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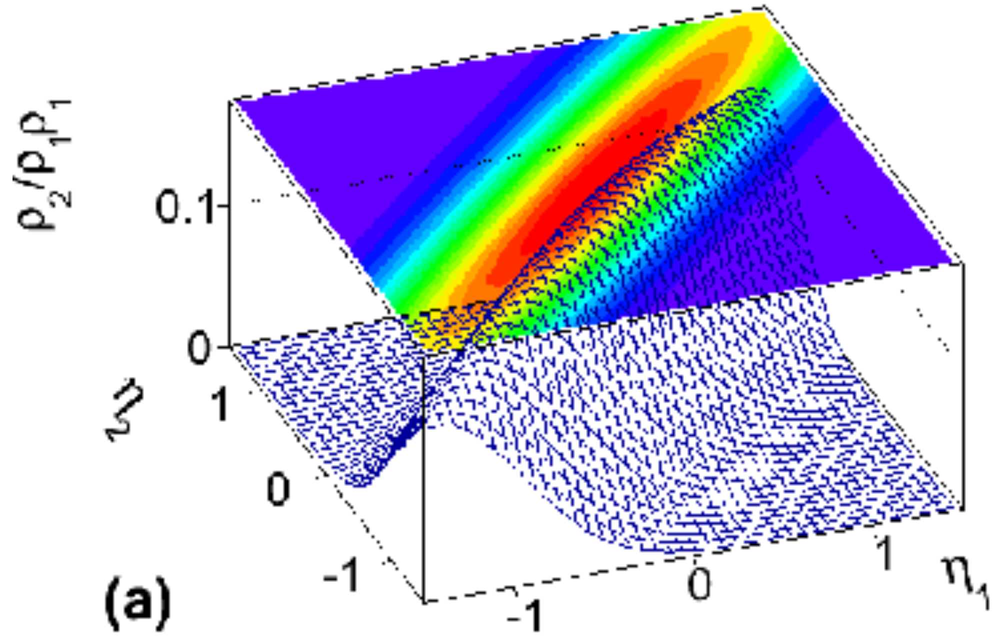


Lessons from Differential Correlation Measurements

EMMI Workshop, Wuhan, China
Oct 2017

Claude A. Pruneau
Wayne State University

Outline

A. Some ***not so random*** thoughts…
B. Controlling errors w/ Differential Correlations
C. **Differential Identity Method**

Definitions (I)

- Definition: Single & Pair Densities

Histogram — number of singles per event normalized per bin width

Single Density: $\rho_1(\phi_i,\eta_i) = \langle N(\phi_i,\eta_i)\rangle / \Delta\phi\Delta\eta$

Histogram — number of pairs per event normalized per bin width

Pair Density: $\rho_2(\phi_1,\eta_1,\phi_2,\eta_2) = \langle N(\phi_1,\eta_1)N(\phi_2,\eta_2)\rangle / \Delta\phi^2\Delta\eta^2$

- Factorize average yield and kinematic dependence

Single Probability Distribution

$$\rho_1(\phi_i,\eta_i) = \langle N\rangle P_1(\phi_i,\eta_i)$$

1st Momeent: Avg Multiplicity

$$\langle N\rangle = \int_{accept} \rho_1(\phi_i,\eta_i)\,d\phi_i\,d\eta_i$$

$$1 = \int_{accept} P_1(\phi_i,\eta_i)\,d\phi_i\,d\eta_i$$

$$\rho_2(\phi_1,\eta_1,\phi_2,\eta_2) = \langle N(N-1)\rangle P_2(\phi_1,\eta_1,\phi_2,\eta_2)$$

2nd Factorial:
Avg Number of Pairs

Pair Probability Distribution

$$\langle N(N-1)\rangle = \int_{accept} \rho_2(\phi_1,\eta_1,\phi_2,\eta_2)\,d\phi_1 d\eta_1 d\phi_2 d\eta_2$$

$$1 = \int_{accept} P_2(\phi_1,\eta_1,\phi_1,\eta_1)\,d\phi_1 d\eta_1 d\phi_2 d\eta_2$$

Definitions (II)

- Two-Particle Cumulant

$$C_2(\phi_1,\eta_1,\phi_2,\eta_2) = \rho_2(\phi_1,\eta_1,\phi_2,\eta_2) - \rho_1(\phi_1,\eta_1)\rho_1(\phi_2,\eta_2)$$

$$C_2(\Delta\phi,\Delta\eta) = \int_{Accept} C_2(\phi_1,\eta_1,\phi_2,\eta_2)\delta(\Delta\phi - \phi_1 + \phi_2)\delta(\Delta\eta - \eta_1 + \eta_2)\, d\phi_1\, d\eta_1\, d\phi_2\, d\eta_2$$

- Normalized Cumulant

Differential:

$$R_2(\phi_1,\eta_1,\phi_2,\eta_2) = \frac{C_2(\phi_1,\eta_1,\phi_2,\eta_2)}{\rho_1(\phi_1,\eta_1)\rho_1(\phi_2,\eta_2)} = \frac{\rho_2(\phi_1,\eta_1,\phi_2,\eta_2)}{\rho_1(\phi_1,\eta_1)\rho_1(\phi_2,\eta_2)} - 1$$

Integral:

$$\overline{R}_2 = \frac{\langle N(N-1)\rangle}{\langle N\rangle^2} - 1$$

Part I: Some not so random thoughts…

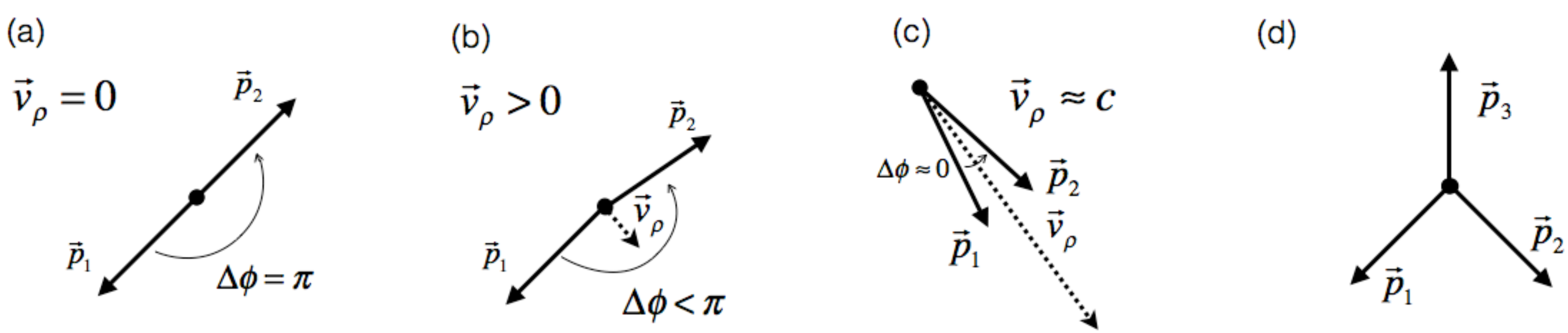
- Role of Conservation Laws
- Role of the Acceptance
- Flow vs Non-Flow:
  - Collective vs. Non-Collective
  - System wide vs. Non-Flow
- “Mediumization”

Role of Conservation Laws (1)

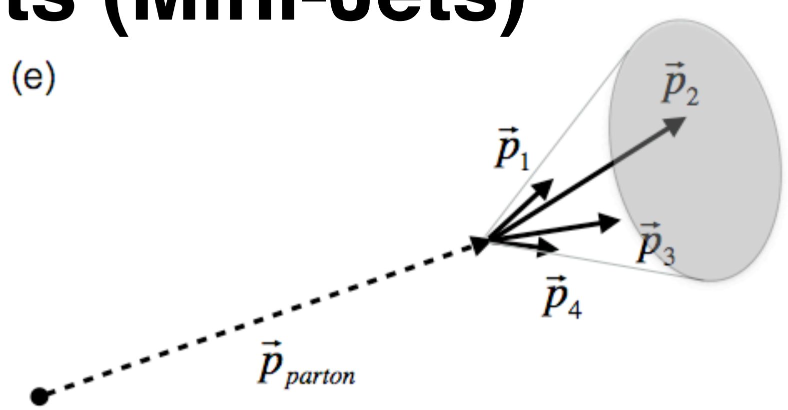
Conservation laws and particle production processes determine correlations and fluctuations .

Energy Momentum Conservation
Charge Conservation
Strangeness Conservation
etc…

Resonance decays, Finite Temperature Emission, Radial Flow:



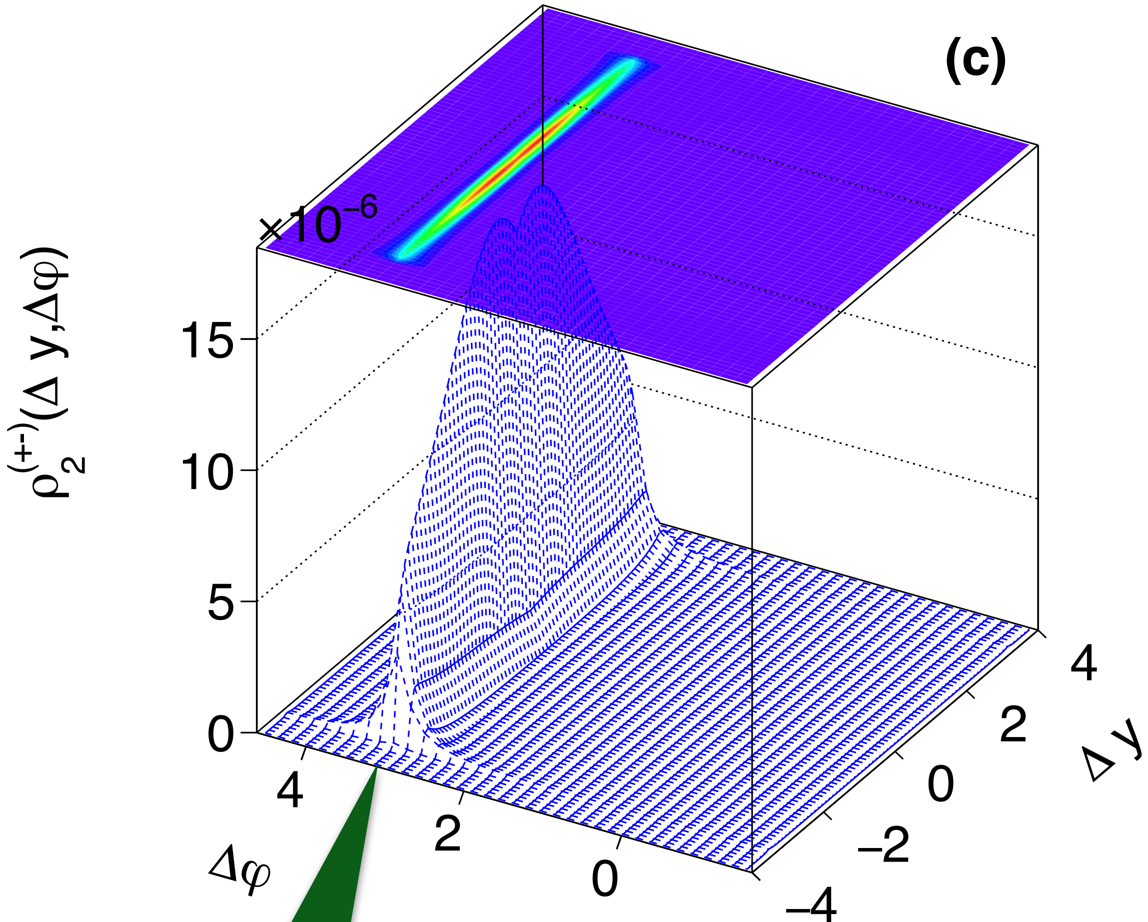
Jets (Mini-Jets)



Number of pairs, triplets, etc within fixed acceptance depends on temperature, flow boost, jet energy, etc

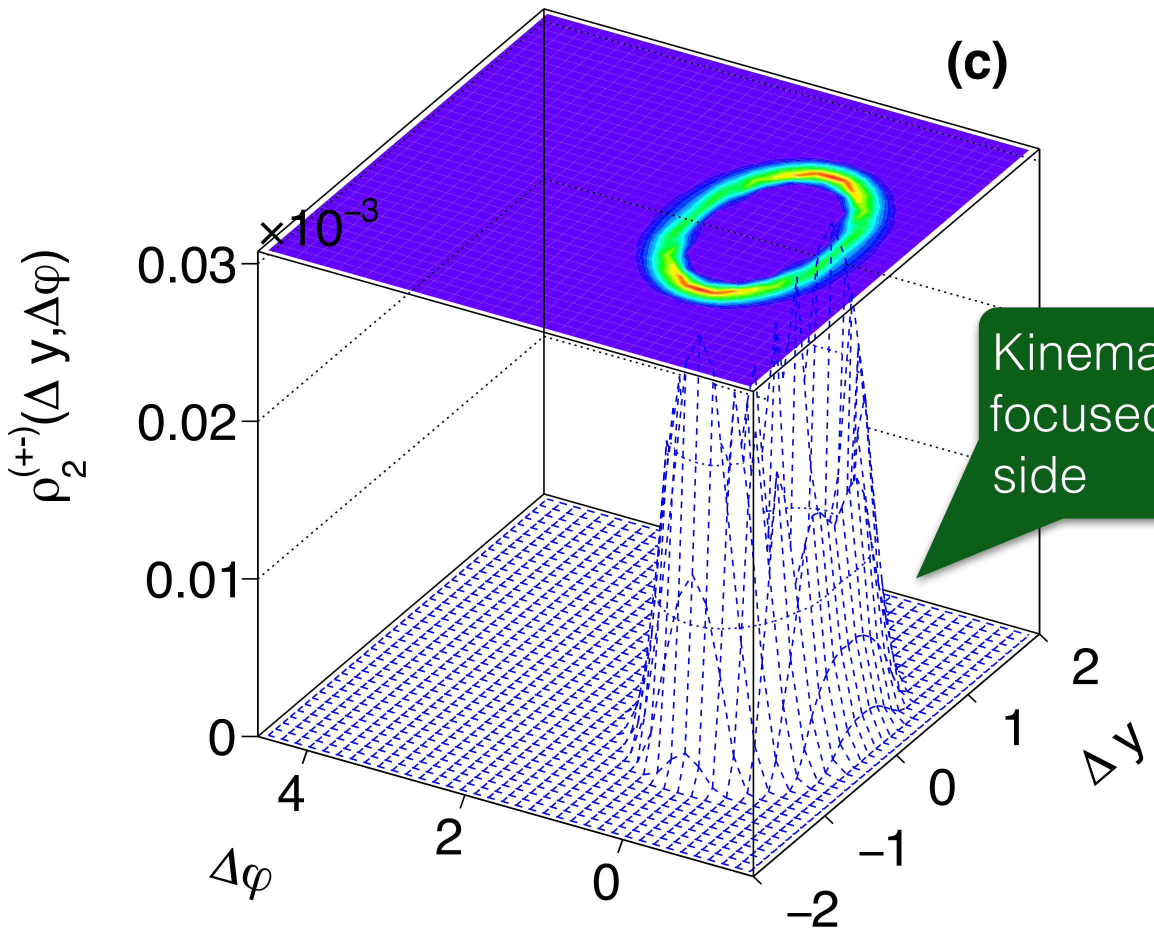
Example: $\rho^o \to \pi^+ + \pi^-$ Decays

Decay at rest:



Back-to-back in azimuth

Transverse Boost: $\beta \sim 0.9$



Kinematically focused to near-side

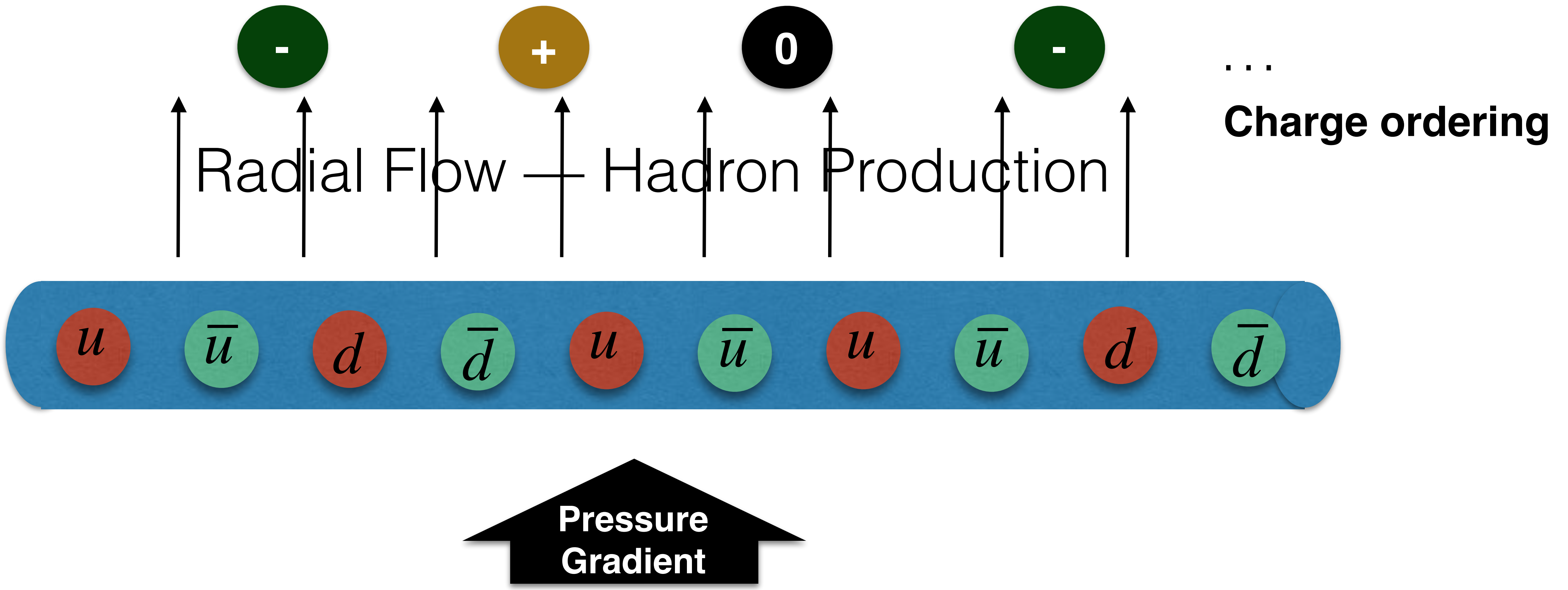
Δy range determined by resonance mass, system temperature, radial boost velocity…

==> Net charge fluctuations.

Role of Conservation Laws (2)

Conservation laws and particle production processes determine correlations.

