Small-x resummation in PDF fits
and implications for high-energy colliders

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Related to work with Simone Marzani, Claudio Muselli, Tiziano Peraro
and the NNPDF collaboration
(Juan, Luca, Richard, StefanoF, Valerio)

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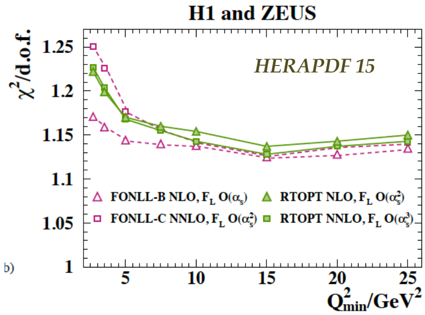
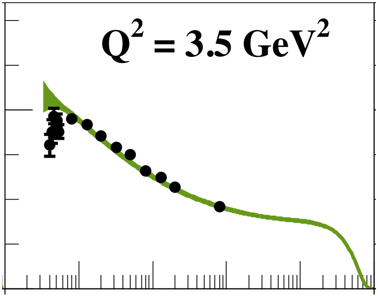
LHeC (FCC-eh) will probe x down to $x \sim 10^{-6}$ (10^{-7}) for $Q^2 \sim 2$ GeV^2

HERA reached $x \sim 2 \times 10^{-5}$ at that scale

Small-x region poorly described by fixed-order perturbative QCD factorization
Tension between HERA data at low Q^2 and low x with theory





NNLO theory happens to describe worse low-Q^2 low-x data than NLO

Small-x resummation

It allows to explain and cure these effects

We are not the first ones to argue this
However, we are probably the first ones to *prove* it!

How?
We use fixed-order perturbative collinear QCD factorization, *supplemented by the resummation of small-x logarithms*, to fit PDFs from data
We are able to successfully describe and fit the low-Q^2 low-x HERA data

A digression on the theory (1)

Collinear QCD factorization:

Observable: $$\sigma = \sigma_0\, C(\alpha_s(\mu)) \otimes f(\mu) \left[\otimes f(\mu) \right]$$

Evolution: $$\mu^2 \frac{d}{d\mu^2} f(\mu) = P(\alpha_s(\mu)) \otimes f(\mu)$$

Any object with a perturbative expansion can potentially contain a logarithmic enhancement:

- coefficient functions $C(\alpha_s(\mu))$ (observable)
- splitting functions $P(\alpha_s(\mu))$ (evolution)

At small x, both objects contain large logarithms $\log \frac{1}{x}$ in the singlet sector and may spoil perturbativity $\to$ resummation

Small-x resummation formalism based on k_t-factorization and BFKL
Developed in the 90s-00s [Catani,Ciafaloni,Colferai,Hautmann,Salam,Stasto] [Altarelli,Ball,Forte] [Thorne,White]

Known at LLx and NLLx since many years, but very limited number of applications. Why?

A digression on the theory (2)

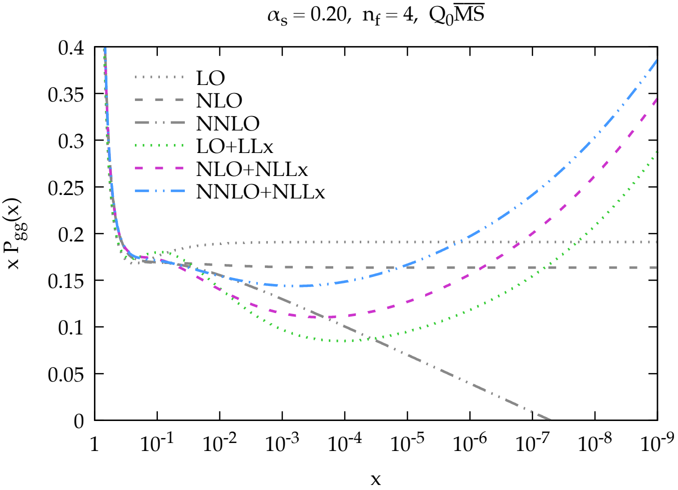
Small-x resummation is a hell!
Various implementations, with different pros and cons, more or less all in agreement but difficult to implement and with no public codes available

