Probing QCD critical fluctuations from light nuclei production in relativistic heavy-ion collisions

EMMI Workshop CCNU(武汉)/17/10/13

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Phys. Lett. B 774, 103 (2017)
arXiv:1702.07620

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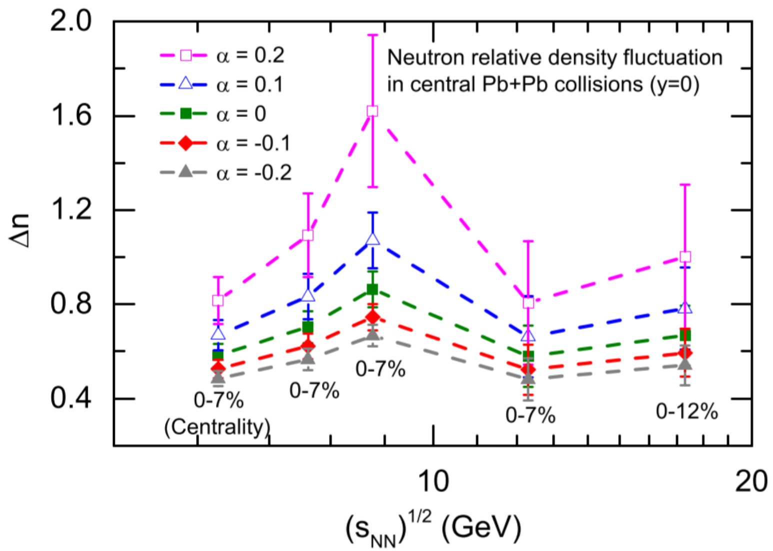
Outline

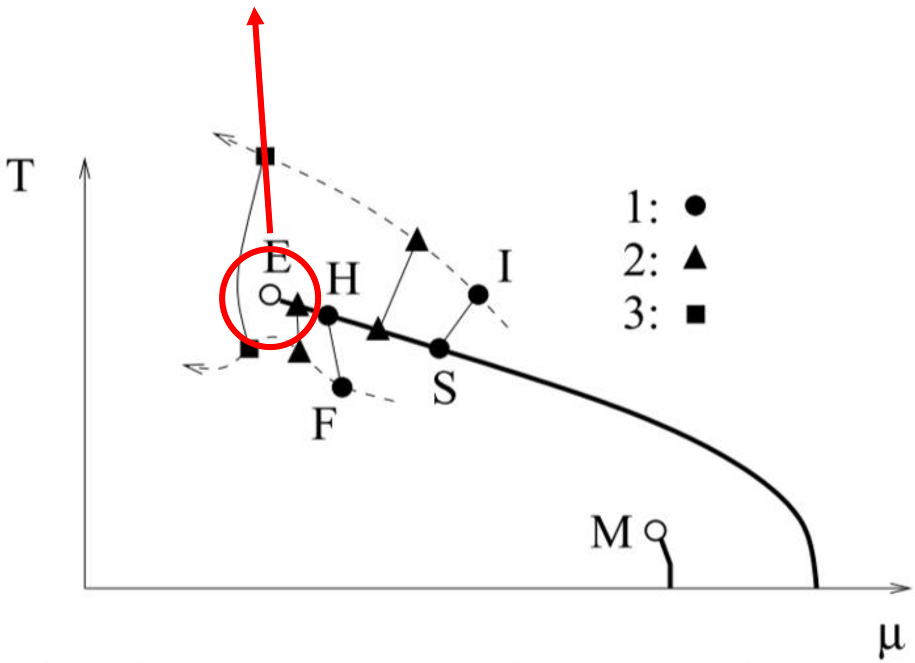
- Background and motivation
- Probing QCD critical fluctuations (critical end point) from light nuclei production
- Summary and outlook



Background and Motivation

Strategy for locating Critical endpoint

Divergence of correlation length leads to divergence of specific heat, compressibility and critical opalescence.

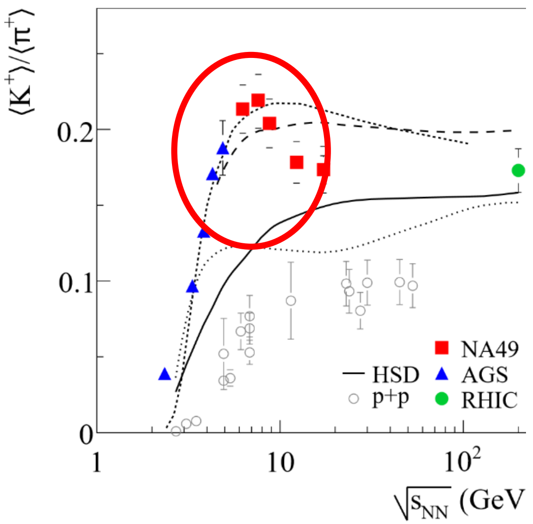


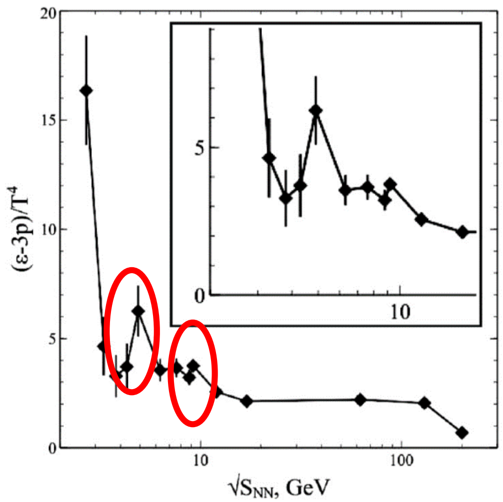
Event-by-event fluctuation of conserved charges

Searching for non-monotonic behaviors of quantities as a function of collision energy

M. Stephanov, K. Rajagopal and E. Shuryak, Phys. Rev. Lett. 81, 4816 (1998). arXiv:hep-ph/0402115v1.

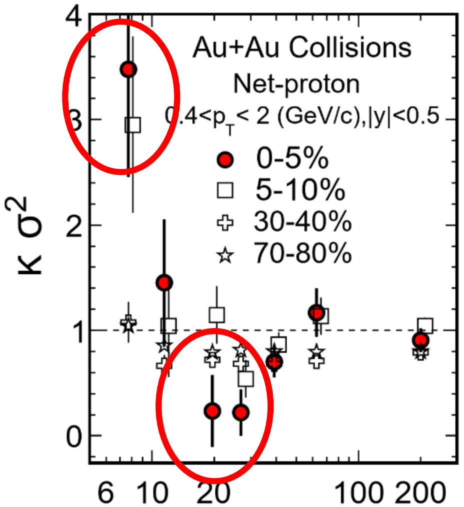
Non-monotonic behaviors

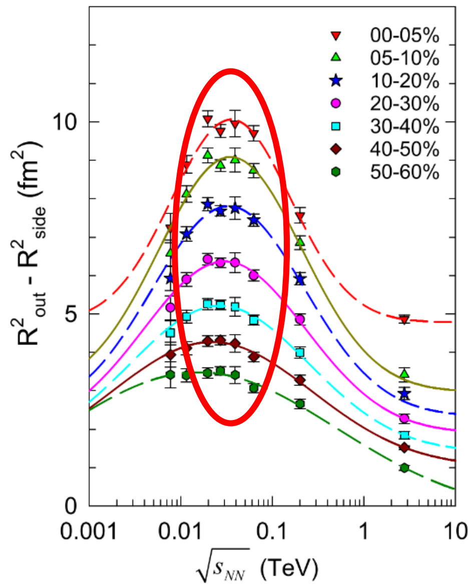


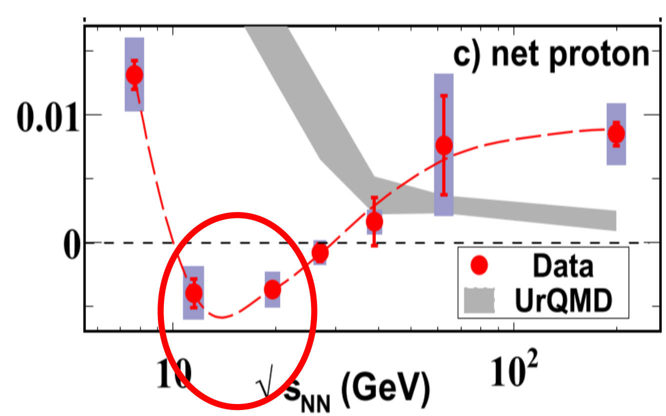


X. F. Luo et al. [STAR Collaboration], PoS CPOD2014, 019 (2015).

C. Alt et al, (NA49 Collaboration), Phys. Rev. C 77, 024903 (2008).
R. A. Lacey, Phys. Rev. Lett. 114, 142301 (2015).
N. -U. Bastian et al, arXiv: 1608.02851 (2016).
L. Adamczyk et al., arXiv: 1401. 3043 (2014).
K.A. Bugaev et al., arXiv:1709.05419 (2017)



