CMB spectroscopy for primordial non-Gaussianity

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A spectral distortion from primordial non Gaussianity

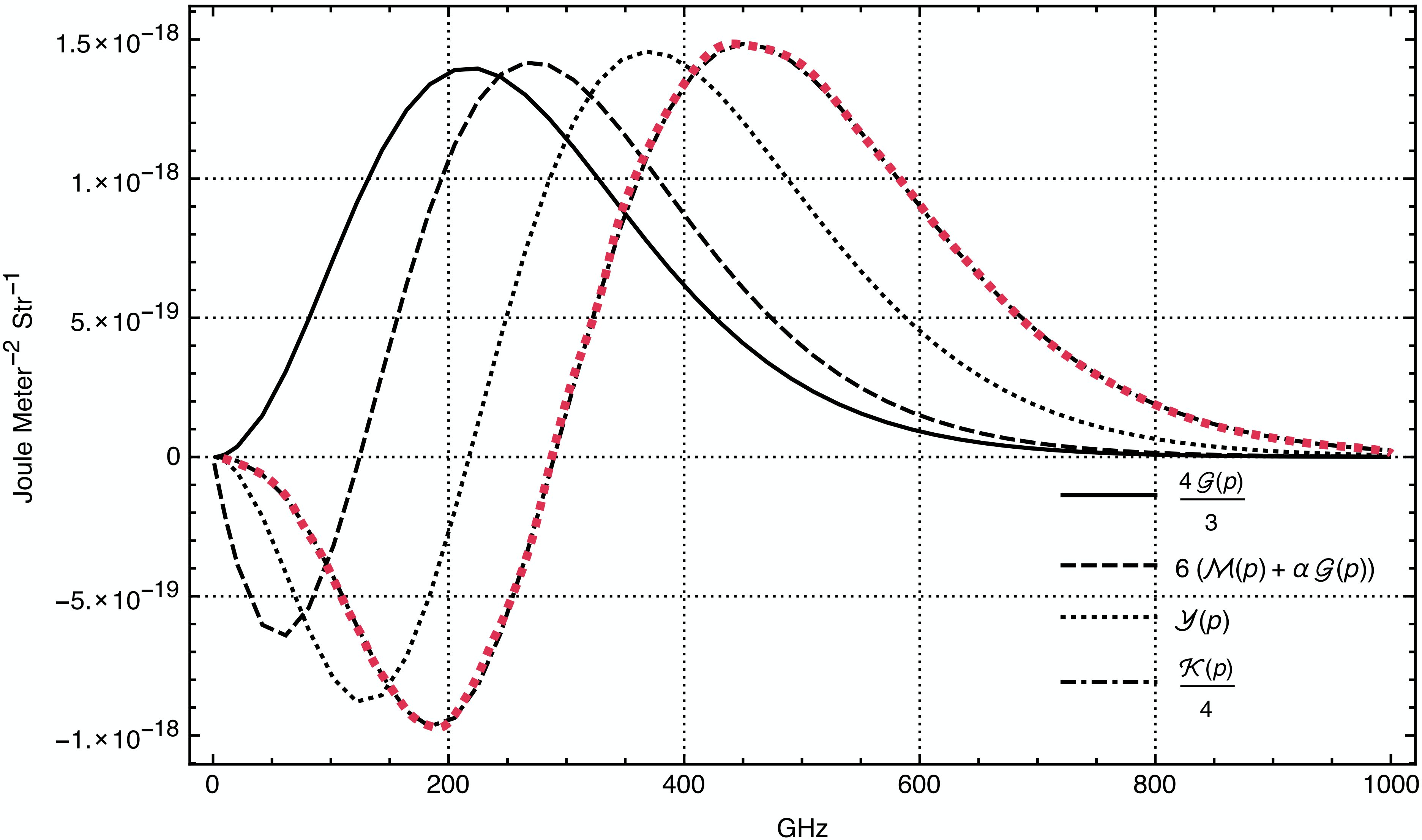
Second order (y)	Cubic order
Powerspectrum	Bispectrum?