String Fragmentation, Color Tube Fragmentation



Multi-particle Longitudinal Long Range Correlations
Anisotropic Flow
Number of pairs, triplets depend on the acceptance…

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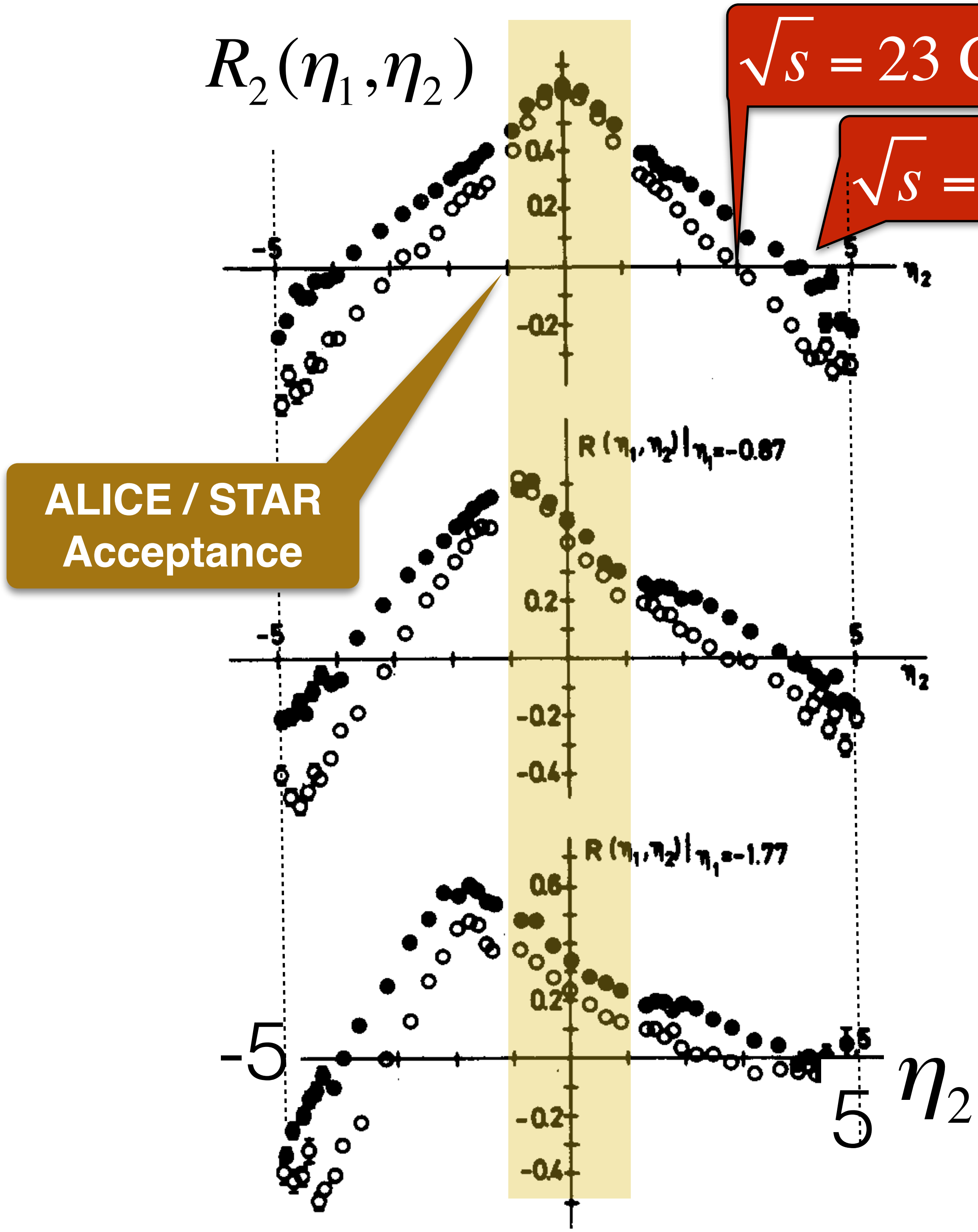
Example: ISR Results (p+p)

- S. R. Amendolia et al., PLB 48 (**1974**)

Charge particles Longitudinal Correlations



- Correlations extend over wide range of rapidities
  - Also in p-pBar @FNAL.
- Such **long range correlations must persist in A+A**
  - Indicator: **Lack of flow plane decor relation vs. eta gap**.
- Maybe modified by nuclear stopping, radial flow, late QGP hadronization, etc BUT they do NOT vanish…
- They underlie fluctuation measurements….
- Note: **Integral correlations for small eta range ARE NOT Poisson.**

Poisson vs. Non-Poisson…

- Fluctuations do not become Poissonian when the acceptance is reduced.
  - Two particle density: $\rho_2(\phi_1,\eta_1,\phi_2,\eta_2) = \langle N(N-1)\rangle P_2(\phi_1,\eta_1,\phi_2,\eta_2)$
  - Poisson realized if: $P_2(\phi_1,\eta_1,\phi_2,\eta_2) = P_1(\phi_1,\eta_1)P_1(\phi_2,\eta_2)$

Statistical Independence

$$\rho_2(\phi_1,\eta_1,\phi_2,\eta_2) = \rho_1(\phi_1,\eta_1)\rho_1(\phi_2,\eta_2)$$

$$\langle N(N-1)\rangle = \langle N\rangle^2$$

  - If the differential correlation functions are **non-poisson, so are the integrals**…
- **But distinguishing non-Poisson from Poisson fluctuations** may be challenging with integral correlations and finite statistics - when the acceptance is reduced.
  - **Go differential!!!**

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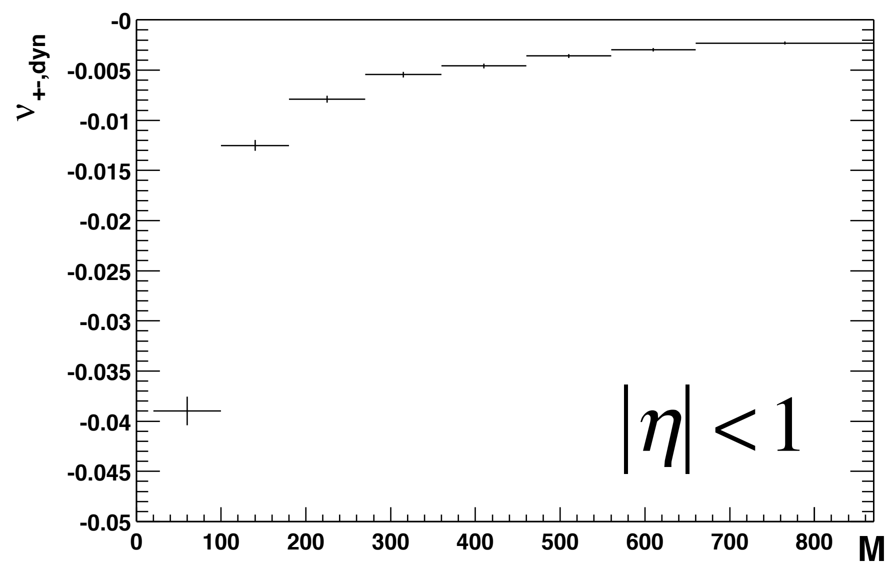
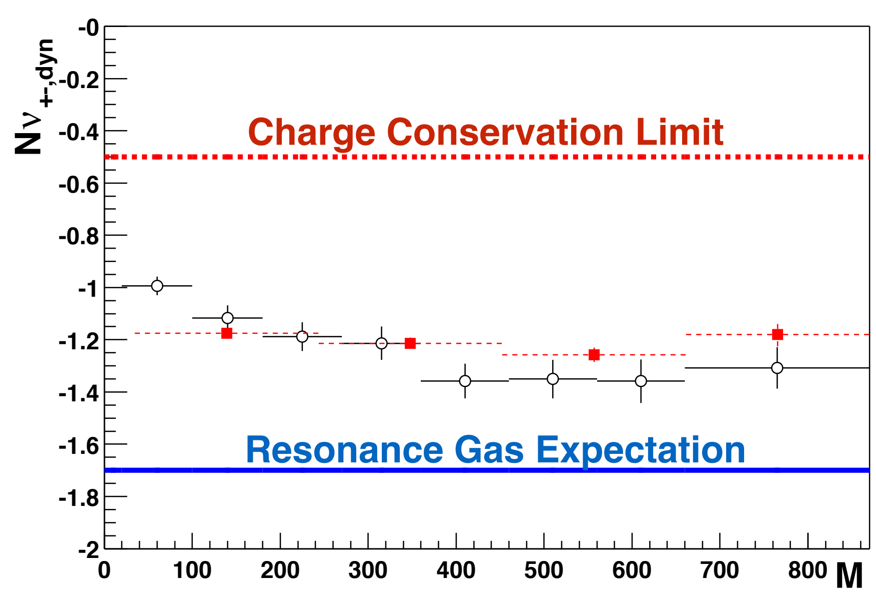
Two-Body Correlations Dilution vs System Size

Multiplicity fluctuations in Au+Au collisions at $\sqrt{s_{NN}} = 130$ GeV

STAR, Phys.Rev. C68 (**2003**) 044905



(a)



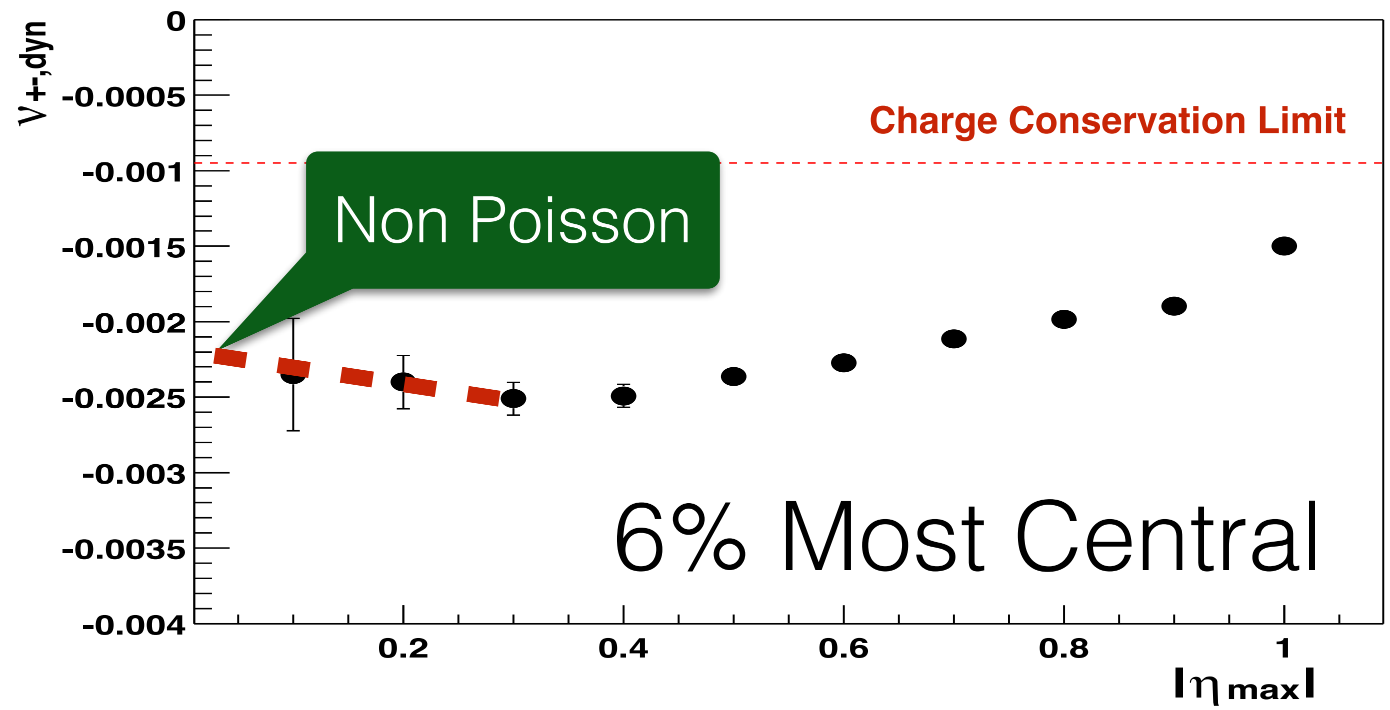
(b)

$$\nu_{dyn}^{(+-)} = \bar{R}_2^{(++)} + \bar{R}_2^{(--)} - 2\bar{R}_2^{(+-)}$$

Two-body correlation dilution:

$$\nu_{dyn}^{(+-)}(AA) \propto \frac{1}{m_s} \nu_{dyn}^{(+-)}(pp)$$

Number of "sources"



Width of the Acceptance Matters

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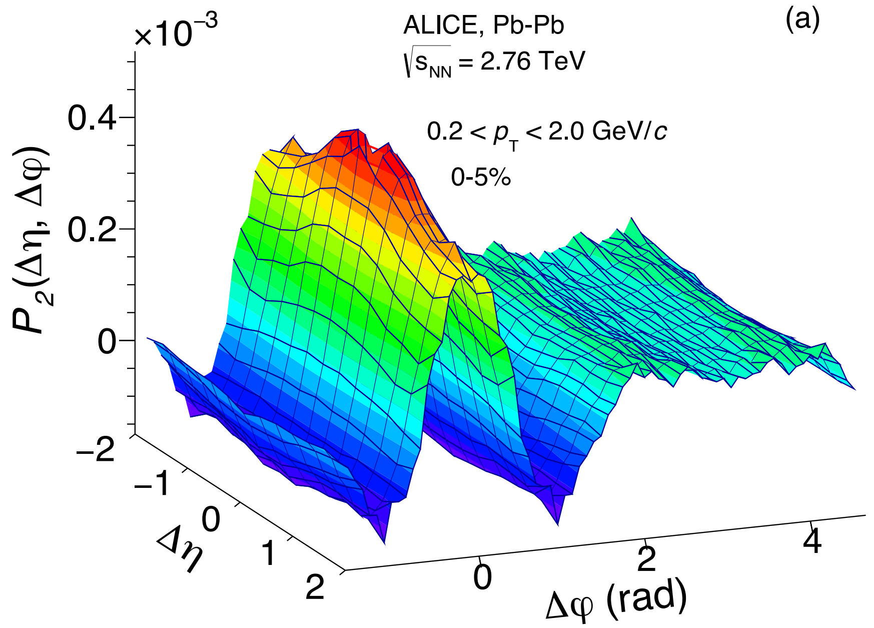
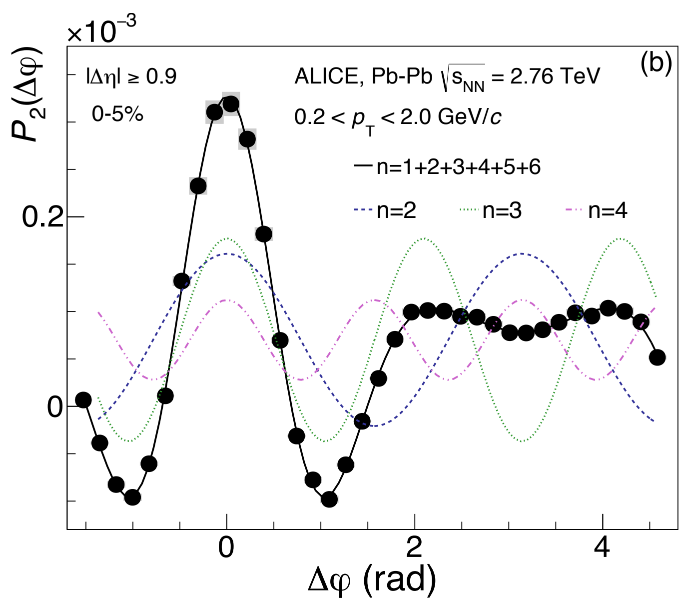
Flow vs. Non-Flow

Collective vs. Non-Collective: System Wide (Properties) vs. Elementary Processes

Flow dominance and factorization of transverse momentum correlations in Pb-Pb collisions at the LHC
ALICE, Phys. Rev. Lett. 118 (2017)162302.





Transverse Momentum Correlations

$$P_2 = \frac{\langle \Delta p_T \Delta p_T \rangle (\Delta\eta, \Delta\varphi)}{\langle p_T \rangle^2} = \frac{1}{\langle p_T \rangle^2} \frac{\int_{p_{T,min}}^{p_{T,max}} \rho_2(\vec{p}_1, \vec{p}_2)\ \Delta p_{T,1} \Delta p_{T,2}\ dp_{T,1} dp_{T,2}}{\int_{p_{T,min}}^{p_{T,max}} \rho_2(\vec{p}_1, \vec{p}_2)\ dp_{T,1} dp_{T,2}}$$

Flow Ansatz

$$v_n[P_2] \cong v_n^{p_T} / \langle p_T \rangle - v_n$$

$$v_n^{p_T} = \int \rho_1 v_n(p_T) p_T dp_T / \int \rho_1 dp_T$$

Pt Correlations more sensitive to v3 than number correlations

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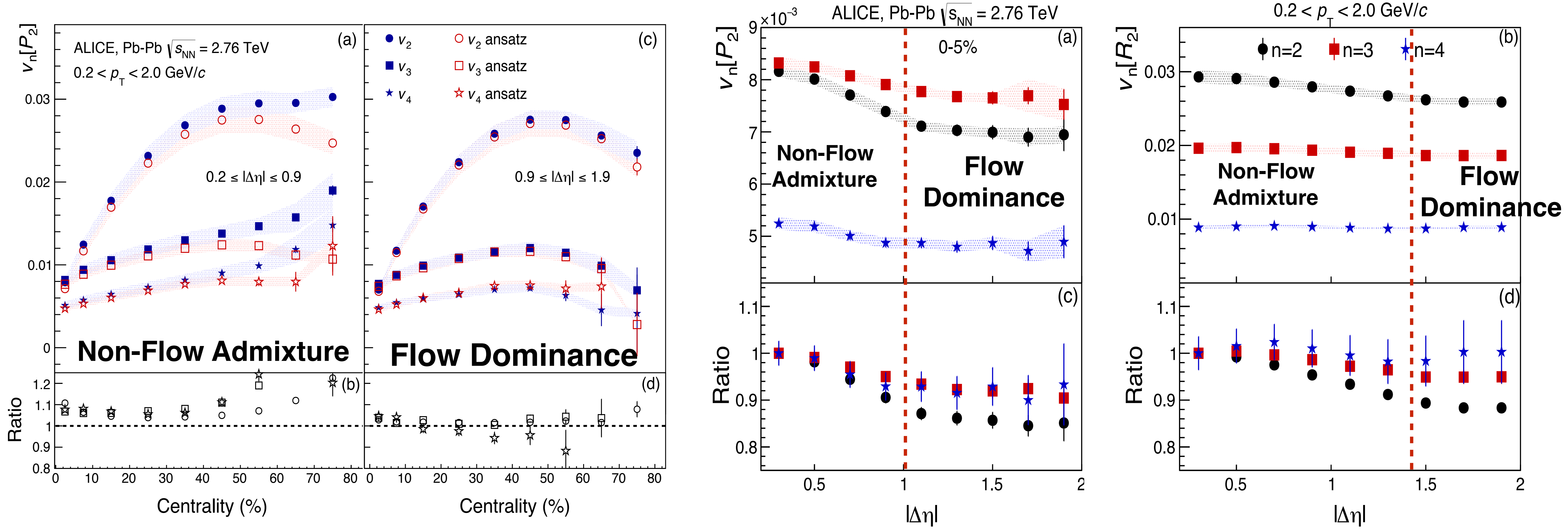
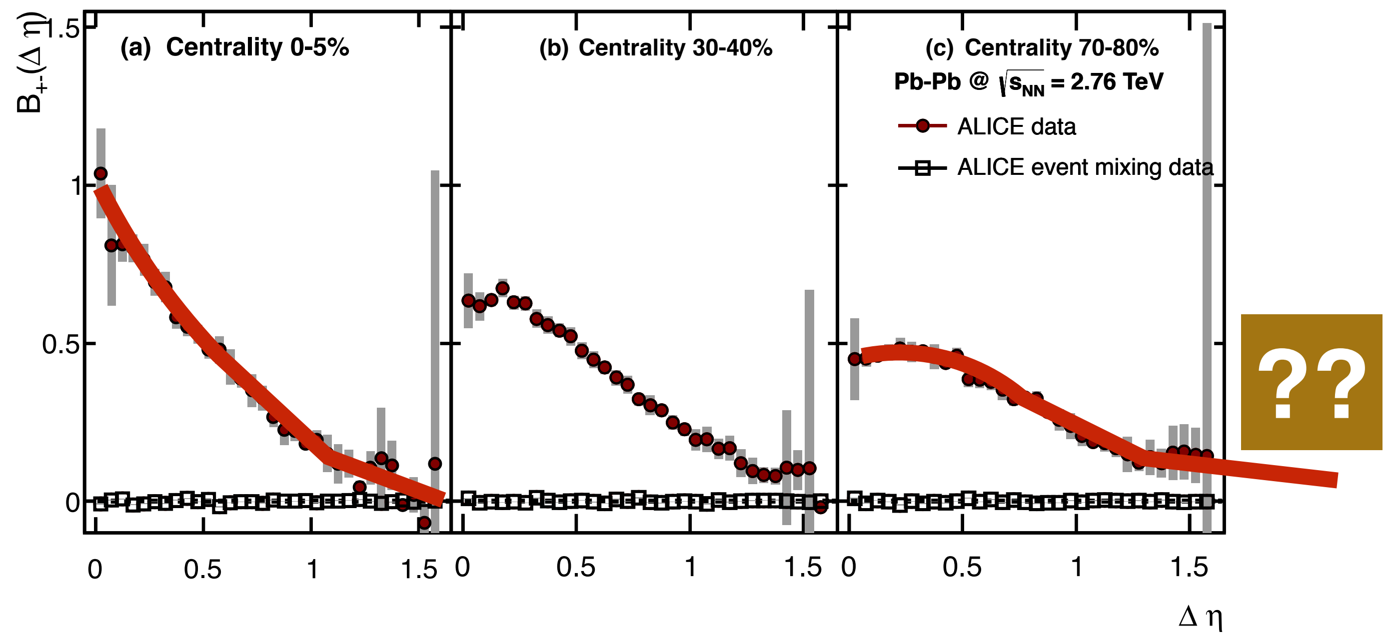


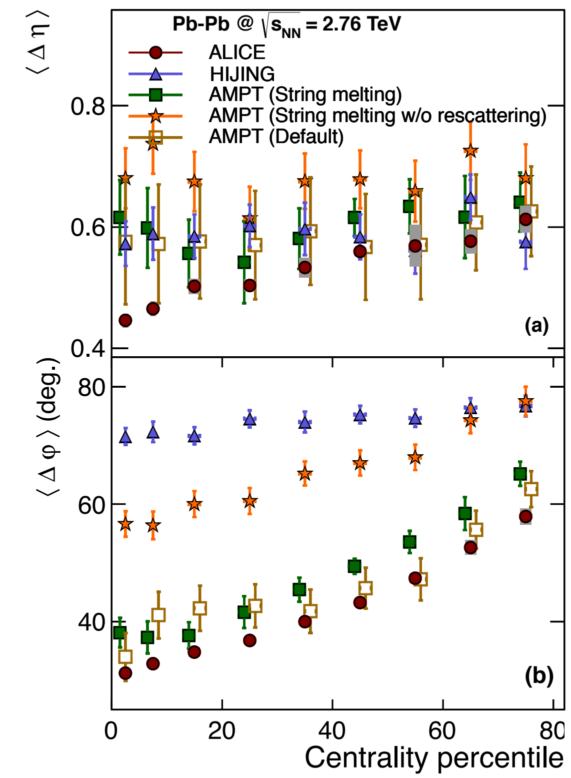
- A pair "eta gap" is required to suppress non-flow effects in differential R_2 or P_2 correlations & flow measurements.
- So why not in integral correlation measurements???