Recent developments: [MB,Marzani,Peraro 1607.02153][MB,Marzani,Muselli 1708.07510]

- we took (and improved) the ABF [Altarelli,Ball,Forte 1995,...,2008] procedure to resum splitting functions and developed a new formalism for coefficient functions
- we have all the ingredients for describing DIS process at small x, including mass effects and heavy flavour matching conditions in DGLAP evolution
- we have been able to match resummation to NNLO, allowing **NNLO+NLLx phenomenology**
- we published (and keep developing) a public code
  `HELL: High-Energy Large Logarithms` www.ge.infn.it/~bonvini/hell
  which delivers resummed splitting functions and coefficient functions
- `HELL` has been interfaced to `APFEL` (apfel.hepforge.org) opening the door to its usage for PDF fitting

A digression on the theory (3)

A representative result



There is much more on the papers...

NNPDF fit with small-x resummation

APFEL+HELL $\to$ make possible a PDF fit with small-x resummation

NNPDF (3.1) framework:

- NeuralNet parametrization of PDFs, MonteCarlo uncertainty, ...
- variable flavour number scheme with mass effects (FONLL)
- charm PDF is fitted
- a large variety of DIS and hadron collider data (~ 4000 datapoints)
- ....

We have performed NLO, NLO+NLLx, NNLO, NNLO+NLLx fits
$\to$ a paper will appear soon! [Ball,Bertone,MB,Forte,Marzani,Rojo,Rottoli 1709.xxxxx]

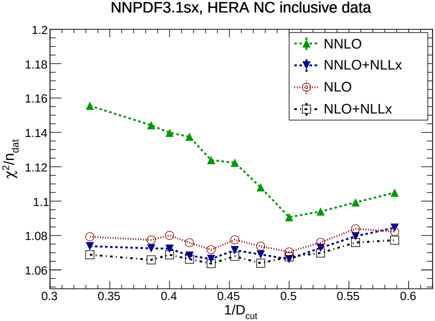
One significant difference in the HERA data we include:

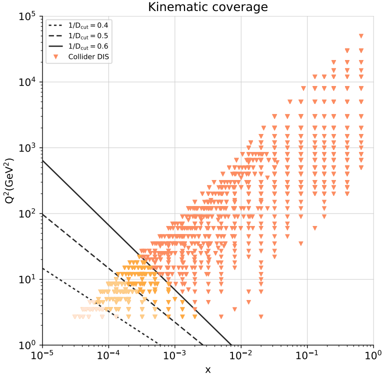
Lowest Q^2 HERA bins	NNPDF3.1	NNPDF3.1sx (these fits)
$Q^2 = 3.5$ GeV^2	included	included
$Q^2 = 2.7$ GeV^2	excluded	included
$Q^2 = 2.0$ GeV^2	excluded	excluded

$\chi^2/N_{\rm dat}$	NLO	NLO+NLLx	NNLO	NNLO+NLLx
DIS-only	**1.117**	**1.118**	**1.126**	**1.104**
Global	**1.122**	**1.127**	**1.135**	**1.106**
	these are similar		largest	smallest

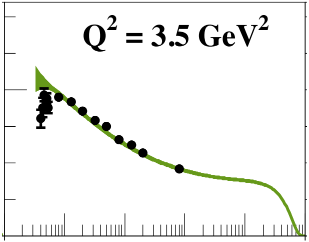
Hierarchy as expected from splitting function behaviour!

Mostly due to HERA data: we study the $\chi^2/N_{\rm dat}$ profile as we cut out HERA data at small x small Q^2





Fit results: description of the HERA data



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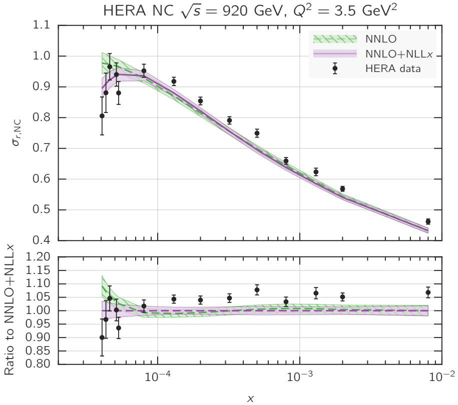