Questions:

-What is the cubic order spectral distortion?

-Is it different from μ & y?

Spectral distortions from primordial non-Gaussianity



CMB spectral distortion from early energy injection

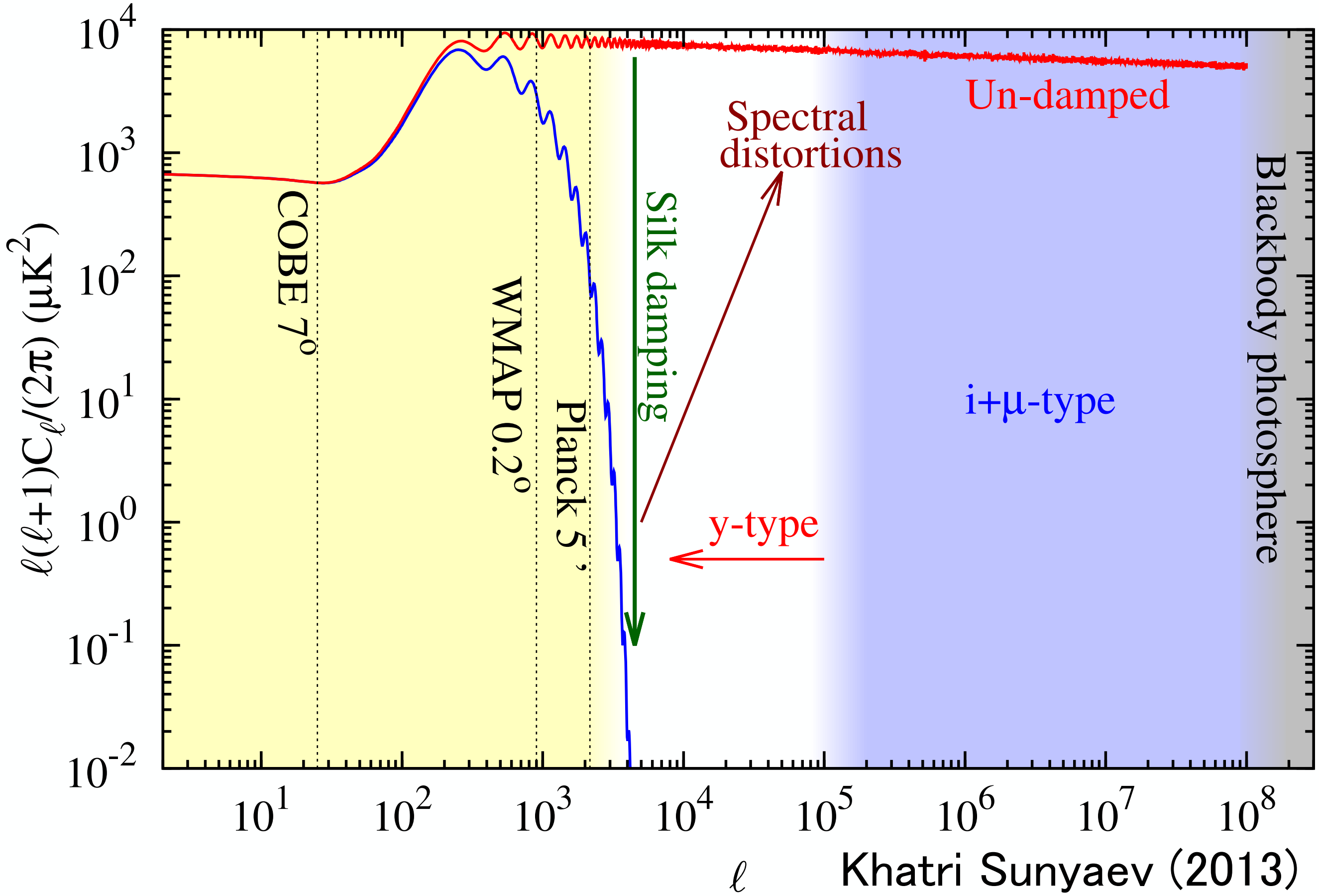
$$\mu = 1.4 \left.\frac{\Delta\rho}{\rho}\right|_{\mu-\mathrm{era}}$$

