QCD Phase Transitions and Baryon Number Fluctuations within FRG

Wei-jie Fu

Dalian University of Technology

EMMI Workshop on Critical Fluctuations Near the QCD Phase Boundary in Relativistic Nuclear Collisions
October 10-13, 2017, Wuhan

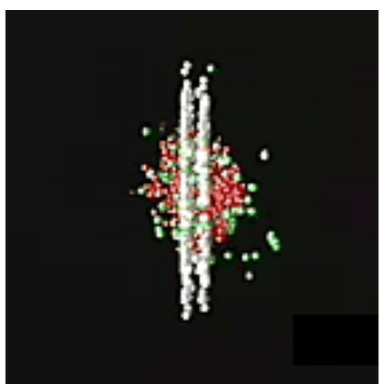
Collaborators:
fQCD collaboration (J. Braun, L. Corell, A. Cyrol, WF, M. Leonhardt, M. Mitter, J.M. Pawlowski, M. Pospiech, F. Rennecke, N. Strodthoff, N. Wink)
Yu-xin Liu (Peking U.), Yue-liang Wu (ITP)……

Outline

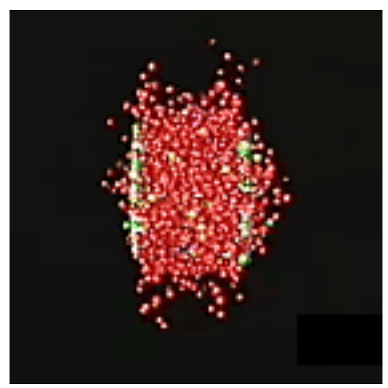
- Introduction
- Fluctuations of Conserved Charges with Mean Field Approximations (MFA)
- Quantum Fluctuations beyond MFA
- Preliminary Results from QCD within FRG
- Summary and outlook

Heavy-ion collision

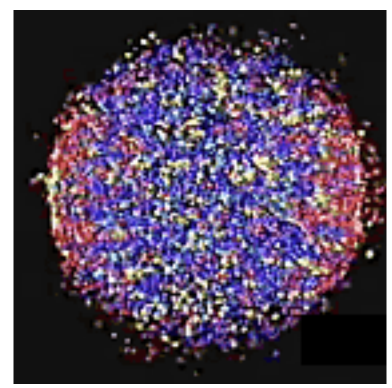
Ions about to collide



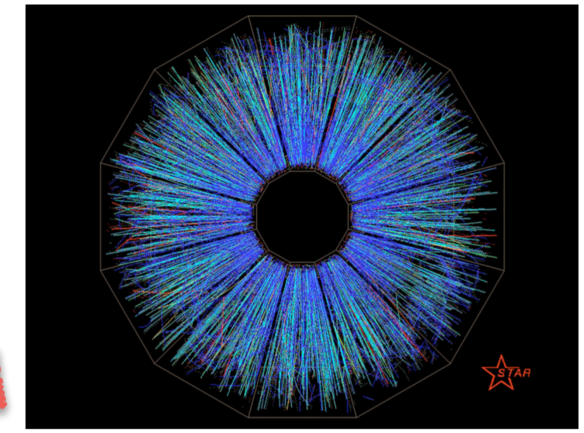
Ion collision



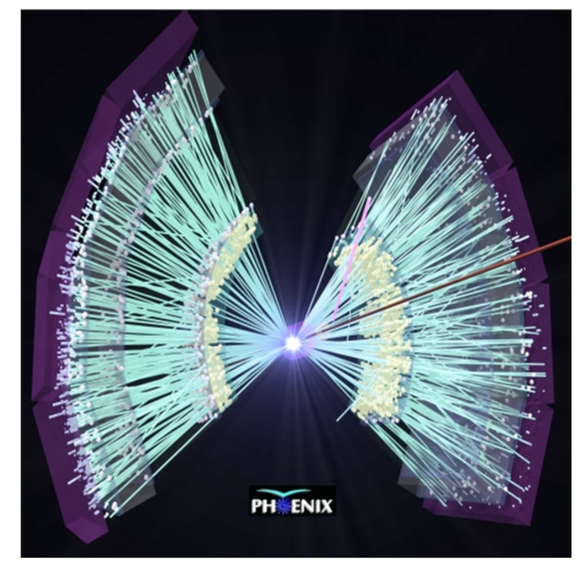
Plasma formation



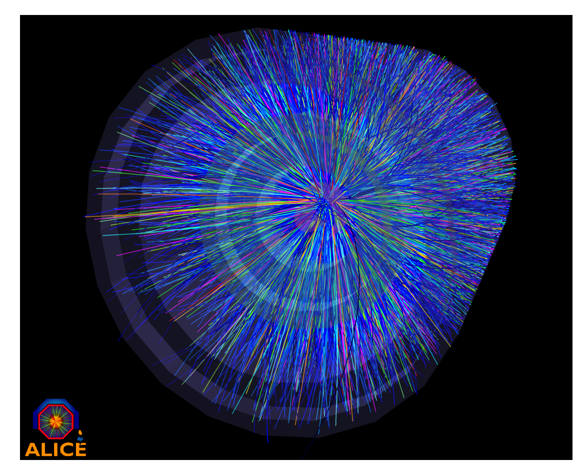
Freeze out



STAR



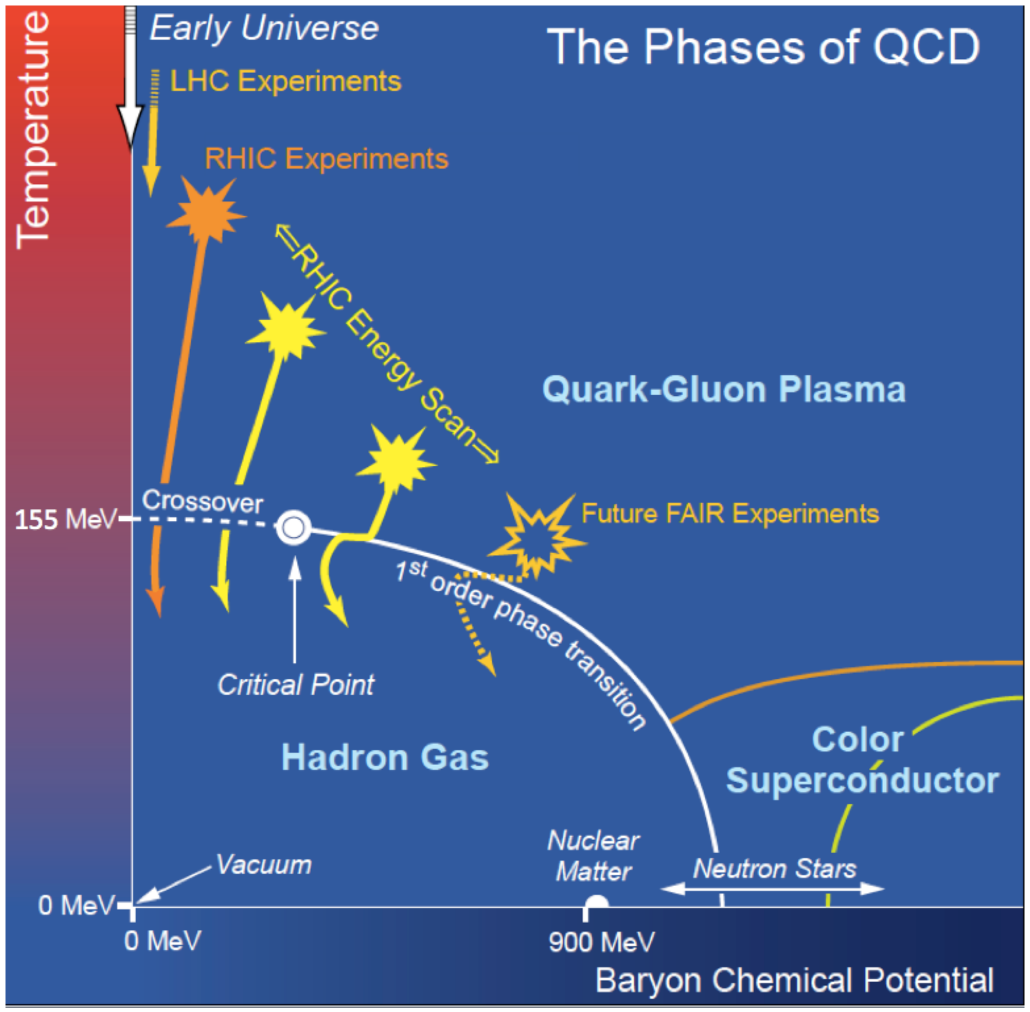
PHENIX



ALICE

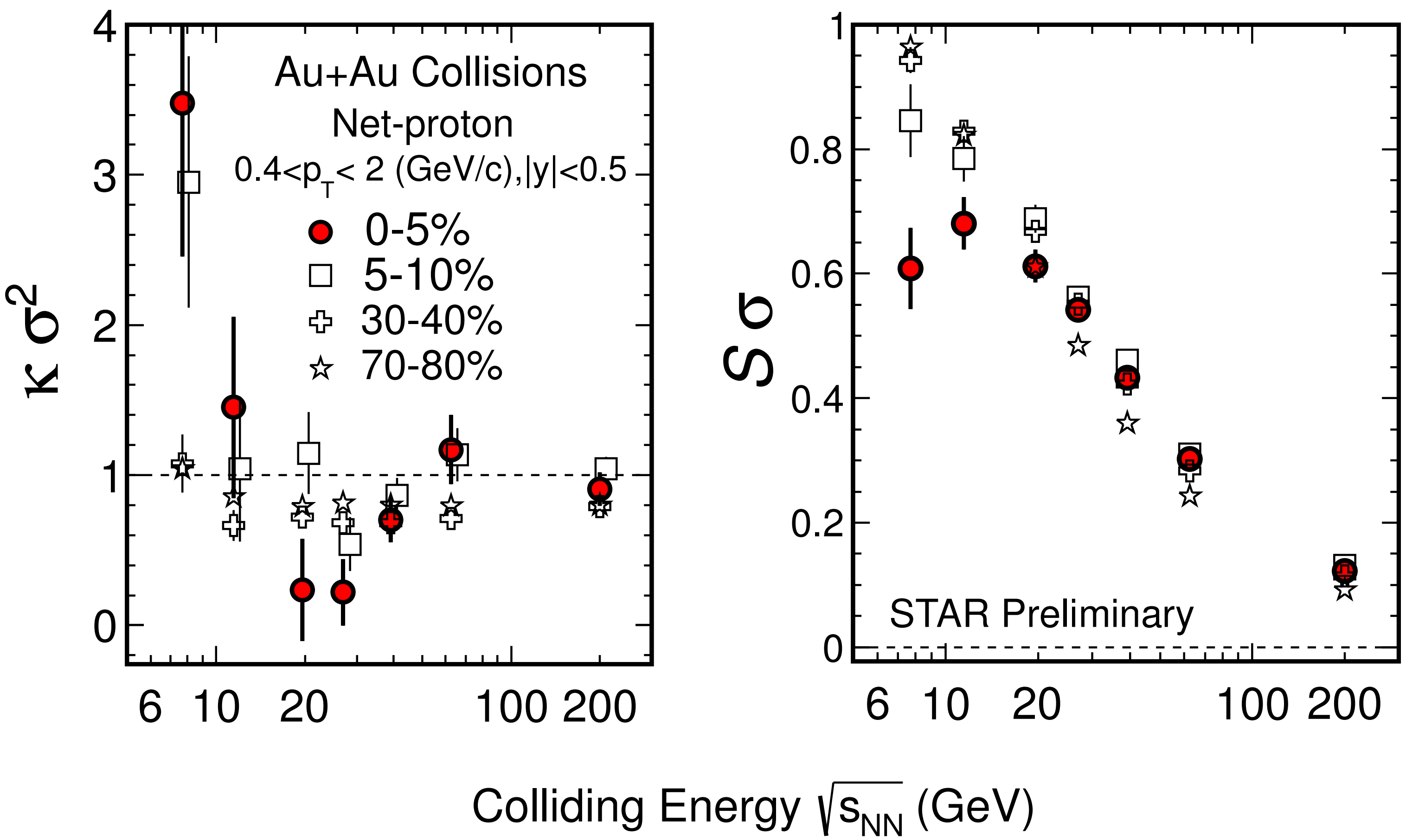
What we see

QCD Phase Structure



The Hot QCD White Paper (2015)

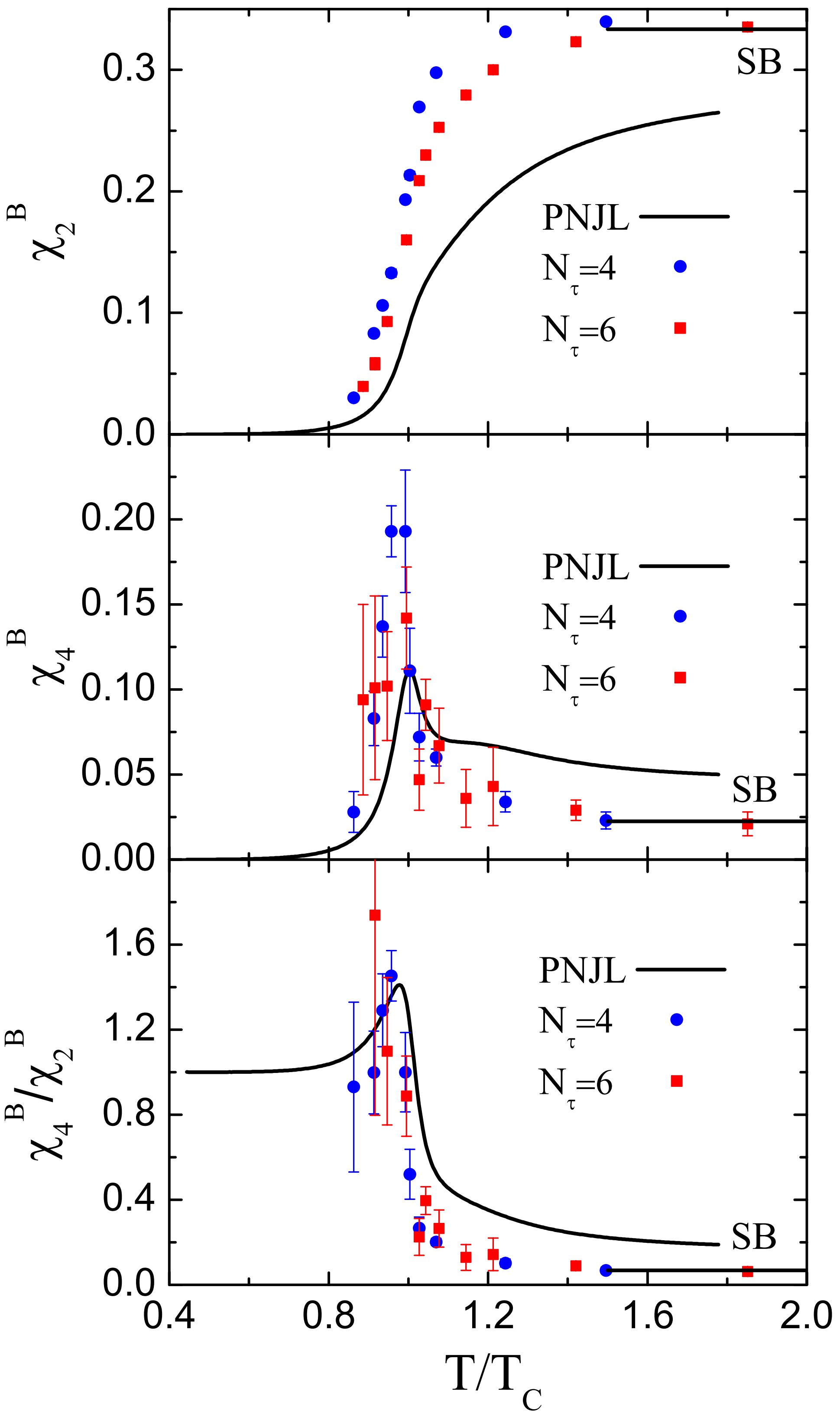
RHIC:



X.Luo (STAR), PoS CPOD2014, 019 (2014)

see talks by Xiao-feng Luo

2+1 Polyakov-loop NJL model



Model:

$$
\begin{aligned}
\mathcal{L}_{PNJL} =& \bar{\psi}\left(i\gamma_\mu D^\mu + \gamma_0\hat{\mu} - \hat{m}_0\right)\psi \\
&+ G\sum_{a=0}^{8}\left[\left(\bar{\psi}\tau_a\psi\right)^2 + \left(\bar{\psi}i\gamma_5\tau_a\psi\right)^2\right] \\
&- K\left[\det{}_f\left(\bar{\psi}\left(1+\gamma_5\right)\psi\right) + \det{}_f\left(\bar{\psi}\left(1-\gamma_5\right)\psi\right)\right] \\
&- \mathcal{U}\left(\Phi, \Phi^*, T\right)
\end{aligned}
$$

Generalized susceptibilities :

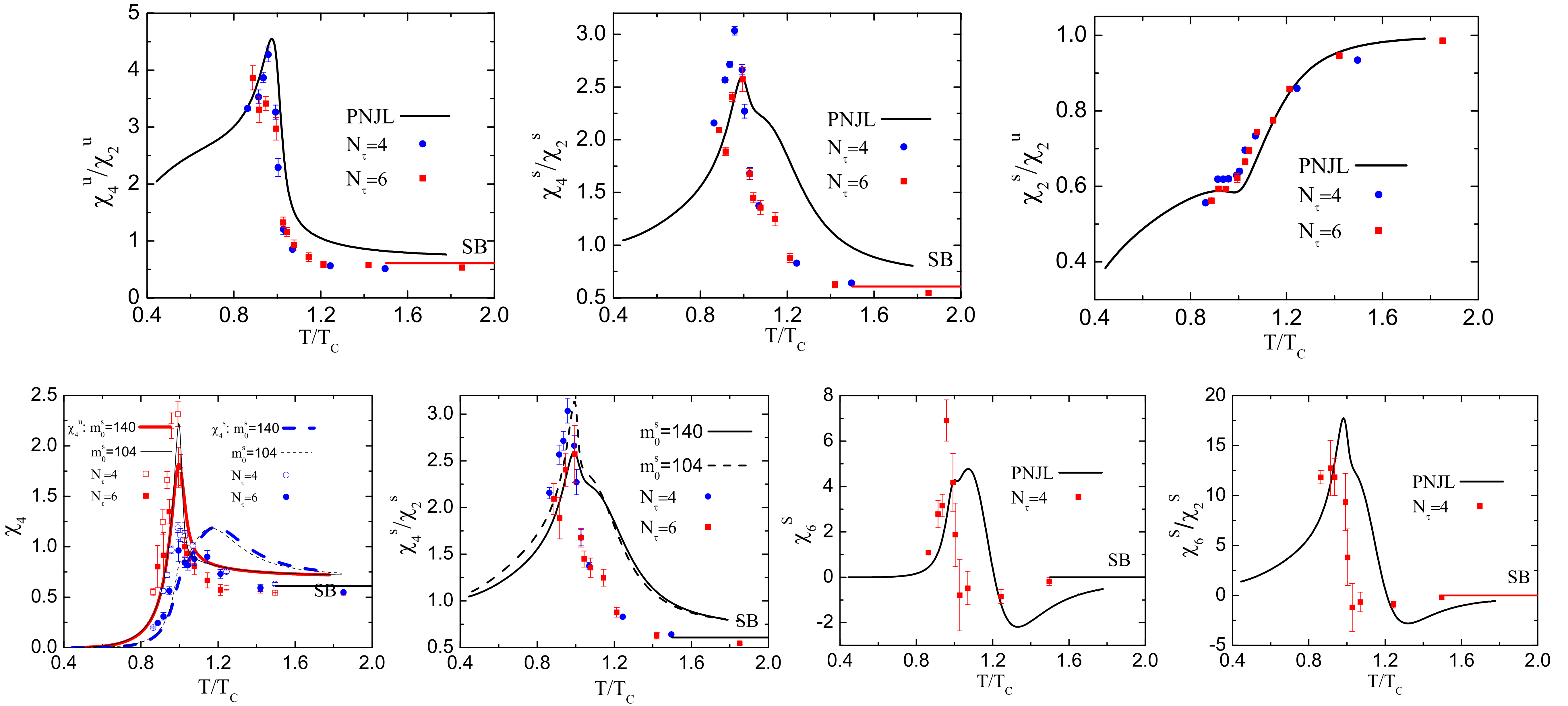
$$
\chi_{ijk}^{BQS} = \left.\frac{\partial^{i+j+k}(P/T^4)}{\partial(\mu_B/T)^i\partial(\mu_Q/T)^j\partial(\mu_S/T)^k}\right|_{\mu_{B,Q,S}=0}
$$

$$
\begin{aligned}
\chi_2^X =& \frac{1}{VT^3}\langle\delta N_X{}^2\rangle \\
\chi_3^X =& \frac{1}{VT^3}\langle\delta N_X{}^3\rangle \\
\chi_4^X =& \frac{1}{VT^3}\left(\langle\delta N_X{}^4\rangle - 3\langle\delta N_X{}^2\rangle^2\right) \\
\chi_6^X =& \frac{1}{VT^3}\left(\langle\delta N_X{}^6\rangle - 15\langle\delta N_X{}^4\rangle\langle\delta N_X{}^2\rangle\right. \\
&\left. - 10\langle\delta N_X{}^3\rangle^2 + 30\langle\delta N_X{}^2\rangle^3\right)
\end{aligned}
$$

$$
\begin{aligned}
\chi_{11}^{XY} =& \frac{1}{VT^3}\langle\delta N_X\delta N_Y\rangle \\
\chi_{12}^{XY} =& \frac{1}{VT^3}\langle\delta N_X\delta N_Y{}^2\rangle \\
\chi_{111}^{XYZ} =& \frac{1}{VT^3}\langle\delta N_X\delta N_Y\delta N_Z\rangle
\end{aligned}
$$