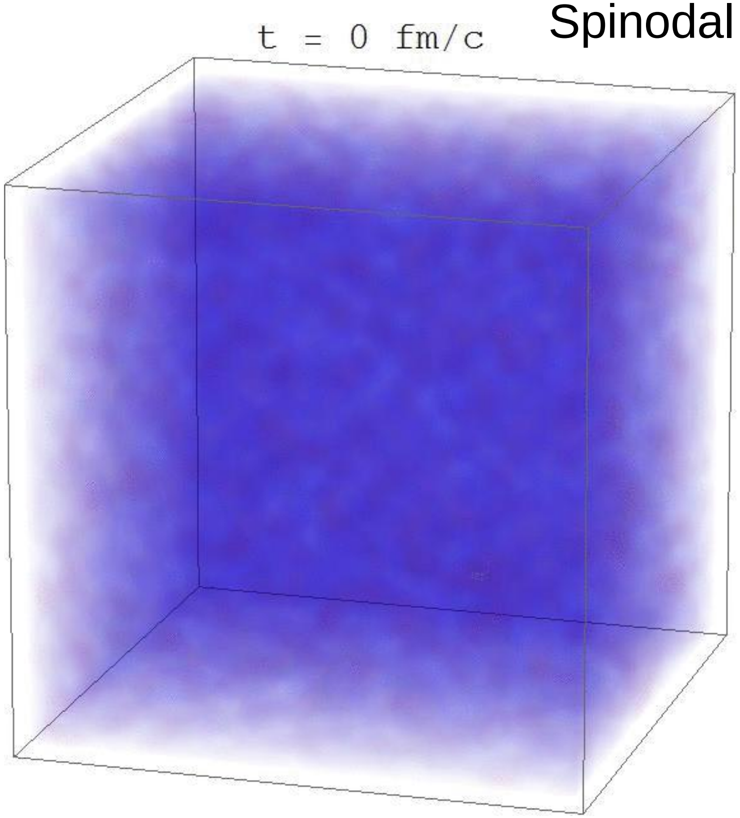




First-order phase transition and baryon density fluctuation

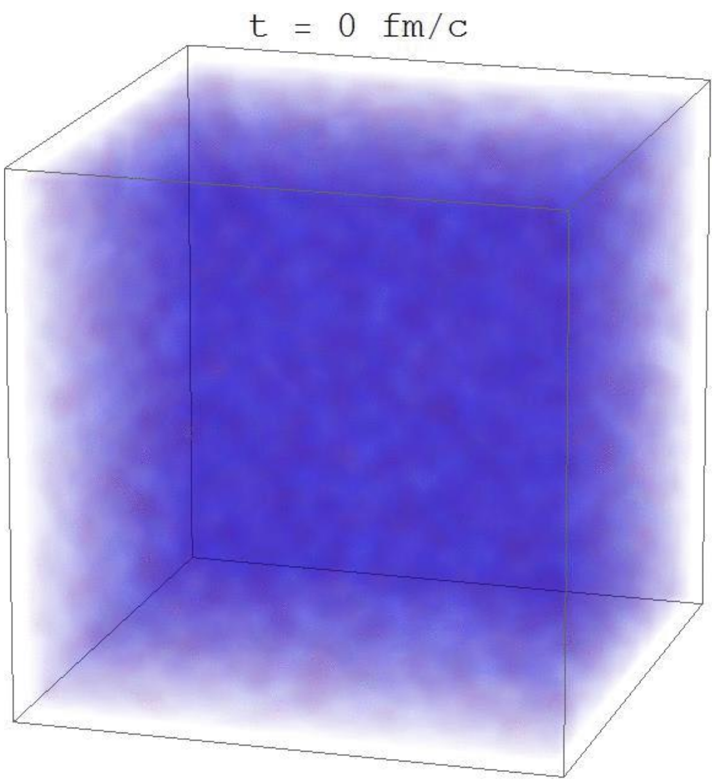
With first-order phase transition

Spinodal instability



$T = 20$ MeV $\quad n = 0.5\ \mathrm{fm}^{-3}$

No first-order phase transition



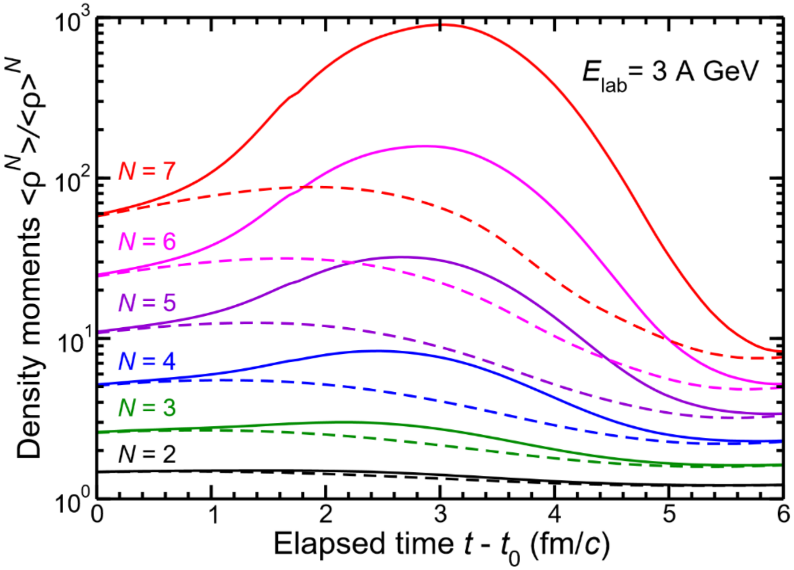
$T = 45$ MeV $\quad n = 0.7\ \mathrm{fm}^{-3}$

Taken from F. Li's talk at Shanghai

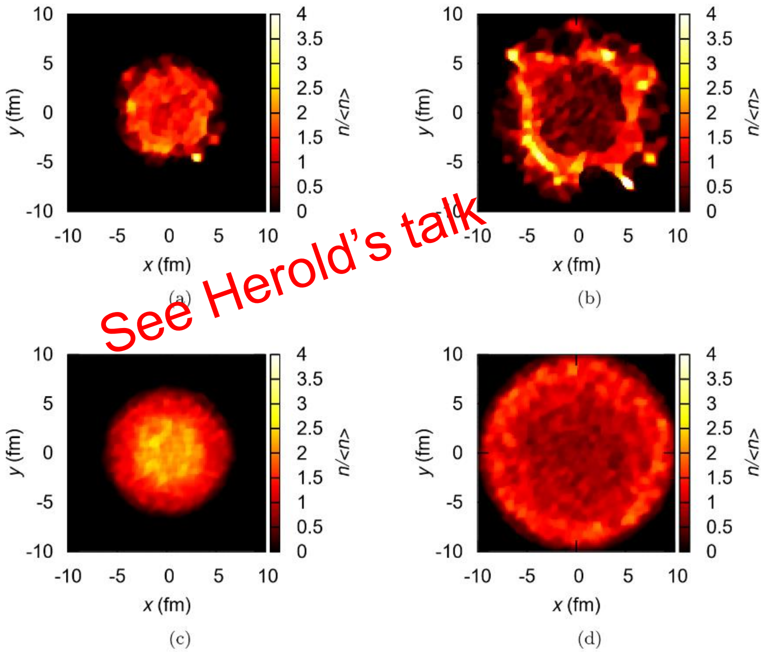
F. Li, C.M. Ko, Phys. Rev. C 93, 035205 (2016).

Spinodal instability in first-order phase transition

Nonequilibrium Chiral Fluid Dynamics **NχFD** model



J. Steinheimer and J. Randrup,
Phys. Rev. Lett. 109, 212301 (2012)



See Herold's talk

C. Herold et al., Nucl. Phys. A 925, 2014

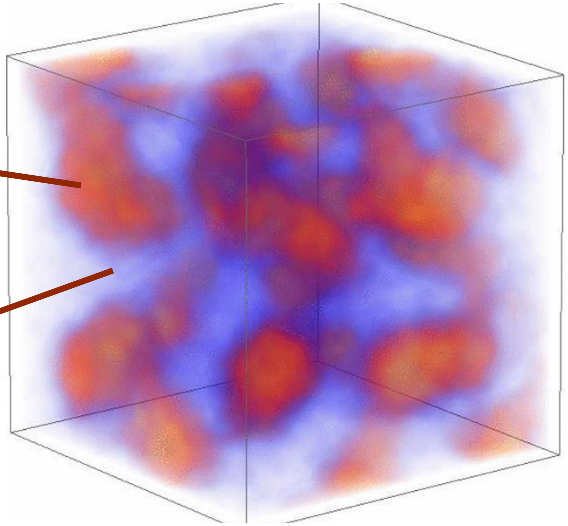
Solid lines: Two-phase equation of state
Dashed lines: One-phase partner