Example (2): Balance Function

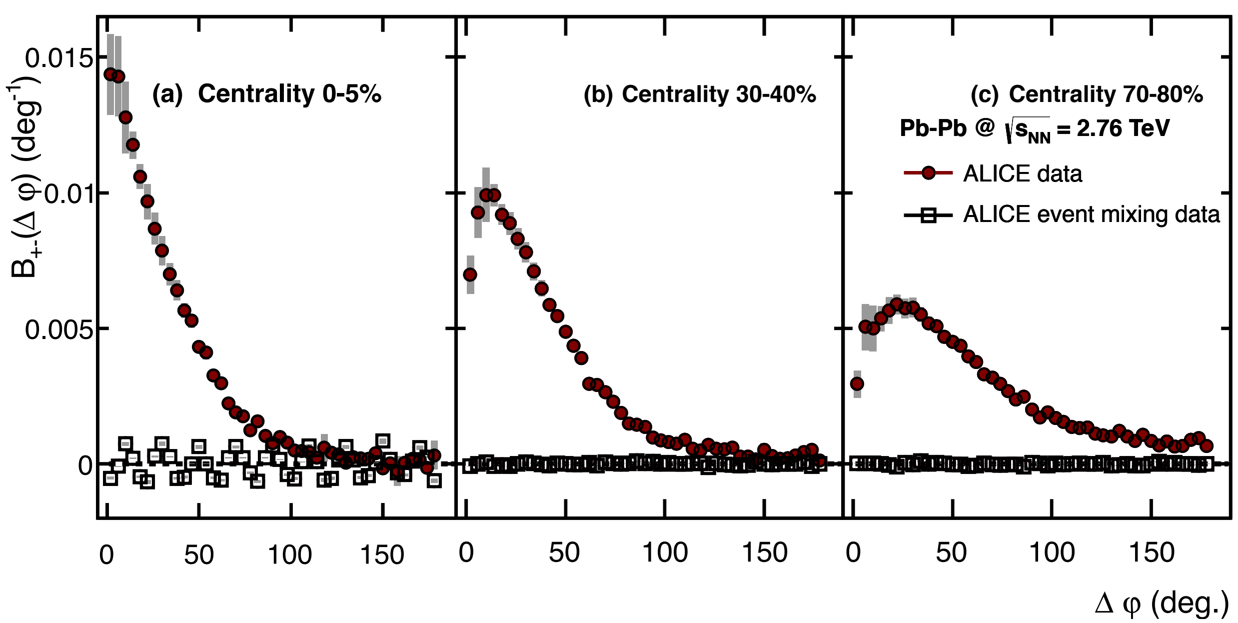
Charge correlations using the balance function in Pb–Pb collisions at 2.76 TeV

ALICE, Phys. Lett. B723 (2013) 267





- Radial Flow
- Delayed Hadronization
- Effective Temperature
- Effective Mass



Width of BF depends on centrality.
Fluctuations depend on radial flow.

Balance Functions

$$R_2^{(CD)}(\Delta\phi,\Delta\eta) = \tfrac{1}{2}\left[R_2^{(US)}(\Delta\phi,\Delta\eta) - R_2^{(LS)}(\Delta\phi,\Delta\eta)\right]$$

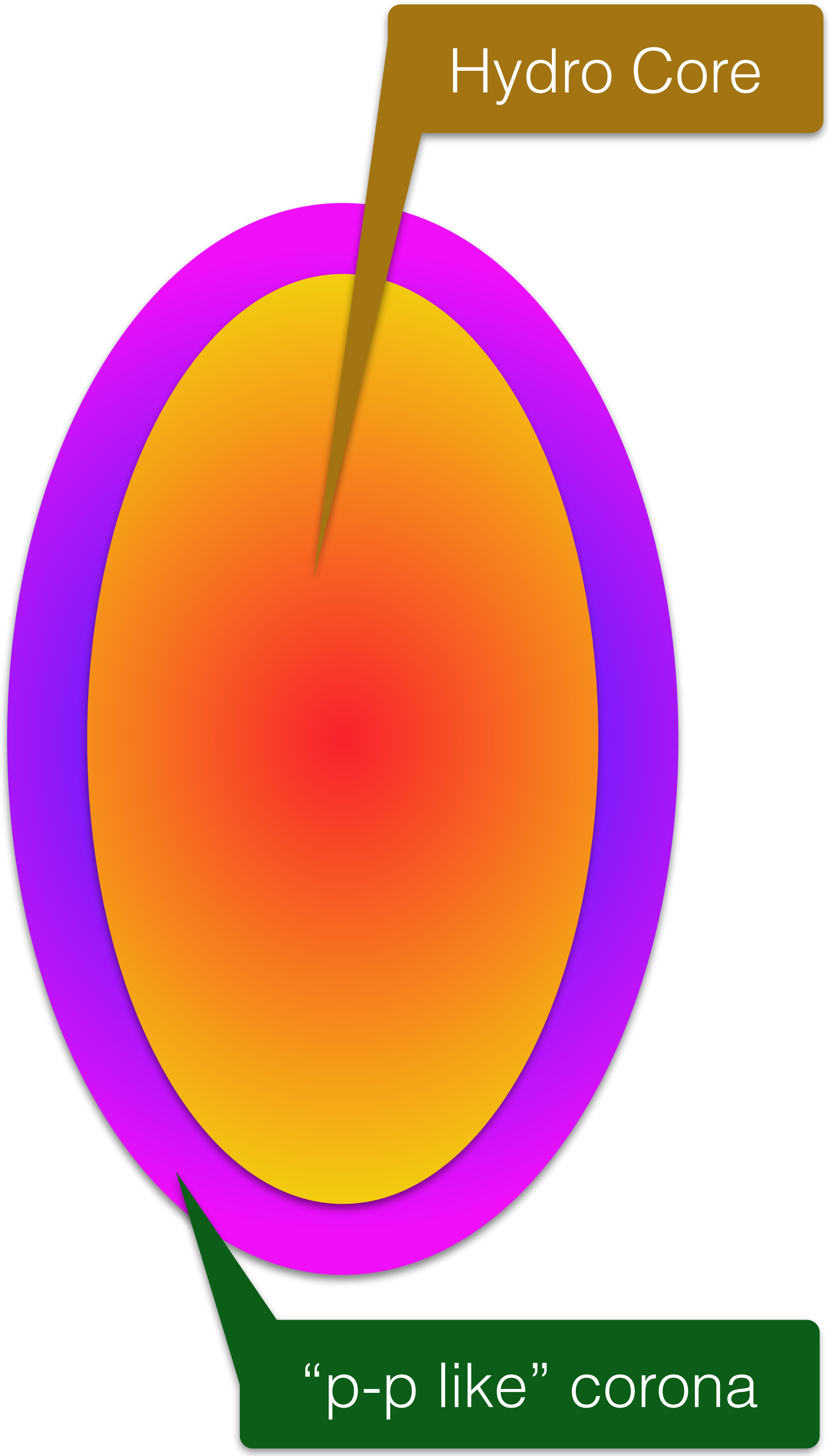
$R_2^{(CD)}(\Delta\phi,\Delta\eta) \propto$ Balance Function

- **Width of Balance Function depends on centrality, radial boost, life time of system (delayed hadronization)**
- **Effective Charge conservation effects change w/ system size, impact parameter, beam energy.**

- **The Acceptance Width Matters But its effects can be measured and modeled.**
- **The model is better if based on a measurement!**

Core/Corona Contributions

- EPOS 3.0 reproduces many observables
  - Particle yields, Species ratios, Average Momentum, Anisotropic flow
  - For small and large systems, collision centrality dependence
- Suggests/Implies
  - Core/Corona **contributions and distinctions** are “real”
  - **Core vs. Corona contributions to charge/strangeness/baryon correlations and fluctuations** may be different and thus must be accounted for.
  - One must understand the **energy dependence of the core vs. corona contributions** to charge correlations/fluctuations.
- But…

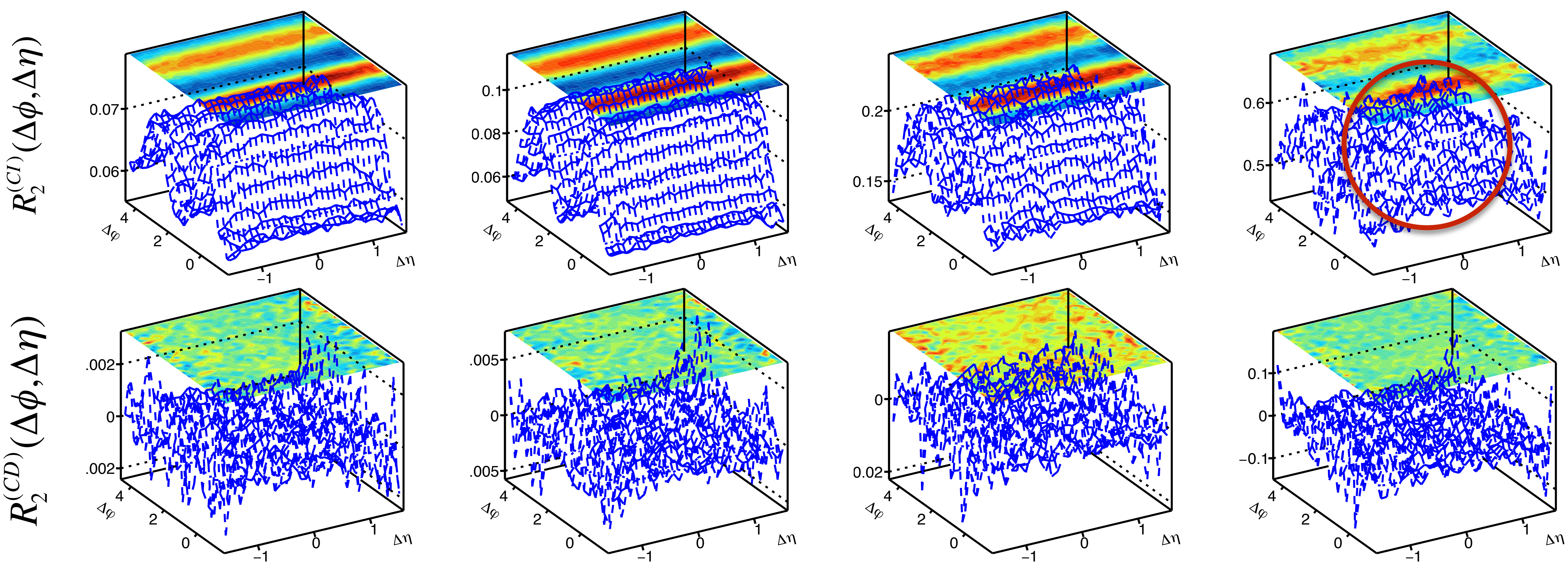


Core/Corona Contributions

Unfortunately **EPOS does not reproduce CI or CD charge R2 correlation functions** in peripheral to central collisions.

Pb-Pb @ 2.76 TeV

S. Basu, V. Gonzalez, C.P., et al., in preparation

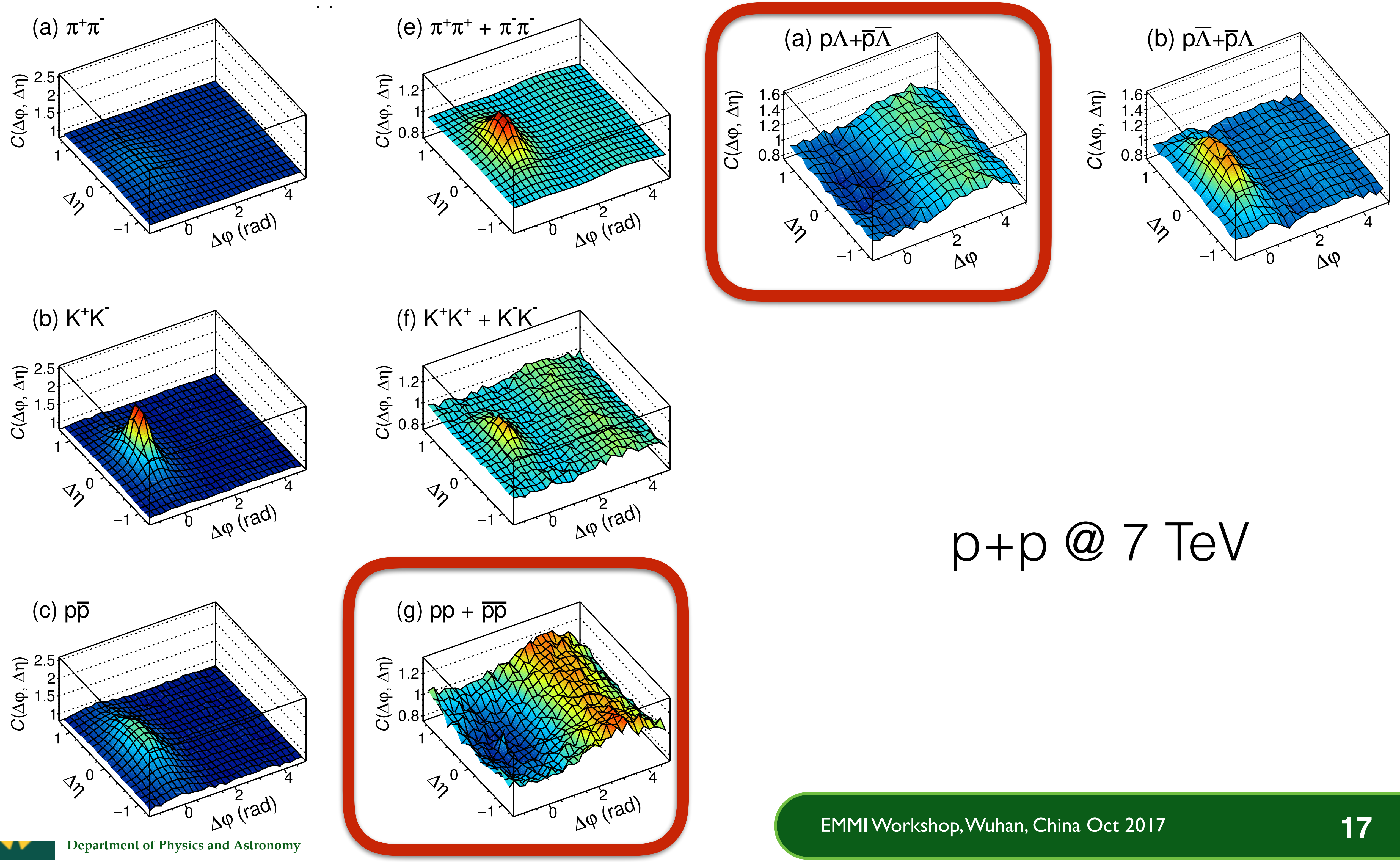


Fails to reproduce ALICE data.
Core contributions mishandled by Cooper-Frye prescription?

Some surprises too

Insight into particle production mechanisms via angular correlations of identified particles in pp collisions at 7 TeV

ALICE, arXiv:1612.08975v1



p+p @ 7 TeV

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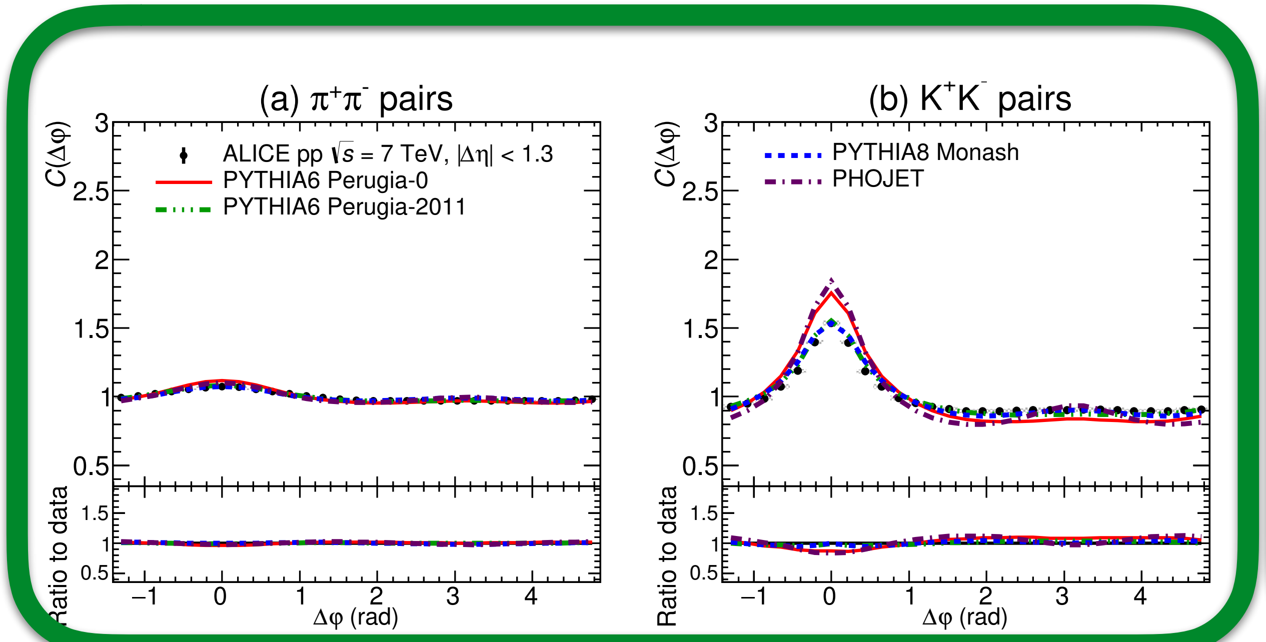
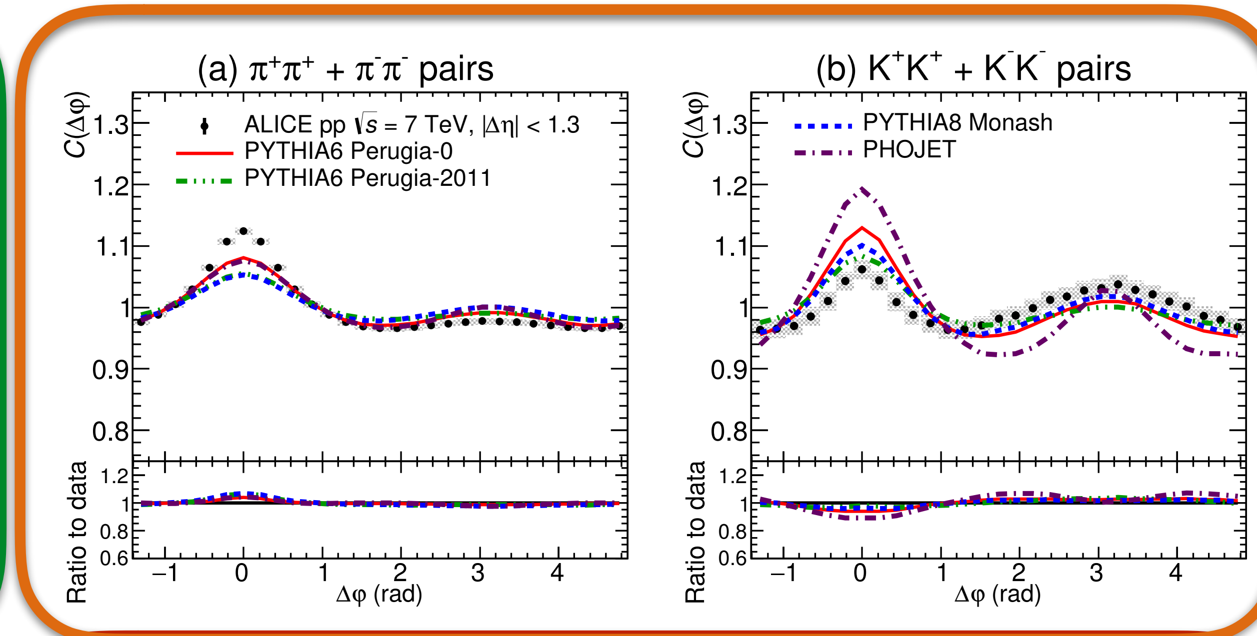
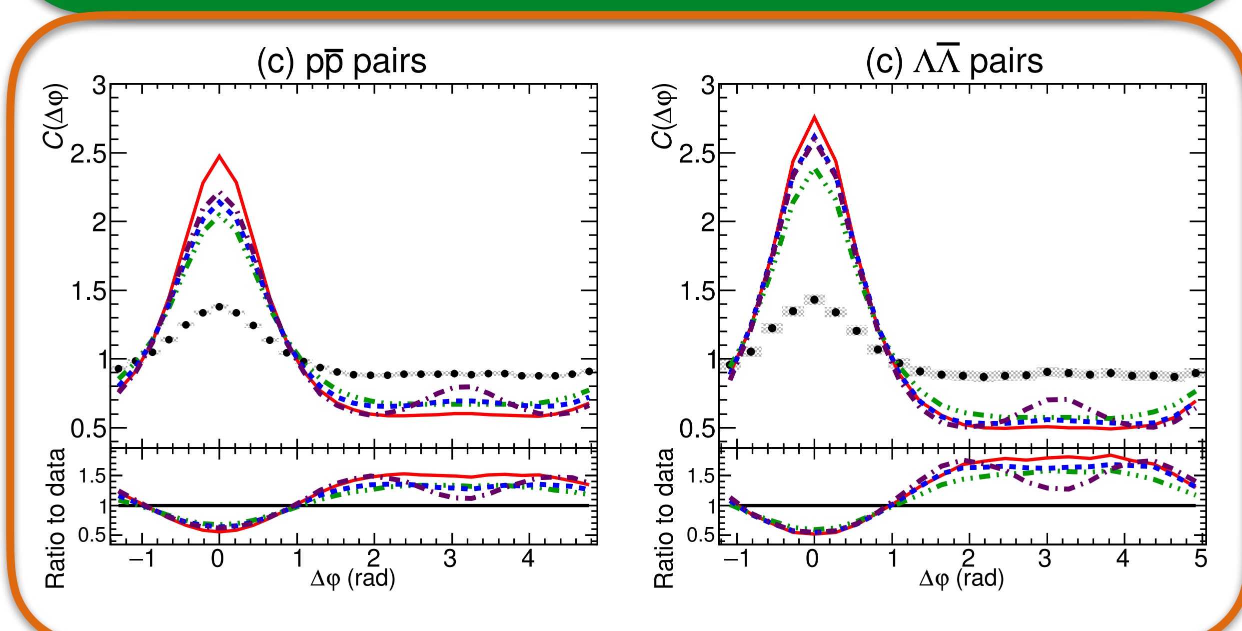
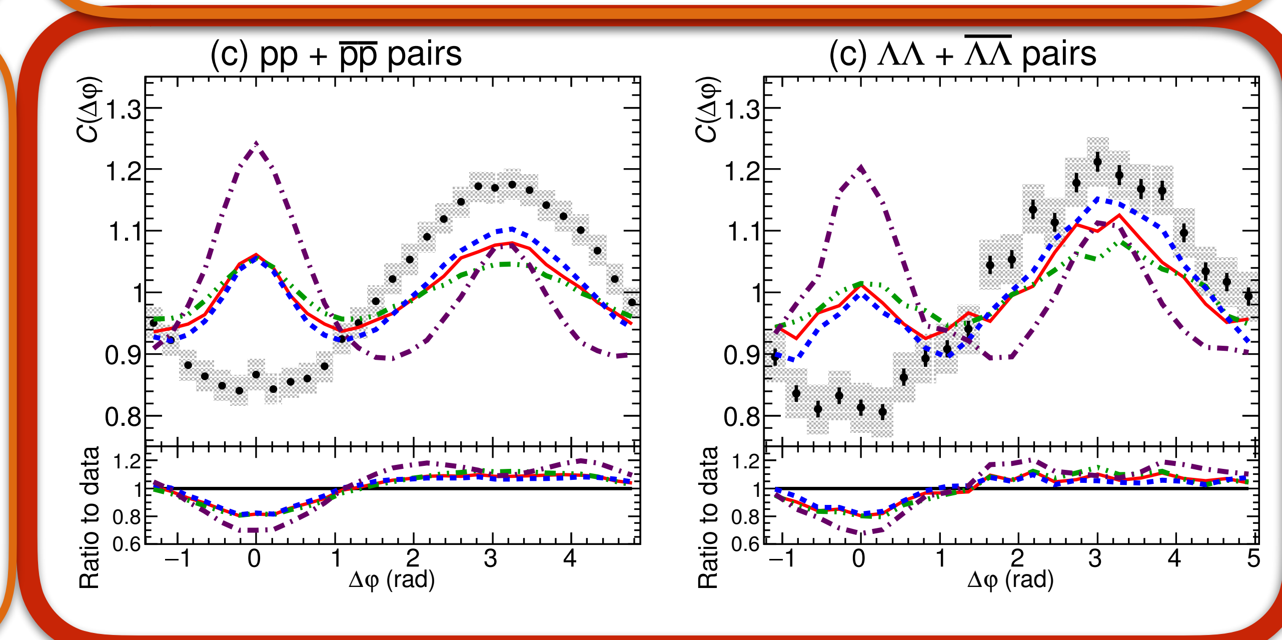
Other models get it wrong too ;-)