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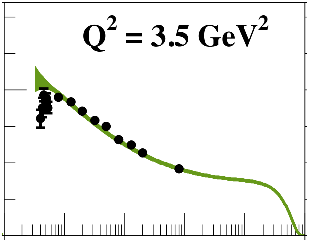
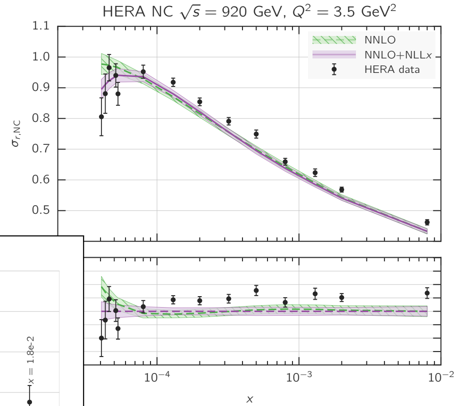
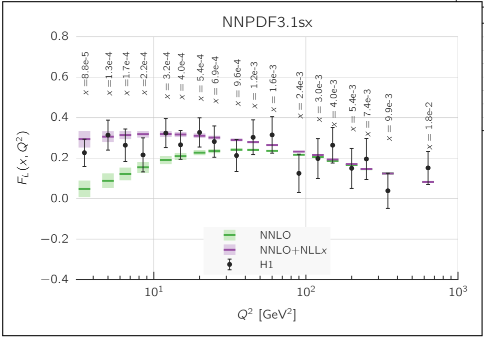
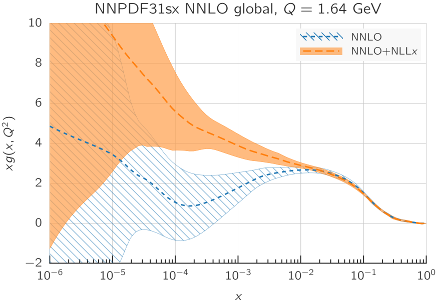


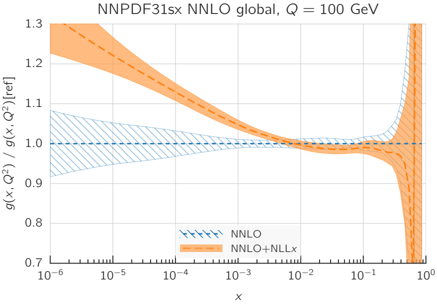


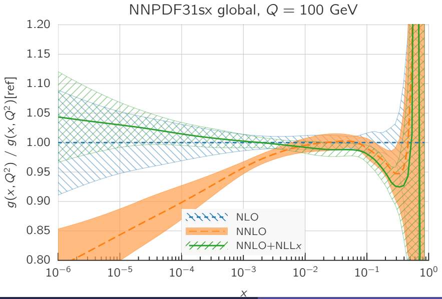


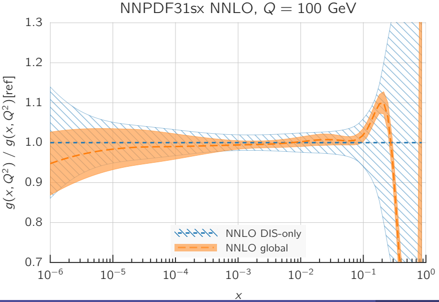
The better description mostly comes from F_L

Fit results: impact on PDFs – the gluon







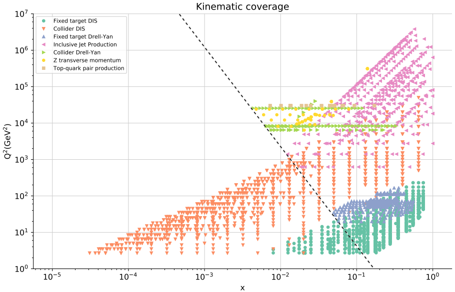


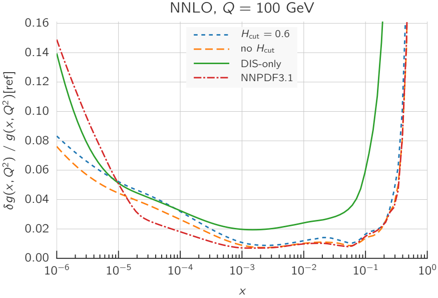
The global fit in greater detail

We have full resummation for DGLAP evolution and DIS structure functions, but not hadron-collider observables (yet)

