

# New developments in low $x$ QCD theory

Anna Staśto



*LHeC and FCC-eh Workshop, CERN, September 11-13, 2017*

# Outline

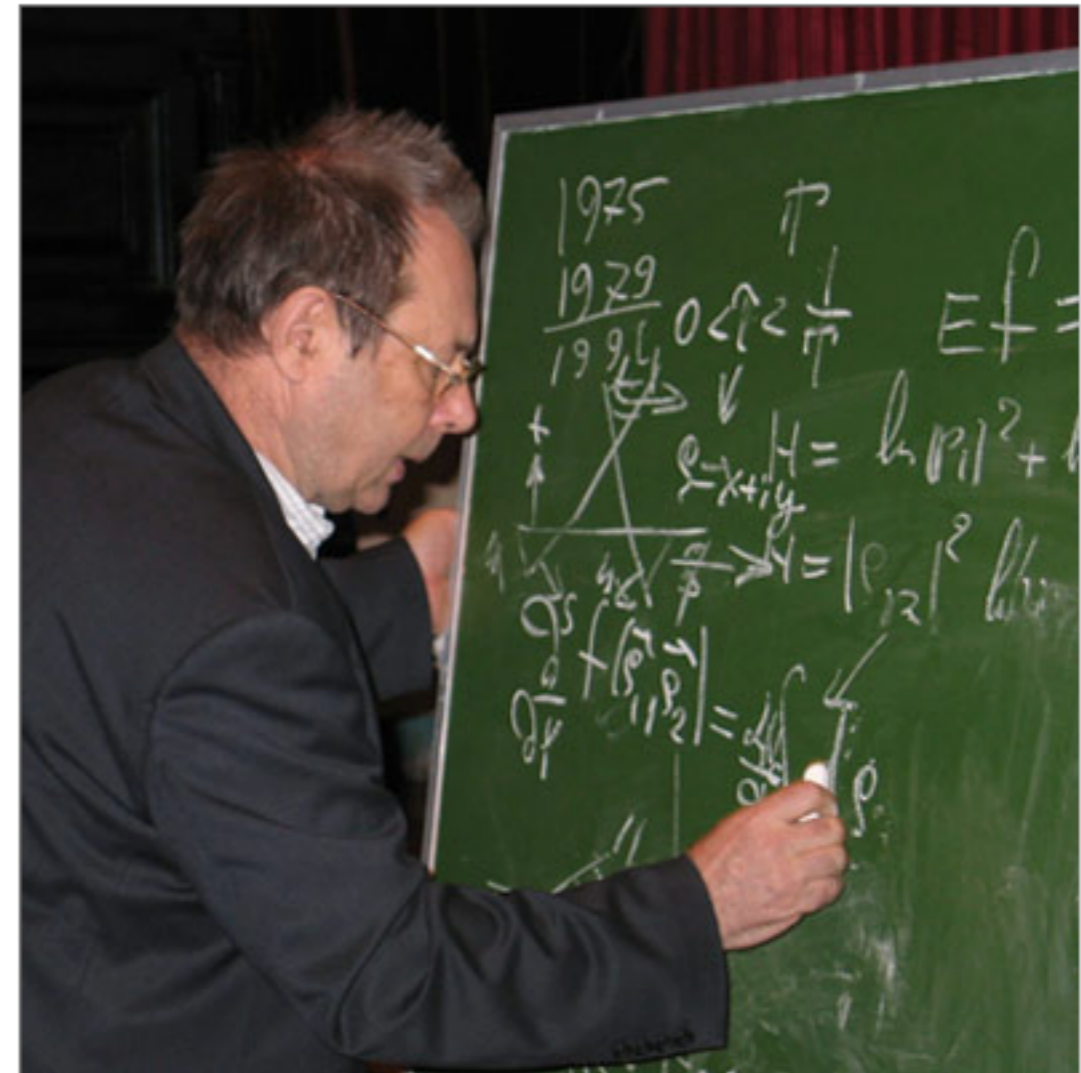
- Introduction: from Regge limit to gluon saturation
- Small  $x$  evolution: higher orders, resummation and saturation
- Impact parameter dependence and low  $x$ : mapping the interaction region at low  $x$

*This presentation will provide with the theoretical background: more simulations and results for LHeC will be presented in the talk by Paul Newman*

# Lev Nikolaevich Lipatov

*2 May 1940 - 4 September 2017*

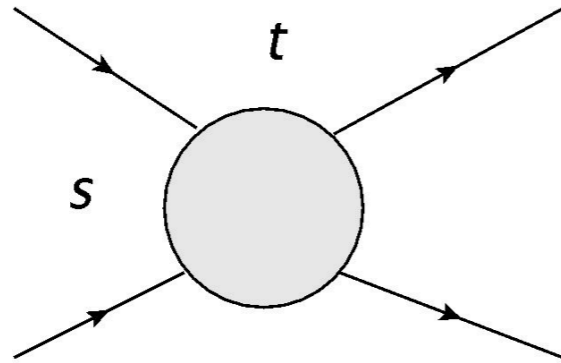
- Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution equations in QCD.
- Seminal paper on Pommeranchuk singularity in QCD: Balitskii-Fadin-Kuraev-Lipatov (BFKL) evolution equation for high energy QCD.
- Effective action for high energy Regge limit in QCD and gravity.
- DGLAP-BFKL duality in N=4 SYM theory.
- Connection between high energy QCD and exactly solvable models.



# Regge limit

Pre QCD...

$A(s, t)$



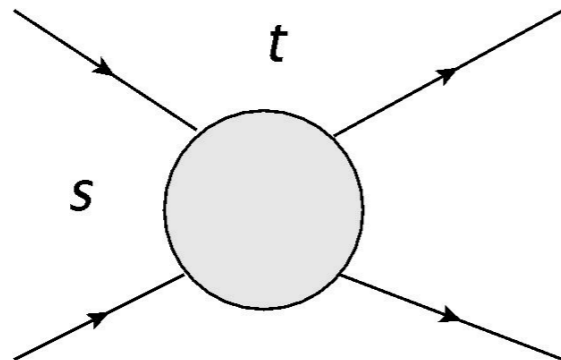
Properties of S matrix:

- Lorentz invariance
- crossing
- unitarity
- analyticity

# Regge limit

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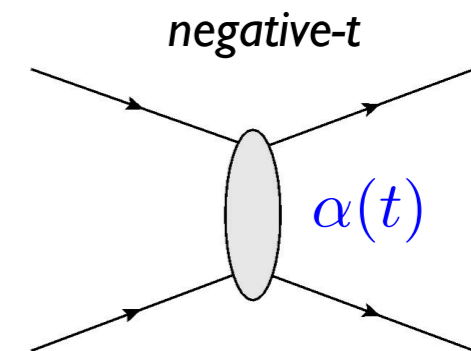


Properties of S matrix:

- Lorentz invariance
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- analyticity

Regge limit:  $s \rightarrow \infty$   $t = \text{const}$

$$A(s, t) \sim \tilde{\beta}(t) s^{\alpha(t)}$$



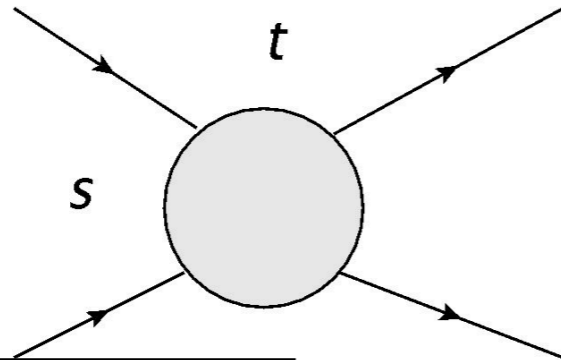
Amplitude dominated by exchange of the Regge trajectory

$$\alpha(t) = \alpha(0) + \alpha' t$$

# Regge limit

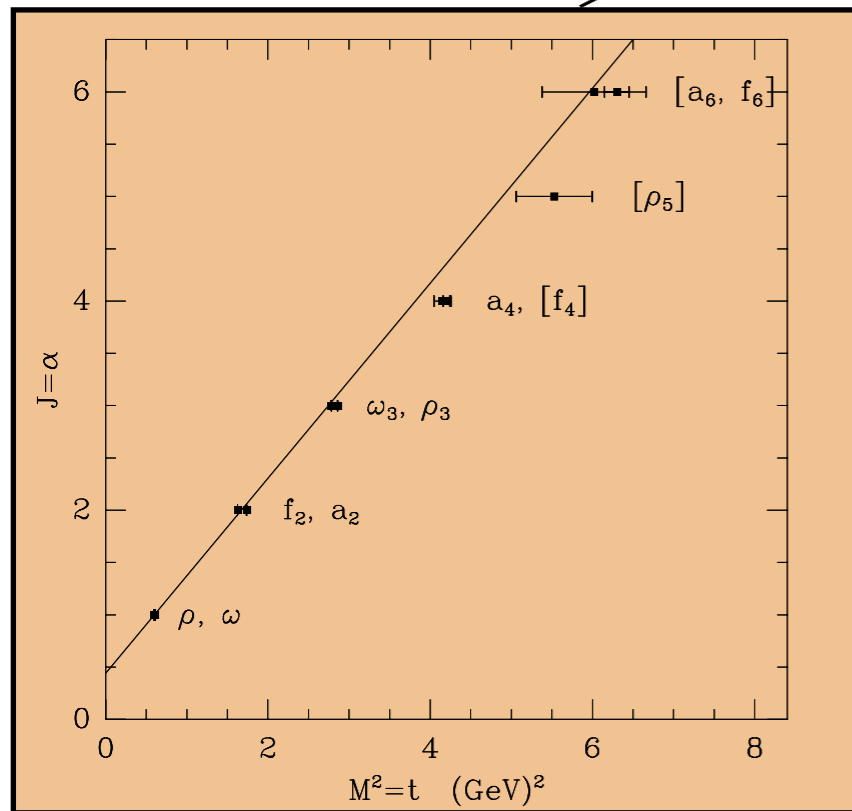
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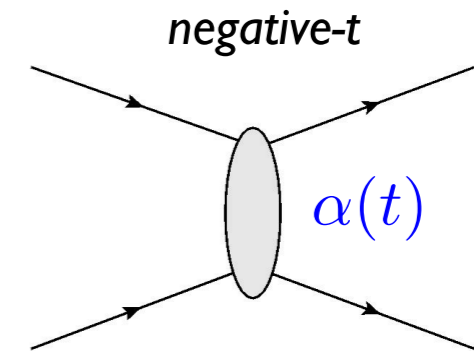
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change of the Regge trajectory

