Signals for critical end point with an *inhomogeneous phase* and *mesonic Gaussian fluctuations*

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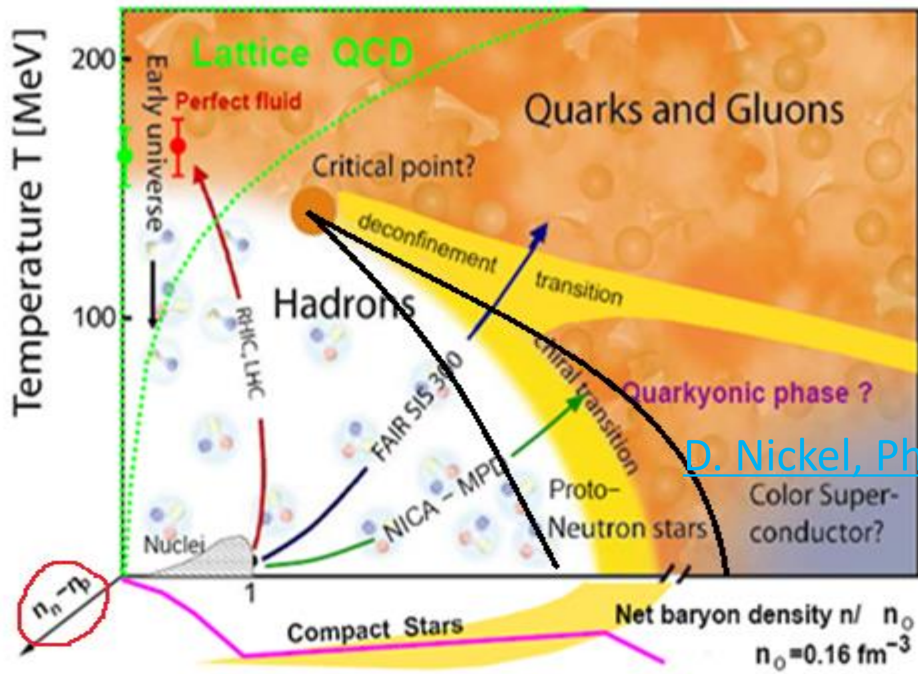
Outline

- Revisit $T-\mu$ phase diagram
- Mesonic Gaussian fluctuations
- NJL model with SM phase and MGF
- Numerical results and discussions
- Conclusions and prospective

