(4): 691-696.

Improved Hadron Resonance Gas Model

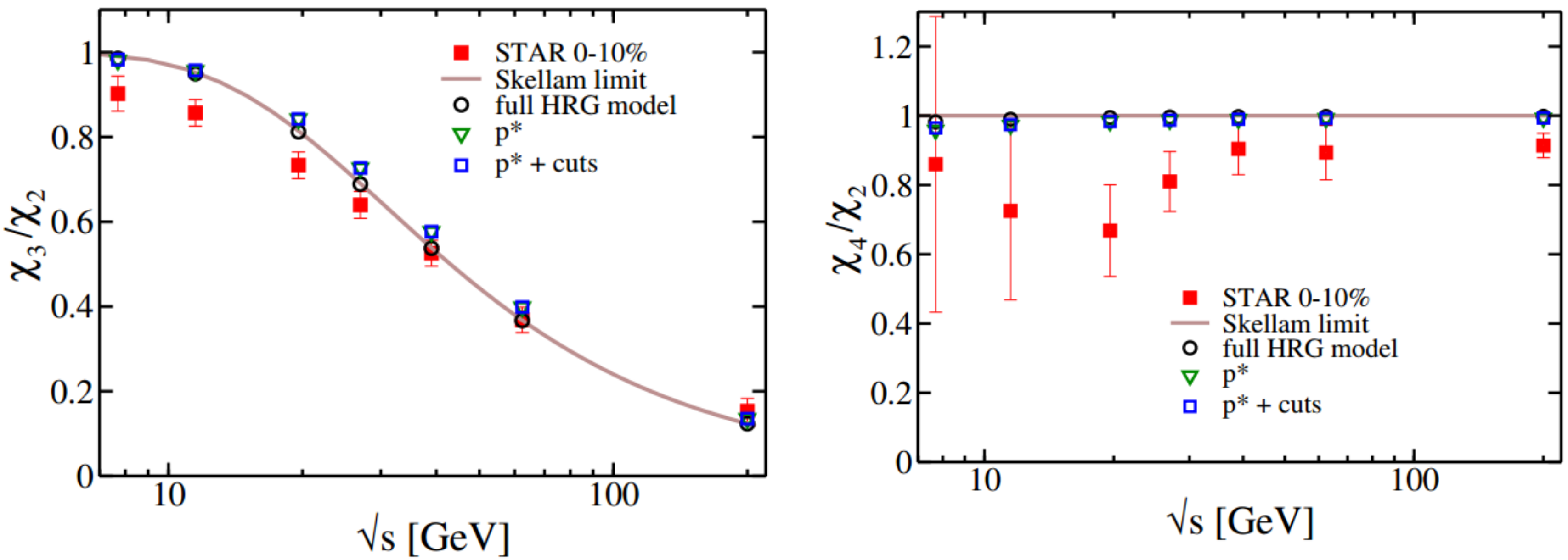
[Nahrgang, et al. EPJC75 (2015) no.12, 573]

-Acceptance cut:

$$n_k(T,\mu_k) = \frac{d_k}{4\pi^2}\int_{-y_{\rm MAX}}^{y_{\rm MAX}} {\rm d}y \int_{p_T^{\rm MIN}}^{p_T^{\rm MAX}} {\rm d}p_T \frac{p_T\sqrt{p_T^2+m_k^2}\,{\rm Cosh}[y]}{(-1)^{B_k+1}+\exp(({\rm Cosh}[y]\sqrt{p_T^2+m_k^2}-\mu_k)/T)}$$

-Resonance decays:

$$VT^3\,\frac{\partial(P/T^4)}{\partial(\mu_h/T)}\bigg|_T = \langle N_h\rangle + \sum_R \langle N_R\rangle\langle n_h\rangle_R$$



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-Acceptance cut:

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-Resonance decays:

$$VT^3 \left.\frac{\partial(P/T^4)}{\partial(\mu_h/T)}\right|_T = \langle N_h\rangle + \sum_R \langle N_R\rangle\langle n_h\rangle_R$$

-For static and equilibrium medium

-Not apple to apple comparison with the data

- A realistic heavy ion collision: dynamical evolutions

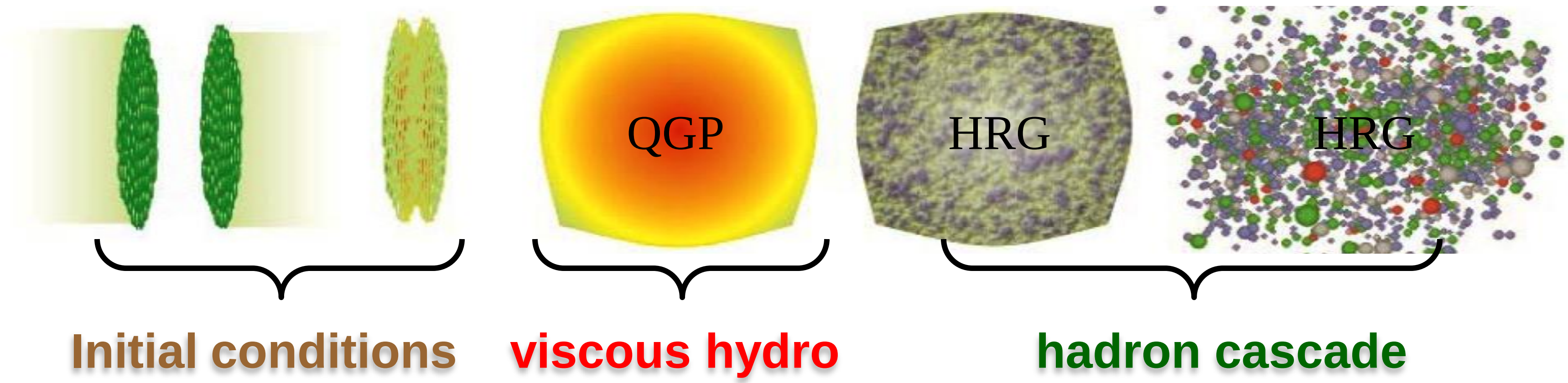
- late hadronic evolution:

