EMMI workshop: Critical Fluctuations Near the QCD Phase Boundary
in Relativistic Nuclear Collisions, CCNU, Oct.10-13th, 2017

Investigate RHIC-BES physics with an extended AMPT model with mean-field potentials

Jun Xu (徐骏)

Shanghai Institute of Applied Physics,

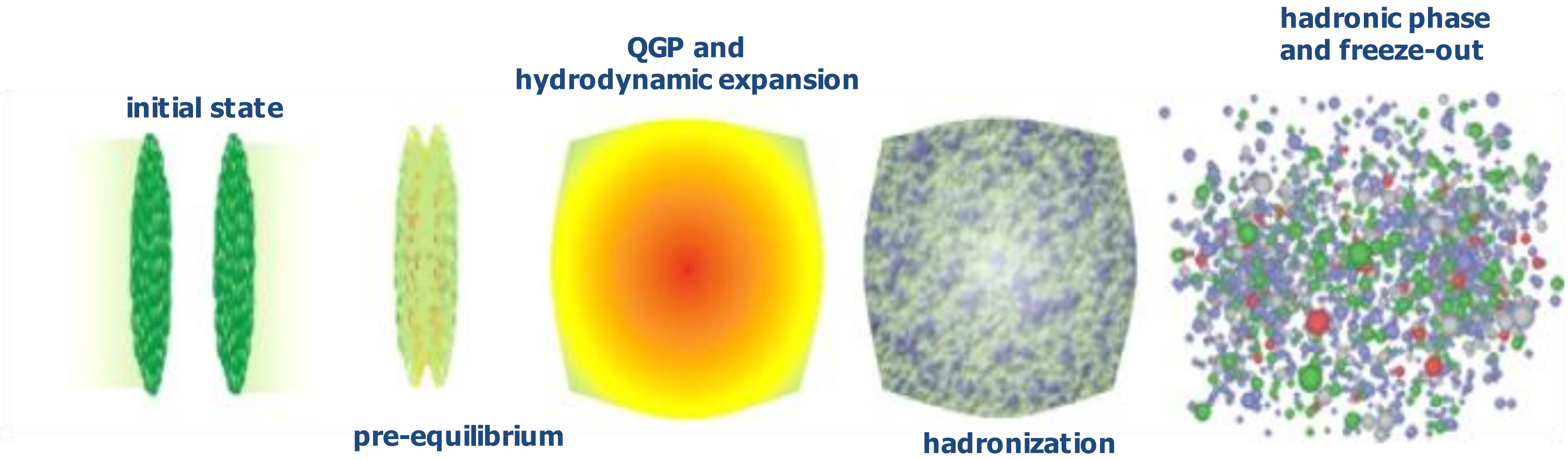
Chinese Academy of Sciences

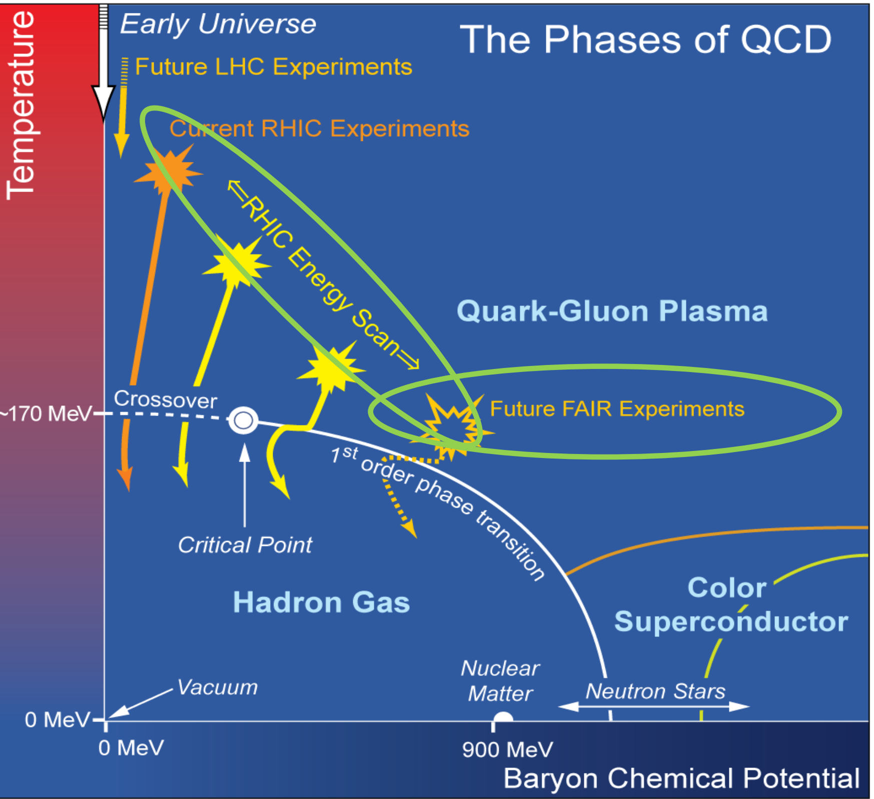
Collaborators：

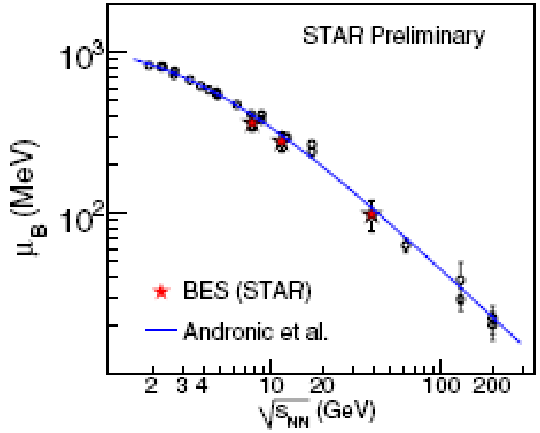
Che Ming Ko (TAMU), Zi-Wei Lin (ECU), Lie-Wen Chen (SJTU), Taesoo Song (Frankfurt am Main U), Feng Li (Frankfurt am Main U), Kai-Jia Sun (SJTU)

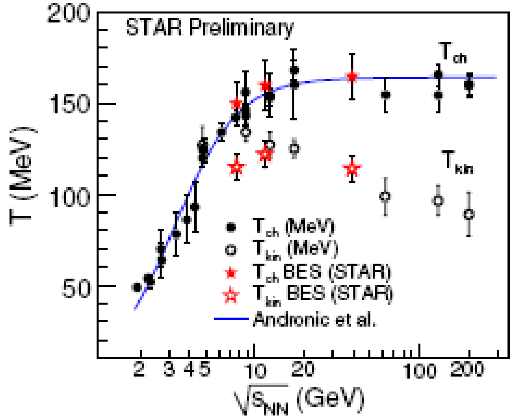
Students：

Chun-Jian Zhang (SINAP), He Liu (SINAP), Zhang-Zhu Han (SINAP), Chong-Qiang Guo (SINAP)







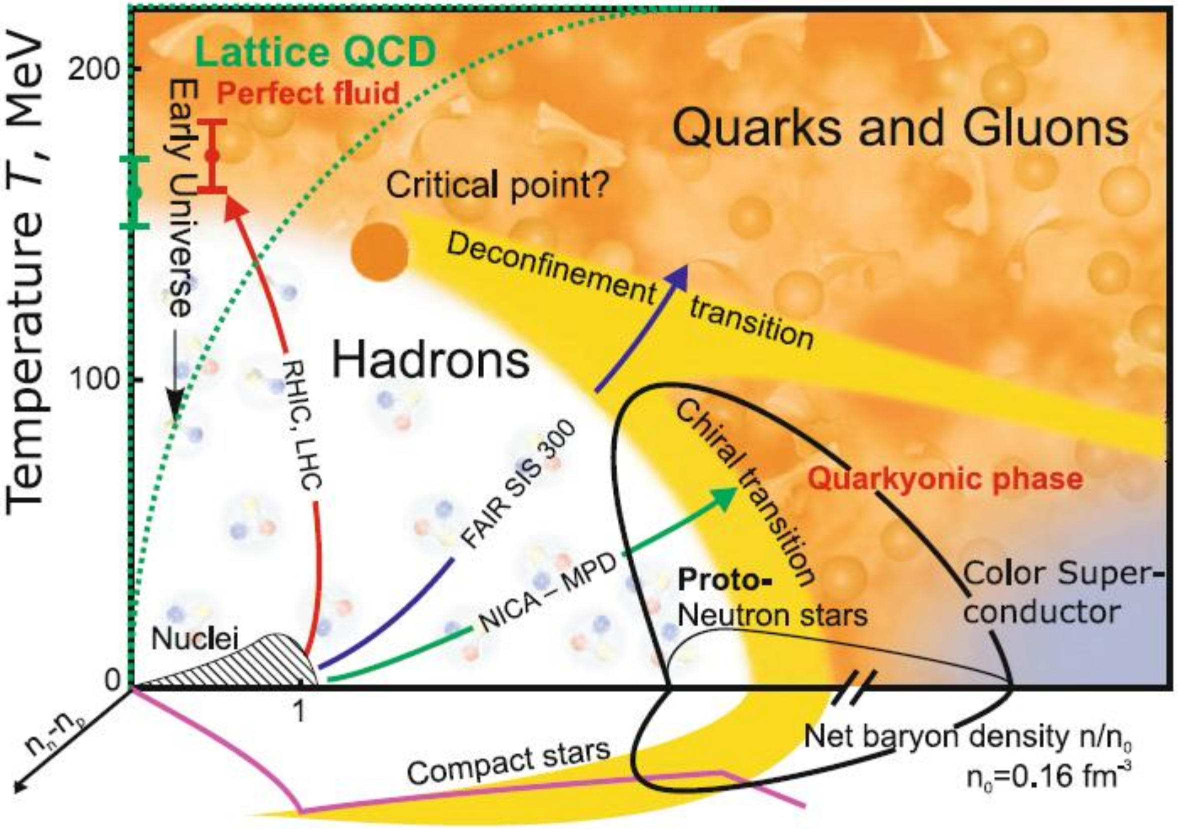


Search for signals of critical point at finite μ_B!

RHIC-BES:
$\sqrt{s}$ ~ 7.7-39 GeV

FAIR-CBM:
$\sqrt{s}$ <12 GeV

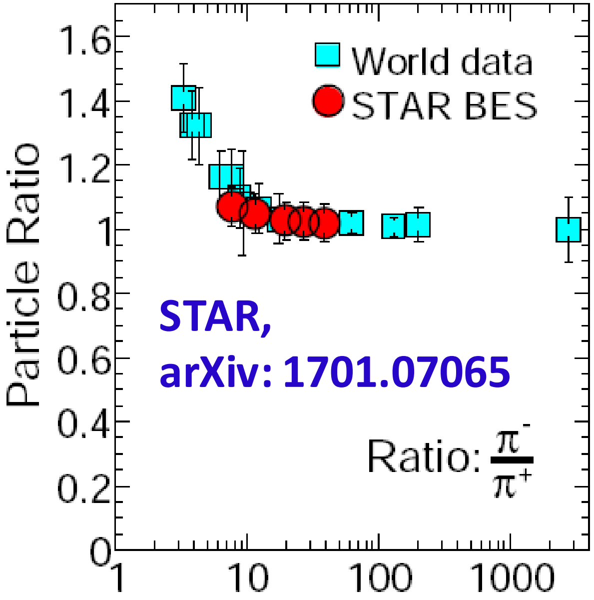
QCD phase structure at finite baryon and isospin chemical potentials



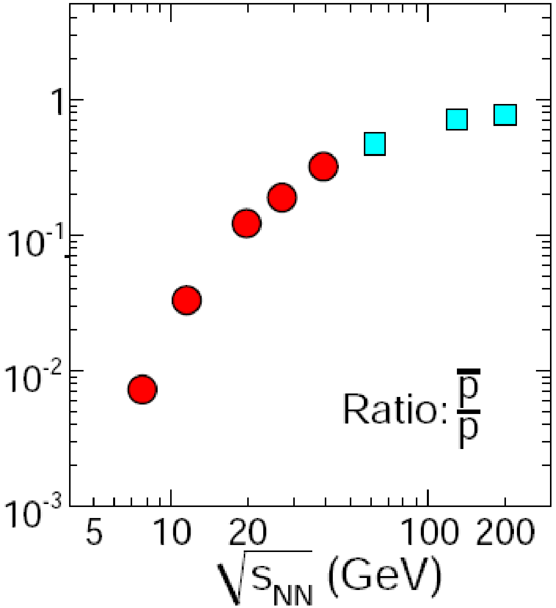
$$\mu_I = -0.0308\mu_B + 2.77\cdot 10^{-8}\mu_B^3$$

$$\mu_I = -0.293 - 0.0264\mu_B$$

Y. Hatta, A. Monnai, and B.W. Xiao, NPA (2016)

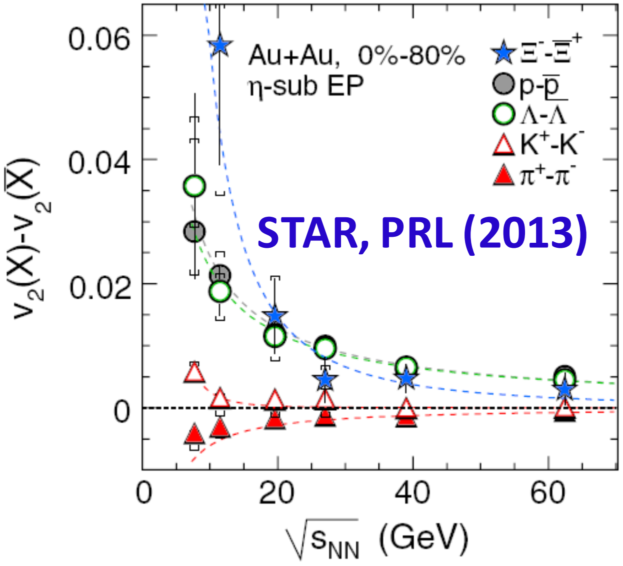


STAR,
arXiv: 1701.07065

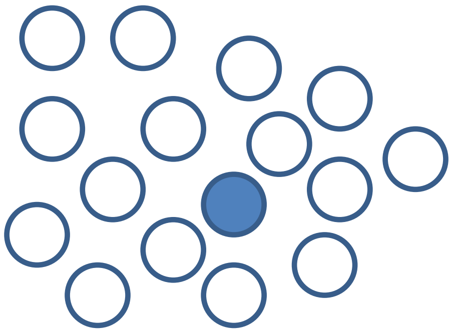


Highlights in RHIC-BES I

v_2 splitting

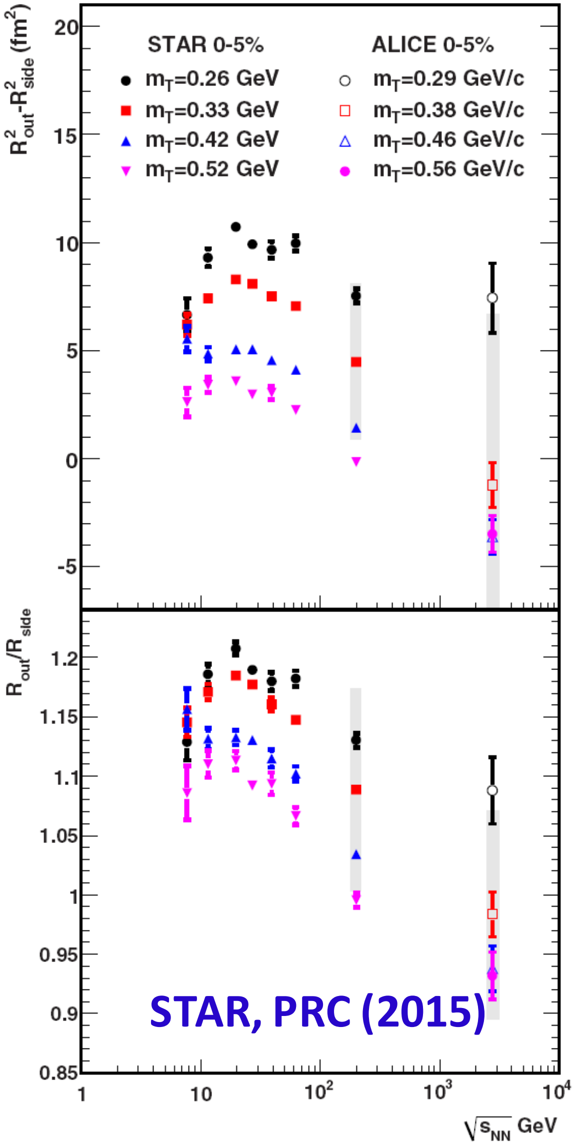


STAR, PRL (2013)

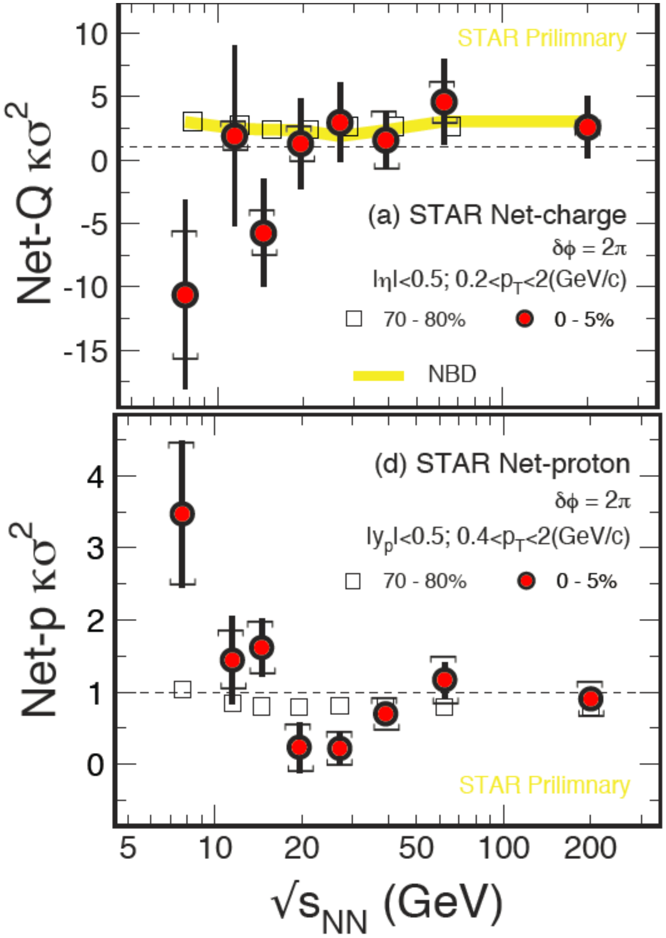


$$U_i \sim \sum_{i \neq j} V_{ij} + \ldots$$

HBT correlation



STAR, PRC (2015)



X.F. Luo's slides

Transport model simulations of intermediate-energy heavy-ion collisions

Heavy-ion experiments

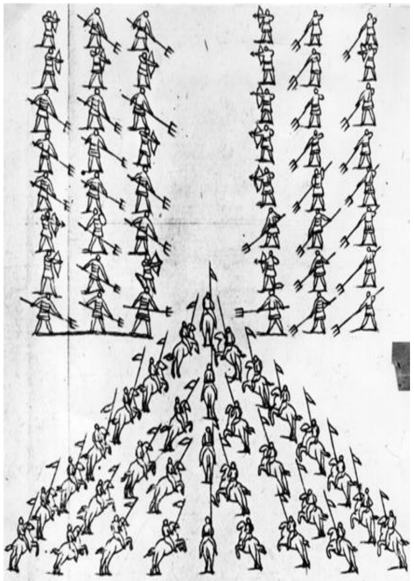
Transport simulations

Mean-field potential

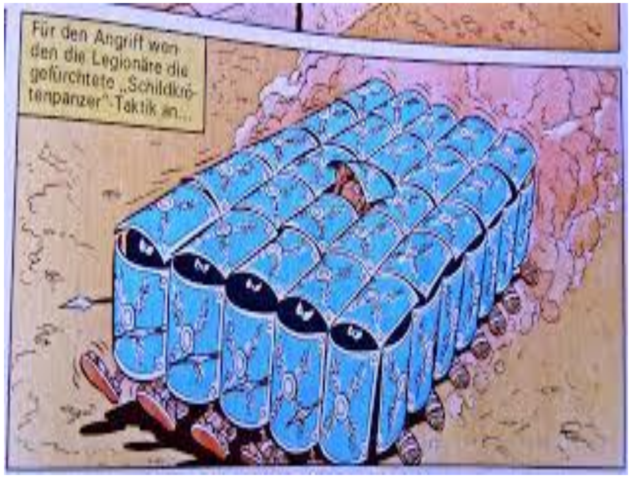
Nuclear EOS, E_{sym}

Main approaches:

Boltzman transport and Quantum Molecular Dynamics



Initialization

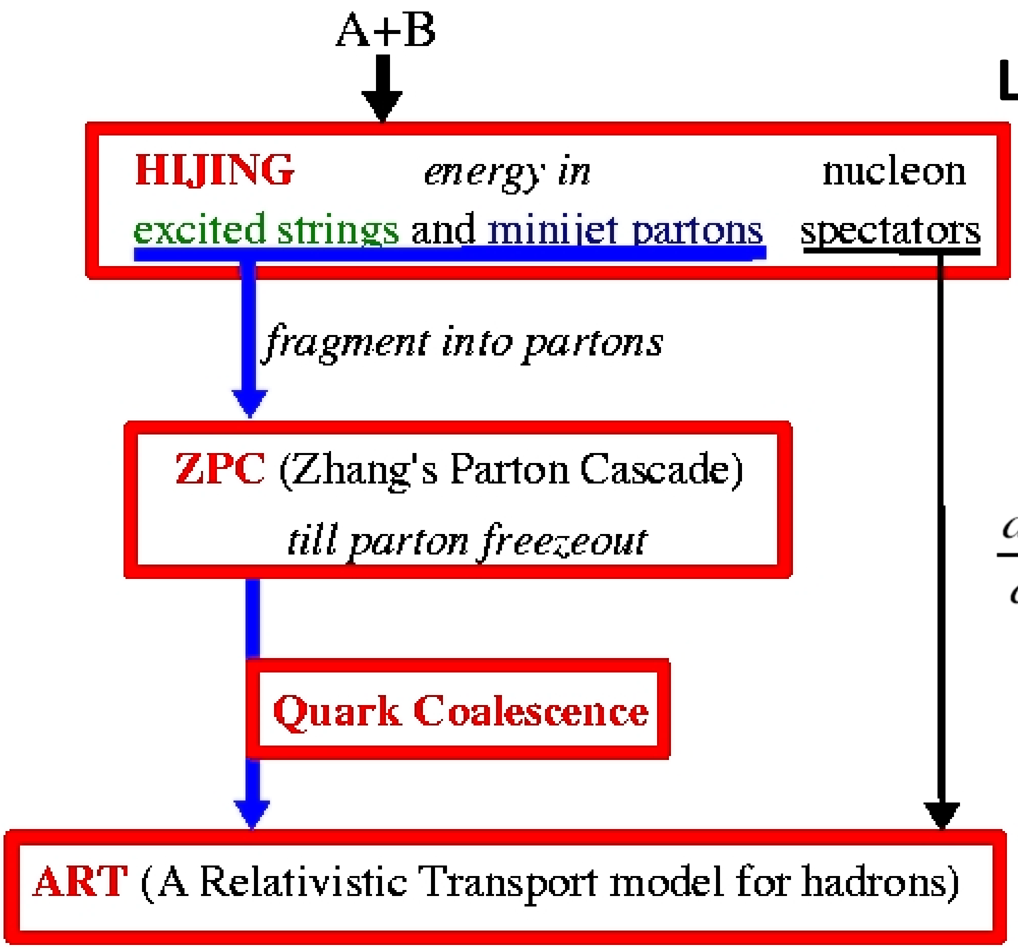


Mean-field potential

NN scatterings

A multiphase transport (AMPT) model with string melting

Structure of AMPT model with string melting



A+B

HIJING *energy in* excited strings and minijet partons

nucleon spectators

fragment into partons

ZPC (Zhang's Parton Cascade) *till parton freezeout*

Quark Coalescence

ART (A Relativistic Transport model for hadrons)

Lund string fragmentation function

$$f(z) \approx z^{-1}(1-z)^{a}\exp\left[-\frac{b\left(m^{2}+p_{t}^{2}\right)}{z}\right]$$

z : light-cone momentum fraction

Parton scattering cross section

$$\frac{d\sigma}{dt} \approx \frac{9\pi\alpha^{2}}{2s^{2}}\left(1+\frac{\mu^{2}}{s}\right)\left(\frac{1}{t-\mu^{2}}\right)^{2}, \quad \sigma \approx \frac{9\pi\alpha^{2}}{2\mu^{2}}$$

α: strong coupling constant

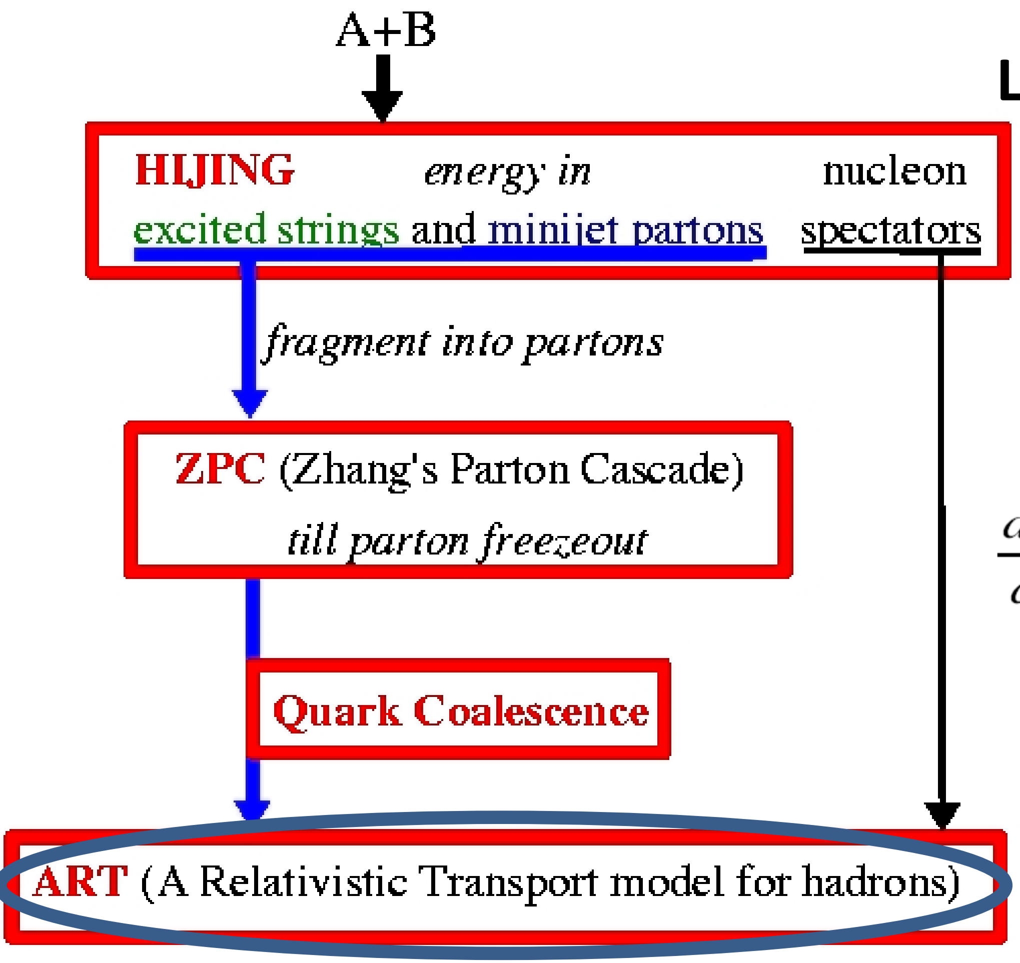
μ: screening mass

a, b: particle multiplicity

α, μ: partonic interaction

A multiphase transport (AMPT) model with string melting

Structure of AMPT model with string melting



A+B

HIJING *energy in* nucleon

excited strings and minijet partons spectators

fragment into partons

ZPC (Zhang's Parton Cascade)

till parton freezeout

Quark Coalescence

ART (A Relativistic Transport model for hadrons)

Turn on hadronic mean-field potentials

Lund string fragmentation function

$$f(z) \approx z^{-1}(1-z)^{a}\exp\left[-\frac{b(m^{2}+p_{t}^{2})}{z}\right]$$

z : light-cone momentum fraction

Parton scattering cross section

$$\frac{d\sigma}{dt} \approx \frac{9\pi\alpha^{2}}{2s^{2}}\left(1+\frac{\mu^{2}}{s}\right)\left(\frac{1}{t-\mu^{2}}\right)^{2}, \quad \sigma \approx \frac{9\pi\alpha^{2}}{2\mu^{2}}$$

α: strong coupling constant

μ: screening mass

a, b: particle multiplicity

α, μ: partonic interaction

hadronic potentials for particles and antiparticles

Nucleon and antinucleon potential

$$\mathcal{L} = \overline{\psi}[i\gamma_\mu\partial^\mu - m - g_\sigma\sigma - g_\omega\gamma_\mu\omega^\mu]\psi + \frac{1}{2}(\partial^\mu\sigma)^2 - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}b\sigma^3 - \frac{1}{4}c\sigma^4 - \frac{1}{4}(\partial_\mu\omega^\nu - \partial_\nu\omega^\mu)^2 + \frac{1}{2}m_\omega^2\omega^{\mu 2},$$

$$\Sigma_s = g_\sigma\langle\sigma\rangle, \qquad \Sigma_{v\mu} = g_\omega\langle\omega_\mu\rangle$$

$$U_{N,\bar{N}} = \Sigma_s(\rho_B, \rho_{\bar{B}}) \pm \Sigma_v^0(\rho_B, \rho_{\bar{B}})$$

$$U_{\Lambda,\overline{\Lambda}} \sim \frac{2}{3}U_{N,\overline{N}}, U_{\Xi,\overline{\Xi}} \sim \frac{1}{3}U_{N,\overline{N}}$$

Vector potential changes sign for antiparticles!

