Primordial non-Gaussianity with CMB spectral distortions: trispectrum and scale dependence

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Based on:
Squeezed bispectra generate couplings between large and small scales i.e., large scale modulation of small scale power.

This modulation couples CMB temperature fluctuations on large scales to spectral distortions arising from acoustic wave dissipation at very small scales.

- Use $T_\mu$ to measure local $f_{NL}$ (Pajer and Zaldarriaga 2013)
- Further constraining power on $f_{NL}$ running (Biagetti et al. 2013, Emami et al. 2015)
- Large S/N increase for “super-squeezed “ shapes (Ganc and Komatsu 2013)

What can we say about trispectra?
Trispectrum

\[ \langle \Phi(k_1)\Phi(k_2)\Phi(k_3)\Phi(k_4) \rangle \propto F(k_1,k_2,k_3,k_4,K_{12},K_{23}) \delta(k_1 + k_2 + k_3 + k_4) \]

Diagonal squeezed trispectrum: \( \tau_{NL} \)

\[ \tau_{NL} < 2800 \quad (95\% \ C.L.) \]

Leg squeezed trispectrum: \( g_{NL} \)

\[ g_{NL}^{local} = (-9.0 \pm 7.7) \times 10^{-4} \]
Trispectrum and spectral distortions

- The $\mu\mu$ spectrum can be used to measure small scale modulation of power due to a squeezed diagonal trispectrum. Sensitivity to $\tau_{\text{NL}}$ (flat $\mu\mu$ spectrum in the Gaussian, stationary case)

- $T(T\mu)$ can measure large scale modulations of the bispectrum -> sensitivity also to $g_{\text{NL}}$

\[
\zeta(x) = \zeta^G(x) + \frac{9}{25} g_{\text{NL}} (\zeta^G(x))^3
\]

\[
\zeta(x) = \zeta_S(x) + \zeta_L(x) = \zeta^G_S(x) + \zeta^G_L(x) + \frac{2L}{2^5} g_{\text{NL}} \zeta^G_S(x) (\zeta^G_L(x))^2
\]

\[
\zeta_S(x) = \zeta^G_S(x) \left[ 1 + \frac{27}{25} g_{\text{NL}} (\zeta^G_L(x))^2 \right]
\]

\[
\frac{\delta \langle \zeta^2 \rangle}{\langle \zeta^2 \rangle} \approx \frac{\delta \mu}{\mu} \approx \frac{54}{25} g_{\text{NL}} (\zeta^G_L(x))^2
\]

\[
\left\langle \frac{\delta T_1}{T} \frac{\delta T_2}{T} \frac{\delta \mu_3}{\mu} \right\rangle \approx \frac{54}{25} g_{\text{NL}} \left\langle \frac{\zeta_1 \zeta_2}{5} (\zeta^G_{L_3})^2 \right\rangle = 108 g_{\text{NL}} \left\langle \frac{\delta T_1}{T} \frac{\delta T_3}{T} \right\rangle \left\langle \frac{\delta T_2}{T} \frac{\delta T_3}{T} \right\rangle
\]

\[
\langle \mu \rangle \approx (9/4) A_S \ln(k_i/k_f) \Rightarrow b_{T1T2\ell_3}^{TT} \approx 108 g_{\text{NL}} \frac{9}{4} A_S \ln \left( \frac{k_i}{k_f} \right) C^TT_{\ell_1} C^TT_{\ell_2}
\]
Same approach can be applied to $\tau_{\text{NL}}$

$$\zeta(x) = \zeta^G(x) + \sqrt{\tau_{\text{NL}}} \sigma(x) \zeta^G(x)$$

$$\frac{\delta \langle \zeta^2 \rangle}{\langle \zeta^2 \rangle} \simeq \frac{\delta \mu}{\mu} \simeq 2 \sqrt{\tau_{\text{NL}}} \sigma(x)$$