Chemical and thermal equilibrium can not be maintained

Multiplicity fluctuations of (net) Charges and (net) protons from iEBE-VISHNU

Li, Xu, Song, 1707.09742

iEBE-VISHNU



Various fluctuations in the hybrid model

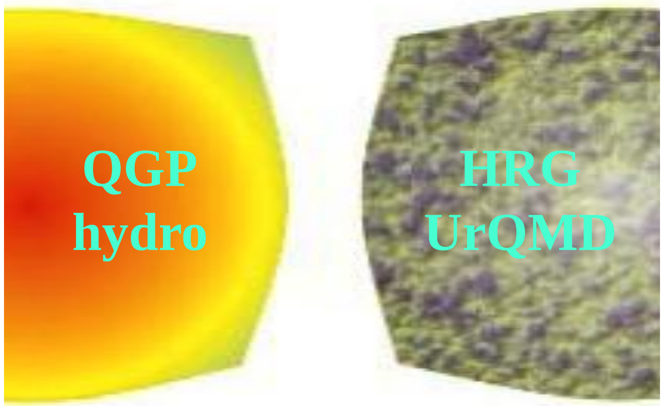
-Initial state fluctuations

-Thermal fluctuations in viscous hydrodynamics

-Thermal fluctuations during the switching between hydro & UrQMD

(statistical hadronization, GCE; → Poisson fluctuations)

-fluctuations from UrQMD hadron cascade



freeze-out/switching

Particle event generator:

$$E\frac{d^3N_i}{d^3p}(x) = \frac{g_i}{(2\pi)^3} p^\mu d^3\sigma_\mu(x) f(x,p),$$

$$f_0 = \frac{1}{\gamma_s^{-|S_i|} e^{(p^\nu \cdot u_\nu - \vec{c}_i \cdot \vec{\mu}_i)/T} \pm 1}$$

Poisson fluctuations:

$$P_i(k) = \frac{\lambda_i^k e^{-\lambda_i}}{k!}, \quad \text{with } \lambda_i = N_i$$

Various fluctuations in the hybrid model

-Initial state fluctuations

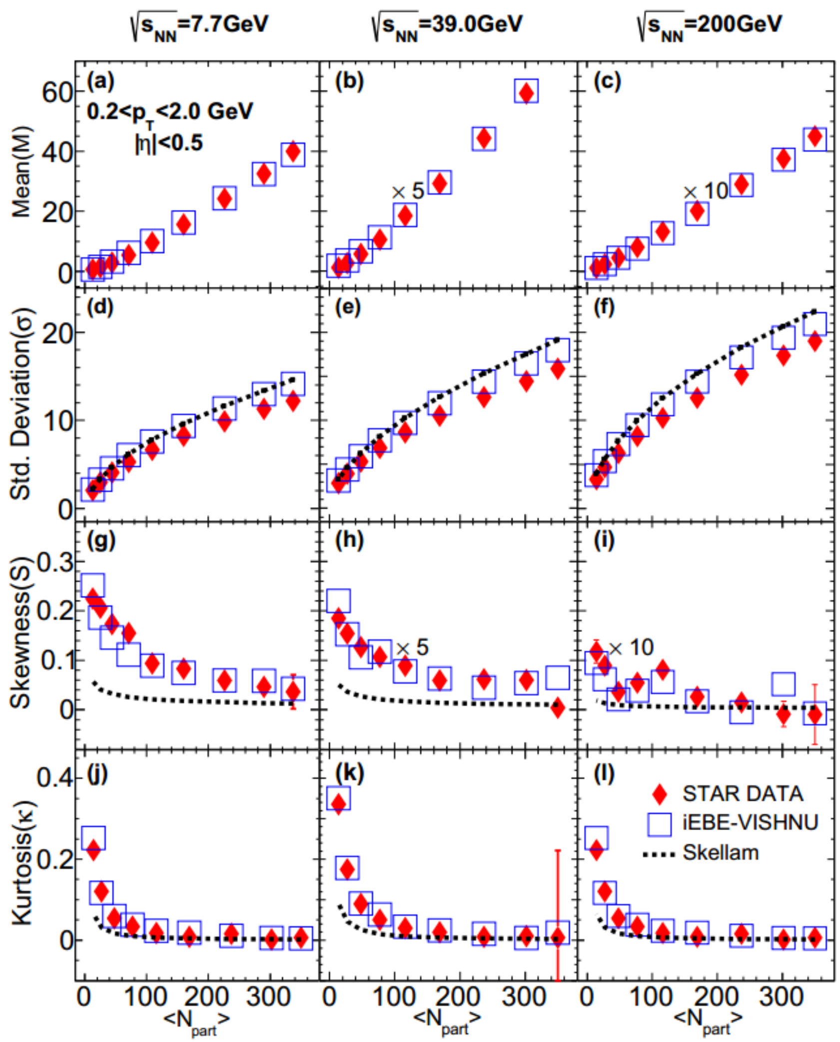
-Thermal fluctuations in viscous hydrodynamics

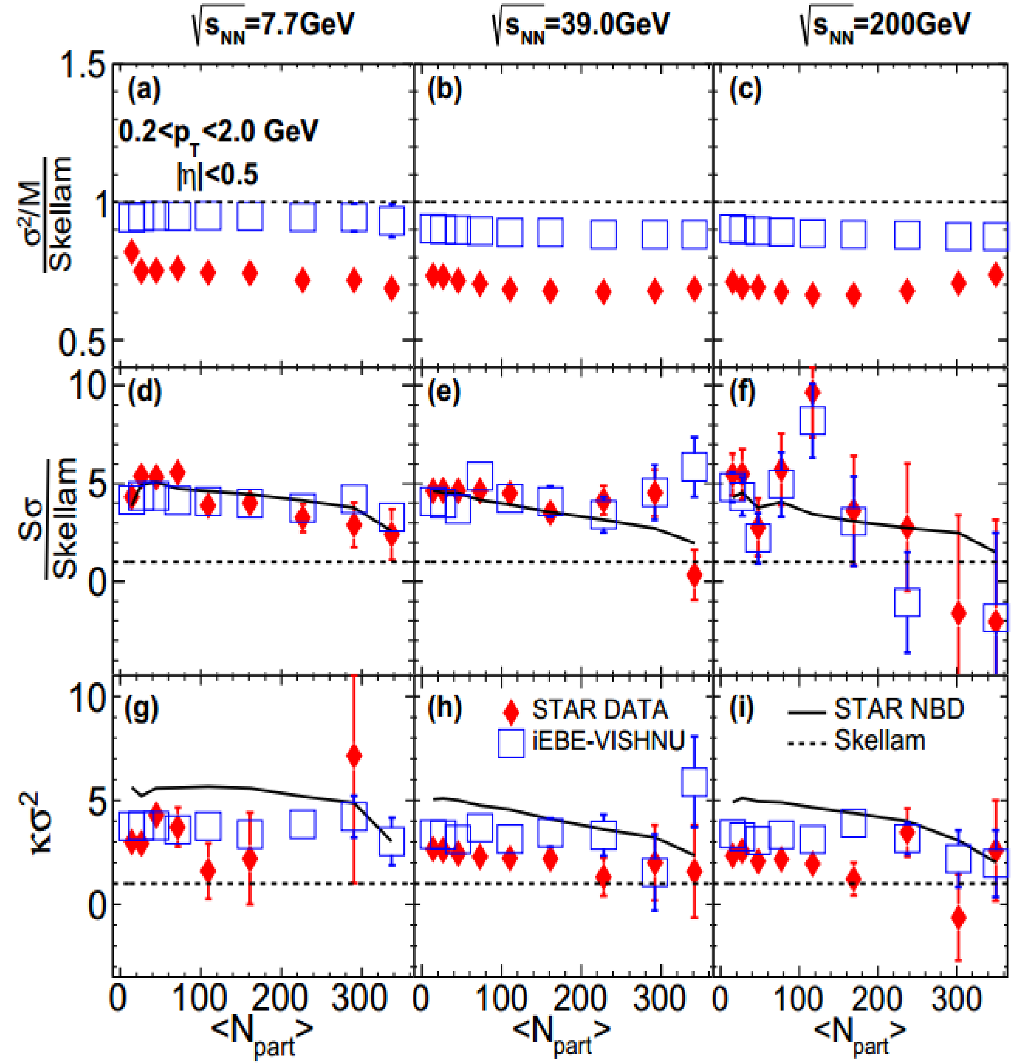
-Thermal fluctuations during the switching between hydro & UrQMD