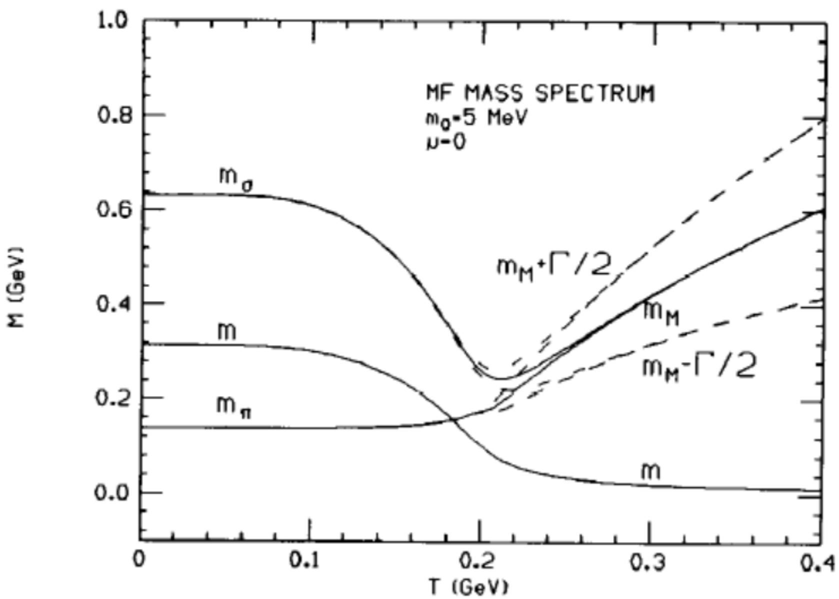
Revisit $T - \mu$ phase diagram of QCD

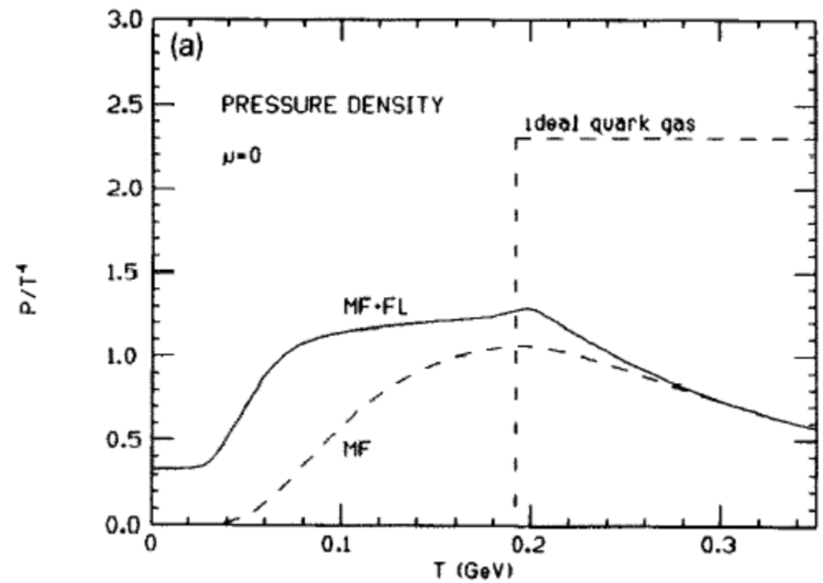


D. Nickel, Phys. Rev. D 80, 074025 (2009).

1) **Crossover at temperature $T_c = 155MeV$ from LQCD;**
2) **First-order transition at moderate chemical potential-might be covered by inhomogeneous solitonic modulation state;**
3) **Finite isospin chemical potential: neutron star and isomorphic with small μ_B at large N_c, no sign problem.**