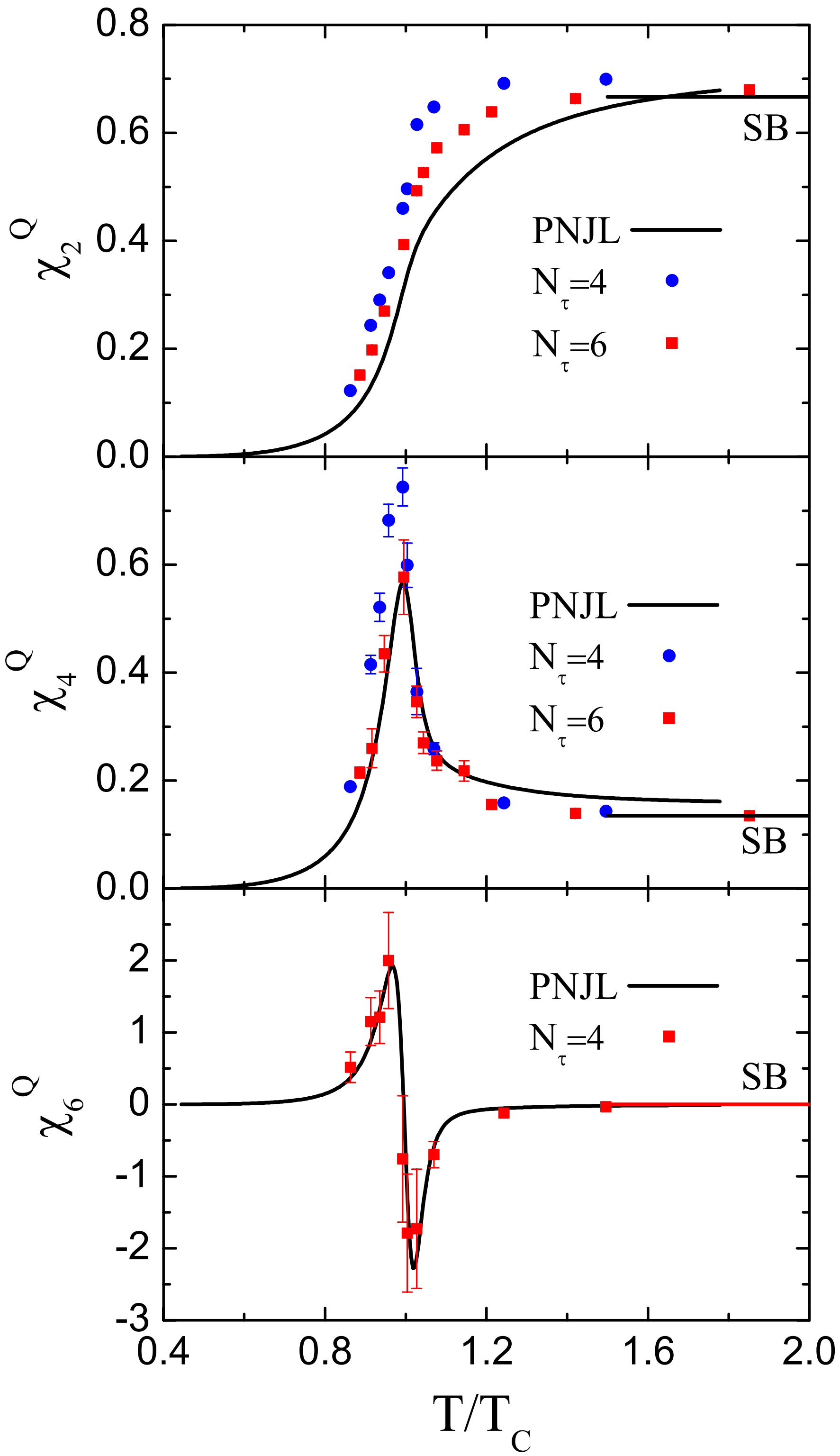
PNJL: WF, Yu-xin Liu, Yue-liang Wu, PRD 81 (2010) 014028
Lattice: M. Cheng *et al.*, PRD 79 (2009) 074505

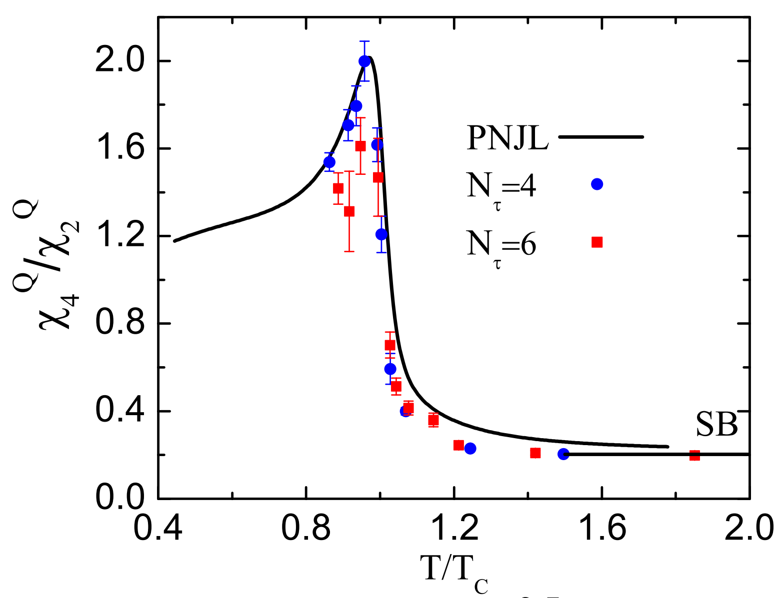
Quark Number Fluctuations

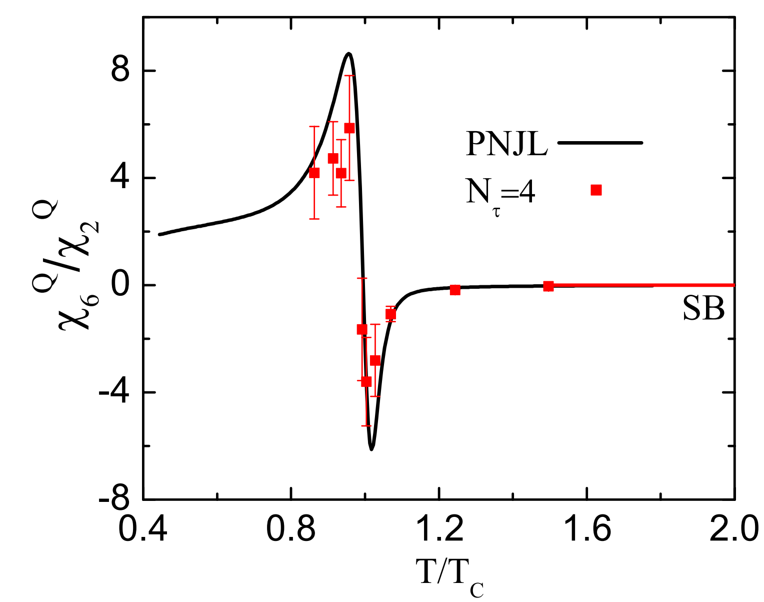


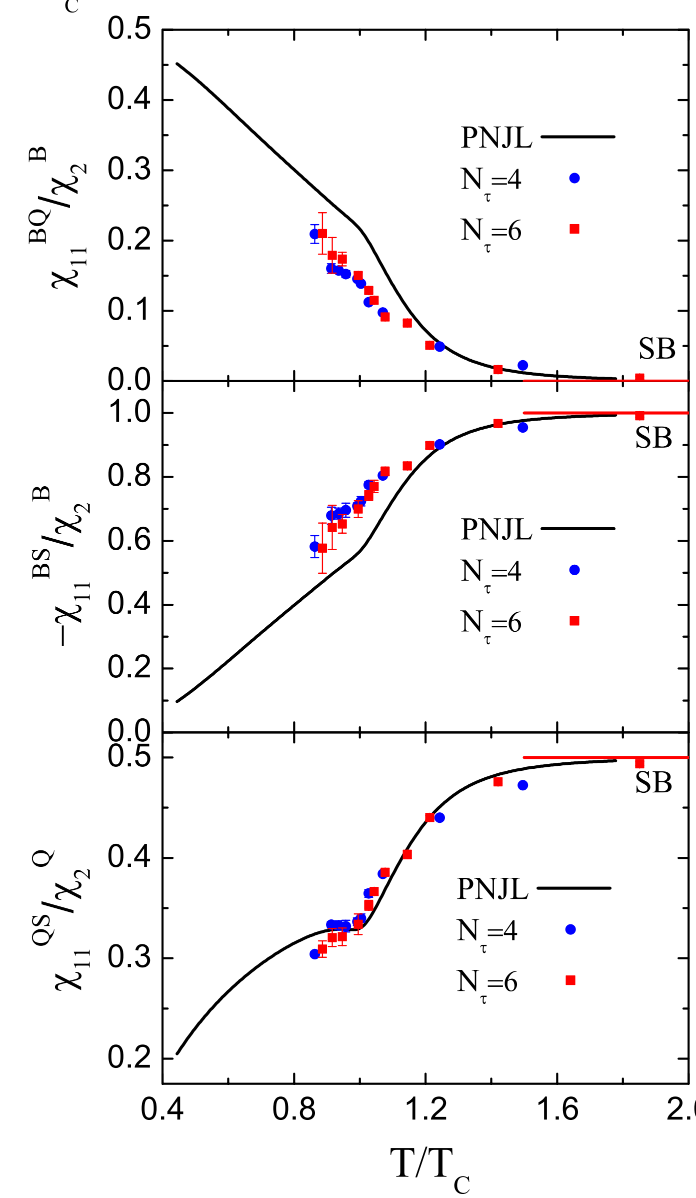
PNJL: WF, Yu-xin Liu, Yue-liang Wu, PRD 81 (2010) 014028
Lattice: M. Cheng *et al.*, PRD 79 (2009) 074505

Fluctuations of Electric Charge and Correlations



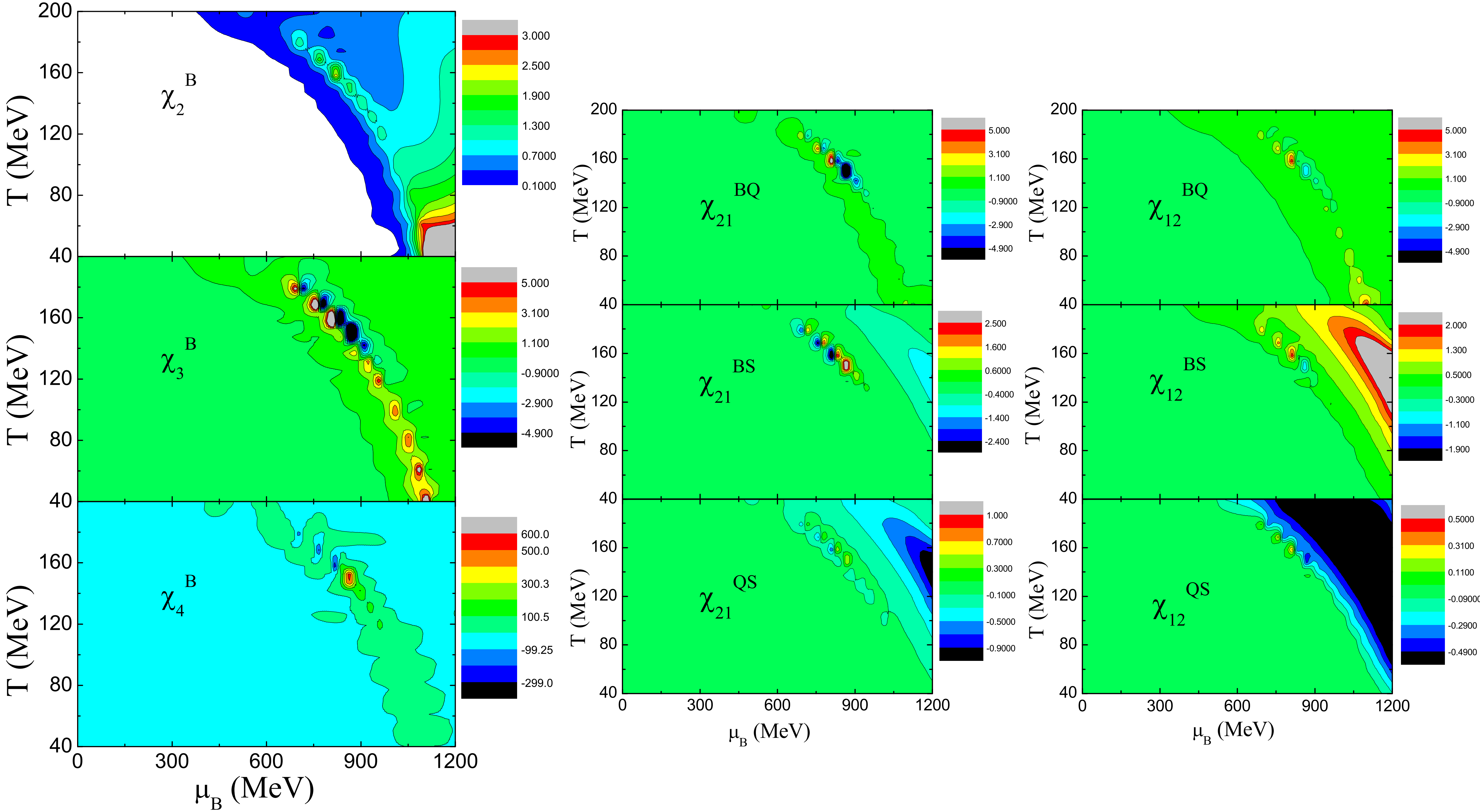






PNJL: WF, Yu-xin Liu, Yue-liang Wu, PRD 81 (2010) 014028
Lattice: M. Cheng *et al.*, PRD 79 (2009) 074505

Fluctuations on the Phase Diagram

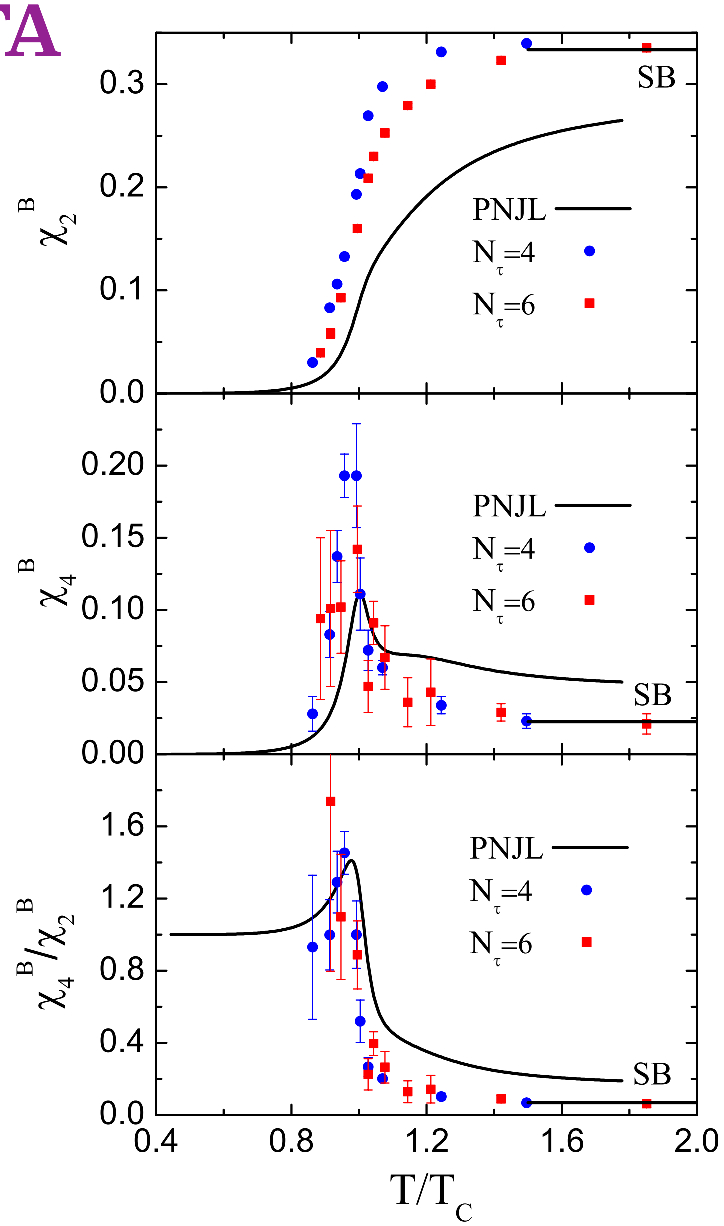


WF, Yue-liang Wu, PRD 82 (2010) 074013

Quantum Fluctuations beyond MFA

Summary on MFA

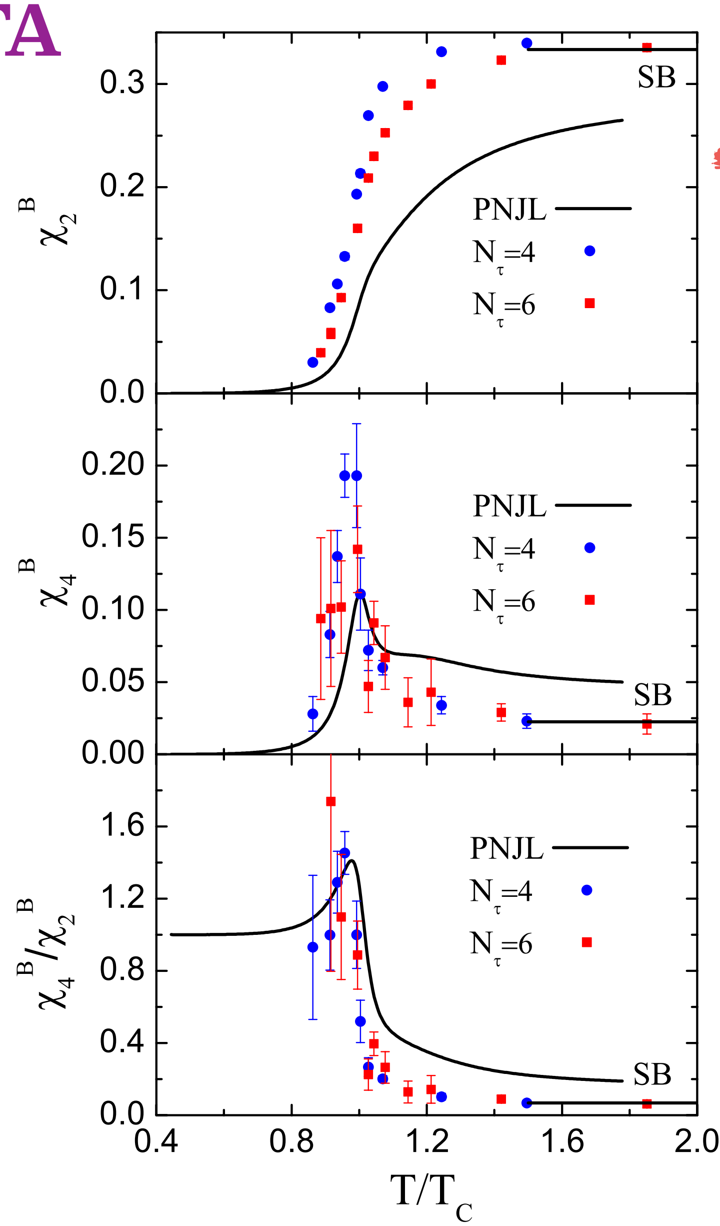
- Easy to calculate
- Agree with lattice results only qualitatively
- Overestimate the phase transition strength in the MFA
- Thus, one has to go beyond the MFA