(statistical hadronization, GCE; → Poisson fluctuations)

-fluctuations from UrQMD hadron cascade

Moments and Moment products of net-charges

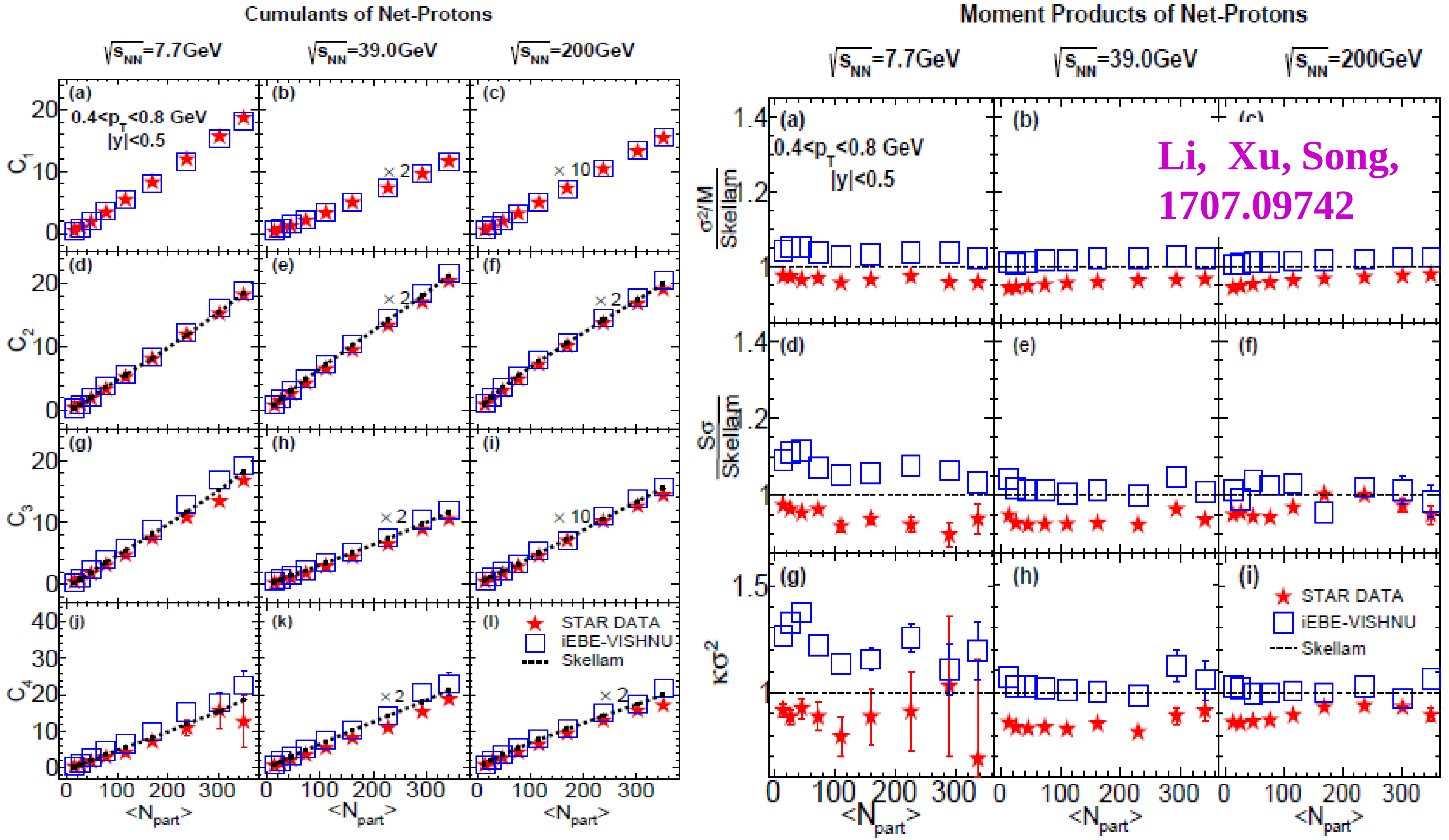




-For net charges, IEBE-VISHNU roughly describes the data of S and κ and the related ratios, shows large deviations from the Poisson baselines.

Li, Xu, Song, 1707.09742

Cumulants and Cumulant ratios of Net-protons



-iEBE-VISHNU: small deviation from the Poisson baselines, roughly describe the data.

-at lower collision energy, larger gap between data and model

--charge conservation, critical fluct. first order phase transition

Multiplicity fluctuations of (net) Charges and (net) protons from iEBE-VISHNU

Li, Xu, Song, 1707.09742

-iEBE-VISHNU roughly describe most of the moments & moments products of (net) charges and (net) protons

- What are dominant factors to influence the multiplicity fluct.

-volume fluctuations?

-resonance decays ?

-hadronic Scatterings ?

… …

What is Volume Fluctuations /Corrections?