MGF in QCD system

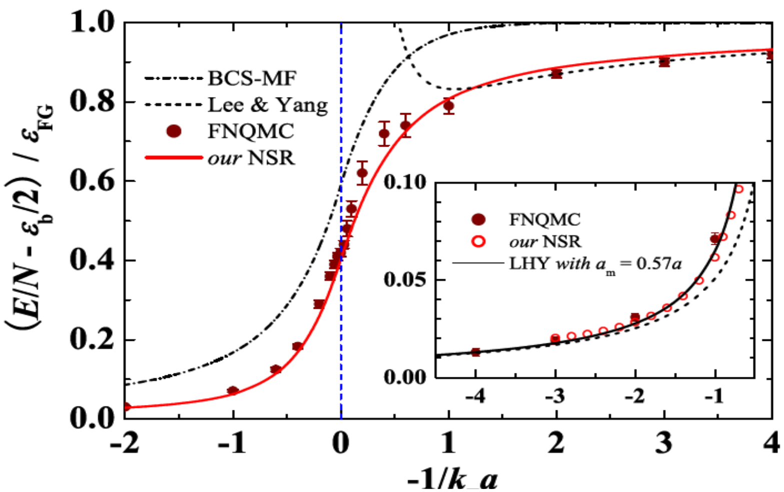


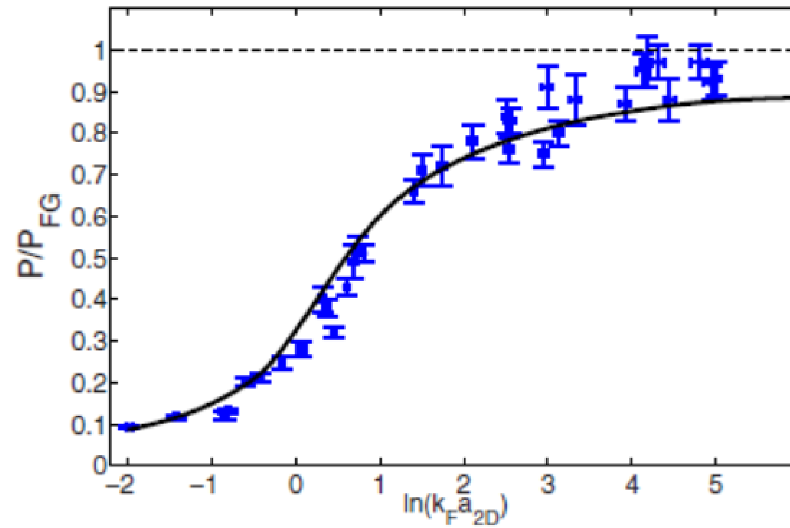


1) **The formalism is able to predict the instability of mesons;**
2) **The reduced pressure is closer to the ideal quark gas by including MGF than that in mean field approximation**

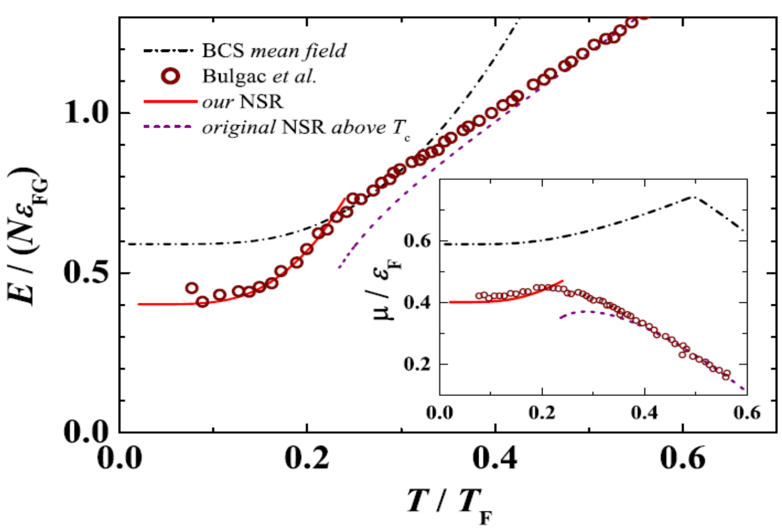
P. Zhuang, J. Hufner and S. P. Klevansky, Nucl. Phys. A 576, 525 (1994).

MGF in Cold atom system





[L. He, H. L, G. Cao, H. Hu and X. J. Liu, Phys. Rev. A 92, no. 2, 023620 (2015) .](#)



[H. Hu, X.-J. Liu, and P. D. Drummond, Europhys. Lett. 74, 574 (2006).](#)

1) At three and two dimensions, the EOSs were well reproduced;

2) The finite temperature results were nicely reproduced.

NJL model with SM phase

The Lagrangian density in Nambu—Jona-Lasinio model:

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m_0 - \mu\gamma^0)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\boldsymbol{\tau}\psi)^2\right]$$

By introducing auxiliary fields, the partition function becomes

$$\mathcal{Z} = \int [\mathcal{D}\sigma][\mathcal{D}\pi]\exp\left\{-i\left[\frac{1}{4G}\int d^4x(\sigma^2+\pi^2) + iN_c\mathrm{Tr}\ln(i\partial\!\!\!/ - m_0 - \sigma - i\gamma_5\pi\cdot\tau - \mu\gamma^0)\right]\right\}$$

Inhomogeneous ansatz： $\langle\sigma(x)\rangle = m_r(x) - m_0,\ \langle\pi^3(x)\rangle = m_i(x)$

The thermodynamic potential becomes：

$$\Omega = \frac{T}{V}\left[-N_c\mathrm{Tr}\ln\left(i\partial\!\!\!/ - \frac{1+\gamma^5\tau^3}{2}m(x) - \frac{1-\gamma^5\tau^3}{2}m^*(x) - \mu\gamma^0\right) + \frac{1}{4G}\int d^4x|m(x)-m_0|^2\right]$$