Sasaki, Friman, Redlich, Phys.Rev. D77 (2008) 034024
V. V. Skokov et al. Nucl.Phys. A828 (2009) 401-438.

$$\langle \rho^N \rangle \equiv \frac{1}{A^N} \int \rho(\boldsymbol{r})^N \rho(\boldsymbol{r})\, d^3\boldsymbol{r} \quad A = \int \rho(\boldsymbol{r}) d^3\boldsymbol{r}$$

Large baryon density fluctuation over space will be developed in the first-order phase transition

Simple Estimation



High density region: V_h, n_h

Low density region: V_l, n_l

$$n_0 = \frac{n_h V_h + n_l V_l}{V} \qquad V = V_h + V_l$$

$$\langle f(r) \rangle = \frac{1}{V} \int f(r) \mathrm{d}r$$

Density fluctuation: $\langle n(r)^2 \rangle = \frac{n_h^2 V_h + n_l^2 V_l}{V}$

Assuming: $n_\mathrm{h} = 1, n_\mathrm{l} = 0.25, n_0 = 0.5$

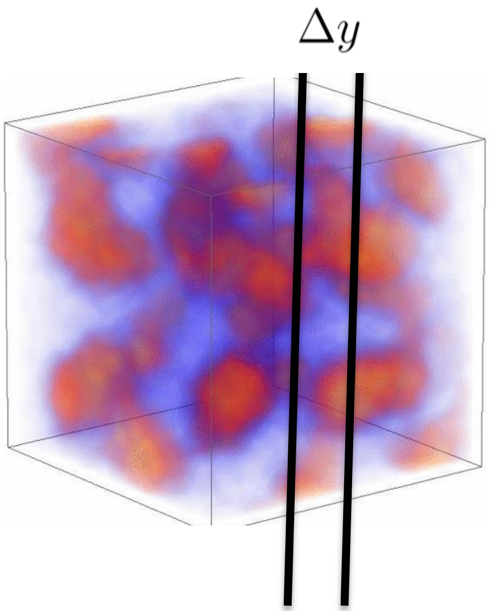
$$\frac{\langle n(r)^2 \rangle}{n_0^2} = 1.5 \qquad \frac{\langle (\delta n(r))^2 \rangle}{n_0^2} = 0.5$$

F. Li, C.M. Ko, Phys. Rev. C 93, 035205 (2016).

Question

How to measure the fluctuations due to phase transition?

1) Measuring event-by-event fluctuations of conserved charges in a sub system
2) Relating event-by-event fluctuations to thermal/critical fluctuations

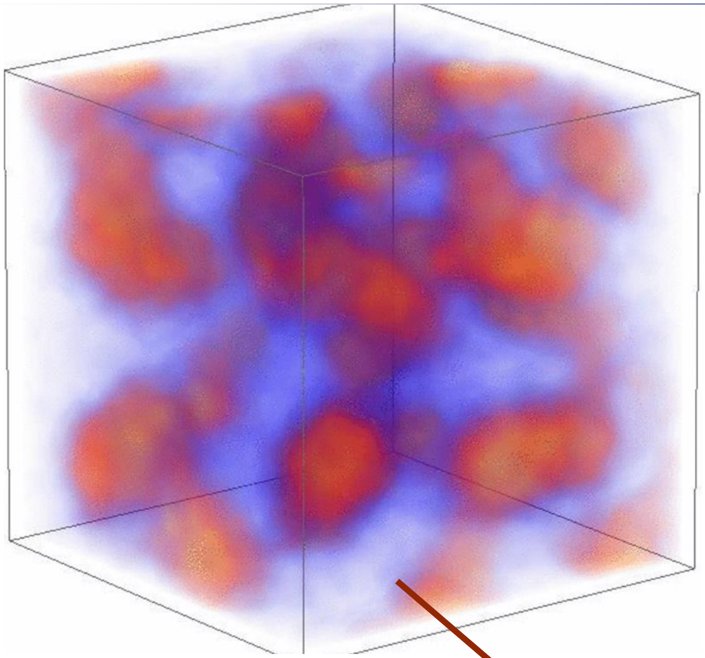


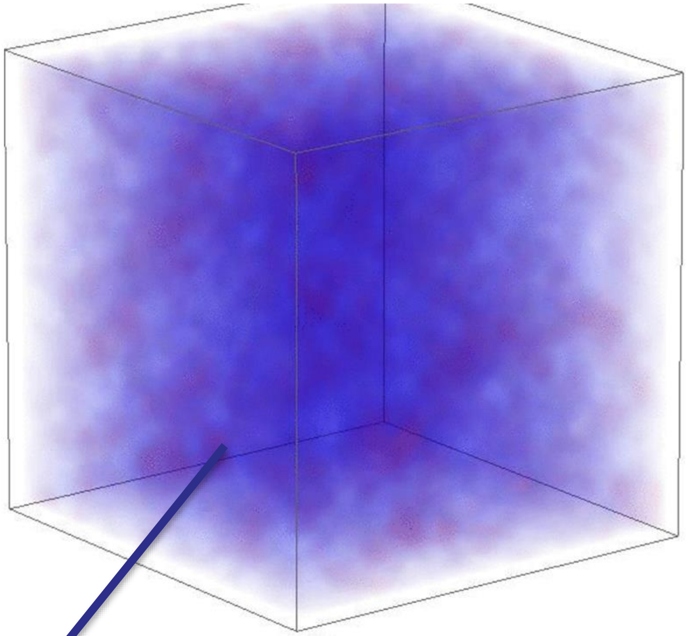
New way ?

Note: there is no fluctuation of conserved quantities for the whole system, but density over space still fluctuates.

Both the existence of critical point and first-order phase transition could induce large baryon density fluctuations

Main idea





Produce different number of light nuclei such as deuteron and triton

Nucleon clusters

Light nuclei can be served as a promising tool.

Density fluctuation equilibrates very slowly for expanding system

Main idea

Baryon density fluctuation is closely related to the correlation between nucleons.

The correlation between nucleons determines the production of light nuclei

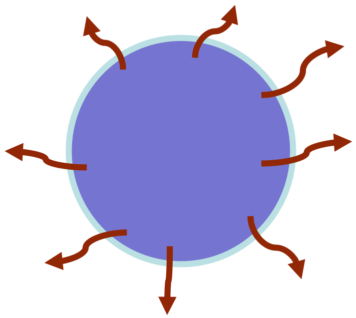
⇨ Baryon density fluctuation in vicinity of first-order phase transition can be deciphered from the production of light nuclei

QCD critical end point and production of light nuclei

$$\mathcal{O}_{\text{p-d-t}} = \frac{N_{^3\text{H}} N_p}{N_{\text{d}}^2} \approx g(1 + \Delta n)$$

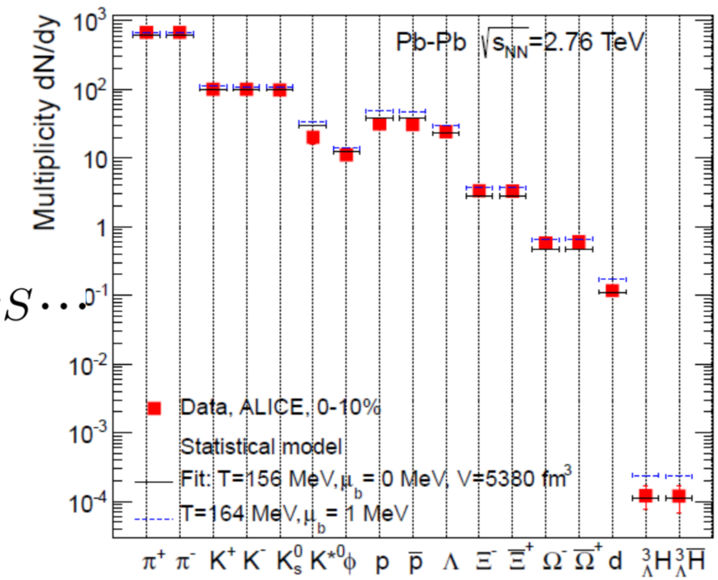
Two scenarios for particle production

Thermal emission:



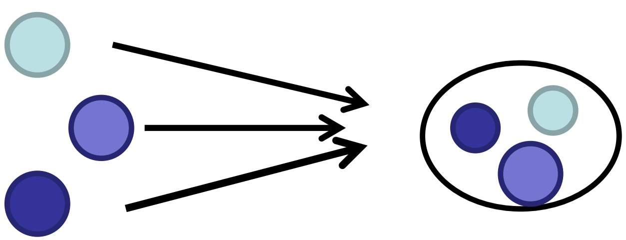
Source: T, V, $\mu_B, \ \mu_S \ldots$

Particle: m, s



P.Braun-Munzinger et al, Phys. Rep. 621,76 (2016).