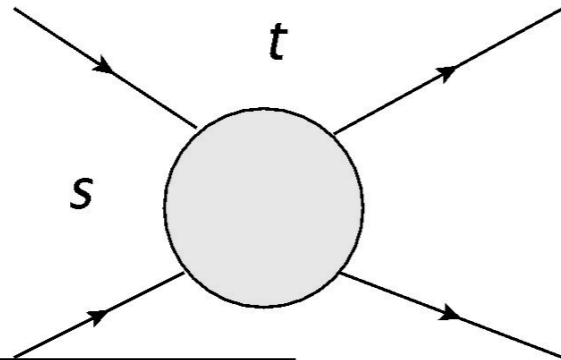


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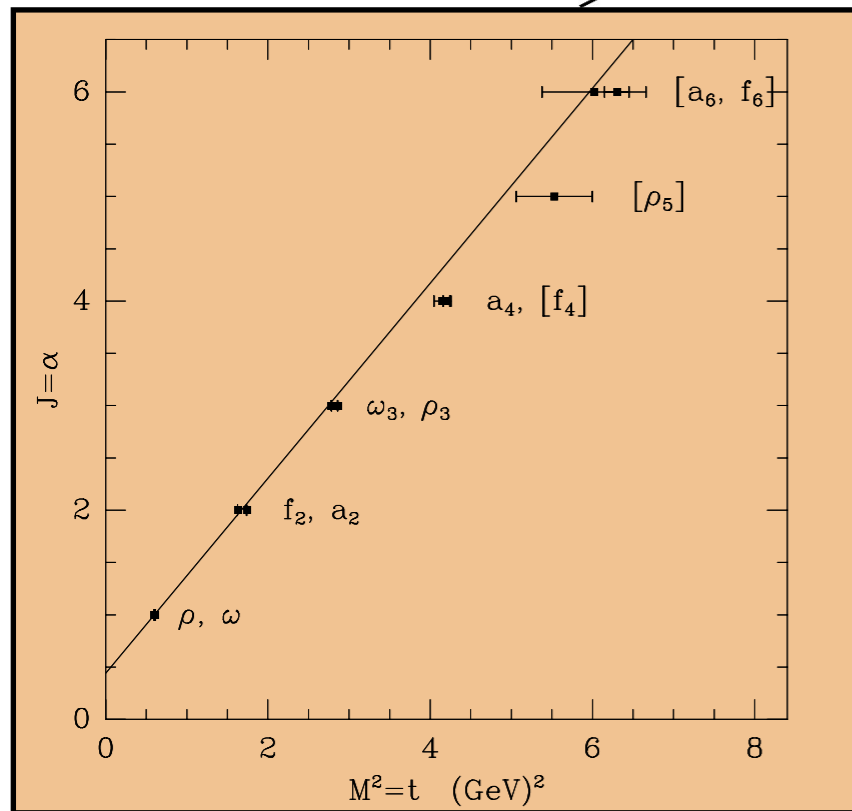
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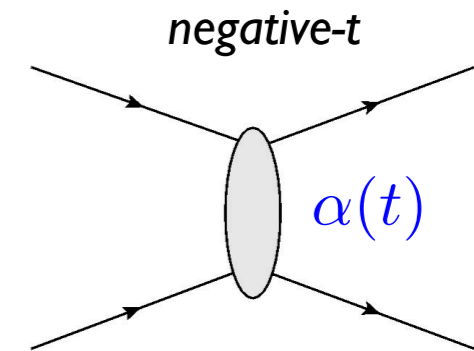
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change of the Regge trajectory

$$\alpha(t) = \alpha(0) + \alpha' t$$

From optical theorem

$$\sigma_{\text{tot}} = s^{-1} \text{Im} A(s, 0) \sim s^{\alpha(0)-1}$$

Intercept  $\alpha(0)$  of Regge trajectory determines the behavior of the cross section

# Pomeron

## *Pomeron:*

*Okun,Pomeranchuk;  
Foldy,Peierls*

- Reggeon with even signature, intercept greater than unity.
- Corresponds to the exchange of the vacuum quantum numbers.
- Dominates the cross section at asymptotically high energies





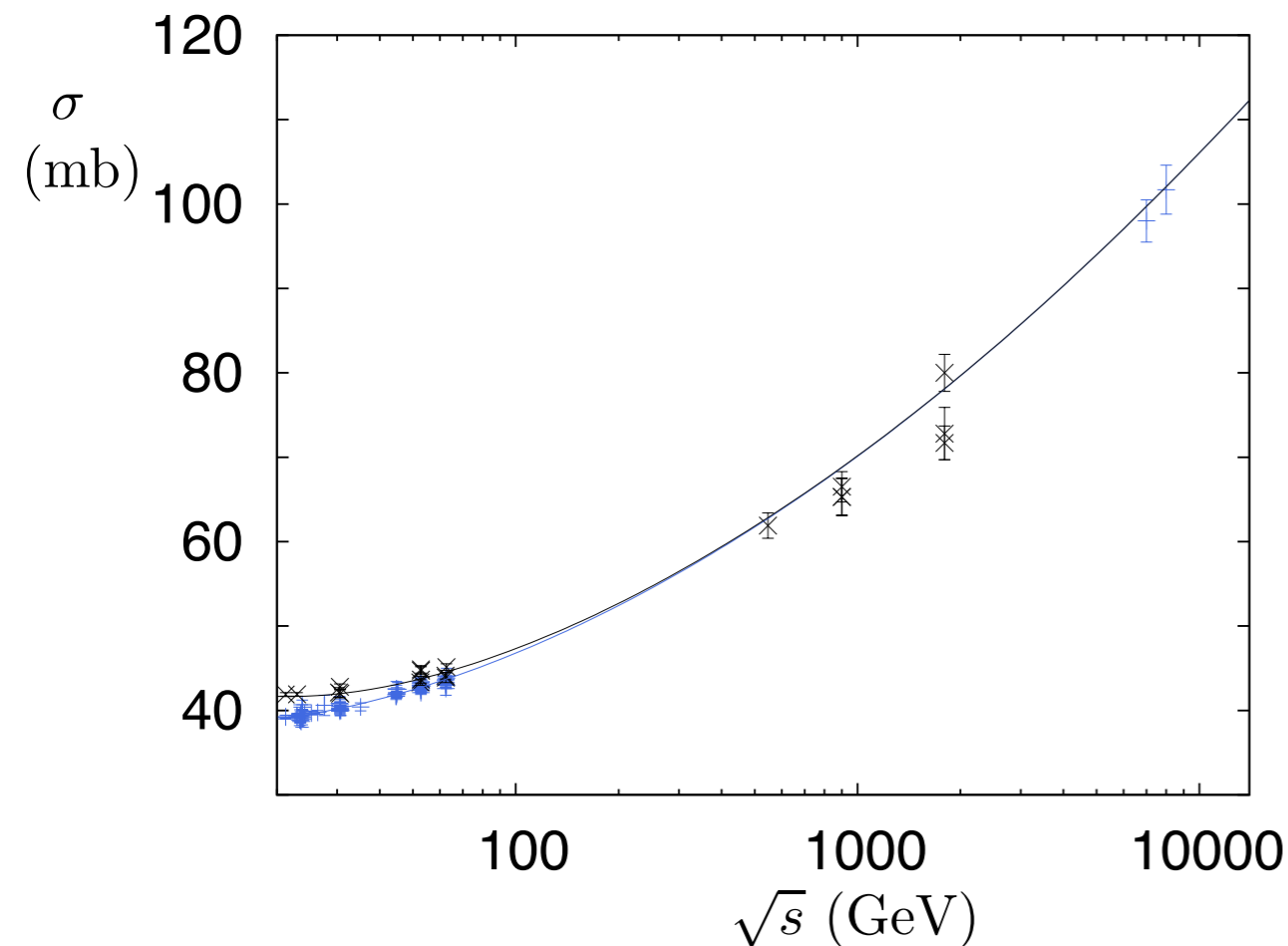
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Donnachie, Landshoff



## Soft Pomeron

$$\alpha_P(t) = 1.11 + 0.165 \text{GeV}^{-2} t$$

(2013 parameters of fit to data including LHC)

$$\sigma_{\text{tot}} \sim s^{\alpha_P(0) - 1}$$

However, such soft pomeron power behavior is potentially in conflict with Froissart bound which stems from unitarity requirements:

$$\sigma^{\text{tot}}(s) \leq C \log^2(s/s_0)$$

Note: the exact value of the constant  $C$  is of crucial importance here.

# Pomeron in QCD

What is a Pomeron in QCD?

High energy limit  
in perturbative QCD:

$$s \gg |t|$$

$$\alpha_s \ll 1$$

$$\alpha_s \log s \sim 1$$

# Pomeron in QCD

What is a Pomeron in QCD?

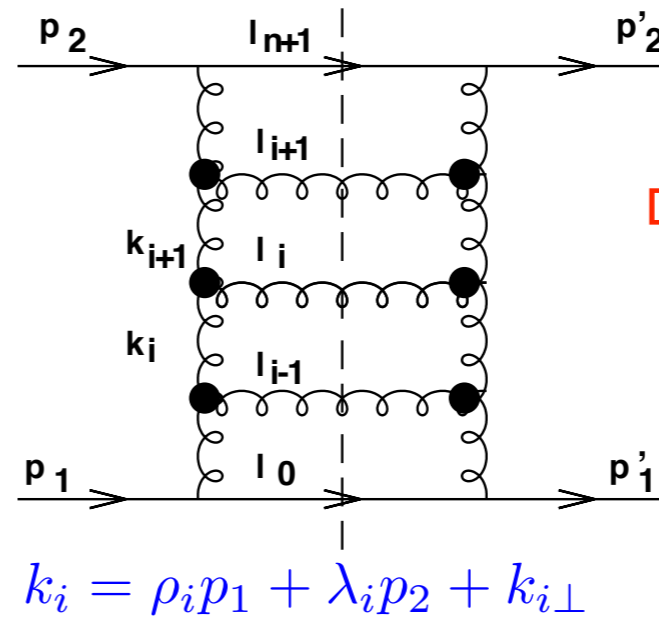
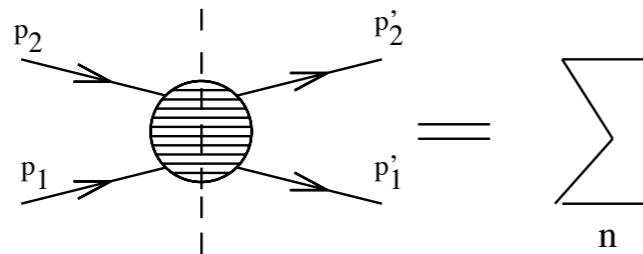
High energy limit  
in perturbative QCD:

$$s \gg |t| \quad \alpha_s \ll 1 \quad \alpha_s \log s \sim 1$$

$$\text{Im}_s A^R(s, t) = \frac{P^R}{2} \sum_n \int d\Phi_{n+2} \mathcal{A}(p_1, p_2; n+2) \mathcal{A}^*(p'_1, p'_2; n+2)$$

$$2p_1 \cdot p_2 = s$$

$$p_1^2 = p_2^2 = 0$$



Dominance of the gluon emissions  
(highest spin elementary quanta)

$$k_i = \rho_i p_1 + \lambda_i p_2 + k_{i\perp}$$

Multi-Regge kinematics:

$$1 \gg \rho_i \gg \rho_{i+1}$$

transverse momenta are of the same order

# BFKL Pomeron in QCD

In general the cross section for scattering of particles A and B:

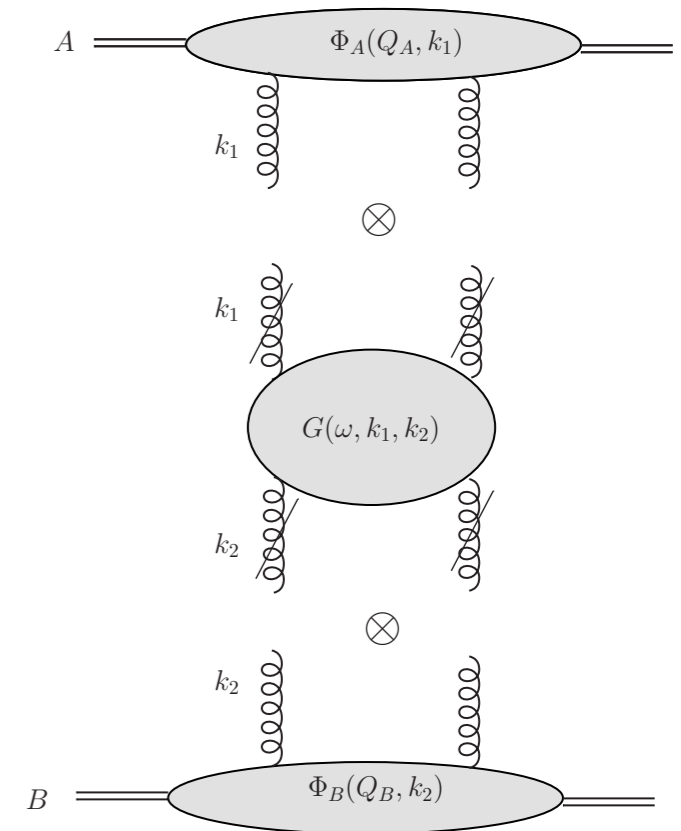
$$\sigma^{AB} = \frac{\text{Im}A(s, 0)}{s}$$

$$\sigma^{AB}(s, Q_A, Q_B) = \int \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^\omega \int \frac{d^2\mathbf{k}_1}{k_1^2} \frac{d^2\mathbf{k}_2}{k_2^2} \Phi_A(Q_A, \mathbf{k}_1) G(\omega; \mathbf{k}_1, \mathbf{k}_2) \Phi_B(Q_B, \mathbf{k}_2)$$

Mellin variable:  $\omega \leftrightarrow \ln s/s_0 \sim Y$

Impact factors:  $\Phi_{A,B}(Q_i, \mathbf{k}_j)$

Gluon Green's function:  $G(\omega; \mathbf{k}_1, \mathbf{k}_2)$   
(Pomeron in QCD)



BFKL equation:

$$\omega G(\omega; \mathbf{k}, \mathbf{k}_0) = \delta^2(\mathbf{k} - \mathbf{k}_0) + \int \frac{d^2\mathbf{k}'}{\pi^2} K(\mathbf{k}, \mathbf{k}') G(\omega; \mathbf{k}', \mathbf{k}_0)$$

$$\frac{d}{dY} G(Y; \mathbf{k}, \mathbf{k}_0) = K(\mathbf{k}, \mathbf{k}') \otimes G(Y; \mathbf{k}', \mathbf{k}_0)$$

Resums gluon emissions strongly ordered in rapidity

Balitskii, Fadin, Kuraev, Lipatov

BFKL kernel:

$$K(\mathbf{k}_1, \mathbf{k}_2) = K_0(\mathbf{k}_1, \mathbf{k}_2) + K_1(\mathbf{k}_1, \mathbf{k}_2) + \mathcal{O}(\alpha^3(\mu^2))$$

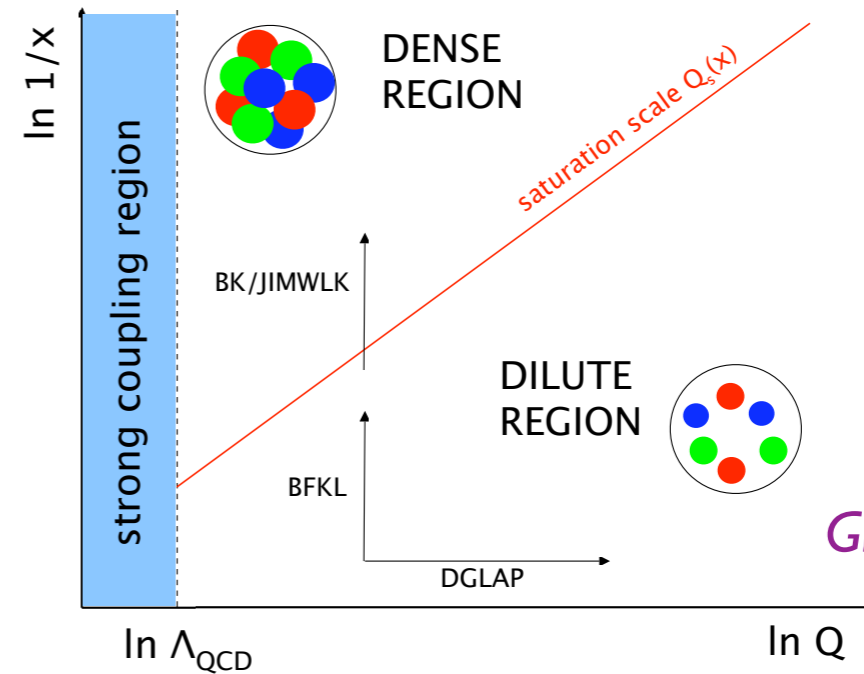
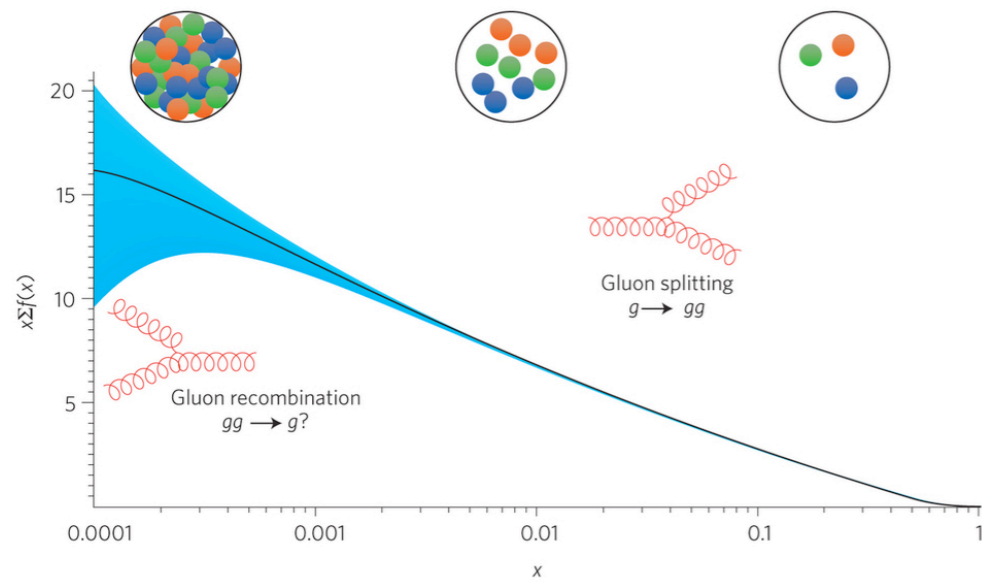
Microscopic realization of Pomeron in perturbative QCD: gluon radiation, strongly ordered in rapidity.

BFKL and Regge factorization is a framework for calculations of processes in the high energy limit.

Question: what is the regime of applicability of this resummation? When will collinear approach break down?

# Parton saturation

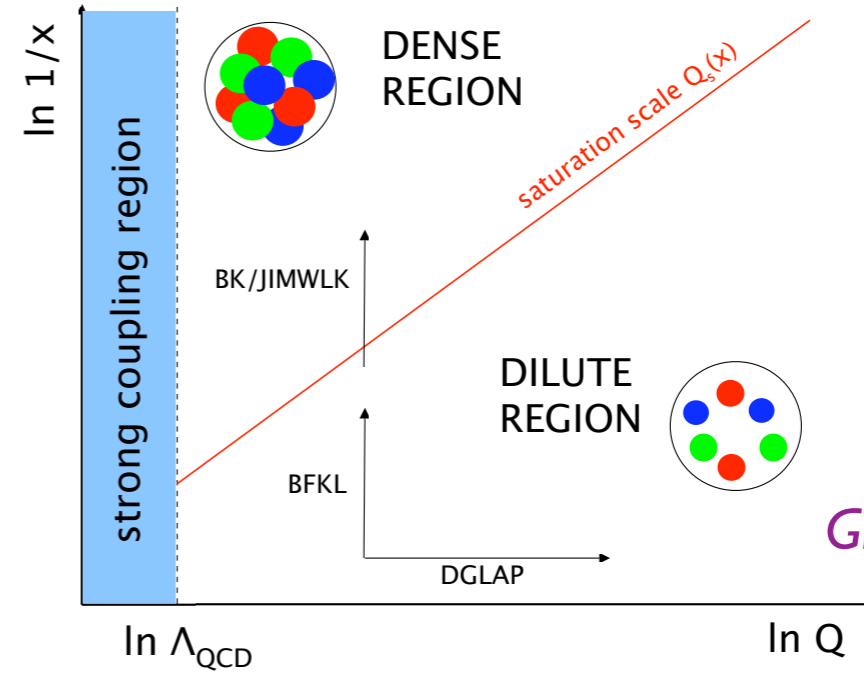
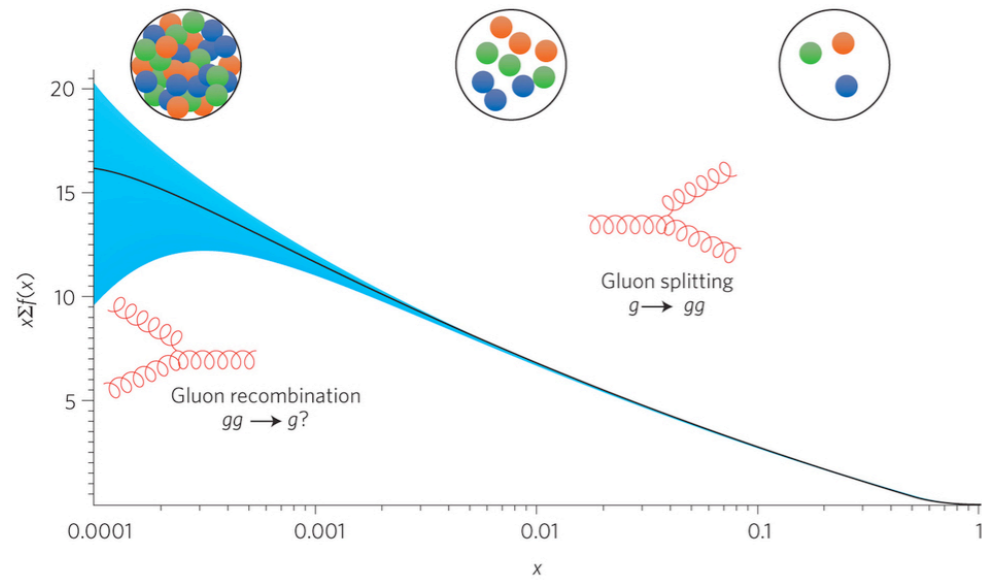
- BFKL predicts strong growth of gluon density with decreasing fraction of longitudinal momentum  $x$ .
- It is too strong at LLx for phenomenology. Nevertheless, the experimental data do confirm strong growth of the proton structure function at small  $x$ .
- The growth of the proton structure function is driven by the growth of the gluon density: “gluon ocean”.
- It is expected that eventually the growth should be tamed in order to satisfy the unitarity bounds.