$$\left\langle \frac{\delta T_1}{T} \frac{\delta T_2}{T} \frac{\delta \mu_3}{\mu} \right\rangle \simeq 2 \left\langle \frac{\zeta_1^G}{5} \frac{\zeta_2^G}{5} \left[1 + \sqrt{\tau_{\text{NL}}} (\sigma_1 + \sigma_2)\right] \sqrt{\tau_{\text{NL}}} \sigma_3 \right\rangle$$

$$b^{TT\mu}_{\ell_1 \ell_2 \ell_3} \simeq 50 \tau_{\text{NL}} \frac{9}{4} A_S \ln \left( \frac{k_i}{k_f} \right) \left[ C^{TT}_{\ell_1} + C^{TT}_{\ell_2} \right] C^{TT}_{\ell_3}$$

Full calculation is cumbersome. Formulae above reproduce quite accurately The exact behavior
$b_{\ell_1 \ell_2 \ell_3}^{TT\mu}$ bispectrum, $g_{NL}$ contributions

\[
b_{\ell_1 \ell_2 \ell_3}^{TT\mu} \approx \frac{54}{25} g_{NL} \sum_{L_1 L_2} \frac{h_{L_1 L_2 \ell_3}^2}{2 \ell_3 + 1} \int_0^\infty r^2 \, dr \nonumber \]

\[
\left[ \beta_{\ell_1}^T (r) \beta_{\ell_2}^T (r) \beta_{L_1}^\mu (r, z) \alpha_{L_2}^\mu (r, z) + \beta_{\ell_1}^T (r) \beta_{L_2}^\mu (r, z) \right] \nonumber \]

\[
\left. + \beta_{\ell_1}^T (r) \alpha_{L_2}^\mu (r, z) \beta_{L_2}^\mu (r, z) \right. \nonumber \]

\[
\left. + \alpha_{\ell_1}^T (r) \beta_{\ell_2}^T (r) \beta_{L_2}^\mu (r, z) \beta_{L_2}^\mu (r, z) \right] \nonumber \]

In SW limit, using asymptotic approximations for integral of product of $j_i$ and for Wigner symbols, we recover previous result, modulo a pre-factor.

\[
\alpha^T_\ell (r) \equiv \frac{2}{\pi} \int_0^\infty k^2 dk T_\ell (k) j_\ell (kr) , \nonumber \]

\[
\alpha^\mu_\ell (r, z) \equiv \frac{3}{\pi} \int_0^\infty k^2 dk j_\ell (k x s) j_\ell (kr) e^{-k^2 / k_D^2 (z)} , \nonumber \]

\[
\beta^T_\ell (r) \equiv \frac{2}{\pi} \int_0^\infty k^2 dk P (k) T_\ell (k) j_\ell (kr) , \nonumber \]

\[
\beta^\mu_\ell (r, z) \equiv \frac{3}{\pi} \int_0^\infty k^2 dk P (k) j_\ell (k x s) j_\ell (kr) e^{-k^2 / k_D^2 (z)} , \nonumber \]

\[
b_{\ell_1 \ell_2 \ell_3}^{TT\mu,SW} \approx \frac{972}{\pi} g_{NL} A_s \ln \left( \frac{k_i}{k_f} \right) C_{\ell_1,SW}^{TT} C_{\ell_2,SW}^{TT} \nonumber \]
Forecasts

Simple Fisher matrix forecasts (assuming perfect component separation)

\[ F_{TT\mu} = \sum_{\ell_1\ell_2\ell_3} \left( \frac{h_{\ell_1\ell_2\ell_3} \hat{b}_{TT\mu}}{2C_{\ell_1}^{TT} C_{\ell_2}^{TT} C_{\ell_3}^{\mu\mu}} \right)^2 \]