(e$^+$e$^-$ exchange γ)

G.Q. Li, C.M. Ko, X.S. Fang, and Y.M. Zheng, PRC (1994)

Kaon and antikaon potential

$$\omega_{K,\bar{K}} = \sqrt{m_K^2 + p^2 - a_K\rho_s + (b_K\rho_B^{\rm net})^2} \pm b_K\rho_B^{\rm net}$$

$$U_{K(\overline{K})} = \omega_{K(\overline{K})} - \omega_0 \qquad \omega_0 = \sqrt{m_K^2 + p^2}$$

G.Q. Li, C.H. Lee, and G.E. Brown, PRL (1997); NPA (1997)

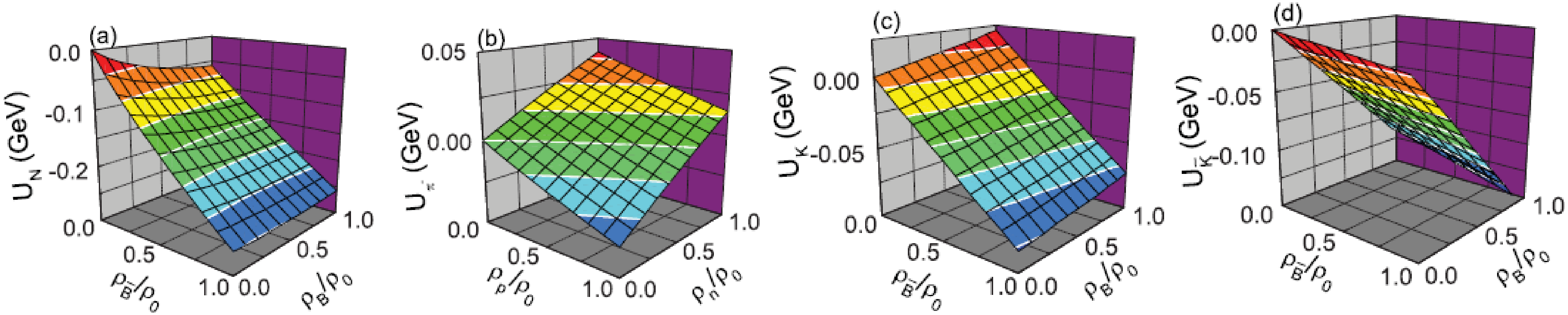
Pion s-wave potential

$$\begin{aligned}\Pi_s^-(\rho_p, \rho_n) &= \rho_n[T_{\pi N}^- - T_{\pi N}^+] - \rho_p[T_{\pi N}^- + T_{\pi N}^+] \\ &\quad + \Pi_{\rm rel}^-(\rho_p, \rho_n) + \Pi_{\rm cor}^-(\rho_p, \rho_n) \\ \Pi_s^+(\rho_p, \rho_n) &= \Pi_s^-(\rho_n, \rho_p) \\ \Pi_s^0(\rho_p, \rho_n) &= -(\rho_p + \rho_n)T_{\pi N}^+ + \Pi_{\rm cor}^0(\rho_p, \rho_n).\end{aligned}$$

$$U_{\pi^{\pm 0}} = \Pi_s^{\pm 0}/(2m_\pi)$$

N. Kaiser and W. Weise, PLB (2001)

hadronic potentials for particles and antiparticles



In baryon-rich and neutron-rich matter:

Introduced with test-particle method

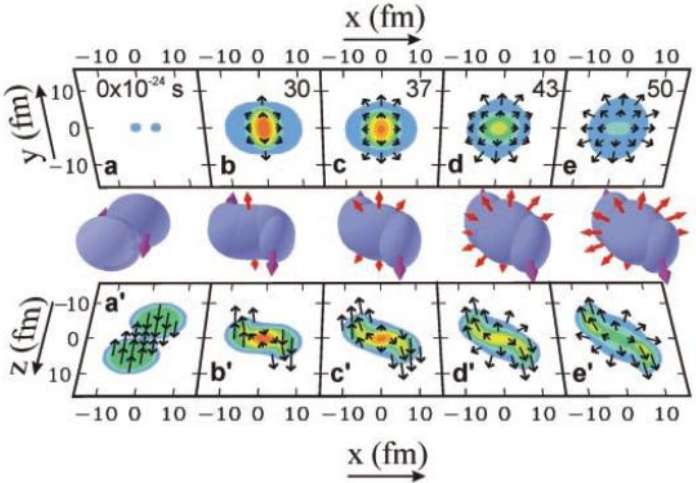
- **Baryon potential: weakly attractive**
- **Antibaryon potential: deeply attractive**
- **K^+ potential: weakly repulsive**
- **K^- potential: deeply attractive**

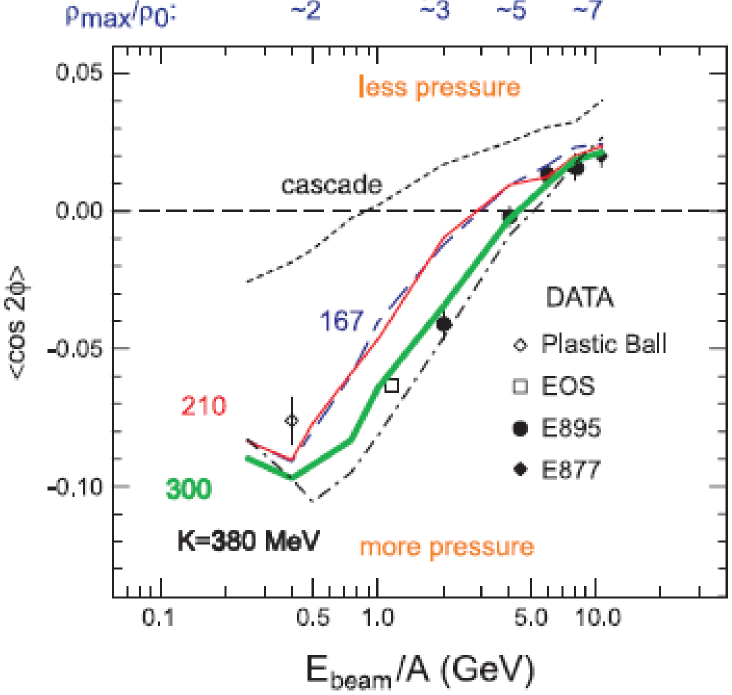
Sub threshold particle production

- **π^+ potential: weakly attractive**
- **π^- potential: weakly repulsive**

Chiral perturbation theory

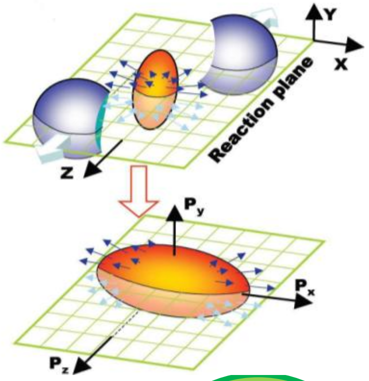
Effects of mean-field potentials on elliptic flow





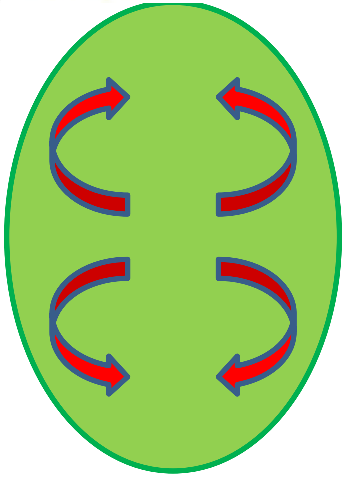
P. Danielewicz, R. Lacey, and W. G. Lynch, Science (2002).

$$v_2 = \frac{\left\langle p_x^2 - p_y^2 \right\rangle}{\left\langle p_x^2 + p_y^2 \right\rangle}$$



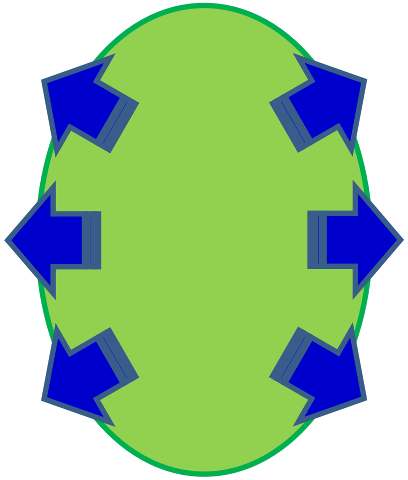
Particles with attractive potentials are more likely to be trapped in the system

v_2 decrease

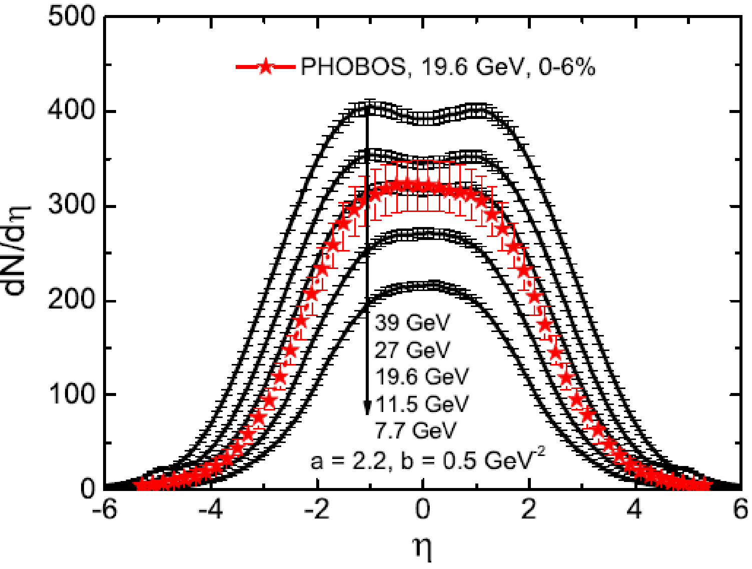


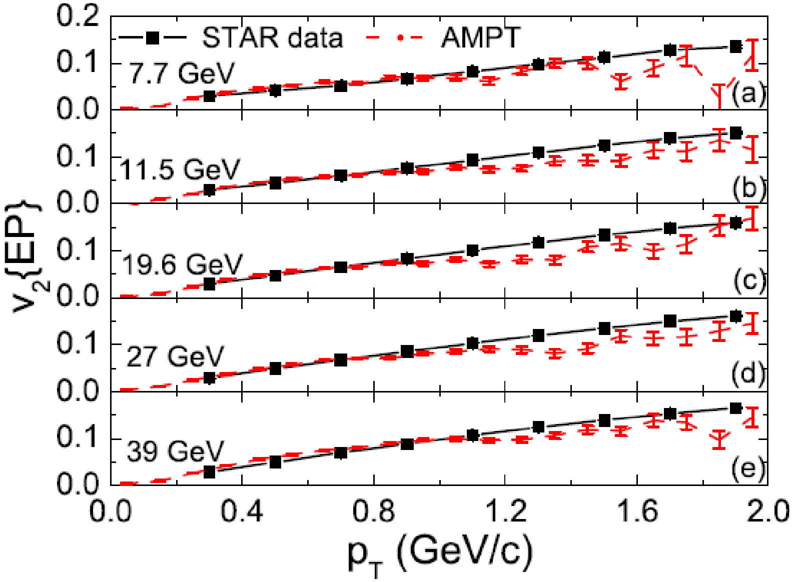
Particles with repulsive potentials are more likely to leave the system

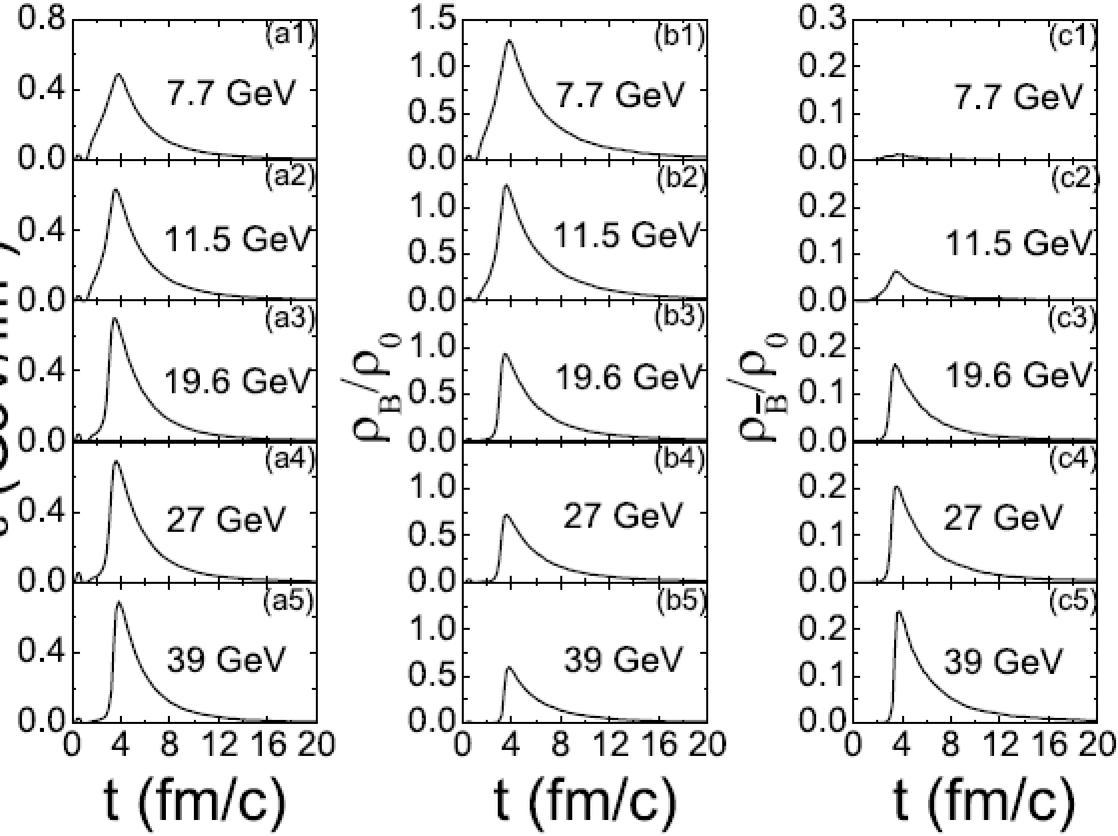
v_2 increase



Fit AMPT parameters at RHIC-BES energies





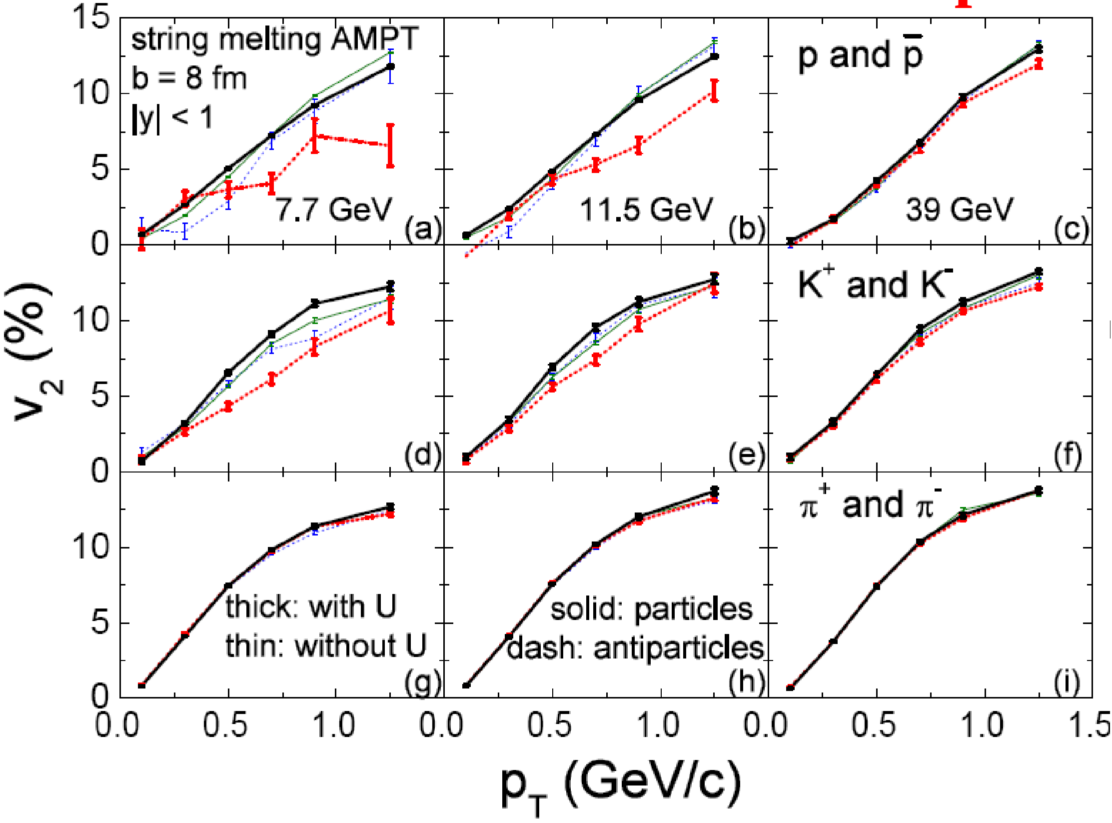


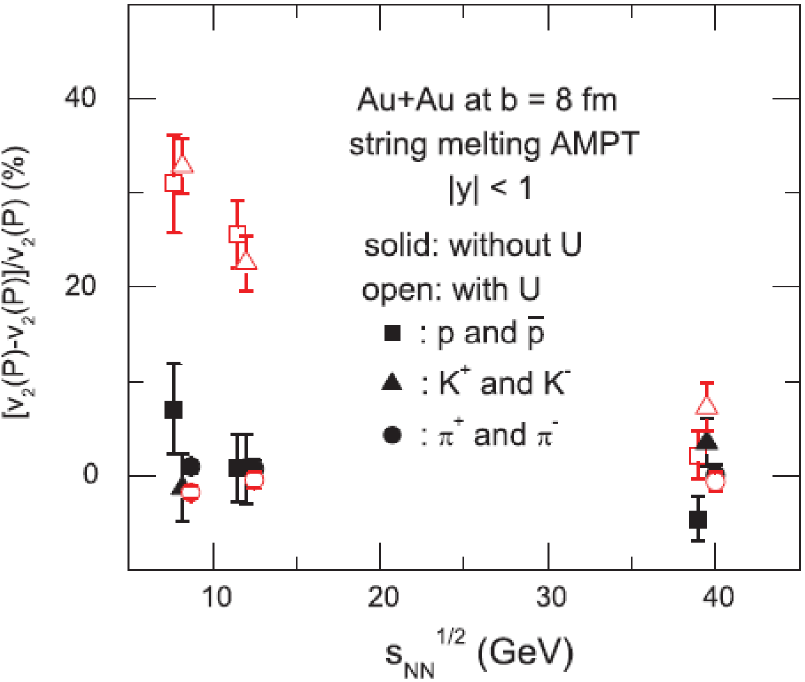
Lund string fragmentation parameters, Parton scattering cross section, parton life time

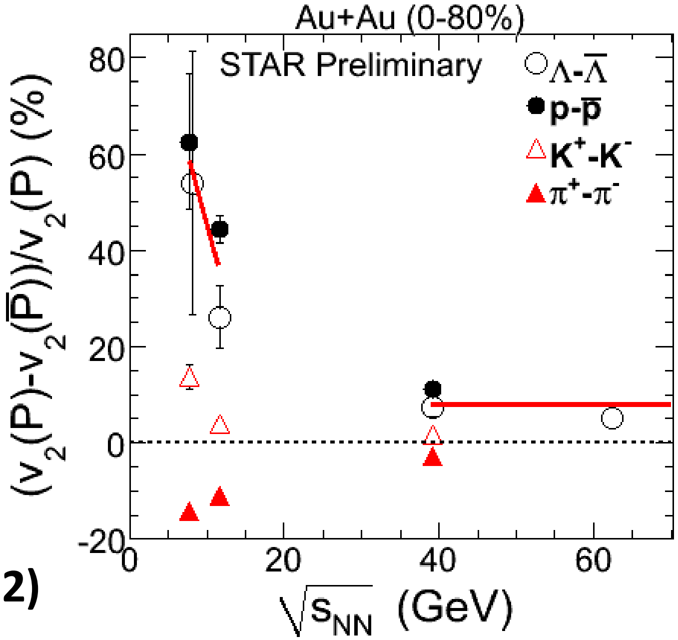
⇓

Pseudorapidity distribution, elliptic flow, energy density at chemical freeze-out

Effects of hadronic mean-field potentials on elliptic flow splitting







Qualitatively consistent

proton and antiproton: underestimate

K^+ and K^-: overestimate

π^+ and π^-: underestimate

JX, L. W. Chen, C. M. Ko, and Z. W. Lin, PRC 85, 041901(R) (2012)

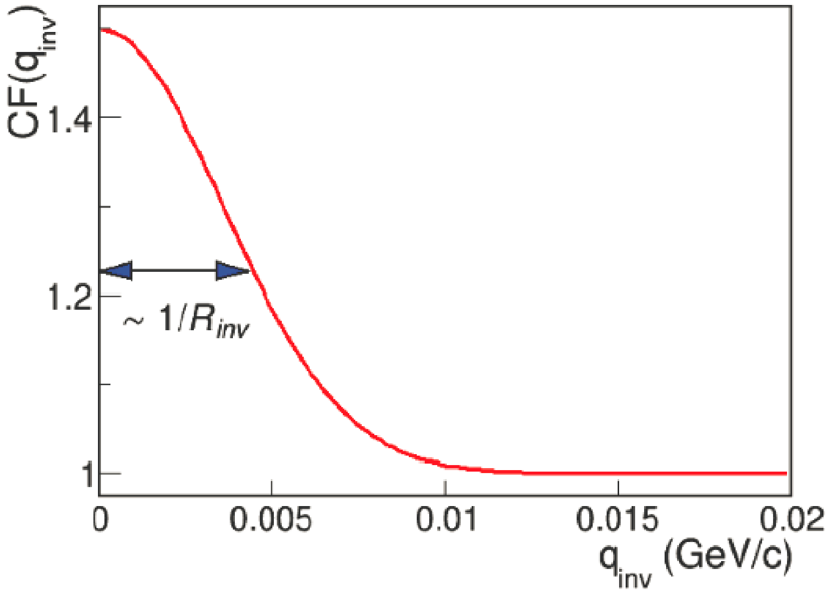
Hanbury-Brown and Twiss (HBT) Correlation