Gribov, Levin, Ryskin

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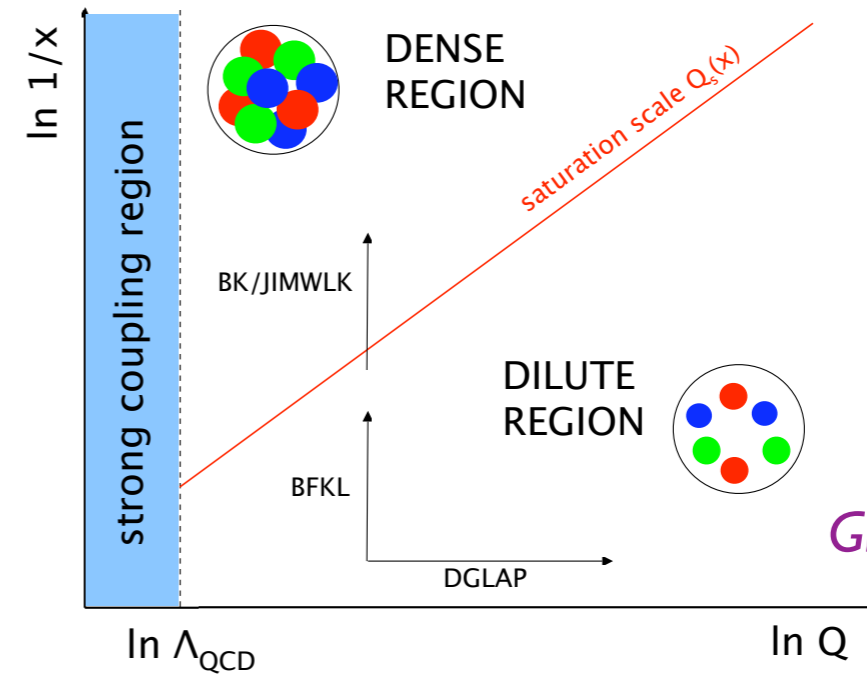
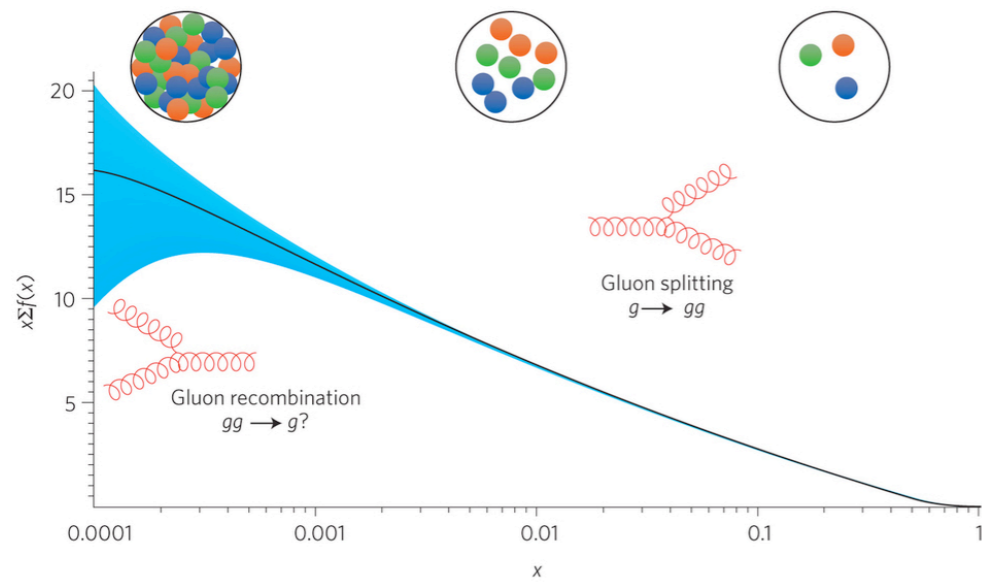
*Gribov, Levin, Ryskin*

Theory predicts the existence of the energy dependent ( $x$  dependent) saturation scale.

$$\frac{A \times xg(x, Q_s^2)}{\pi A^{2/3}} \times \frac{\alpha_s(Q_s^2)}{Q_s^2} \sim 1 \quad Q_s^2 \sim A^{1/3} Q_0^2 \left(\frac{1}{x}\right)^\lambda$$

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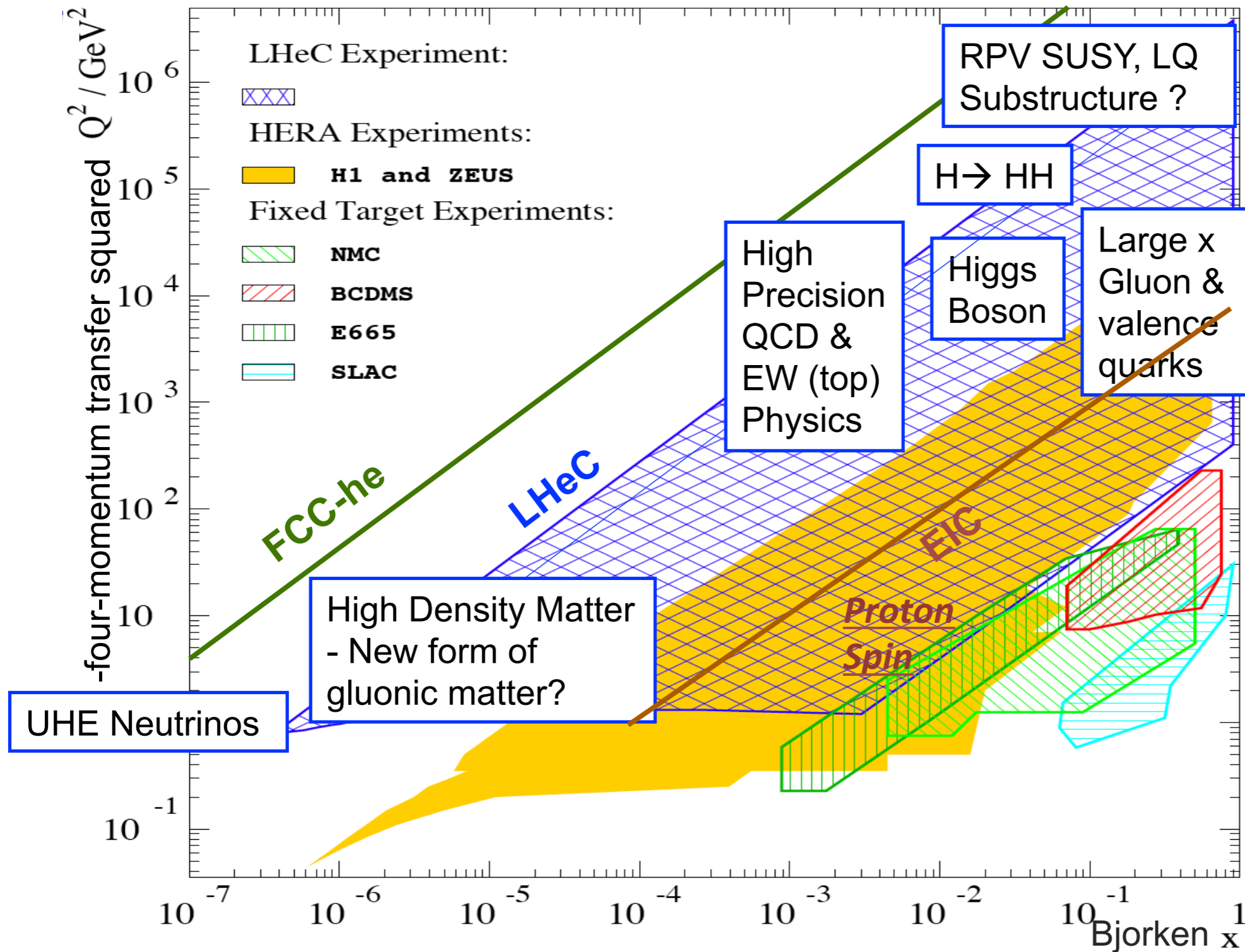
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Theoretical frameworks of parton saturation at small  $x$ :

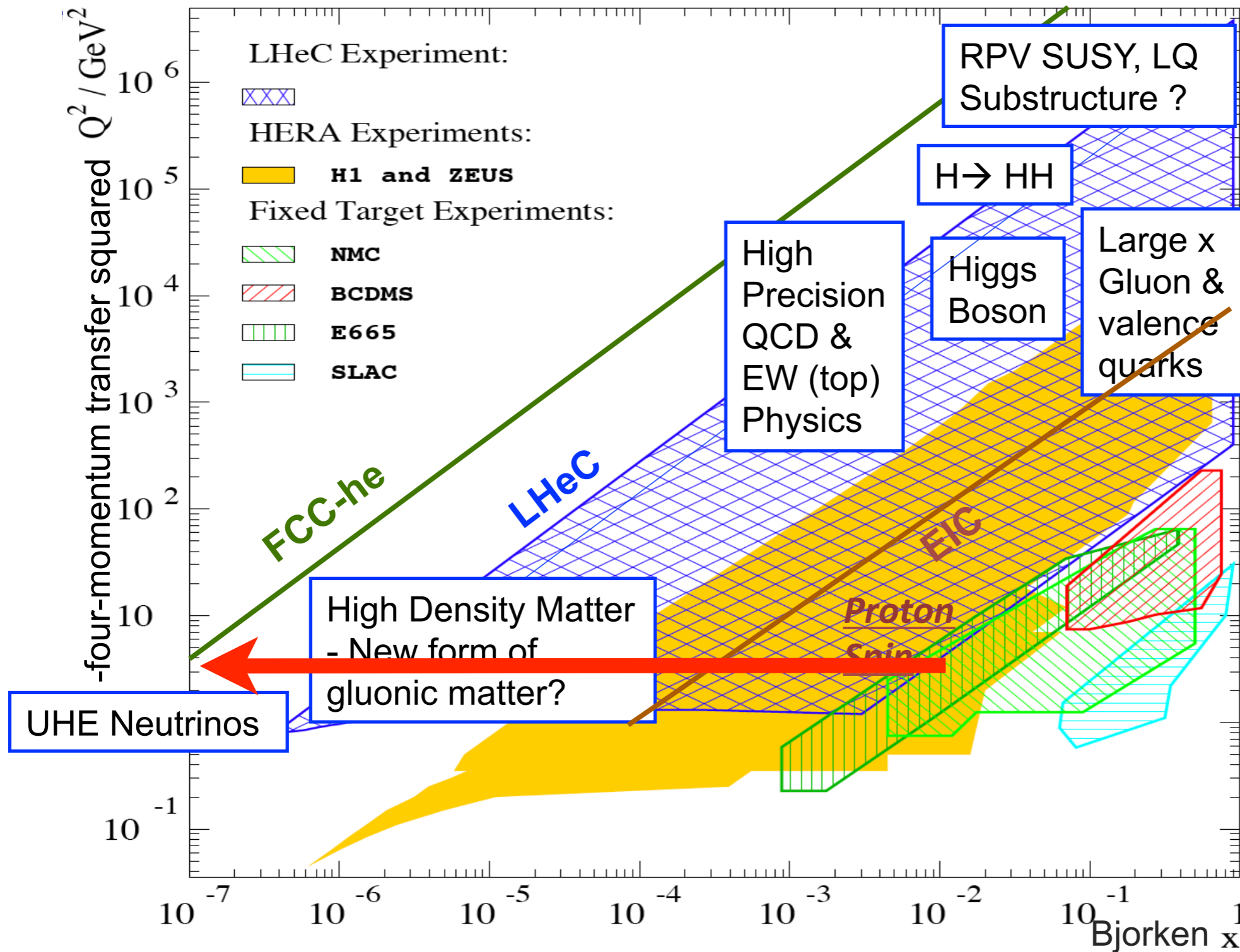
- *Gribov-Levin-Ryskin* nonlinear equation
- *Mueller-Qiu* equation
- *Kovchegov* nonlinear equation for dipoles
- *Balitsky* hierarchy for correlators of Wilson lines.
- Color Glass Condensate (*McLerran-Venugopalan*) with JIMWLK (*Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner*) renormalization group equation for high density QCD.