Quantum Fluctuations beyond MFA

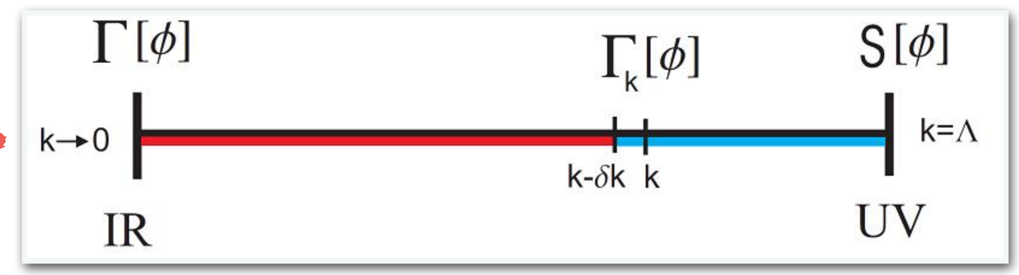
Summary on MFA

- Easy to calculate
- Agree with lattice results only qualitatively
- Overestimate the phase transition strength in the MFA
- Thus, one has to go beyond the MFA



FRG

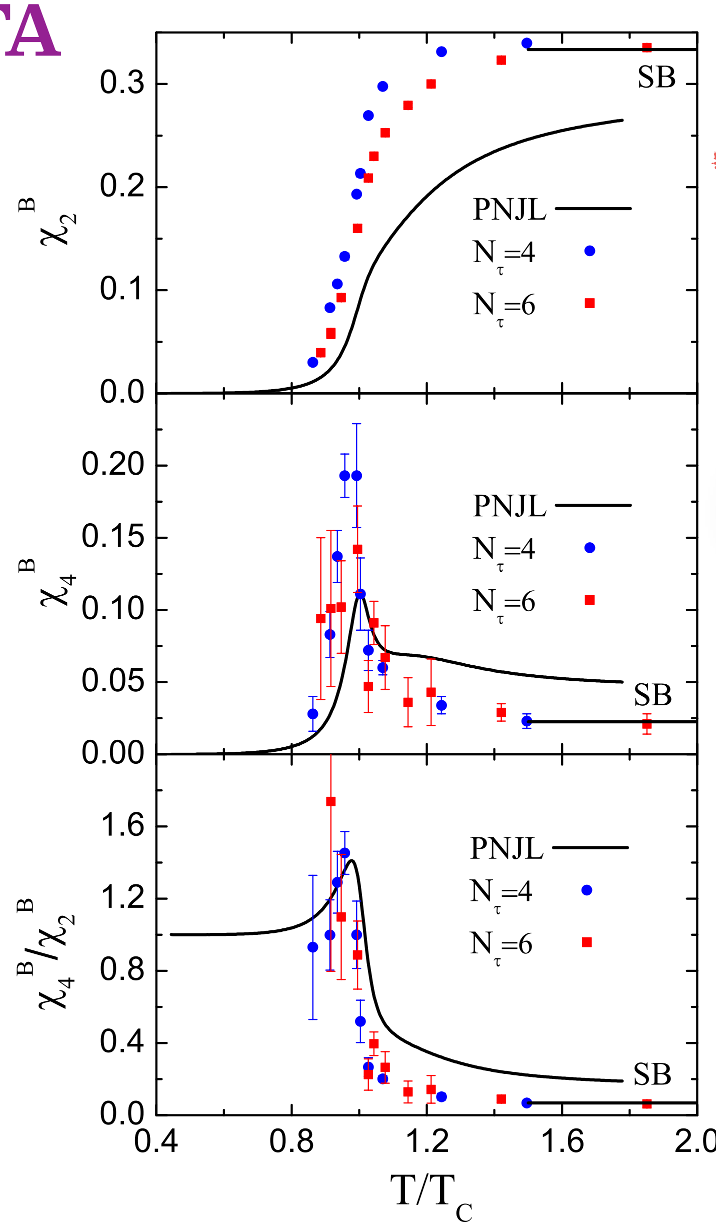
see talks by Jan, Defu



Quantum Fluctuations beyond MFA

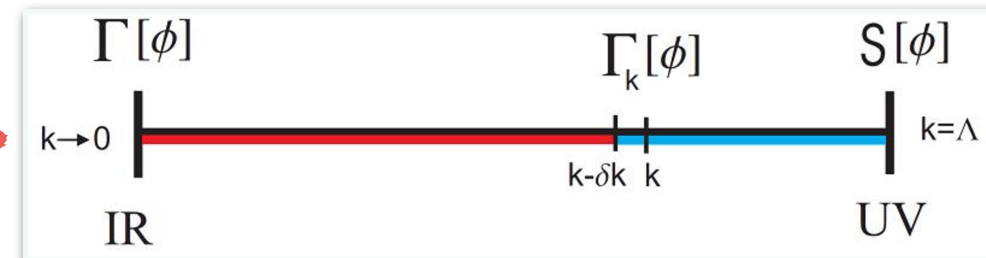
Summary on MFA

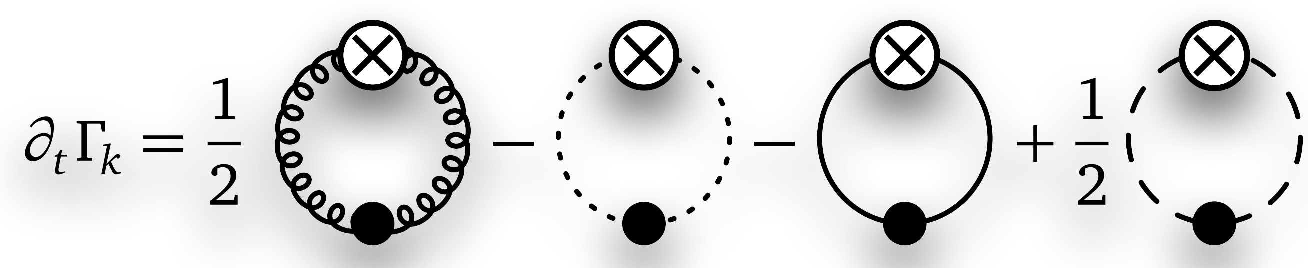
- Easy to calculate
- Agree with lattice results only qualitatively
- Overestimate the phase transition strength in the MFA
- Thus, one has to go beyond the MFA



FRG

see talks by Jan, Defu

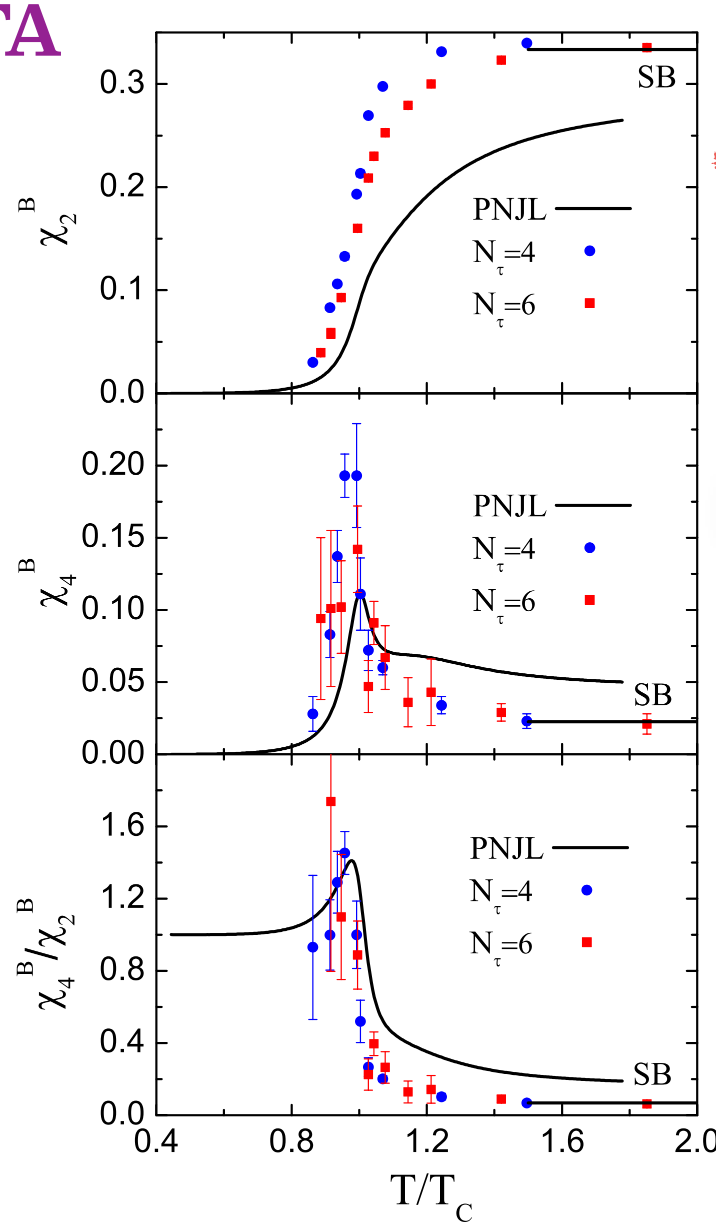




Quantum Fluctuations beyond MFA

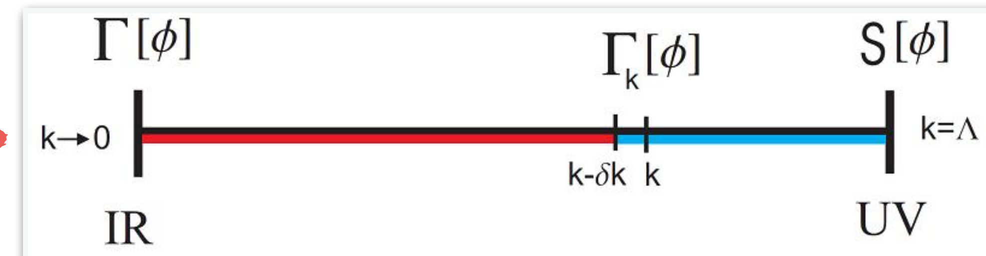
Summary on MFA

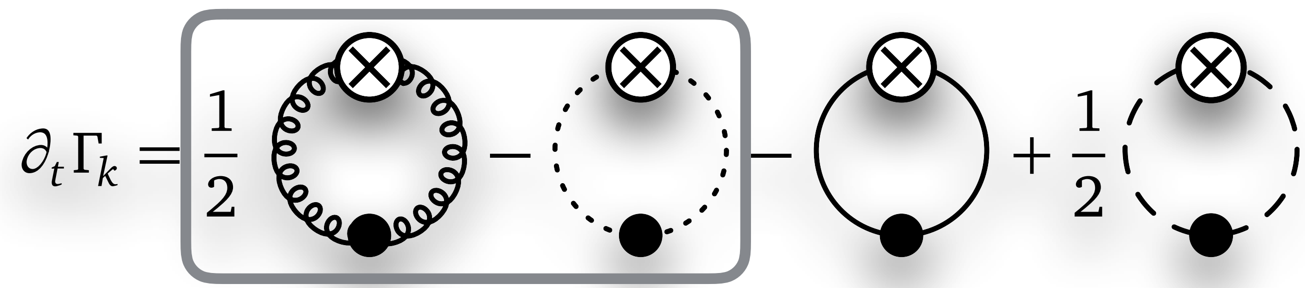
- Easy to calculate
- Agree with lattice results only qualitatively
- Overestimate the phase transition strength in the MFA
- Thus, one has to go beyond the MFA



FRG

see talks by Jan, Defu



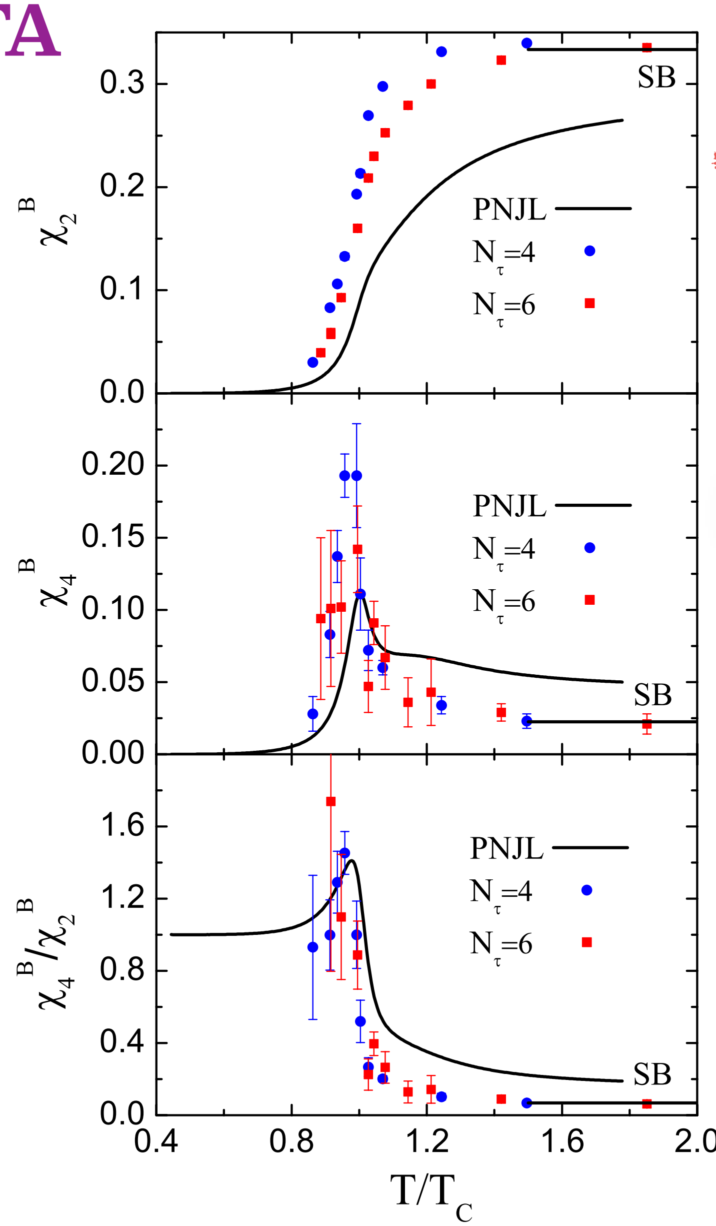


glue sector

Quantum Fluctuations beyond MFA

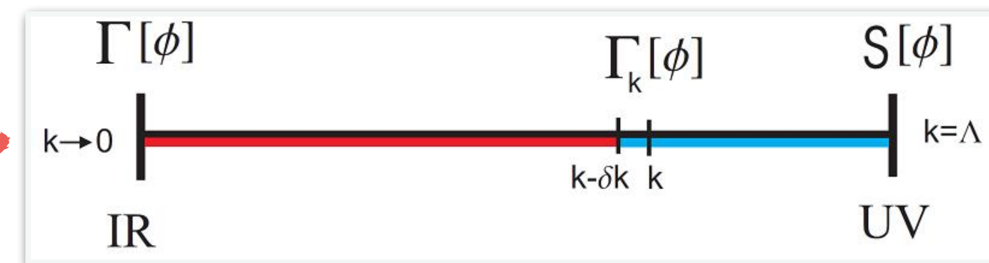
Summary on MFA

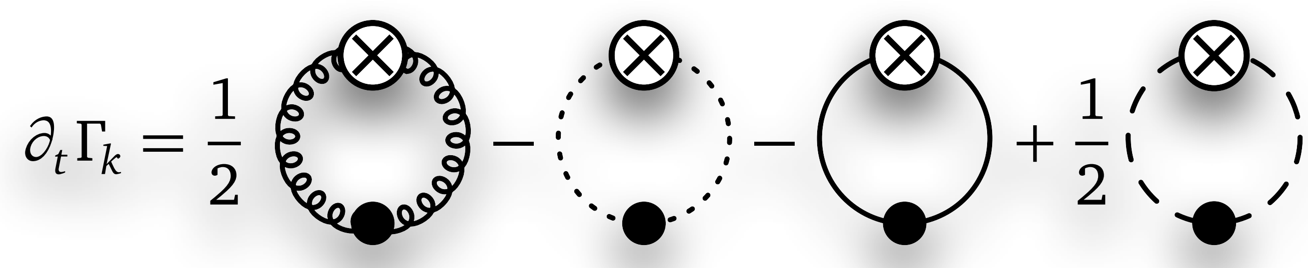
- Easy to calculate
- Agree with lattice results only qualitatively
- Overestimate the phase transition strength in the MFA
- Thus, one has to go beyond the MFA



FRG

see talks by Jan, Defu

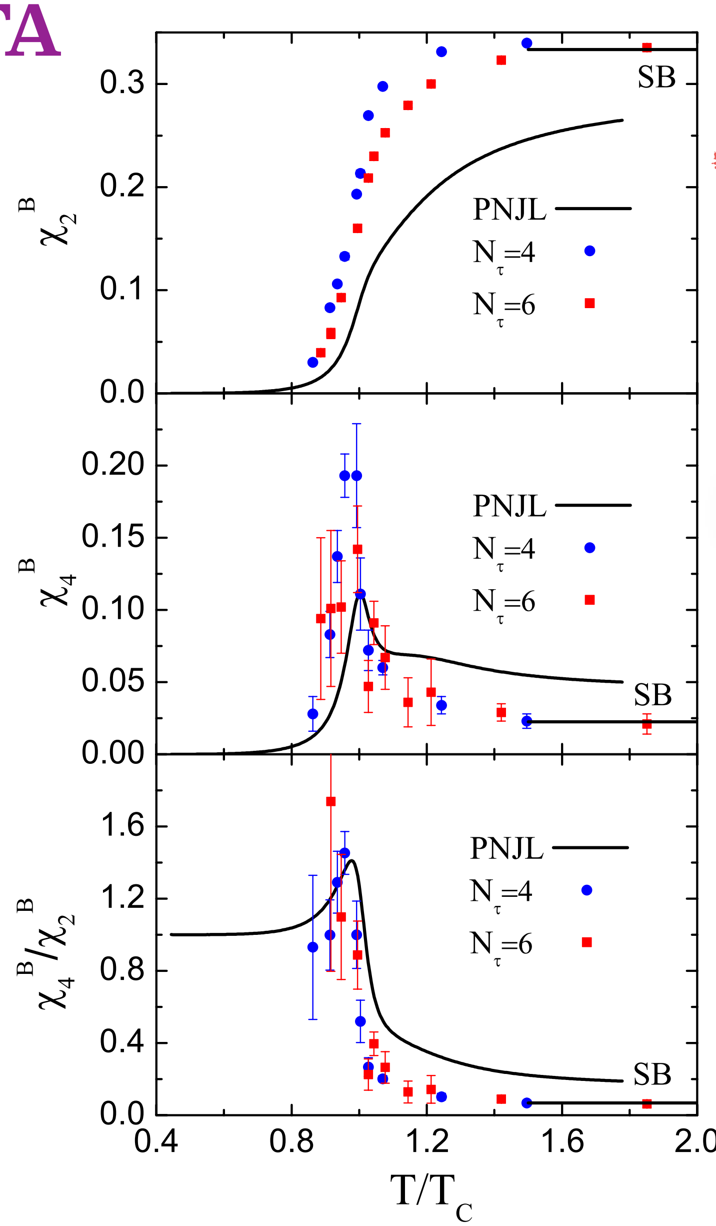




Quantum Fluctuations beyond MFA

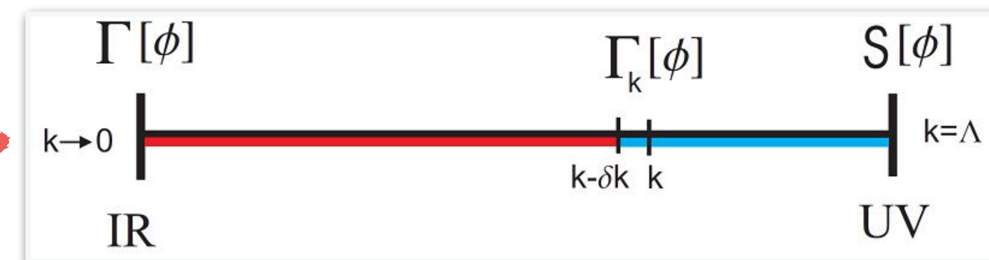
Summary on MFA

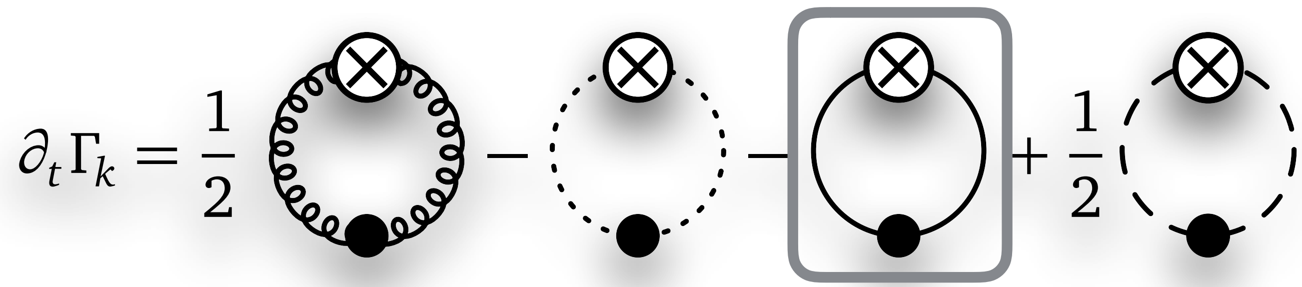
- Easy to calculate
- Agree with lattice results only qualitatively
- Overestimate the phase transition strength in the MFA
- Thus, one has to go beyond the MFA