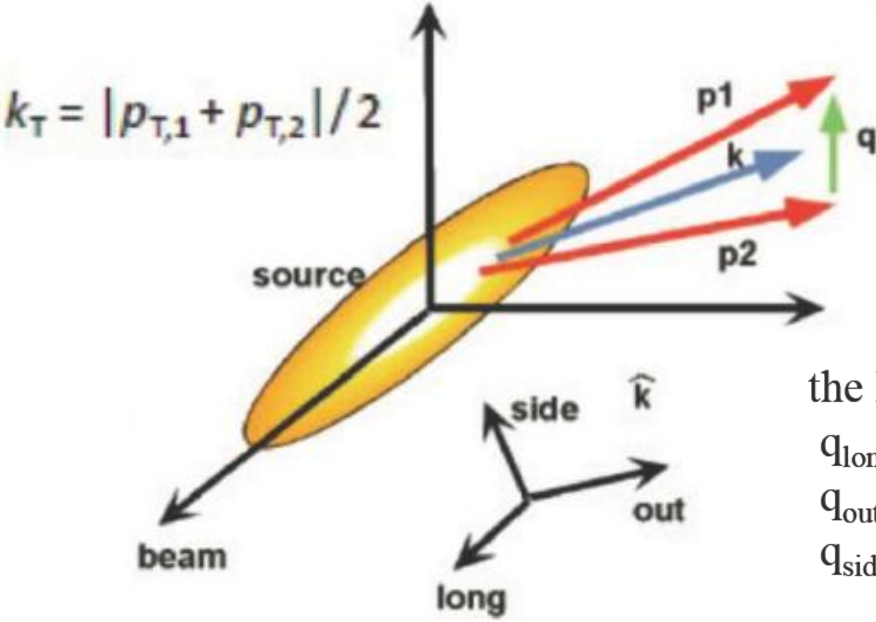
Two-particle correlation function:

$$C(\mathbf{k}^*)=\frac{\int S(\mathbf{r}^*,\mathbf{k}^*)\left|\Psi^{S(+)}_{-\mathbf{k}^*}(\mathbf{r}^*)\right|^2 d^4\mathbf{r}^*}{\int S(\mathbf{r}^*,\mathbf{k}^*)\, d^4\mathbf{r}^*}$$

where $\mathbf{r}^*=\mathbf{x}_1-\mathbf{x}_2$ and $\mathbf{k}^*=\mathrm{q}_{\mathrm{inv}}/2=(\mathbf{p}_1-\mathbf{p}_2)/2$

R.H. Brown and R.Q. Twiss, Nature (1956)
S. Pratt, PRD (1986)





One Dimension :

$$\mathrm{C}(\mathrm{q}_{\mathrm{inv}})=(1\text{-}\lambda)+\lambda\mathrm{K}_{\mathrm{coul}}(\mathrm{q}_{\mathrm{inv}})\left(1+e^{-\mathrm{q}_{\mathrm{inv}}^2\mathrm{R}_{\mathrm{inv}}^2}\right)$$

Three Dimension:

$$\mathrm{C}(\vec{\mathrm{q}})=(1\text{-}\lambda)+\lambda\mathrm{K}_{\mathrm{coul}}(\mathrm{q}_{\mathrm{inv}})\times\left(1+e^{-\mathrm{q}_o^2\mathrm{R}_o^2-\mathrm{q}_s^2\mathrm{R}_s^2-\mathrm{q}_l^2\mathrm{R}_l^2-2\mathrm{q}_o\mathrm{q}_s\mathrm{R}_{os}^2-2\mathrm{q}_o\mathrm{q}_l\mathrm{R}_{ol}^2}\right)$$

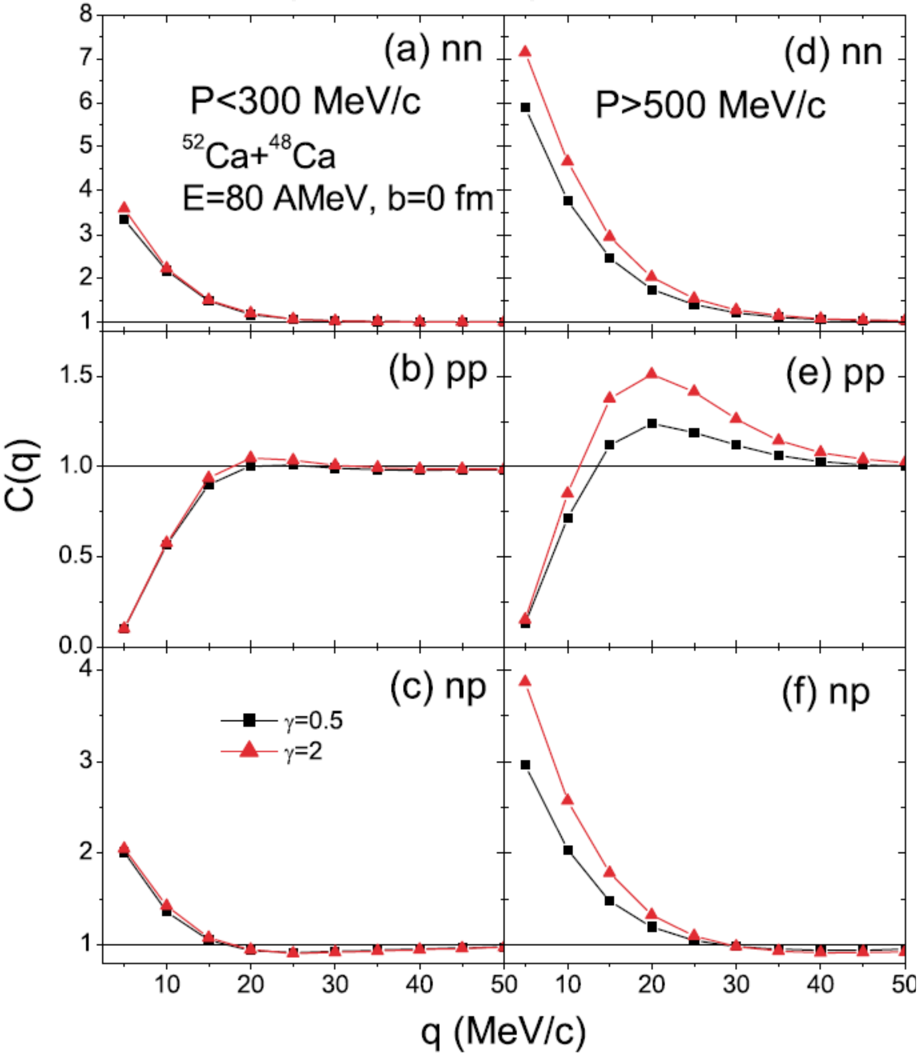
the Bertsch-Pratt, out-side-long system:

$\mathrm{q}_{\mathrm{long}}$ - along the beam direction
$\mathrm{q}_{\mathrm{out}}$ – along the transverse momentum of the pair
$\mathrm{q}_{\mathrm{side}}$ – perpendicular to longitudinal and outwards directions

Effects of mean-field potentials on HBT correlation

a probe of neutron-proton U difference
based on IBUU

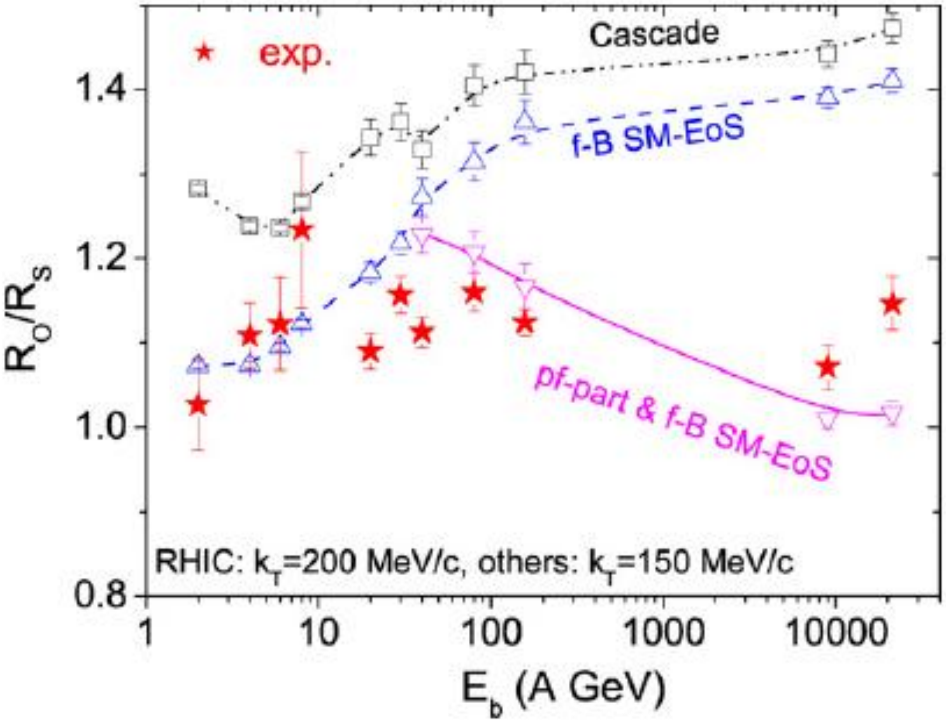
$$E_{\mathrm{sym}}(\rho) = E_{\mathrm{sym}}(\rho_0) \cdot u^{\gamma}$$



L.W. Chen, V. Greco, C.M. Ko, and B.A. Li, PRL (2003)

$$C(\vec{q}) = (1-\lambda) + \lambda K_{\mathrm{coul}}(q_{\mathrm{inv}}) \times \left(1 + e^{-q_o^2 R_o^2 - q_s^2 R_s^2 - q_l^2 R_l^2 - 2q_o q_s R_{os}^2 - 2q_o q_l R_{ol}^2}\right)$$

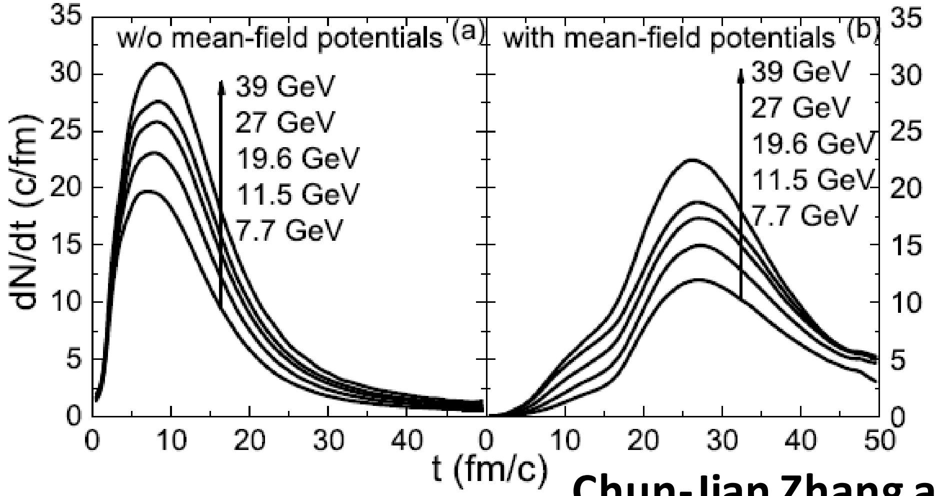
affect the HBT radii
based on UrQMD



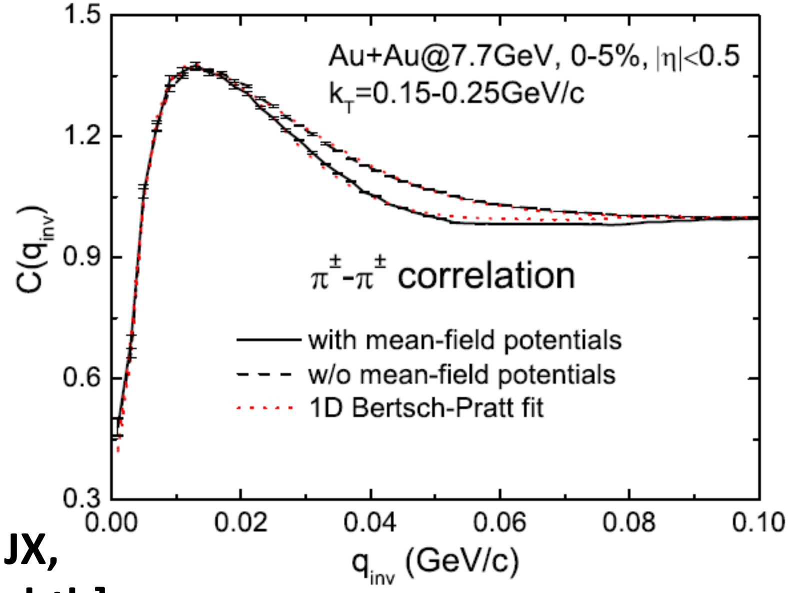
Q.F. Li, M. Bleicher, and H. Stocker, PLB (2008)

Effects of hadronic mean-field potentials on HBT correlation

later emission and broader emission time distribution

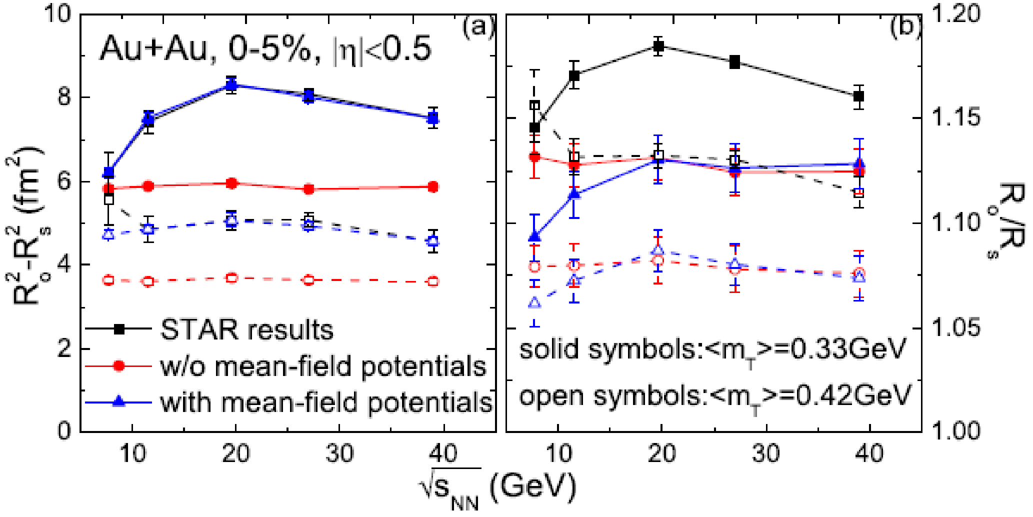


larger HBT radius

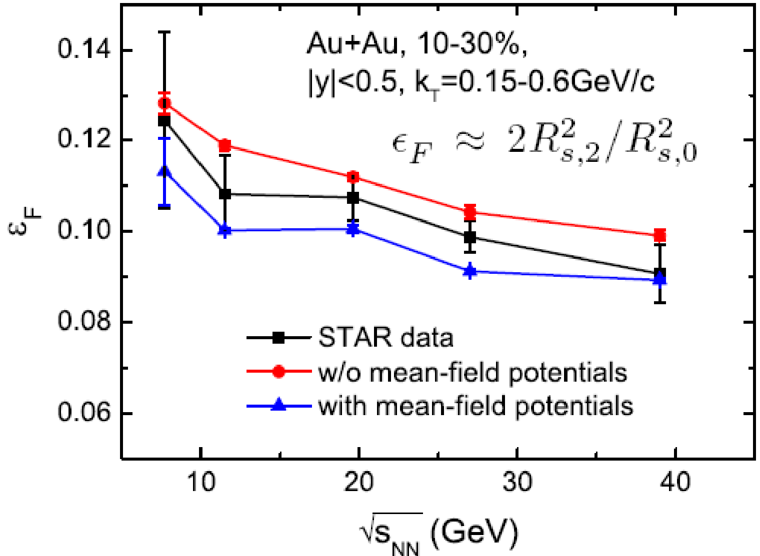


Chun-Jian Zhang and JX, arXiv:1707.07272 [nucl-th]

Affect R_{out} and R_{side}

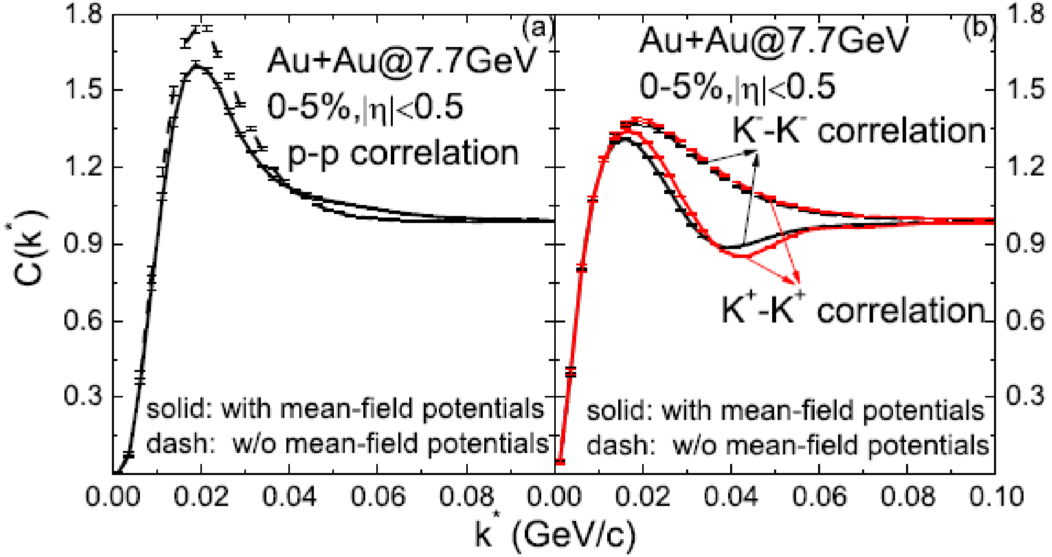


Lead to a smaller freeze-out eccentricity



Effects of hadronic mean-field potentials on HBT correlation

Affect correlation for indentified particles

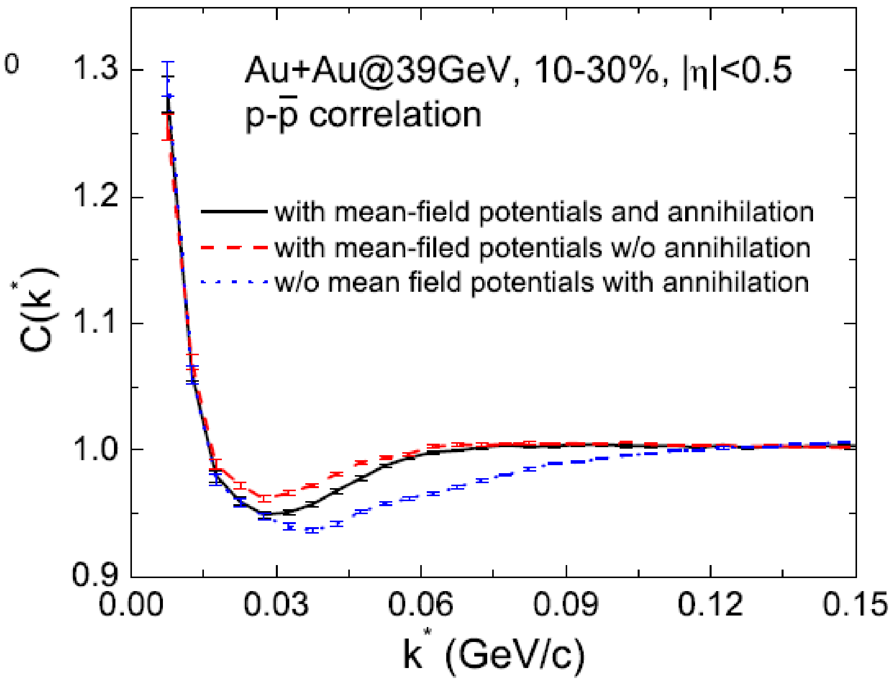


Affect global system evolution

+

Affect emission of individual particles

Interplay with p-pbar annihilation



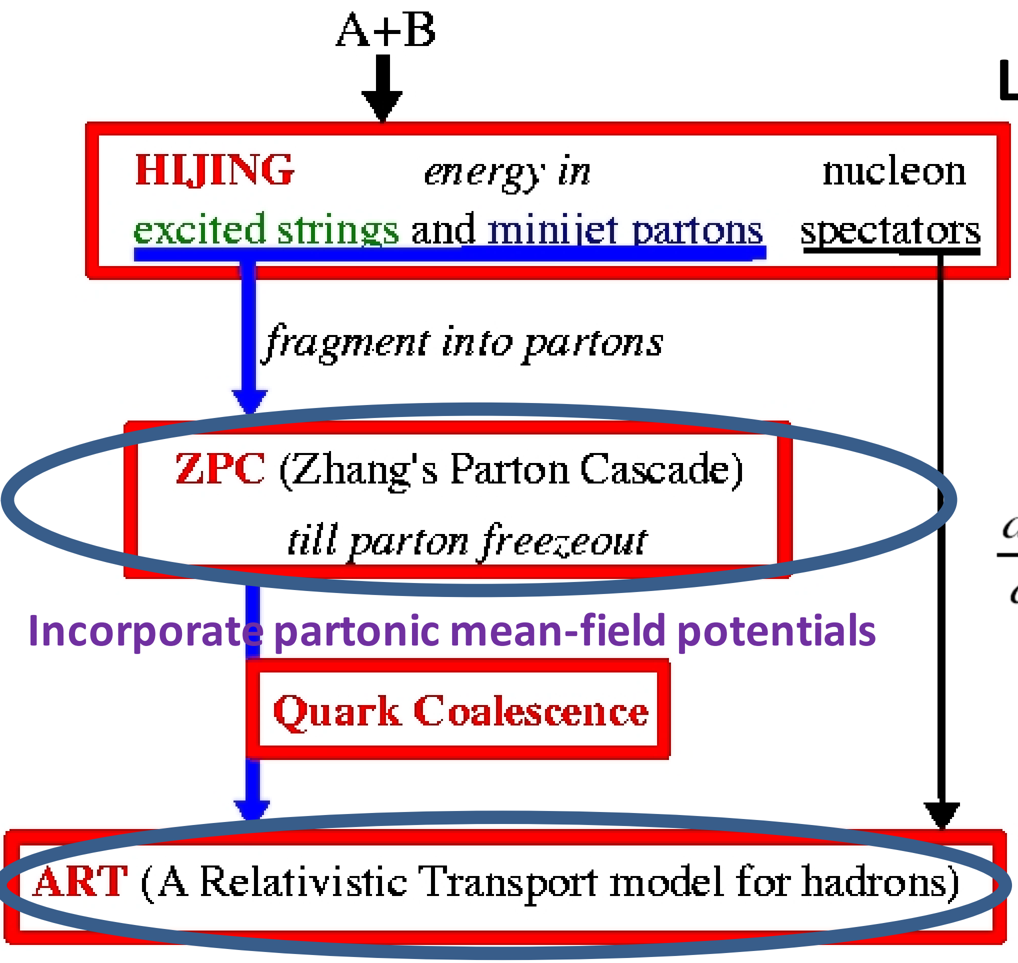
Attractive potential and later emission
=> weaken anti-correlation

Annihilation
=> enhance anti-correlation

Chun-Jian Zhang and JX,
arXiv:1707.07272 [nucl-th]

A multiphase transport (AMPT) model with string melting

Structure of AMPT model with string melting



A+B

HIJING *energy in* excited strings and minijet partons

nucleon spectators

fragment into partons

ZPC (Zhang's Parton Cascade) *till parton freezeout*

Incorporate partonic mean-field potentials

Quark Coalescence

ART (A Relativistic Transport model for hadrons)

Turn on hadronic mean-field potentials

Lund string fragmentation function

$$f(z) \approx z^{-1}(1-z)^{a} \exp\left[-\frac{b(m^2+p_t^2)}{z}\right]$$

z : light-cone momentum fraction

Parton scattering cross section

$$\frac{d\sigma}{dt} \approx \frac{9\pi\alpha^2}{2s^2}\left(1+\frac{\mu^2}{s}\right)\left(\frac{1}{t-\mu^2}\right)^2, \quad \sigma \approx \frac{9\pi\alpha^2}{2\mu^2}$$

α: strong coupling constant

μ: screening mass

a, b: particle multiplicity

α, μ: partonic interaction

3-flavor Nambu-Jona-Lasinio transport model

Lagrangian:

scalar pseudoscalar vector pseudovector

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - M)\psi + \frac{G}{2}\sum_{a=0}^{8}[(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma_5\lambda^a\psi)^2] - \sum_{a=0}^{8}\Big[\frac{G_V}{2}(\bar{\psi}\gamma_\mu\lambda^a\psi)^2 + \frac{G_A}{2}(\bar{\psi}\gamma_\mu\gamma_5\lambda^a\psi)^2\Big]$$

Kobayashi-Maskawa-t'Hooft interaction

$$- K[\det_f(\bar{\psi}(1+\gamma_5)\psi) + \det_f(\bar{\psi}(1-\gamma_5)\psi)]$$

Parameters taken from
M. Lutz, S. Klimt, and W. Weise, NPA (1992)
to reproduce meson properties

Yoichiro *Nambu*

2008年
Nobel Prize
For Physics

$$\rho^\mu = \langle\bar{\psi}\gamma^\mu\psi\rangle$$

Boltzmann equation:

$$\frac{\partial}{\partial t}f + \vec{v}\cdot\nabla_x f - \nabla_x H\cdot\nabla_p f = \mathcal{C}$$

$$\langle\bar{\psi}\gamma^\mu\psi\rangle = 2N_c\sum_{i=u,d,s}\int\frac{d^3\mathbf{k}}{(2\pi)^3E_i}\,k^\mu[f_i(k) - \bar{f}_i(k)]$$

$$\rho^0 \equiv \langle\bar{\psi}\gamma^0\psi\rangle$$

Net quark density

Single-quark Hamiltonian:

$$H = \sqrt{M^{*2} + p^{*2}}\boxed{\pm}g_V\rho^0$$

$$\begin{aligned} M_u &= m_u - 2G\langle\bar{u}u\rangle + 2K\langle\bar{d}d\rangle\langle\bar{s}s\rangle \\ M_d &= m_d - 2G\langle\bar{d}d\rangle + 2K\langle\bar{s}s\rangle\langle\bar{u}u\rangle \\ M_s &= m_s - 2G\langle\bar{s}s\rangle + 2K\langle\bar{u}u\rangle\langle\bar{d}d\rangle \end{aligned}$$

$$\mathbf{p}^* = \mathbf{p} \mp g_V\boldsymbol{\rho} \qquad g_V \equiv (2/3)G_V$$

Equations of motion:

$$\begin{aligned} \frac{dx_i}{dt} &= \frac{\partial H}{\partial p_i} = \frac{p_i^*}{E^*} \\ \frac{dp_i}{dt} &= -\frac{\partial H}{\partial x_i} \\ &= -\frac{M^*}{E^*}\frac{\partial M^*}{\partial x_i} \pm g_V\left(v_j\frac{\partial\rho_j}{\partial x_i} - \frac{\partial\rho_0}{\partial x_i}\right) \end{aligned}$$

Solve with
test particle method

Quark condensate:

$$\langle \bar{q}_i q_i \rangle = -2M_i N_c \int \frac{d^3\mathbf{k}}{(2\pi)^3 E_i} \; [1 - f_i(k) - \bar{f}_i(k)],$$