1) Single hydro + many many UrQMD

- Poisson fluctuations, no volume fluct/correc

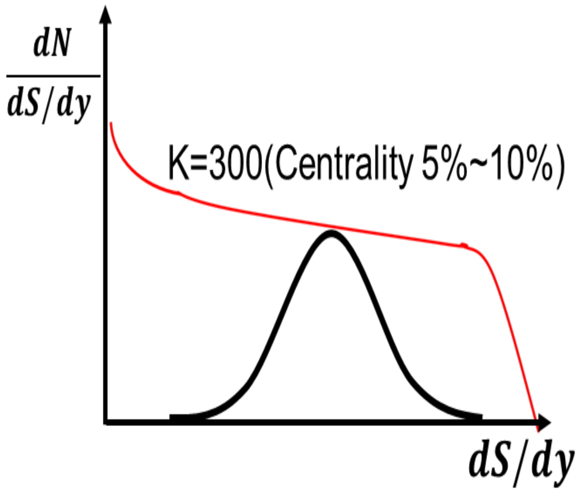
2) In a certain centrality bins:

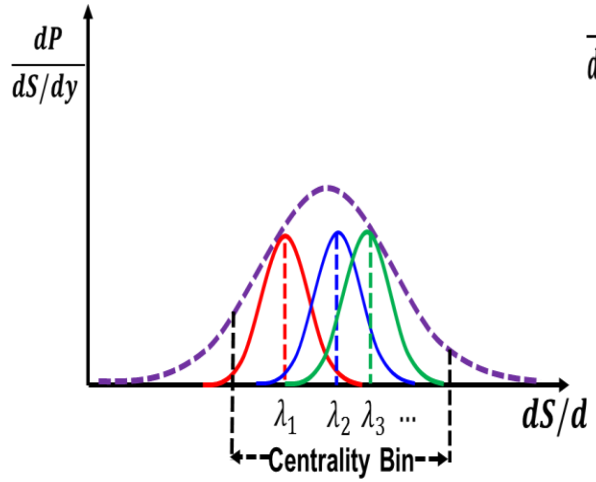
Many (Single hydro + many many UrQMD)

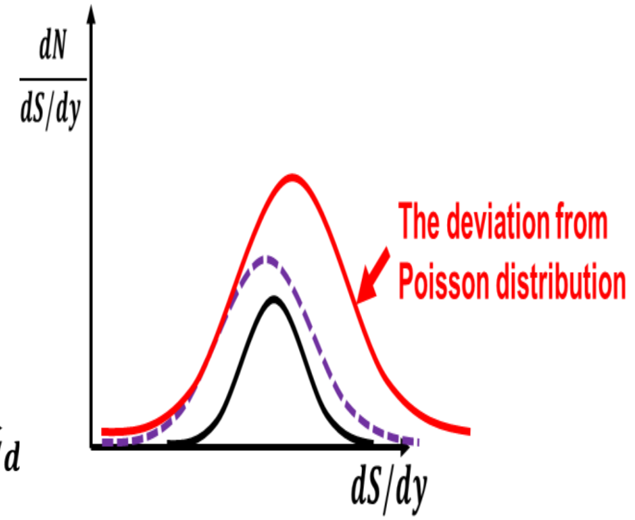
- many Poisson fluctuations are superimposed together

--> volume fluct /correc & wide centrality bin effect

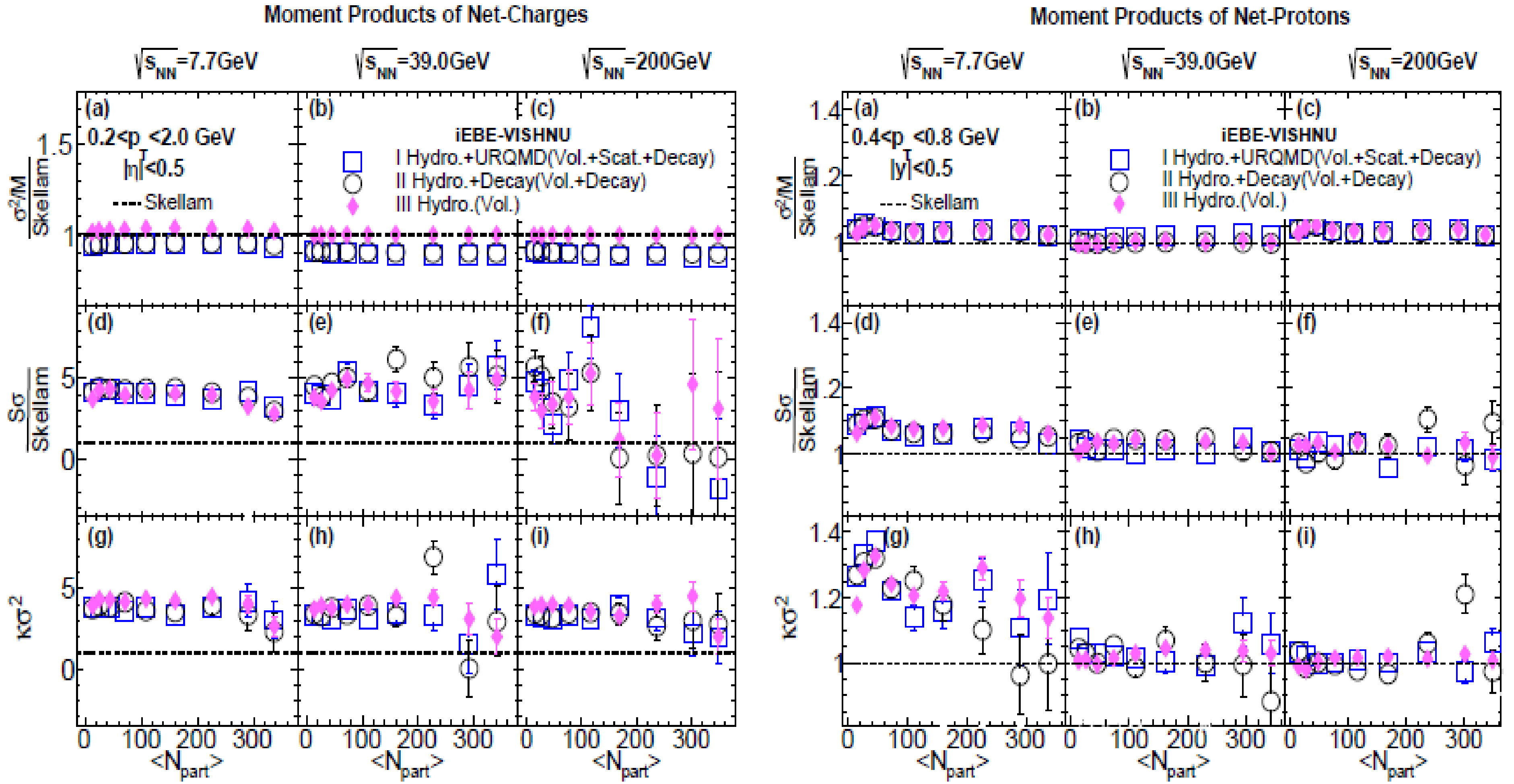
3) In a fine centrality bins: **--> volume fluct /correc**