PYTHIA, p+p @ 7 TeV





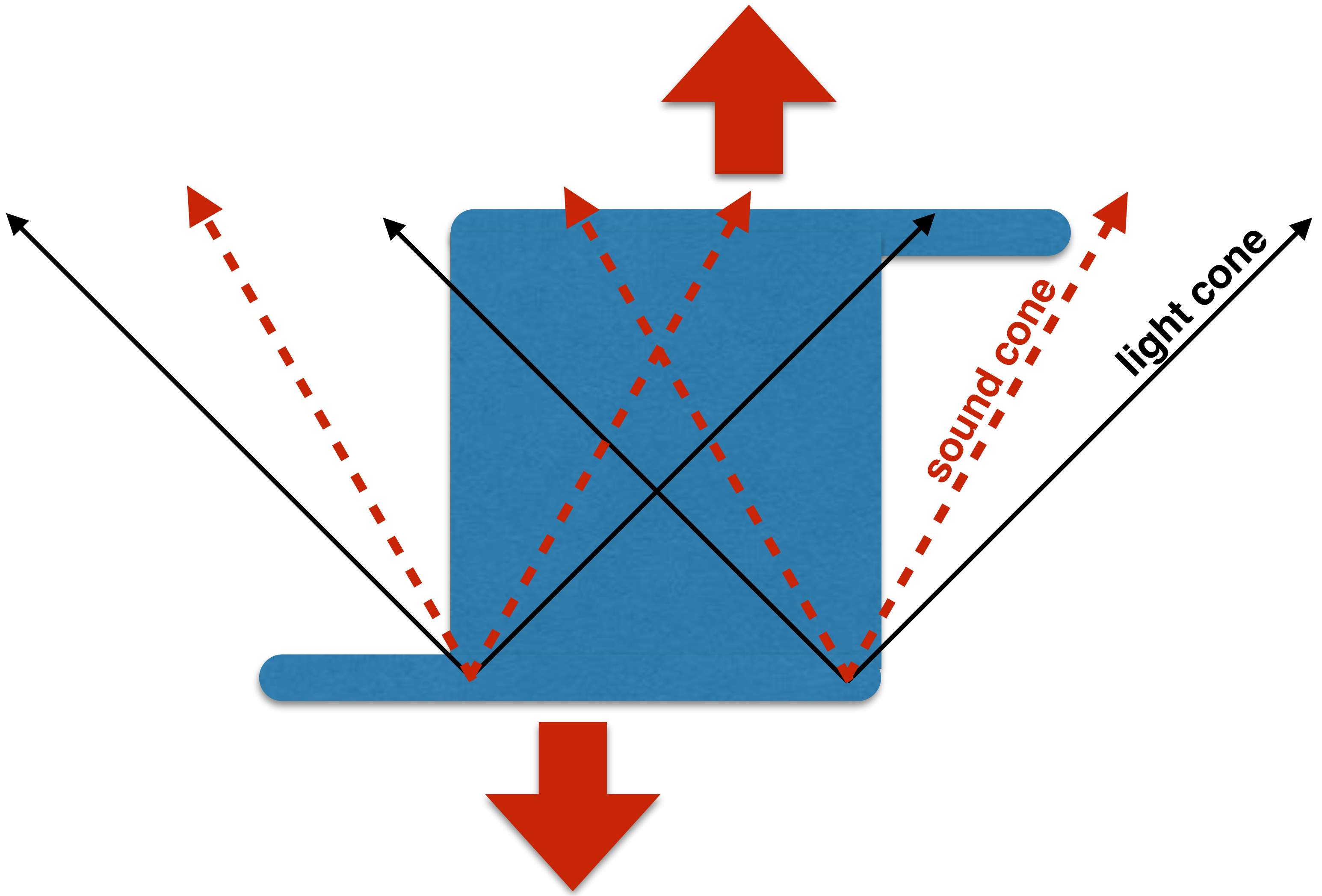




p-p “base-line” rather unexpected even at 7 TeV!!!

System Size & Lifespan

- Inclusive particle production, e.g., strangeness saturation, suggests local thermalization is achieved rather rapidly.
- Yet, if one probes the long distance scale behavior, there has to be time for the system to relax into a medium, i.e., to “**mediumize**.”
  - This should be particularly important for the spatial correlation length one wishes to probe with fluctuations.
  - But even at the speed of light, opposing sides of the collision systems remain causally disconnected until freeze out — and the **speed of sound is likely more relevant in this context…**



- The left end side is not “informed” of the right end side until the system freezes out.
- Transverse long range behavior does not materialize before system break-up
- The correlation length actually probed is probably quite short.

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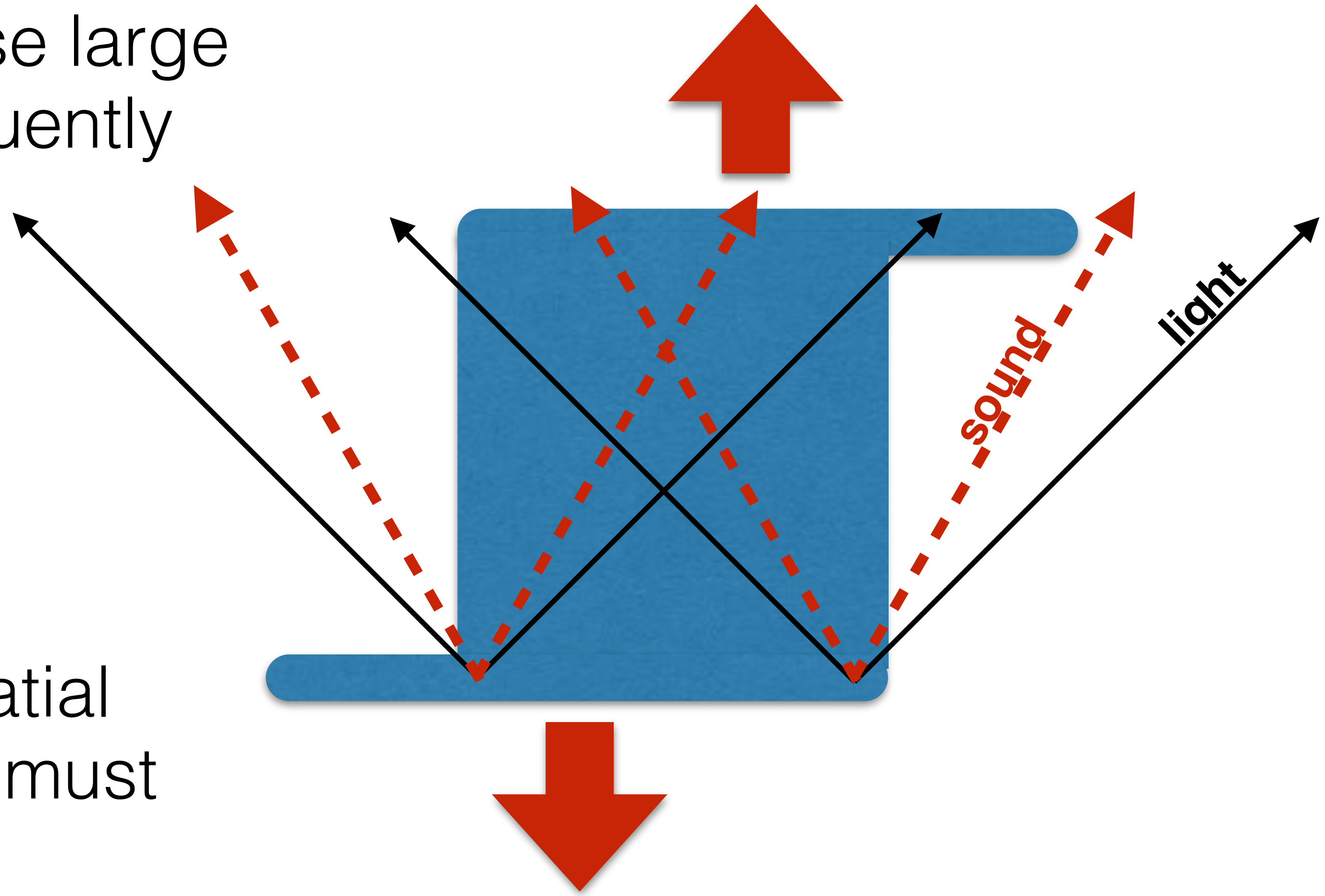
System Size & Lifespan (2)

- Capacity to mediumize and sense large correlation lengths must consequently depend on
  - collision centrality
  - system size
  - system lifetime
- Sensitivity to variations of the spatial correlation length consequently must depend on
  - produced multiplicity
  - system size
  - beam energy
  - as well as EOS and proximity to C.P.



Berdnikov-Rajagopal (2000): $\xi \sim 2 - 3\ \mathrm{fm}$

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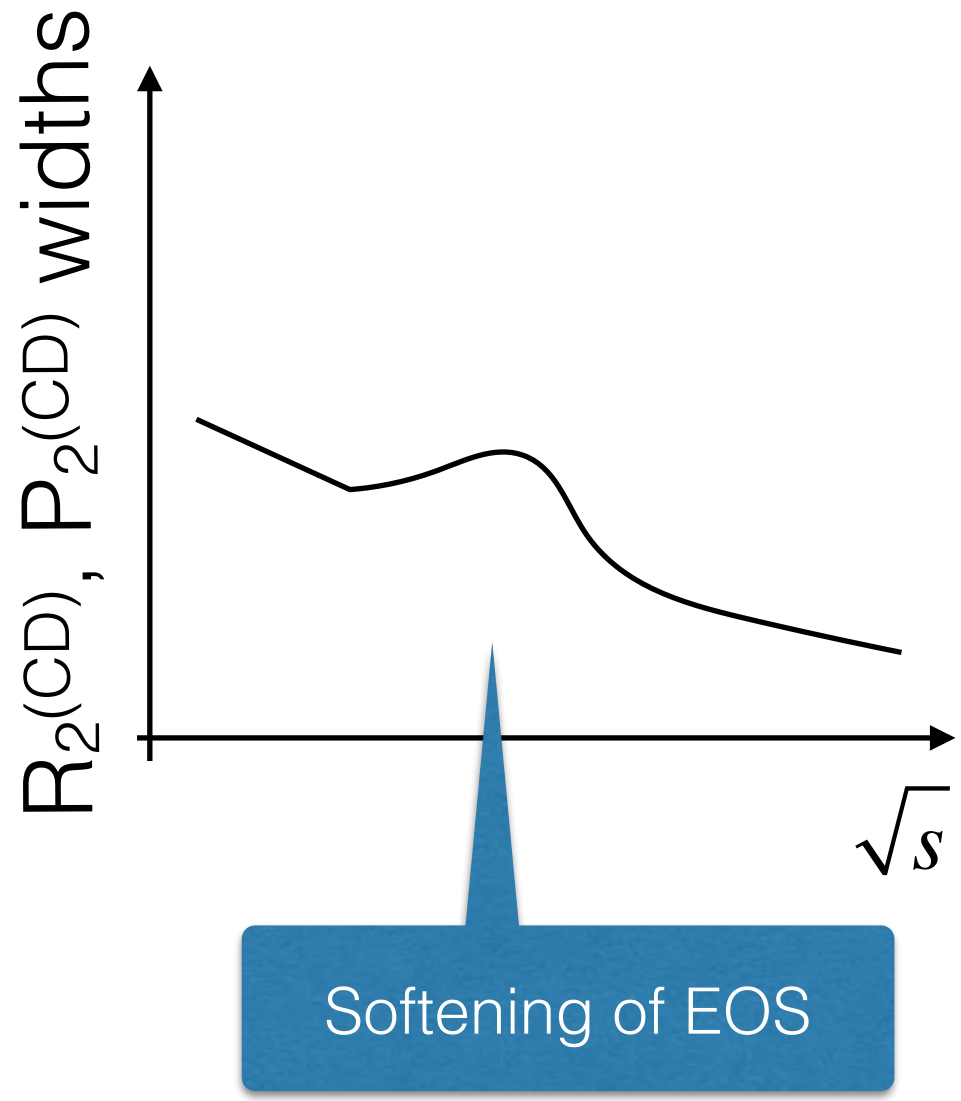
Measure Factorial Moments and Corr Fct.

- Integral correlations are
  - (slow) **functions of the acceptance**
  - PARTLY determined by non-flow phenomena, not just system wide properties.
- **Eliminate/Suppress non-flow effects**
  - Require an eta-gap in measurement of cumulants?
  - Better still: measure differential correlations
    - e.g., R_4 (Differential version of Volker/Adam's C_4) suppress non-flow thereby focusing on system wide properties.
    - Side benefit: differential correlations are more easily corrected for detector effects.
    - **Factorial moments are more robust** than integral cumulants
  - **Note: Puzzling features of STAR's R2 Measurement at 19.6 GeV … show R2 is more sensitive to details of the detector response BUT when these details are understood, the integral of R2 shall be more reliable.**

Diversify Types of Measurements

- For instance:
    - Width and strength of Balance Functions of R_2 and P_2 vs. beam energy
- Softening of the equation implies change in radial flow which implies
    - changes in kinematic focusing
    - broadening of the width of balance function
- Caveat:
    - reduction in beam energy implies changes in particle production mechanisms that may feature different rapidly spans.



Study R2, P2 amplitude also

Part II: Some Technical Considerations

- Acceptance:
  - **Cannot be corrected** for but can be varied or averaged across, e.g., vs. $\bar{\eta} = (\eta_1 + \eta_2)/2$
- Efficiency:
  - Single and Pair Efficiencies can be accounted for with robust ratios, weight technique, etc.
- Contamination:
  - Secondaries contamination identifiable as extra features or correlation shape changes in differential correlation functions.

S. Ravan, P. Pujahari, S. Prasad, C.A.P., Phys.Rev. C89 (2014) 024906

Efficiency & Robustness (I)

- Model the Probability of observing n particles given N (in a given “bin”) were produced with binomial distribution.

$$P_{\det}(n \mid N;\varepsilon) = \frac{\varepsilon^{N}(1-\varepsilon)^{N-n}}{n!(N-n)!}$$

- Model the Probability of observing particle fluctuations…
  - Singles

$$P_M(n(\eta_1) \mid N(\eta_1);\varepsilon_1,) = \sum_{N_1=1}^{\infty} P_T(N(\eta_1)) \frac{\varepsilon_1^{N(\eta_1)}(1-\varepsilon_1)^{N(\eta_1)-n(\eta_1)}}{n(\eta_1)!(N(\eta_1)-n(\eta_1))!}$$

Measured Probability distribution

True Probability distribution

  - Pairs

$$P_M(n(\eta_1),n(\eta_2) \mid N(\eta_1),N(\eta_2);\varepsilon_1,\varepsilon_2) = \sum_{N_1,N_2=1}^{\infty} P_T(N(\eta_1),N(\eta_2)) \frac{\varepsilon_1^{N(\eta_1)}(1-\varepsilon_1)^{N(\eta_1)-n(\eta_1)}}{n(\eta_1)!(N(\eta_1)-n(\eta_1))!} \frac{\varepsilon_2^{N(\eta_2)}(1-\varepsilon_2)^{N(\eta_2)-n(\eta_2)}}{n(\eta_2)!(N(\eta_2)-n(\eta_2))!}$$

Measured Probability distribution

True Probability distribution

Efficiency & Robustness (1)

- Singles Average
  - True

$$\langle N \rangle = \int P_T(N) N dN$$

  - Measured

$$\langle n \rangle = \int P_M(n) n dn$$

$$\langle n \rangle = \int P_T(N) dN \int n P_{\det}(n \mid N; \varepsilon) dn = \varepsilon \int P_T(N) N \, dN$$

$$\langle n \rangle = \varepsilon \langle N \rangle$$

- Pair Averages
  - True

$$\langle N_1 N_2 \rangle = \int P_p(N_1, N_2) N_1 N_2 dN_1 dN_2$$

  - Measured

$$\langle n_1 n_2 \rangle = \int P_m(n_1, n_2) n_1 n_2 dn_1 dn_2$$

$$\langle n_1 n_2 \rangle = \varepsilon_1 \varepsilon_2 \langle N_1 N_2 \rangle$$

Correct for any True PDF
Only requires binomial sampling.