- Fiducial \( f_{NL} = 0 \). No \( T\mu \) terms in the variance
- \( C_{\mu\mu} \) depends on \( \tau_{NL} \). \( C_{\mu\mu,\tau_{NL}} \sim (5 \times 10^{-23} \tau_{NL} \, \text{l}^{-2}) \) dominates over Gaussian part, \( C_{\mu\mu,G} \sim 10^{-30} \), if \( \tau_{NL} \) does not vanish
- We fix several values of \( \tau_{NL} \) and compute \( \Delta g_{NL} \) in each case
- S/N scaling, from flat sky approximation

\[ \Delta g_{NL} |_{\tau_{NL}=0} \sim \left( \frac{C_{\mu\mu,G}}{10^{-30}} \right)^{1/2} \left[ \ln \left( \frac{\ell_{\text{max}}}{2} \right) \right]^{-1} \]
Expected errors on $g_{NL}$ (top panel) and $\tau_{NL}$ (bottom panel) estimated from $TT_{\mu}$ (colored lines) and $TT_{TT}$ (black lines) in the cosmic-variance dominated case (i.e., $N_{\mu \mu} = 0$). Solid and dashed lines are the full radiation transfer case (Eqs. (3.25) and (3.31) for $TT_{\mu}$) and the SW case (Eqs. (3.26) and (3.34) for $TT_{\mu}$), respectively. In the $TT_{\mu}$ cases, we consider several nonzero $\tau_{NL}$'s with $f_{NL} = 0$.

For $\tau_{NL} = 0$, $g_{NL}$ and $\tau_{NL}$ obtained from $TT_{\mu}$ scale like $1/\ln(l_{\max}/2)$ and $1/l_{\max}$, respectively (see Eqs. (4.4) and (4.7)). It is apparent that, if $\tau_{NL} \lesssim 1000$, for $l_{\max} \lesssim 1000$, $TT_{\mu}$ always outperforms $TT_{TT}$, because $C_{\mu \mu}, G_{l_{\max}} + C_{\mu \mu}, \tau_{NL} \tau_{C_{TT}}$. At larger $l_{\max}$, $TT_{\mu}$ remains clearly superior to $TT_{TT}$ for $g_{NL}$ measurements. For $\tau_{NL}$ estimation the comparison is instead dependent on the fiducial value of $\tau_{NL}$; see main text for further discussion.
Figure 3. Expected errors on $g_{NL}$ computed from $TT_{\mu}$ (colored lines) for noise-levels representative of Planck, PIXIE and CMBpol. For comparison, we also plot the errors computed from $TTTT$ (black lines) for a noiseless CMB survey, which are almost the same as the errors obtained in the Planck temperature data analysis [5, 6]. Solid and dashed lines correspond to the results including full CMB transfer function (Eqs. (3.25) and (3.31) for $TT_{\mu}$) and those in the SW limit (Eqs. (3.26) and (3.34)) for $TT_{\mu}$, respectively. We here assume $f_{NL} = \Delta f_{NL} = 0$. For $l_{\mu}$, the scaling agrees with expectations from Eqs. (4.1) and (4.4):

$$g_{NL} = \left( N_{\mu} / 10^{30} \right)^{1/2} \ln \left( l_{\mu} / 2 \right).$$

At large $l_{\mu}$, when $N_{\mu}$ starts dominating, the $TT_{\mu}$ sensitivity falls below $TTTT$. $f_{NL}$ deviates drastically from $1 / \ln \left( l_{\mu} / 2 \right)$, because of non-negligible contributions of $C_{\mu\mu}, f_{NL}$ to the denominator of the Fisher matrix. For comparison, in Fig. 2, we also plot our expected uncertainties estimated in a noiseless, cosmic-variance dominated measurement of the CMB temperature trispectrum ($TTTT$), which agree with results in previous literature [54–58]. This level of sensitivity is essentially already achieved using current Planck data [5, 6]. As shown in this figure, since the cosmic variance uncertainty for $\mu$-distortions is smaller than that for temperature anisotropies (i.e., $C_{\mu\mu}, G_{\mu}$ + $C_{\mu\mu}, f_{NL}$), $TT_{\mu}$ allows to achieve better sensitivity to both $g_{NL}$ and $f_{NL}$ than $TTTT$ does, for $l_{\mu} \approx 1000$. However, given the difference in scaling with $l_{\mu}$ of the two quantities – i.e., $f_{NL} / l_{\mu}^{2}$ vs. $f_{NL} / l_{\mu}^{4}$ [54] – $TTTT$ might become better than $TT_{\mu}$ at measuring $f_{NL}$ for higher $l_{\mu}$ and large values of $f_{NL}$.