# LHeC and FCC-eh as low x machines



# LHeC and FCC-eh as low x machines



LHeC and FCC-eh are ideal machines to study low x phenomena.

Can keep  $Q^2$  fixed in semi-hard but pQCD regime, and go to low x: Regge limit

# BFKL at NLLx

*Fadin, Lipatov; Camici, Ciafaloni*

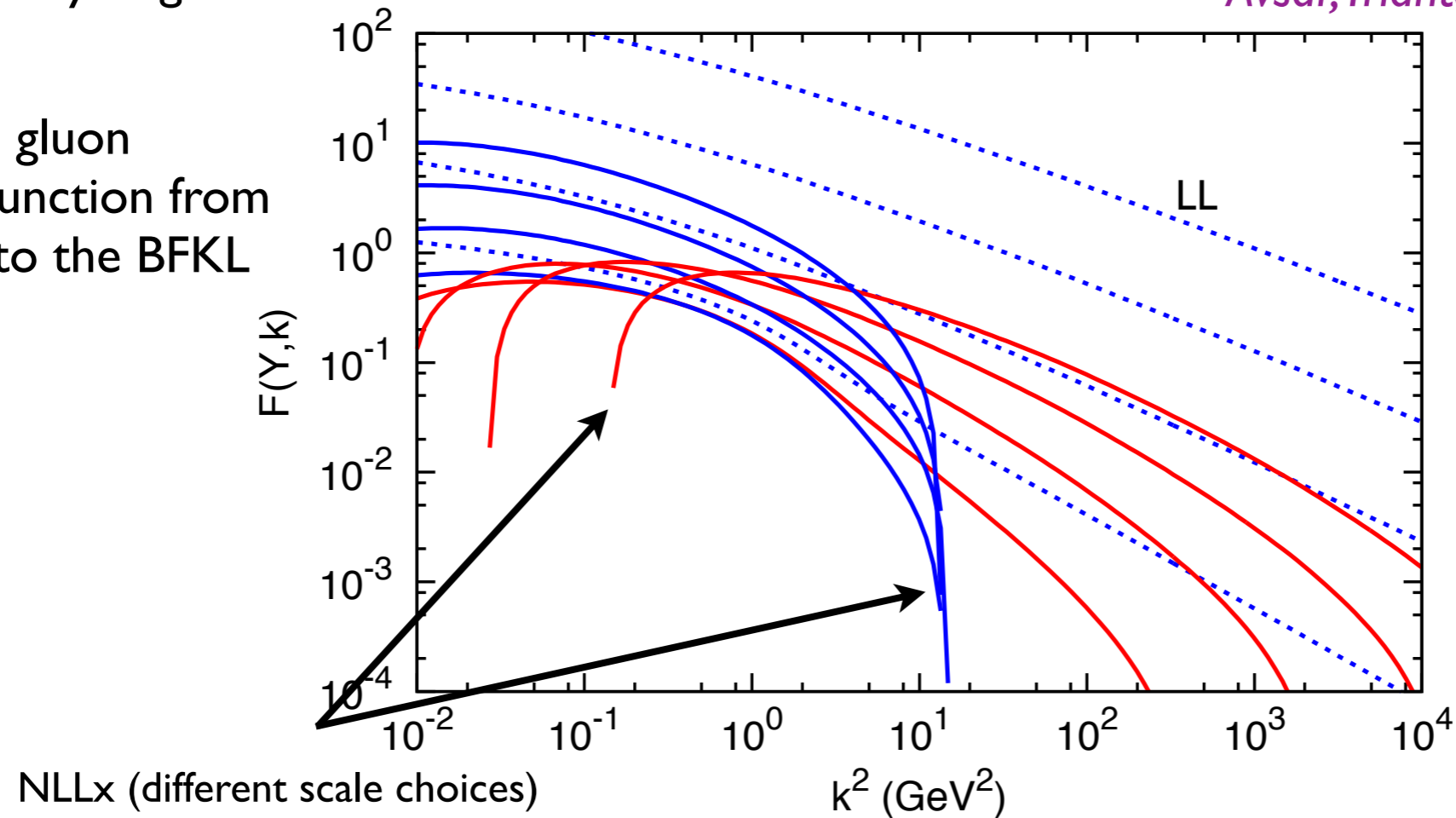
Calculation of BFKL at NLLx

*Ross*

NLLx is a very large correction and leads to some instabilities:

*Avsar, Triantafyllopoulos, Zaslavsky, AS*

Unintegrated gluon  
distribution function from  
the solution to the BFKL  
equation



Resummation is necessary...

*Are NLL instabilities also present in the nonlinear equation?  
Or do they get 'cured' by saturation?*

# Balitsky-Kovchegov equation at LLx

Balitsky; Kovchegov

linear BFKL part

nonlinear : saturation part

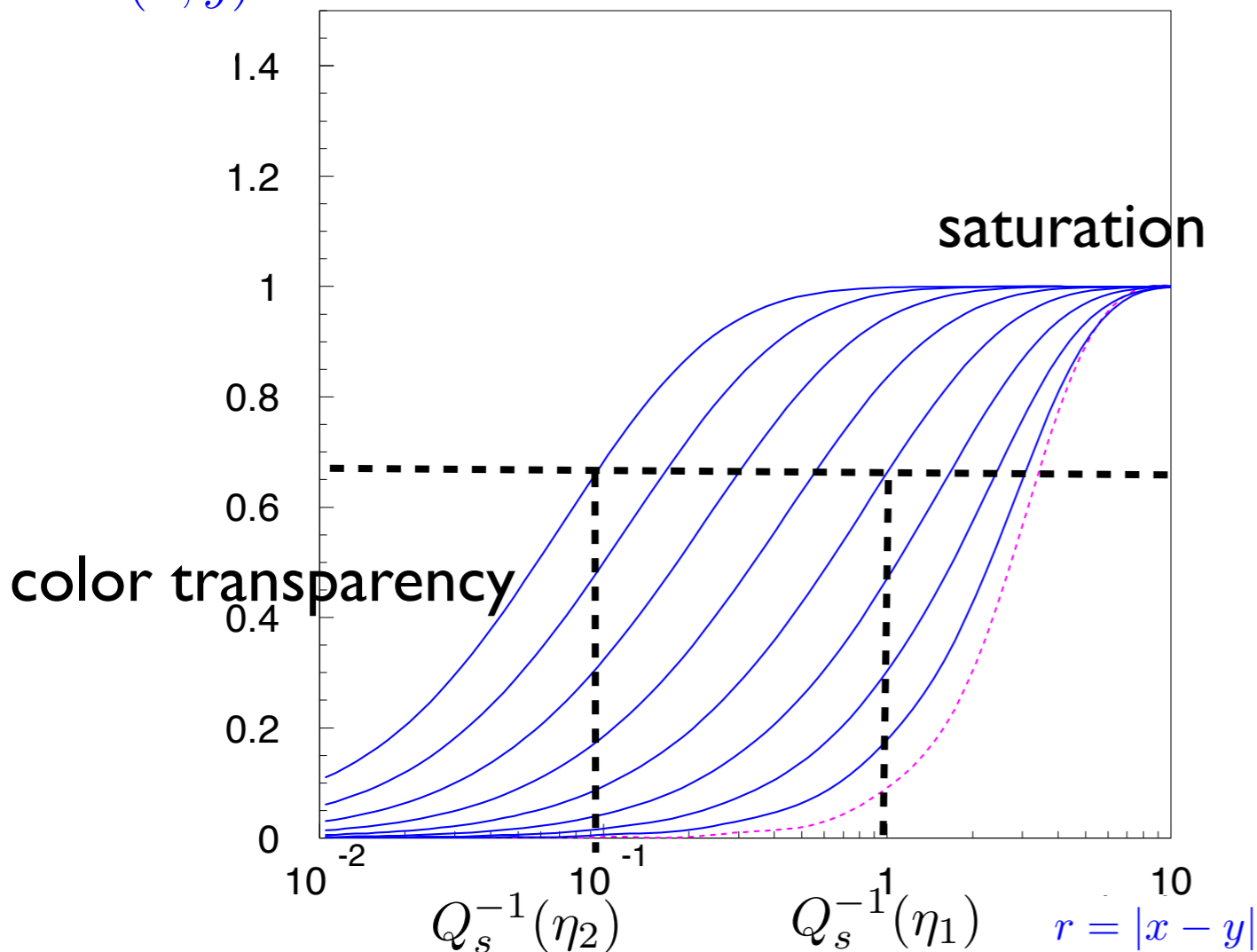
$$\frac{d}{d\eta} \hat{U}(x, y) = \frac{\alpha_s N_c}{2\pi^2} \int d^2z \frac{(x-y)^2}{(x-z)^2(z-y)^2} [\hat{U}(x, z) + \hat{U}(y, z) - \hat{U}(x, y) - \hat{U}(x, z)\hat{U}(z, y)]$$

Dipole operator:  $\hat{U}(x, y) \equiv \hat{U}^\eta(x, y)$

Can obtain unintegrated gluon distribution from the dipole operator

$$r = |x - y| \leftrightarrow \frac{1}{k}$$

$\hat{U}^\eta(x, y)$



$$\eta = \ln \frac{1}{x_B} \quad \text{rapidity}$$

$$x \equiv x_\perp$$

$$y \equiv y_\perp$$

$$z \equiv z_\perp$$

transverse coordinates

What about interplay of NLL and saturation?