$$y = \frac{1}{4} \left.\frac{\Delta\rho}{\rho}\right|_{y-\mathrm{era}}$$

$\Delta\rho$: Energy injection to the CMB

- Dark matter
- Magnetic field
- Gravitational waves
- Density perturbations

Energy injection from Silk damping



Khatri Sunyaev (2013)

"Energy in fluctuation is redistributed homogeneously"

Energy injection from Silk damping

$$\frac{\Delta\rho}{\rho} = 4\frac{\delta T}{T} + 6\left(\frac{\delta T}{T}\right)^2 + \cdots$$

$$\rho \propto (T + \delta T)^4$$

$$\left.\frac{\Delta\rho}{\rho}\right|_{\mathrm{Silk}} = 6\left\langle\left(\frac{\delta T}{T}\right)^2\right\rangle \sim \langle\zeta^2\rangle$$

"Spectral distortions are sensitive to the powerspectrum of ζ"

Weak points of the traditional way

- Gauge invariance?
- Does whole energy go to the Photon sector?
- Anisotropy of distortion?

✓ Let us employ cosmological perturbation theory and solve the Boltzmann equation directly!

$$\frac{df}{d\eta} = \mathcal{C}[f]$$

Assumption: we only consider y era

Spectral distortions in cosmological perturbation theory

First order Boltzmann equation:

$$\frac{d\Theta}{d\eta} - \frac{d\ln p}{d\eta} = n_{\mathrm{e}}\sigma_{\mathrm{T}}a\left(\Theta_0 - \Theta + V + \frac{1}{10}\sum_{m=-2}^{2} Y_{2m}\Theta_{2m}\right)$$

$$V = \mathbf{n}\cdot\mathbf{v}$$

- ✓ Explicit p dependence is canceled.
- ✓ Temperature perturbations have no p dependence.
- ✓ CMB is in “local equilibrium”.

$$\Theta^{(1)} = \Theta^{(1)}(\eta, \mathbf{x}, \mathbf{n})$$

Spectral distortions in cosmological perturbation theory

- ✓ Higher order: complicated frequency dependence
- ✓ Continuous infinite number of equations for each $(\eta, \mathbf{x}, \mathbf{n})$

$$\Theta^{(2)} = \Theta^{(2)}(\eta, \mathbf{x}, p\mathbf{n})$$

Number of parameters for the CMB distribution function

✓ Number of parameters for each $(\eta, \mathbf{x}, \mathbf{n})$

Order in ζ	0	1	2	3
Parameters	0	Θ	Θ, y?	?

✓ Full p dependence would be re-parameterized by a few spectral distortions

→Moment expansion in terms of spectral distortions

Spectral distortions in a cosmological perturbation theory

E.g.) Static electromagnetics

$$\phi(\mathbf{x}) = \frac{q}{4\pi\varepsilon_0|\mathbf{x}|} + \frac{\mathbf{p}\cdot\mathbf{x}}{4\pi\varepsilon_0|\mathbf{x}|^3} + \frac{3\mathbf{x}\cdot Q\mathbf{x}}{8\pi\varepsilon_0|\mathbf{x}|^5} + \cdots$$

Drop the complicated part by moment expansion

Electro magnetics (1/|x|, P_l)	monopole	dipole	quadrupole	octpole
CMB (ζ, ?)	Planck dist.	temperature pert.	y distortion	?