Quank 4-dimensional density:

$$\langle \bar{\psi} \gamma^\mu \psi \rangle = 2N_c \sum_{i=u,d,s} \int \frac{d^3\mathbf{k}}{(2\pi)^3 E_i} \; k^\mu [f_i(k) - \bar{f}_i(k)],$$

$$E_i(p) = \sqrt{p^2 + M_i^2}$$

$$\begin{aligned} M_u &= m_u - 2G\langle \bar{u}u \rangle + 2K\langle \bar{d}d \rangle \langle \bar{s}s \rangle, \\ M_d &= m_d - 2G\langle \bar{d}d \rangle + 2K\langle \bar{s}s \rangle \langle \bar{u}u \rangle, \\ M_s &= m_s - 2G\langle \bar{s}s \rangle + 2K\langle \bar{u}u \rangle \langle \bar{d}d \rangle, \end{aligned}$$

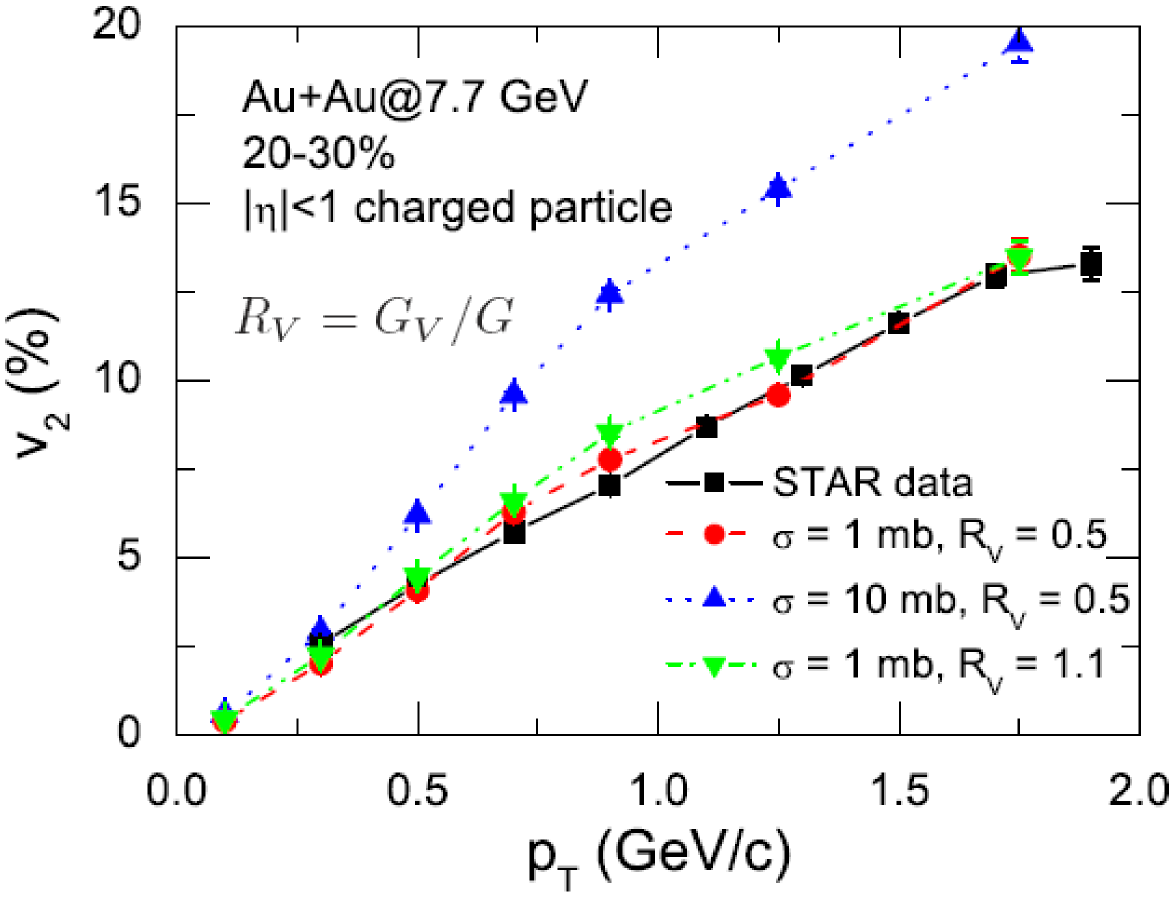
iteration needed

test particle method:

$$f(\vec{x}, \vec{k}) = \frac{1}{N_{test}} \sum_i g(\vec{x} - \vec{x}_i) g'(\vec{k} - \vec{k}_i)$$

Fit the parton scattering cross section with charged-particle v_2

Hadronization happens when chiral symmetry is broken, i.e., $M^*>M_{vac}/2$



(global hadronization,
can be improved by local hadronization)

Total v_2 is less sensitive to R_V

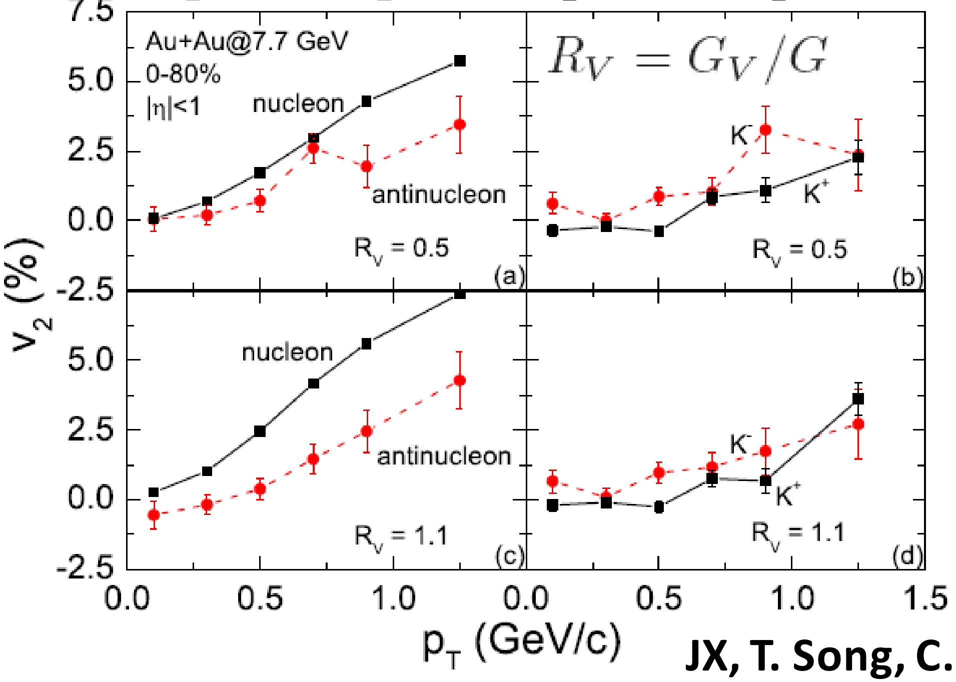
Fierz transformation: $R_V = 0.5$

Vector meson-mass spectrum: $R_V = 1.1$

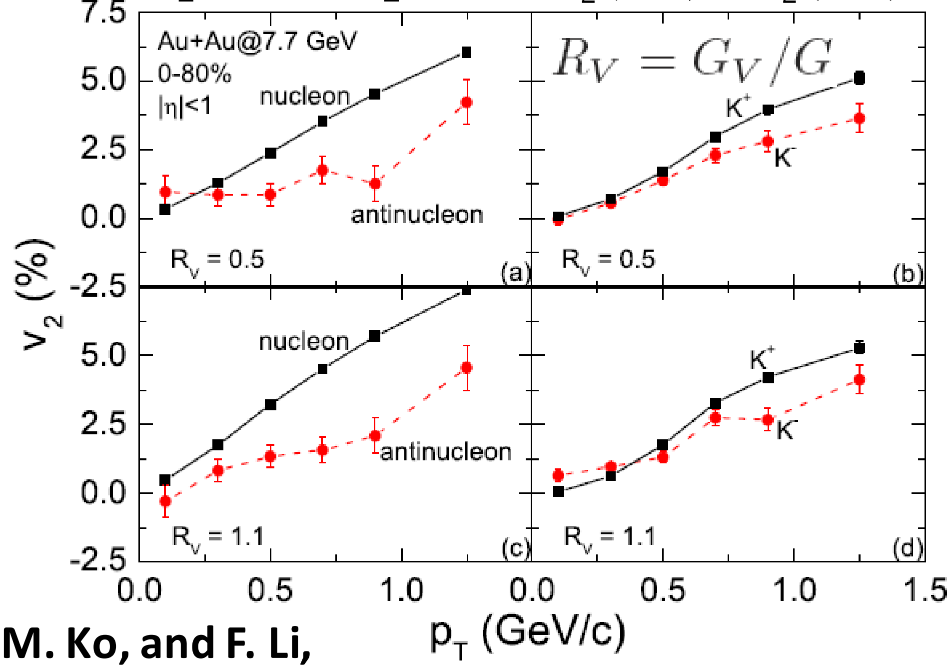
v_2 right after hadronization

Final v_2

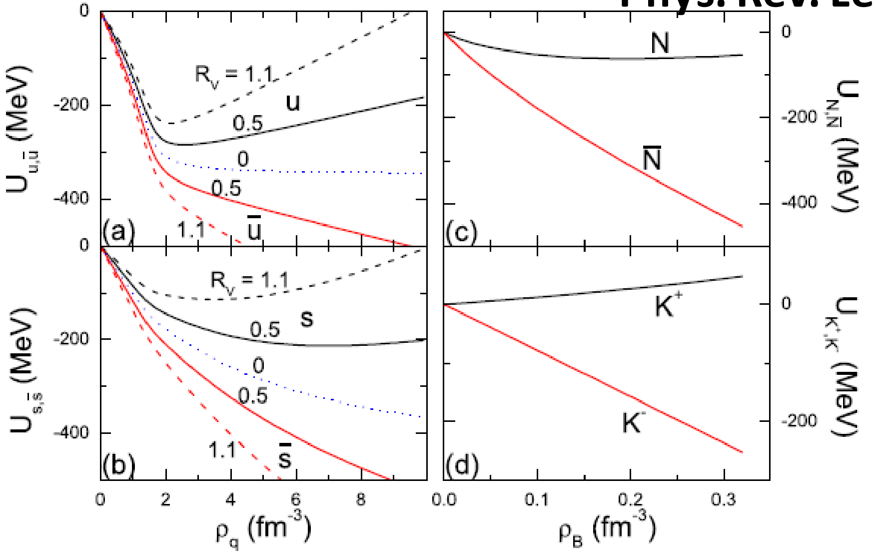
$v_2(N) > v_2(\bar{N})$ $v_2(K^+) < v_2(K^-)$

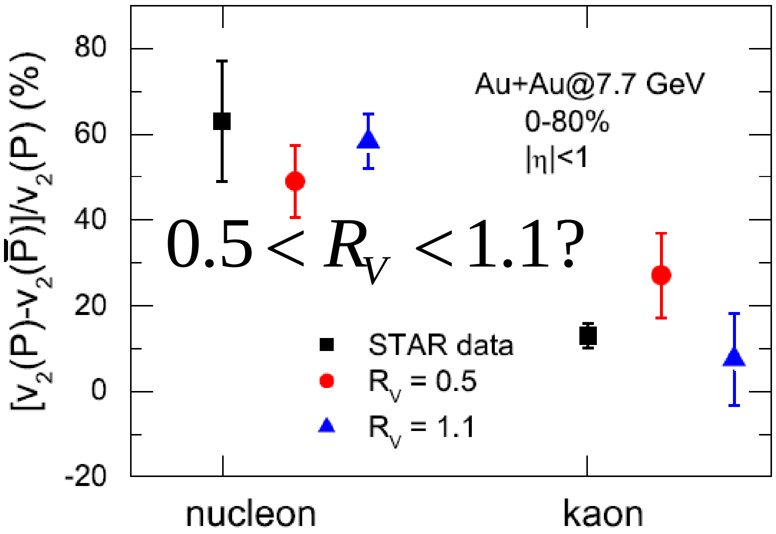


$v_2(N) > v_2(\bar{N})$ $v_2(K^+) > v_2(K^-)$



JX, T. Song, C. M. Ko, and F. Li,
Phys. Rev. Lett. 110, 012301 (2014)





$0.5 < R_V < 1.1?$

Phase diagram from NJL model

Lagrangian:

$$L = \bar{q}(i\gamma^{\mu}\partial_{\mu} - M)q + \frac{G}{2}\sum_{a=0}^{8}\left[\left(\bar{q}\lambda^{a}q\right)^{2} + \left(\bar{q}i\gamma_{5}\lambda^{a}q\right)^{2}\right] - \frac{G_{V}}{2}\sum_{a=0}^{8}\left[\left(\bar{q}\gamma_{\mu}\lambda^{a}q\right)^{2} + \left(\bar{q}\gamma_{\mu}\gamma_{5}\lambda^{a}q\right)^{2}\right]$$

$$- K\left[\det{}_{f}\left(\bar{q}(1+\gamma_{5})q\right) + \det{}_{f}\left(\bar{q}(1-\gamma^{5})q\right)\right]$$

After mean-field approximation:

$$L_{MF} = \sum_{q=u,d,s}\bar{q}(\gamma^{\mu}i\partial'_{\mu} - M'_{q})q + L_{den}$$

$$M_{u} = m_{u} - 2G\phi_{u} + 2K\phi_{d}\phi_{s}$$

$$M_{d} = m_{d} - 2G\phi_{d} + 2K\phi_{u}\phi_{s}$$

$$M_{s} = m_{s} - 2G\phi_{s} + 2K\phi_{u}\phi_{d}$$

$$i\partial'_{\mu} = i\partial_{\mu} - \frac{2}{3}G_{V}\rho$$

Quark condensate $\phi_{q} = \langle\bar{q}q\rangle$

Quark density $\rho_{q}^{\mu} = \langle\bar{q}\gamma^{\mu}q\rangle$

Net quark density $\rho = \rho_{u}^{0} + \rho_{d}^{0} + \rho_{s}^{0}$

$$L_{den} = -G\left(\phi_{u}^{2} + \phi_{d}^{2} + \phi_{s}^{2}\right) + 4K\phi_{u}\phi_{d}\phi_{s} - \frac{1}{3}G_{V}\rho^{2}$$

Free Fermions:

$$L=\bar{\psi}(\gamma^{\mu}i\partial_{\mu}-m)\psi \qquad H=\pi\frac{\partial\psi}{\partial t}-L \qquad \pi=\frac{\partial L}{\partial(\partial\psi/\partial t)}$$

Partition function:

$$Z=Tr[e^{-\beta(H-\boxed{\mu\bar{\psi}\gamma^{0}\psi)}}]$$

converse baryon charge

$$\ln Z=2V\int\frac{d^{3}p}{(2\pi)^{3}}\left[\beta\omega+\ln(1+e^{-\beta(\omega-\mu)})+\ln(1+e^{-\beta(\omega+\mu)})\right] \qquad \Omega=-\frac{T}{V}\ln Z$$

$$\omega=\sqrt{p^{2}+m^{2}}$$

NJL system

$$L_{MF}=\sum_{q=u,d,s}\bar{q}(\gamma^{\mu}i\partial'_{\mu}-M_{q})q+L_{den} \qquad i\partial'_{\mu}=i\partial_{\mu}-\frac{2}{3}G_{V}\rho$$

$$H-\mu\bar{q}\gamma^{0}q=\tilde{H}-\tilde{\mu}\bar{q}\gamma^{0}q$$

reduced chemical potential

$$\boxed{\tilde{\mu}=\mu-\frac{2}{3}G_{V}\rho}$$

Thermal potential

$$\Omega=\Omega_{cond}+\Omega_{quark}+\Omega_{zero}$$

$$\Omega_{cond}=-L_{den}$$

$$\frac{\partial\Omega}{\partial\phi_{q}}=0$$

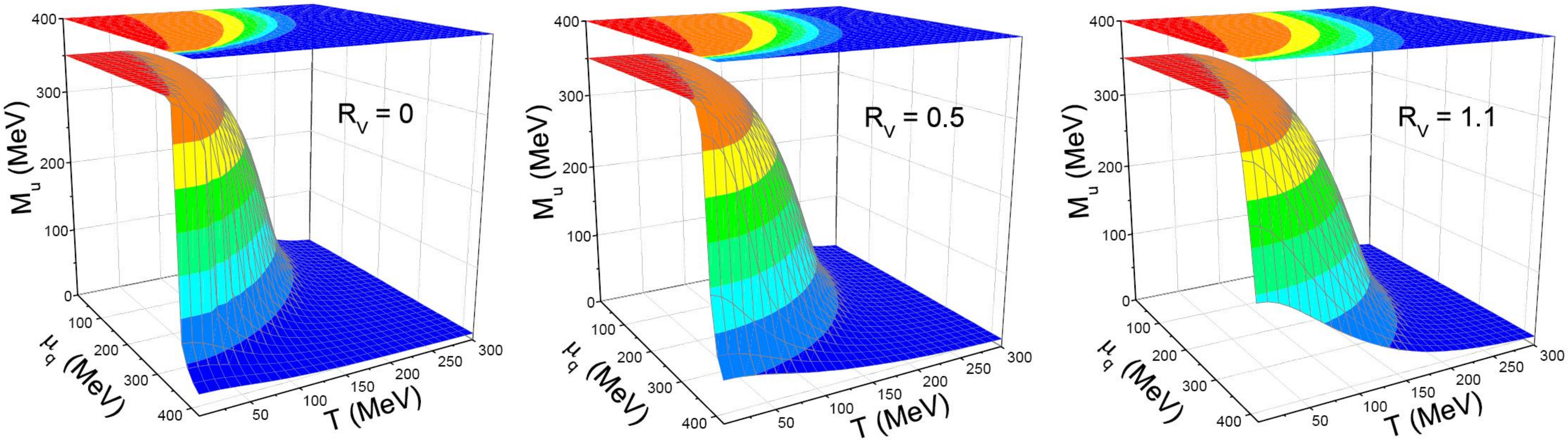
$$\Omega_{quark}=-2TN_{c}\sum_{q=u,d,s}\int\frac{d^{3}p}{(2\pi)^{3}}\left[\ln(1+e^{-\beta(E_{q}-\tilde{\mu})})+\ln(1+e^{-\beta(E_{q}+\tilde{\mu})})\right]$$

with cut-off parameter Λ

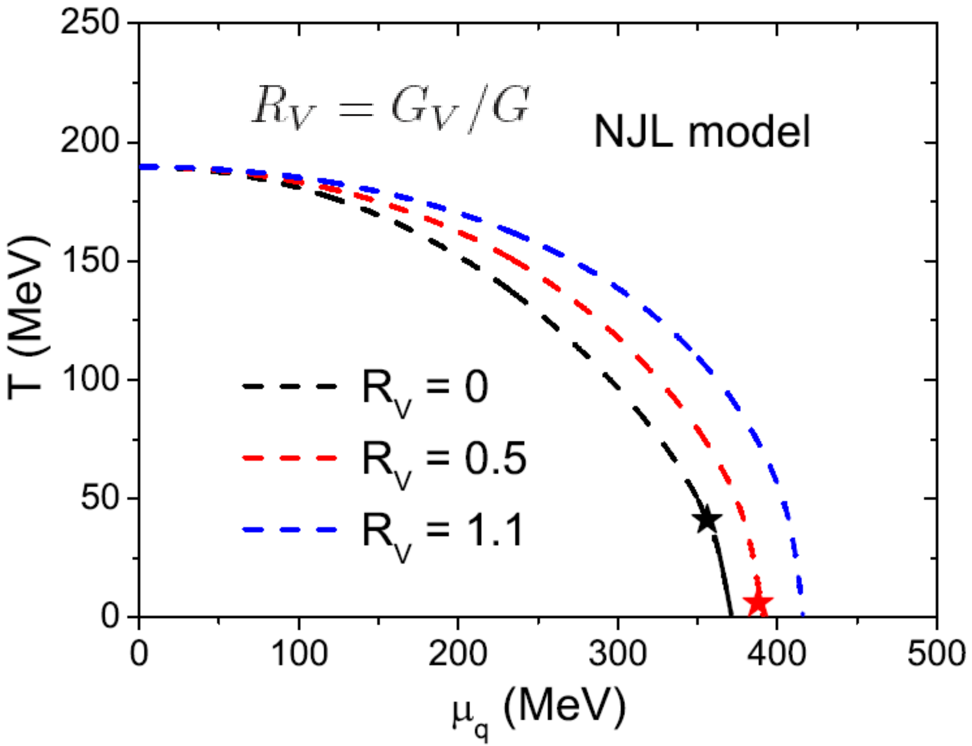
$$\Omega_{zero}=-2N_{c}\sum_{q=u,d,s}\int\frac{d^{3}p}{(2\pi)^{3}}E_{q} \qquad E_{q}=\sqrt{p^{2}+M_{q}^{2}}$$

Quark mass (Quark condensate)

$R_V = G_V / G$



NJL Phase diagram (chiral transition)



NJL model with polyakov loop

Polyakov loop:

$$L(x) = P\exp\left[-ig\int_0^{\beta} dx_4 A_4(x, x_4)\right]$$

A_4: gauge field

$$l = \frac{1}{N_c} Tr[L]$$

Thermal potential

$$\Omega = \Omega_{cond} + \Omega_{quark} + \Omega_{zero} + \Omega_{polyakov}$$

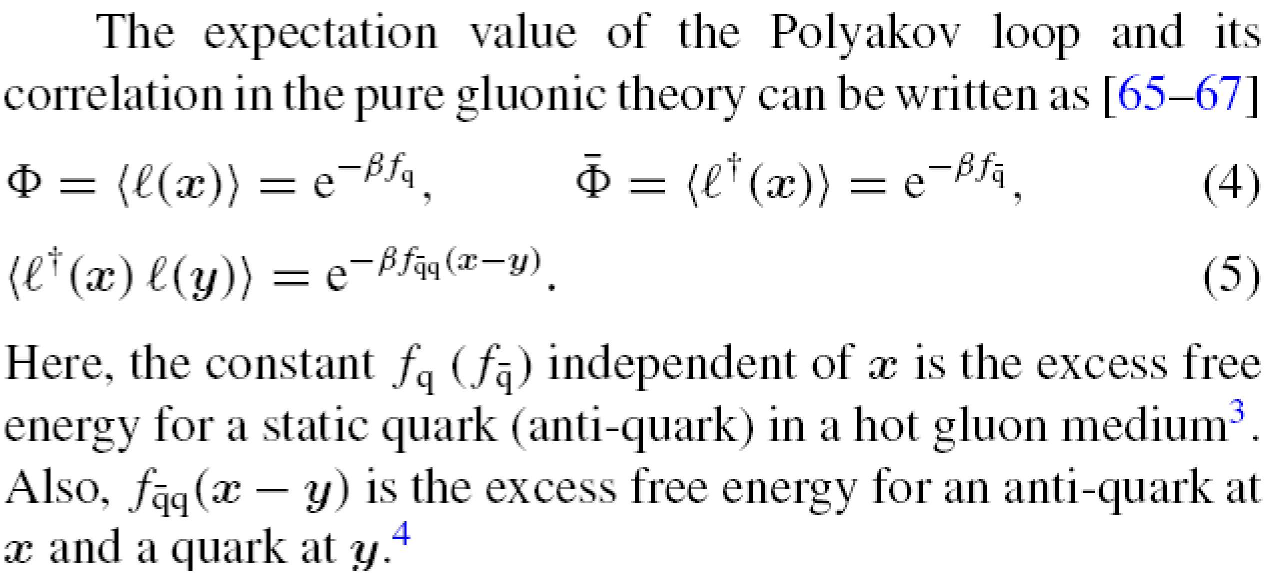
$$\Omega_{quark} = -2T \sum_{q=u,d,s} \int \frac{d^3p}{(2\pi)^3} \left[\ln\det(1 + Le^{-\beta(E_q - \tilde{\mu})}) + \ln\det(1 + L^{+} e^{-\beta(E_q + \tilde{\mu})})\right]$$

$$\Omega_{polyakov} = -bT\left\{54e^{-a/T} l\bar{l} + \ln\left[1 - 6l\bar{l} - 3\left(l\bar{l}\right)^2 + 4\left(l^3 + \bar{l}^3\right)\right]\right\}$$

Taken from Kenji Fukushima, PRD (2008)

$$\frac{\partial\Omega}{\partial\phi_q} = \frac{\partial\Omega}{\partial l} = \frac{\partial\Omega}{\partial\bar{l}} = 0$$

Polyakov loop: order parameter of deconfinement



The expectation value of the Polyakov loop and its correlation in the pure gluonic theory can be written as [65–67]

$$\Phi = \langle \ell(\boldsymbol{x}) \rangle = \mathrm{e}^{-\beta f_{\mathrm{q}}}, \qquad \bar{\Phi} = \langle \ell^{\dagger}(\boldsymbol{x}) \rangle = \mathrm{e}^{-\beta f_{\bar{\mathrm{q}}}}, \tag{4}$$

$$\langle \ell^{\dagger}(\boldsymbol{x})\, \ell(\boldsymbol{y}) \rangle = \mathrm{e}^{-\beta f_{\bar{\mathrm{q}}\mathrm{q}}(\boldsymbol{x}-\boldsymbol{y})}. \tag{5}$$

Here, the constant f_{q} ($f_{\bar{\mathrm{q}}}$) independent of $\boldsymbol{x}$ is the excess free energy for a static quark (anti-quark) in a hot gluon medium[3]. Also, $f_{\bar{\mathrm{q}}\mathrm{q}}(\boldsymbol{x}-\boldsymbol{y})$ is the excess free energy for an anti-quark at $\boldsymbol{x}$ and a quark at $\boldsymbol{y}$.[4]