NJL model with SM phase

Solitonic modulation along z direction

$$M_{\rm SM} = m\left[\nu\,\mathrm{sn}(b|\nu)\,\mathrm{sn}(mz|\nu)\,\mathrm{sn}(mz+b|\nu) + \frac{\mathrm{cn}(b|\nu)\,\mathrm{dn}(b|\nu)}{\mathrm{sn}(b|\nu)}\right]$$

sn, cn, dn **are Jacobi elliptic functions with** $b \in (0, \mathbf{K}(\nu))$

and elliptic modulus $\sqrt{\nu} \in (0, 1)$

The explicit form of the thermodynamic potential with Pauli-Villars regularization:

$$\Omega_{\rm SM} = \frac{\langle (M_{\rm SM}-m_0)^2\rangle}{4G} - \frac{2N_c}{\pi}\int_0^\infty p\,dp\int_0^\infty d\varepsilon\;\rho_{\rm SM}(\varepsilon;m,\nu)\,F_{\rm PV}\Big(T,\mu,p,(\varepsilon+\beta\,m^2)^{1/2}\Big)$$

DOS: $\rho_{\rm SM}(\varepsilon;m,\nu) = \dfrac{1}{\pi}\dfrac{\varepsilon^2 - m^2\mathbf{E}(\nu)/\mathbf{K}(\nu)}{\sqrt{(\varepsilon^2-m^2)(\varepsilon^2-(1-\nu)m^2)}}\Big[\theta(\varepsilon^2-m^2) - \theta\Big(-\varepsilon^2+(1-\nu)m^2\Big)\Big]$

NJL model with SM phase

PV regularized function:

$$F_{\mathrm{PV}}(T,\mu,p,\varepsilon)=\sum_{i=0}^{3} C_{\mathrm{i}}\ \epsilon(p,\varepsilon_i)+T\sum_{s=\pm}\ln\left(1+e^{-(\epsilon(p,\varepsilon)-s\mu)/T}\right) \qquad \varepsilon_i = \sqrt{\varepsilon^2+i\Lambda^2}$$

The advantage of PV regularization：

The **chiral symmetry breaking** phase can be reproduced by taking $\boldsymbol{\nu} \to \mathbf{1}, \boldsymbol{b} \to \infty$;
the **chiral symmetry restoration** phase can be reproduced by taking $\boldsymbol{\nu} \to \mathbf{0}$ in both chiral limit $\mathrm{m}_0 = 0$ and **physical case $\mathbf{m}_0 \neq \mathbf{0}$.**

The MGF in homogeneous phase

The MGF contribution to thermodynamic potential:

$$\Omega_M = N_M \int \frac{d^3p}{(2\pi)^3} \int_{-p^2}^{\infty} ds \left[\frac{1}{2}\sqrt{s+p^2} + \frac{1}{\beta}\ln\left(1-\exp\left(-\beta\sqrt{s+p^2}\right)\right)\right] D_M(s),$$

P. Zhuang, J. Hufner and S. P. Klevansky, Nucl. Phys. A 576, 525 (1994).

$$D_M(s) = \frac{1}{\pi}\frac{d}{ds}(\phi + \phi_M)$$

The phases

$$\tan(\phi) = -\frac{I^{i}_{PV}(s)}{I^{r}_{PV}(s)}\theta(s-4m^2), \quad \tan(\phi_M) = \frac{\frac{m_0 I^{i}_{PV}(s)}{2mG}\theta(s-4m^2)}{(s-\varepsilon_M^2)|I_{PV}(s)|^2 + \frac{m_0 I^{r}_{PV}(s)}{2mG}},$$