We cut those hadronic data potentially sensitive to small-x resummation, i.e.

$$\alpha_s(Q^2) \log\frac{1}{x} > H_{\rm cut} = 0.6 \quad \text{(default cut)}$$





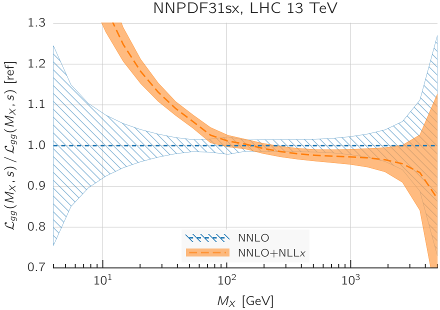
The most important missing observable is Drell-Yan

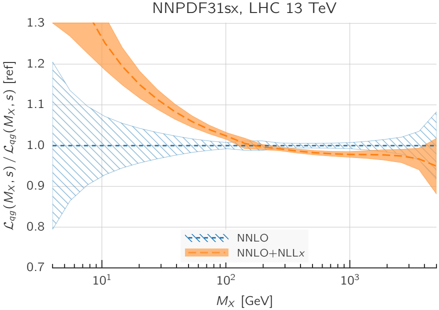
Work in progress to include it in `HELL` with the new formalism

PDF uncertainties are anyway competitive with those of a fully global fit!

Impact at the LHC

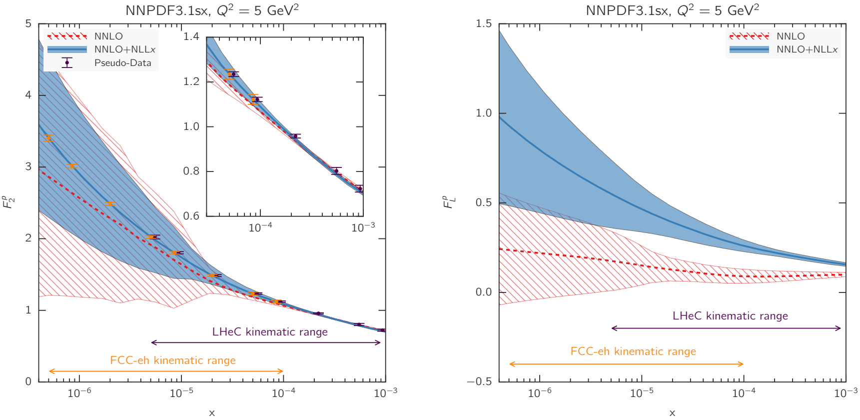
Parton luminosities (gg and qg)





Significant impact for $M_X \lesssim 100$ GeV, but also non-negligible impact above.

Small-x resummed phenomenology at the LHC is now starting....



Prediction in the LHeC and FCC-eh kinematic regions for F_2 and F_L

Uncertainties are large (extrapolation region)

Pseudo data show a small error — significant constraining power!

Conclusions

What's new:

- public HELL code implements small-x resummation with many improvements
- NNLO+NLLx results now available
- DIS-only and global PDF fits performed with NNPDF3.1 methodology/setting
- theorywise, **NNLO+NLLx fit is the best PDF fit ever** (limited by reduced dataset)
- sizeable impact — relevant for future collider phenomenology

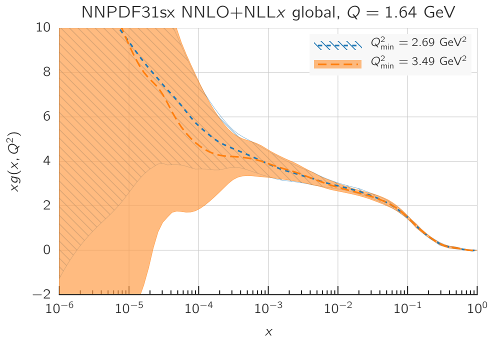
Key Message: At small x we need small-x resummation, and with small-x resummation we can successfully describe the small-x region