Search for the frequency basis

p derivatives in the Boltzmann equations:

✓ Liouville operator:

$$\frac{d}{d\eta} = \frac{d\mathbf{x}}{d\eta} \cdot \nabla + \frac{d\mathbf{n}}{d\eta} \cdot \nabla_{\mathbf{n}} - \frac{d \ln p}{d\eta} \left(-p \frac{\partial}{\partial p} \right)$$

✓ Collision operator (the weak Compton scattering)

$$\left(-p \frac{\partial}{\partial p} \right), \ \mathcal{O}\left(\cancel{\frac{T_\mathrm{e}}{m_\mathrm{e}}} \right) \left(-p \frac{\partial}{\partial p} \right), \ \mathcal{O}\left(\cancel{\frac{T_\gamma}{m_\mathrm{e}}} \right) \left(-p \frac{\partial}{\partial p} \right), \ \mathcal{O}\left(\cancel{\frac{p}{m_\mathrm{e}}} \right) \left(-p \frac{\partial}{\partial p} \right),$$

Operators on f: $\left(-p \frac{\partial}{\partial p} \right)^n$

Search for the frequency basis

$$f(\eta, \mathbf{x}, p\mathbf{n}) = \frac{1}{e^{\frac{p}{T_{\mathrm{rf}}\, e^{\tilde{\Theta}(\eta, \mathbf{x}, p\mathbf{n})}}} - 1}$$

$$= \sum_n \frac{\tilde{\Theta}^n}{n!} \left(-p \frac{\partial}{\partial p} \right)^n f^{(0)}(p)$$

p dependence always arises as

$$\left(-p \frac{\partial}{\partial p} \right)^n f^{(0)}(p)$$

Construction of the CMB distribution function

Momentum basis

$$f^{(0)} = \frac{1}{e^{p/T_{\rm rf}} - 1}$$

$$\mathcal{G} = \left(-p\frac{\partial}{\partial p}\right) f^{(0)}$$

$$\mathcal{Y} = \left(-p\frac{\partial}{\partial p}\right)^2 f^{(0)} - 3\mathcal{G}$$

$$\mathcal{K} = \left(-p\frac{\partial}{\partial p}\right)^3 f^{(0)} - 9\mathcal{G} - 3\mathcal{Y}$$

Cubic order ansatz

$$f = f^{(0)} + [\Theta + \cdots]\mathcal{G} + \left[y + \frac{1}{2}\Theta^2 + \cdots\right]\mathcal{Y} + \left[\frac{1}{3!}\Theta^3 + \kappa\right]\mathcal{K},$$

Spectral distortions in cosmological perturbation theory

Boltzmann equation for the distribution function:

$$\frac{df}{d\eta} = \mathcal{C}[f] \rightarrow \begin{array}{c} (\cdots) \times \mathcal{G} = 0 \\ (\cdots) \times \mathcal{Y} = 0 \\ (\cdots) \times \mathcal{K} = 0 \end{array}$$

$$\frac{d\Theta}{d\eta} - \frac{d\ln p}{d\eta} + \cdots = \mathcal{A},$$

$$\frac{dy}{d\eta} + \Theta\left(\frac{d\Theta}{d\eta} - \frac{d\ln p}{d\eta}\right) + \cdots = \mathcal{B},$$

$$\frac{d\kappa}{d\eta} - y\frac{d\ln p}{d\eta} + \frac{1}{2}\Theta^2\left(\frac{d\Theta}{d\eta} - \frac{d\ln p}{d\eta}\right) = \mathcal{D}$$

Spectral y distortion from the second order Boltzmann equation

Boltzmann equation for the y distortion:

$$\frac{dy}{d\eta} = -\Theta\mathcal{A} + \mathcal{B} + \cdots .$$

$$
\begin{aligned}
(n_{\mathrm{e}}\sigma_{\mathrm{T}}a)^{-1}\frac{dy}{d\eta} = &-\Theta\Theta_0 + \Theta^2 - V\Theta - \frac{1}{10}\Theta\sum_{m=-2}^{2} Y_{2m}\Theta_{2m} \\
&+ y_0 - y + \frac{1}{10}\sum_{m=-2}^{2} Y_{2m}y_{2m} \\
&+ V\Theta_0 - [V\Theta]_0 + \frac{1}{2}V^2 + \frac{1}{2}[V^2]_0 \\
&+ \frac{1}{10}\sum_{m=-2}^{2} Y_{2m}\Big[V\Theta_{2m} - [V\Theta]_{2m} + [V^2]_{2m}\Big].
\end{aligned}
$$