Efficiency & Robustness (III)

- Correlation function measurement
- Goal:

$$C_2^{(True)}(\eta_1,\eta_2) = \rho_2(\eta_1,\eta_2) - \rho_1(\eta_1)\rho_1(\eta_2)$$

True

- "Raw" Measurement

Measured

$$C_2^{(measured)}(\eta_1,\eta_2) = \frac{1}{\Delta\eta^2}\langle n(\eta_1)n(\eta_2)\rangle - \langle n(\eta_1)\rangle\langle n(\eta_2)\rangle$$

$$= \frac{1}{\Delta\eta^2}\varepsilon_1(\eta_1)\varepsilon_2(\eta_2)\left\{\langle N(\eta_1)N(\eta_2)\rangle - \langle N_1(\eta_1)\rangle\langle N_2(\eta_2)\rangle\right\}$$

- Ratio Fct

$$R_2^{(Measured)}(\eta_1,\eta_2) = \frac{\langle n(\eta_1)n(\eta_2)\rangle}{\langle n(\eta_1)\rangle\langle n(\eta_2)\rangle} - 1$$

$$= \frac{\varepsilon_1(\eta_1)\varepsilon_1(\eta_2)\langle N(\eta_1)N(\eta_2)\rangle}{\varepsilon_1(\eta_1)\varepsilon_1(\eta_2)\langle N(\eta_1)\rangle\langle N(\eta_2)\rangle} - 1 = \frac{\langle N(\eta_1)N(\eta_2)\rangle}{\langle N(\eta_1)\rangle\langle N(\eta_2)\rangle} - 1$$

$$R_2^{(Measured)}(\eta_1,\eta_2) = R_2^{(True)}(\eta_1,\eta_2)$$

Efficiencies cancel >>> Robust Observable

Folding of Singles vs Event Mixing

- Ratio R requires product of single yields
  - Can be obtained from actual singles

$$R_M(\eta_1,\eta_2)=\frac{\langle n_1(\eta_1)n_2(\eta_2)\rangle}{\langle n_1(\eta_1)\rangle\langle n_2(\eta_2)\rangle}-1$$

No event mixing required

  - Can be obtained from mixed events

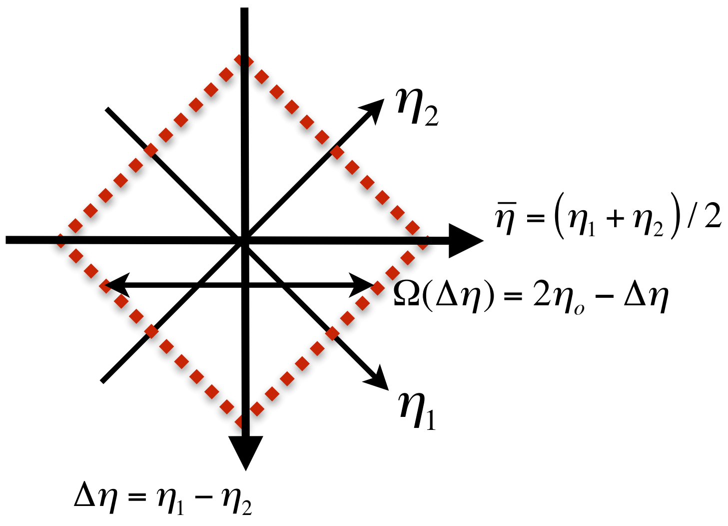
$$R_M^{(mixed)}(\eta_1,\eta_2)=\frac{\langle n_1 n_2(\eta_1,\eta_2)\rangle}{\langle n_1(\eta_1)\rangle\langle n_2(\eta_2)\rangle}=\frac{\langle n_1 n_2(\eta_1,\eta_2)\rangle_{\text{same}}}{\langle n_1 n_2(\eta_1,\eta_2)\rangle_{\text{mixed}}}-1$$

Greater flexibility w/ cuts

Evaluation of Normalized Cumulants Two Methods

- Method 1: Ratio of averages (Common Approach)
  - Measure pair yields (same and mixed) directly vs Δη.
  - Calculate R(Δη) by taking the ratio of same to mixed.

$$R_M(\Delta\eta) = \frac{\frac{1}{\Omega(\Delta\eta)}\int_{accept} \rho_2(\Delta\eta,\bar{\eta})\,d\bar{\eta}}{\frac{1}{\Omega(\Delta\eta)}\int_{accept} \rho_1 \otimes \rho_1(\Delta\eta,\bar{\eta})\,d\bar{\eta}} - 1$$



- **Method 2: Average of Ratio**
  - Measure R(η_1,η_2) by taking the ratio of same to mixed.
  - Average out $\bar{\eta}$ dependence, i.e. project onto Δη to get R(Δη)

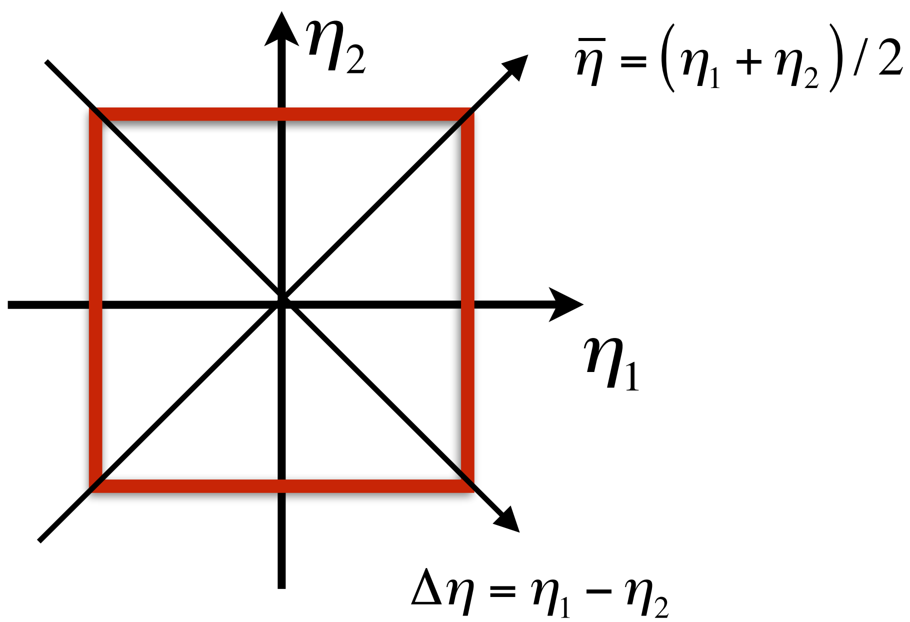
$$R_M(\Delta\eta) = \frac{1}{\Omega(\Delta\eta)}\int_{accept} R_2(\Delta\eta,\bar{\eta})\,d\bar{\eta} = \frac{1}{\Omega(\Delta\eta)}\int_{accept}\left(\frac{\rho_2(\Delta\eta,\bar{\eta})}{\rho_1 \otimes \rho_1(\Delta\eta,\bar{\eta})} - 1\right)d\bar{\eta}$$

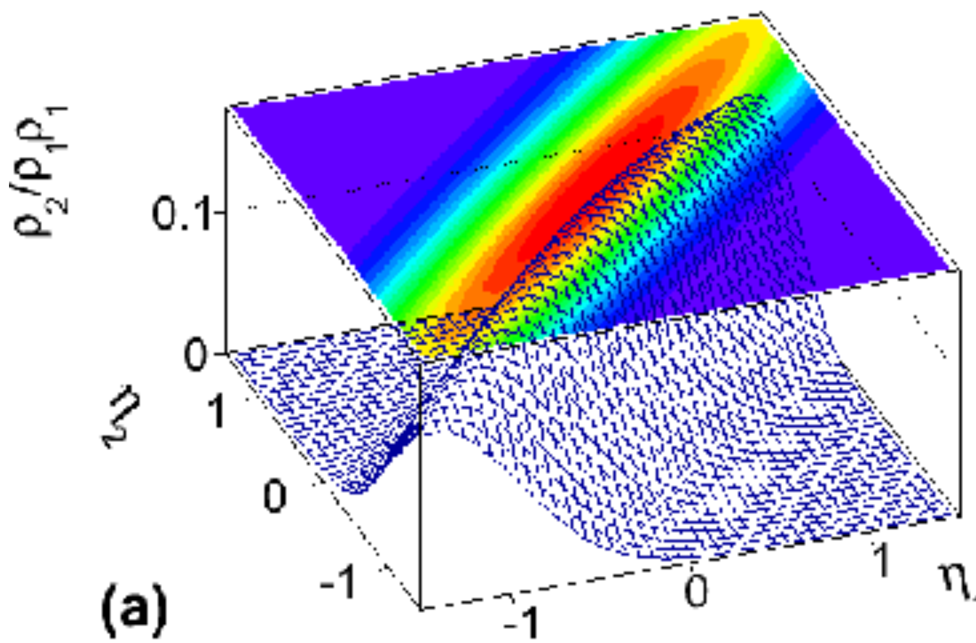
Method 1 vs. Method 2: Correlation Model

- Correlation Model:
  - Longitudinal Model w/ Two-particle emission correlated vs.

$$C(\Delta\eta,\bar{\eta}) \propto \exp\left(-\frac{\Delta\eta^2}{2\sigma^2_{\Delta\eta}}\right)\exp\left(-\frac{\bar{\eta}^2}{2\sigma^2_{\bar{\eta}}}\right)$$

- Assumed factorization of the dependence on the relative and average pseudorapidity.
- **Factorization may not be realized in practice**



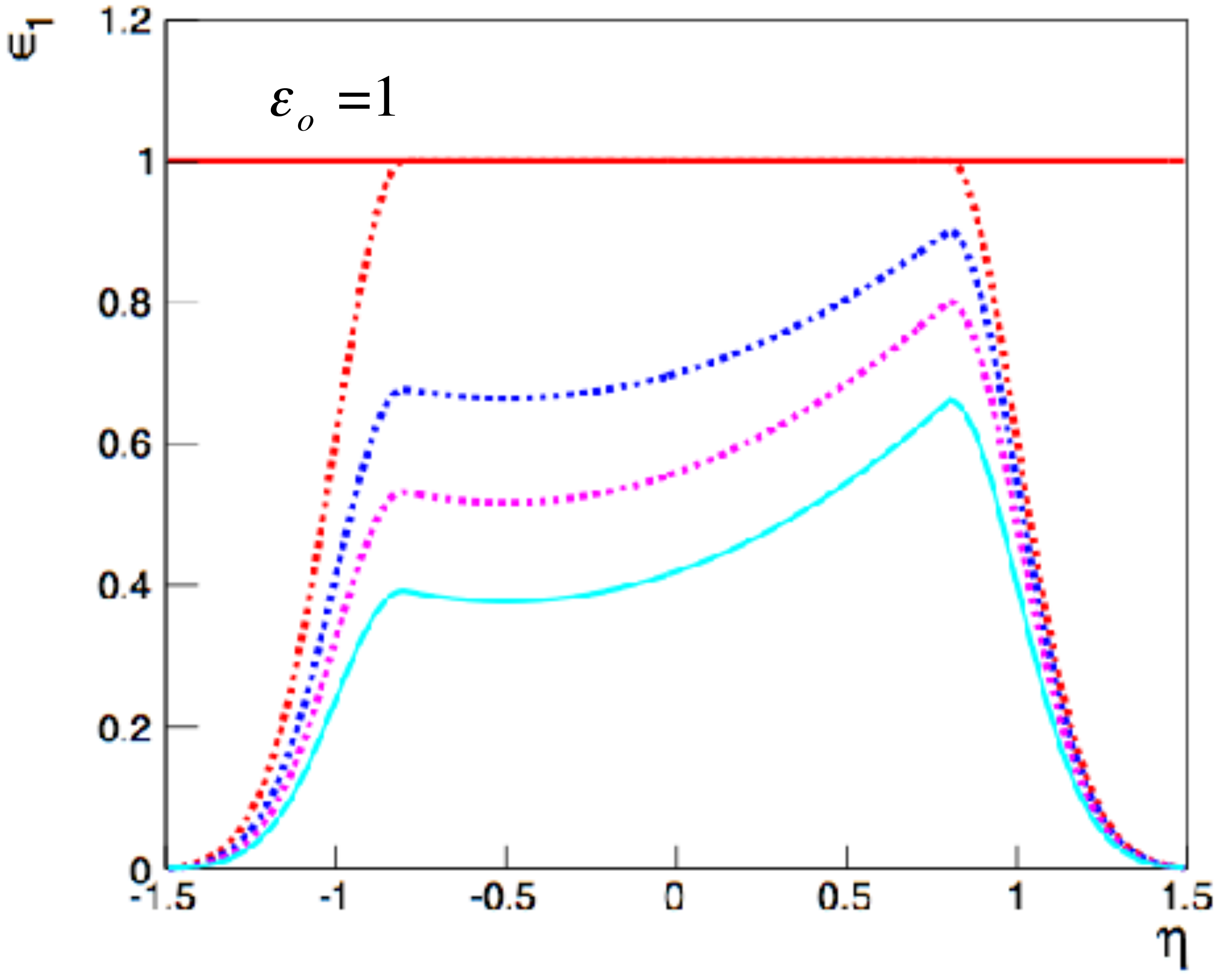


Method 1 vs. Method 2: Efficiency Model

- Use a simple but non trivial correlation model
- Use a simple model of the detection efficiency and edge effects.

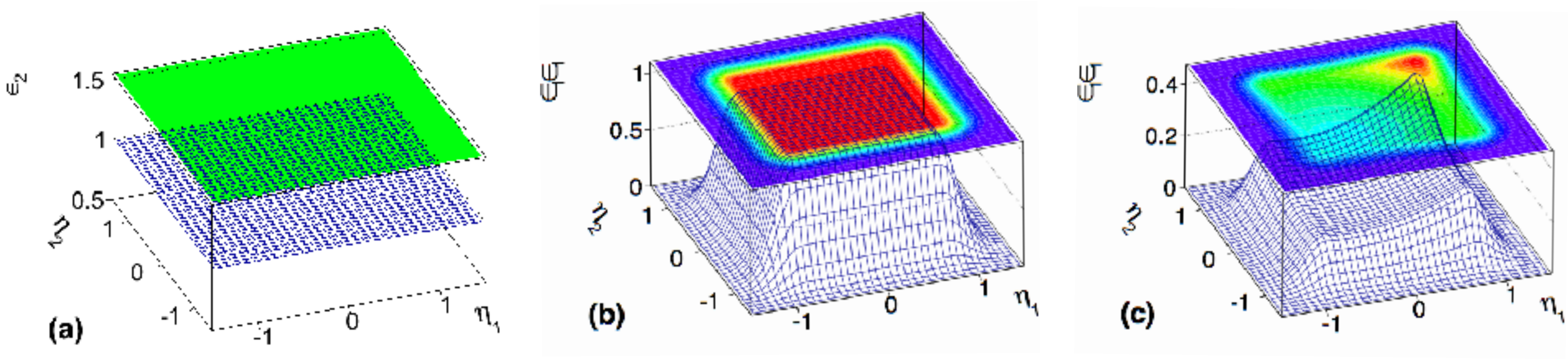
$$
\begin{aligned}
\varepsilon(\eta) &= \varepsilon_q(\eta)\exp\left(-\frac{(\eta-\eta_<)^2}{2\sigma_\varepsilon^2}\right) && \text{for} \quad \eta < \eta_< \\
&= \varepsilon_q(\eta) && \text{for} \quad \eta_< < \eta < \eta_> \\
&= \varepsilon_q(\eta)\exp\left(-\frac{(\eta-\eta_>)^2}{2\sigma_\varepsilon^2}\right) && \text{for} \quad \eta > \eta_>
\end{aligned}
$$

$$
\varepsilon_q(\eta) = 1 + \alpha(\eta-\eta_o) + \beta(\eta-\eta_o)^2
$$

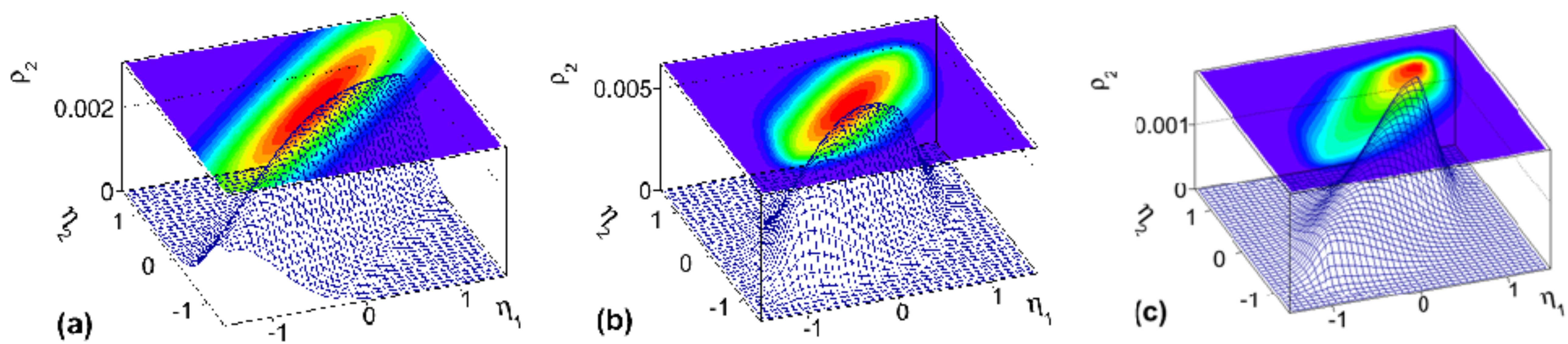


Efficiency, Pair Yield

- Efficiency

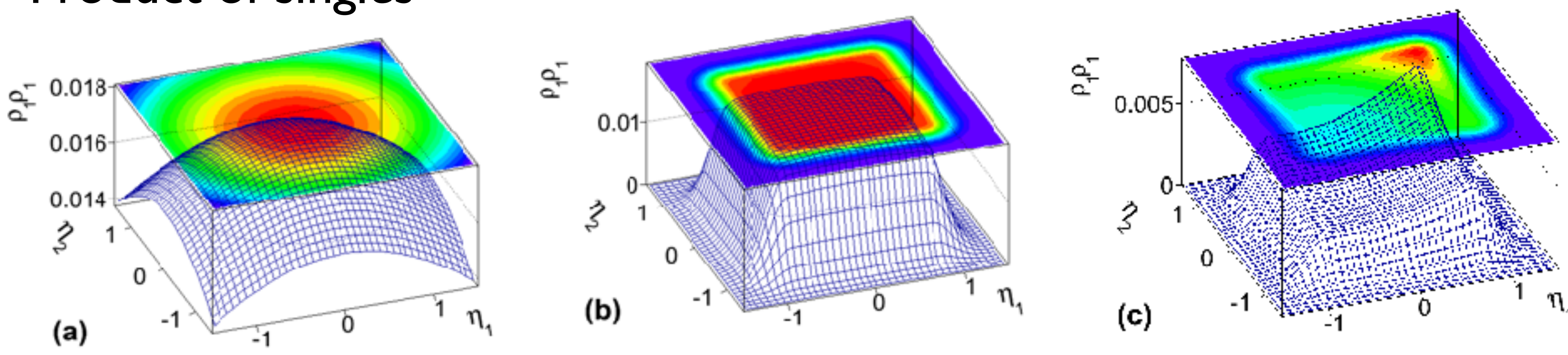


- Pair Yield



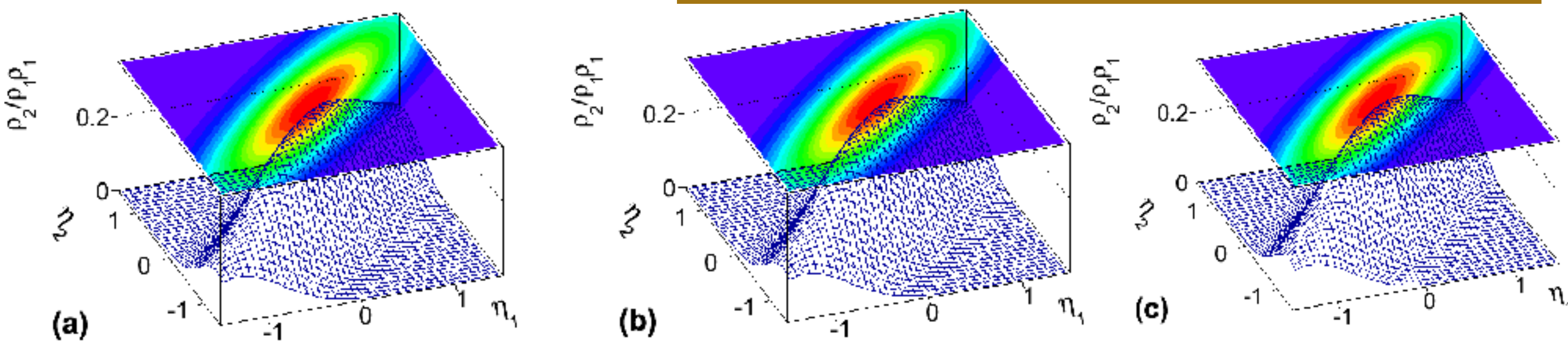
Method 2: Results

Product of singles

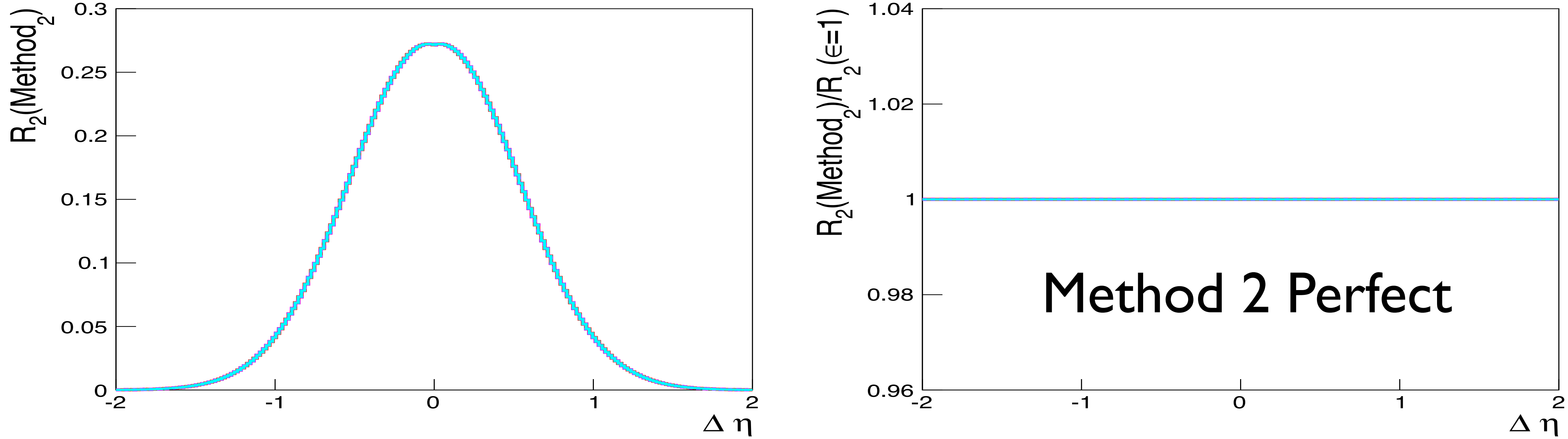


R2 (Method 2)

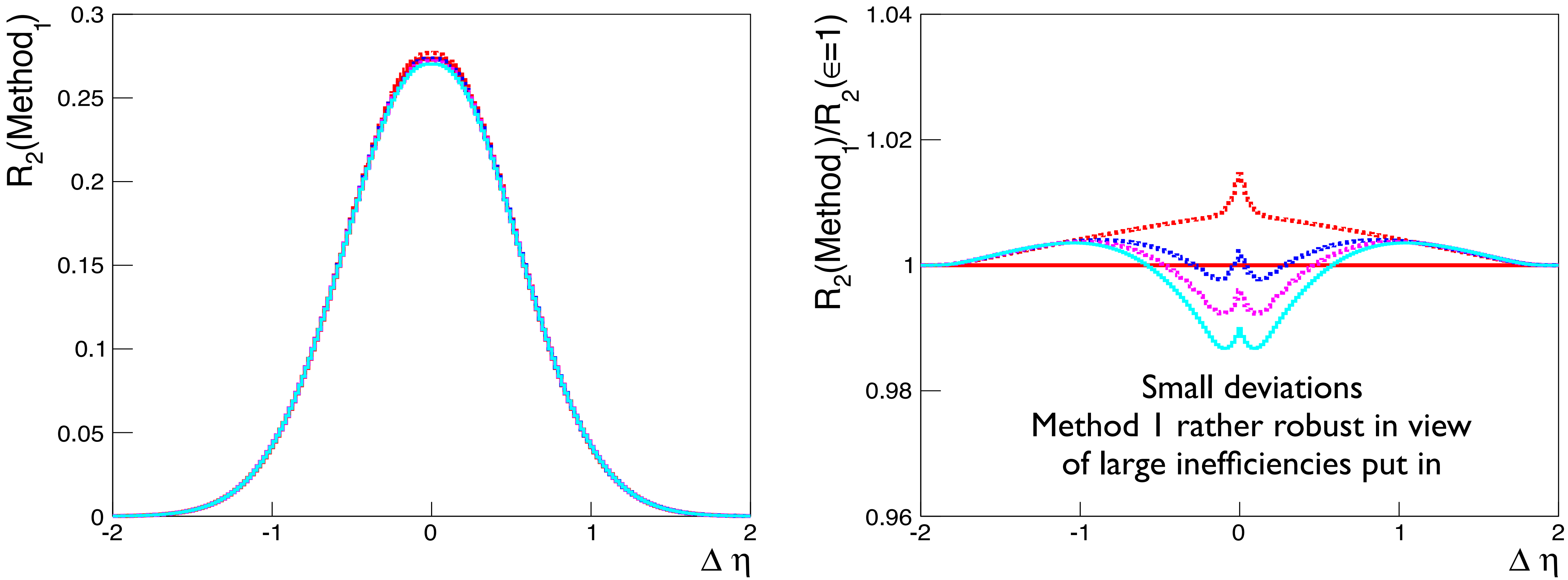
Perfect Reconstruction for any factorized efficient model w/ sufficient statistics



R2(Δη) Method 2



R2(Δη) Method 1



Why?