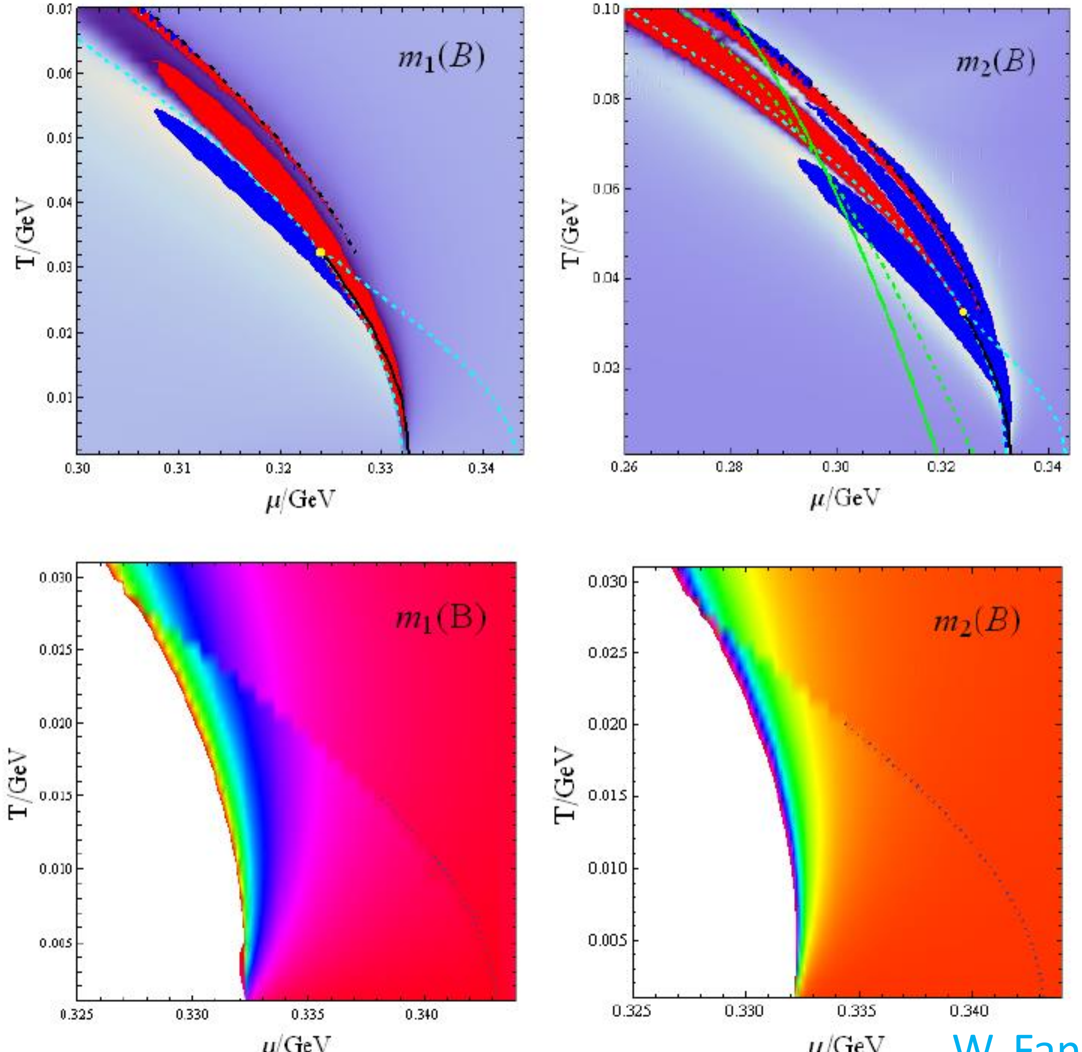
The real and imaginary parts

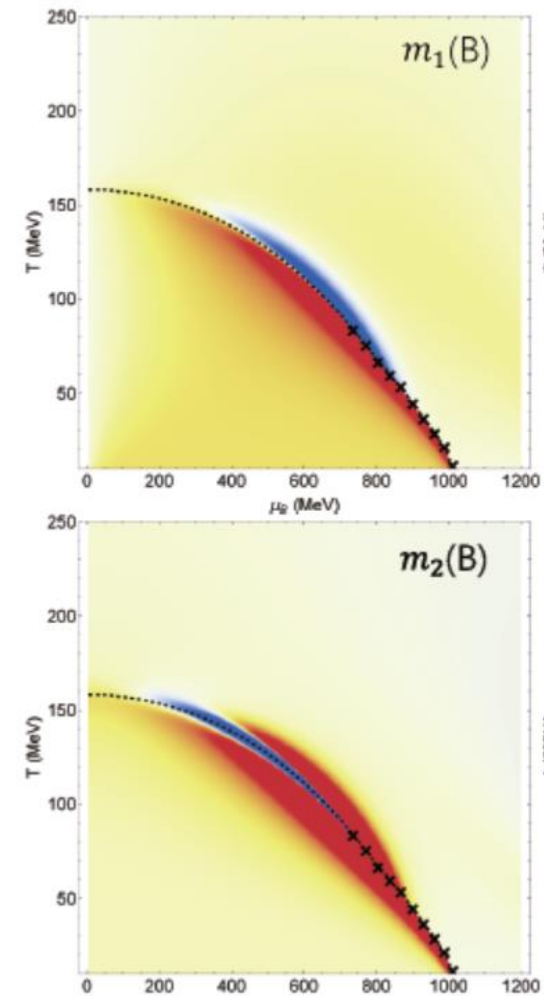
$$I^{r}_{PV}(s) = \frac{N_f}{2}\Bigg\{N_c\int\frac{d^3q}{(2\pi)^3}\sum_{t=\pm}\frac{f(E_q - t\mu)}{E_q^2(\sqrt{s}+2E_q)} + \frac{N_c}{4\pi^2}\int_0^{\frac{\sqrt{s}}{2}-m}dx\sum_{t,t'=\pm}\frac{\left[\left(tx+\frac{\sqrt{s}}{2}\right)^2-m^2\right]^{1/2}}{tx\left(tx+\frac{\sqrt{s}}{2}\right)}f\left(tx+\frac{\sqrt{s}}{2}-t'\mu\right) + \frac{N_c}{4\pi^2}\int_{\frac{\sqrt{s}}{2}-m}^{\infty}dx$$
$$\sum_{t=\pm}\frac{\left[\left(x+\frac{\sqrt{s}}{2}\right)^2-m^2\right]^{1/2}}{x\left(x+\frac{\sqrt{s}}{2}\right)}f\left(x+\frac{\sqrt{s}}{2}-t\mu\right) + \frac{N_c}{2\pi^2}\sum_{i=0}^{3}C_i\,\Re\left(\sqrt{\frac{s_i}{s}}\,\mathrm{arccoth}\sqrt{\frac{s_i}{s}}+\log M_i\right)\Bigg\},$$

$$I^{i}_{PV}(s) = \frac{N_f}{8\pi}N_c\left\{\sqrt{\frac{s_0}{s}}\sum_{t=\pm}f\left(\frac{\sqrt{s}}{2}-t\mu\right) - \sum_{i=0}^{3}C_i\sqrt{\frac{s_i}{s}}\,\theta(s_i)\right\}$$