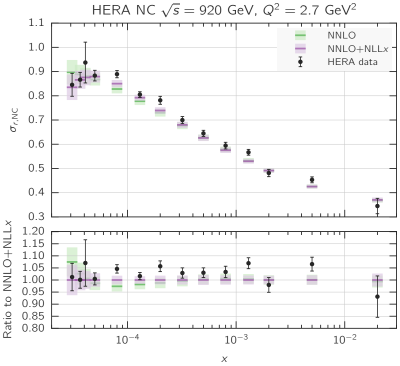
Outlook:

- resum Drell-Yan and update the global resummed fits
- phenomenological applications (LHC, FCC, ultra high energy astrophysics, ...)

Backup slides

The small-Q^2 HERA bin

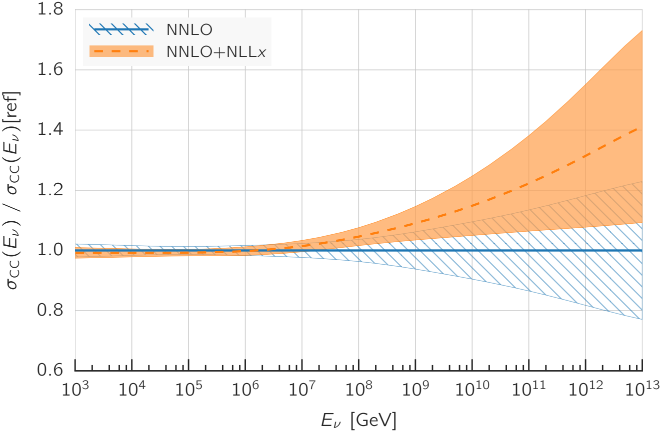




Including the $Q^2 = 2.7$ GeV^2 bin reduces the uncertainty on the small-x gluon, and does not deteriorate the fit quality — we perfectly describe those data

Ultra high energy astrophysics

UHE neutrino cross section, relevant for IceCube and KM3NET experiments



Sizeable impact, though uncertainties still large

DGLAP: $$\mu^2 \frac{d}{d\mu^2} f(x,\mu^2) = \int \frac{dz}{z} P\left(\frac{x}{z}, \alpha_s(\mu^2)\right) f(z,\mu^2)$$

BFKL: $$x \frac{d}{dx} f(x,\mu^2) = \int \frac{d\nu^2}{\nu^2} K\left(x, \frac{\mu^2}{\nu^2}, \alpha_s(\cdot)\right) f(x,\nu^2)$$

double Mellin transform $$f(N,M) = \int dx\, x^N \int \frac{d\mu^2}{\mu^2} \left(\frac{\mu^2}{\mu_0^2}\right)^{-M} f(x,\mu^2)$$

DGLAP: $$M f(N,M) = \gamma(N, \alpha_s(\cdot)) f(N,M) + \text{boundary}$$

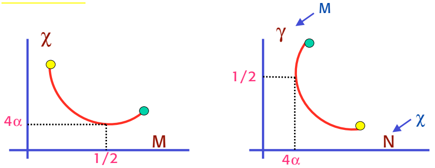
BFKL : $$N f(N,M) = \chi(M, \alpha_s(\cdot)) f(N,M) + \text{boundary}$$

When both are valid (small x, large μ^2), consistency between the solutions gives (at fixed coupling)

$$\chi(\gamma(N,\alpha_s), \alpha_s) = N$$

duality relation

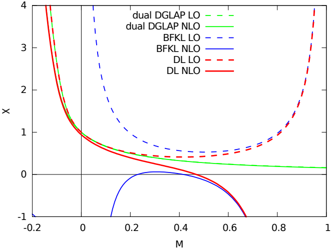
For $\chi(M,\alpha_s) = \alpha_s \chi_0(M)$
the dual γ contains all orders in α_s/N



What do we get?

- LL: strong growth at small x (not observed)
- NLL: no enhancement at small x (!!)