FRG

see talks by Jan, Defu



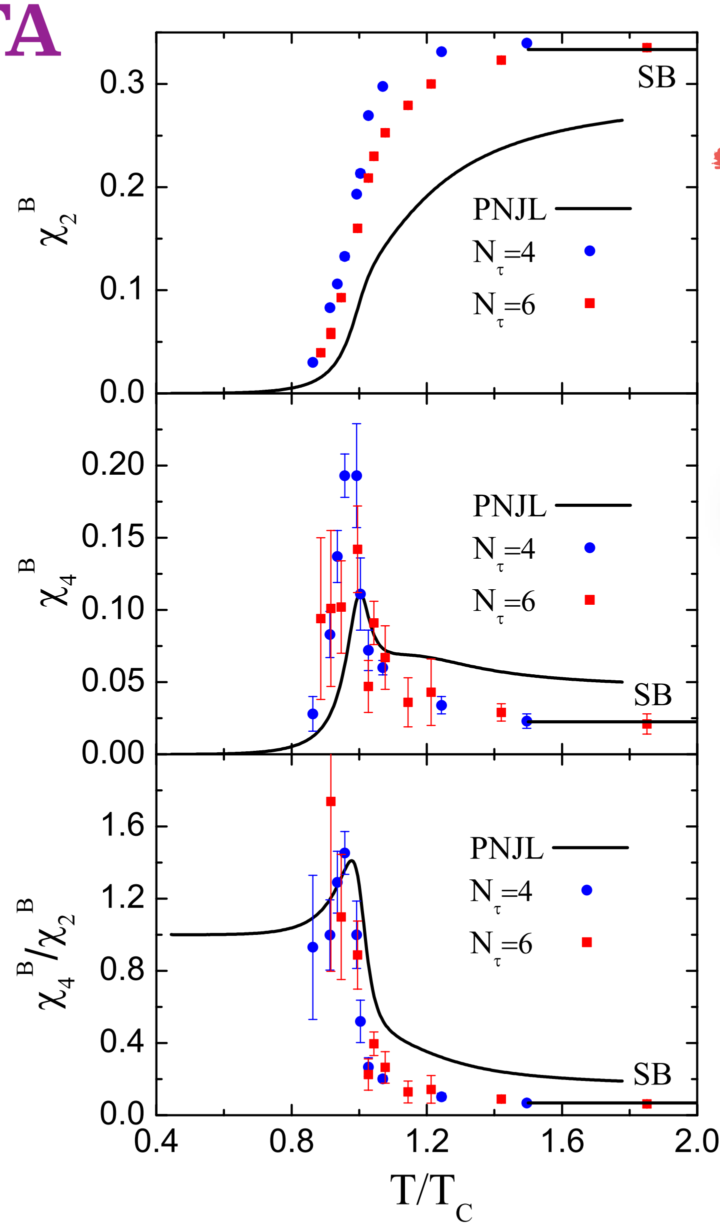


MFA

Quantum Fluctuations beyond MFA

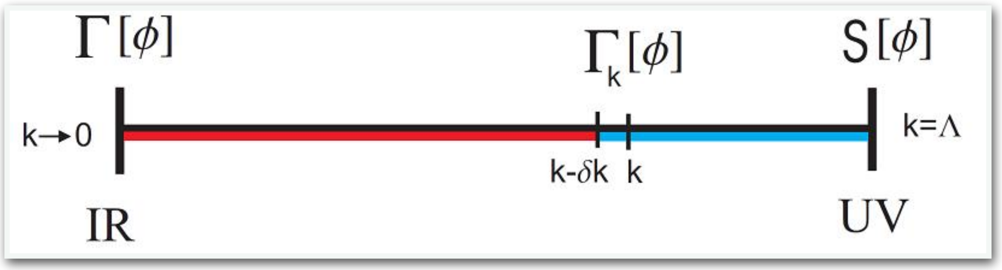
Summary on MFA

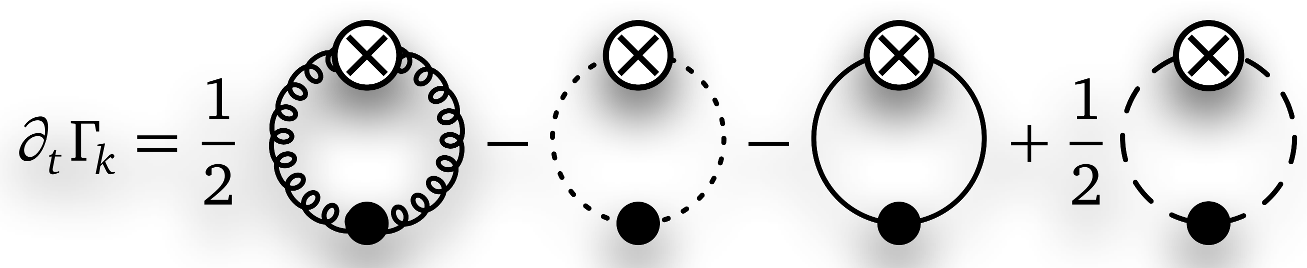
- Easy to calculate
- Agree with lattice results only qualitatively
- Overestimate the phase transition strength in the MFA
- Thus, one has to go beyond the MFA



FRG

see talks by Jan, Defu

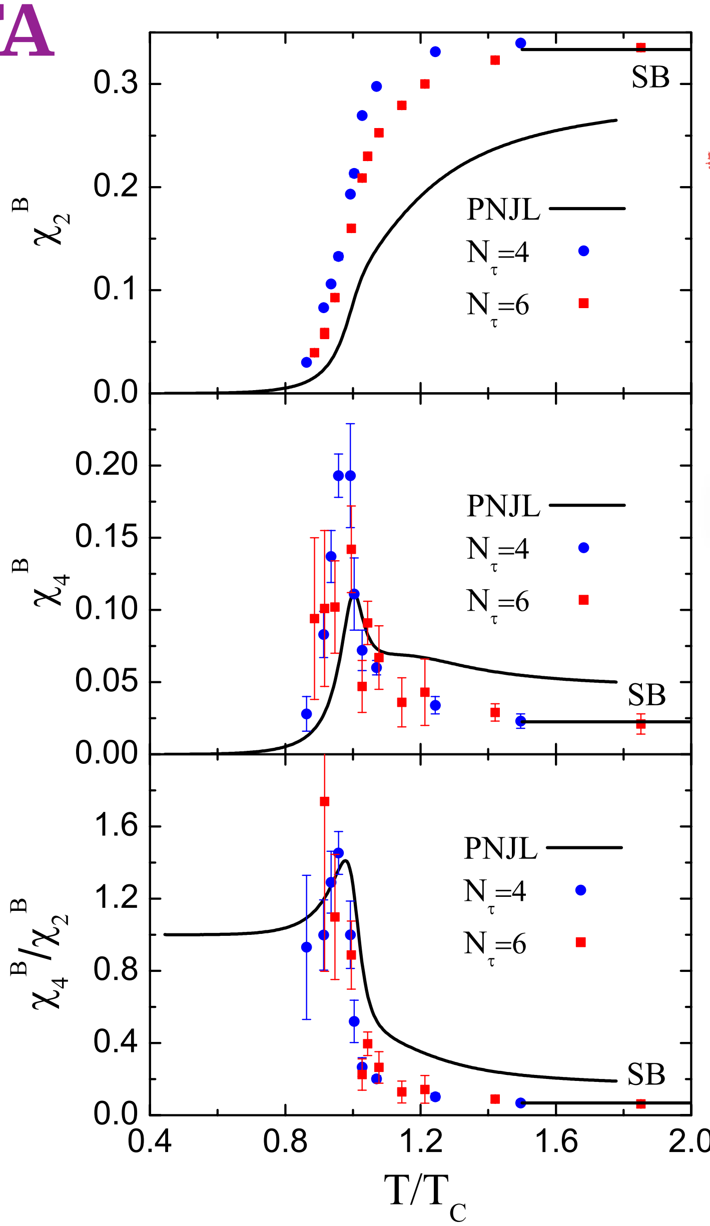




Quantum Fluctuations beyond MFA

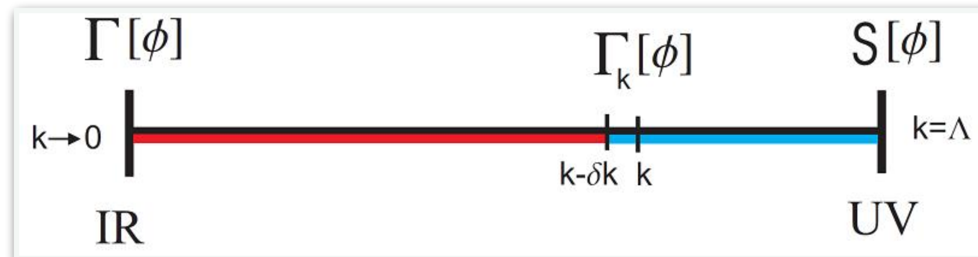
Summary on MFA

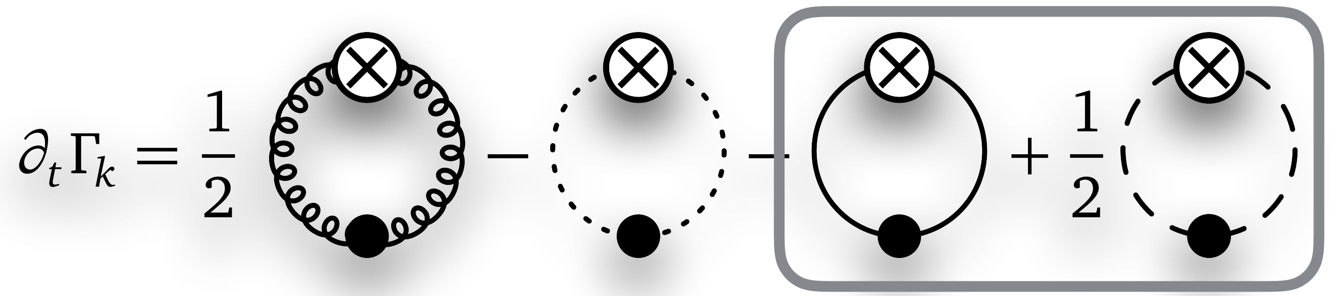
- Easy to calculate
- Agree with lattice results only qualitatively
- Overestimate the phase transition strength in the MFA
- Thus, one has to go beyond the MFA



FRG

see talks by Jan, Defu



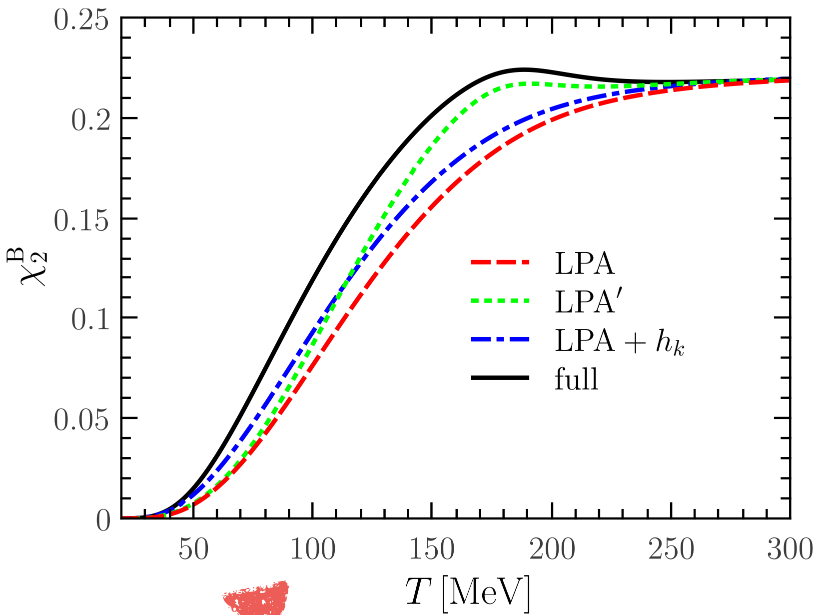


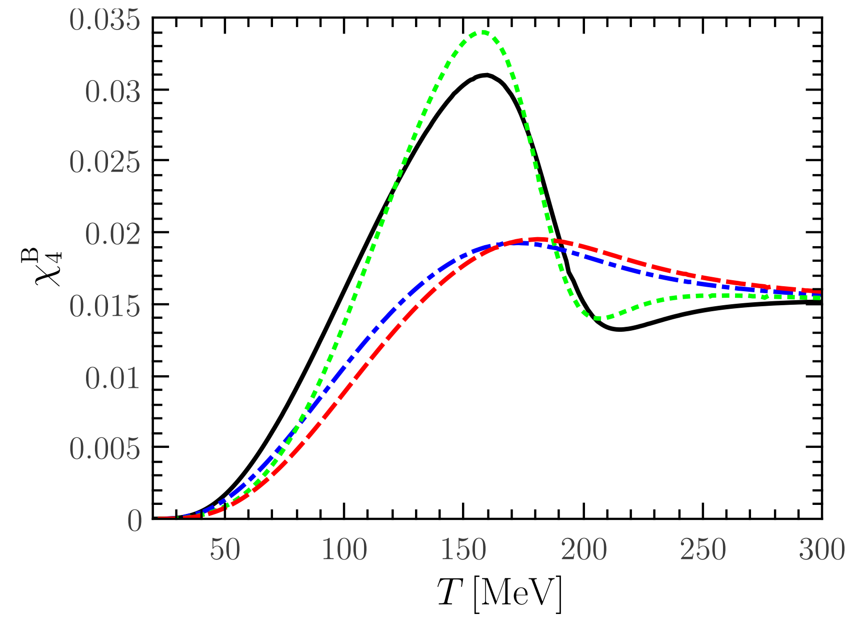
Matter part, the effective model:

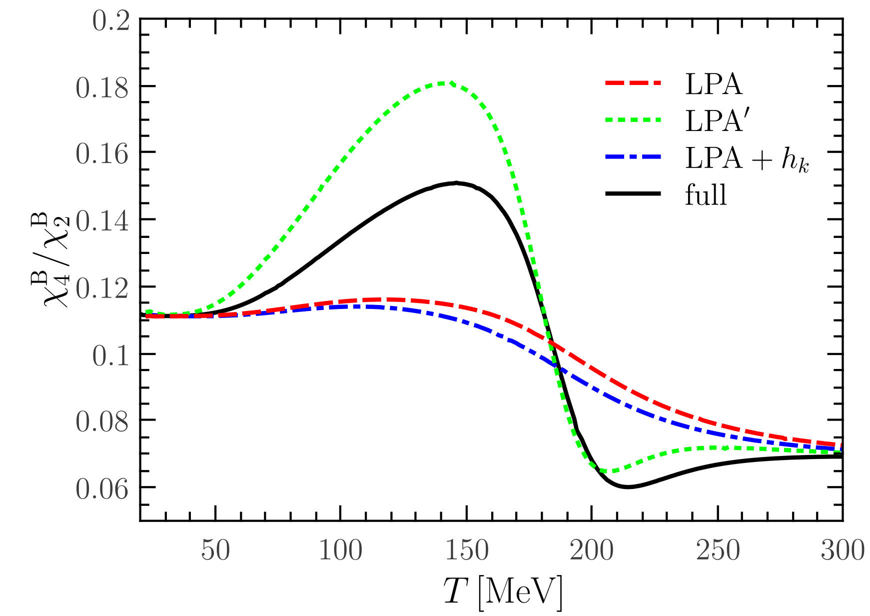
$$\Gamma_k = \int_x \Big\{ Z_{q,k}\bar{q}(\gamma_\mu \partial_\mu - \gamma_0 \mu) q + \frac{1}{2} Z_{\phi,k}(\partial_\mu \phi)^2$$

$$+ h_k\, \bar{q} \left(T^0 \sigma + i\gamma_5 \vec{T} \cdot \vec{\pi} \right) q + V_k(\rho) - c\sigma \Big\} + \cdots$$

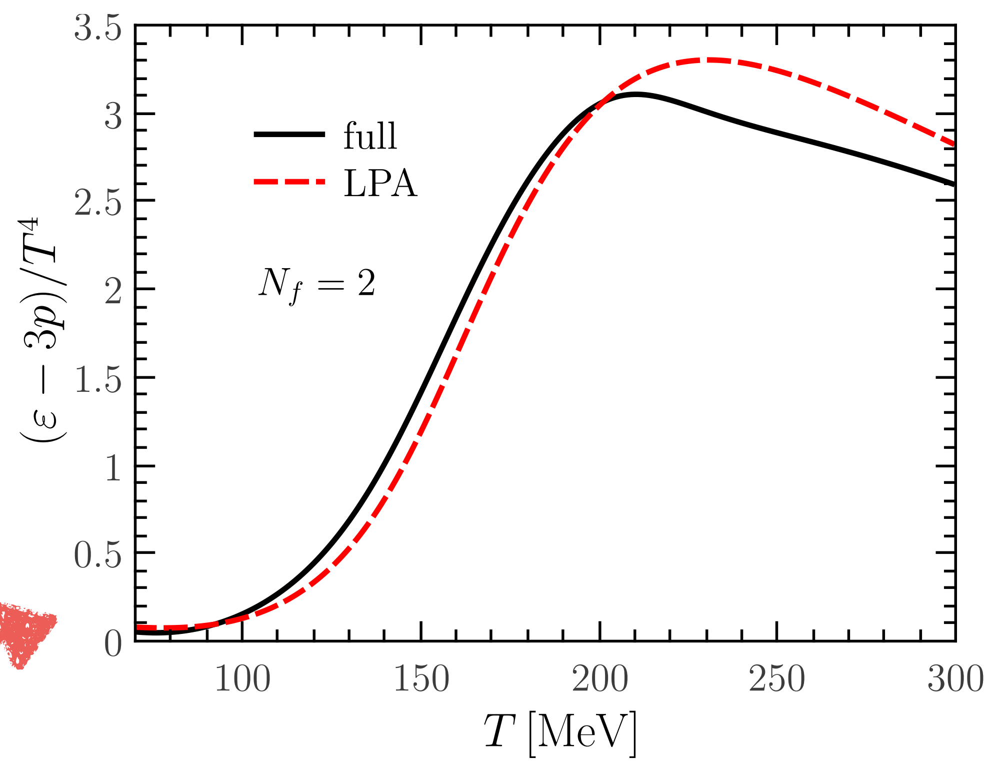
Thermodynamics of the effective model within FRG