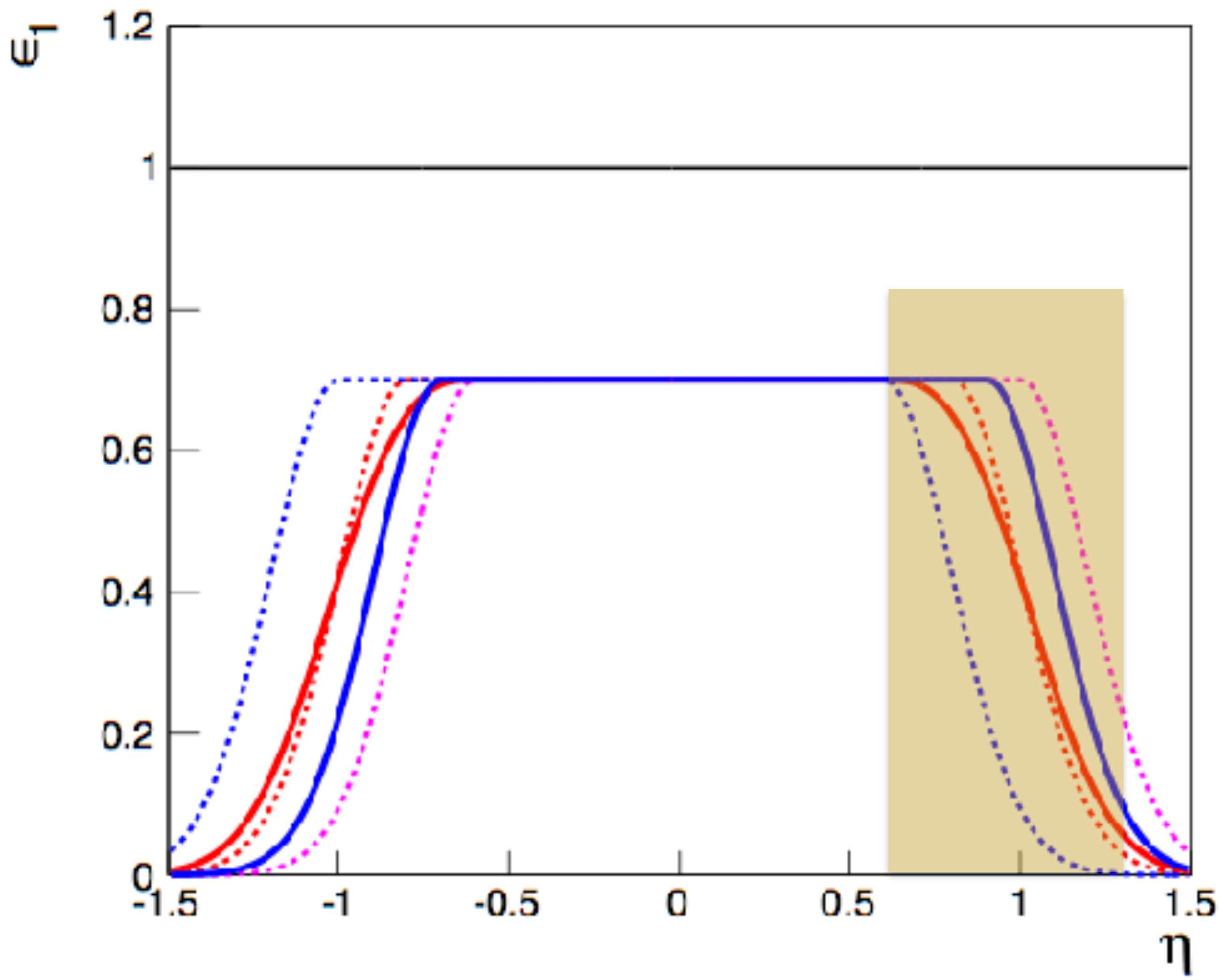
$$R_2(\Delta\eta)^{Method1} = \frac{\int g(\Delta\eta,\overline{\eta}) R_2^{true}(\Delta\eta,\overline{\eta}) d\overline{\eta}}{\int g(\Delta\eta,\overline{\eta}) d\overline{\eta}}$$

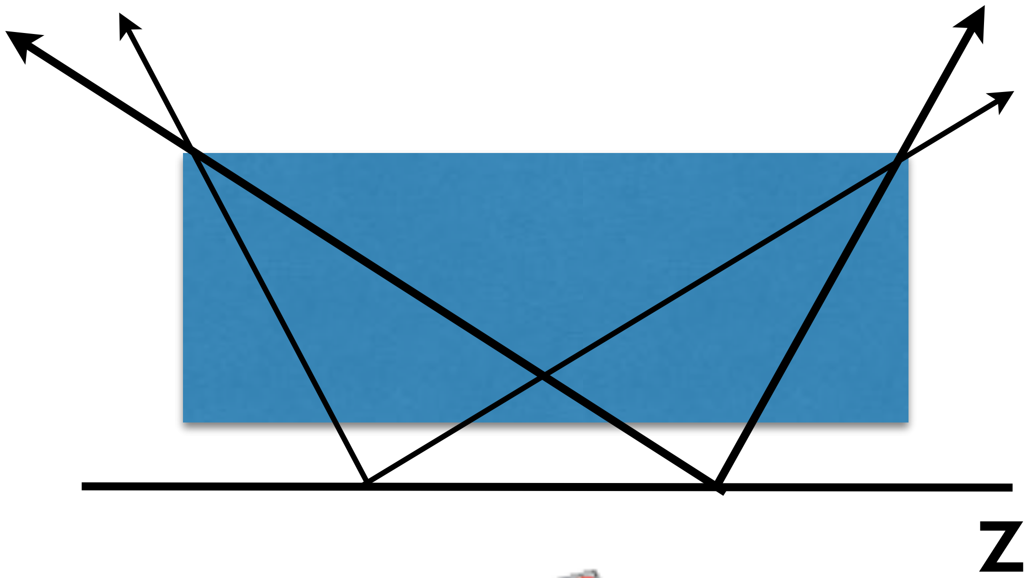
$$g(\Delta\eta,\overline{\eta}) = \varepsilon_1(\eta_1)\rho_1(\eta_1) \otimes \varepsilon_1(\eta_2)\rho_1(\eta_2)$$

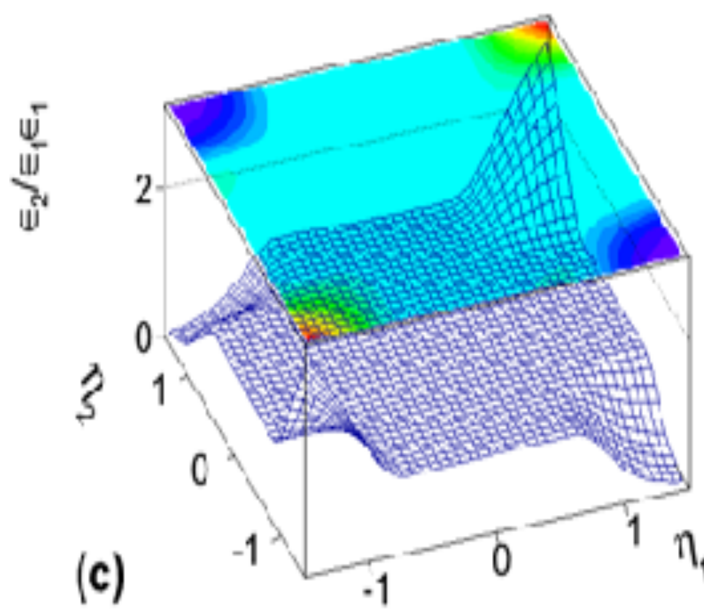
- If efficiency, yield, or correlation varies with avg-rapidity, then g or R2 cannot be factorized out of the integrals.
  - The numerator and denominator are in general NOT equal.
- Method 1 is only approximately robust - for slow varying functions
- Note: not a problem in azimuthal correlation because of periodic boundary conditions.

Dependence on z-vertex

- ALICE, STAR Acceptances are functions of the vertex position.
- Use a simple model as before...







S. Ravan, P. Pujahari, S. Prasad, C.A.P., Phys.Rev. C89 (2014) 024906

Efficiency Factorization???

- Local Factorization:

$$\epsilon_{pair}(\eta_1, \eta_2|z) = \epsilon_1(\eta_1|z) \times \epsilon_1(\eta_2|z)$$

- Loss of "Global" Factorization:

$$\begin{aligned}
\langle n_1(\eta_1)\rangle &= K\int_{z_{min}}^{z_{max}} P_c(z)\epsilon(\eta_1|z)\langle N_1(\eta_1)\rangle dz = \langle N_1(\eta_1)\rangle f_1(\eta_1) \\
\langle n_2(\eta_1,\eta_2)\rangle &= K\int_{z_{min}}^{z_{max}} P_c(z)\epsilon(\eta_1|z)\times\epsilon(\eta_2|z)\langle N_2(\eta_1,\eta_2)\rangle dz = \langle N_2(\eta_1,\eta_2)\rangle f_2(\eta_1,\eta_2)
\end{aligned}$$

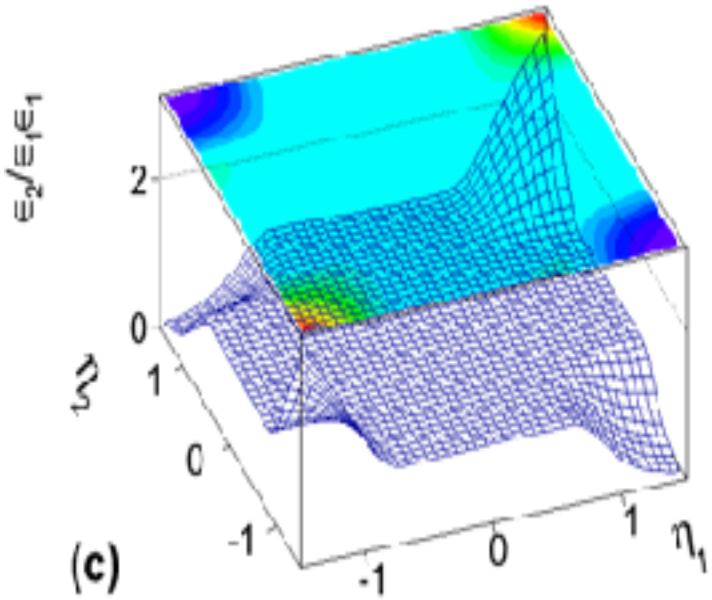
$$\begin{aligned}
f_1(\eta_1) &= K\int_{z_{min}}^{z_{max}} P_c(z)\epsilon(\eta_1|z)dz \\
f_2(\eta_1,\eta_2) &= K\int_{z_{min}}^{z_{max}} P_c(z)\epsilon(\eta_1|z)\times\epsilon(\eta_2|z)dz
\end{aligned}$$

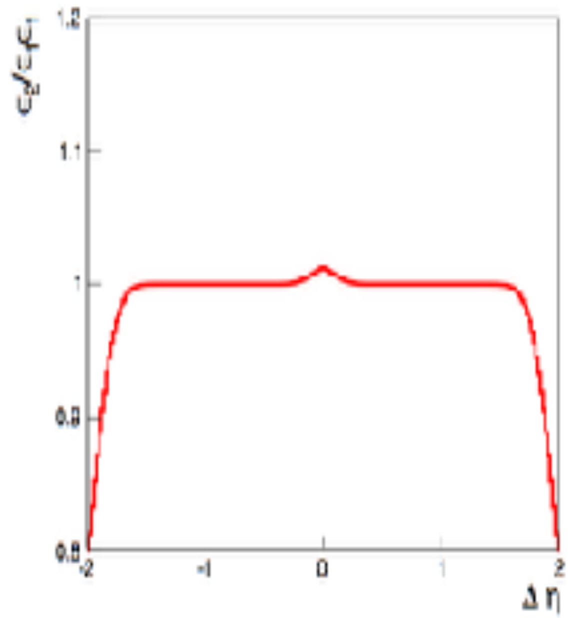
$$K^{-1} = \int_{z_{min}}^{z_{max}} P_c(z)dz$$

$$R_2(\eta_1,\eta_2) = \frac{f_2(\eta_1,\eta_2)}{f_1(\eta_1)f_1(\eta_2)}\frac{\langle N_2(\eta_1,\eta_2)\rangle}{\langle N_1(\eta_1)\rangle\langle N_1(\eta_2)\rangle}$$

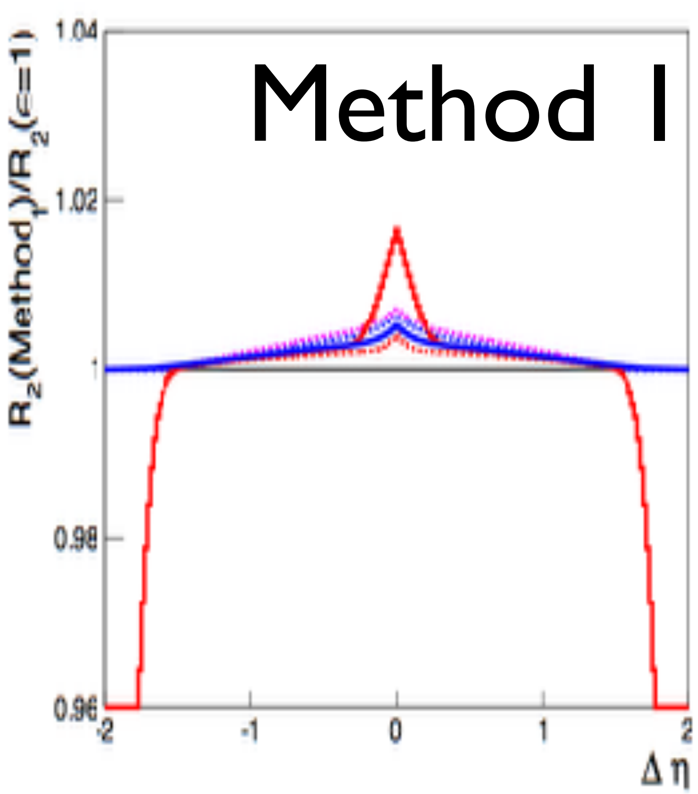
Neither Methods Robust

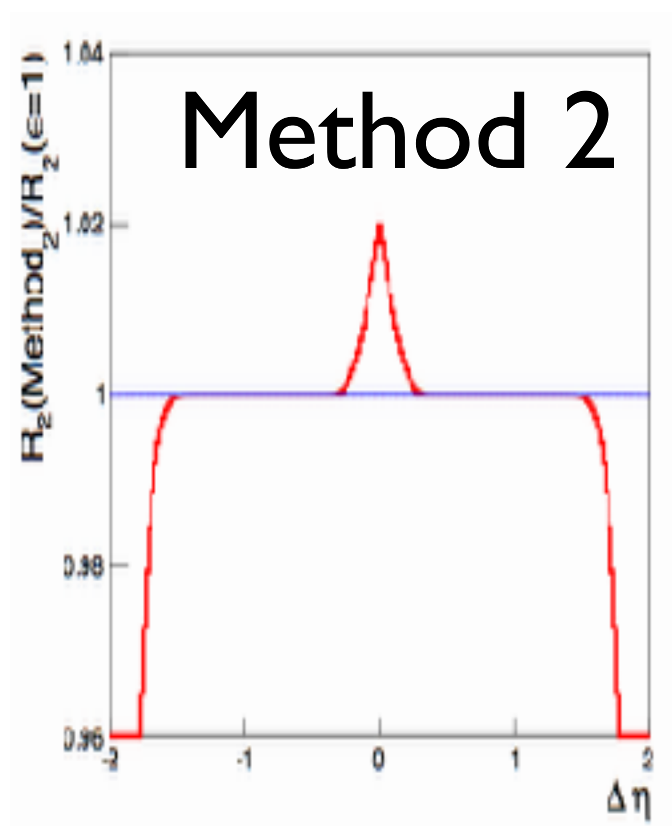
How this works…





Efficiency dependence on “z-vertex”, with gaussian edges, but flat in the fiducial volume.





Recovery...

- Method 2:
  - Carry analysis in fine (narrow) z bins.
  - Apply local efficiency factorization.