Kenji Fukushima and Tetsuo Hatsuda, Rep. Prog. Phys. 2011

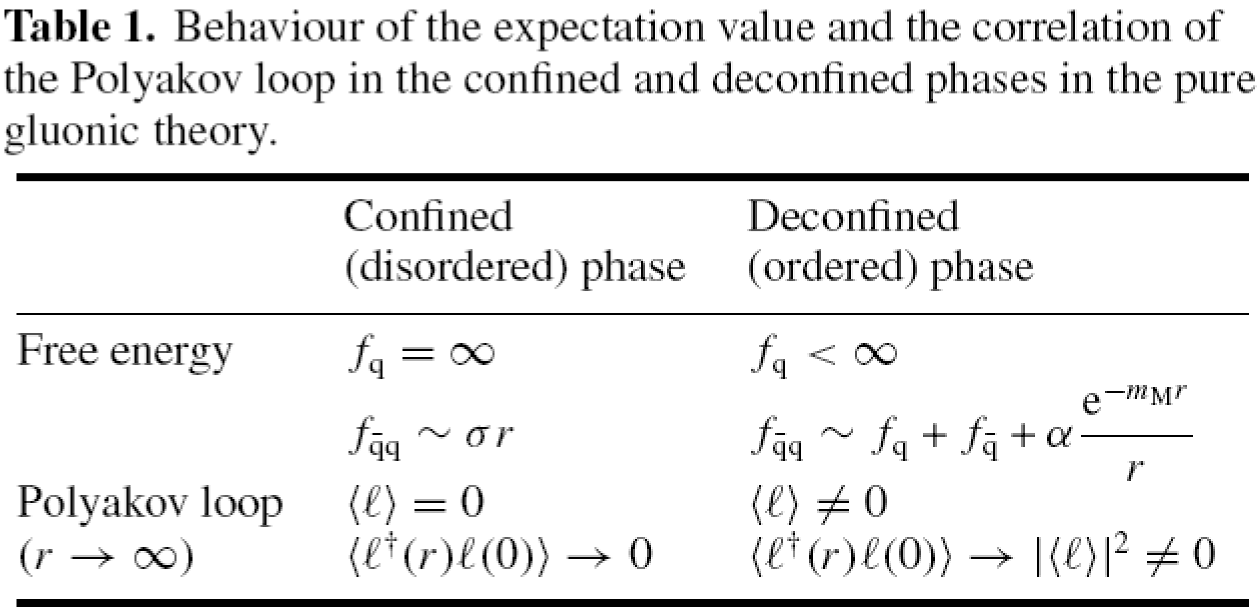
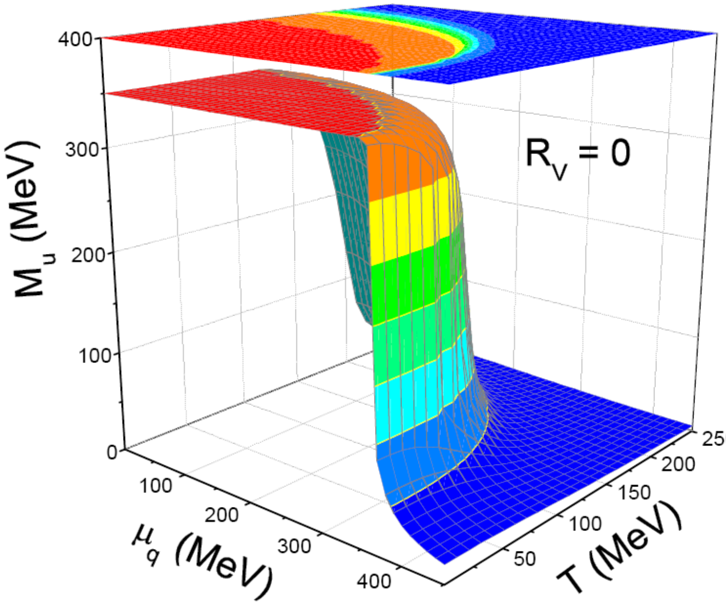


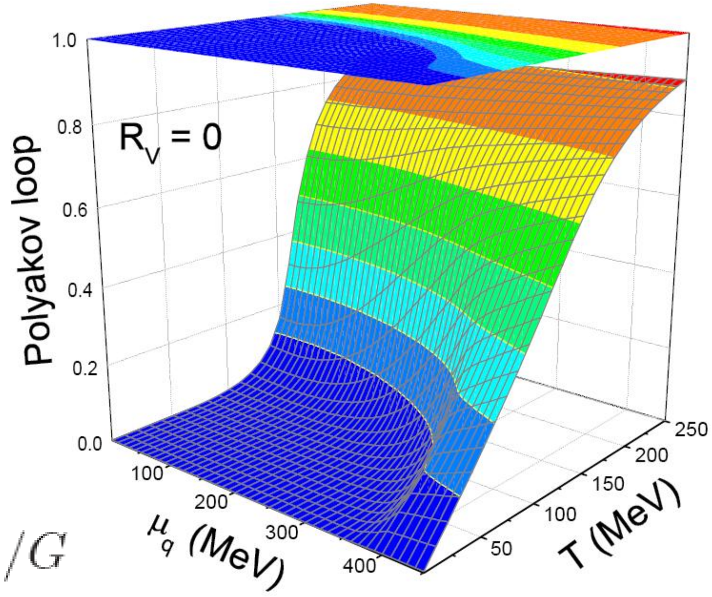
Table 1. Behaviour of the expectation value and the correlation of the Polyakov loop in the confined and deconfined phases in the pure gluonic theory.

	Confined (disordered) phase	Deconfined (ordered) phase
Free energy	$f_{\mathrm{q}} = \infty$	$f_{\mathrm{q}} < \infty$
	$f_{\bar{\mathrm{q}}\mathrm{q}} \sim \sigma r$	$f_{\bar{\mathrm{q}}\mathrm{q}} \sim f_{\mathrm{q}} + f_{\bar{\mathrm{q}}} + \alpha \dfrac{\mathrm{e}^{-m_{\mathrm{M}} r}}{r}$
Polyakov loop	$\langle \ell \rangle = 0$	$\langle \ell \rangle \neq 0$
$(r \to \infty)$	$\langle \ell^{\dagger}(r)\ell(0) \rangle \to 0$	$\langle \ell^{\dagger}(r)\ell(0) \rangle \to \|\langle \ell \rangle\|^2 \neq 0$

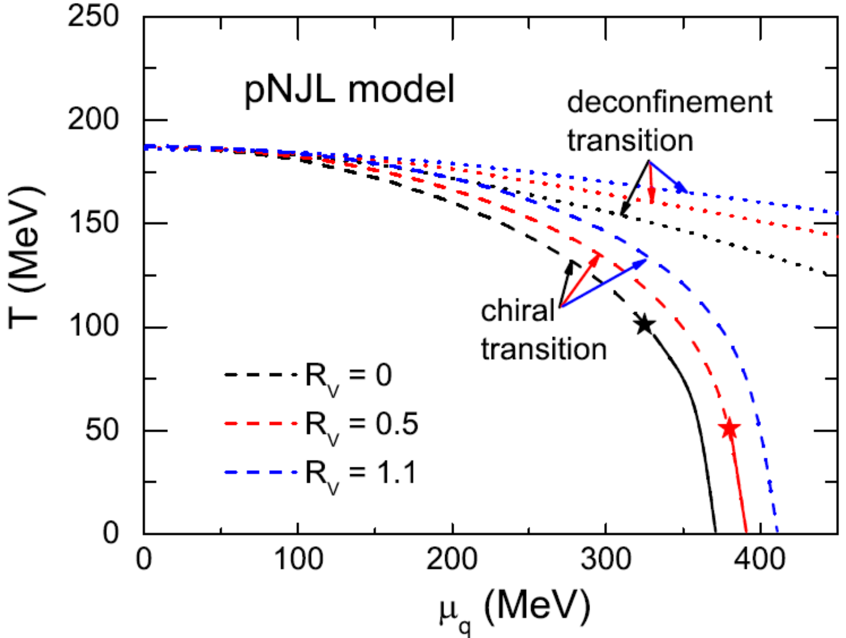
Quark mass/condensate (chiral transition)



Polyakov loop (deconfinement transition)



$$R_V = G_V/G$$



pNJL Phase diagram Chiral&Deconfinement

Isovector couplings in NJL model

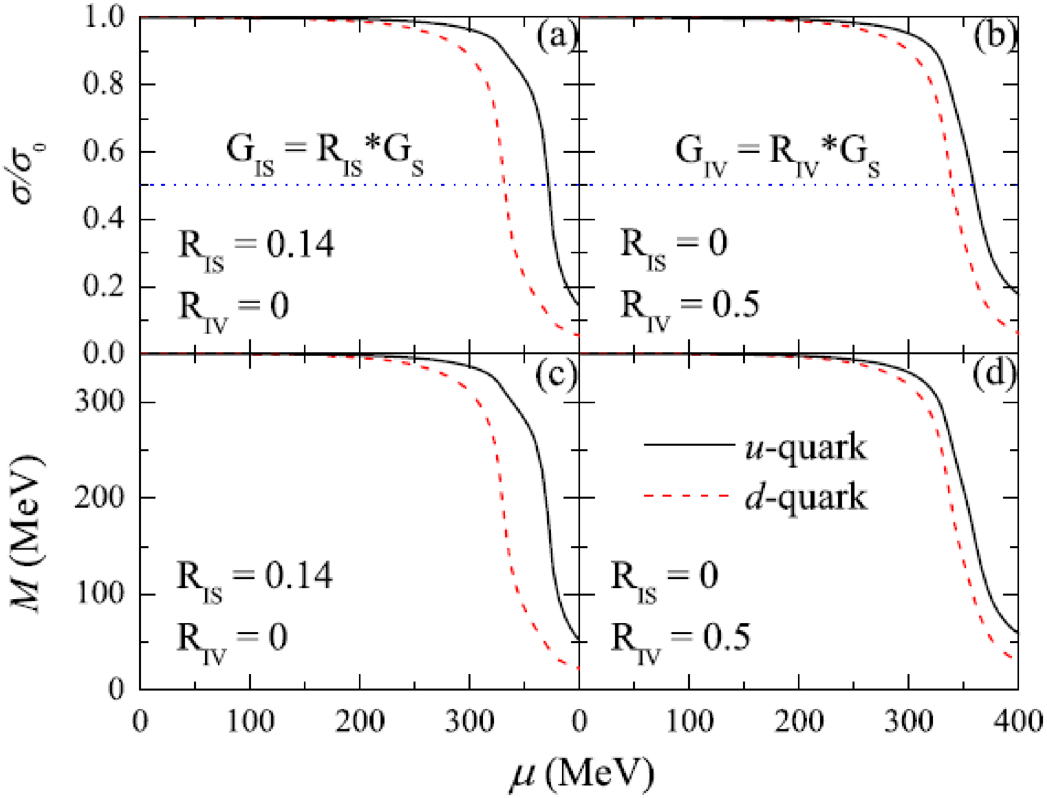
Scalar-isovector coupling $G_{IS}\sum_{a=1}^{3}[(\bar{q}\lambda_a q)^2+(\bar{q}i\gamma_5\lambda_a q)^2]$

Vector-isovector coupling $G_{IV}\sum_{a=1}^{3}[(\bar{q}\gamma_\mu\lambda_a q)^2+(\bar{q}\gamma_5\gamma_\mu\lambda_a q)^2]$

a=1~3
Pauli Matrics
In isospin space

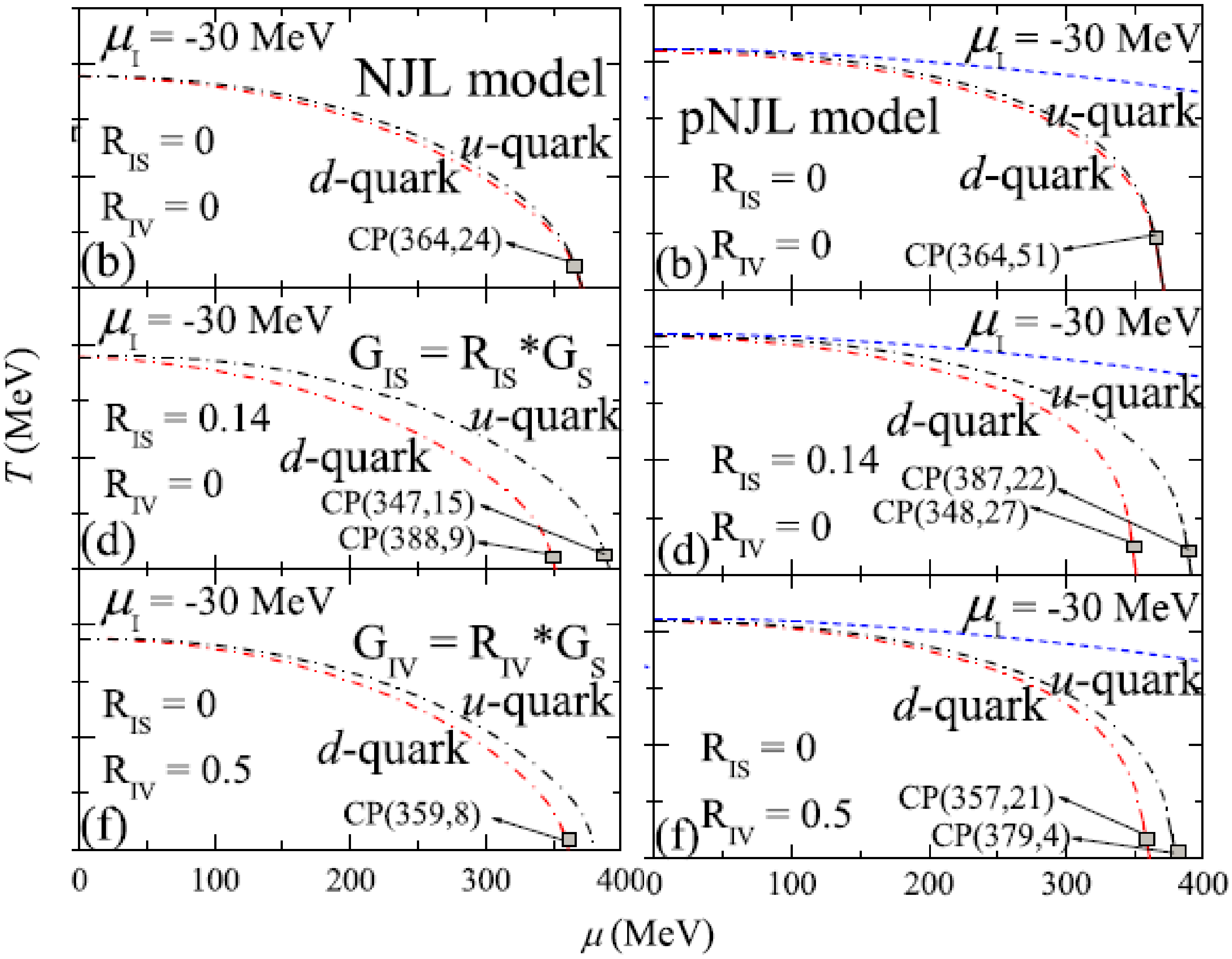
Dynamical mass $M_i=m_i-2G_S\sigma_i+2K\sigma_j\sigma_k-2G_{IS}\tau_{3i}(\sigma_u-\sigma_d)$

Mass splitting
for u and d quarks,
especially near
the phase boundary



H. Liu, JX, L.W. Chen,
and K.J. Sun, PRD (2016)

Phase diagram from (p)NJL model

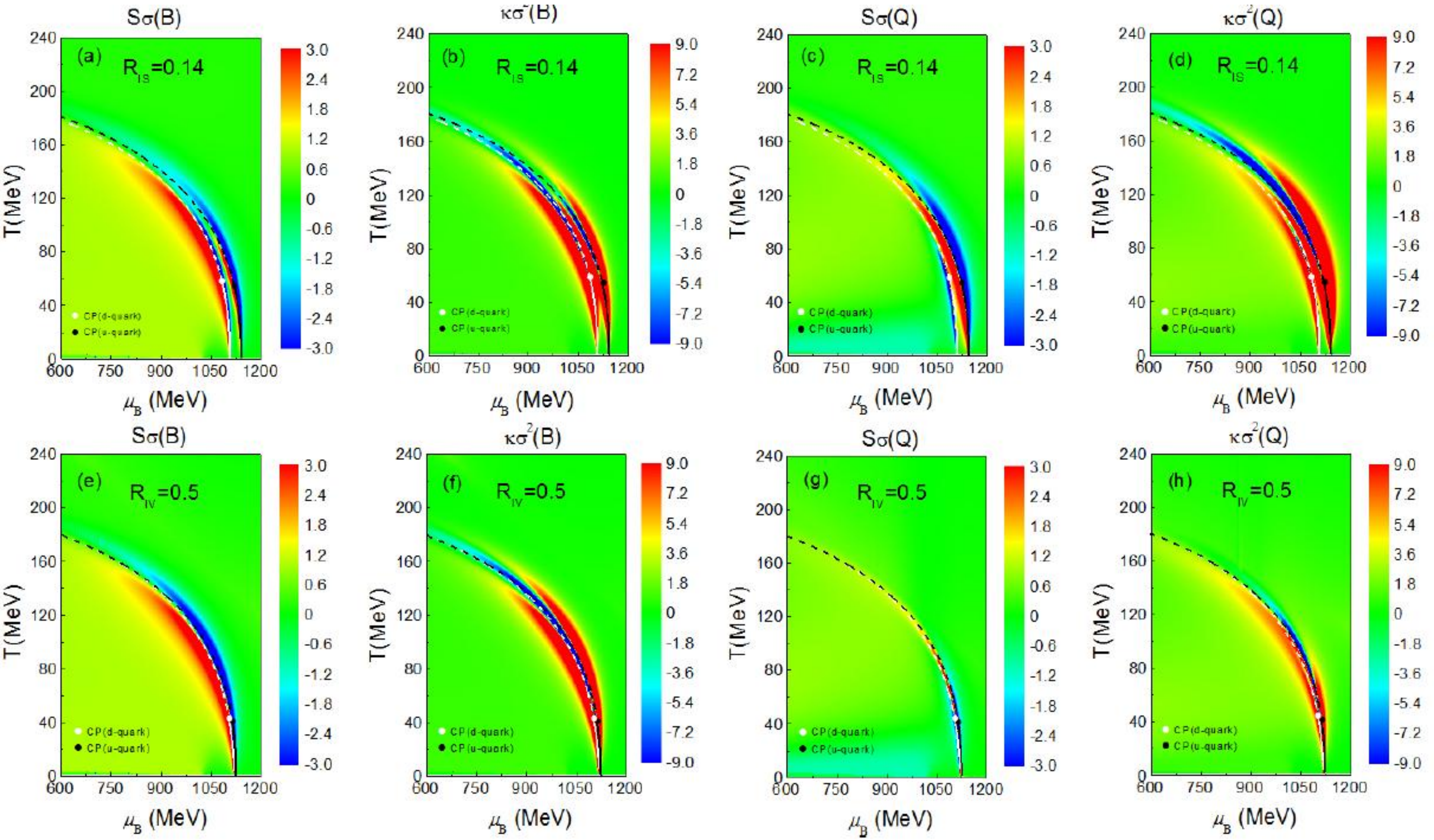


Polyakov potential:

$$\mathcal{U}(\Phi,\bar{\Phi},T) = -b\cdot T\{54e^{-a/T}\Phi\bar{\Phi} + \ln[1-6\Phi\bar{\Phi} - 3(\Phi\bar{\Phi})^2 + 4(\Phi^3+\bar{\Phi}^3)]\}$$

H. Liu, JX, L.W. Chen, and K.J. Sun, PRD (2016)

Isospin effect on susceptibility from pNJL



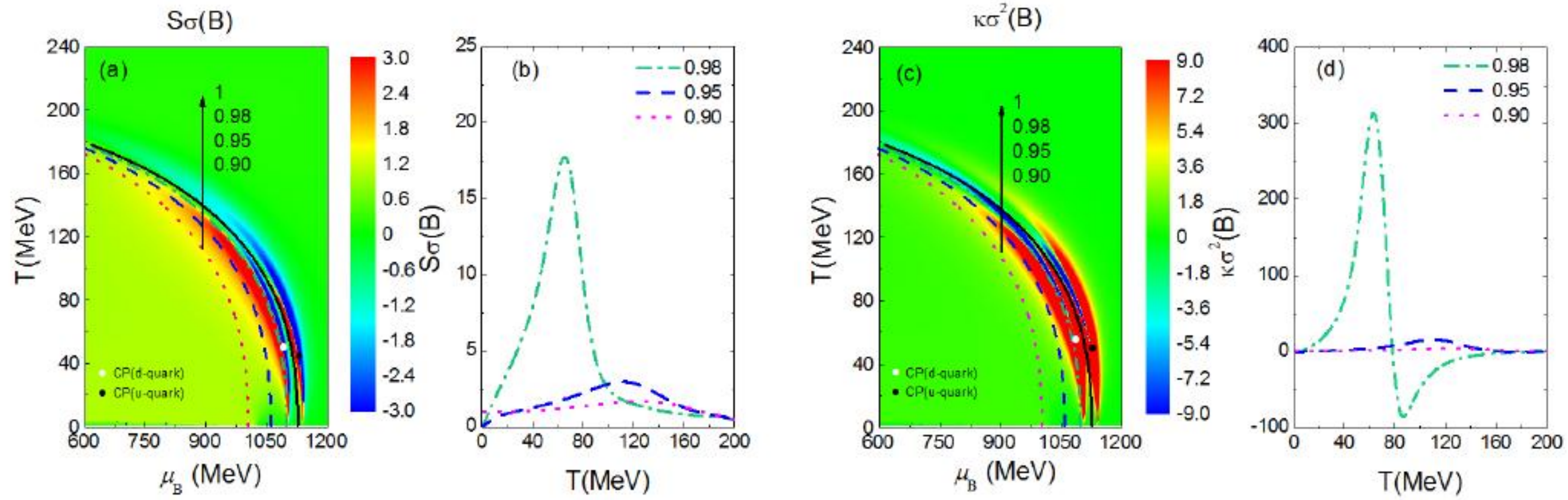
$$\chi_X^{(n)} = \frac{\partial^n(-\Omega/T)}{\partial(\mu_X/T)^n}$$

$$S\sigma = \frac{\chi^{(3)}}{\chi^{(2)}}, \quad \kappa\sigma^2 = \frac{\chi^{(4)}}{\chi^{(2)}}$$

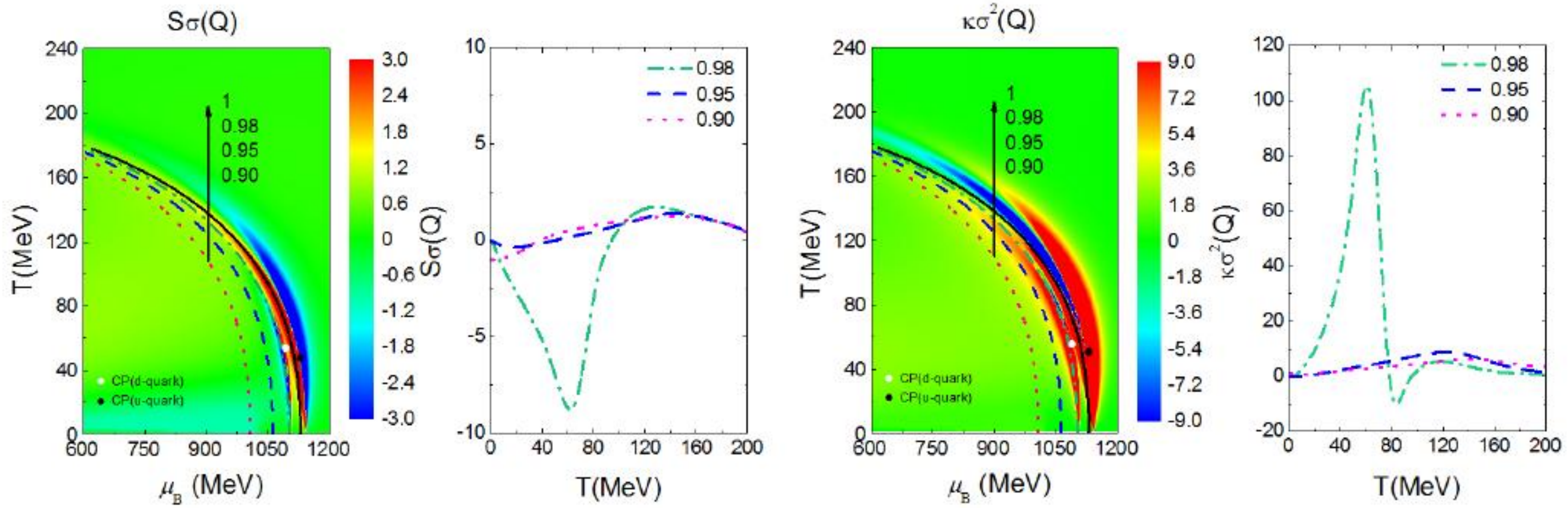
$$\mu_I = -0.293 - 0.0264\mu_B \ (\mathrm{MeV})$$

H. Liu and JX, arXiv: 1709.05178

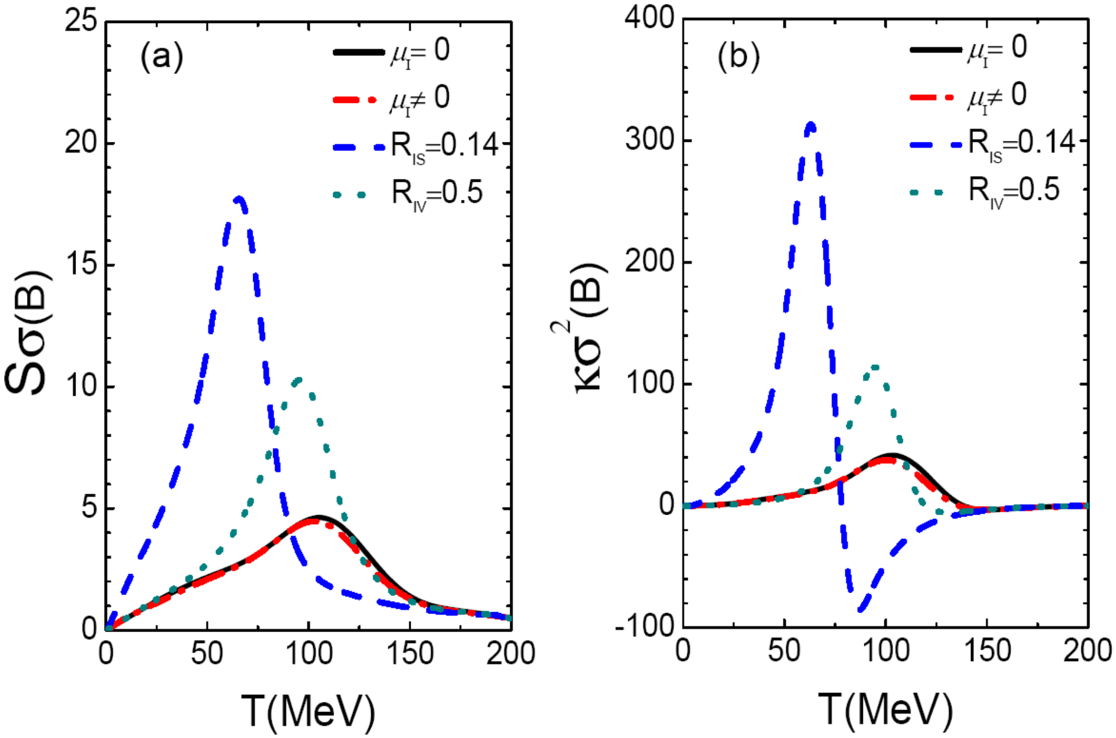
Susceptibility from different assumptions of chemical freeze-out lines