Volume corrections, resonance decays & hadronic evolution



-The effects of hadronic scatterings and resonance decays are very small

-Volume fluctuations plays the dominant role for multiplicity fluctuations

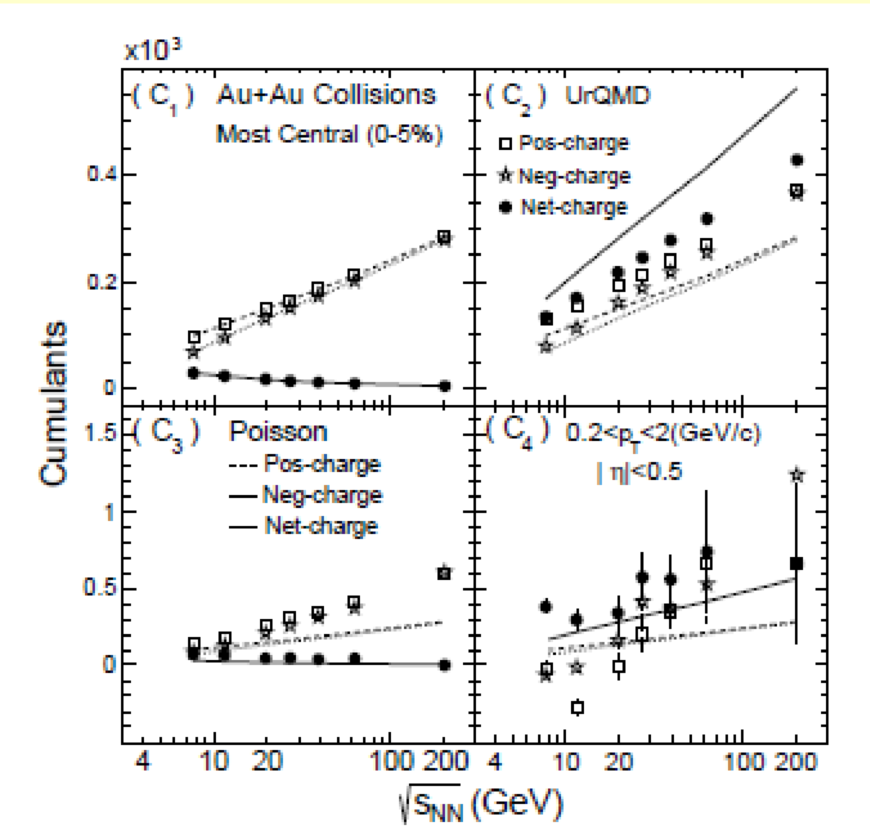
-For net protons, the effects of volume fluctuations are relatively small

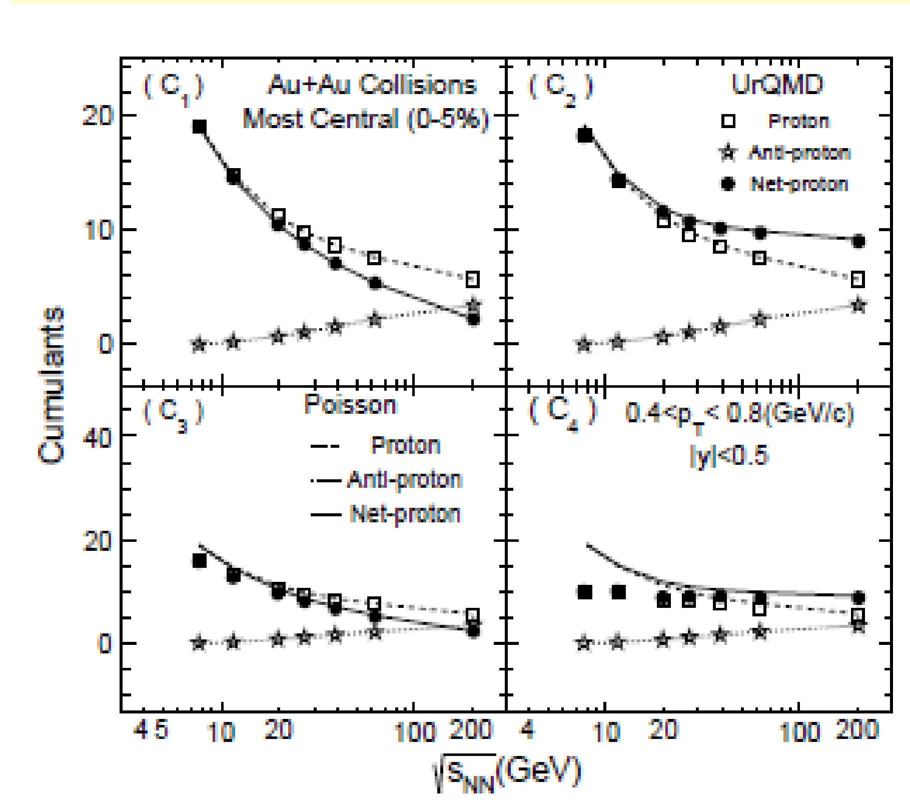
–>close to Poisson fluctuations

Li, Xu, Song, 1707.09742

Non-Critical fluctuations

-results from UrQMD



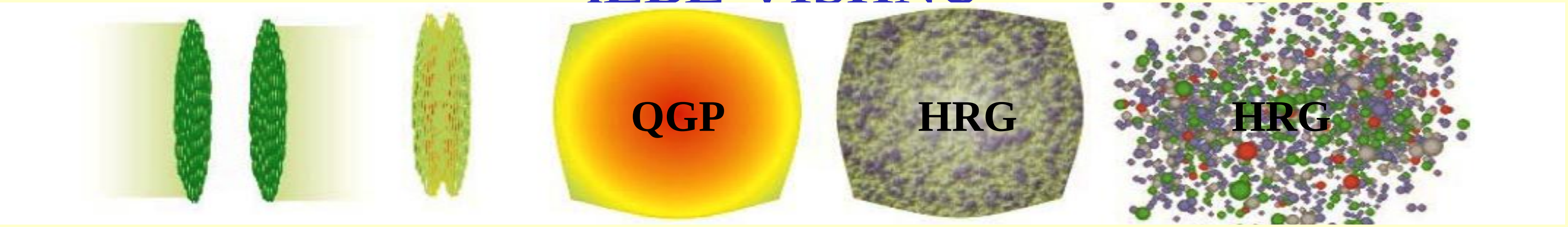


J. Xu, S. Yu, F. Liu and X. Luo, Phys. Rev. C 94, no. 2, 024901 (2016); S. He and X. Luo, arXiv:1704.00423 [nucl-ex].Z. Yang, X. Luo and B. Mohanty, Phys. Rev. C 95, no. 1, 014914 (2017)