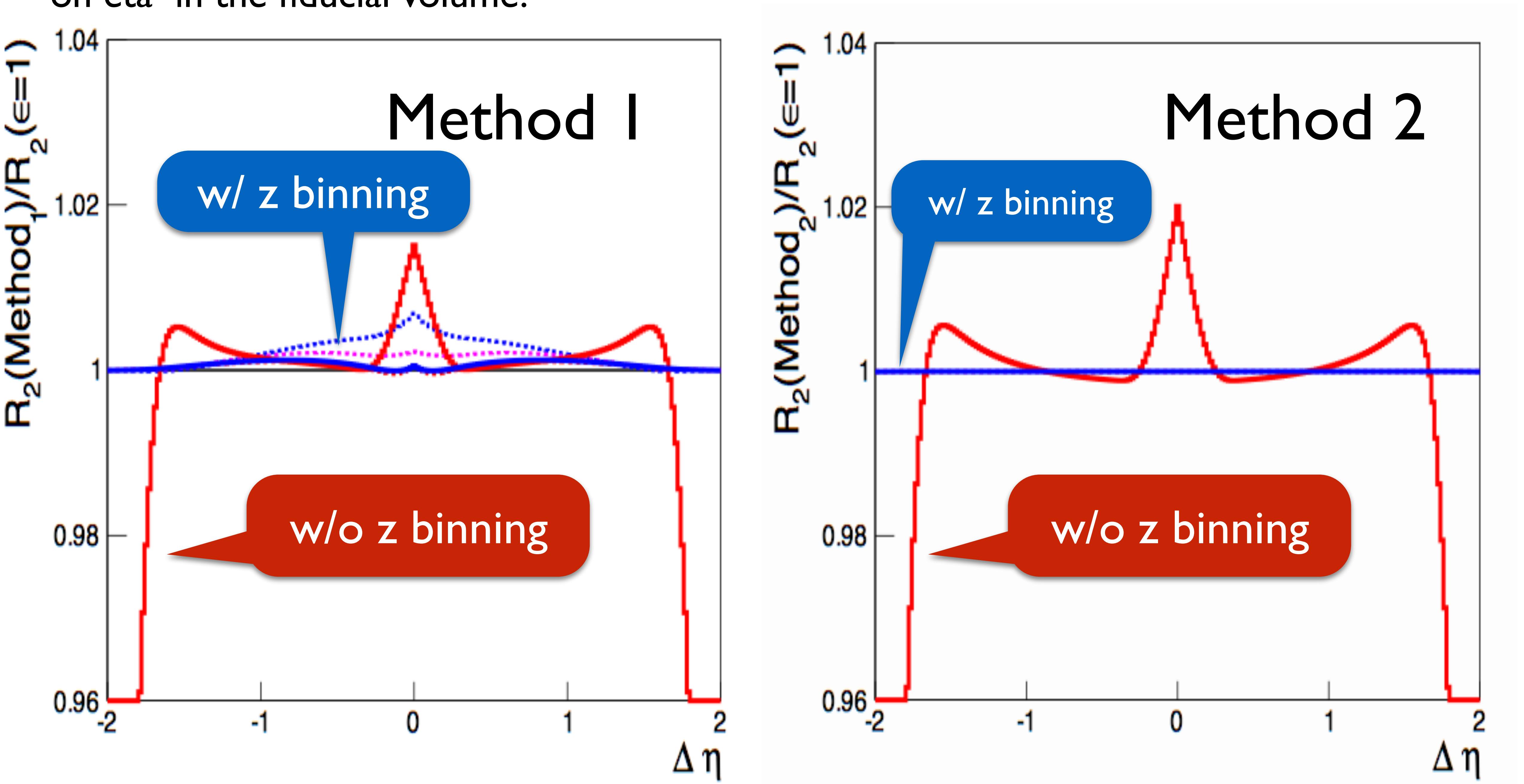
$$R_2(\eta_1,\eta_2) = K\int_{z_{min}}^{z_{max}} P_c(z) \times \frac{\epsilon(\eta_1|z)\times\epsilon(\eta_2|z)\langle N_2(\eta_1,\eta_2)\rangle}{\epsilon(\eta_1|z)\times\epsilon(\eta_2|z)\langle N_1(\eta_1)\rangle\langle N_1(\eta_2)\rangle} dz \qquad (49)$$

$$R_2(\eta_1,\eta_2) = K\frac{\langle N_2(\eta_1,\eta_2)\rangle}{\langle N_1(\eta_1)\rangle\langle N_1(\eta_2)\rangle}\int P_c(z)dz = \frac{\langle N_2(\eta_1,\eta_2)\rangle}{\langle N_1(\eta_1)\rangle\langle N_1(\eta_2)\rangle}$$

- Method 1:
  - This "recipe" not strictly valid for method 1

Method 1 and 2

Efficiency dependence on “z-vertex”, with gaussian edges, but **quadratic** dependence on eta in the fiducial volume.



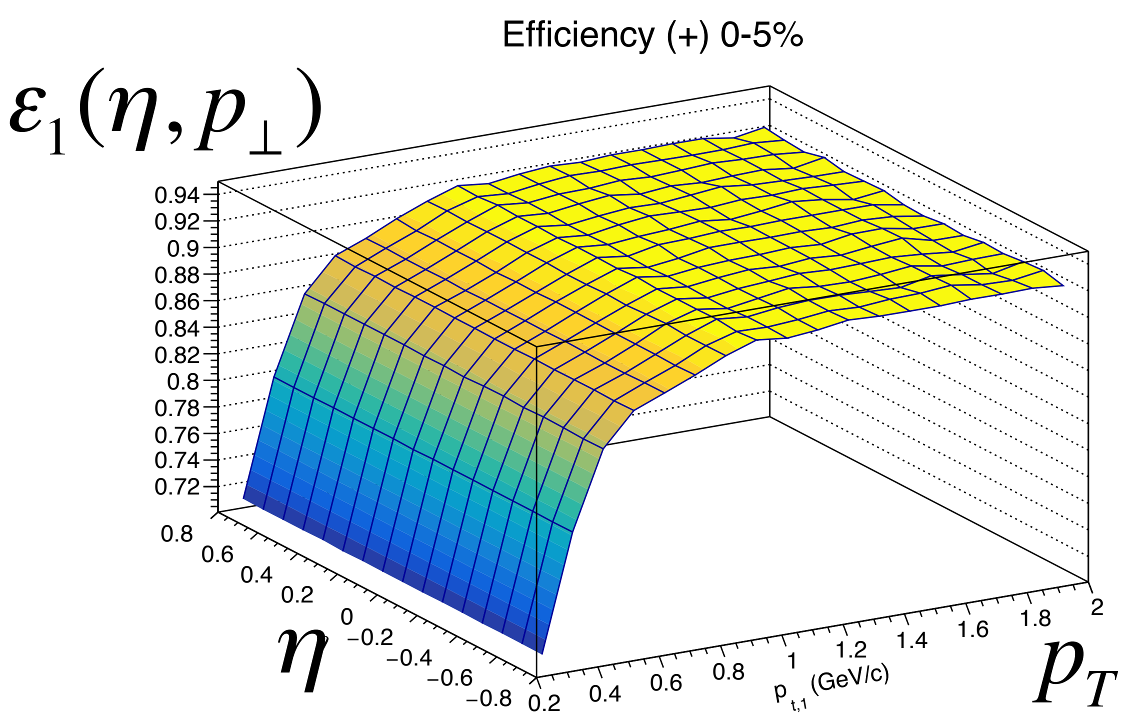
Both methods fail if efficiency is dependent on “z”.

Approximate recovery with fine z-bins using Method 1

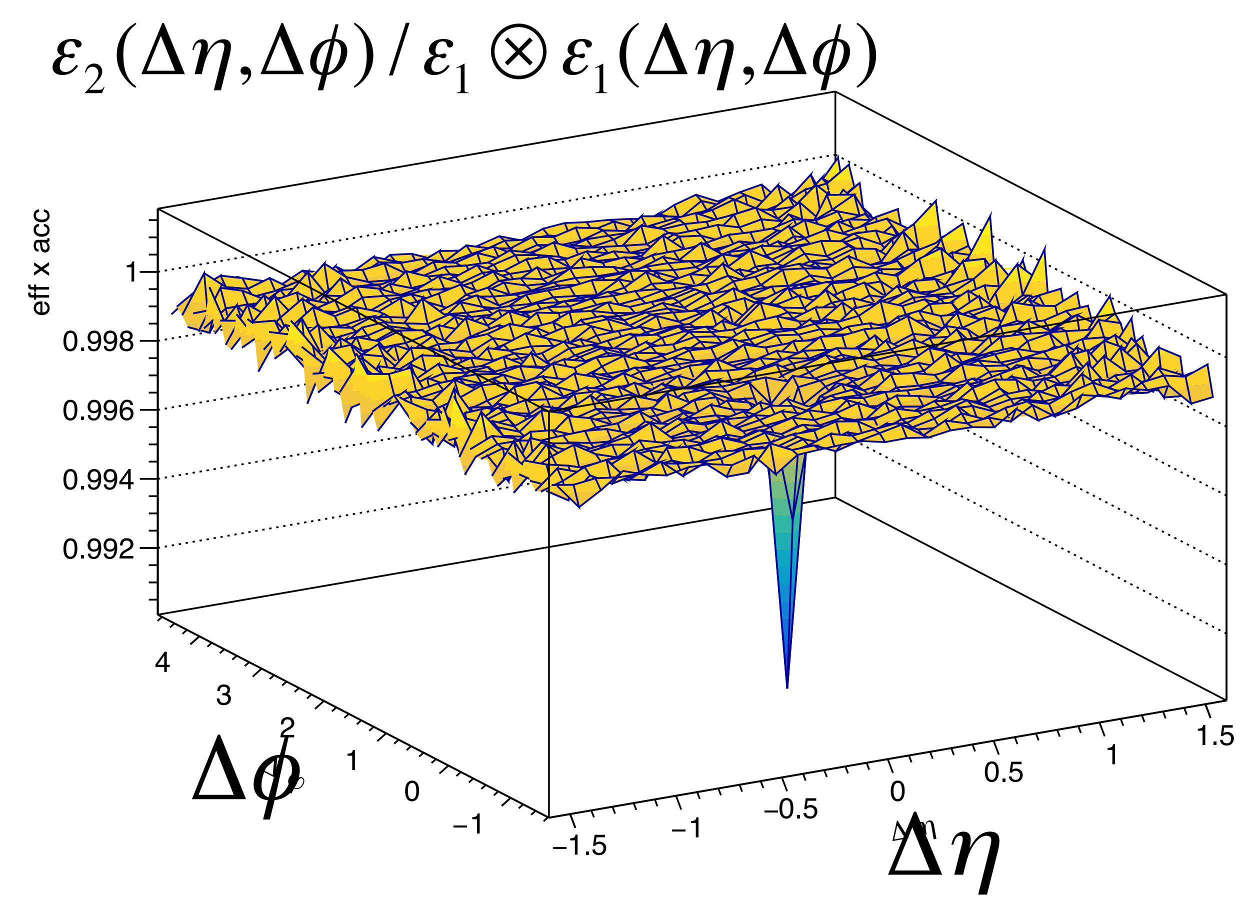
Complete recovery with fine z-bins using Method 2

What about momentum dependence and pair losses

- Example from ALICE detector
- Determined from HIJING events propagated through detector simulation with GEANT and detector response simulator



Single Particle Efficiency



Pair Efficiency

- **Technique:**
  - Discretize the densities in pT also
  - Use eta, phi, pT dependent weights proportional to inverse of efficiencies.

Part II: Conclusions

- Method 2 Robust
  - unless efficiency has dependence on z-vertex
  - but recovery possible for analysis in narrow z-bins
- Method 1 Only Approximately Robust
  - Robustness lost if singles, correlation, or efficiency are function of avg-eta
  - Approximate Robustness lost if dependence on z-vertex
  - "Partial" recovery possible for analysis in narrow z-bins

- **Bigger Point:**
  - With differential correlations, it is possible to identify detector features more readily than with integral correlations.
  - Integral correlations average over detector issues, they DO NOT eliminate them.

Part III: The Identity Method - Revisited/Extended

C.A.P, arXiv:1706.01333

- Context:
  - Identity Method developed by **Marek Gazdzicki** to study fluctuations of the multiplicity of particle species w/o loss of statistics and in cases where the PID signals (dE/dx, TOF, or mass) associated with different species might overlap significantly and lead to considerable ambiguities.
  - Method developed first for PID problems involving two species (A Method to study chemical equilibration in nucleus-nucleus collisions), Eur. Phys. J. C8 (1999) 131.
  - Extended to "n" species by **M. Gorenstein** (Identity Method for Particle Number Fluctuations and Correlations) Phys. Rev. C84 (2011) 024902.
  - Extended to "n" species and arbitrary orders by **A. Rustamov** and **M. Gorenstein** (Identity Method for Moments of Multiplicity Distribution) Phys. Rev. C86 (2012) 044906.
  - BUT: Method outlined in these papers does not explicitly account for particle losses.
- C.A.P, **arXiv:1706.01333**
  - **Identity method extended to account for particle losses.**
- **This presentation:**
  - **Extending identity method to differential correlation functions.**

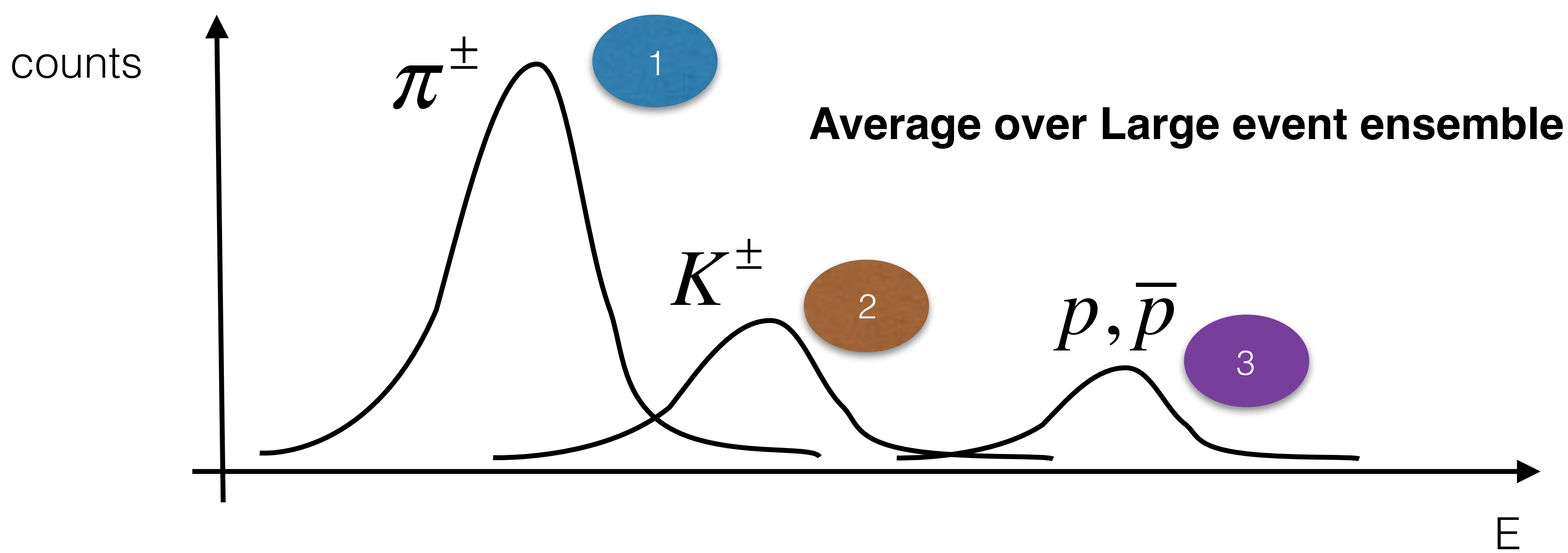
PID Disambiguation ….

- Goal: Measure moments of e-by-e yield fluctuations of different species

$$\nu_{dyn} = \frac{\langle N_1(N_1-1)\rangle}{\langle N_1\rangle^2} + \frac{\langle N_2(N_2-1)\rangle}{\langle N_2\rangle^2} - \frac{\langle N_1 N_2\rangle}{\langle N_1\rangle\langle N_2\rangle}$$

  - where N_1, N_2: number of particles of types “1” and “2” measured e-by-e,

- Species identified with some PID detector that produces a signal E with distributions that are nominally distinct for different species but some overlap.



- Tracks may be lost because of detector or algorithm artifacts. The detection efficiency ε_i is species dependent - and momentum dependent.

Moments of produced/measured multiplicities

- Probability of producing multiplicities $N_1, N_2, \ldots N_k$ of all species is denoted by $P_T(N_1, N_2, \ldots N_k)$
- Probability of measuring multiplicities $n_1, n_2, \ldots n_k$ of all species is denoted by $P_M(n_1, n_2, \ldots n_k)$
- Moments are calculated according to

$$\langle N_p \rangle \equiv \sum_{N_1, N_2, \ldots, N_K}^{\infty} P_T(N_1, N_2, \ldots, N_K) N_p \qquad \langle n_p \rangle \equiv \sum_{n_1, n_2, \ldots, n_K}^{\infty} P_M(n_1, n_2, \ldots, n_K) n_p$$

$$\langle N_p^2 \rangle \equiv \sum_{N_1, N_2, \ldots, N_K}^{\infty} P_T(N_1, N_2, \ldots, N_K) N_p^2 \qquad \langle n_p^2 \rangle \equiv \sum_{n_1, n_2, \ldots, n_K}^{\infty} P_M(n_1, n_2, \ldots, n_K) n_p^2$$

$$\langle N_p N_q \rangle \equiv \sum_{N_1, N_2, \ldots, N_K}^{\infty} P_T(N_1, N_2, \ldots, N_K) N_p N_q \qquad \langle n_p n_q \rangle \equiv \sum_{n_1, n_2, \ldots, n_K}^{\infty} P_M(n_1, n_2, \ldots, n_K) n_p n_q$$

and similarly for higher moments …

Moments of measured multiplicities (2)

- Particle Loss Model:

$$P_M(n_1, n_2, \ldots, n_k) = \sum_{N_1=0}^{\infty} \sum_{N_2=0}^{\infty} \cdots \sum_{N_k=0}^{\infty} P_T(N_1, N_2, \ldots, N_k) B(n_1|N_1, \varepsilon_1) B(n_2|N_2, \varepsilon_2) \times \cdots \times B(n_k|N_k, \varepsilon_k).$$

- Calculations yields:

$$\begin{aligned} \langle n_p \rangle &= \varepsilon_p \langle N_p \rangle \\ \langle n_p^2 \rangle &= \varepsilon_p (1 - \varepsilon_p) \langle N_p \rangle + \varepsilon_p^2 \langle N_p^2 \rangle \\ \langle n_p n_q \rangle &= \varepsilon_p \varepsilon_q \langle N_p N_q \rangle. \end{aligned}$$

$$\langle n_p (n_p - 1) \rangle = \varepsilon_p^2 \langle N_p (N_p - 1) \rangle.$$

Factorial Moments are well behaved

$$R_2 = \frac{\langle n_p (n_p - 1) \rangle}{\langle n_p \rangle^2} = \frac{\langle N_p (N_p - 1) \rangle}{\langle N_p \rangle^2}$$

Robust Ratio (R2)

$$\nu_{dyn}^{(measured)} = \nu_{dyn}^{(true)}$$

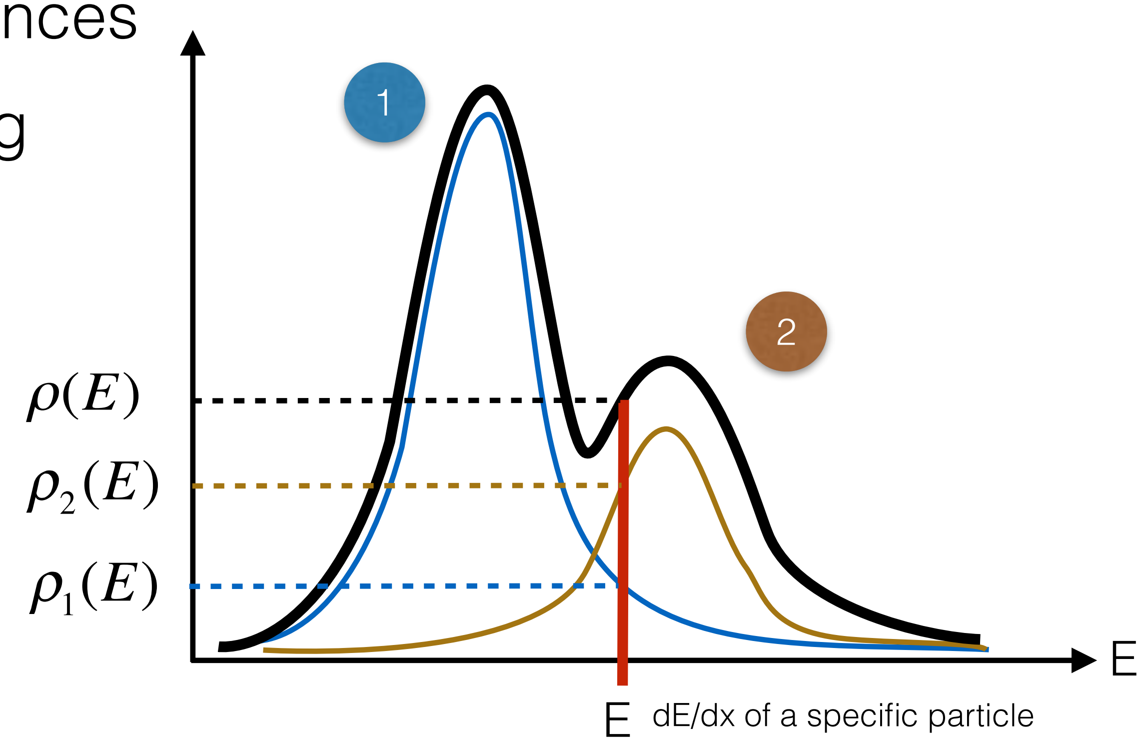
Nu-dyn is ROBUST

However: Must discretize measurement to correct for efficiency momentum dependence (weight technique)

Identity Method: PID weight

- Counting probabilities rather than integer instances
- Assume one knows the probability of observing **signal E** given **species "p"**: $P_p(E)$
- Define the Line Shape (Density):

$$\rho_p(E) = \langle n_p \rangle P_p(E)$$



- Normalization:

$$\rho(E) = \sum_{p=1}^{K} \rho_p(E) = \sum_{p=1}^{K} \langle n_p \rangle P_p(E)$$

- Probability of signal being due to species "p" given PID signal E.

$$P(p \mid E) = \frac{\rho_p(E)}{\rho(E)} \equiv \omega_p(E)$$

For each particle and each candidate species

In the absence of overlap (unambiguous PID): $\omega_p(E) = 0 \text{ or } 1$

In most cases (ambiguous PID): $0 \le \omega_p(E) \le 1$

Identity Variable

- Definition: Identity Variable (particle and event):

$$W_p = \sum_{i=1}^{n} \omega_p(E_i)$$

For each of the K candidate species: sum weights of all n particles of an event.