Spectral y distortion from the second order Boltzmann equation

✓ Ensemble average of the monopole is easier:

$$\langle y_0 \rangle = \int d\eta (n_{\rm e} \sigma_{\rm T} a) \int \frac{dk}{k} \frac{k^3 P_\zeta(k)}{2\pi^2} \left(\frac{9}{2} {\Theta_2}^2 + 3(V_1 - \Theta_1)^2 + \cdots \right)$$

✓ Shear viscosity & heat conduction produce y
✓ Gauge invariant form (local process)
✓ Directly related to the power spectrum.

We do the same procedures at cubic order!

Spectral κ distortion from the cubic order Boltzmann equation

Preliminary

The Boltzmann equation for the new distortion:

$$\frac{d}{d\eta}\left(\kappa - \Theta y\right) = \frac{1}{2}\Theta^2 \mathcal{A} - y\mathcal{A} - \Theta\mathcal{B} + \mathcal{D}.$$

- ✓ Angular dependence is complicated in general.
- ✓ Let us consider only the monopole component!
- ✓ For simplicity, we assume local type non-Gaussianity

$$\langle \kappa_0 \rangle \approx [\langle \Theta y \rangle]_0 + \frac{24}{5} f_{\rm NL}^{\rm loc.} \int d\eta (n_{\rm e}\sigma_{\rm T} a) \int \frac{dk}{k} \frac{k^3 P_\zeta(k)}{2\pi^3} \left[9\Theta_2 y_2 + 3(\Theta_1 - V_1) y_1\right]$$

Spectral κ distortion from the cubic order Boltzmann equation

Preliminary

$$\langle \kappa_0 \rangle \approx [\langle \Theta y \rangle]_0 + \frac{24}{5} f_{\rm NL}^{\rm loc.} \int d\eta (n_{\rm e} \sigma_{\rm T} a) \int \frac{dk}{k} \frac{k^3 P_\zeta(k)}{2\pi^3} [9\Theta_2 y_2 + 3(\Theta_1 - V_1) y_1]$$

- ✓ No triple products of the linear perturbations.
- ✓ No metric perturbations
- ✓ Shear & heat conduction from temperature & y
- ✓ Local process is related to the bispectrum.
- ✓ Contains gauge dependent $[\Theta y]_0$

$$\langle \kappa_0^{\rm loc.} \rangle = \langle \kappa_0 \rangle - [\langle \Theta y \rangle]_0$$

κ value depends on the choice of frame
= Parameterization of the cubic order distortion is not unique

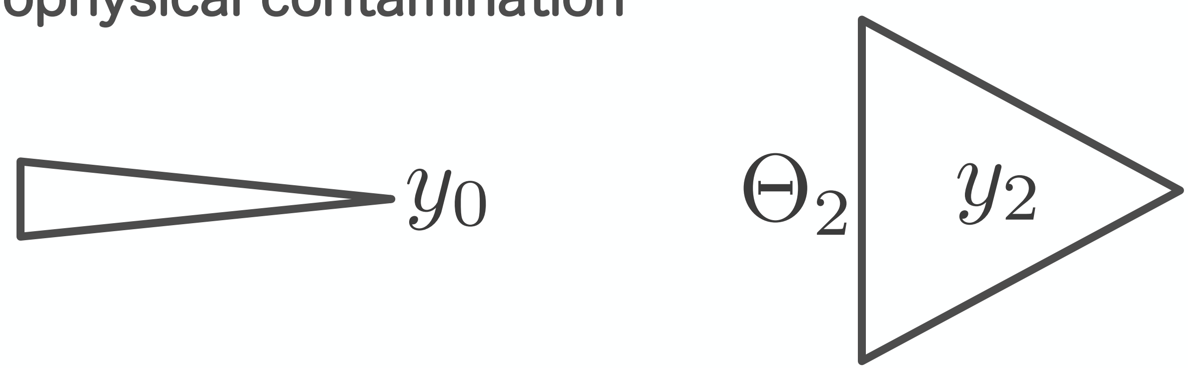
Summary & future directions

Summary

- ✓ A new cubic order spectral distortion is formulated.
- ✓ It is sensitive to the primordial bispectrum.
- ✓ Gauge feature of cubic order distortion is discussed