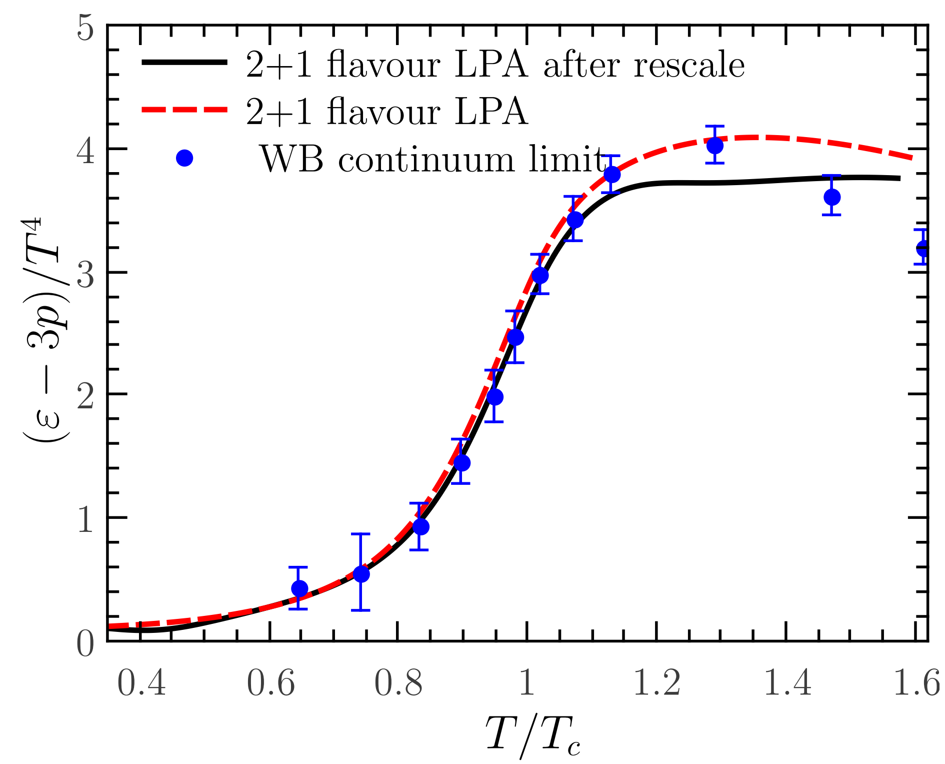




QM

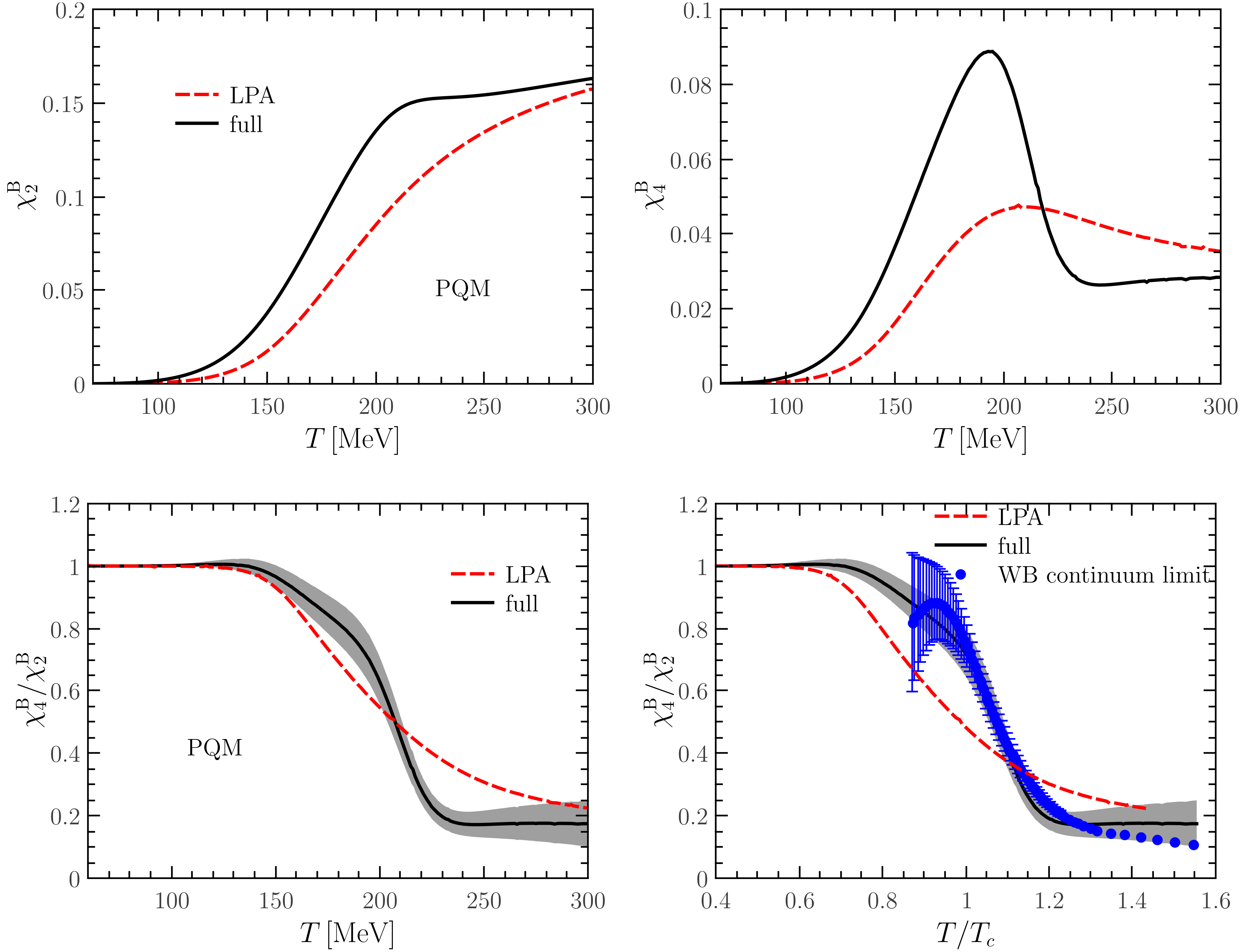


PQM



WF, J.M. Pawlowski, PRD 92 (2015) 116006

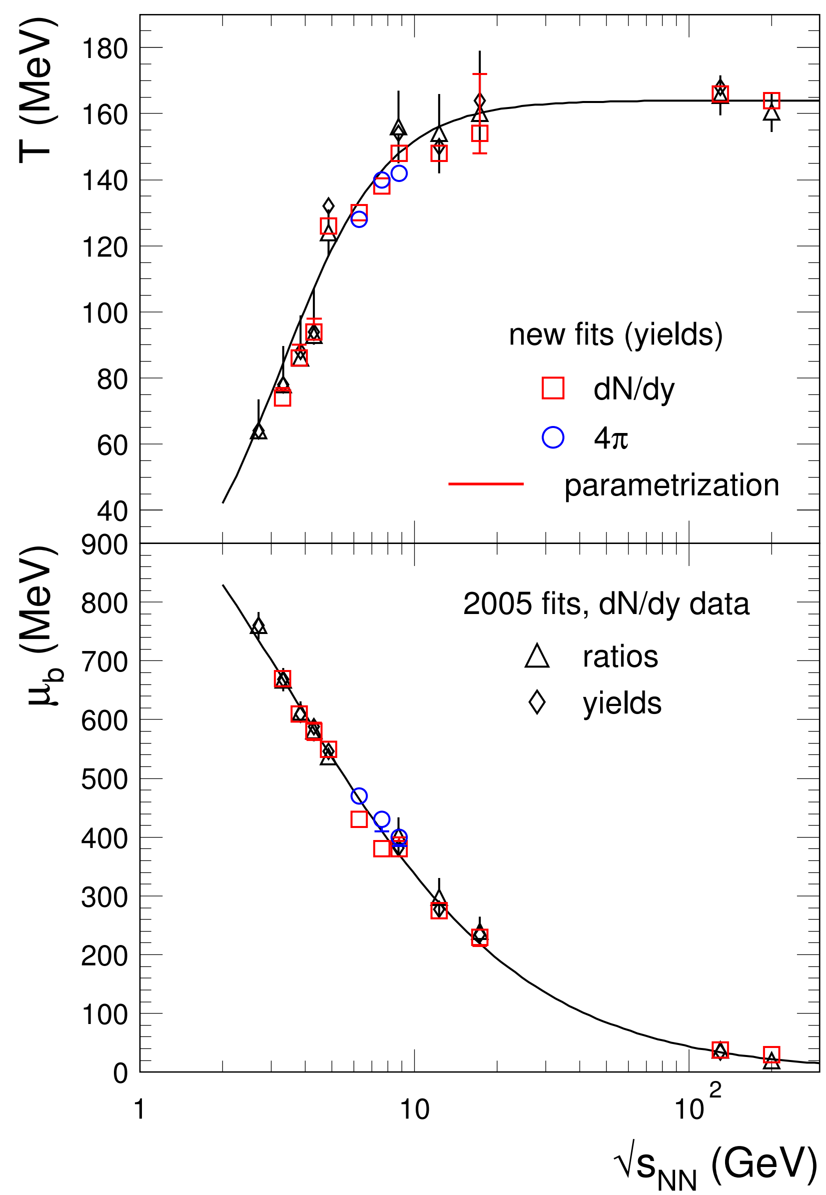
Baryon number fluctuations



WF, J.M. Pawlowski, PRD 92 (2015) 116006

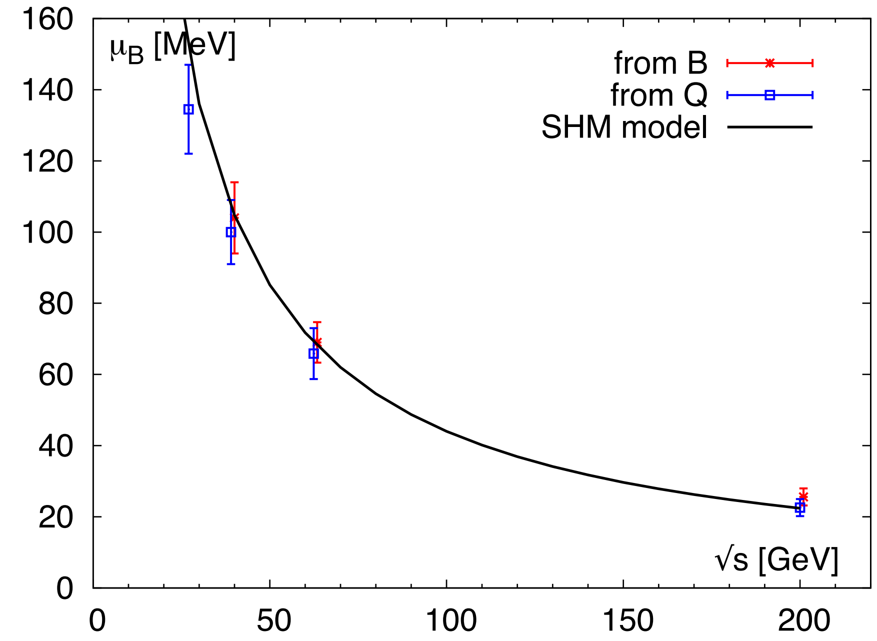
Freeze-out line

see talk by Peter Braun-Munzinger



Freeze-out temperature and chemical potential obtained from the Statistical Hadronization Model

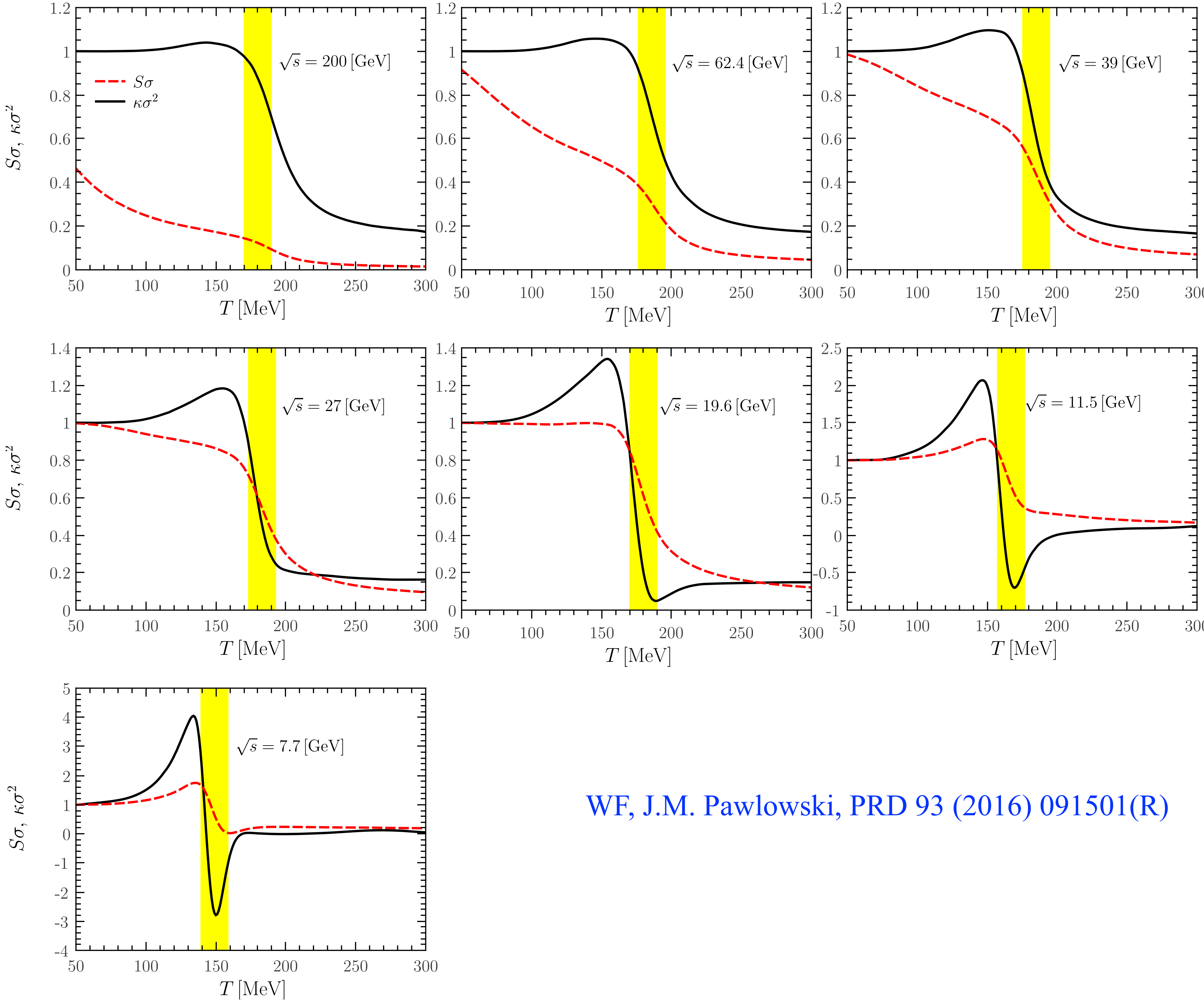
A. Andronic, P. Braun-Munzinger, J. Stachel, Phys.Lett. B673 (2009) 142



Freeze-out chemical potential obtained from lattice simulations

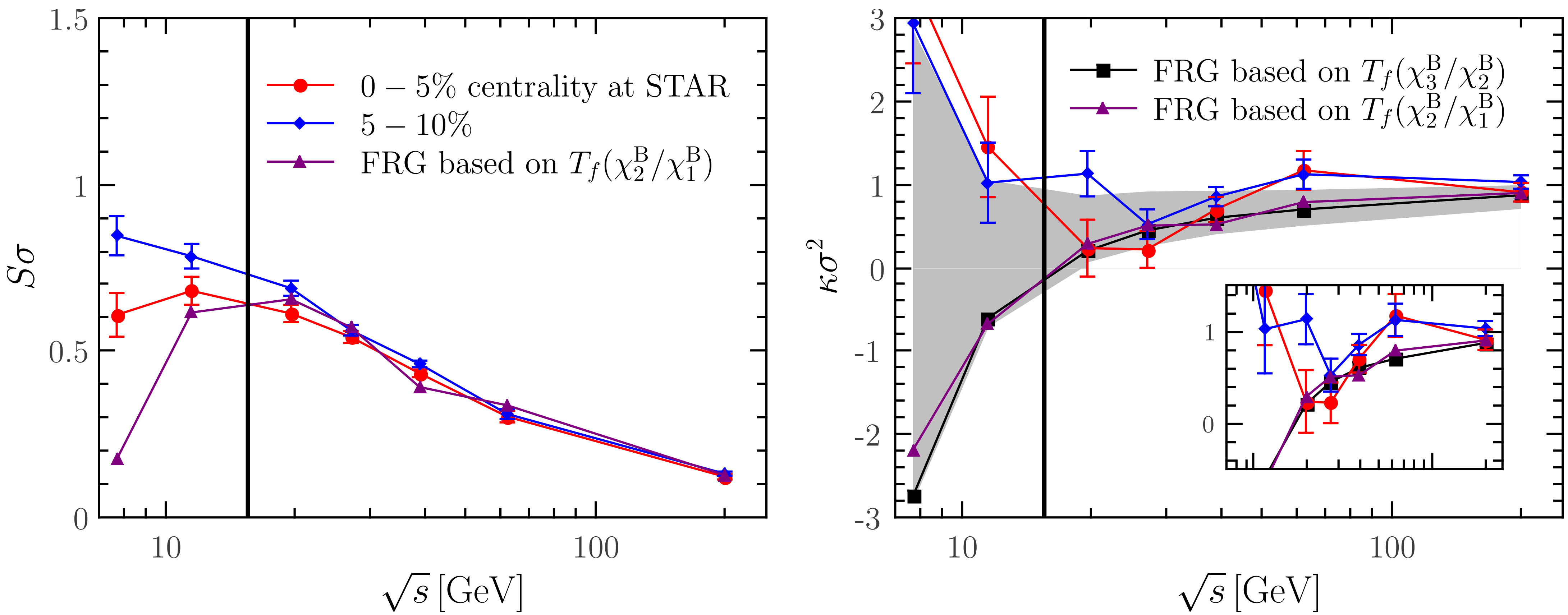
S. Borsanyi *et al.*, Phys.Rev.Lett. 113 (2014) 052301

Correlating the skewness and kurtosis of baryon number distributions



WF, J.M. Pawlowski, PRD 93 (2016) 091501(R)

Comparison with experimental measurements



WF, J.M. Pawlowski, PRD 93 (2016) 091501(R)

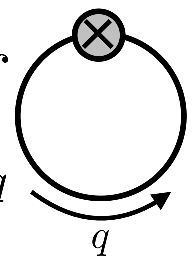
Silver Blaze Property and the Frequency Dependence

Frequency-dependent quark anomalous dimension:

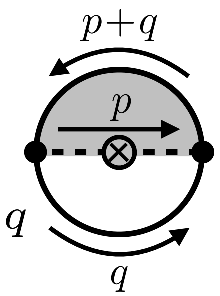
$$\eta_{q,k}(p) = \frac{1}{Z_{q,k}(p)} \frac{1}{4N_c N_f} \frac{\partial^2}{\partial|\vec{p}|^2} \mathrm{Tr}\Big(i\vec{\gamma}\cdot\vec{p}\,\partial_t \tilde{\Gamma}^{(2)}_{q\bar{q},k}(p) \Big)$$