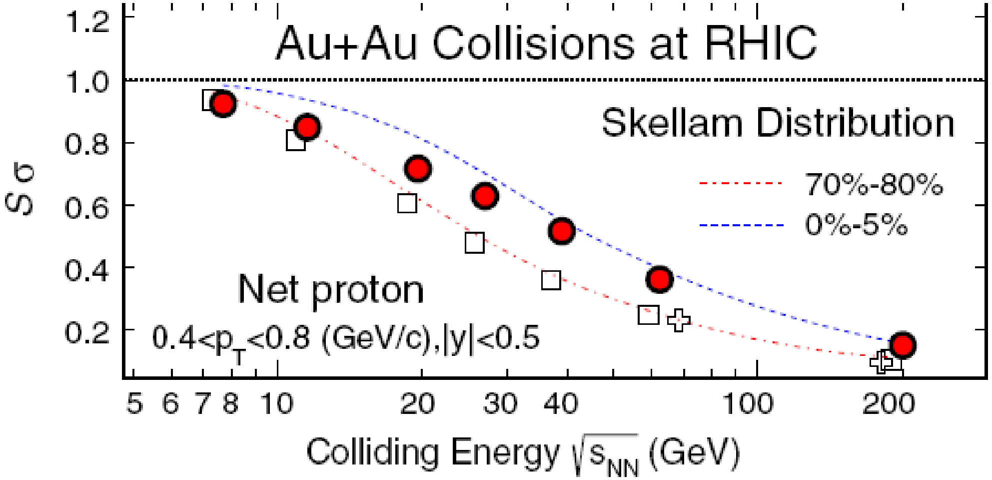


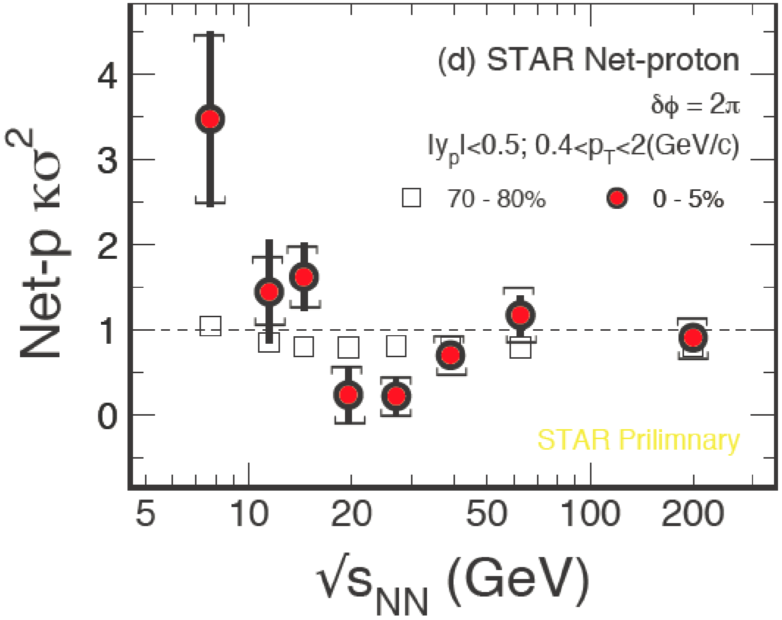
H. Liu and JX, arXiv: 1709.05178



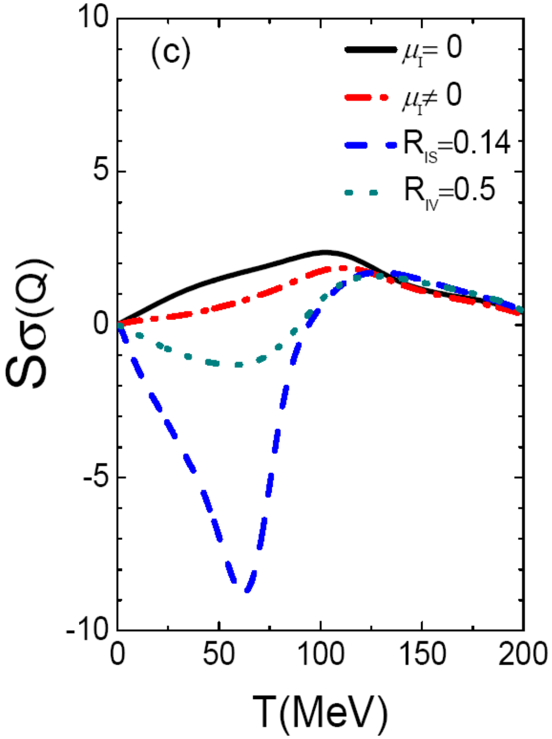
What if the chemical potential line is very close to the phase boundary (0.98)

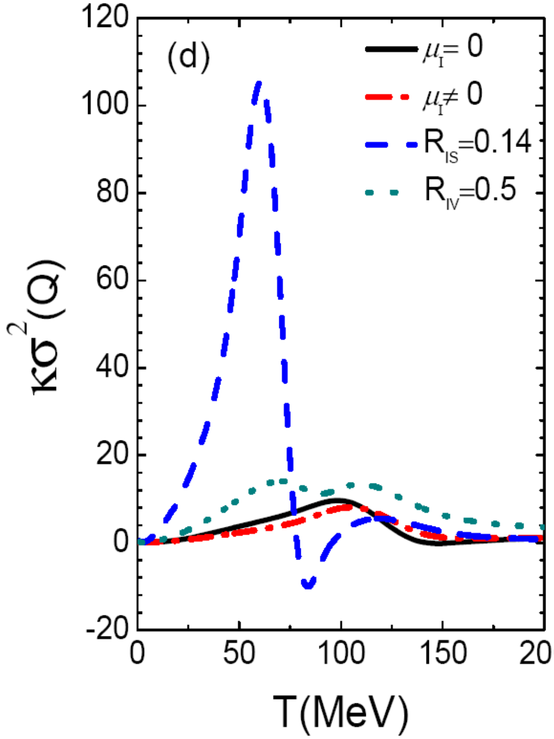


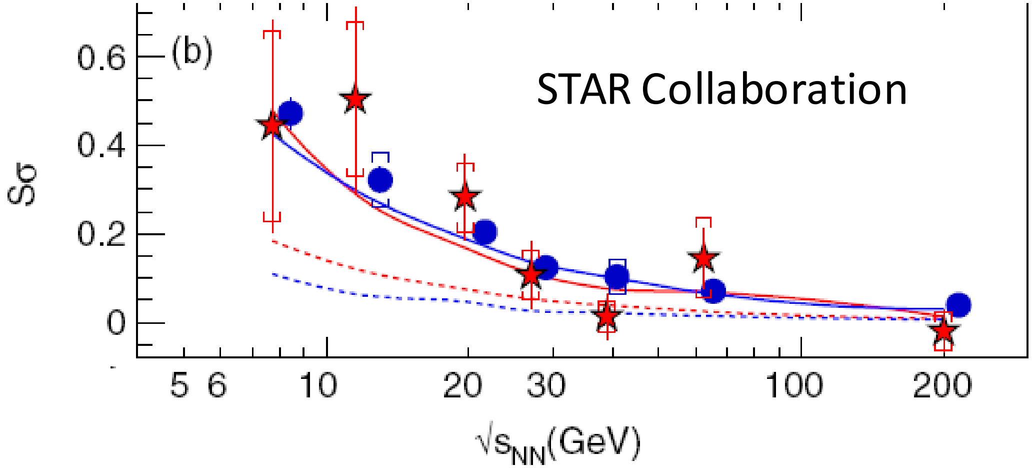


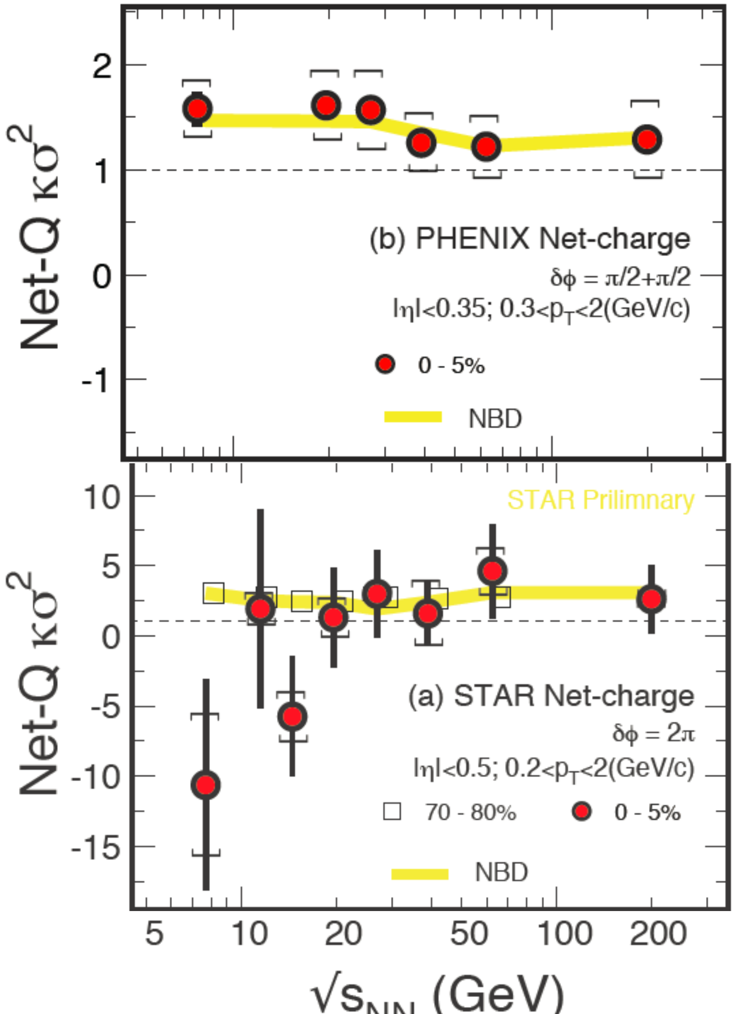


What if the chemical potential line is very close to the phase boundary (0.98)





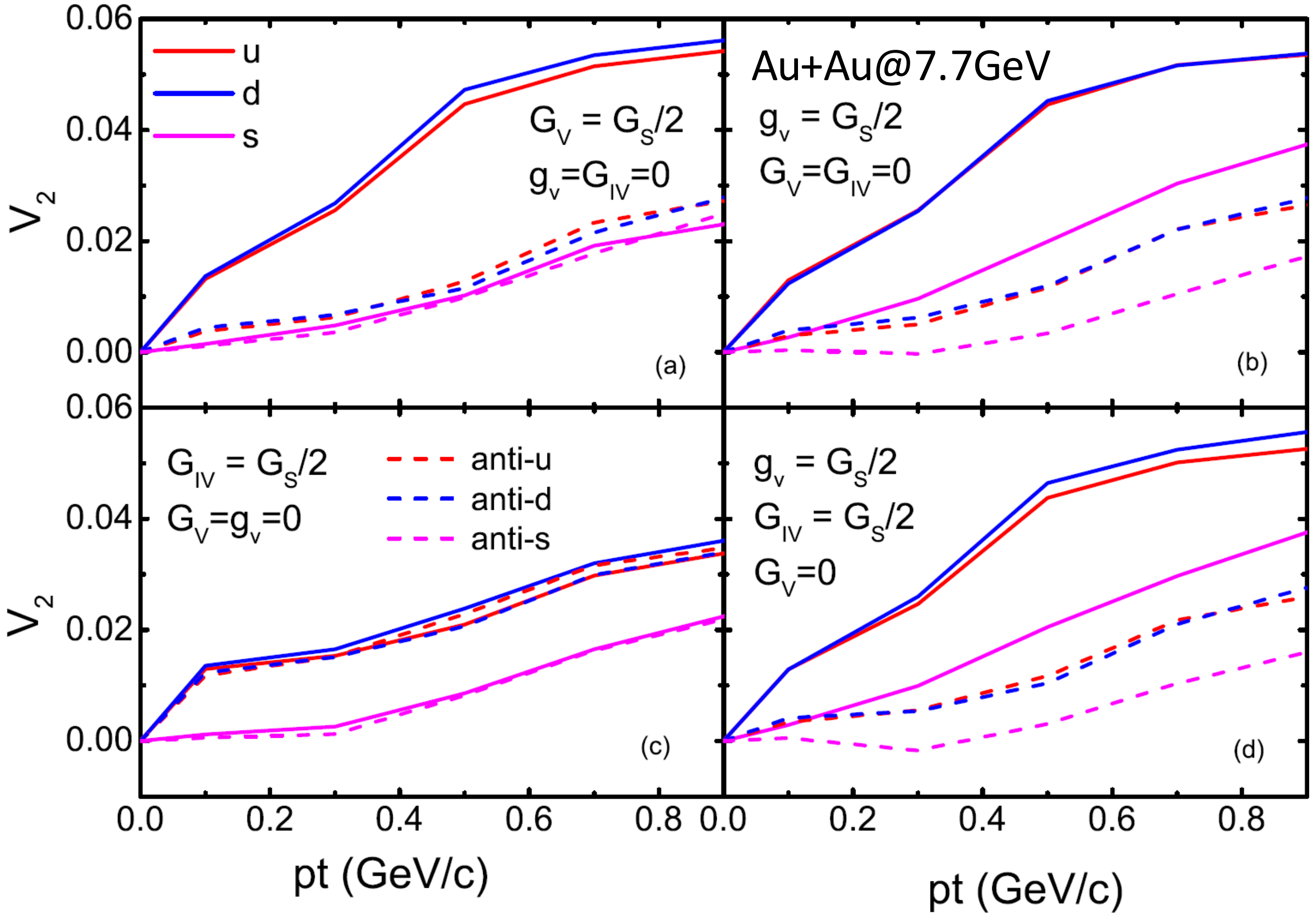




Isospin splitting of parton elliptic flow

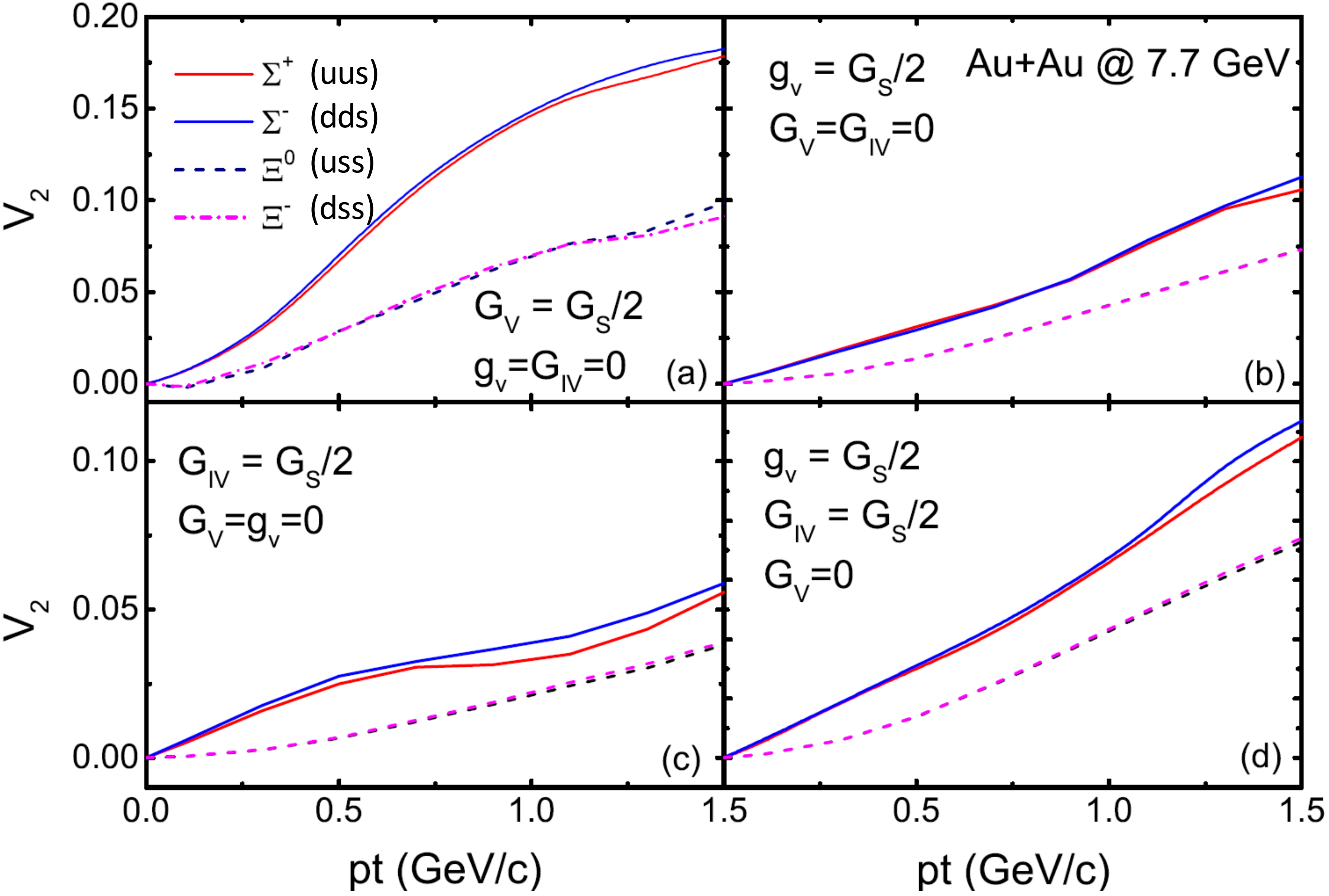
$$H_i = \sqrt{p_i^{*2} + M_i^2} \pm G_V \rho_i^0 \pm g_V \rho^0 + G_{IV} \tau_{3i} \left(\rho_u^0 - \rho_d^0 \right)$$

$$\vec{p}_i^{\,*} = \vec{p} \mp G_V \vec{\rho}_i \mp g_V \vec{\rho} \mp G_{IV} \tau_{3i} \left(\vec{\rho}_u - \vec{\rho}_d \right) \qquad M_i = m_i - 2G_S\sigma_i + 2K\sigma_j\sigma_k - 2G_{IS}\tau_{3i}(\sigma_u - \sigma_d)$$



K.J. Sun et al., in preparation

Isospin splitting of baryon elliptic flow

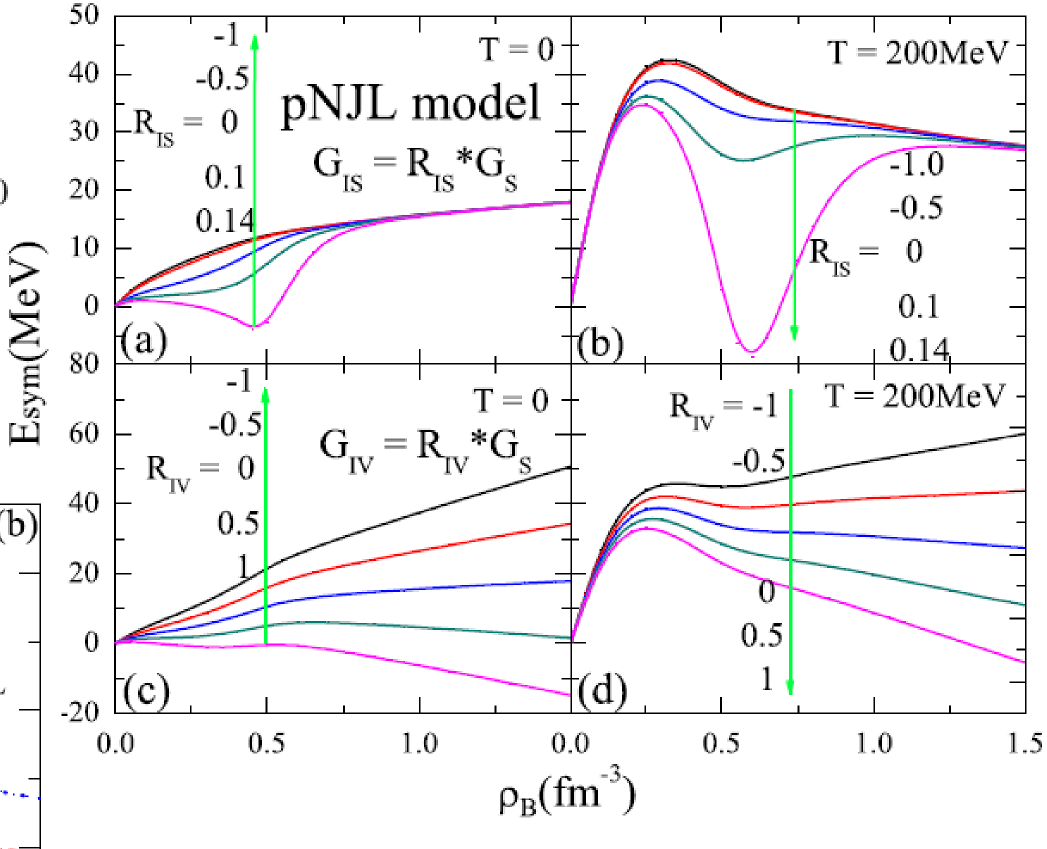


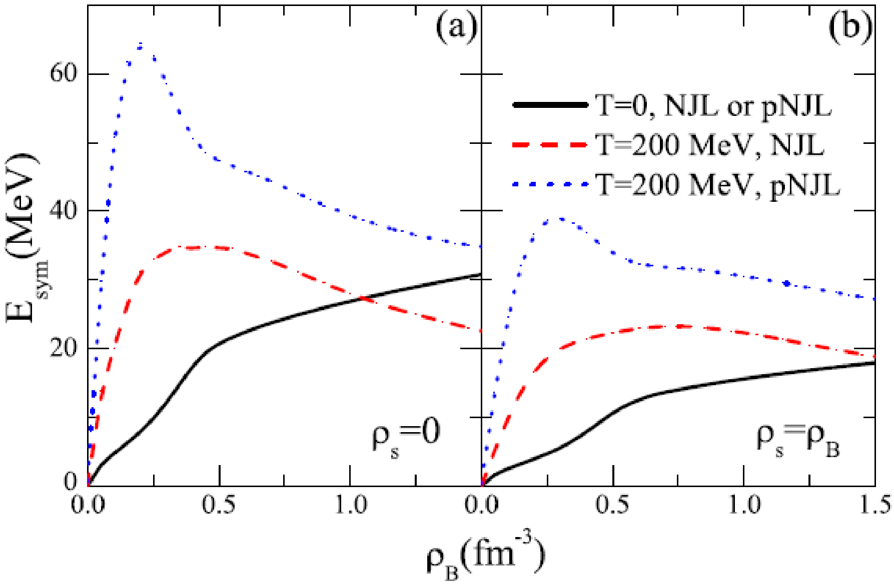
K.J. Sun et al., in preparation

Symmetry energy from NJL model

$$E(\rho_B, \delta, \rho_s) = E_0(\rho_B, \rho_s) + E_{\rm sym}(\rho_B, \rho_s)\delta^2 + \vartheta(\delta^4).$$

$$E_{\rm sym}(\rho_B, \rho_s) = \frac{1}{2!}\frac{\partial^2 E(\rho_B, \delta, \rho_s)}{\partial \delta^2}\bigg|_{\delta=0}$$

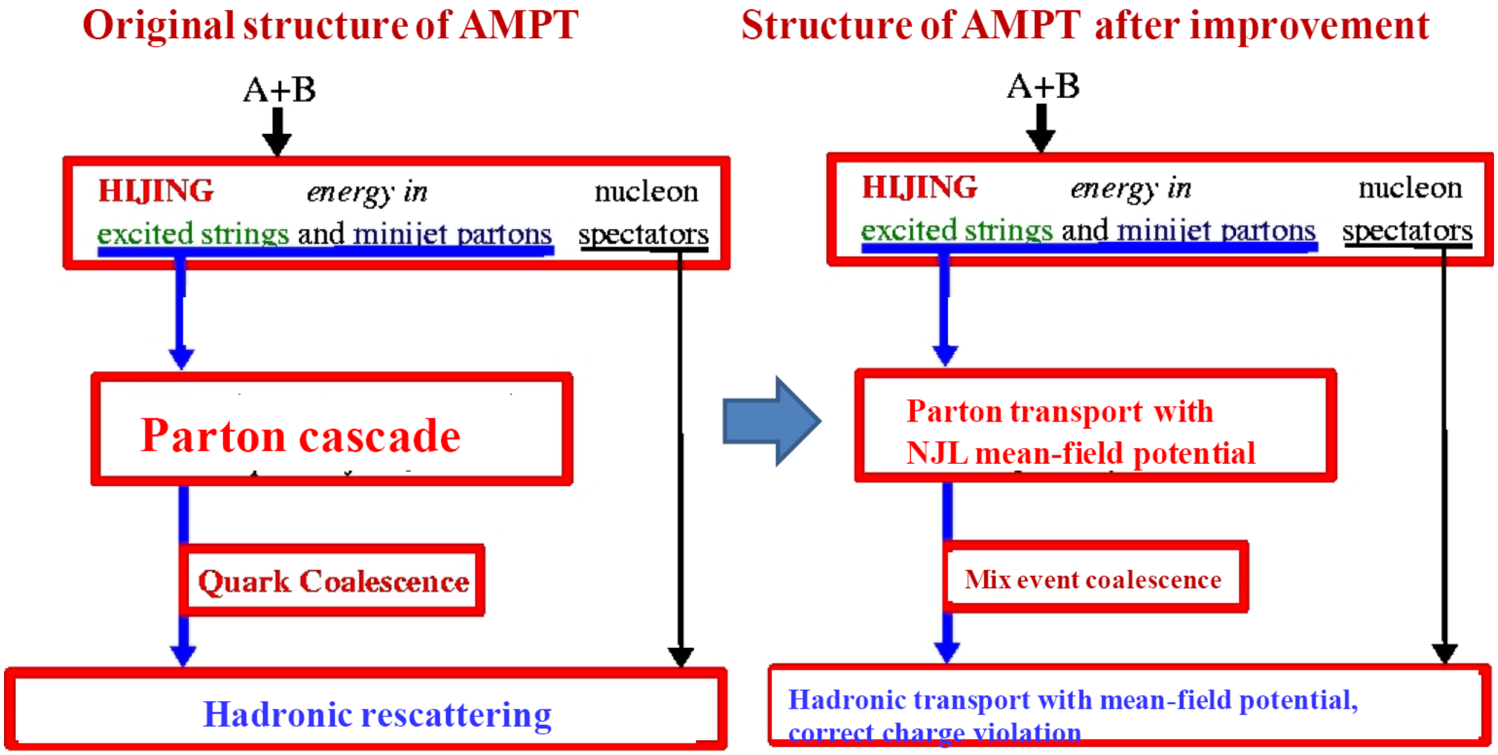




H. Liu, JX, L.W. Chen, and K.J. Sun, PRD (2016)

Summary

- Elliptic flow splitting between particles and antiparticles
- HBT correlation, radii, and chaoticity parameter
- Isospin effect on phase diagram and susceptibility