# Balitsky-Kovchegov equation at LLx

Balitsky; Kovchegov

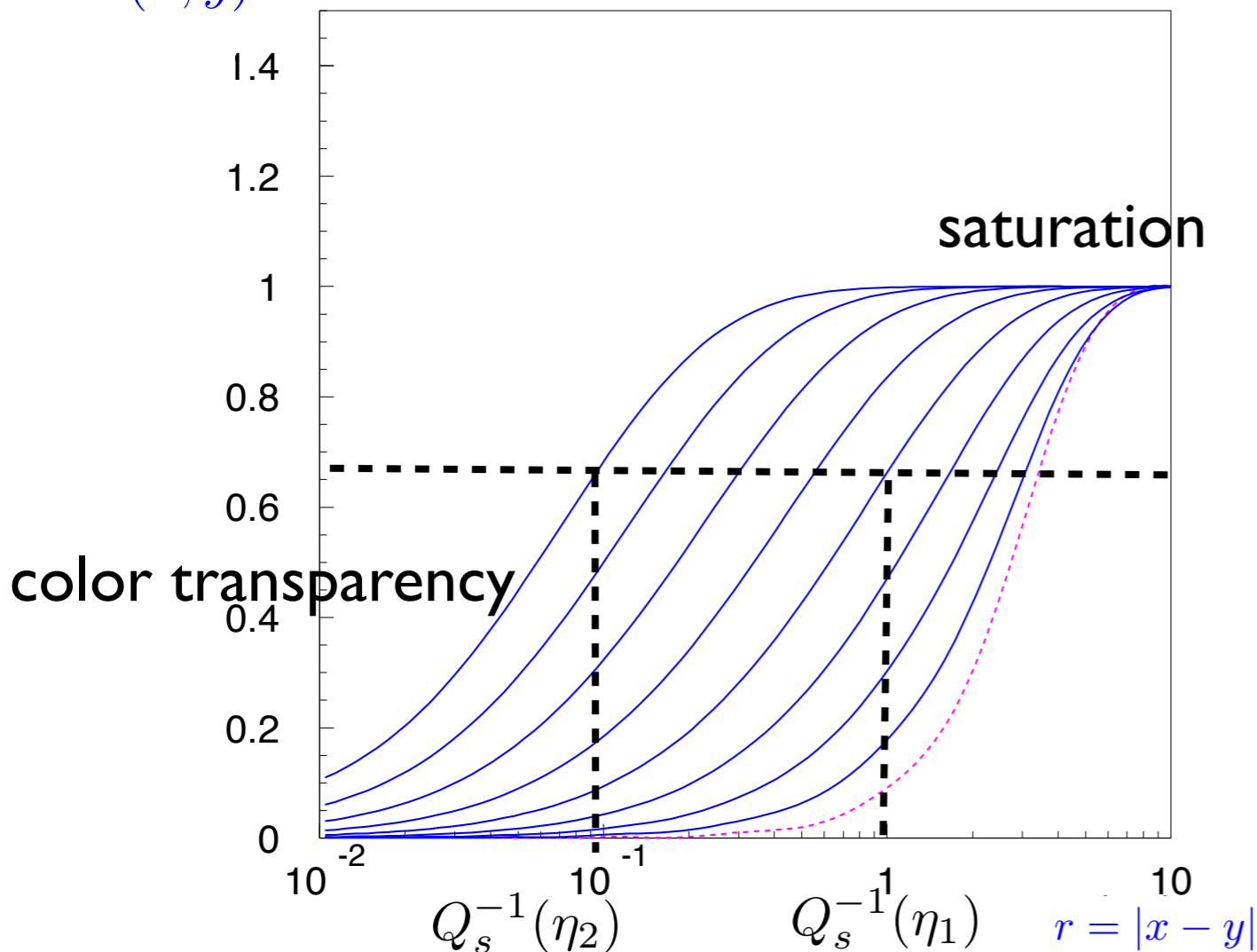
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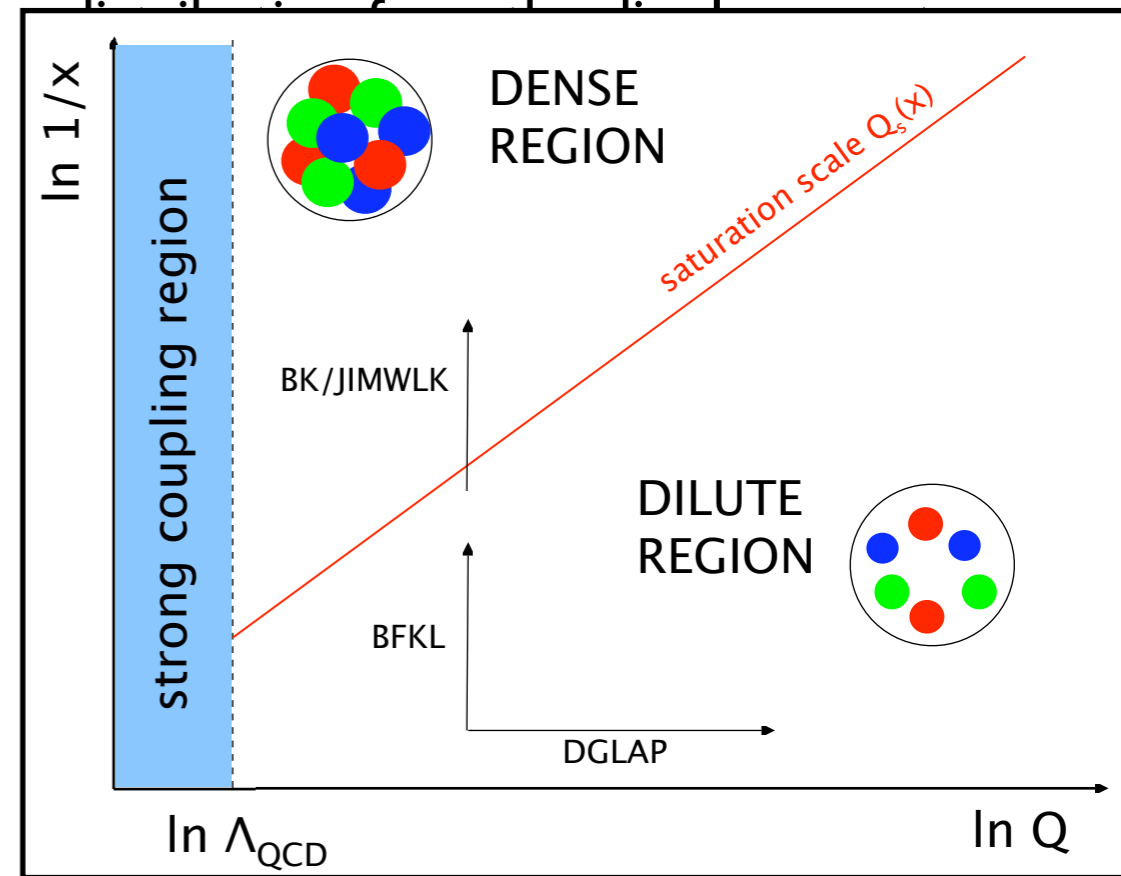
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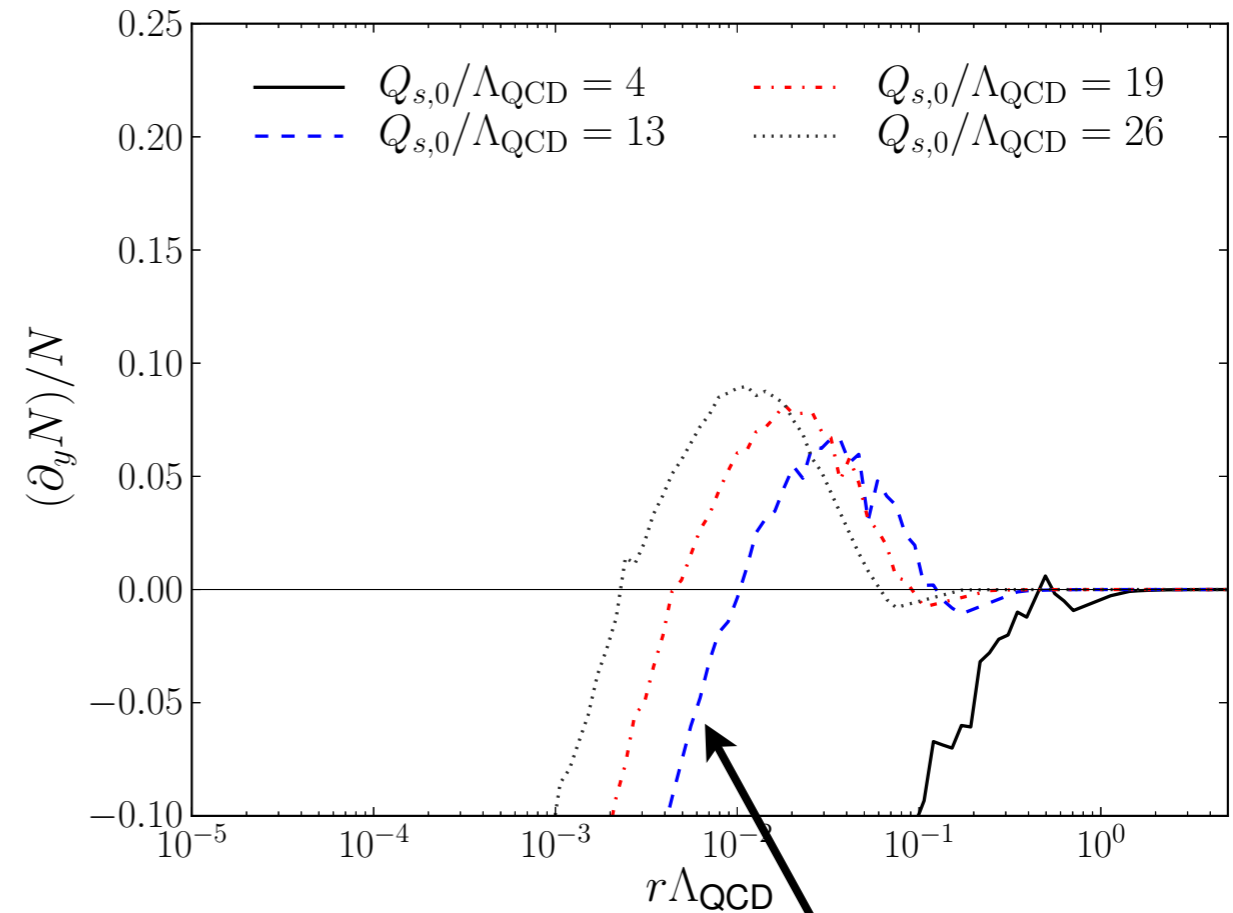
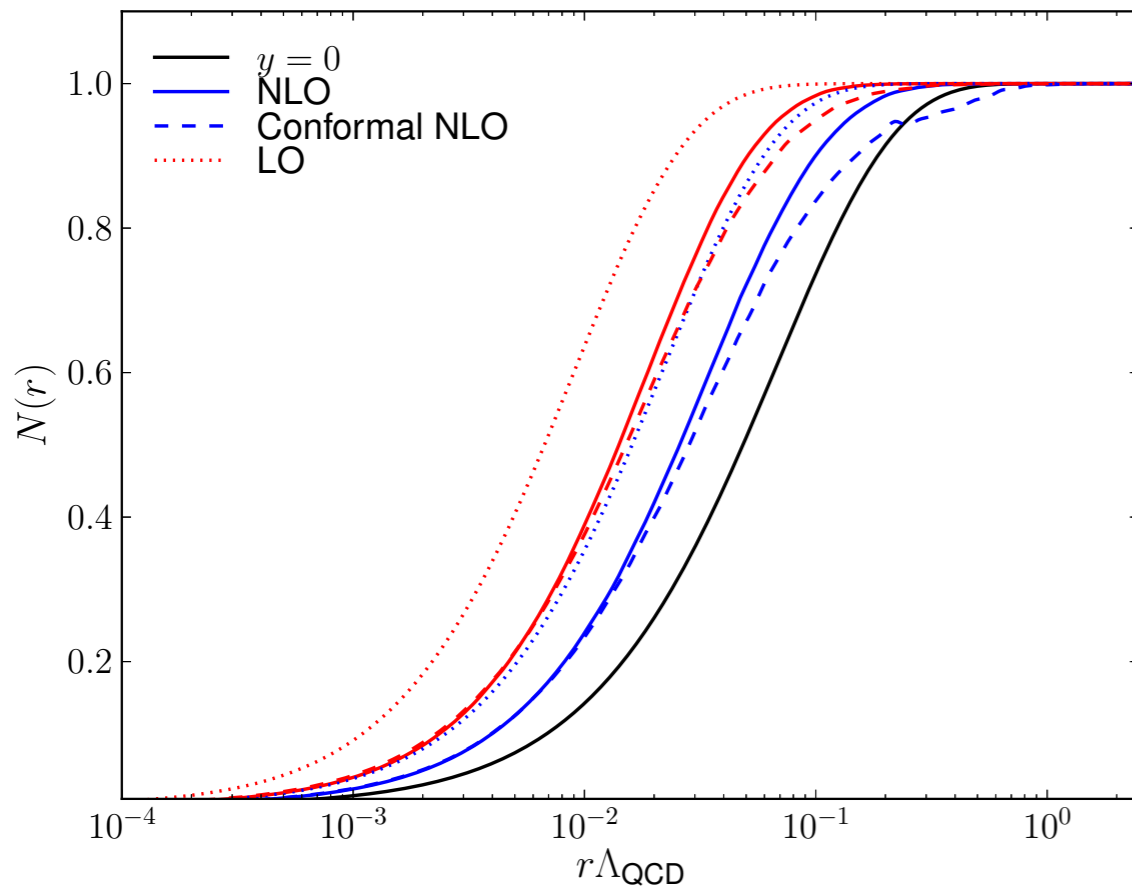


What about interplay of NLL and saturation?