Insert this into the flow of effective potential, and perform the two-loop summation

$$\partial_t V_k^q = -\int_q$$



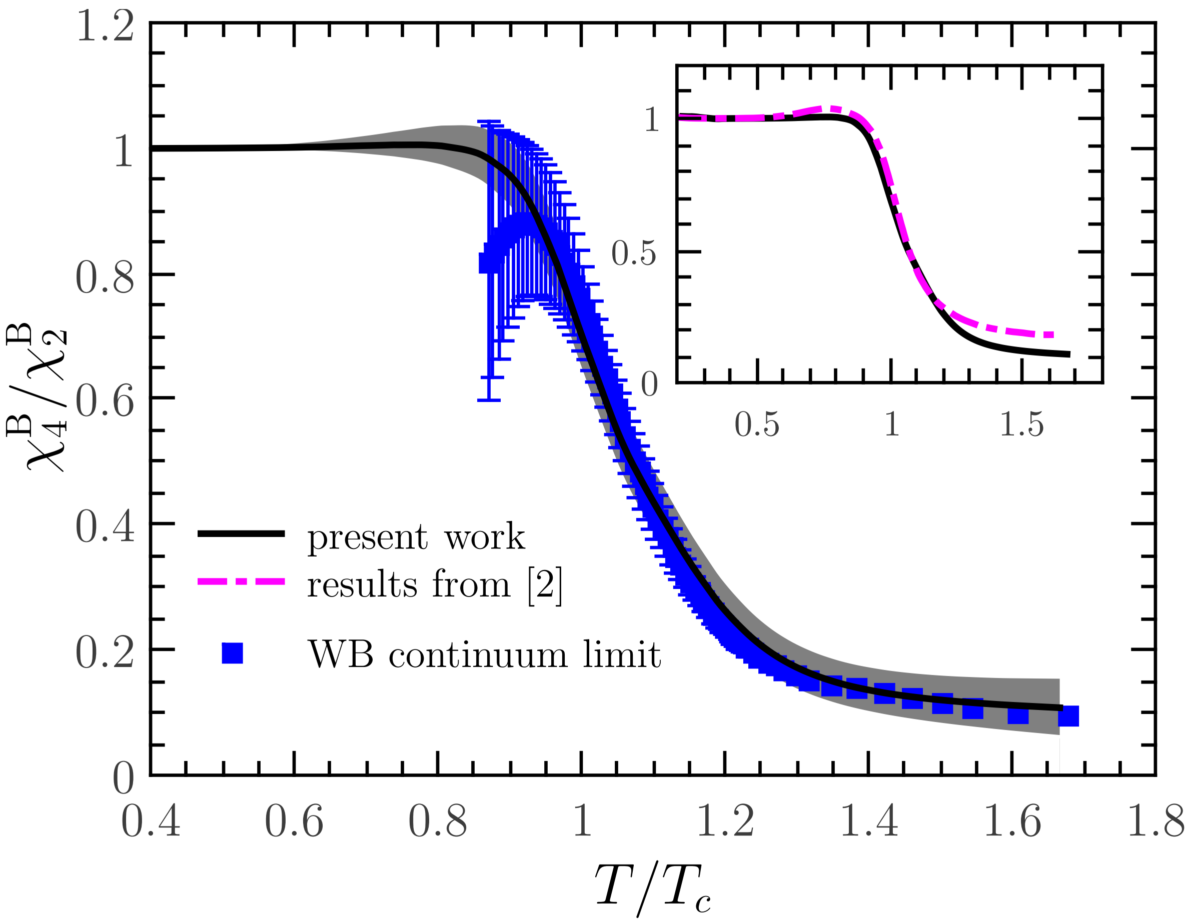
$$\approx -\iint_{p,q}$$

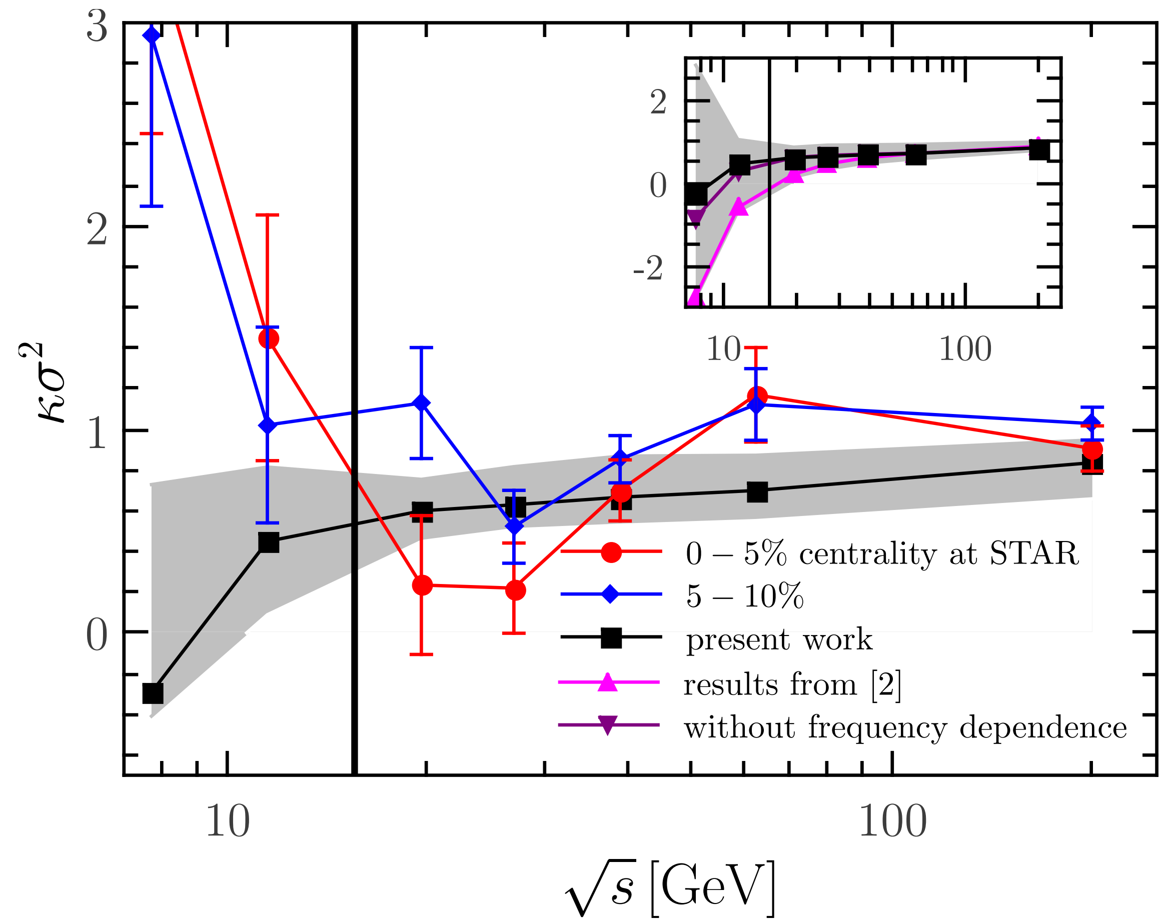


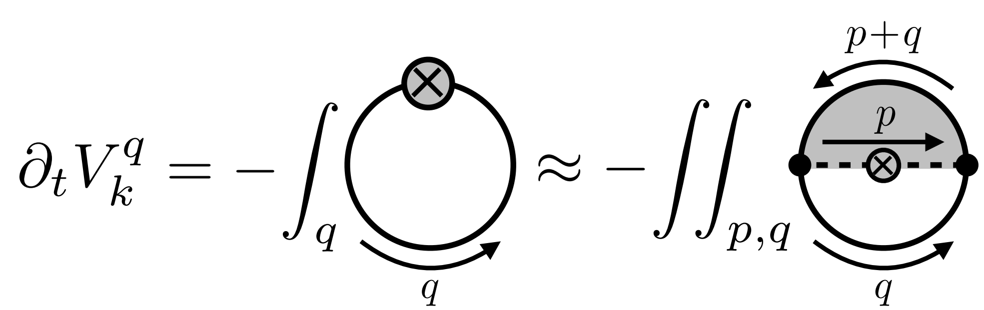
One obtains

$$\partial_t V_k(\rho) = \frac{k^4}{360\pi^2}\Big\{ 12(5-\eta_{\phi,k})\big[(N_f^2-1)\mathcal{B}_{(1)}(\bar{m}^2_{\pi,k}) + \mathcal{B}_{(1)}(\bar{m}^2_{\sigma,k})\big] - 5N_c\Big(48N_f\mathcal{F}_{(1)}(\bar{m}^2_{F,k}) + \frac{1}{2\pi^2}(-4+\eta_{\phi,k})\bar{h}_k^2\Big[\mathcal{FFB}_{(1,1,2)}(\bar{m}^2_{F,k},\bar{m}^2_{\sigma,k}) + (N_f^2-1)\mathcal{FFB}_{(1,1,2)}(\bar{m}^2_{F,k},\bar{m}^2_{\pi,k})\Big]\Big)\Big\},$$

Two-loop Results







WF, J.M. Pawlowski, F. Rennecke, Bernd-Jochen Schaefer, PRD 94 (2016) 116020

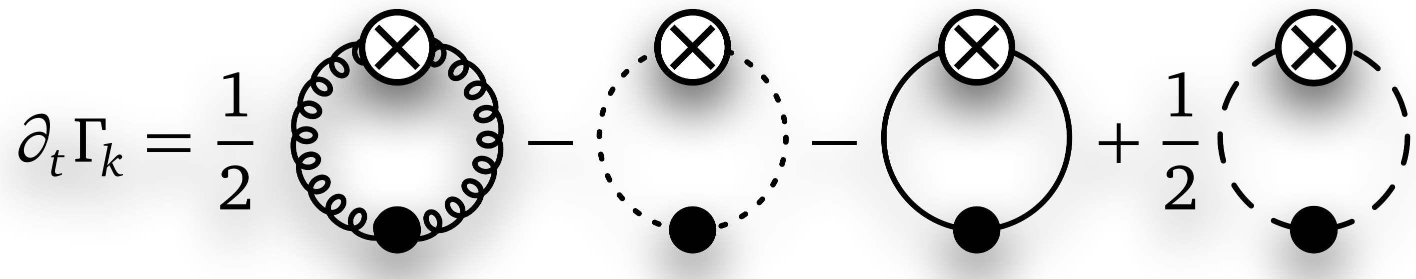
From effective models to QCD

Summary on effective model including mesonic fluctuations

- Quantitative agreement with lattice results below ~1.2 Tc
- No bump on the baryon number kurtosis
- Discrepancy observed at large temperature or density because of the UV cutoff effect
- UV cutoff should be pushed up higher, and glue quantum fluctuations should be included

From effective models to QCD

Summary on effective model including mesonic fluctuations



- Quantitative agreement with lattice results below ~1.2 Tc

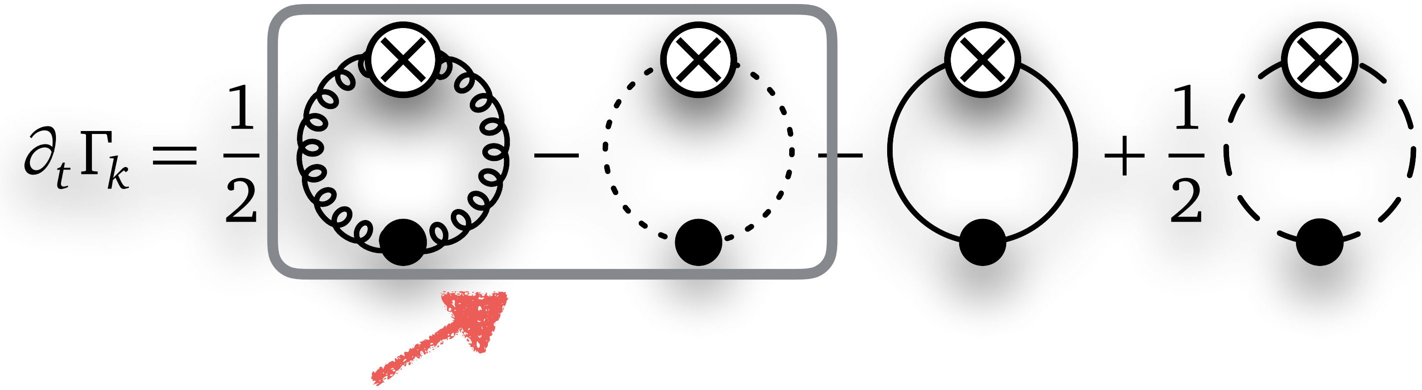
- No bump on the baryon number kurtosis

- Discrepancy observed at large temperature or density because of the UV cutoff effect

- UV cutoff should be pushed up higher, and glue quantum fluctuations should be included

From effective models to QCD

Summary on effective model including mesonic fluctuations



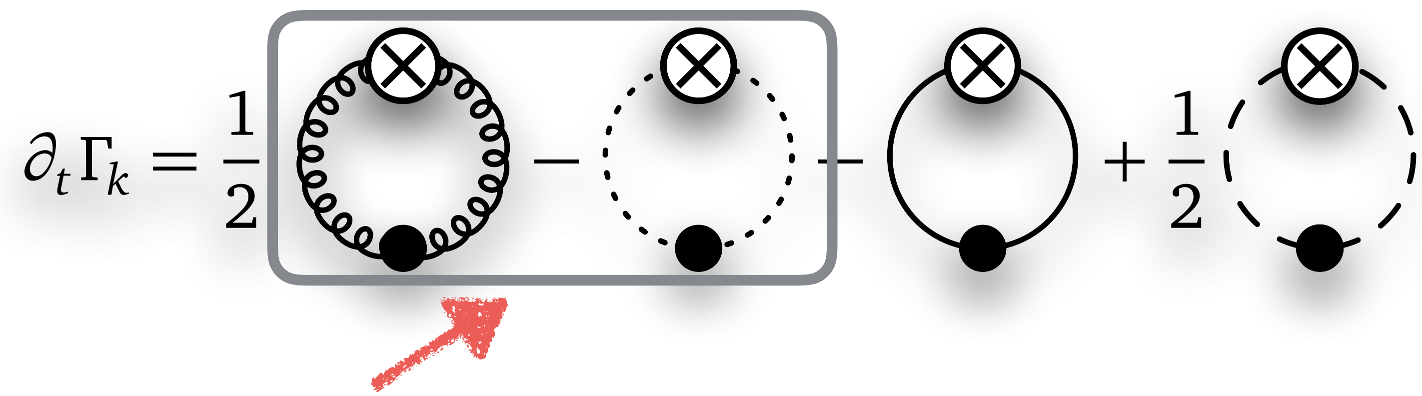
Come back!

- Quantitative agreement with lattice results below ~1.2 Tc
- No bump on the baryon number kurtosis
- Discrepancy observed at large temperature or density because of the UV cutoff effect
- UV cutoff should be pushed up higher, and glue quantum fluctuations should be included

From effective models to QCD

Summary on effective model including mesonic fluctuations

- Quantitative agreement with lattice results below ~1.2 Tc
- No bump on the baryon number kurtosis
- Discrepancy observed at large temperature or density because of the UV cutoff effect
- UV cutoff should be pushed up higher, and glue quantum fluctuations should be included



Come back!

Effective model:

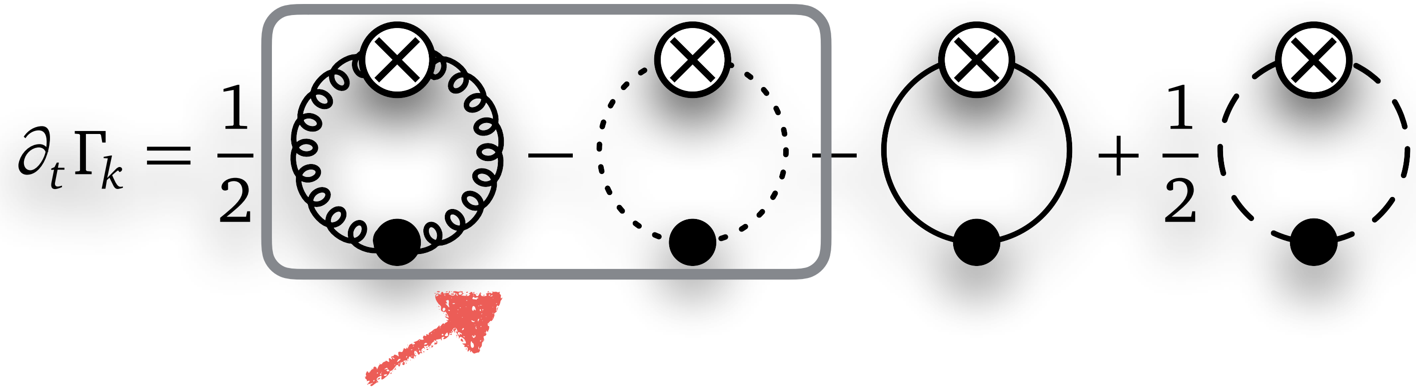
$$\Gamma_k = \int_x \Big\{ Z_{q,k}\bar{q}(\gamma_\mu\partial_\mu - \gamma_0\mu)q + \frac{1}{2}Z_{\phi,k}(\partial_\mu\phi)^2$$

$$+ h_k\,\bar{q}\left(T^0\sigma + i\gamma_5\vec{T}\cdot\vec{\pi}\right)q + V_k(\rho) - c\sigma\Big\} + \cdots$$

From effective models to QCD

Summary on effective model including mesonic fluctuations

- Quantitative agreement with lattice results below ~1.2 Tc
- No bump on the baryon number kurtosis
- Discrepancy observed at large temperature or density because of the UV cutoff effect
- UV cutoff should be pushed up higher, and glue quantum fluctuations should be included



Come back!

Effective model:

$$\Gamma_k = \int_x \Big\{ Z_{q,k} \bar{q}(\gamma_\mu \partial_\mu - \gamma_0 \mu) q + \frac{1}{2} Z_{\phi,k} (\partial_\mu \phi)^2$$

$$+ h_k \, \bar{q} \left(T^0 \sigma + i\gamma_5 \vec{T} \cdot \vec{\pi} \right) q + V_k(\rho) - c\sigma \Big\} + \cdots$$

Rebosonized QCD:

$$\Gamma_k = \int_x \Big\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + Z_{c,k} (\partial_\mu \bar{c}^a) D^{ab}_\mu c^b + \frac{1}{2\xi} (\partial_\mu A^a_\mu)^2$$

$$+ Z_{q,k} \bar{q}(\gamma_\mu D_\mu) q - \lambda_{q,k} \left[(\bar{q} T^0 q)^2 - (\bar{q} \gamma_5 \vec{T} q)^2 \right]$$

$$+ h_k \left[\bar{q}(i\gamma_5 \vec{T} \vec{\pi} + T^0 \sigma) q \right] + \frac{1}{2} Z_{\phi,k} (\partial_\mu \phi)^2$$

$$+ V_k(\rho) - c\sigma \Big\} + \Delta\Gamma_{\text{glue}}$$

Flow Equations

Wetterich equation:

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2}\mathrm{STr}\Big\{\partial_t R_k \big(\Gamma_k^{(2)}[\Phi] + R_k\big)^{-1}\Big\}$$

$$= \frac{1}{2}\mathrm{STr}\Big\{\tilde{\partial}_t \ln\big(\Gamma_k^{(2)}[\Phi] + R_k\big)\Big\}$$

with $t = \ln(k/\Lambda)$ and

$$\big(\Gamma_k^{(2)}[\Phi]\big)_{ij} := \frac{\overrightarrow{\delta}}{\delta\Phi_i}\Gamma_k[\Phi]\frac{\overleftarrow{\delta}}{\delta\Phi_j} \qquad \Gamma_k^{(2)} + R_k = \mathcal{P} + \mathcal{F}$$

Vertex expansion:

$$\partial_t \Gamma_k = \frac{1}{2}\mathrm{STr}\{\tilde{\partial}_t \ln(\mathcal{P}+\mathcal{F})\} = \frac{1}{2}\mathrm{STr}\tilde{\partial}_t \ln \mathcal{P}$$

$$+ \frac{1}{2}\mathrm{STr}\tilde{\partial}_t\Big(\frac{1}{\mathcal{P}}\mathcal{F}\Big) - \frac{1}{4}\mathrm{STr}\tilde{\partial}_t\Big(\frac{1}{\mathcal{P}}\mathcal{F}\Big)^2 + \cdots$$

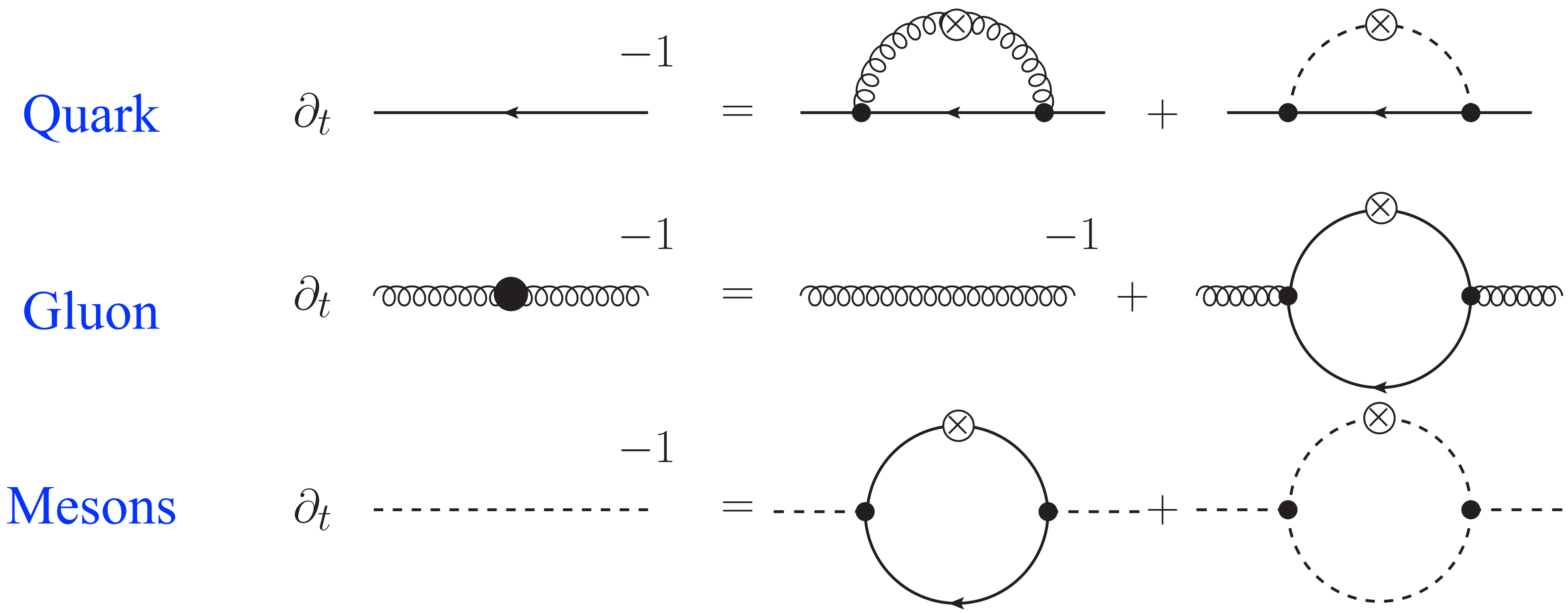
$3d$ optimized regulator:

$$R_{\mathrm{F},k}(q) = Z_{q,k} i\vec{q}\cdot\vec{\gamma}\, r_F(\frac{\vec{q}^2}{k^2}), \quad \text{with} \quad r_F(x) = (\frac{1}{\sqrt{x}} - 1)\Theta(1-x)$$