Original structure of AMPT

A+B

HIJING *energy in* excited strings and minijet partons nucleon spectators

Parton cascade

Quark Coalescence

Hadronic rescattering

Structure of AMPT after improvement

A+B

HIJING *energy in* excited strings and minijet partons nucleon spectators

Parton transport with NJL mean-field potential

Mix event coalescence

Hadronic transport with mean-field potential, correct charge violation

Suitable for top RHIC and LHC **Suitable for RHIC-BES and FAIR-CBM**

$\sqrt{s_{NN}}$ (GeV)		λ	R_o (fm)	R_s (fm)	R_l (fm)
7.7	cascade	0.673 ± 0.007	5.58 ± 0.03	4.75 ± 0.03	4.39 ± 0.03
	mean-field	0.719 ± 0.007	6.16 ± 0.04	5.53 ± 0.04	5.51 ± 0.04
	expt.	0.532 ± 0.007	5.57 ± 0.13	4.93 ± 0.10	5.01 ± 0.11
11.5	cascade	0.663 ± 0.007	5.59 ± 0.03	4.72 ± 0.03	4.39 ± 0.03
	mean-field	0.704 ± 0.007	6.33 ± 0.04	5.54 ± 0.04	5.67 ± 0.04
	expt.	0.508 ± 0.004	5.68 ± 0.07	4.79 ± 0.05	5.43 ± 0.07
19.6	cascade	0.656 ± 0.007	5.60 ± 0.03	4.78 ± 0.03	4.43 ± 0.03
	mean-field	0.698 ± 0.008	6.39 ± 0.04	5.48 ± 0.04	5.76 ± 0.04
	expt.	0.498 ± 0.002	5.84 ± 0.05	4.84 ± 0.03	5.80 ± 0.05
27	cascade	0.651 ± 0.007	5.60 ± 0.03	4.78 ± 0.03	4.49 ± 0.03
	mean-field	0.689 ± 0.008	6.41 ± 0.04	5.51 ± 0.04	5.88 ± 0.04
	expt.	0.492 ± 0.002	5.82 ± 0.03	4.89 ± 0.02	5.99 ± 0.04
39	cascade	0.655 ± 0.007	5.63 ± 0.03	4.79 ± 0.03	4.55 ± 0.03
	mean-field	0.678 ± 0.008	6.42 ± 0.04	5.53 ± 0.04	5.95 ± 0.04
	expt.	0.491 ± 0.004	5.86 ± 0.07	4.97 ± 0.05	6.18 ± 0.08

Equations of motion for solving Boltzmann equation

Substitute

$$f(rp,t) = \int \frac{\mathrm{d}r_0\, \mathrm{d}p_0\, \mathrm{d}s}{(2\pi\hbar)^3} \exp\{\mathrm{i}s \cdot [p - P(r_0 p_0 s, t)]/\hbar\}\, \delta[r - R(r_0 p_0 s, t)]\, f(r_0 p_0, t_0)$$

into the Boltzmann-Vlasov equation

$$\frac{\partial f(rp,t)}{\mathrm{d}t} + \frac{p}{m} \cdot \nabla_r f(rp,t) - \frac{2}{\hbar} \sin\left\{\frac{\hbar}{2} \nabla_r^V \cdot \nabla_p^f\right\} V(r,t) f(rp,t) = 0$$

First term:

$$\begin{aligned} \frac{\partial f(rp,t)}{\partial t} = \int \frac{\mathrm{d}r_0\, \mathrm{d}p_0\, \mathrm{d}s}{(2\pi\hbar)^3} f(r_0 p_0, t_0) \Bigg[& \frac{(-\mathrm{i}s)}{\hbar} \cdot \frac{\partial P}{\partial t} \\ & \times \exp\{\mathrm{i}s \cdot [p - P(r_0 p_0 s, t)]/\hbar\} \delta[r - R(r_0 p_0 s, t)] \\ & + \exp\{\mathrm{i}s \cdot [p - P(r_0 p_0 s, t)]/\hbar\} \\ & \times (\nabla_R \delta[r - R(r_0 p_0 s, t)]) \cdot \partial R(r_0 p_0 s, t)/\partial t \Bigg]. \end{aligned}$$

Noting that

$$\nabla_R \delta[r - R(r_0 p_0 s, t)] = -\nabla_r \delta[r - R(r_0 p_0 s, t)]$$

So

$$\begin{aligned}
&\int \frac{\mathrm{d}r_0\,\mathrm{d}p_0\,\mathrm{d}s}{(2\pi\hbar)^3} f(r_0 p_0, t_0) \exp\{\mathrm{i}s\cdot[p - P(r_0 p_0 s, t)]/\hbar\} \\
&\quad \times (-\nabla_r \delta[r - R(r_0 p_0 s, t)]) \cdot \partial R(r_0 p_0 s, t)/\partial t \\
&= -\frac{\partial R(r_0 p_0 s, t)}{\partial t} \cdot \nabla_r f(rp, t),
\end{aligned}$$

The potential term:

$$\begin{aligned}
&\frac{2}{\hbar} \sin\left\{\frac{\hbar}{2} \nabla_r^V \cdot \nabla_p^f\right\} V(r, t) f(rp, t) \\
&\quad = \frac{1}{\hbar} \int \frac{\mathrm{d}r_0\,\mathrm{d}p_0\,\mathrm{d}s}{(2\pi\hbar)^3} f(r_0 p_0, t_0) \\
&\qquad \times \exp\{\mathrm{i}s\cdot[p - P(r_0 p_0 s, t)]/\hbar\} \delta[r - R(r_0 p_0 s, t)] \\
&\qquad \times \frac{[V(r - \frac{s}{2}, t) - V(r + \frac{s}{2}, t)]}{\mathrm{i}}.
\end{aligned}$$

Put everything together:

$$\left[-\frac{\partial R(r_0p_0s,t)}{\partial t}+\frac{p}{m}\right]\cdot\nabla_r f(rp,t)$$
$$+\int\frac{\mathrm{d}r_0\,\mathrm{d}p_0\,\mathrm{d}s}{(2\pi\hbar)^3}f(r_0p_0,t_0)\left[\frac{(-\mathrm{i}s)}{\hbar}\cdot\frac{\partial P}{\partial t}\right.$$
$$\left.-\frac{[V(r-\frac{s}{2},t)-V(r+\frac{s}{2},t)]}{\mathrm{i}\hbar}\right]$$
$$\times\ \exp\{\mathrm{i}s\cdot[p-P(r_0p_0s,t)]/\hbar\}$$
$$\times\ \delta[r-R(r_0p_0s,t)]=0.$$

Equations of motion:

$$\frac{\partial R}{\partial t}=\frac{p}{m},$$

$$s\cdot\frac{\partial P}{\partial t}=V\left(R-\frac{s}{2},t\right)-V\left(R+\frac{s}{2},t\right).$$

Momentum-dependent potential:
One more term in BV equation

$$+\frac{2}{\hbar}\sin\left(\frac{\hbar}{2}\nabla_p^V\cdot\nabla_r^f\right)V(R,P,t)\,f(R,P,t)$$

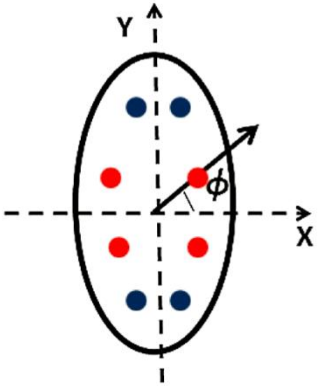
$$\frac{\partial R}{\partial t}=\frac{p}{m}+\nabla_p V$$

$$\frac{\partial P}{\partial t}\approx-\nabla_R V(R,t)$$

Calculate phase-space distribution function $f(\vec{r},\vec{p};t)=\frac{1}{N_{\mathrm{TP}}}\sum_{i=1}^{N_{\mathrm{TP}}A}g[\vec{r}-\vec{r}_i(t)]\tilde{g}[\vec{p}-\vec{p}_i(t)]$

Explanations for v_2 splitting

$v_2(\pi^+) < v_2(\pi^-)$



- Chiral magnetic wave

  => electric quadrupole moment

  Y. Burnier, D. E. Kharzeev, J. F. Liao, and H. U. Yee, PRL (2011)

- Different v_2 of transported and produced partons

  J. C. Dunlop, M. A. Lisa, and P. Sorensen, PRC (2011)

- Different rapidity distributions of quarks and antiquarks

  V. Greco, M. Mitrovski, and G. Torrieri, PRC (2012)

- Conservation of baryon charge, strangeness, and isospin

  J. Steinheimer, V. Koch, and M. Bleicher, PRC (2012)

- Different mean-field potentials for particles and their antiparticles

  JX, L. W. Chen, C. M Ko, and Z. W. Lin, PRC (2012);
  T. Song, S. Plumari, V. Greco, C. M. Ko, and F. Li, arXiv:1211.5511 [nucl-th];
  JX, T. Song, C. M. Ko, and F. Li, PRL (2014)

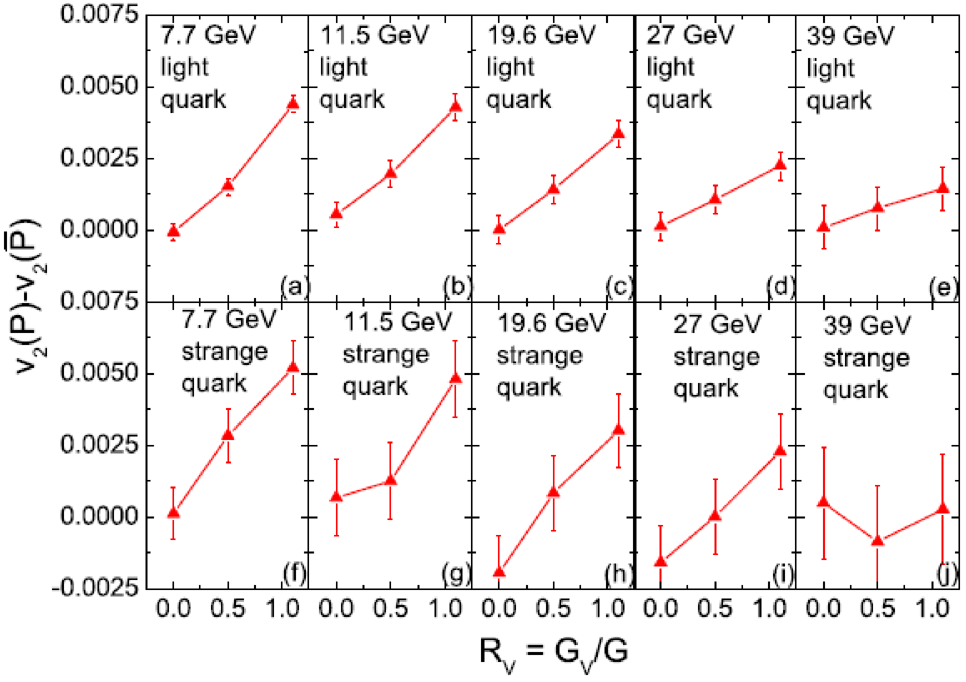
- Different radial flows of protons and antiprotons

  X. Sun, H. Masui, A.M. Poskanzer, and A. Schmach, PRC (2015)

- Hydrodyanmics at finite baryon chemical potential

  Y. Hatta, A. Monnai, and B.W. Xiao, arXiv: 1505.04226 [nucl-th]; 1507.04909 [nucl-th]

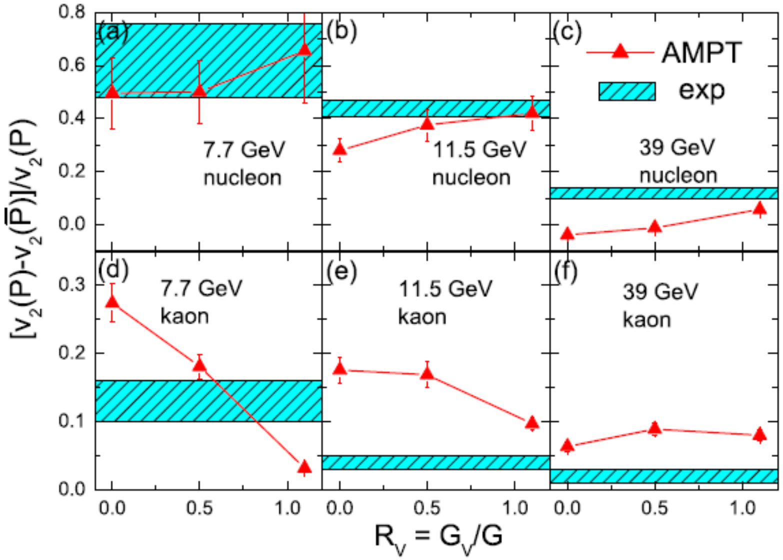
Collision energy dependence of v_2 splitting



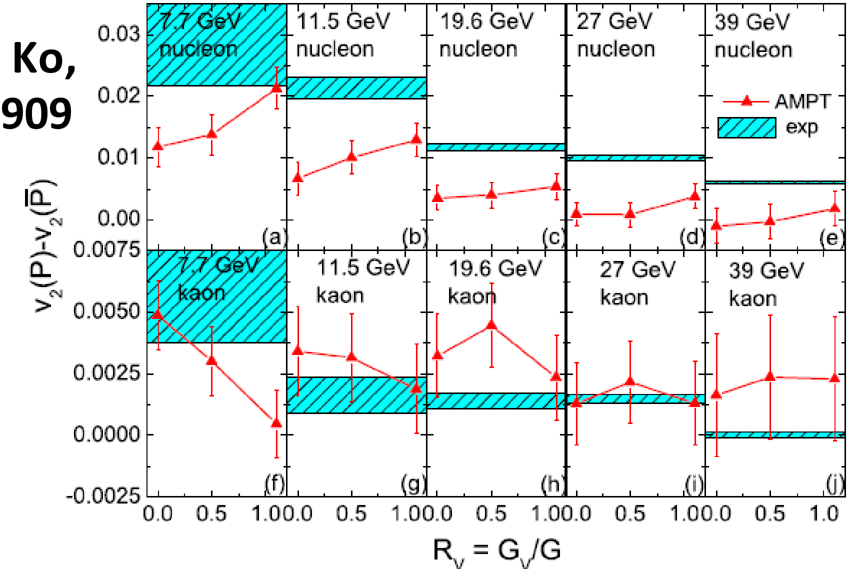
Difficult to reproduce quantitatively v_2 splitting at all collision energies at RHIC-BES

Effect of vector potential still holds

Energy dependence of v_2 splitting is qualitatively consistent with experimental observation



JX and C.M. Ko, PRC 94, 054909 (2016)



Formulae of HBT correlation

$$C(\mathbf{k}^*) = \frac{\int S(\mathbf{r}^*, \mathbf{k}^*) \left|\Psi_{\mathbf{k}^*}(\mathbf{r}^*)\right|^2 d^4\mathbf{r}^*}{\int S(\mathbf{r}^*, \mathbf{k}^*) d^4\mathbf{r}^*}$$

$$\begin{aligned} \Psi_{\mathbf{k}^*}(\mathbf{r}^*) &= e^{i\delta_c}\sqrt{A_c(\eta)} \\ &\times \left[e^{-i\mathbf{k}^*\mathbf{r}^*} F(-i\eta, 1, i\xi) + f_c(\mathbf{k}^*)\frac{\tilde{G}(\rho,\eta)}{\mathbf{r}^*}\right] \end{aligned}$$

Coulomb penetration factor $A_c(\eta) = 2\pi\eta\left[\exp(2\pi\eta) - 1\right]^{-1}$

$\eta = (k^* a_c)^{-1}$ a_c Bohr radius

Coulomb s-wave phase shift $\delta_c = arg\Gamma(1 + i\eta)$

$\xi = \mathbf{k}^*\mathbf{r}^* + k^* r^*$ $\rho = k^* r^*$

$$f_c(k^*) = \left[\frac{1}{f_0} + \frac{1}{2}d_0 k^{*2} - \frac{2}{a_c}h(\eta) - ik^* A_c(\eta)\right]^{-1}$$

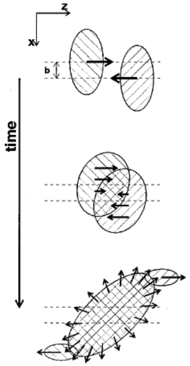
Strong interaction

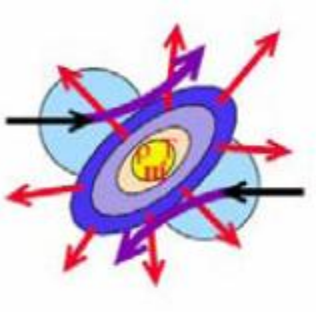
Coulomb force

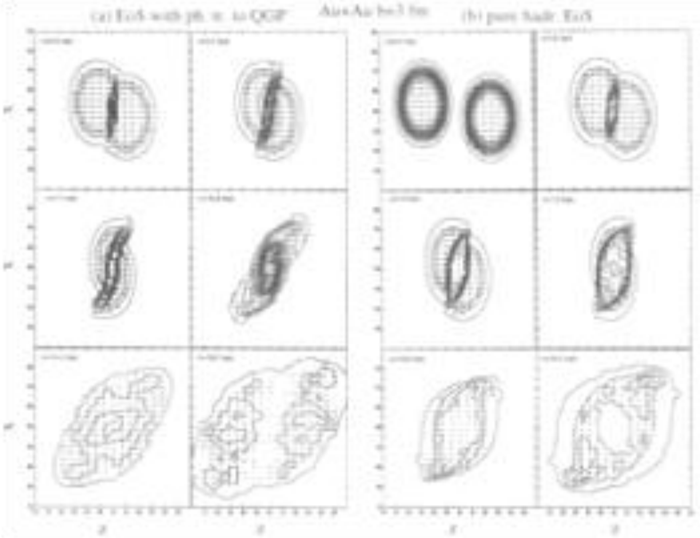
F, G, h are special functions

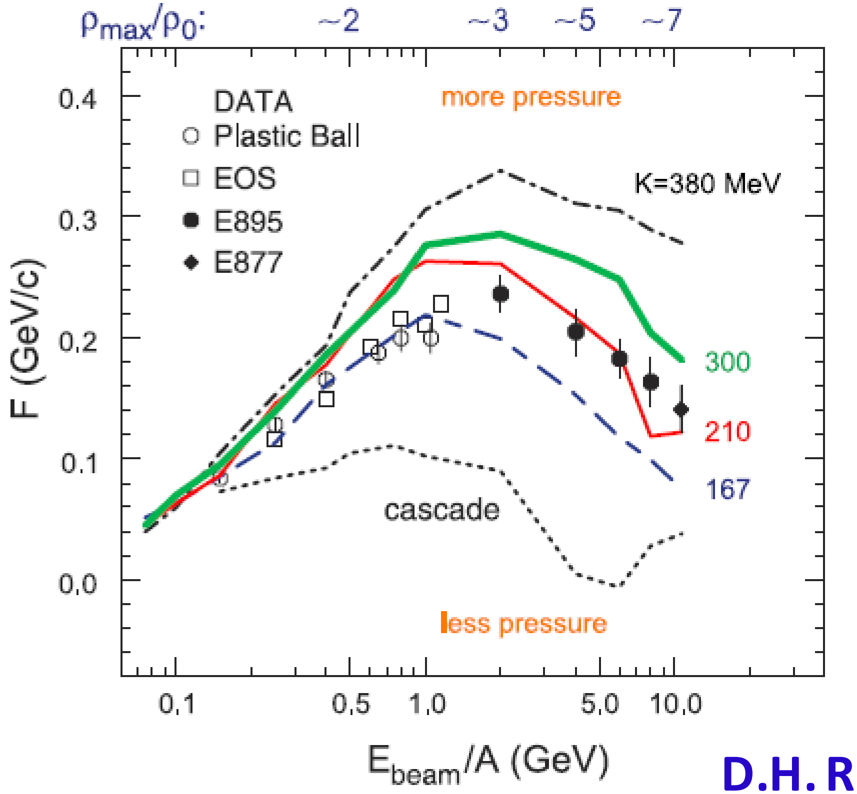
EOS effects on the directed flow

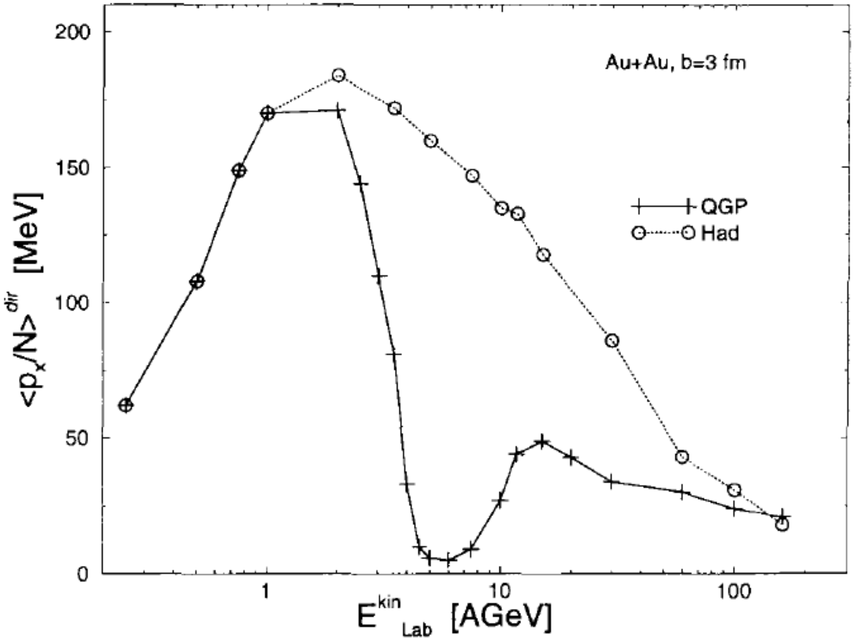
$$v_1 = \left\langle \frac{p_x}{p_T} \right\rangle$$











D.H. Rischke et al., APH N.S. Heavy Ion Physics (1995)

P. Danielewicz, R. Lacey, and W.G. Lynch, Science (2002)

EOS of quark phase from NJL

$$\Omega_{\mathrm{NJL}} = -2N_c \sum_{i=u,d,s} \int_0^{\Lambda} \frac{d^3p}{(2\pi)^3} [E_i + T\ln(1 + e^{-\beta(E_i-\tilde{\mu}_i)})$$

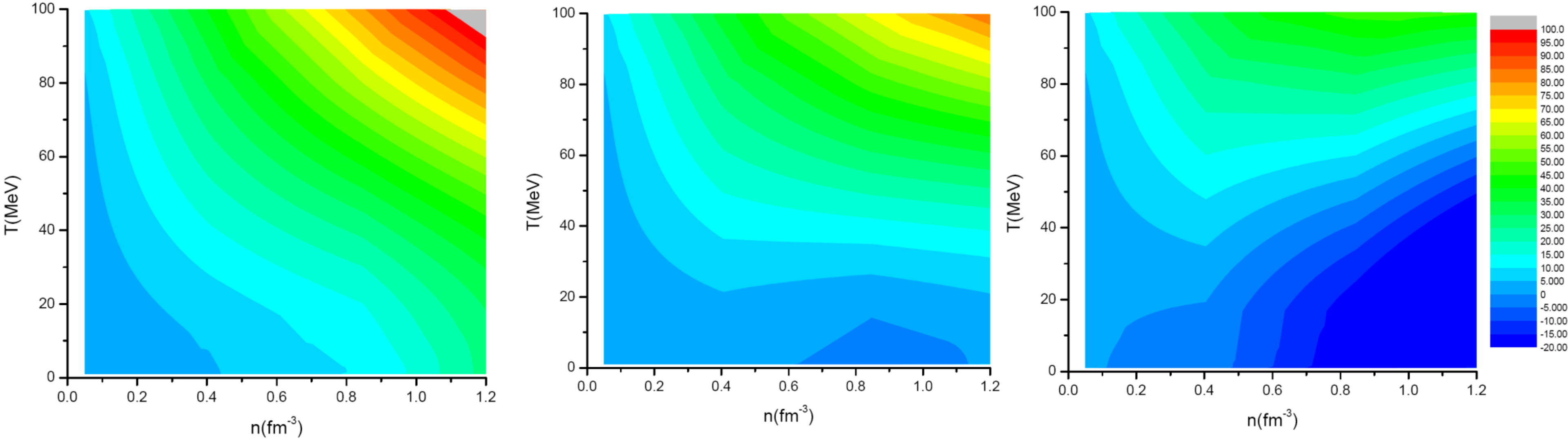
$$+ T\ln(1 + e^{-\beta(E_i+\tilde{\mu}_i)})] + G_S(\sigma_u^2 + \sigma_d^2 + \sigma_s^2)$$

$$-4K\sigma_u\sigma_d\sigma_s - \frac{1}{3}G_V(\rho_u + \rho_d + \rho_s)^2$$

$$P = -\Omega_{NJL}$$

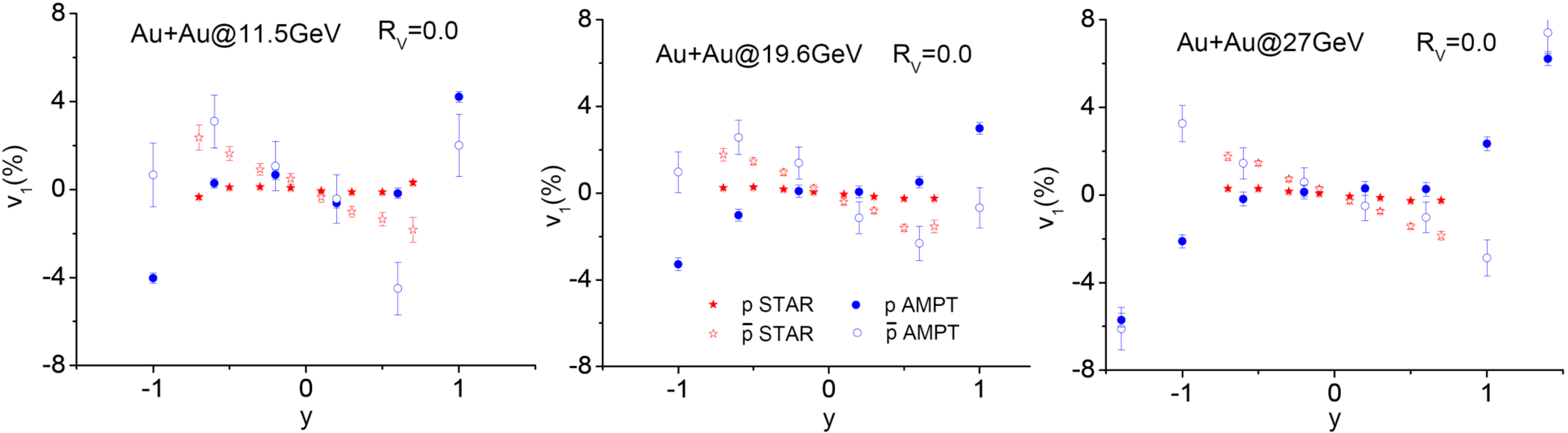
Pressure in the temperature-density (T-n) plane