Coalescence formation:



$$N_c = g_c \int \left(\prod_{i=1}^{N} \mathrm{d}N_i \right) \rho_c^W (x_1, ..., x_N; p_1, ..., p_N)$$

$$\propto Tr(\hat{\rho}_i \hat{\rho}_f)$$

Takes the internal structure into consideration

Source: T, V, $N_1, N_2 \ldots$

Cluster: $m_i, s_i; l_i, r_{\rm rms}(w)$

Wigner function:

$$\rho^W(\vec{x}, \vec{p}) = \frac{1}{(2\pi\hbar)^3} \int < x + y | \hat{\rho} | x - y > e^{-2i\vec{p}\vec{y}/\hbar} d\vec{y}$$

Particle coalescence production

Deuteron:

$$N_{\rm d} = g_{\rm d}\int d^3\mathbf{x}_1\int d^3\mathbf{k}_1\int d^3\mathbf{x}_2\int d^3\mathbf{k}_2 f_n(\mathbf{x}_1,\mathbf{k}_1) f_p(\mathbf{x}_2,\mathbf{k}_2)W_{\rm d}(\mathbf{x}_1-\mathbf{x}_2,(\mathbf{k}_1-\mathbf{k}_2)/2),$$

$$f(\mathbf{x},\mathbf{k}) = \frac{2\xi}{(2\pi)^3}\, e^{-\frac{k^2}{2mT}}$$

$$W_{\rm d}(\mathbf{x},\mathbf{k}) = 8\, e^{-\frac{x^2}{\sigma^2}}\, e^{-\sigma^2 k^2}$$

$$\mathbf{X} = \frac{\mathbf{x}_1+\mathbf{x}_2}{2},\quad \mathbf{x}=\mathbf{x}_1-\mathbf{x}_2,$$

$$\mathbf{K} = \mathbf{k}_1+\mathbf{k}_2,\quad \mathbf{k}=\frac{\mathbf{k}_1-\mathbf{k}_2}{2},$$

$$N_{\rm d} = \frac{32 g_{\rm d}\xi_1\xi_2}{(2\pi)^6}\int d^3\mathbf{X}\int d^3\mathbf{x}\, e^{-\frac{x^2}{\sigma^2}}\int d^3\mathbf{K}\, e^{-\frac{K^2}{4mT}} \int d^3\mathbf{k}\, e^{-k^2(\sigma^2+\frac{1}{mT})}$$

$$= \frac{32 g_{\rm d}\xi_1\xi_2}{(2\pi)^6} V\, (\pi\sigma^2)^{3/2}\, (4\pi mT)^{3/2}\left(\frac{\pi}{\sigma^2+\frac{1}{mT}}\right)^{3/2}$$

$$= \frac{3}{2^{1/2}}\left(\frac{2\pi}{mT}\right)^{3/2}\frac{1}{(1+\frac{1}{mT\sigma^2})^{3/2}}\frac{N_n N_p}{V}.$$

$mT \gg 1/\sigma^2$ ⟹

$$N_{\rm d} \approx \frac{3}{2^{1/2}}\left(\frac{2\pi}{mT}\right)^{3/2}\frac{N_n N_p}{V}$$

A general formula (COAL-SH) with boost invariance:

$$\frac{dN_c}{dy} \approx g_{\rm rel} g_{\rm size} g_c \mu_0^{\frac{3}{2}}\left[\prod_{i=1}^{N}\frac{\frac{dN_i}{dy_i}}{m_i^{\frac{3}{2}}}\right]\times \prod_{i=1}^{N-1}\frac{(\frac{4\pi}{w})^{\frac{3}{2}}}{V(\frac{2T}{w})^{\frac{1}{2}}(1+\frac{2T}{w})}\left(\frac{\frac{2T}{w}}{\frac{2T}{w}+1}\right)^{l_i} G\left(l_i,(\frac{2T}{w})^{\frac{1}{2}}\right).$$

K. J. Sun and L. W. Chen, arXiv:1701.01935 (2017).

No density fluctuation

Coalescence model:

$$N_{\mathrm{d}} = \frac{3}{2^{1/2}}\left(\frac{2\pi}{m_0 T}\right)^{3/2}\frac{N_p N_n}{V}, \quad \sim \int d\vec{r}\, n(\vec{r}) n_p(\vec{r})$$

$$N_{^3\mathrm{H}} = \frac{3^{3/2}}{4}\left(\frac{2\pi}{m_0 T}\right)^{3}\frac{N_p N_n^2}{V^2}. \quad \sim \int d\vec{r}\, n(\vec{r})^2 n_p(\vec{r})$$

K. J. Sun and L. W. Chen, Che Ming Ko, and Zhangbu Xu, Phys. Lett. B774, 103 (2017).

In vicinity of density fluctuation

Density fluctuation over space:

$$n(\vec{r}) = \frac{1}{V}\int n(\vec{r})\mathrm{d}\vec{r} + \delta n(\vec{r}) = \langle n\rangle + \delta n(\vec{r})$$

When: $\delta n \neq 0$

Neutron:

$$n(\vec{r}) = \langle n\rangle + \delta n(\vec{r}),$$

Proton:

$$n_p(\vec{r}) = \langle n_p\rangle + \delta n_p(\vec{r}),$$

$$\langle \delta n\rangle = 0,\ \langle \delta n_p\rangle = 0,$$

$$N_n = \int d\vec{r}\, n = V\langle n\rangle,\ N_p = \int d\vec{r}\, n_p = V\langle n_p\rangle.$$

Strictly speaking, one needs to introduce fluctuation from the beginning in coalescence formulism

Approximately:

$$\begin{aligned} N_{\mathrm{d}} &= \frac{3}{2^{1/2}}\left(\frac{2\pi}{m_0T}\right)^{3/2}\int d\vec{r}\, n(\vec{r})n_p(\vec{r}) \\ &= \frac{3}{2^{1/2}}\left(\frac{2\pi}{m_0T}\right)^{3/2}\int d\vec{r}\,(\langle n\rangle + \delta n(\vec{r}))(\langle n_p\rangle + \delta n_p(\vec{r})) \\ &= \frac{3}{2^{1/2}}\left(\frac{2\pi}{m_0T}\right)^{3/2}\int d\vec{r}\,(\langle n\rangle\langle n_p\rangle + \delta n(\vec{r})\delta n_p(\vec{r})) \\ &= \frac{3}{2^{1/2}}\left(\frac{2\pi}{m_0T}\right)^{3/2}(N_p\langle n\rangle + \int d\vec{r}\,\delta n(\vec{r})\delta n_p(\vec{r})). \end{aligned}$$

Cross terms vanish

In vicinity of density fluctuation

$$
\begin{aligned}
N_{^3\mathrm{H}} &= \frac{3^{3/2}}{4}\left(\frac{2\pi}{m_0 T}\right)^3 \int d\vec{r}\, n(\vec{r})^2 n_p(\vec{r}) \\
&= \frac{3^{3/2}}{4}\left(\frac{2\pi}{m_0 T}\right)^3 \int d\vec{r}\, (\langle n\rangle + \delta n(\vec{r}))^2 (\langle n_p\rangle + \delta n_p(\vec{r})) \\
&= \frac{3^{3/2}}{4}\left(\frac{2\pi}{m_0 T}\right)^3 \int d\vec{r}\, \{(\langle n\rangle^2 + (\delta n(\vec{r}))^2)\langle n_p\rangle + (2\langle n\rangle \delta n(\vec{r}) + (\delta n(\vec{r}))^2)\delta n_p(\vec{r})\} \\
&= \frac{3^{3/2}}{4}\left(\frac{2\pi}{m_0 T}\right)^3 \{(\langle n\rangle^2 + \langle(\delta n(\vec{r}))^2\rangle)\langle n_p\rangle V + \int d\vec{r}\, (2\langle n\rangle \delta n(\vec{r}) + (\delta n(\vec{r}))^2)\delta n_p(\vec{r})\}
\end{aligned}
$$

$$
N_{\mathrm{d}} = \frac{3}{2^{1/2}}\left(\frac{2\pi}{m_0 T}\right)^{3/2} \left(N_p\langle n\rangle + \int d\vec{r}\, \delta n(\vec{r})\delta n_p(\vec{r})\right)
$$

$$
\langle n(\vec{r})\rangle = \frac{1}{V}\int d\vec{r} n(\vec{r}) \qquad \langle(\delta n(\vec{r}))^2\rangle = \frac{1}{V}\int d\vec{r}(\delta n(\vec{r}))^2
$$

Neglecting correlation: $\int d\vec{r}\, \delta n(\vec{r})\delta n_p(\vec{r}) = 0.$ $\qquad \int d\vec{r}\, \delta n(\vec{r})^2 \delta n_p(\vec{r}) = 0.$

In vicinity of density fluctuation

$$N_{\rm d} = \frac{3}{2^{1/2}}\left(\frac{2\pi}{m_0 T}\right)^{3/2} N_p \langle n\rangle,$$

$$N_{^3{\rm H}} = \frac{3^{3/2}}{4}\left(\frac{2\pi}{m_0 T}\right)^{3} N_p \langle n\rangle^2 (1+\Delta n),$$

Relative neutron density fluctuation: $\Delta n = \frac{<(\delta n)^2>}{<n>^2}$

$$\langle n(\vec{r})\rangle = \frac{1}{V}\int d\vec{r} n(\vec{r})$$

$$\langle (\delta n(\vec{r}))^2\rangle = \frac{1}{V}\int d\vec{r} (\delta n(\vec{r}))^2$$

Density fluctuation has more effect on ^{3}H than d

Let's take ratios !