Non-Critical fluctuations

---iEBE-VISHNU vs. UrQMD

iEBE-VISHNU



UrQMD

Non-Critical fluctuations

---iEBE-VISHNU vs. UrQMD

iEBE-VISHNU

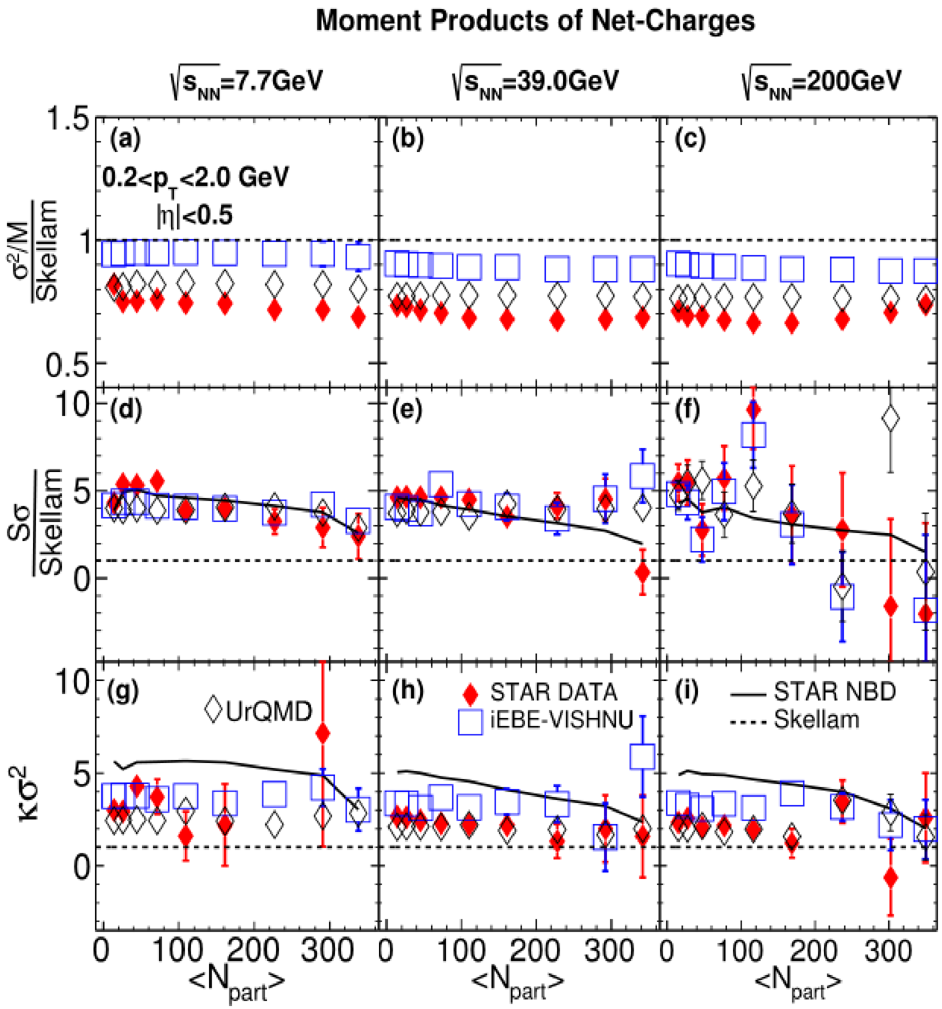
<u>Initialization of UrQMD:</u> statistical hadronization, independent particle production, Poisson fluctuations

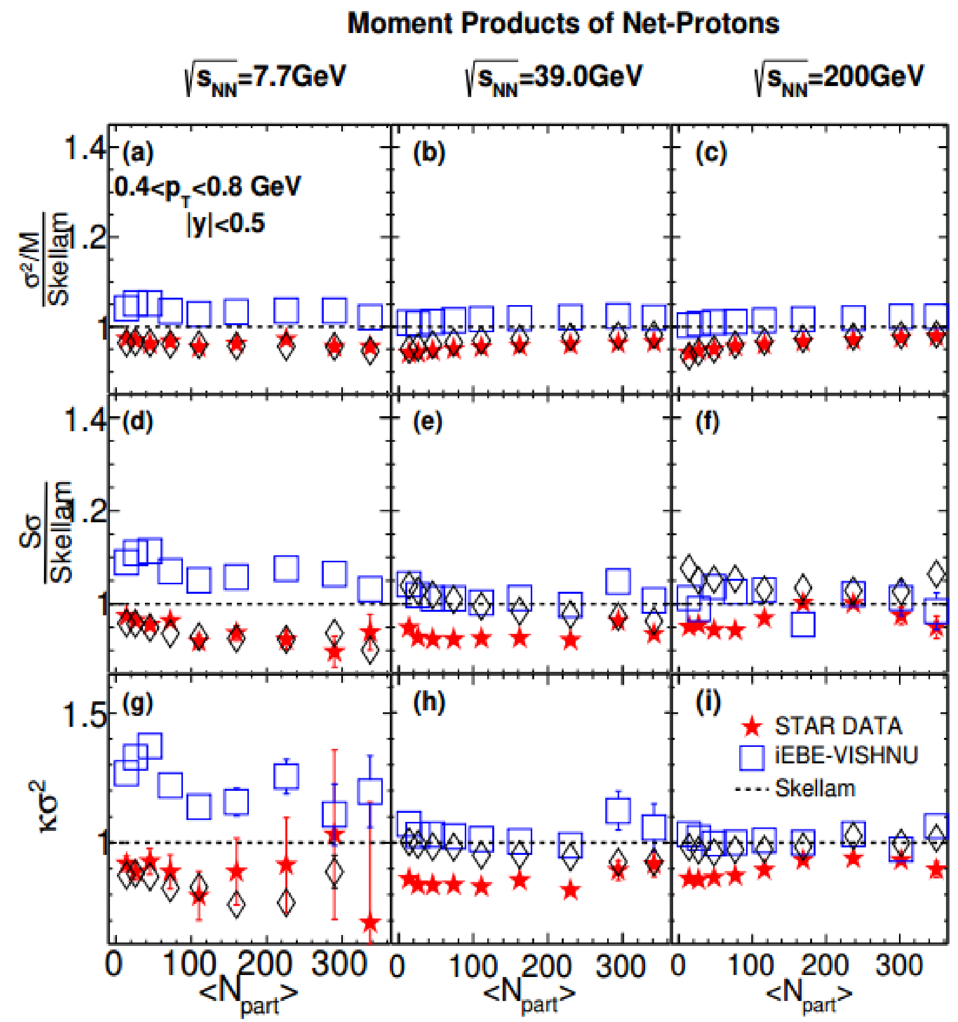
<u>UrQMD</u>

<u>Initialization of UrQMD: projectile & target nuclei</u>, charge conservation

iEBE-VISHNU vs UrQMD

-Li, Luo, Xu & Song, unpublished notes





-Volume fluctuations is the main factors to influence the multiplicity fluct. of (net) charges and (net) protons