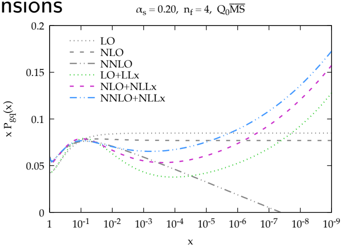
Totally unstable,
due to perturbative instability of the BFKL kernel



ABF solution [Altarelli,Ball,Forte 1995,...,2008]

- use duality to resum BFKL kernel
- exploit symmetry $M \to 1 - M$ of χ
- impose momentum conservation
- reuse duality to get resummed anomalous dimensions

The result is perturbatively stable!



Finally

- resum running coupling contributions (changes the nature of the small-N singularity: branch-cut to pole)

Singlet:

$$P(x,\alpha_s) = \sum_{n=0}^{\infty} \alpha_s^{n+1} \left[\sum_{k=0}^{n} a_{nk} \frac{\log^k x}{x} + \sum_{k=0}^{2n} b_{nk} \log^k x + \dots \right]$$

$$\gamma(N,\alpha_s) = \sum_{n=0}^{\infty} \alpha_s^{n+1} \left[\sum_{k=0}^{n} \frac{a_{nk}}{(N-1)^{k+1}} + \sum_{k=0}^{2n} \frac{b_{nk}}{N^{k+1}} + \dots \right]$$

Single log enhancement at leading small x, in the singlet sector

$$P_{\text{singlet}} = \begin{pmatrix} P_{gg} & P_{gq} \\ P_{qg} & P_{qq} \end{pmatrix} = \begin{pmatrix} \text{LL} & \text{LL} \\ \text{NLL} & \text{NLL} \end{pmatrix}$$

Non-singlet:

$$P(x,\alpha_s) = \sum_{n=0}^{\infty} \alpha_s^{n+1} \left[\sum_{k=0}^{2n} b_{nk} \log^k x + \dots \right]$$

is double log enhanced but subleading.

Small-x resummation of coefficient functions

High-energy (k_T) factorization:

$$\sigma \sim \int \frac{dz}{z} \int d^2\boldsymbol{k}\; \hat{\sigma}_g\left(\frac{x}{z}, \frac{Q^2}{\boldsymbol{k}^2}, \alpha_s(Q^2)\right) \mathcal{F}_g(z, \boldsymbol{k})$$

$$\begin{cases} \mathcal{F}_g(x, \boldsymbol{k}) : \text{unintegrated PDF} \\ \hat{\sigma}_g\left(z, \frac{Q^2}{\boldsymbol{k}^2}, \alpha_s\right) : \text{off-shell xs} \end{cases}$$

Collinear factorization

$$\sigma \sim \int \frac{dz}{z}\, C_g\left(\frac{x}{z}, \alpha_s(Q^2)\right) f_g(z, Q^2)$$

$$\begin{cases} f_g(x, Q^2) : \text{standard PDF} \\ C_g(z, \alpha_s) : \text{on-shell coefficient function} \end{cases}$$

Defining

$$\mathcal{F}_g(N, \boldsymbol{k}) = U\left(N, \frac{\boldsymbol{k}^2}{Q^2}\right) f_g(N, Q^2)$$

we get

[MB,Marzani,Peraro 1607.02153]

$$C_g(N, \alpha_s) = \int d^2\boldsymbol{k}\; \hat{\sigma}_g\left(N, \frac{Q^2}{\boldsymbol{k}^2}, \alpha_s\right) U\left(N, \frac{\boldsymbol{k}^2}{Q^2}\right)$$

At LLx accuracy, U has a simple form, in terms of small-x resummed anom dim γ

$$U\left(N, \frac{\boldsymbol{k}^2}{Q^2}\right) \approx \boldsymbol{k}^2 \frac{d}{d\boldsymbol{k}^2} \exp \int_{Q^2}^{\boldsymbol{k}^2} \frac{d\nu^2}{\nu^2} \gamma(N, \alpha_s(\nu^2))$$

- Only known at LLx
- Just uses the off-shell cross sections $\hat{\sigma}(N, Q^2/\boldsymbol{k}^2, \alpha_s)$ (one for each process)
- Formally equivalent to ABF (practically easier and numerically stabler)