Observable

The observable we propose is:

$$\mathcal{O}_{\text{p-d-t}} = \frac{N_{^3\text{H}} N_p}{N_{\text{d}}^2} = g(1+\Delta n)$$
$$g = 4/9 \times (3/4)^{3/2} \approx 0.29$$

$$\Delta n = \frac{<(\delta n)^2>}{<n>^2}$$

$$\langle n(\vec{r})\rangle = \frac{1}{V}\int d\vec{r} n(\vec{r})$$

$$\langle (\delta n(\vec{r}))^2\rangle = \frac{1}{V}\int d\vec{r} (\delta n(\vec{r}))^2$$

One can see that it has a linear dependence on neutron relative density fluctuation and it has no dependence on T, V, or other parameters.

This is different from what has been done in the measurements of fluctuation of yields of net protons within a specific phase-space in momentum so far.

K. J. Sun and L. W. Chen, Che Ming Ko, and Zhangbu Xu, Phys. Lett. B774, 103 (2017).

Correction from correlation

Correlation between density fluctuation of proton and neutron

$$\langle \delta n \delta n_p \rangle = \frac{1}{V}\int d\vec{r}\delta n(\vec{r})\delta n_p(\vec{r}) = \frac{1}{V}\int d\vec{r}\, c(\vec{r})(\delta n(\vec{r}))^2$$

with $\delta n_p(\vec{r}) = c(\vec{r})\delta n(\vec{r})$

One can always express: $\langle \delta n \delta n_p \rangle = \alpha \frac{\langle n_p \rangle}{\langle n \rangle} \langle (\delta n)^2 \rangle$

$$N_{\mathrm{d}} = \frac{3}{2^{1/2}}\left(\frac{2\pi}{m_0 T}\right)^{3/2} N_p \langle n \rangle (1+\alpha\Delta n),$$

$$N_{^3\mathrm{H}} = \frac{3^{3/2}}{4}\left(\frac{2\pi}{m_0 T}\right)^{3} N_p \langle n \rangle^2 (1+(1+2\alpha)\Delta n)$$

$$\mathcal{O}_{\text{p-d-t}} = \frac{N_{^3\mathrm{H}} N_p}{N_{\mathrm{d}}^2} = g\frac{1+(1+2\alpha\Delta n)}{(1+\alpha\Delta n)^2} \approx g(1+\Delta n+\mathcal{O}((\alpha\Delta n)^2))$$

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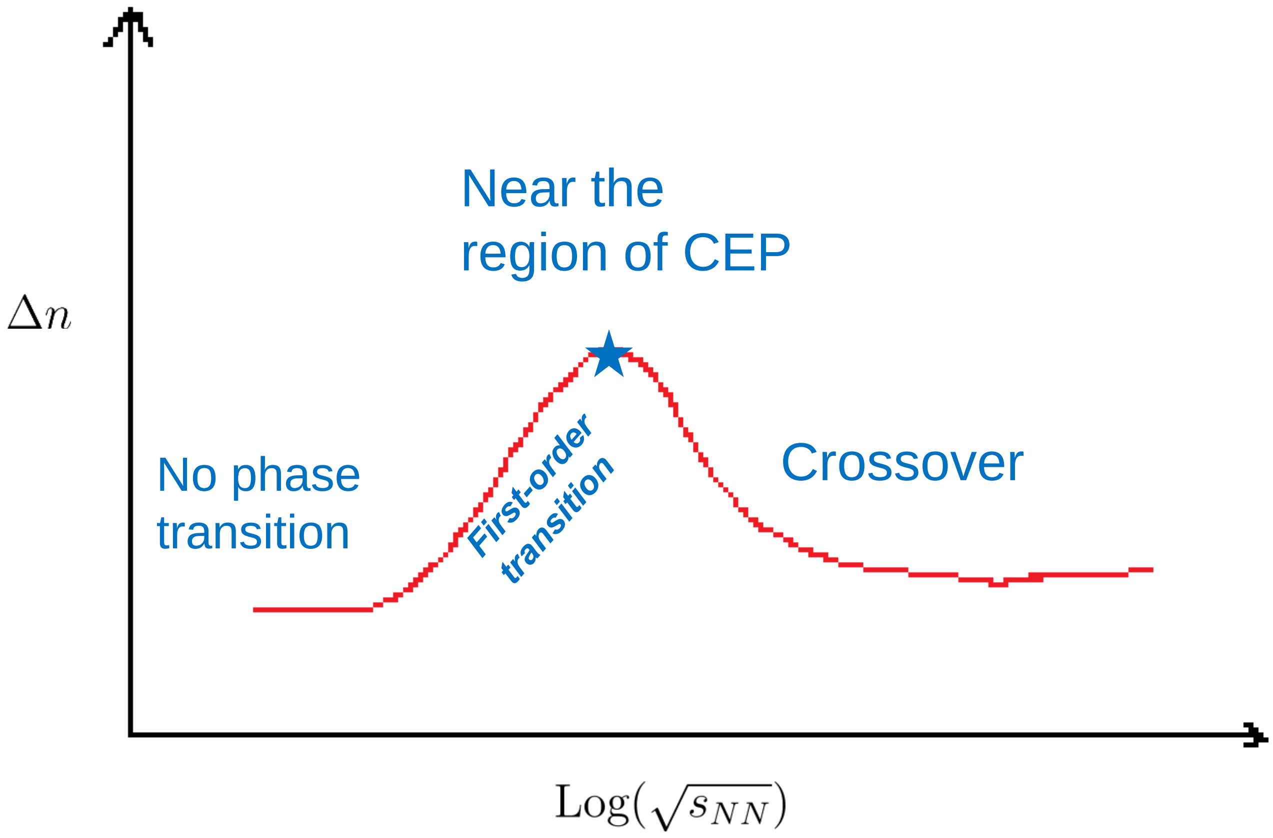
Naive expectation



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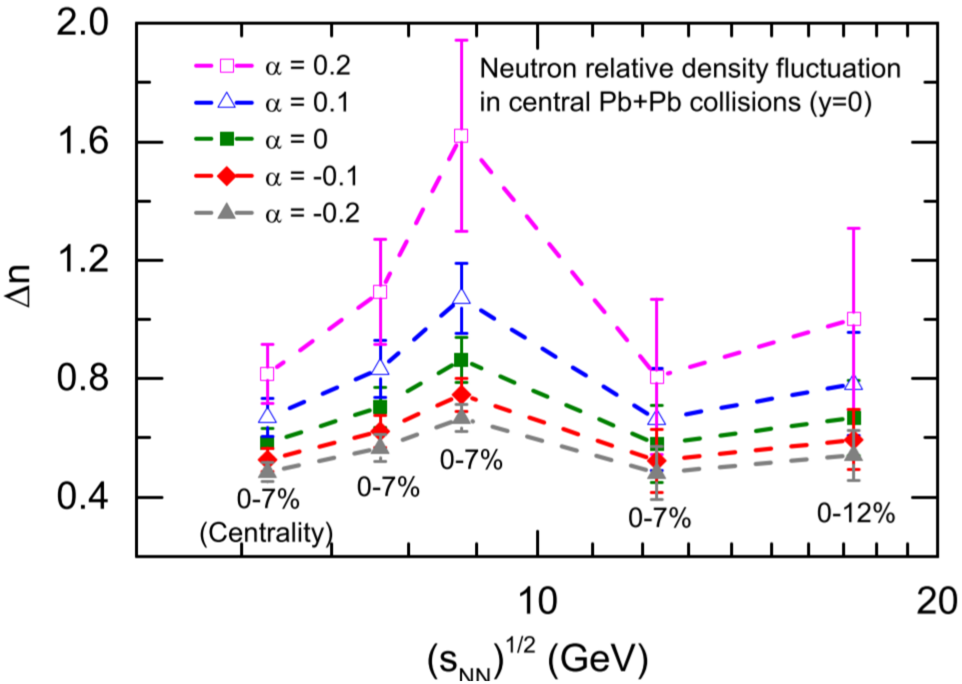
Peak structure



$$\mathcal{O}_{\text{p-d-t}} = \frac{N_{^3\text{H}} N_p}{N_{\text{d}}^2} = g\frac{1+(1+2\alpha)\Delta n}{(1+\alpha\Delta n)^2}$$