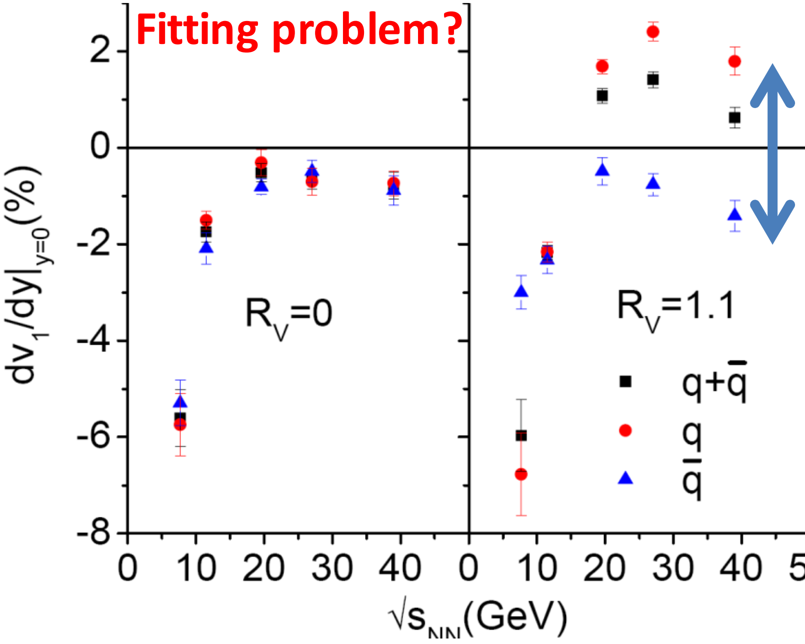
G_V = 1.1G **G_V = 0** **G_V = -1.1G?**



Directed flow from AMPT+NJL

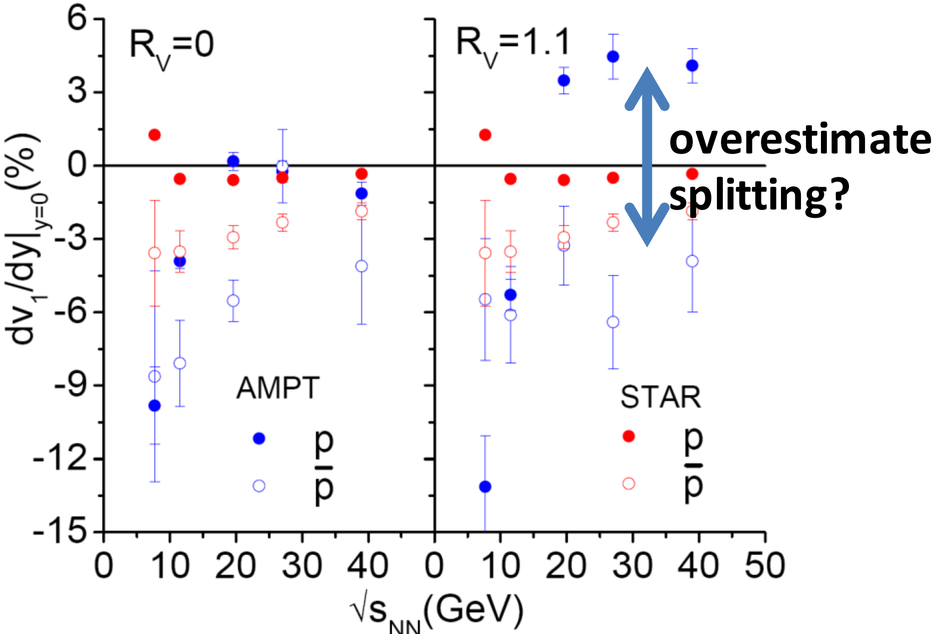
preliminary





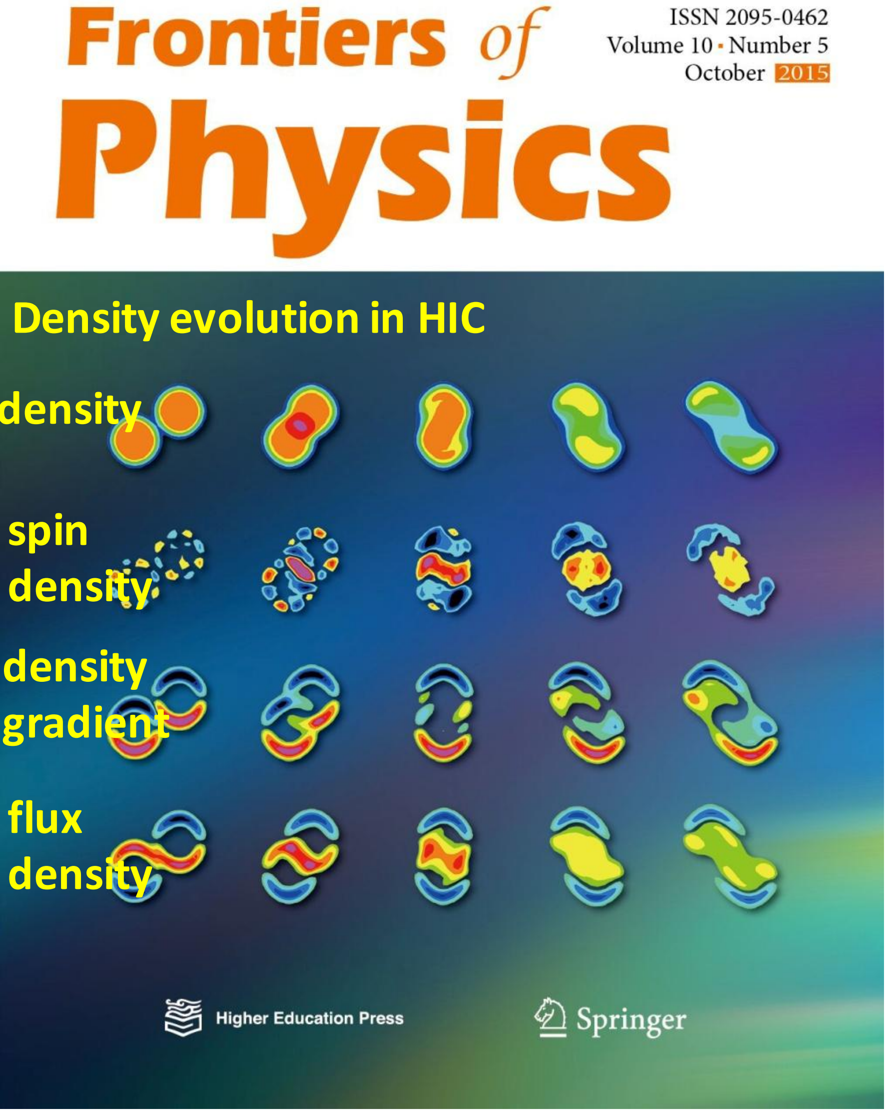
Fitting problem?

large splitting



overestimate splitting?

Spin dynamics in intermediate- and low-energy heavy-ion collisions



invited review, selected as cover story

JX, B.A. Li, W.Q. Shen, and Y. Xia, Front. Phys. (2015)

SIBUU12

Boltzmann-Uehling-Uhlenbeck equation

$$\frac{\partial f}{\partial t}+v\cdot\nabla_r f-\nabla U\cdot\nabla_p f=-\int\frac{d^3p_2d^3p_1'd^3p_2'}{(2\pi)^9}\sigma v_{12}[ff_2(1-f_{1'})(1-f_{2'})$$

$$-f_{1'}f_{2'}(1-f)(1-f_2)](2\pi)^3\delta^3(p+p_2-p_1'-p_2')$$

test-particle method

C.Y. Wong, PRC 25, 1460 (1982)

equations of motion

$$\frac{d\vec{r}}{dt}=\frac{\vec{p}}{m} \qquad \frac{d\vec{p}}{dt}=-\nabla U$$

Spin-dependent Boltzmann-Uehling-Uhlenbeck eq

$$\hat{\varepsilon}(\vec{r},\vec{p}) = \varepsilon(\vec{r},\vec{p})\hat{I}+\vec{h}(\vec{r},\vec{p})\cdot\vec{\sigma},$$

$$\hat{f}(\vec{r},\vec{p}) = f_0(\vec{r},\vec{p})\hat{I}+\vec{g}(\vec{r},\vec{p})\cdot\vec{\sigma}.$$

$$\frac{\partial\hat{f}}{\partial t}+\frac{i}{\hbar}\left[\hat{\varepsilon},\hat{f}\right]+\frac{1}{2}\left(\frac{\partial\hat{\varepsilon}}{\partial\vec{p}}\cdot\frac{\partial\hat{f}}{\partial\vec{r}}+\frac{\partial\hat{f}}{\partial\vec{r}}\cdot\frac{\partial\hat{\varepsilon}}{\partial\vec{p}}\right)-\frac{1}{2}\left(\frac{\partial\hat{\varepsilon}}{\partial\vec{r}}\cdot\frac{\partial\hat{f}}{\partial\vec{p}}+\frac{\partial\hat{f}}{\partial\vec{p}}\cdot\frac{\partial\hat{\varepsilon}}{\partial\vec{r}}\right)=I_c$$

test-particle method

Y. Xia, JX, B.A. Li, and W.Q. Shen, Phys. Lett. B (2016)

spin-dependent equations of motion

$$\frac{d\vec{r}}{dt}=\frac{\vec{p}}{m}+\nabla_p(\varepsilon+\vec{h}\cdot\vec{n}) \qquad \frac{d\vec{p}}{dt}=-\nabla(\varepsilon+\vec{h}\cdot\vec{n})$$

$$\frac{d\vec{n}}{dt}=2\vec{h}\times\vec{n}$$

$\vec{n}$ spin expectation direction

Spin-dependent Hamiltonian from NJL model

Euler-Lagrange equation

$$[\gamma^\mu(i\partial_\mu - A_\mu) - M_i]\psi_i = 0. \qquad (8)$$

Space and time components of the vector potential

$$\begin{aligned} A_0 &= B_i g_V \rho_0 + Q_i e\varphi, & (9)\\ \vec{A} &= B_i g_V \vec{\rho} + Q_i e\vec{A}_m, & (10)\end{aligned}$$

with $g_V = \frac{2}{3}G_V$, $\rho_0 = \langle\bar{\psi}\gamma^0\psi\rangle$ and

$\vec{\rho} \equiv \langle\bar{\psi}\vec{\gamma}\psi\rangle$

The scalar and vector potential of the real electromagnetic field

$$\begin{aligned} e\varphi(t,\vec{r}) &= \frac{e^2}{4\pi}\sum_n Z_n \frac{1}{R_n - \vec{v}_n\cdot\vec{R}_n}, & (11)\\ e\vec{A}_m(t,\vec{r}) &= \frac{e^2}{4\pi}\sum_n Z_n \frac{\vec{v}_n}{R_n - \vec{v}_n\cdot\vec{R}_n}, & (12)\end{aligned}$$

Calculated from Eq. (8)

$$i\partial_t\psi_i = [\gamma^0\gamma^k(-i\partial_k + A_k) + \gamma^0 M_i + A_0]\psi_i. \qquad (13)$$

Hamiltonian operator

$$\hat{H} = \gamma^0\gamma^k(-\hat{p}_k + A_k) + \gamma^0 M_i + A_0, \qquad (14)$$

eigenvalue

Single-particle Hamiltonian

$$H = \sqrt{(\vec{p} - \vec{A})^2 + M_i^2 - \vec{\sigma}\cdot(\nabla\times\vec{A})} + A_0. \qquad (15)$$

$$\vec{\sigma}\cdot(\nabla\times\vec{A}) \ll (\vec{p}-\vec{A})^2 + M_i^2$$

Single-particle Hamiltonian can be further expressed as

$$H \approx \sqrt{M_i^2 + (\vec{p} - \vec{A})^2} + A_0 - \frac{\vec{\sigma}\cdot(\nabla\times\vec{A})}{2\sqrt{M_i^2 + (\vec{p} - \vec{A})^2}}. \qquad (16)$$

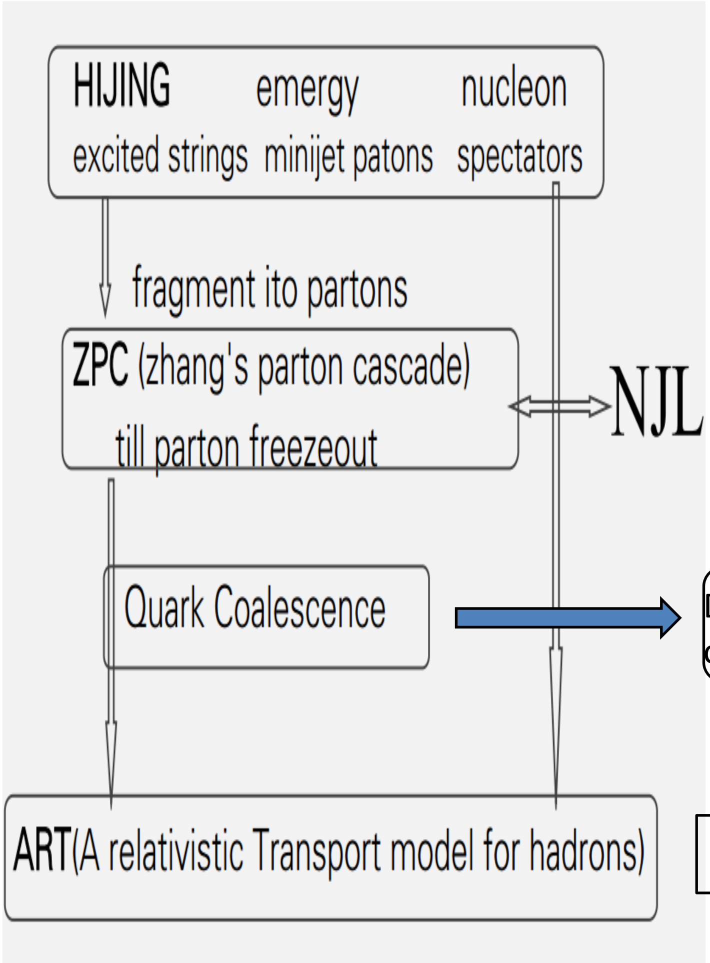
Equations of motion for partons and the extended AMPT model

$$
\begin{aligned}
\dot{\vec{r}} &= \vec{\nabla}_{\vec{p}} H, \\
\dot{\vec{p}} &= -\vec{\nabla} H, \\
\dot{\vec{\sigma}} &= -i[\vec{\sigma}, H].
\end{aligned}
$$

$$
\begin{aligned}
\frac{dr_k}{dt} &= \frac{p_k^*}{E_i^*} + \frac{1}{2}\frac{p_k^*}{E_i^{*3}}[\vec{\sigma}\cdot(\nabla\times\vec{A})], \\
\frac{dp_k^*}{dt} &= -\frac{M_i}{E_i^*}\frac{\partial M_i}{\partial r_k} + \frac{p_j^*}{E_i^*}\frac{\partial A_j}{\partial r_k} - \frac{\partial A_0}{\partial r_k} - \frac{\partial A_k}{\partial t} \\
&\quad - \dot{r}_j\frac{\partial A_k}{\partial r_j} - \frac{1}{2}[\vec{\sigma}\cdot(\nabla\times\vec{A})]\frac{M_i}{E_i^{*3}}\frac{\partial M_i}{\partial r_k} \\
&\quad + \frac{1}{2}[\vec{\sigma}\cdot(\nabla\times\vec{A})]\frac{p_j^*}{E_i^{*3}}\frac{\partial A_j}{\partial r_k} \\
&\quad + \frac{\vec{\sigma}}{2E_i^*}\cdot(\nabla\times\frac{\partial\vec{A}}{\partial r_k}) \\
\frac{d\vec{\sigma}}{dt} &= \frac{\vec{\sigma}\times(\nabla\times\vec{A})}{E_i^*},
\end{aligned}
$$

Detailed EOMs

EXTENDED AMPT MODEL



Dynamical coalescence

neglect

Spin polarization from vector interactions

Consider quark spin in NJL Hamiltonian

$$H \approx \sqrt{M_i^2 + (\vec{p} - \vec{A})^2} + A_0 \boxed{-\frac{\vec{\sigma}\cdot(\nabla\times\vec{A})}{2\sqrt{M_i^2 + (\vec{p}-\vec{A})^2}}}$$

spin-orbit coupling

$$\begin{aligned} A_0 &= B_i g_V \rho_0 + Q_i e \varphi, \\ \vec{A} &= B_i g_V \vec{\rho} + Q_i e \vec{A}_m, \end{aligned}$$

effective magnetic field

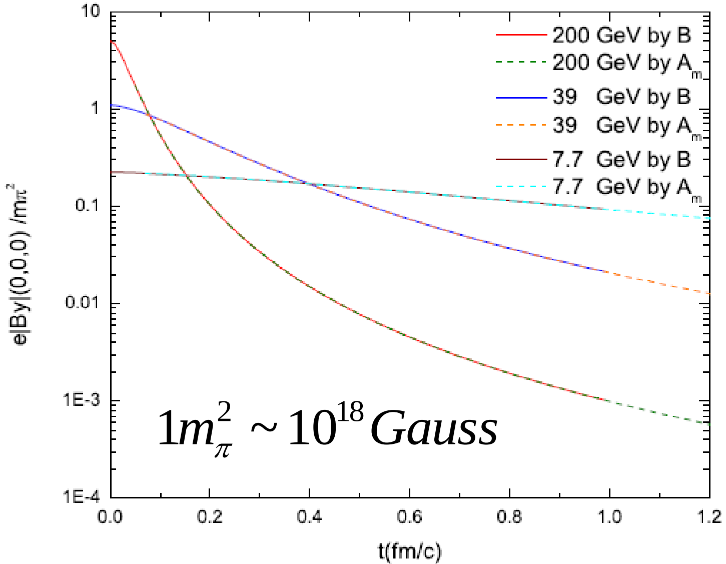
real magnetic field

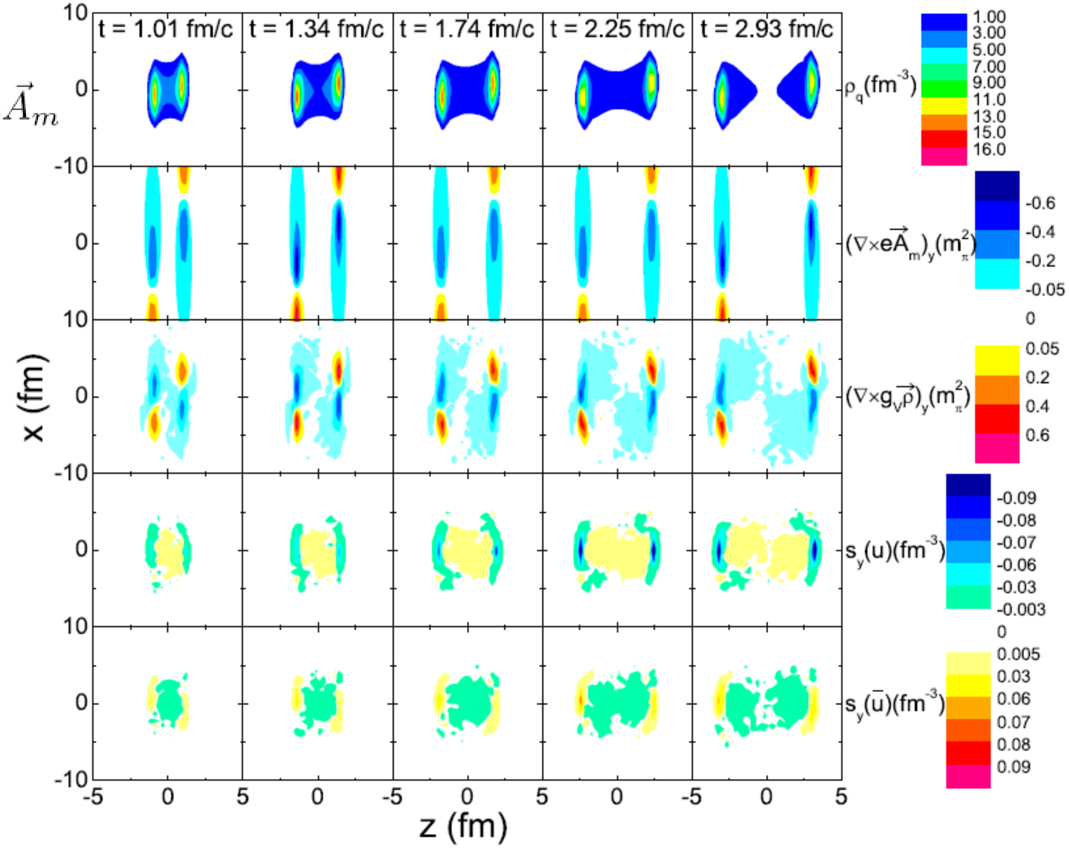
B_i: baryon number; Q_i: charge number

Comparing the real magnetic field from two calculation methods

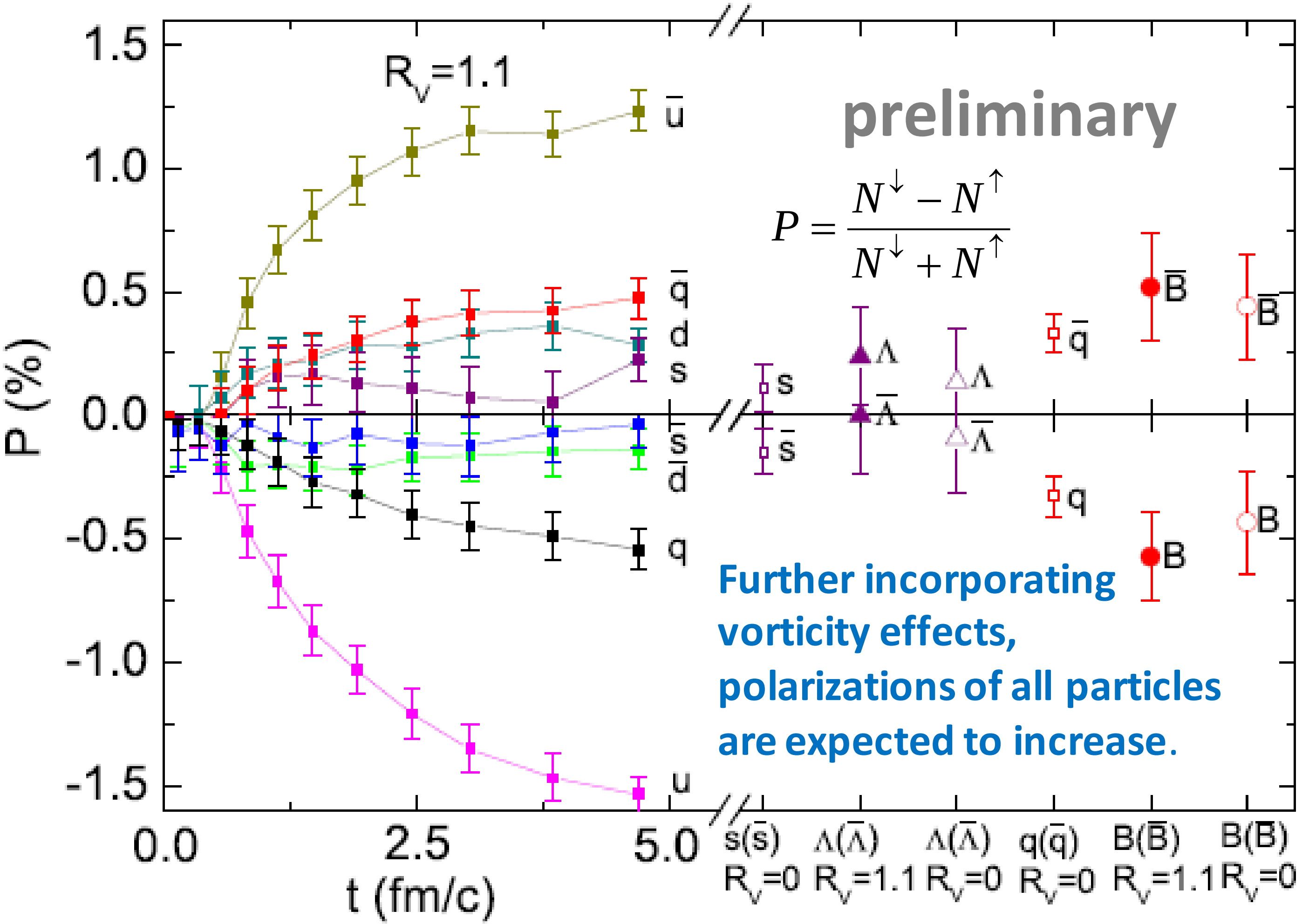
$$e\vec{A}_m(t,\vec{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\vec{v}_n}{R_n - \vec{v}_n\cdot\vec{R}_n} \qquad e\vec{B} = \nabla\times e\vec{A}_m$$

$$\vec{B}(t,\vec{r}) = \frac{e}{4\pi}\sum_n Z_n \frac{\vec{v}_n\times\vec{R}_n}{(R_n - \vec{v}_n\cdot\vec{R}_n)^3}(1-v_n^2)$$





Effects on the different baryon and antibaryon polarizations



$$P = \frac{N^{\downarrow} - N^{\uparrow}}{N^{\downarrow} + N^{\uparrow}}$$

Further incorporating vorticity effects, polarizations of all particles are expected to increase.

$$\Lambda^{\uparrow(\downarrow)} \sim uds^{\uparrow(\downarrow)}$$

$$\overline{\Lambda}^{\uparrow(\downarrow)} \sim \bar{u}\bar{d}\bar{s}^{\uparrow(\downarrow)}$$

$$P_{\Lambda} > P_{\overline{\Lambda}}$$

$$B^{\uparrow(\downarrow)} \sim qqq^{\uparrow(\downarrow)}$$

$$\overline{B}^{\uparrow(\downarrow)} \sim \bar{q}\bar{q}\bar{q}^{\uparrow(\downarrow)}$$

$$P_{B} < P_{\overline{B}}$$

Baryon spin: quark spin, gluon spin, quark angular momentum, …

Zhang-Zhu Han and JX, arXiv:1707.07262 [nucl-th]