# Numerical solution to BK at NLLx

Lappi, Mantysaari

$$Q_{s,0}/\Lambda_{\text{QCD}} = 19, \gamma = 0.6$$



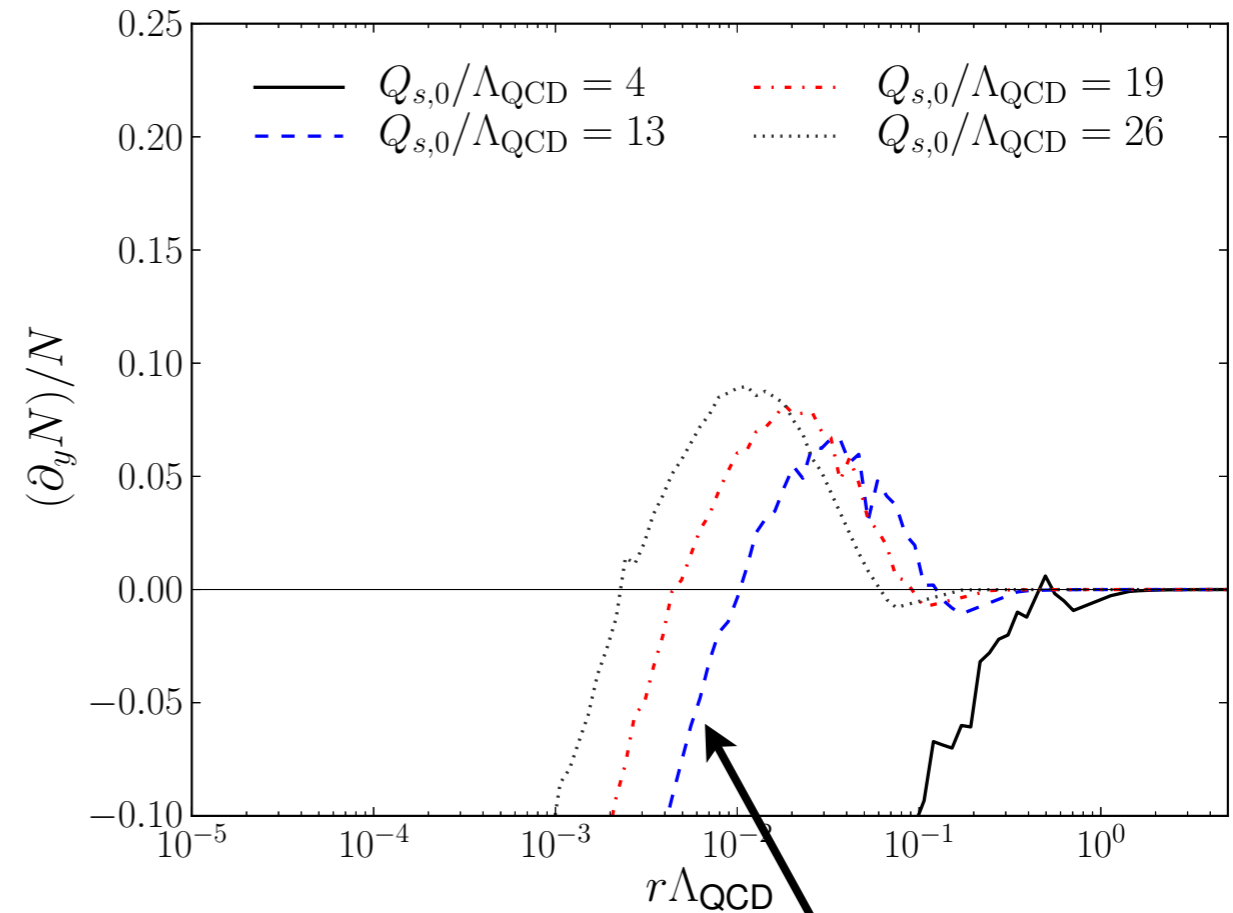
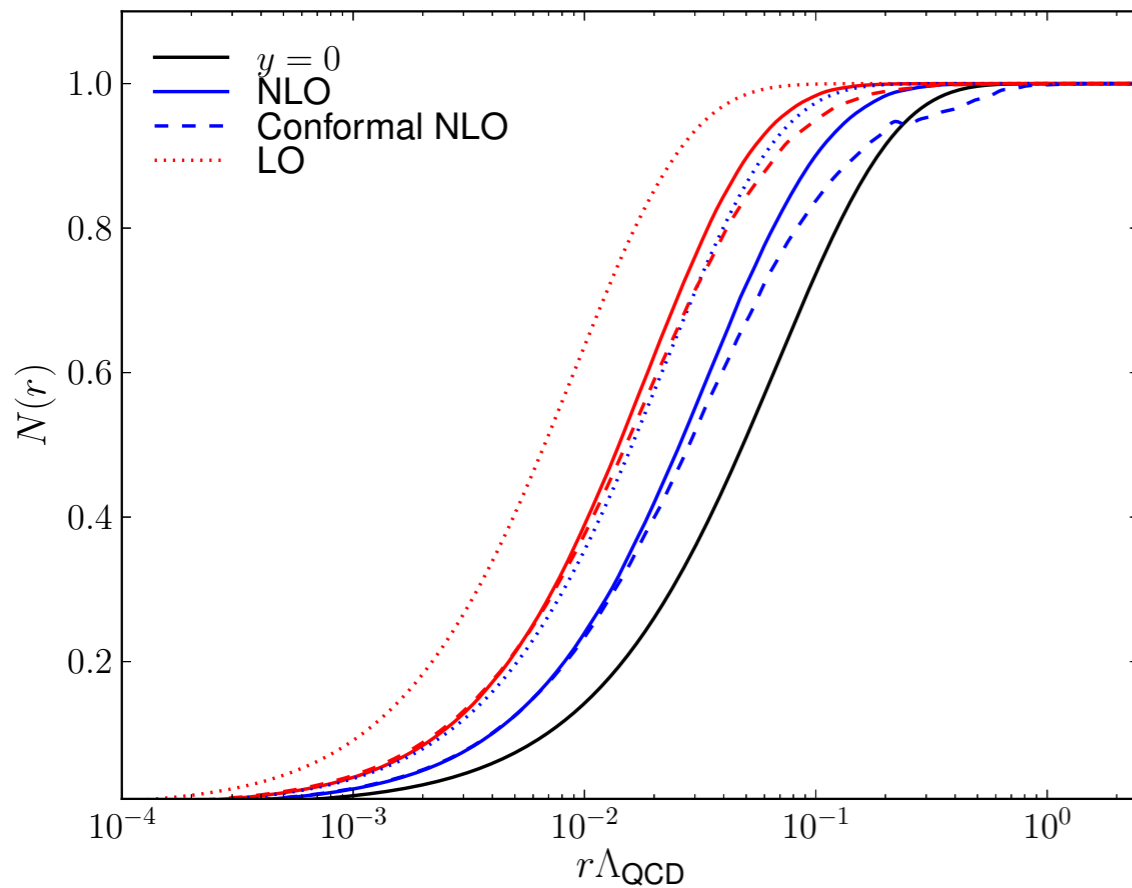
- Evolution speed significantly slowed down with at NLLx order.
- The solution is unstable for some initial conditions.
- Evolution speed turns negative for small dipoles.
- Amplitude becomes negative and unphysical.

$$\frac{\partial_y N(r)}{N(r)} \sim \ln r$$

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*Negativity at NLLx is also present in the calculation with saturation. Low x resummation needed also for the nonlinear case.*

# General setup of resummation for linear BFKL

Andersson, Gustafson, Kharraziha, Samuelsson

Kwiecinski, Martin, Sutton

Kwiecinski, Martin, AS

Salam

Ciafaloni, Colferai, Salam, AS

Altarelli, Ball, Forte

Sabio-Vera

Thorne

also Brodsky, Kim, Lipatov, Pivovarov

BLM scheme

*collinear resummation*

- Kinematical constraint.
- DGLAP splitting function at LO and NLO.
- NLLx BFKL with suitable subtraction of terms included above.
- Momentum sum rule.
- Running coupling.

Resummation yields results which are stable. Comparisons with phenomenology are favorable.