-Charge/baryon conservation should be further included in iEBE-VISHNU

Summary

It is important to study both critical and non-critical fluctuations for BES and the search of the critical point

Critical fluctuations near the QCD critical point

-Static (equilibrium) critical fluctuation

-qualitatively explain the acceptance dependence of critical fluctuations

-C_4 and $\kappa\sigma^2$ can be reproduced through tuning the parameters of the model

-C_2 , C_3 are well above the poisson baselines, which can NOT describe the data

-dynamical critical fluctuation

-Sign of the C_3, C_4 cumulants can be different from the equilibrium one due to the memory effects

-model A , model B, model H …

Non-critical (thermal) Fluctuations

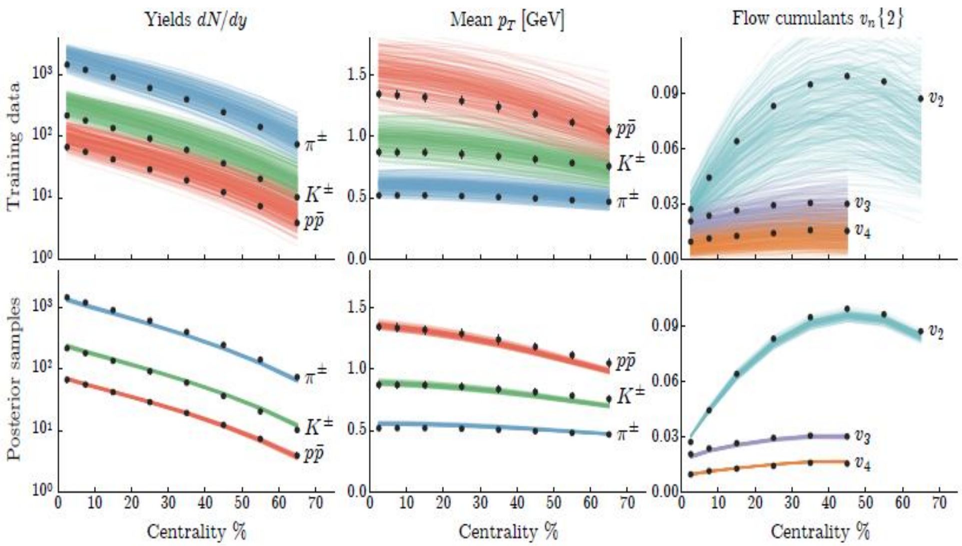
-At higher collision energy, (net) charge distributions deviate from Poisson, (net) protons distributions are pretty close to Poisson

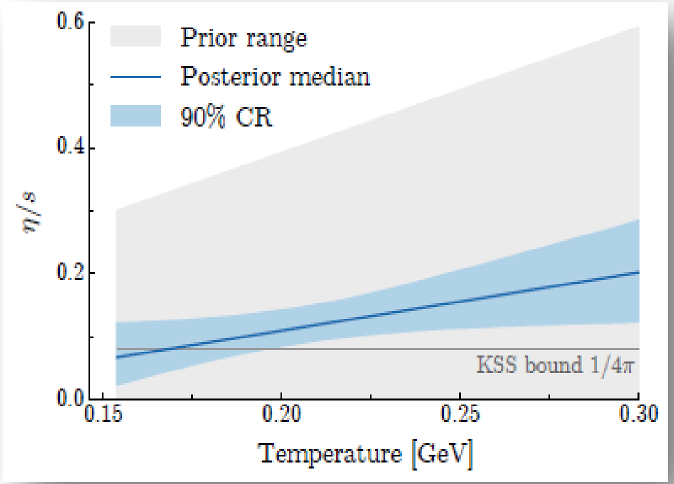
-Volume correction is the main factors to influence the multiplicity fluct. Of (net) charges and (net) protons

- Charge conservations need to be further included in iEBE-VISHNU

Initial State Fluctuations

- Hydrodynamics and hybrid model has been fully developed
-Lots of efforts from both exp and theory to study the initial state fluctuations and final state correlations
-the QGP shear viscosity has been extracted with massive data evaluations!