- In the absence of overlap (unambiguous PID):

$$W_1 = n_1$$ n_1 particles of type 1

$$W_2 = n_2$$ n_2 particles of type 2

$\ldots$ $\quad n = n_1 + n_2 + n_K$

- w/ ambiguous PID:

$$0 \le W_p \le n$$

Identity Variables Moments

Calculation of 1st, 2nd, and covariances of the identity variables

1st Moments: $$\langle W_p \rangle = \frac{1}{N_{events}} \sum_{i=1}^{N_{events}} W_p^{(i)}$$

2nd Moments: $$\langle W_p{}^2 \rangle = \frac{1}{N_{events}} \sum_{i=1}^{N_{events}} \left(W_p^{(i)} \right)^2$$

Covariances: $$\langle W_p W_q \rangle = \frac{1}{N_{events}} \sum_{i=1}^{N_{events}} W_p^{(i)} W_q^{(i)}$$

These moments are **linear combinations of moments of the measured multiplicities** — **proportional to moments of the produced multiplicities**.

Identity Method w/o Losses (1)

- One finds:

$$\langle W_p \rangle = \sum_{i_1=1}^{n_1}\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty} P_T(n_1,n_2)u_{p1} + \sum_{i_2=1}^{n_2}\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty} P_T(n_1,n_2)u_{p2}$$

$$= \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty} P_T(n_1,n_2)n_1 u_{p1} + \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty} P_T(n_1,n_2)n_2 u_2$$

$$= u_{p1}\langle n_1 \rangle + u_{p2}\langle n_2 \rangle$$

Linear Equations

$$\langle W_1 \rangle = u_{11}\langle n_1 \rangle + u_{12}\langle n_2 \rangle$$
$$\langle W_2 \rangle = u_{21}\langle n_1 \rangle + u_{22}\langle n_2 \rangle$$

Example for two species

Matrix Form

$$\begin{pmatrix} \langle W_1 \rangle \\ \langle W_2 \rangle \end{pmatrix} = \begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix} \begin{pmatrix} \langle n_1 \rangle \\ \langle n_2 \rangle \end{pmatrix}$$

Matrix Inversion

$$\begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix}^{-1} \begin{pmatrix} \langle W_1 \rangle \\ \langle W_2 \rangle \end{pmatrix} = \begin{pmatrix} \langle n_1 \rangle \\ \langle n_2 \rangle \end{pmatrix}$$

Identity Method with Efficiency Losses (2)

$$\langle W_p \rangle = \sum_{N_1=0}^{\infty}\sum_{N_2=0}^{\infty} P_T(N_1,N_2)\left[u_{p1}\varepsilon_1 N_1 + u_{p2}\varepsilon_2 N_2\right] = u_{p1}\varepsilon_1\langle N_1\rangle + u_{p2}\varepsilon_2\langle N_2\rangle$$

$$\langle W_p^2 \rangle = \sum_{j=1}^{2} u_{pj}^2 \varepsilon_j \langle N_j\rangle + \sum_{j=1}^{2}(u_{pj})^2 \varepsilon_j^2 \langle N_j(N_j-1)\rangle + 2u_{p1}u_{p2}\varepsilon_1\varepsilon_2\langle N_1 N_2\rangle$$

$$\langle W_p W_q \rangle = \sum_{j=1}^{2} u_{pqj}\varepsilon_j\langle N_j\rangle + \sum_{j=1}^{2} u_{pj}u_{qj}\varepsilon_j^2\langle N_j(N_j-1)\rangle + (u_{p1}u_{q2} + u_{q1}u_{p2})\varepsilon_1\varepsilon_2\langle N_1 N_2\rangle$$

Trick is to absorb the efficiency and define N' quantities

$$\langle W_p^2 \rangle = \sum_{j=1}^{2} u_{pj}^2 \langle N'_j\rangle + \sum_{j=1}^{2}(u_{pj})^2 \langle N_j(N_j-1)'\rangle + 2u_{p1}u_{p2}\langle N'_1 N'_2\rangle$$

$$\langle W_p W_q \rangle = \sum_{j=1}^{2} u_{pqj}\langle N'_j\rangle + \sum_{j=1}^{2} u_{pj}u_{qj}\langle N_j(N_j-1)'\rangle + (u_{p1}u_{q2} + u_{q1}u_{p2})\langle N'_1 N'_2\rangle$$

One can then write

$$b_p = \langle W_p^2\rangle - \sum_{j=1}^{\kappa} u_{pj}^2\langle N'_j\rangle,$$

$$b_{pq} = \langle W_p W_q\rangle - \sum_{j=1}^{k} u_{pqj}\langle N'_j\rangle,$$

$$\begin{aligned}
a_j^i &= (u_{ij})^2, && 1 \le i, j, \le k;\\
a_i^{pq} &= 2u_{ip}u_{iq}, && 1 \le p < q \le k, i = 1,\dots,k;\\
a_{pq}^i &= u_{pi}u_{qi}, && 1 \le p < q \le k, i = 1,\dots,k;\\
a_{pq}^{ij} &= u_{pi}u_{qj} + u_{pj}u_{qi}, && 1 \le p < q \le k, 1 \le i < jk.
\end{aligned}$$

$$\mathbf{N} = \begin{pmatrix} \langle N_1(N_1-1)'\rangle \\ \vdots \\ \langle N_k(N_k-1)'\rangle \\ \langle N_1{}' N_2{}'\rangle \\ \vdots \\ \langle N_{k-1}{}' N_k{}'\rangle \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} b_1 \\ \vdots \\ b_k \\ b_{12} \\ \vdots \\ b_{(k-1)k} \end{pmatrix}$$

$$\mathbf{N} = \mathbf{A}^{-1}\mathbf{B}.$$

Differential Correlations (Discretization)

- Discretize momentum space (single particle level)
  - Rapidity: $-\eta_0 \le \eta \le \eta_0 \rightarrow \alpha = 1,\ldots,m_\alpha$
  - Azimuth: $0 \le \phi \le 2\pi \rightarrow \beta = 1,\ldots,m_\beta$
  - Transverse Momentum: $p_{\perp,\min} \le p_\perp \le p_{\perp,\max} \rightarrow \gamma = 1,\ldots,m_\gamma$
- Single Particle Density: $\rho_1^{(p)}(\eta,\phi,p_\perp) \rightarrow \hat{\rho}_1^{(p)}(\alpha,\beta,\gamma)$
- Pair Density: $\rho_2^{(pq)}(\eta_1,\phi_1,p_{\perp,1},\eta_2,\phi_2,p_{\perp,2}) \rightarrow \hat{\rho}_2^{(pq)}(\alpha_1,\beta_1,\gamma_1,\alpha_2,\beta_2,\gamma_2)$

$$\hat{\rho}_1^{(p)}(\alpha,\beta,\gamma) = \frac{\frac{1}{N_{events}} \sum_{i=1}^{N_{events}} \frac{N_{1,(i)}^{(p)}(\alpha,\beta,\gamma)}{\varepsilon_1^{(p)}(\alpha,\beta,\gamma)}}{\Delta\eta\Delta\phi\Delta p_\perp}$$

Detection Efficiency for Species (p)

$$\hat{\rho}_2^{(pq)}(\alpha_1,\beta_1,\gamma_1,\alpha_2,\beta_2,\gamma_2) = \frac{\frac{1}{N_{events}} \sum_{i=1}^{N_{events}} \frac{N_{1,(i)}^{(p)}(\alpha_1,\beta_1,\gamma_1)}{\varepsilon_1^{(p)}(\alpha_1,\beta_1,\gamma_1)} \frac{N_{1,(i)}^{(q)}(\alpha_2,\beta_2,\gamma_2)}{\varepsilon_1^{(q)}(\alpha_2,\beta_2,\gamma_2)}}{\Delta\eta^2\Delta\phi^2\Delta p_\perp{}^2}$$

use N(N-1) is all indices are equal.

Marginalization (integrating pT)

- Integrals replaced by sums…

$$\hat{\rho}_1^{(p)}(\alpha,\beta)=\sum_{\gamma=1}^{N_\gamma}\hat{\rho}_1^{(p)}(\alpha,\beta,\gamma)$$

$$\hat{\rho}_2^{(pq)}(\alpha_1,\beta_1,\alpha_2,\beta_2)=\sum_{\gamma_1,\gamma_2=1}^{m_\gamma}\hat{\rho}_2^{(pq)}(\alpha_1,\beta_1,\gamma_1,\alpha_2,\beta_2,\gamma_2)=\sum_{\gamma_1=1}^{m_\gamma}\hat{\rho}_2^{(pq)}(\alpha_1,\beta_1,\gamma_1,\alpha_2,\beta_2,\gamma_1)+\sum_{\gamma_1\neq\gamma_2=1}^{m_\gamma}\hat{\rho}_2^{(pq)}(\alpha_1,\beta_1,\gamma_1,\alpha_2,\beta_2,\gamma_2)$$

$$R_2^{(pq)}(\Delta\alpha,\Delta\beta)=\frac{\sum_{\alpha_1,\alpha_2}^{m_\alpha}\sum_{\beta_1,\beta_2}^{m_\beta}\hat{\rho}_2^{(pq)}(\alpha_1,\beta_1,\alpha_2,\beta_2)\delta\left(\Delta\alpha-\alpha_1+\alpha_2\right)\delta\left(\Delta\beta-\beta_1+\beta_2\right)}{\sum_{\alpha_1,\alpha_2}^{m_\alpha}\sum_{\beta_1,\beta_2}^{m_\beta}\hat{\rho}_1^{(p)}(\alpha_1,\beta_1)\hat{\rho}_1^{q}(\alpha_2,\beta_2)\delta\left(\Delta\alpha-\alpha_1+\alpha_2\right)\delta\left(\Delta\beta-\beta_1+\beta_2\right)}-1$$

Differential Identity Method w/ Losses

Moments of the identity variables in differential bins

$$\left\langle W_p(\alpha,\beta,\gamma)\right\rangle = \sum_{j=1}^{K} u_{pj}(\alpha,\beta,\gamma)\varepsilon_1^{(j)}(\alpha,\beta,\gamma)\left\langle N_1^{(j)}(\alpha,\beta,\gamma)\right\rangle$$

$$\begin{aligned}\left\langle W_p(\alpha,\beta,\gamma)^2\right\rangle = &\sum_{j=1}^{K} u_{pj}^{(2)}(\alpha,\beta,\gamma)\varepsilon_1^{(j)}(\alpha,\beta,\gamma)\left\langle N_1^{(j)}(\alpha,\beta,\gamma)\right\rangle \\ &+\sum_{j=1}^{K}\left[u_{pj}(\alpha,\beta,\gamma)\varepsilon_1^{(j)}(\alpha,\beta,\gamma)\right]^2\left\langle N_1^{(j)}(\alpha,\beta,\gamma)\left[N_1^{(j)}(\alpha,\beta,\gamma)-1\right]\right\rangle \\ &+\sum_{j\neq j'=1}^{K} u_{pj}(\alpha,\beta,\gamma)\varepsilon_1^{(j)}(\alpha,\beta,\gamma)u_{pj'}(\alpha,\beta,\gamma)\varepsilon_1^{(j')}(\alpha,\beta,\gamma)\left\langle N_1^{(j')}(\alpha,\beta,\gamma)\right\rangle\end{aligned}$$

$$\begin{aligned}\left\langle W_p(\alpha_1,\beta_1,\gamma_1)W_q(\alpha_2,\beta_2,\gamma_2)\right\rangle = &\sum_{j=1}^{K} u_{pqj}(\alpha,\beta,\gamma)\varepsilon_1^{(j)}(\alpha,\beta,\gamma)\left\langle N_1^{(j)}(\alpha,\beta,\gamma)\right\rangle \\ &+\sum_{j=1}^{K} u_{pj}(\alpha,\beta,\gamma)\varepsilon_1^{(j)}(\alpha,\beta,\gamma)u_{pj}(\alpha,\beta,\gamma)\varepsilon_1^{(j)}(\alpha,\beta,\gamma)\left\langle N_1^{(j)}(\alpha,\beta,\gamma)\left[N_1^{(j)}(\alpha,\beta,\gamma)-1\right]\right\rangle \\ &+\sum_{j\neq j'=1}^{K} u_{pj}(\alpha,\beta,\gamma)\varepsilon_1^{(j)}(\alpha,\beta,\gamma)u_{pj'}(\alpha,\beta,\gamma)\varepsilon_1^{(j')}(\alpha,\beta,\gamma)\left\langle N_1^{(j')}(\alpha,\beta,\gamma)\right\rangle\end{aligned}$$

Matrix Inversion provides differential moments: $N = A^{-1}B$

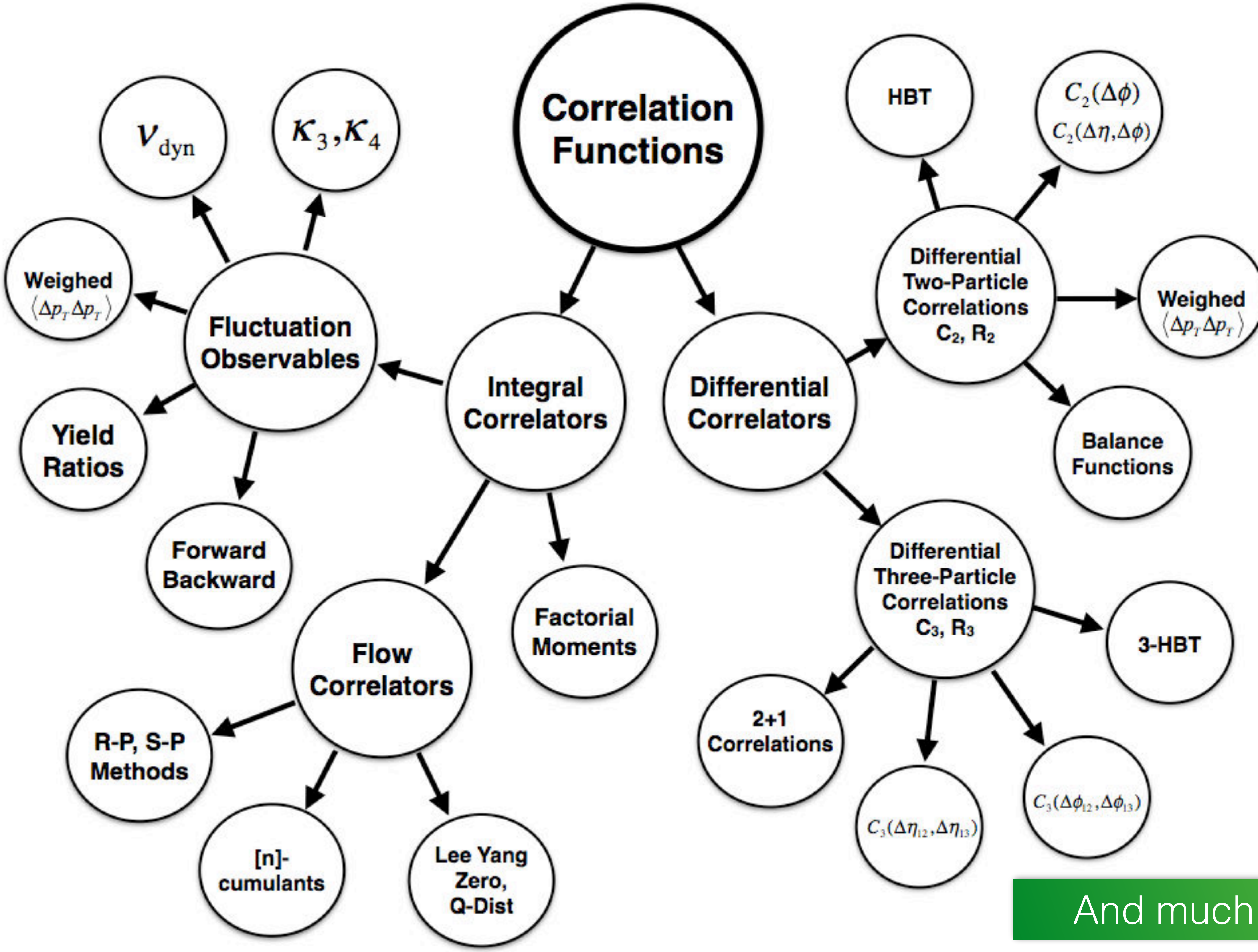
Part III: Summary

- The Identity method extended to account for rapidity, azimuth, and pT dependent detection efficiency…
  - for fluctuation measurements
  - for differential correlation measurements
  - flow measurements (not discussed here).

Summary

- Observing anomalies in the BES of fluctuations is rather challenging.
- I argued to
  - diversify observables used in the search
  - **include differential observables and their basic characteristics (amplitude, width) with system size, beam energy, etc.**
- I showed that differential correlations can be measured robustly — even when the efficiency is a complicated function of momentum or detector conditions..
- I extended the identity method
  - to account for particle losses — that vary across the acceptance.
  - to differential correlation measurements
    - maximize use of the accumulated statistics
    - extend pT range of differential correlation measurements

The Multiple Facets of Correlation Functions



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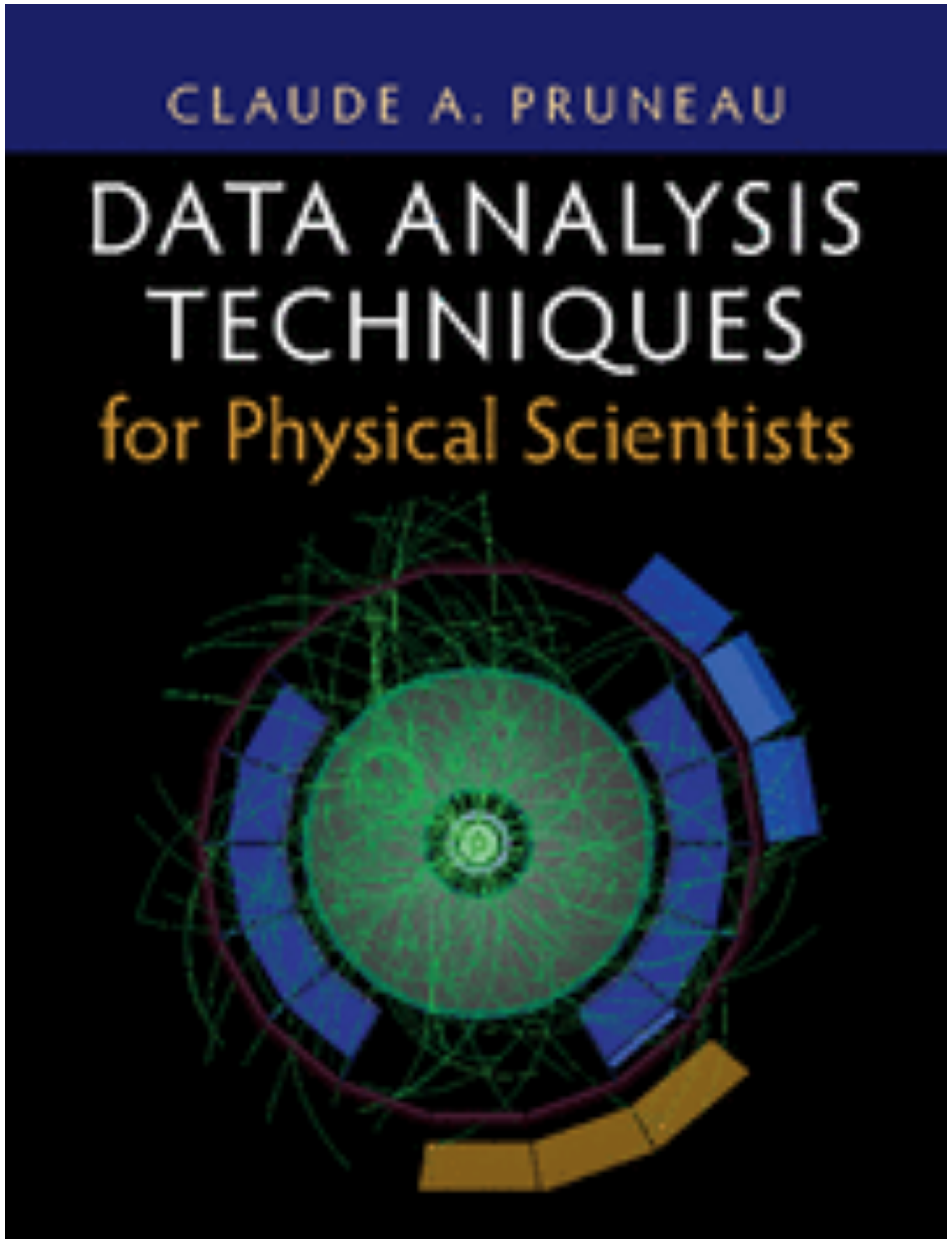

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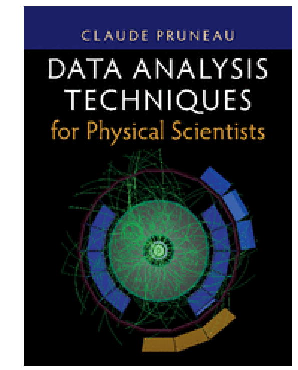
Claude A. Pruneau

Wayne State University, Michigan



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Preface; How to read this book; 1. The scientific method; Part I. Foundation in Probability and Statistics: 2. Probability; 3. Probability models; 4. Classical inference I: estimators; 5. Classical inference II: optimization; 6. Classical inference III: confidence intervals and statistical tests; 7. Bayesian inference; Part II. Measurement Techniques: 8. Basic measurements; 9. Event reconstruction; 10. Correlation functions; 11. The multiple facets of correlation functions; 12. Data correction methods; Part III. Simulation Techniques: 13. Monte Carlo methods; 14. Collision and detector modeling; List of references; Index.

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Peter Jacobs,
Lawrence Berkeley National Laboratory

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