4.2 Effects of experimental uncertainties

Besides the ideal, cosmic-variance dominated case, we consider also several different noise levels, corresponding to experiments like Planck [59], PIXIE [60] and CMBpol [61]. For $\mu$-$\mu$ noise spectra, we assume $N_{\mu\mu} = N_{\mu} \exp \left( l_{\mu} / 2 l_{\mu} \right)$, with $(N_{\mu}, l_{\mu}) = (10^{15}, 861)$ (Planck), (10$^{17}$, 84) (PIXIE) and (2$\times$10$^{18}$, 1000) (CMBpol) [28, 32]. As it is typical for this type of analysis, we see that current and forthcoming surveys, such as Planck and PIXIE, are expected to produce error bars on relevant NG parameters which are much worse than what is achievable with the current Planck measurements or cosmic-variance dominated CMB measurements.
Figure 4. Expected errors on $g_{NL}$ (top panel) and $\tau_{NL}$ (bottom panel) estimated from $TT\mu$ (colored lines) at $l_{\text{max}} = 1000$, a function of the magnitude of instrument noise $N\mu$, keeping $\mu = 1000$ angular resolution fixed. Black lines show the expected errors, at $l_{\text{max}} = 2000$, obtained from $TTTT$ in a noiseless CMB measurement, very close to the error bars obtained from the Planck temperature data \cite{5,6}. In the $TT\mu$ cases, we consider several nonzero $\tau_{NL}$'s with $f_{NL} = 0$. The $TT\mu$ bispectrum used in this estimation is computed from Eqs. (3.25) and (3.31), including the full CMB transfer function dependence.

If we focus on $g_{NL}$, and consider the fiducial case $\tau_{NL} = 0$ (resulting in $C_{\mu\mu} = C_{\mu\mu} + N_{\mu\mu}$), we find that Planck can achieve a level of...
Testing $f_{NL}$ running with $\mu$ and $y$ distortions

- Can use $T_\mu$ and $T_y$ to measure $f_{NL}$ at different scales and test running.
- Main problem: SZ effect generates large $T_y$ (bias) and $yy$ (noise) (Emami et al. 2015, Dimastrogiovanni & Emami 2016, Creque-Sarbinowski, Bird, Kamionkowski 2016)
Remove bias: $yE$ correlation

- SZ-CMB temperature correlation is due to late ISW.
- $yE$ does not contain this spurious, non-primordial signal.
Mitigate yy noise from SZ

- Mask resolved clusters, exploiting external datasets (e.g. e-Rosita)
- For diffuse yy component from unresolved cluster: exploit cross-correlation with external tracers (lensing) to build a SZ yy-map template, and subtract
  - Use halo model to calculate SZ power spectrum
  - Use halo model to calculate Lensing-SZ cross-correlation
  - Consider ML estimator of SZ yy-map, given observed lensing map

\[
p(a_{\ell m}^{\text{SZ}}|t_{\ell m}) = \mathcal{N}(C^T_{\ell}B^{-1}_{\ell}d_{\ell m}, C^{\text{SZSZ}}_{\ell} - C^T_{\ell}B^{-1}_{\ell}C_{\ell}) \Rightarrow \hat{a}_{\ell m}^{\text{SZ}} = C^T_{\ell}B^{-1}_{\ell}d_{\ell m}
\]