Future directions

- ✓ Contribution from the equilateral type non Gaussianity
- ✓ κ anisotropy and trispectrum
- ✓ Astrophysical contamination



Spectral distortions in a cosmological perturbation theory

Hint: Boltzmann hierarchy at linear order in Fourier space:

$$\frac{\partial\Theta}{\partial\eta} + \mathbf{n}\cdot i\mathbf{k}\Theta - \frac{d\ln p}{d\eta} = n_\mathrm{e}\sigma_\mathrm{T}a\left(\Theta_0 - \Theta + V + \frac{1}{10}\sum_{m=-2}^{2} Y_{2m}\Theta_{2m}\right)$$

Multipole expansion:

$$\Theta = \sum_l (-i)^l (2l+1) P_l(\mathbf{n}\cdot\hat{k})\Theta_l$$

Infinite number of ODEs → a few ODEs

Drop the too much complicated part

Spectral distortions in a cosmological perturbation theory

Boltzmann equation for the distribution function:

$$\frac{df}{d\eta} = \left[\frac{d\Theta}{d\eta} - \frac{d\ln p}{d\eta} + \cdots\right]\mathcal{G}$$
$$+ \left[\frac{dy}{d\eta} + \Theta\left(\frac{d\Theta}{d\eta} - \frac{d\ln p}{d\eta}\right) + \cdots\right]\mathcal{Y}$$
$$+ \left[\frac{d\kappa}{d\eta} - y\frac{d\ln p}{d\eta} + \frac{1}{2}\Theta^2\left(\frac{d\Theta}{d\eta} - \frac{d\ln p}{d\eta}\right) + \cdots\right]\mathcal{K},$$

$$\mathcal{C}_{\mathrm{T}}[f] = \mathcal{A}\mathcal{G} + \mathcal{B}\mathcal{Y} + \mathcal{D}\mathcal{K}.$$

Spectral distortions in a cosmological perturbation theory

Boltzmann equation for the distribution function:

$$\frac{dy}{d\eta} = -\Theta\mathcal{A} + \mathcal{B} + \cdots .$$

$$\begin{aligned}
&(n_{\rm e}\sigma_{\rm T}a)^{-1}\mathcal{B}\\
=&y_0 - y + \frac{1}{10}\sum_{m=-2}^{2} Y_{2m}y_{2m}\\
&+\frac{1}{2}[\Theta^2]_0 - \frac{1}{2}\Theta^2 + \frac{1}{20}\sum_{m=-2}^{2} Y_{2m}[\Theta^2]_{2m}\\
&+V\Theta_0 - [V\Theta]_0 + \frac{1}{2}V^2 + \frac{1}{2}[V^2]_0\\
&+\frac{1}{10}\sum_{m=-2}^{2} Y_{2m}\left[V\Theta_{2m} - [V\Theta]_{2m} + \frac{1}{2}[V^2]_{2m}\right]
\end{aligned}$$

$$\begin{aligned}
&(n_{\rm e}\sigma_{\rm T}a)^{-1}\mathcal{A}\\
&= \Theta_0 - \Theta + V + \frac{1}{10}\sum_{m=-2}^{2} Y_{2m}\Theta_{2m} + \cdots ,
\end{aligned}$$

Spectral distortions in a cosmological perturbation theory

Boltzmann equation for the distribution function:

$$\left.\frac{\Delta\rho}{\rho}\right|_{y-\mathrm{era}} = 4\int d\eta (n_{\mathrm{e}}\sigma_{\mathrm{T}}a)\int \frac{dk}{k}\frac{k^3 P_\zeta(k)}{2\pi^2}\left(\frac{9}{2}{\Theta_2}^2 + \frac{1}{3}(iv_g)^2\right)$$

Θ_2 : Shear viscosity of photon fluid

v_g : Velocity difference btw. Baryon and photon fluid

☆ The source is Gauge invariant.