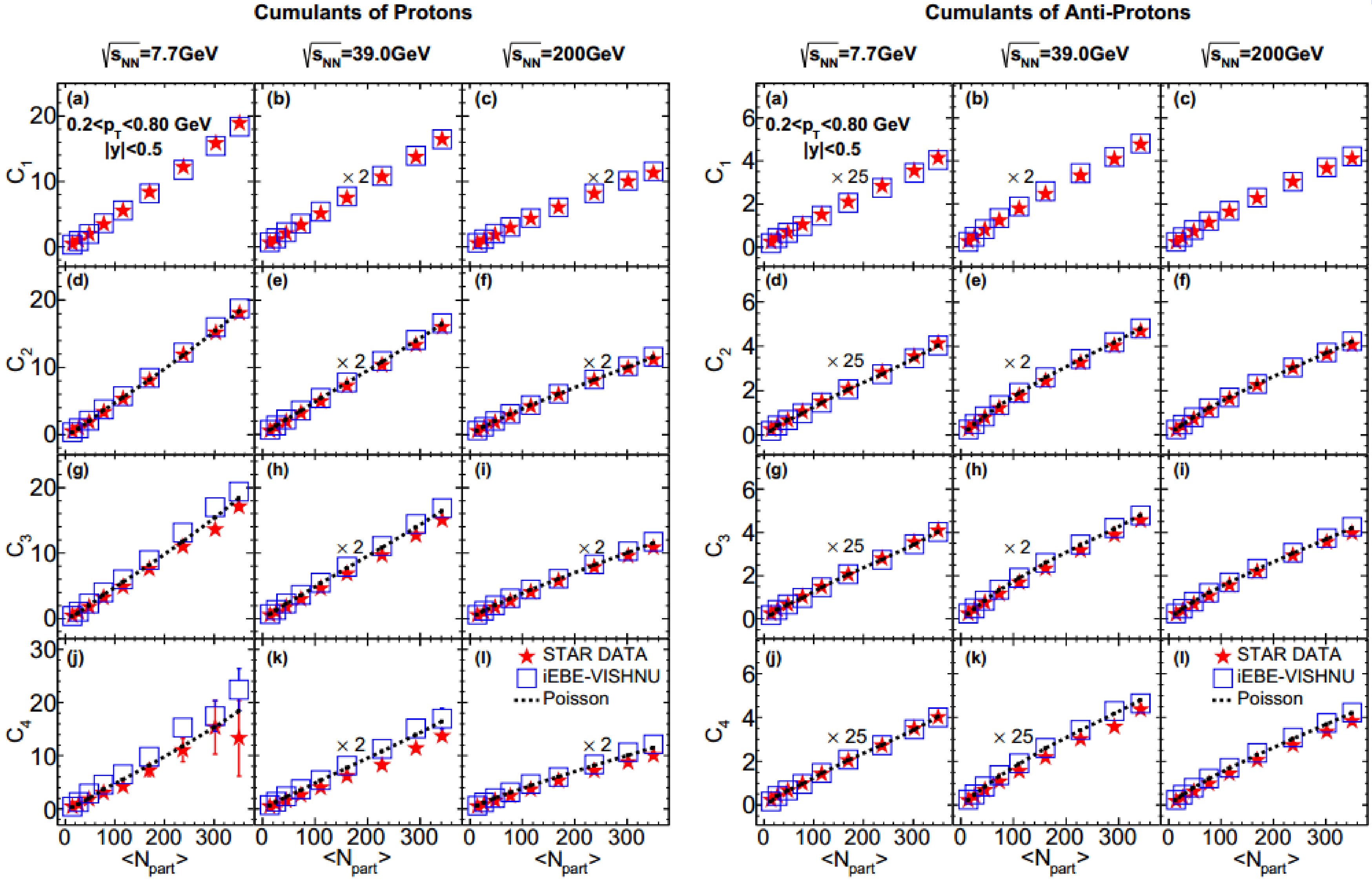


To do list of critical Fluctuations

-Better understanding for dynamical (non-equilibrium) critical fluctuations
- Full development of the dynamical model near the critical point
-More realistic non-critical fluct baselines
-Interactions between critical & non-critical fluctuations
-where is the critical points located in the (T μ) plane ?
-what is the effective correlation length ξ ?

Thank You

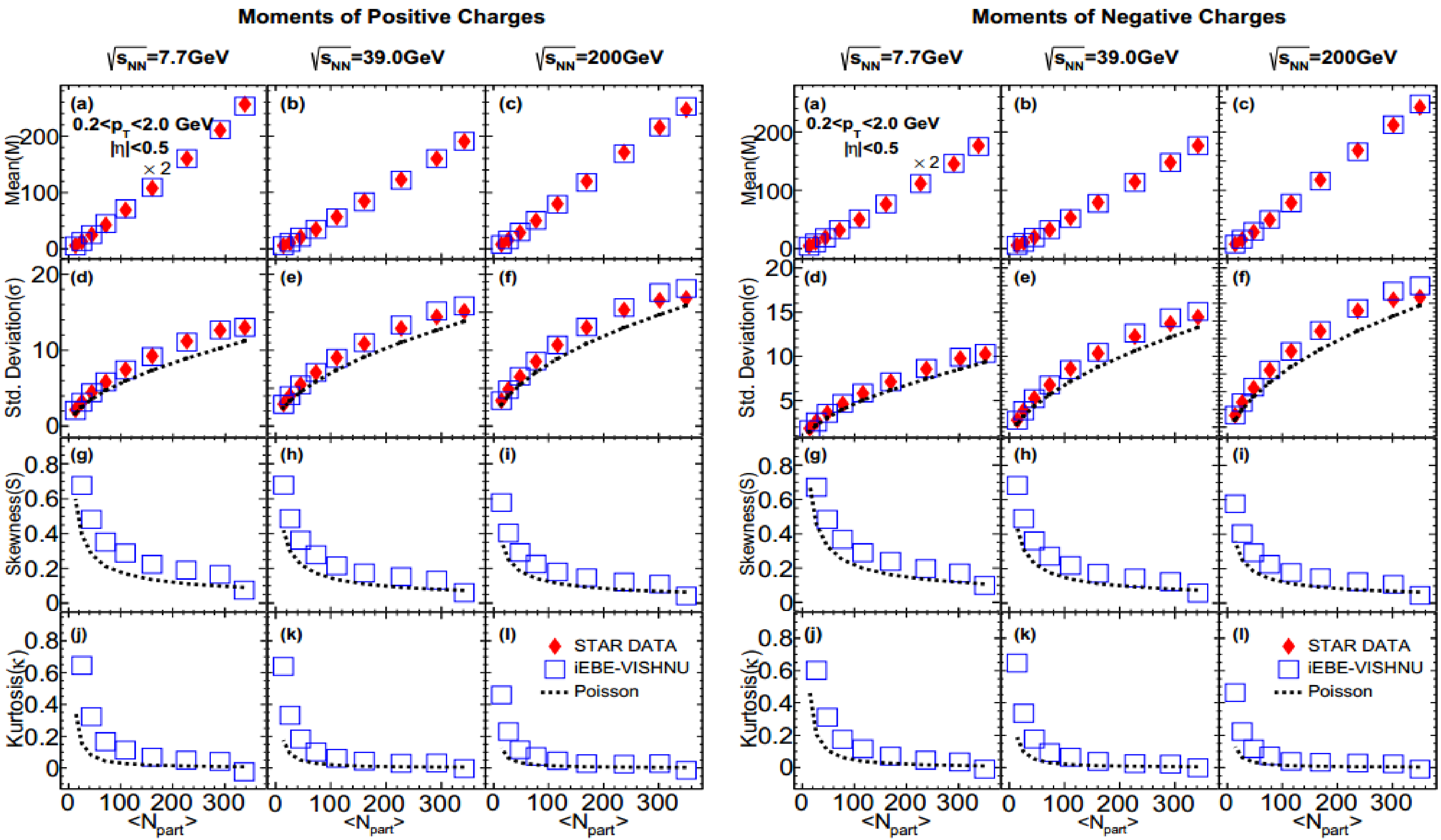
Cumulants of Protons/Anti-protons



-iEBE-VISHNU roughly describe the experimental data.

-Small deviation from the Poisson baselines. **Li, Xu, Song, paper in preparation**

Moments of Positive/Negative Charges



-iEBE-VISHNU model give a good description of M and σ

-For S and κ, the iEBE-VISHNU results shows cetrain deviations from the Poisson baseline.

Li, Xu, Song, paper in preparation

Boltzmann approach with external field

Stephanov PRD 2010

$$\mathcal{S} = \int d^3x \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma - U(\sigma)) - \int ds M(\sigma),$$

$$\begin{cases} \partial^2\sigma + dU/d\sigma + (dM/d\sigma)\int_p f/\gamma = 0. \\ \frac{p^\mu}{M}\frac{\partial f}{\partial x^\mu} + \partial^\mu M \frac{\partial f}{\partial p^\mu} + \mathcal{C}[f] = 0, \end{cases}$$

-analytical solution with perturbative expansion, please refer to Stephanov PRD 2010

Stationary solution for the Boltamann equation with external field

$$f_\sigma(\boldsymbol{p}) = e^{\mu/T} e^{-\gamma(\boldsymbol{p})M/T}.$$

Effective particle mass: $M = M(\sigma) = g\sigma$