- Subtract

\[
a_{\ell m}^{\text{clean}} = a_{\ell m}^{\text{obs}} - \hat{a}_{\ell m}^{\text{SZ}}
\]

\[
p(a_{\ell m}^{\text{clean}}|t_{\ell m}) = \mathcal{N}(-C^T_{\ell}B^{-1}_{\ell}t_{\ell m}, \text{Var}(a_{\ell m}^{\text{obs}}) + \text{Var}(\hat{a}_{\ell m}^{\text{SZ}}) - 2\text{Cov}(a_{\ell m}^{\text{obs}}, \hat{a}_{\ell m}^{\text{SZ}}))
\]
Non-linear kinetic SZ

- After reionization, free electrons source $y$-distortions. Non-linear kinetic SZ due to bulk motion of free electrons. Proportional to $v_b^2$

- Correlation with polarization sourced at reionization

- Leading order $\sim$ trispectrum

$$y_{\text{reio}} E = y_{\text{reio}} E^{(1)} + y_{\text{reio}} E^{(2)} + \ldots$$

$$y_{\text{reio}}^{(2)} - E^{(2)} = (-1)^{\ell'-m'} 64\pi \int \frac{q_1^2 dq_1 q_2^2 dq_2}{(2\pi)^3} \int \frac{k_1^2 dk_1}{(2\pi)^3} P(q_1) P(q_2) \delta^{\ell'} \delta_{m'}$$

$$+ \sum_{L_1} \sum_{L_1} \sum_{m_1} \frac{1}{(2\pi)^3} T_X^{(2)} (q_1, q_2, k_1) \frac{11\pi}{45} I_{\ell', m_1}^{(2)} (q_1, q_2, k_1) (-1)^{L_1+1} L_1^{L+1} L_1^{L+1} (-1)^{3m_1}$$

$$\left( \begin{array}{ccc} L_1 & 1 & L \\ 0 & 0 & 0 \end{array} \right) \left( \begin{array}{ccc} L_1 & 1 & L \\ n & -n & 0 \end{array} \right) \left( \begin{array}{ccc} |m_1| & 1 \\ -n & m_1 & -m_1 + n \end{array} \right)$$

$$3(2L + 1)(2L_1 + 1) \frac{\alpha_{n,m}}{4\pi} \int x^2 dx j_L (xk_1) j_L (xq_1) j_1 (xq_2) \right].$$

- We computed the correlation numerically. 2nd order $E$ transfer functions computed with SONG (Pettinari et al. 2014)
Andrea Ravenni (Unipd & INFN)
Early Universe physics with CMB spectral distortions
ASI/COSMOS Padova, 22/2/18

Signals

The cross correlations with $E$ are $\approx 100$ times smaller than those with $T$...

WAIT TO SEE THE S/N!

13/16

All primordial $y_T$, $y_E$

Forecast results

- $>3x$ improvement w.r.t. previous methods for $y$
- $20\%$ improvement for $\mu$

PIXIE

<table>
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<th>$T \oplus E$, clean.</th>
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PRISM

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Cosmic Variance Limited

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Overall factor $\sim 16$ improvement (PRISM)
From masking + template cleaning + $yE$

20% improvement on $\mu T$ when adding $\mu E$ (see also Ota 2016)

M. Liguori (Unipd, INFN) "Probing fundamental Physics with CMB spectral dist.”
CERN, 12-16 March 2018
Conclusions

• The $\mu\mu T$ bispectrum carries interesting extra information about PNG, with respect to $\mu\mu$, $\mu T$. It allows building (unbiased) $g_{NL}$ estimators.

• As usual, since one integrates over lots of modes up to very high $k$, the potential of an ideal survey is impressive: $g_{NL} < 1$.

• In practice, requires $\sim 1000$ smaller noise power than a CMBpol-like survey to improve over current Planck bounds (very optimistic, neglects foregrounds and systematics).

• Correlating spectral distortions with polarization is also interesting. Besides obvious error bars improvements, $yE$ does not display SZ contamination. Useful especially for $f_{NL}$ running.

• Combining $yT + yE$ with cluster masking and $y$-map template removing we achieve a factor $\sim 16$ improvement on $y$-based $f_{NL}$ forecasts.