$$g = 4/9 \times (3/4)^{3/2} \approx 0.29$$

$$\Delta n = \frac{<(\delta n)^2>}{<n>^2}$$

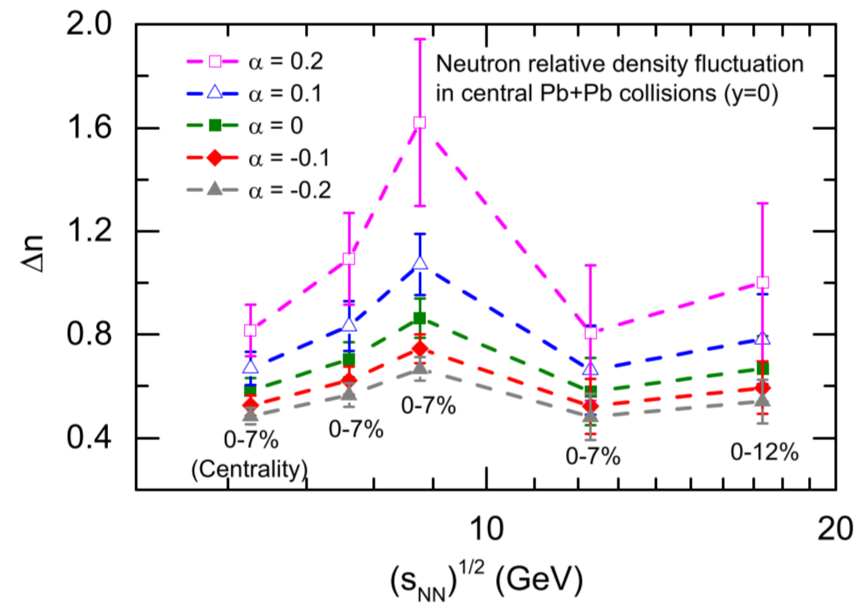
From NA49 Collaboration

T. Anticic et al. (NA49 Collaboration), Phys. Rev. C 94, 044906 (2016).

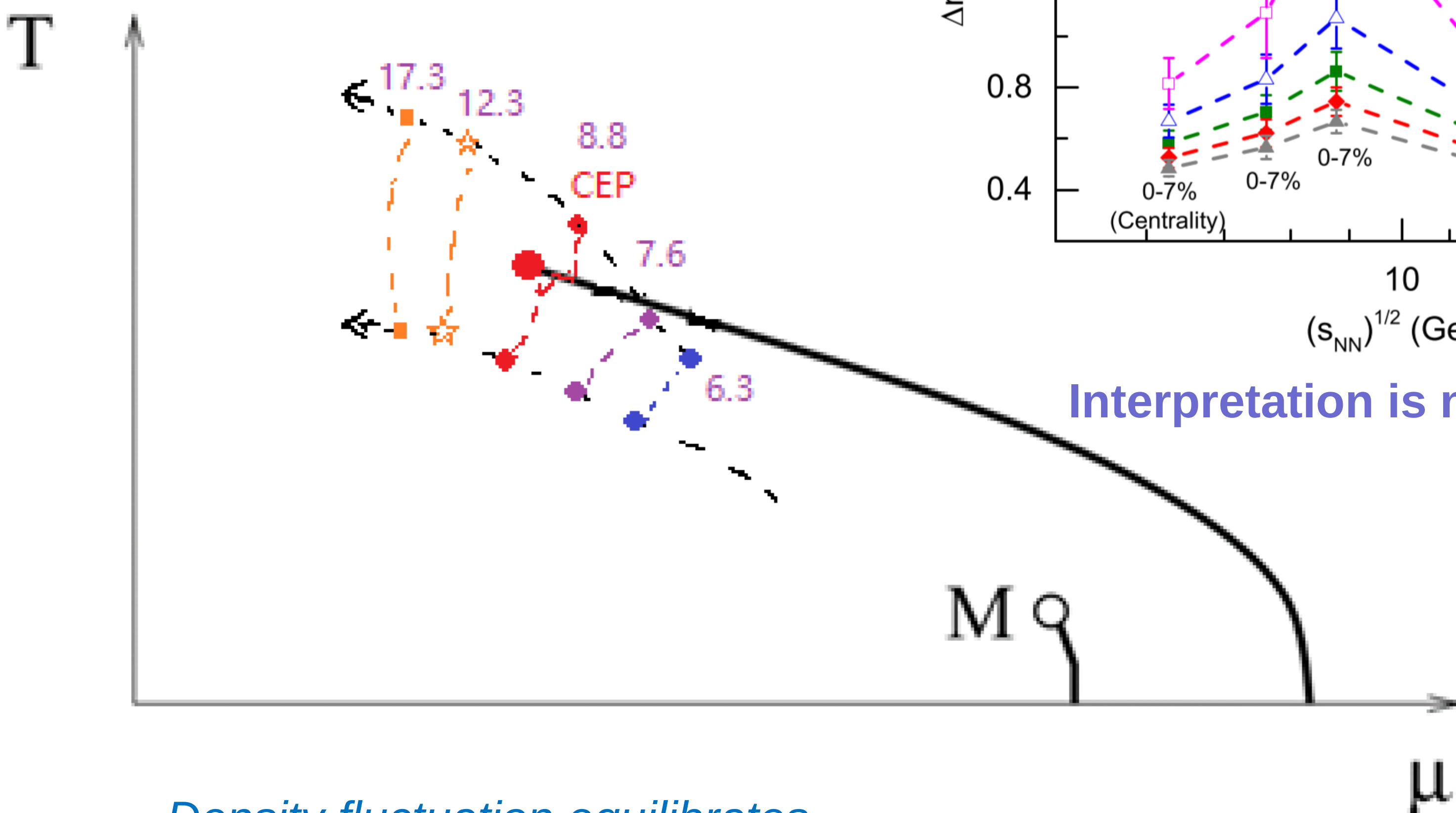
TABLE I: Yields (dN/dy at midrapidity) of p, d, ^{3}He and ^{3}H as well as the yield ratio ^{3}H/^{3}He measured in Pb+Pb collisions at SPS energies [47] together with the derived yield ratio $\mathcal{O}_{\text{p-d-t}}$. The units for E and $\sqrt{s_{NN}}$ are AGeV and GeV, respectively.

E	$\sqrt{s_{NN}}$	centrality	p	d	^{3}He	^{3}H/^{3}He	^{3}H	$\mathcal{O}_{\text{p-d-t}}$
20	6.3	0 – 7%	46.1±2.1	2.094±0.168	$3.58(\pm0.43)\times10^{-2}$	1.22±0.10	$4.37(\pm0.64)\times10^{-2}$	0.459±0.014
30	7.6	0 – 7%	42.1±2.0	1.379±0.111	$1.89(\pm0.23)\times10^{-2}$	1.18±0.11	$2.23(\pm0.34)\times10^{-2}$	0.494±0.020
40	8.8	0 – 7%	41.3±1.1	1.065±0.086	$1.28(\pm0.15)\times10^{-2}$	1.16±0.15	$1.48(\pm0.26)\times10^{-2}$	0.541±0.022
80	12.3	0 – 7%	30.1±1.0	0.543±0.044	$3.90(\pm0.50)\times10^{-3}$	1.15±0.19	$4.49(\pm0.94)\times10^{-3}$	0.458±0.038
158	17.3	0 – 12%	23.9±1.0	0.279±0.023	$1.50(\pm0.20)\times10^{-3}$	1.05±0.15	$1.58(\pm0.31)\times10^{-3}$	0.484±0.037

