Non-Gaussianity discussion
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Inflation: We observe so much yet see so little…

• It is remarkable and disappointing that we can explain the statistical property of $10^7$ CMB pixels with just two primordial numbers (+ background parameters)

• We have only measured the amplitude and spectral index of the power spectrum

• Is this evidence that inflation was simple? Bayesian model comparison is inconclusive.

• We need more data => go to smaller scales
Current power spectrum constraints

Fig. 26. Bayesian reconstruction of the primordial power spectrum averaged over different values of $N_{\text{int}}$ (as shown in Fig. 24), weighted according to the Bayesian evidence. The region $30 < \ell < 2300$ is highly constrained, but the resolution is lacking to say anything precise about higher $\ell$. At lower $\ell$, cosmic variance reduces our knowledge of $P_R(k)$. The weights assigned to the lower $N_{\text{int}}$ models outweigh those of the higher models, so no oscillatory features are visible here.

- Featureless power law over 1 decade in scales (or $\log(2300/30)=4.3$ e-folds)
- Could the perturbations grow dramatically on small scales?
Spectral distortions
Testing inflation with 17 efoldings

Khatri 2013
The slow-roll hierarchy is not observationally required

Figure 1. Constraints from Planck 2015 TT+lowTEB [1] and BICEP-Keck [22] on a constant $\alpha_s$ (left) and a constant $\beta_s$ (marginalized over $\alpha_s$ at the pivot scale; right), both against $n_s$ at the pivot scale. Dashed black contours assume a null tensor-to-scalar ratio, $r$, whereas blue contours marginalize over it. The light shaded region corresponds to the part of the parameter space where the quantity in the vertical axis becomes greater than $|n_s - 1|^2$ and the dark shaded region is where it becomes greater than $|n_s - 1|$. The naive expectation is that the true value of $\alpha_s$ (left) should be close to the boundary of the unshaded region and far away from the dark shaded region, whereas that of $\beta_s$ (right) should be well inside the unshaded region. Current constraints allow a much greater area of the parameter space.
Allowed power spectra assuming the “standard” parametrisation

**Figure 6.** Consequences of the imposition of slow roll (defined by the smallness of $g$) for the power spectrum scaled by $e^{-2\tau}$, where $\tau$ is the optical depth (whose value affects the amplitude of the spectrum, but not its shape). The blue contours represent the 68% (dark blue) and 95% (light blue) limits on the allowed values of the power spectrum (rescaled by a factor of $e^{-2\tau}$) extrapolated from *Planck 2015 TT, TE, lowTEB* constraints (over gray shaded scales) assuming a constant $\alpha_s$ (left) and a constant $\beta_s$ (right), for different values of $k$. The solid and dashed red contours represent the 68% and 95% limits on the fraction of these spectra for which $|g| < 0.2$ for the range of scales corresponding to $10^{-3}\text{Mpc}^{-1} < k < 10^4\text{Mpc}^{-1}$. The solid and dashed black contours represent the 68% and 95% limits on the fraction of these spectra corresponding to the unshaded regions in figure 1 (note that for the plot on the right the limits of this region already violate the naive expectation for the magnitude of $\beta_s$).
Distortions provide general power spectrum constraints!

- Amplitude of power spectrum rather uncertain at $k > 3 \text{ Mpc}^{-1}$
- improved limits at smaller scales can rule out many inflationary models
- CMB spectral distortions would extend our lever arm to $k \sim 10^4 \text{ Mpc}^{-1}$
- very complementary piece of information about early-universe physics

- Slide by Jens Chluba - Ultra compact minihalo constraints are unreliable (Gosenca et al, Delos et al, Nakama et al; all 2017). Spectral distortion constraints are tighter than PBH constraints for $M > 10^3 \text{M}_\odot$
- Non-Gaussian perturbations can evade these constraints, e.g. Nakama, Carr & Silk 2017

PBHs: Juan and Yacine talks on Thursday
Probing the tail

Local non-Gaussianity (chi-squared)

$$\zeta = \zeta_g + \frac{3}{5} f_{NL} (\zeta_g^2 - \sigma^2)$$

$$\mathcal{P}_\zeta = 10^{-2}$$

Young and CB 2013
Local (squeezed limit) $f_{\text{NL}}$ forecasts

Assuming scale invariance. If power spectrum grows by 3 orders of magnitude then $f_{\text{NL}} \sim 1$ becomes detectable!

If \textit{any} solar mass PBHs exist then such a growth in the power spectrum is expected, quite possibly accompanied by non-Gaussianity.

Some popular inflationary models generate scale independent $f_{\text{NL}}$ even if the power spectrum amplitude massively changes.

Hence if power spectrum grows by more than $220/5$ on relevant scales then distortions provide the best constraint on scale-invariant $f_{\text{NL}}$. 

$$f_{\text{NL}}^\gamma \approx 220 \left( \frac{y_{\text{min}}}{2 \times 10^{-10}} \right) \left( \frac{\langle y \rangle}{4 \times 10^{-9}} \right)^{-1}$$

$$f_{\text{NL}}^\mu \approx 220 \left( \frac{\mu_{\text{min}}}{10^{-9}} \right) \left( \frac{\langle \mu \rangle}{2 \times 10^{-8}} \right)^{-1}$$

Emami++ 2015
Scale dependent non-Gaussianity

- Just like the primordial power spectrum, non-Gaussianity is generically expected to have some scale dependence.

- This is typically slow-roll suppressed but can be made arbitrarily large (in which case using a spectral index is inadequate).

- Scale dependence can arise from:
  1. Self-interactions of a spectator field
  2. Multiple-field perturbations present in the power spectrum
  3. A non-trivial field space metric
  4. Non-canonical kinetic term with a scale/time dependent sound speed
Strongly self-interacting curvaton scenario

\[ n_{f_{NL}} = \frac{d \ln |f_{NL}|}{d \ln k}, \quad n_{g_{NL}} = \frac{d \ln |g_{NL}|}{d \ln k} \]

\[ \eta_\sigma = \frac{m_\sigma^2}{3H^2} \sim 10^{-2} \]

CB, Enqvist, Nurmi, Takahashi 2011
Elevator pitch?

- Lots of science can be done with distortions

- But what are the key selling points? Much harder to summarise than the search for primordial tensors!

- If Pixie flew and did not detect any primordial distortions, what have we learnt? Is the situation similar to DE/MG experiments, loads will be learnt if the cosmological constant is ruled out, but not so much if everything remains consistent with the simplest models.