regularized parts

Numerical results: higher moments



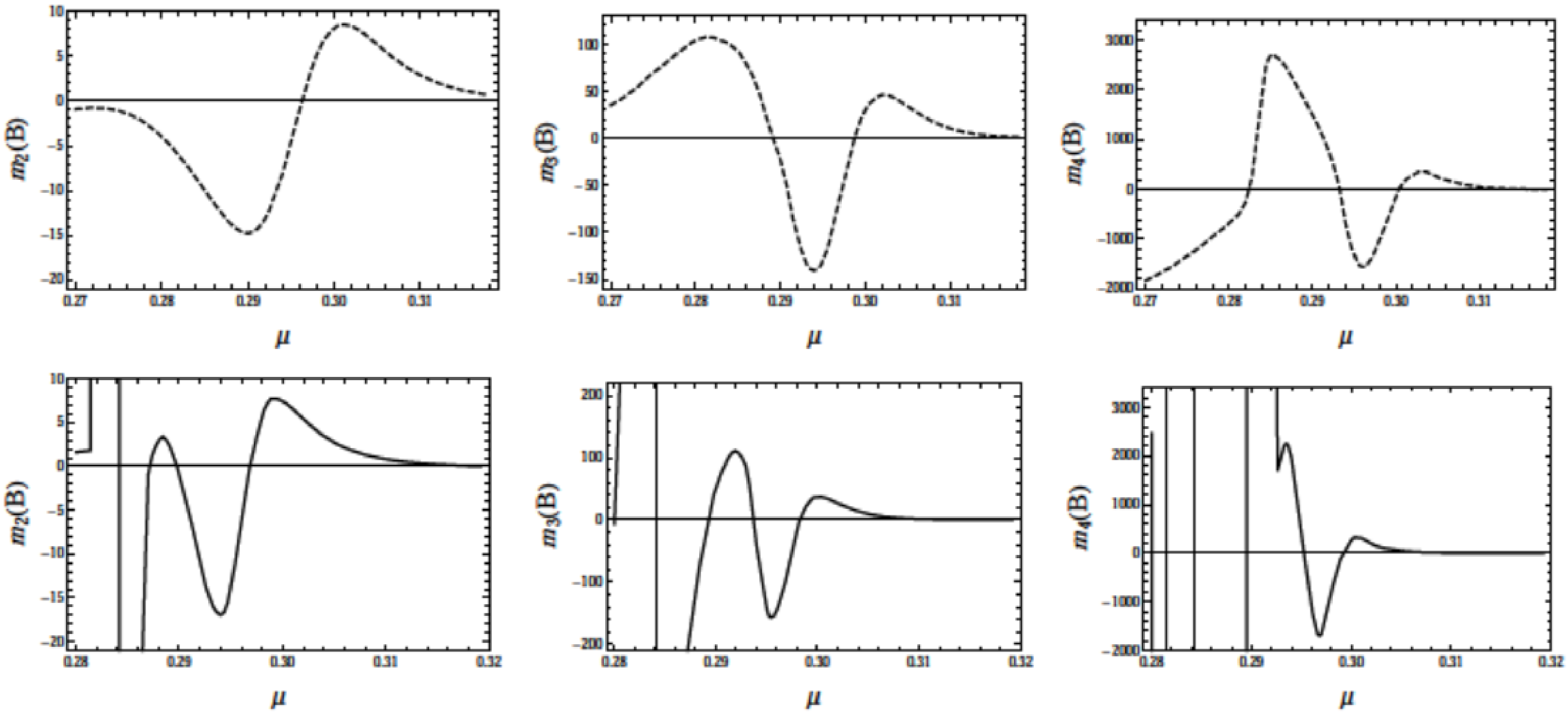


W. Fan, X. Luo and H. S. Zong, Int. J. Mod. Phys. A 32, no. 11, 1750061 (2017) .

Numerical results: high moments

1) The calculation precision is low for high temperature;
2) extra highly fluctuated regions show up – instability of pions, but probably unphysical;
3) Mean field results are almost not affected by MGF;
3) First order and second order transitions are more distinguishable for lower moments -- hard in experiments

Numerical results: along freeze-out processes



For the one crosses pion instable region (lower panel), extra larger fluctuations in the low chemical potential region

Conclusions and perspective

- The signals for CEP is restudied in NJL model by taking the inhomogeneous SM phase and MGF contribution into account;
- Except for the emerging of unphysical pion instable region with high fluctuations, the mean field results are almost not affected by MGF;
- The first order and second order transitions can be distinguished in the lower moments in principal;
- The work can be extended to three flavor case and finite isospin chemical potential can be considered

Thank you very much!