Peak structure and CEP

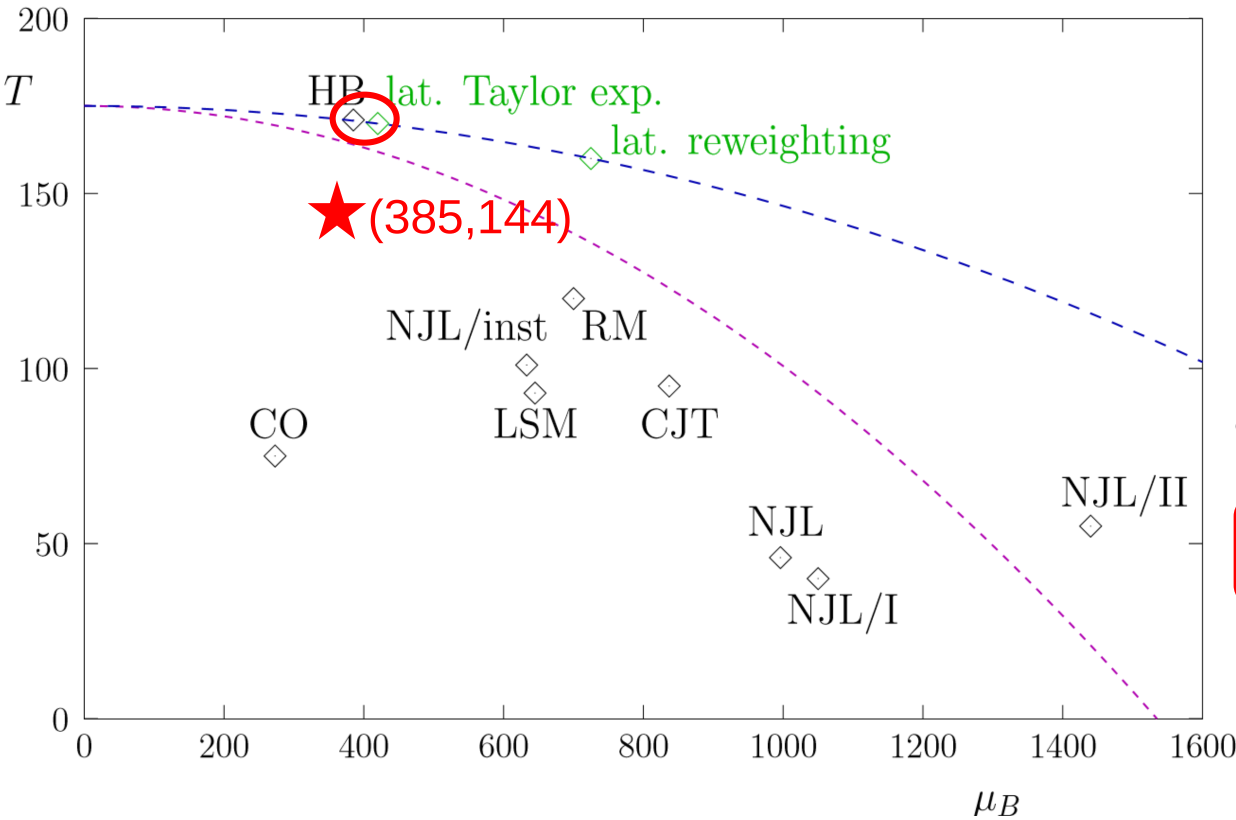


Interpretation is not unique



Density fluctuation equilibrates very slowly for expanding system

Estimation of location of CEP



$$T(\mu_B) = a - b\mu_B^2 - c\mu_B^4$$

$$\mu_B(\sqrt{s}) = \frac{d}{1 + e\sqrt{s}}$$

$a = 0.166 \pm 0.002$ GeV, $b = 0.139 \pm 0.016$ GeV^{-1}

$c = 0.053 \pm 0.021$ GeV^{-3}

$d = 1.308 \pm 0.028$ GeV and $e = 0.273 \pm 0.008$ GeV^{-1}

$\sqrt{s_{NN}} = 8.8$ GeV

$T \sim 144$ MeV and $\mu_B \sim 385$ MeV

TABLE I: Locations of the QCD critical point from Lattice QCD and DSE, respectively.

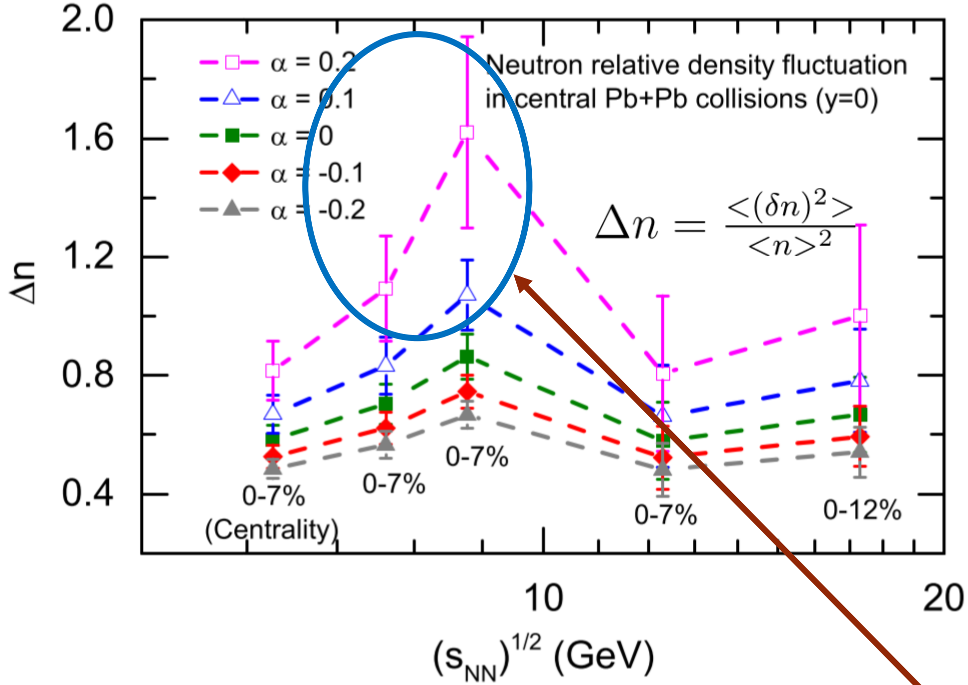
	Lattice			DSE		
(μ_B^E, T^E)	I [33]	II [34, 35]	III [36–39]	I [40]	II [41]	III [42]
MeV	(360,162)	(285,155)	$\mu_B^E/T^E > 2$	(372,129)	(405,127)	(504,115)

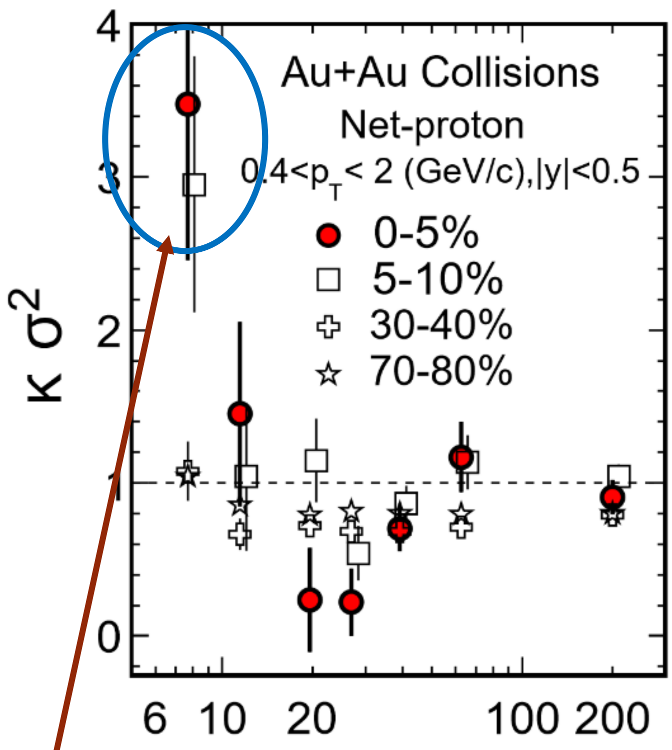
J. Cleymans et al, Phys. Rev. C 73, 034905 (2006).

M.A. Stephanov et al, Int.J.Mod.Phys. A20 (2005) 4387-4392.

K. J. Sun and L. W. Chen, Che Ming Ko, and Zhangbu Xu, Phys. Lett. B774, 103 (2017).

Nucleon cluster ?