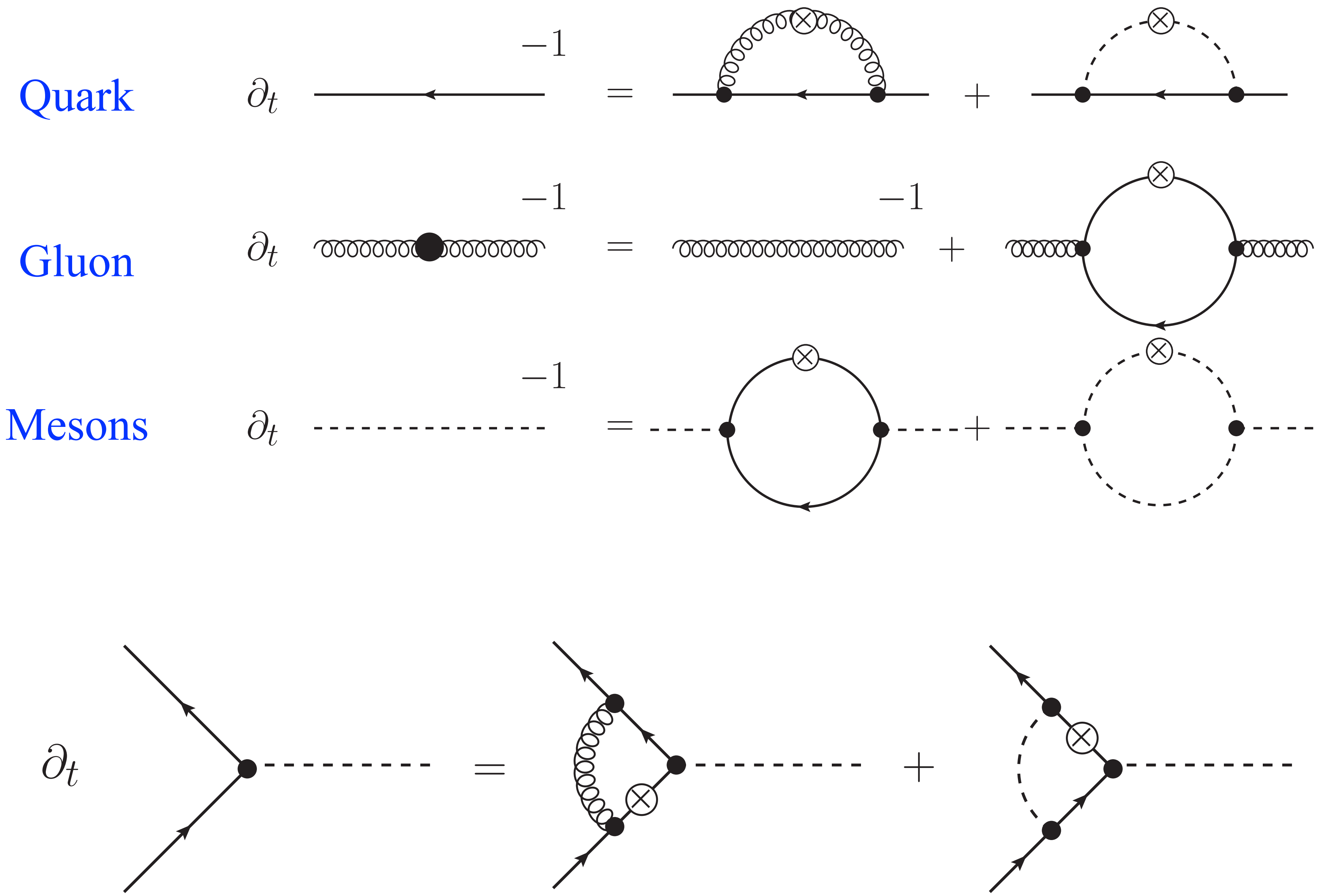
$$R_{\mathrm{B},k}(q) = Z_{\phi,k}\vec{q}^2\, r_B(\frac{\vec{q}^2}{k^2}), \quad \text{with} \quad r_B(x) = (\frac{1}{x} - 1)\Theta(1-x)$$