Construction of the CMB distribution function

1st order ansatz

CMB is in equilibrium locally.

$$f = f^{(0)} + \Theta \left(-p \frac{\partial}{\partial p} \right) f^{(0)}$$

"Local equilibrium"

$$\Theta = \Theta(\eta, \mathbf{x}, p\mathbf{n})$$

$$\frac{df}{d\eta} = \mathcal{C}[f] \rightarrow (\cdots) \times \left(-p \frac{\partial}{\partial p} \right) f^{(0)} = 0$$

Construction of the CMB distribution function

Cubic order

CMB is in equilibrium locally.

$$f = \frac{1}{e^{\frac{p}{T(1+\Theta)}} - 1} + (y + 3\Theta y)\mathcal{Y} + \kappa\mathcal{K}$$

"No more local equilibrium" $\quad \Theta = \Theta(\eta, \mathbf{x}, p\mathbf{n})$

$$\frac{df}{d\eta} = \mathcal{C}[f] \rightarrow \begin{matrix} (\cdots) \times \mathcal{G} = 0 \\ (\cdots) \times \mathcal{Y} = 0 \\ (\cdots) \times \mathcal{K} = 0 \end{matrix}$$

Spectral distortions in a cosmological perturbation theory

$$\frac{d}{d\eta}\left(\kappa - \Theta y\right) = \frac{1}{2}\Theta^2\mathcal{A} - y\mathcal{A} - \Theta\mathcal{B} + \mathcal{D}.$$

$$(n_{\rm e}\sigma_{\rm T}a)^{-1}\langle\mathcal{D}_0\rangle$$
$$= \frac{3}{2}\Theta_0 V_1^2 - 3\Theta_0^2 V_1 + 3\Theta_2 V_1^2 - 6\Theta_1\Theta_2 V_1$$
$$+ \frac{1}{20\cdot 4\pi}\sum_{m=-2}^{2}[V^2]_{2m}\Theta_{2m} + \frac{1}{2}[V^2]_0\Theta_0 - [Vy]_0,$$

$$\frac{1}{2}(n_{\rm e}\sigma_{\rm T}a)^{-1}\int\frac{d\mathbf{n}}{4\pi}\langle\Theta^2\mathcal{A}\rangle$$
$$= -\frac{9}{2}\Theta_0\Theta_2^2 - \frac{45}{14}\Theta_2^3 + 3\Theta_0\Theta_1 V_1 - 3\Theta_0\Theta_1^2$$
$$- \frac{27}{10}\Theta_1^2\Theta_2 + 6\Theta_1\Theta_2 V_1 - 6\Theta_1^2\Theta_2.$$

$$- (n_{\rm e}\sigma_{\rm T}a)^{-1}\int\frac{d\mathbf{n}}{4\pi}\langle\Theta\mathcal{B}\rangle$$
$$= -\frac{3}{2}\Theta_0 V_1^2 - 3\Theta_2 V_1^2 + \frac{15}{2}\Theta_0\Theta_2^2 + \frac{25}{7}\Theta_2^3$$
$$+ \frac{9}{2}\Theta_0\Theta_1^2 + \frac{1}{2}\Theta_0^3 + 9\Theta_1^2\Theta_2$$
$$- \Theta_0 y_0 + [\Theta y]_0 - \frac{1}{2}\Theta_0[V^2]_0 - \frac{1}{2}\Theta_0[\Theta^2]_0$$
$$- \frac{1}{10\cdot 4\pi}\sum_{m=-2}^{2}\Theta_{2m}\left(y_{2m} + \frac{1}{2}[V^2 + \Theta^2]_{2m}\right).$$