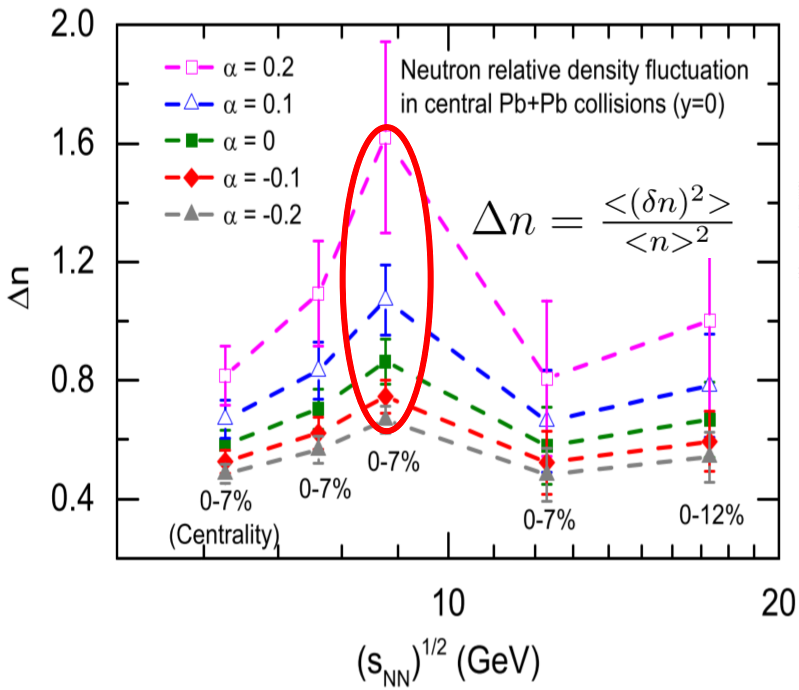


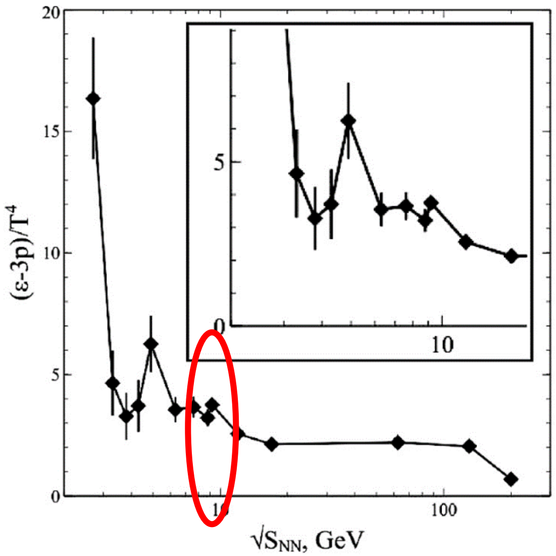


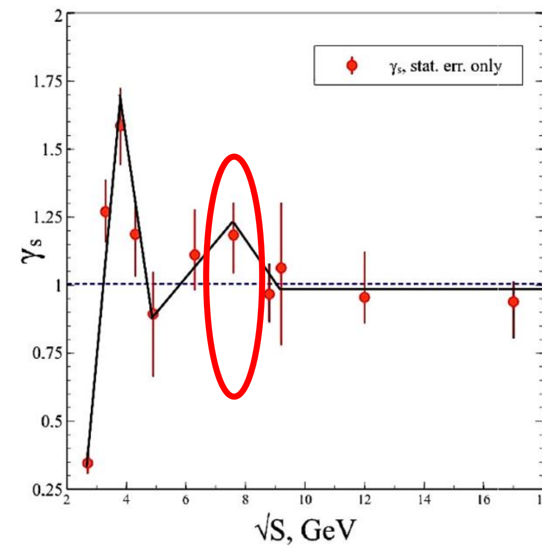
Nucleon cluster?

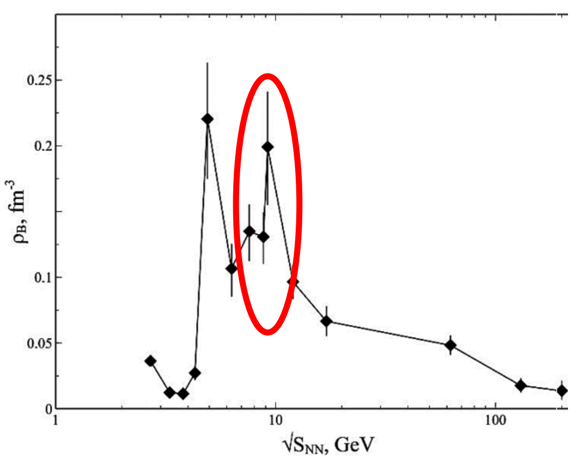
A. Bzdak, V. Koch and V. Skokov, Eur. Phys. J. C77 (2017) no.5, 288.

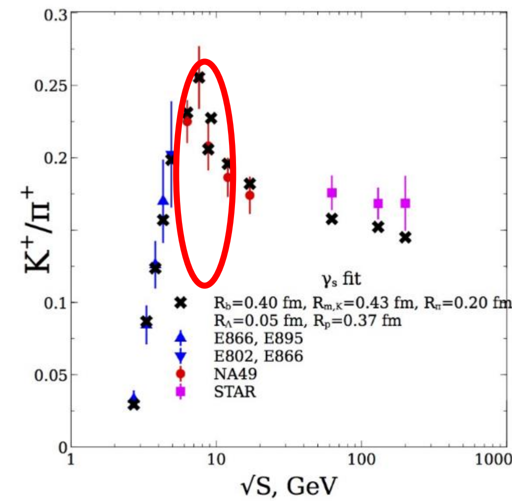
3-CEP ?











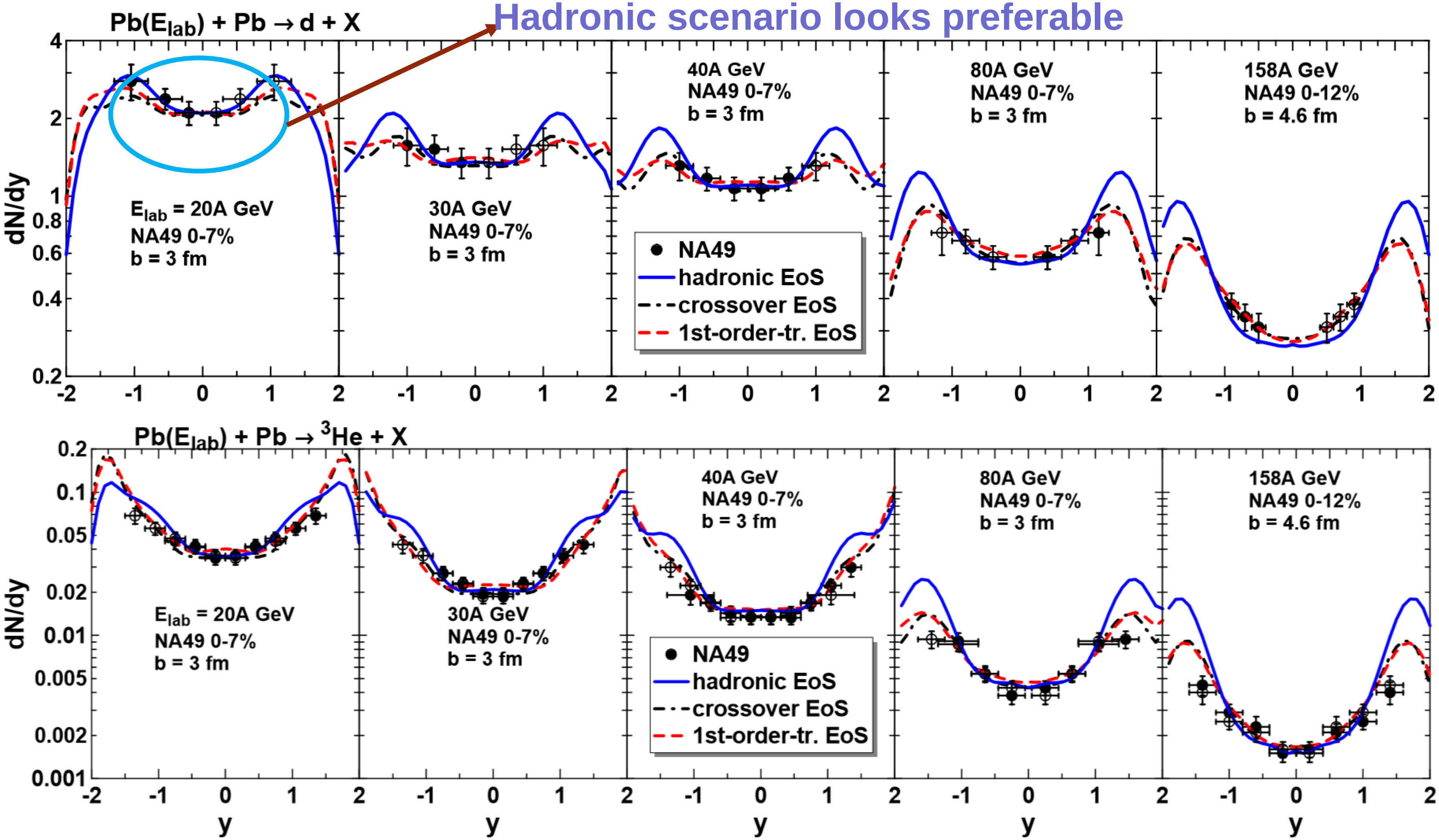
6-12 GeV is very interesting

K. A. Bugaev et al., arXiv: 1709. 05419 (2017).

Three-fluid dynamics

Three-fluid dynamics + coalescence

Hadronic scenario looks preferable



Y. B. Ivanov and A. A. Sodatov, arXiv: 1703. 05040 (2017).

Summary and outlook

1. Light nuclei provide a natural tool to probe the nucleon density fluctuations at kinetic freezeout. We have constructed a specific yield ratio $\frac{N_{^3\mathrm{H}} N_p}{N_\mathrm{d}^2}$ to do the job.

2. The extracted relative neutron density fluctuation in central Pb+Pb collisions at CERN SPS energies measured by NA49 collaboration exhibits a non-monotonic behavior with a peak at $\sqrt{s_{NN}} = 8.8\mathrm{GeV}$, suggesting that the CEP in the QCD phase diagram may have been reached in these collisions with its temperature and baryon chemical potential estimated to be $T^{\mathrm{CEP}} \sim 144\mathrm{MeV}$ and $\mu^{\mathrm{CEP}} \sim 385\mathrm{MeV}$.

3. Further investigations from experiments, such as the BES program at RHIC, and theoretical modeling of nucleus production and its connection to baryon density fluctuations are required.

Thank You !