Feynman Diagrams

Feynman Diagrams

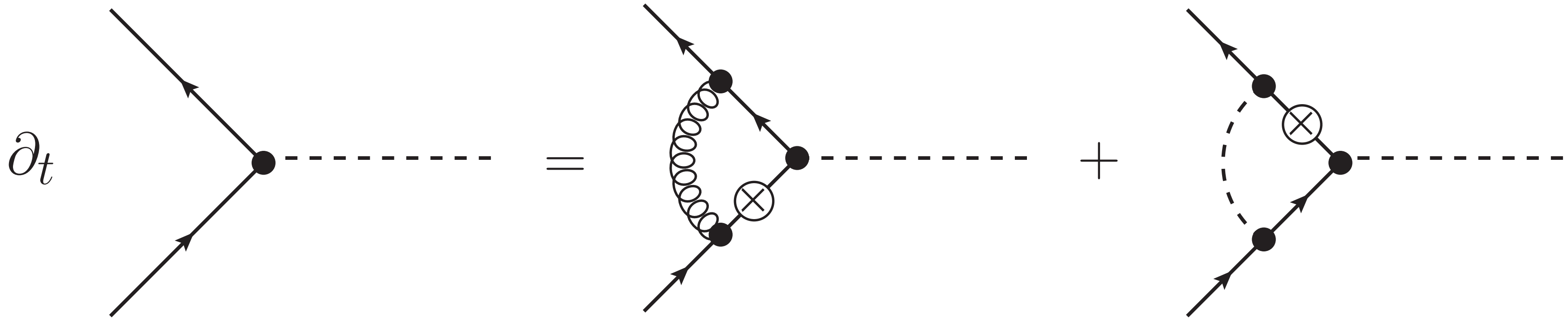


Feynman Diagrams



Yukawa coupling

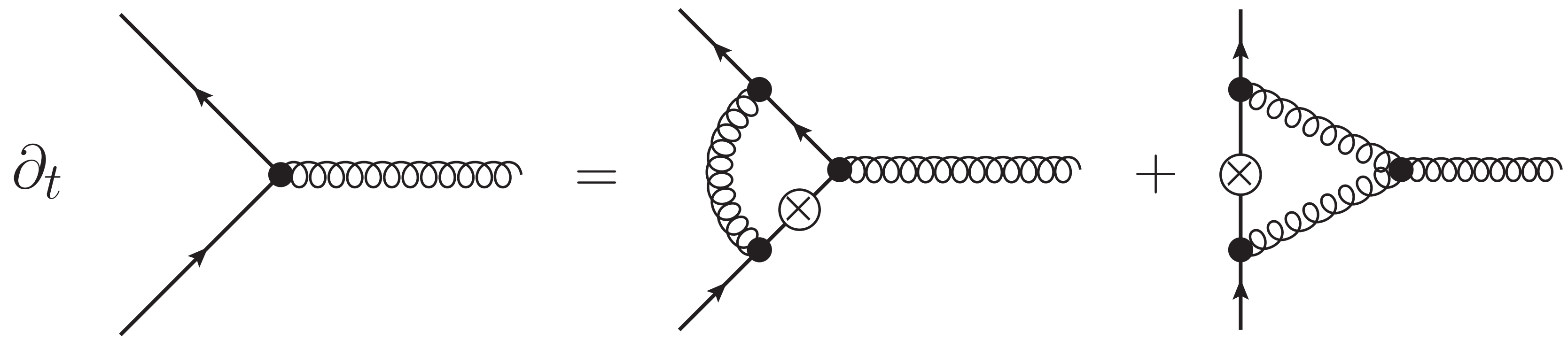
Feynman Diagrams



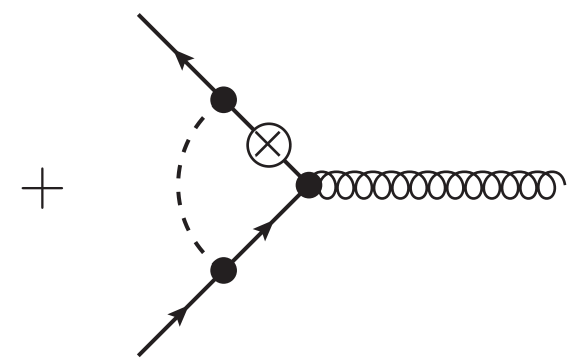
Yukawa coupling

Feynman Diagrams

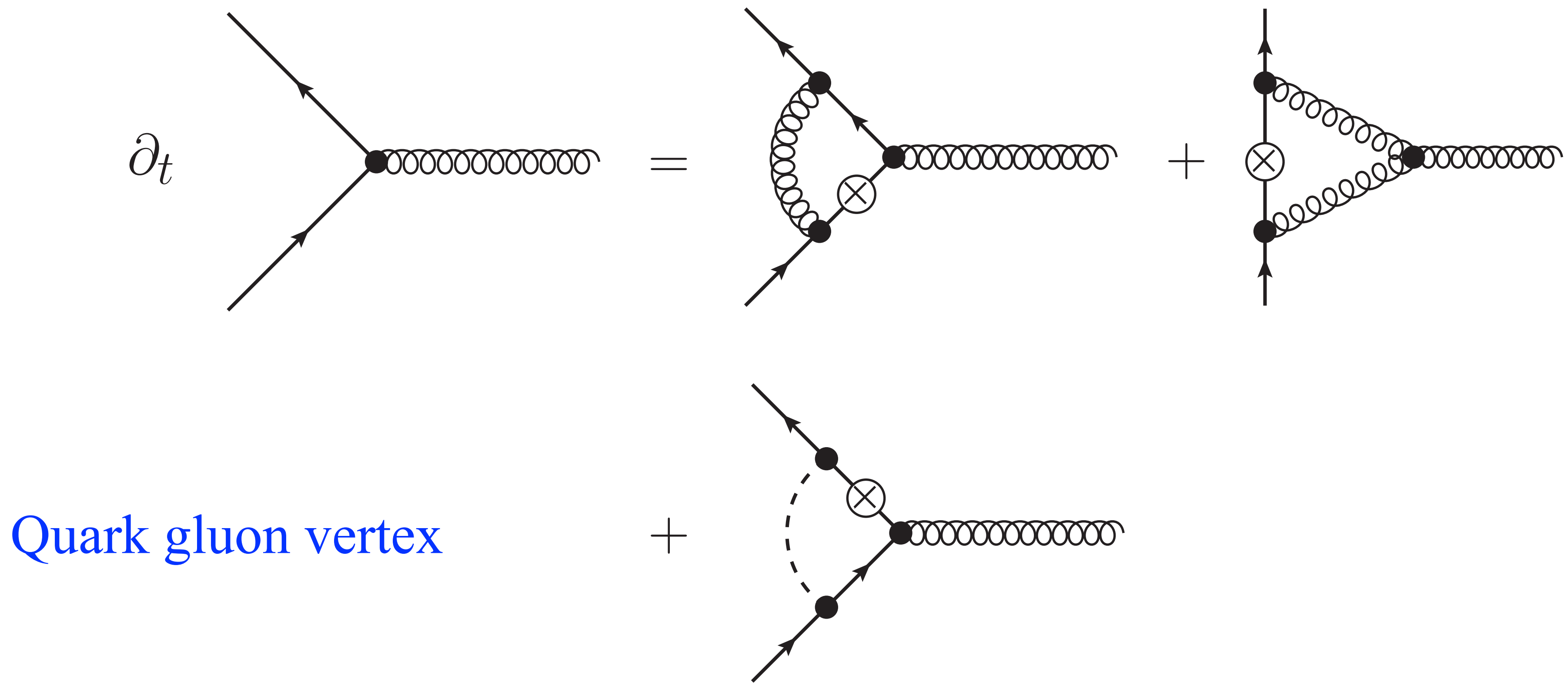
Feynman Diagrams

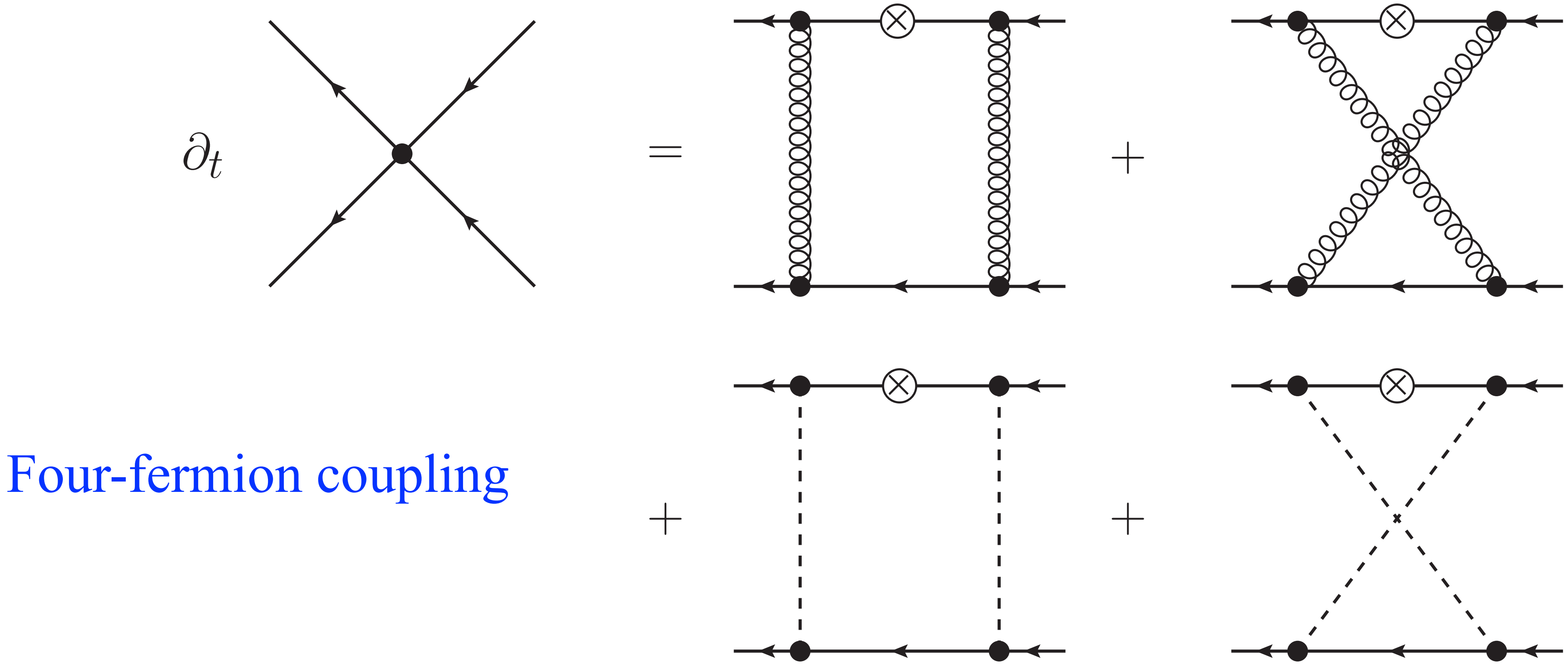


Quark gluon vertex

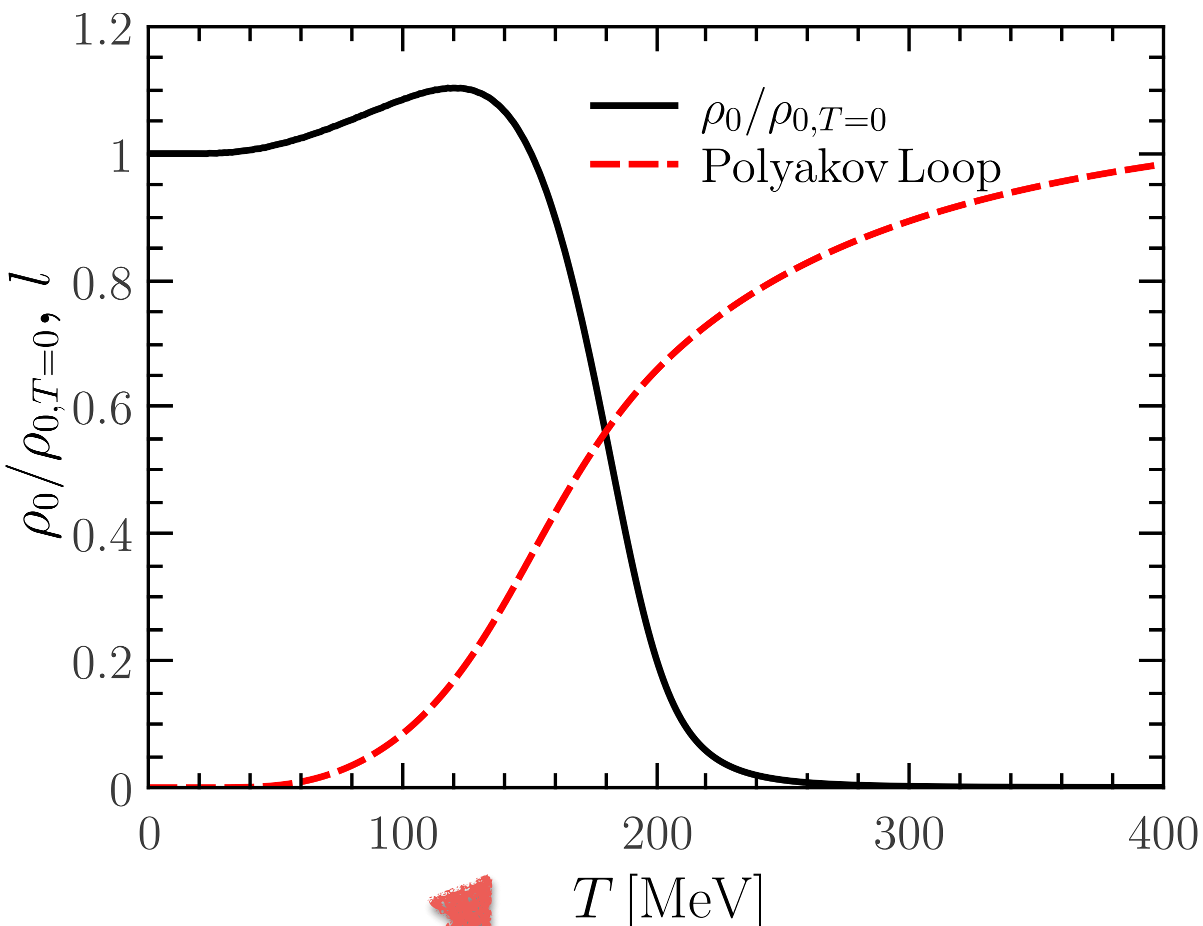


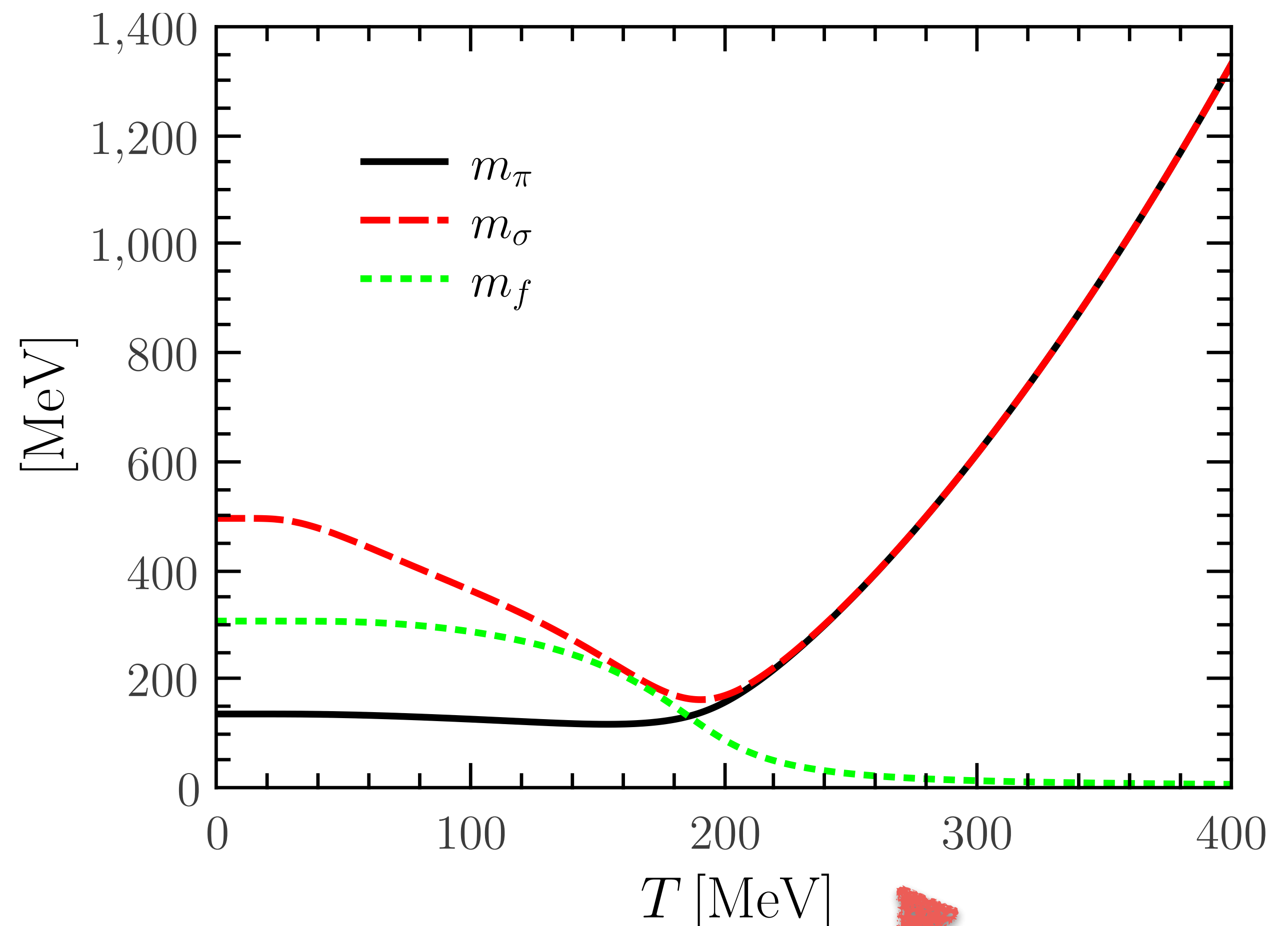
Feynman Diagrams





QCD Phase Transition with T



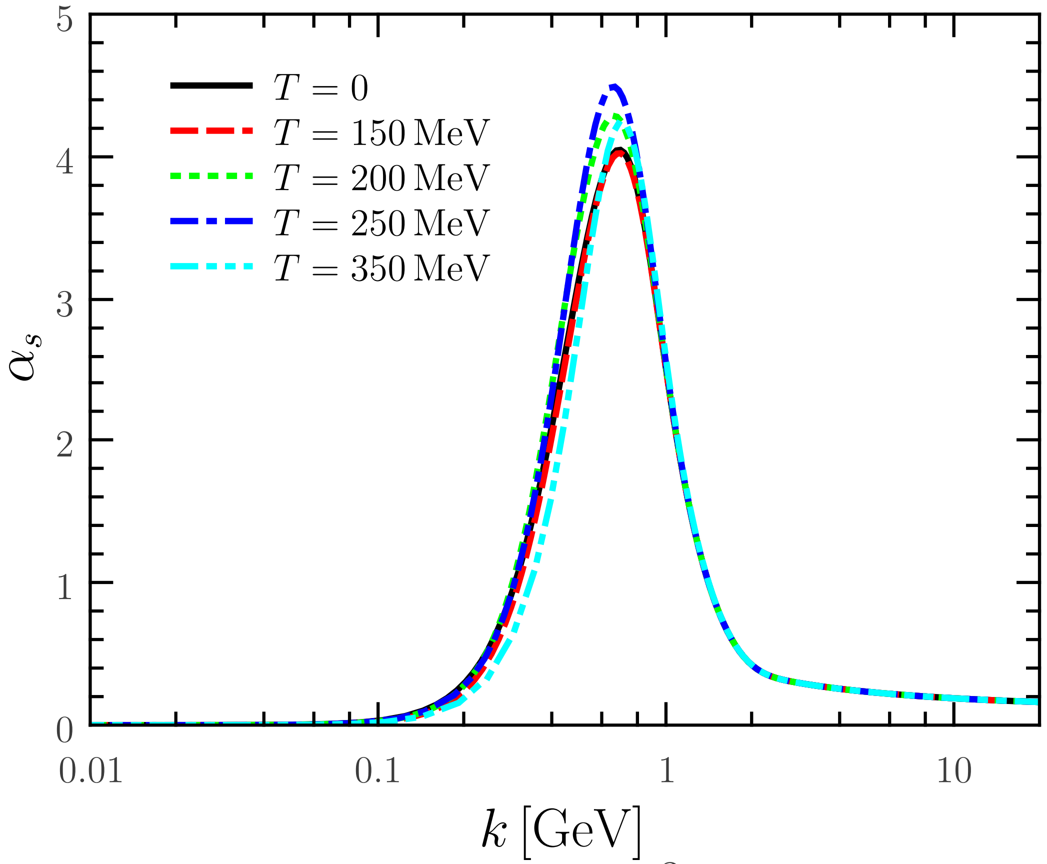


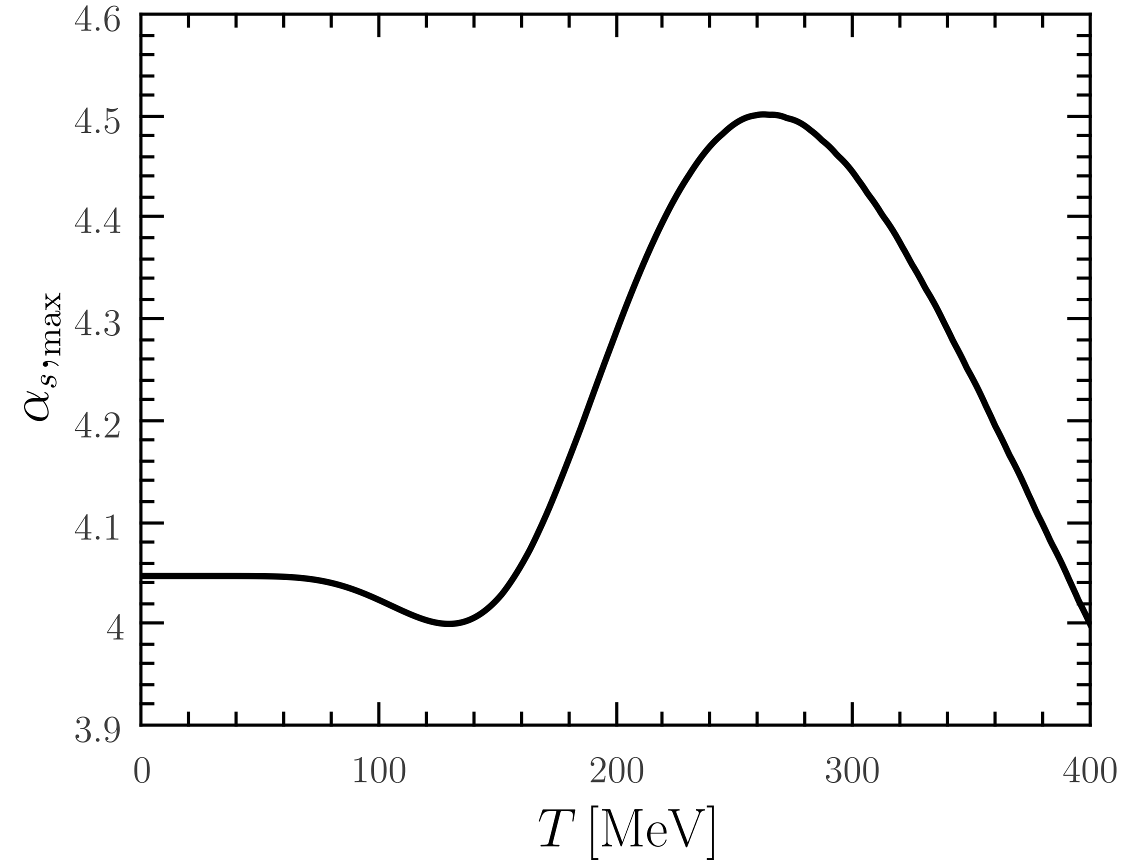
Order parameter

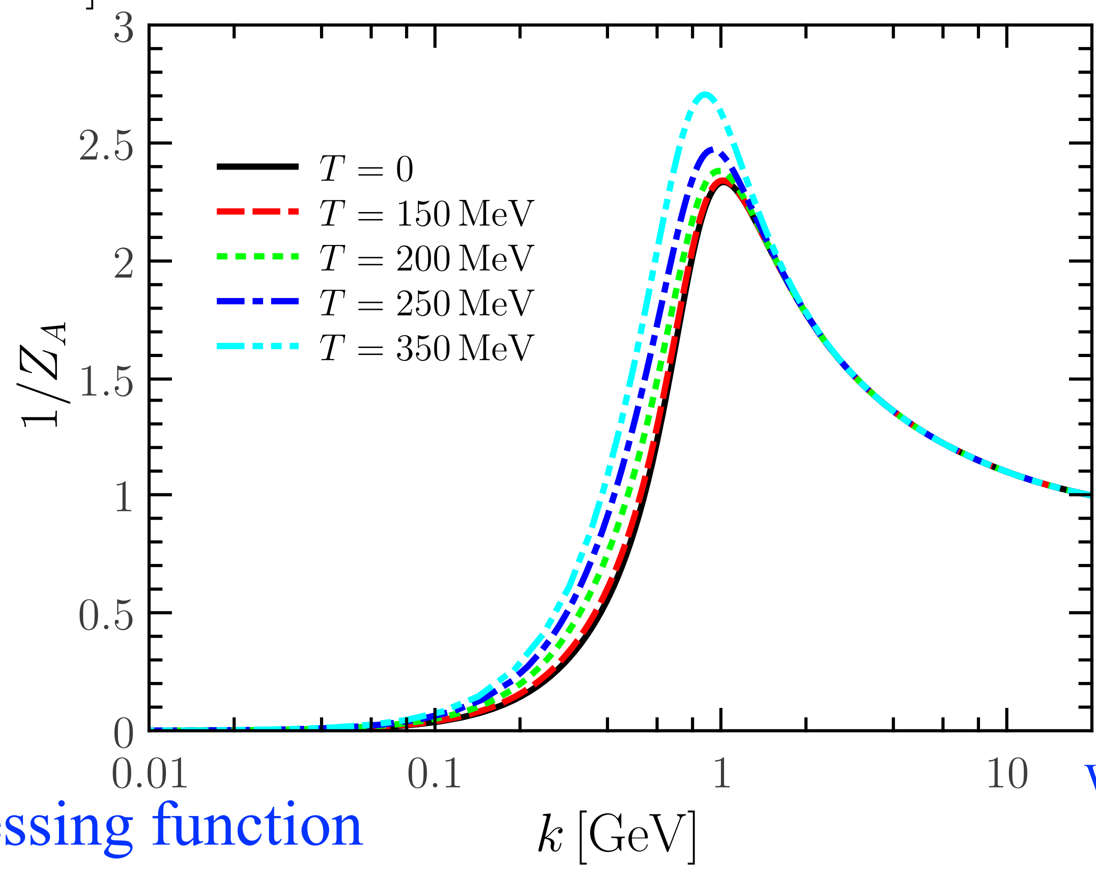
Meson and quark mass

WF, J.M. Pawlowski, F. Rennecke, in preparation

Running of the Strong Coupling



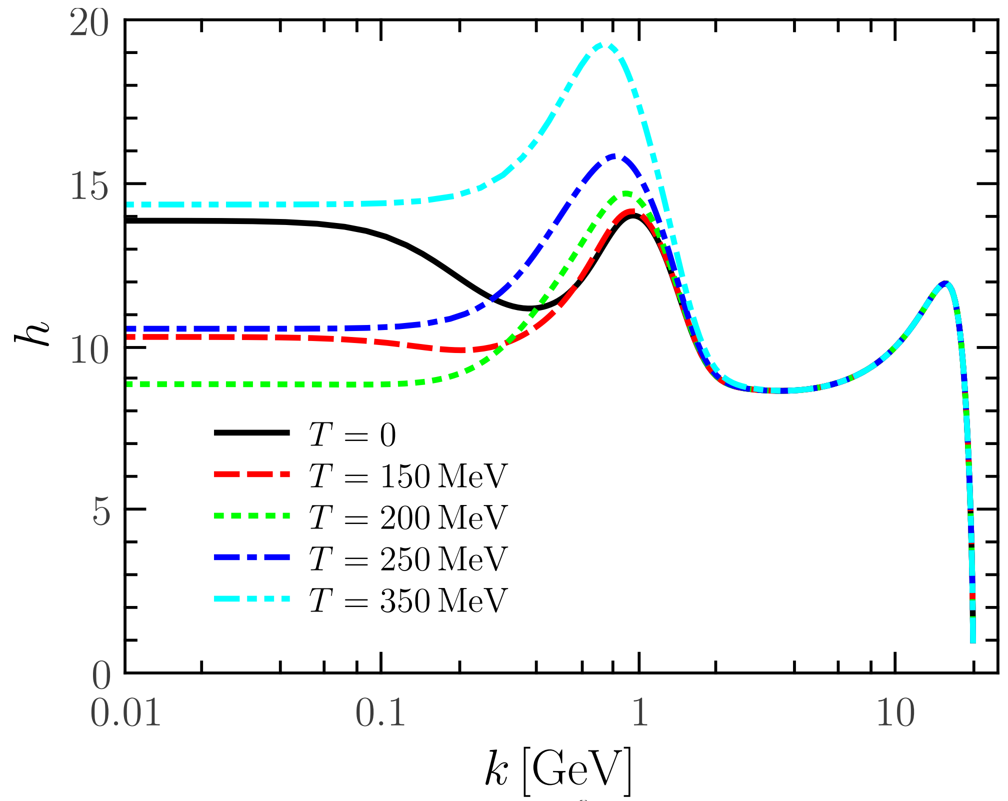


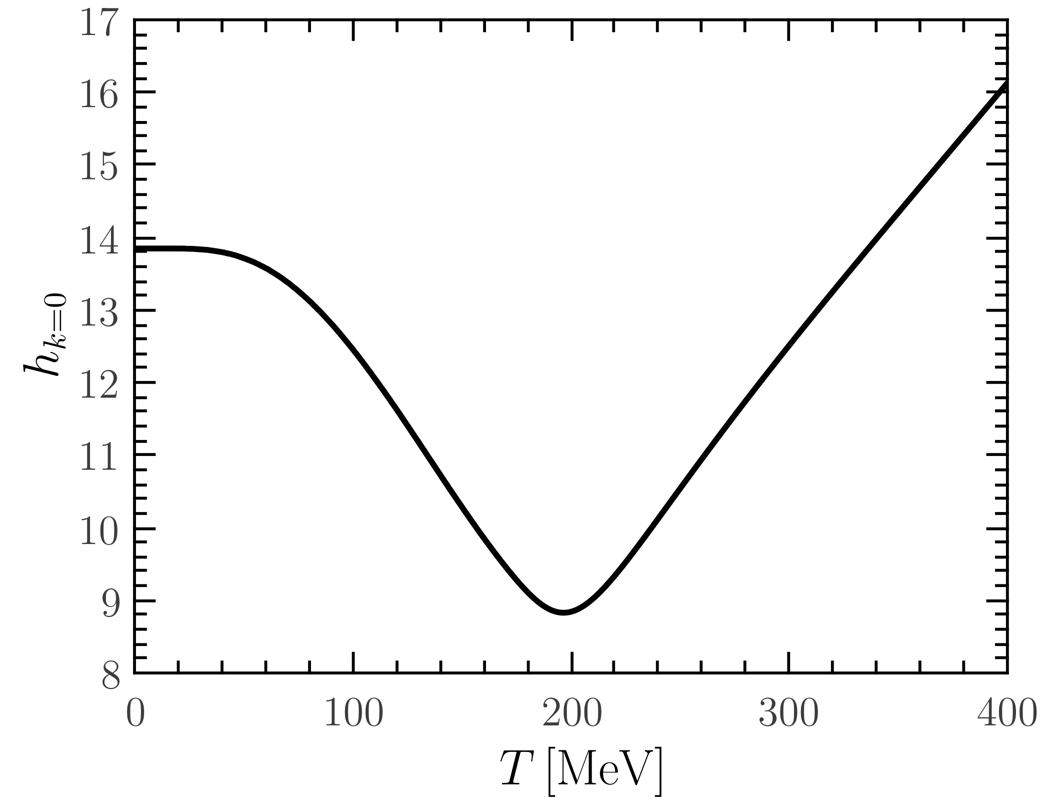


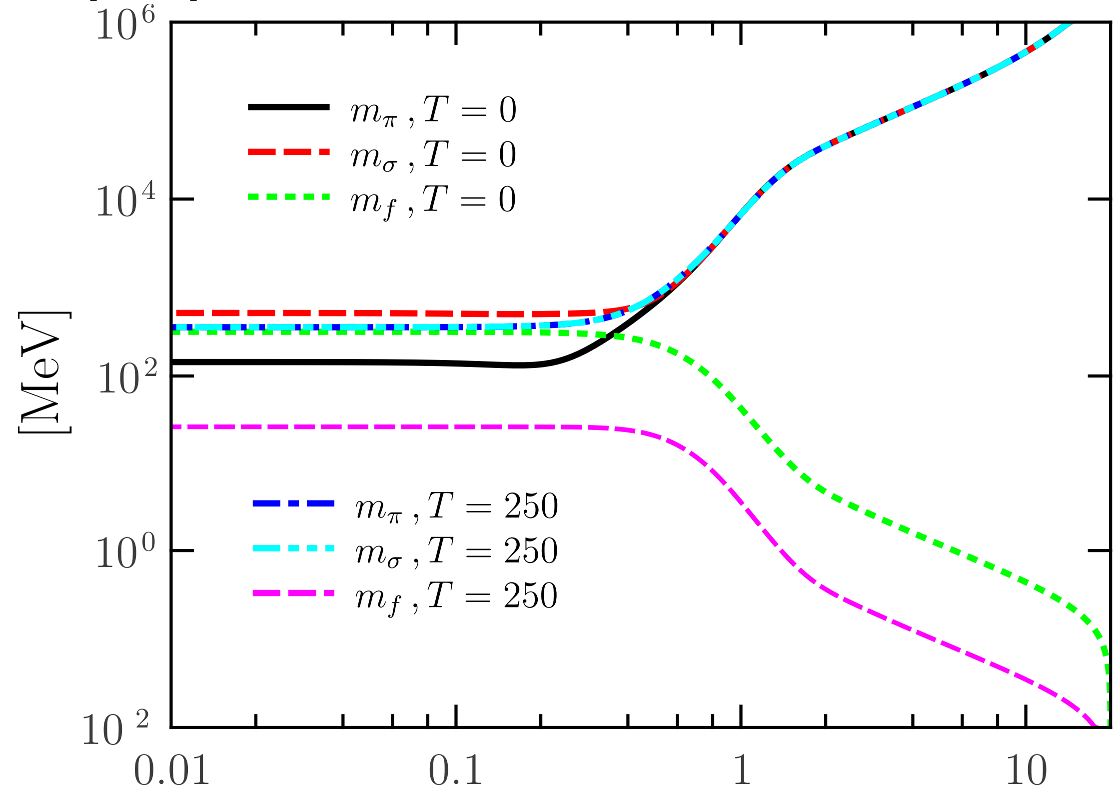
Gluon dressing function

WF, J.M. Pawlowski, F. Rennecke, in preparation

Yukawa coupling and Masses







Meson and quark masses

WF, J.M. Pawlowski, F. Rennecke, in preparation

Summary and outlook

Summary and outlook

- ★ Baryon number fluctuations and other conserved charge fluctuations have been investigated in the effective models with MFA, beyond MFA with quantum fluctuations included through the FRG approach.

- ★ Better agreement with lattice simulations and experiments is observed when more quantum fluctuations are included.

- ★ We have also performed FRG QCD calculations at finite temperature. The QCD phase transitions have been investigated.

- ★ Thermodynamics and fluctuations of conserved charges calculated in the FRG QCD approach are in progress.

Summary and outlook

- ★ Baryon number fluctuations and other conserved charge fluctuations have been investigated in the effective models with MFA, beyond MFA with quantum fluctuations included through the FRG approach.

- ★ Better agreement with lattice simulations and experiments is observed when more quantum fluctuations are included.

- ★ We have also performed FRG QCD calculations at finite temperature. The QCD phase transitions have been investigated.

- ★ Thermodynamics and fluctuations of conserved charges calculated in the FRG QCD approach are in progress.

Thank you very much for your attentions!