Spectral κ distortion from the cubic order Boltzmann equation

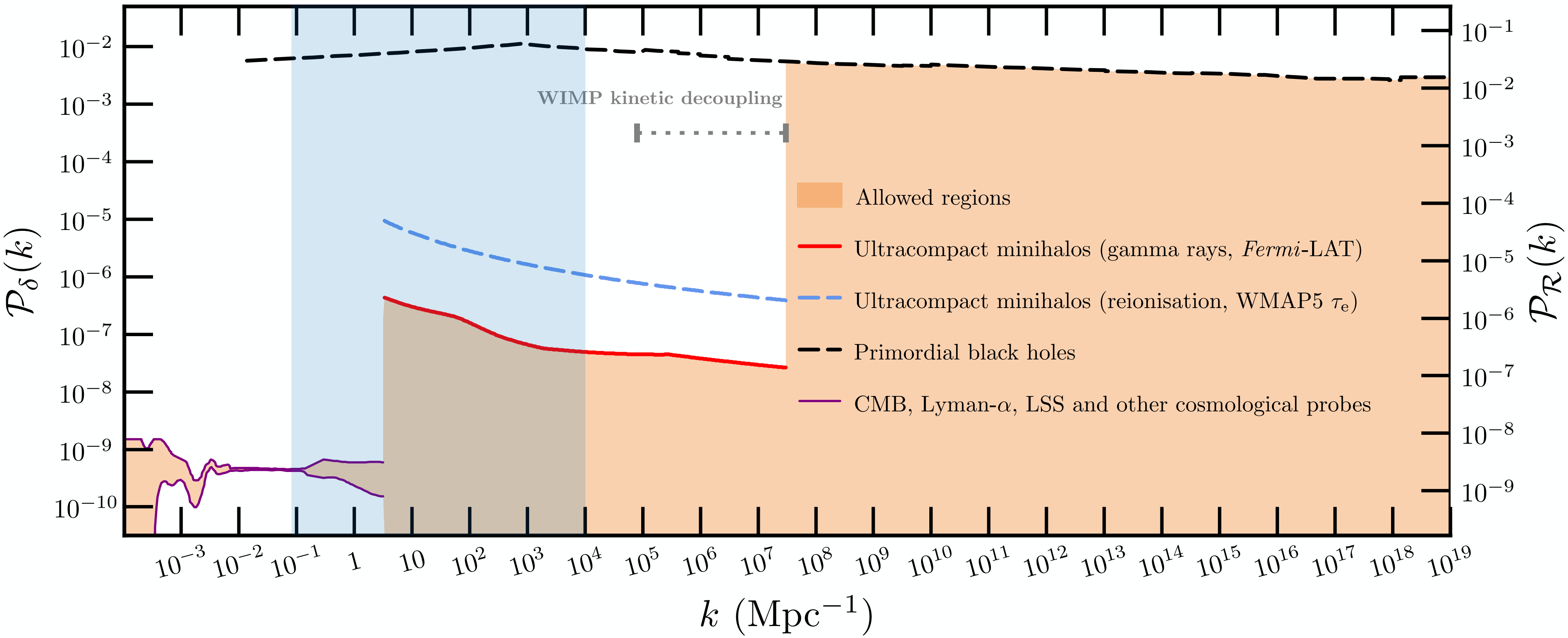
$$f = \frac{1}{e^{\frac{p}{T(1+\Theta)}} - 1} + (y + 3\Theta y)\mathcal{Y} + \kappa\mathcal{K}$$

Black body part spectral distortion

- ✓ y distortion also contains gauge dependent part at cubic order.
- ✓ Observation should be done where

$$[\langle\Theta y\rangle]_0 = 0$$

Constraints on the primordial curvature perturbations



Torsten Bringmann, Pat Scott, Yashar Akrami (2011)

Spectral distortion constraints short wavelength perturbations