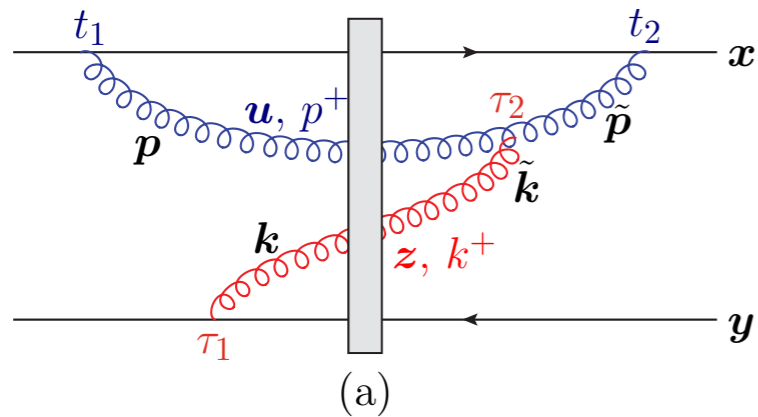
see talk by Marco Bonvini



# Resummation in nonlinear equation

*Iancu, Madrigal, Mueller, Soyez, Triantafyllopoulos*

Gluon emission in dipole formalism



Resummation extended to BK: BFKL with saturation

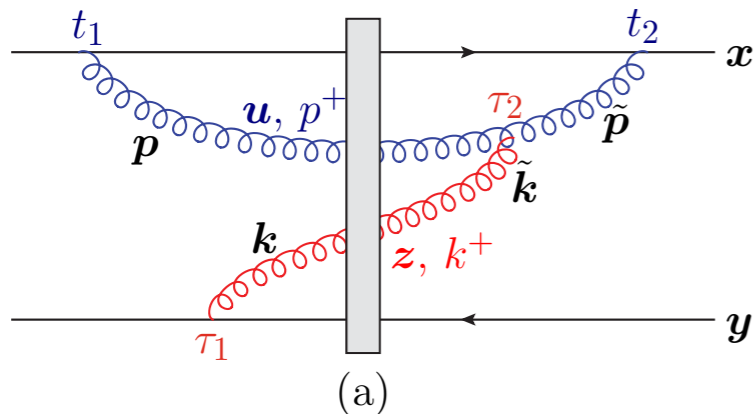
Ordering in fluctuation lifetime for the gluon emissions

$$\Theta(\tau_p - \tau_k) = \Theta(p^+ - k^+ (p^2 / k^2))$$

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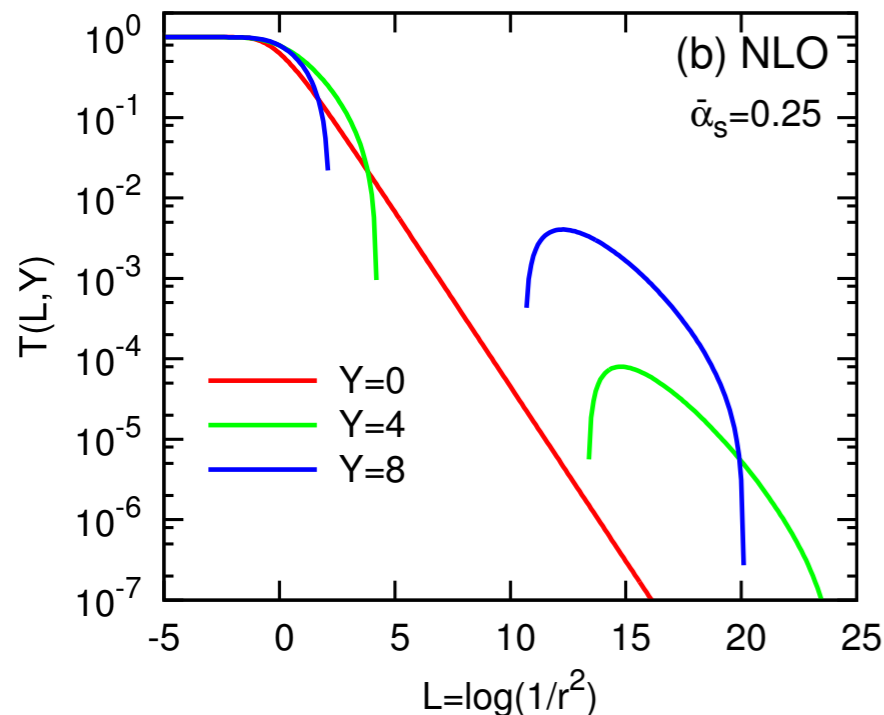
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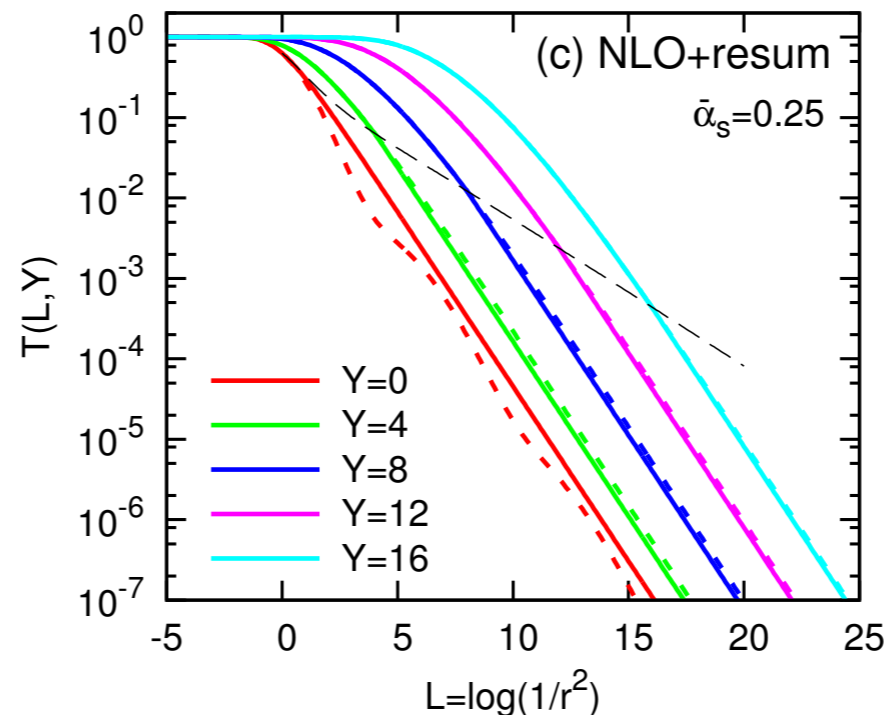
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Kinematical constraint which resums important double logs.

*see also Motyka, AS; Beuf*



NLLx (double logs only)

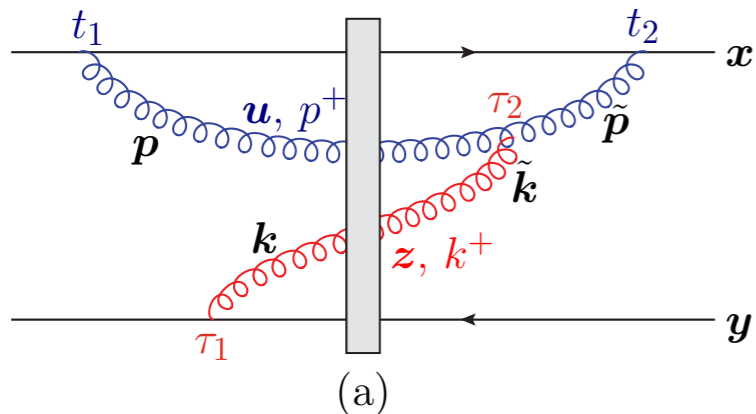


Resummation (rc LLx + including k.c.)

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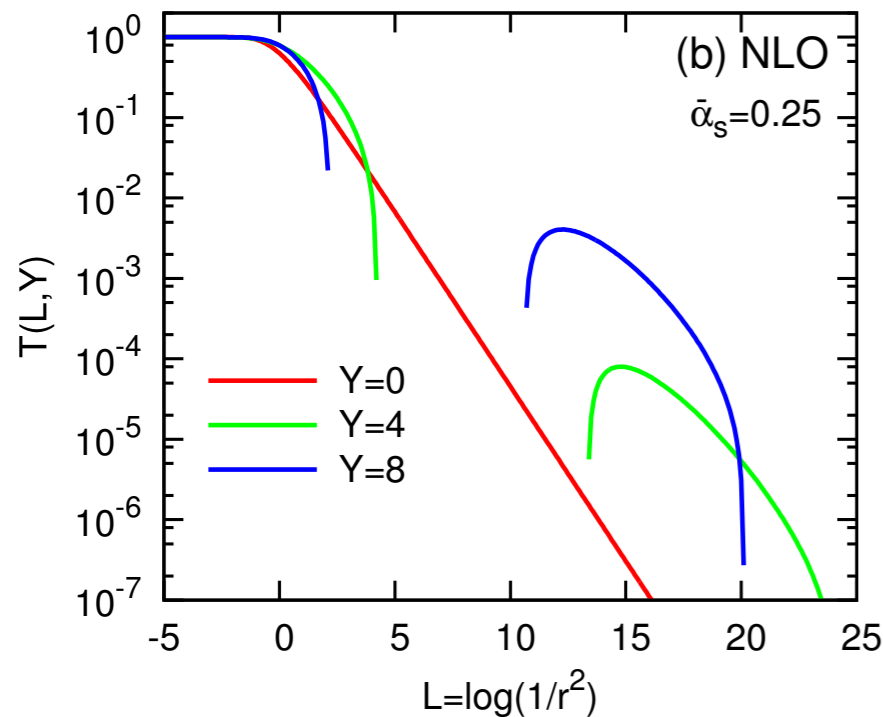
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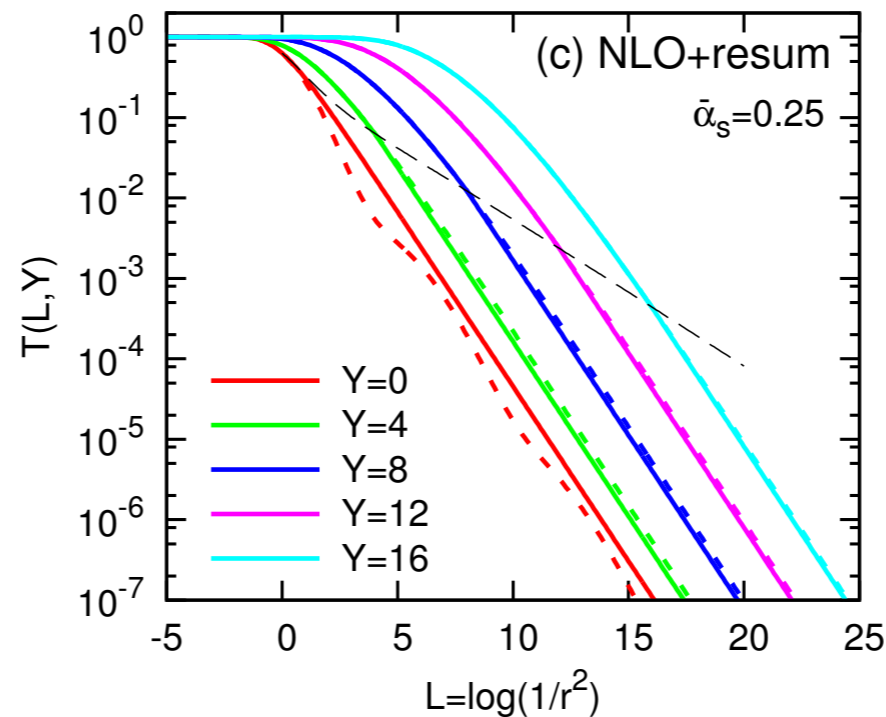
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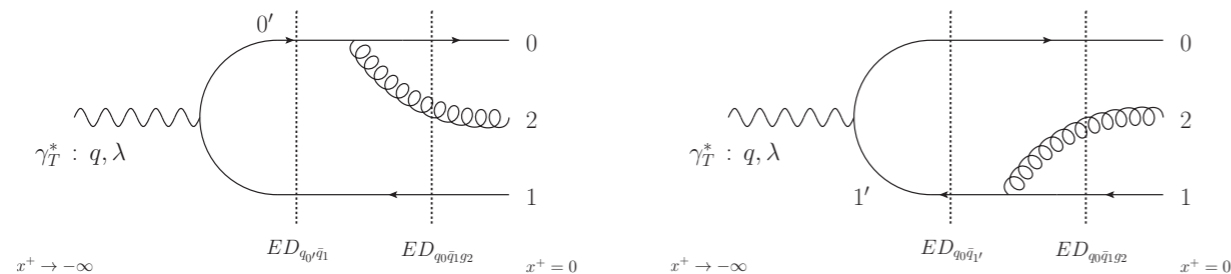
Resummation (rc LLx + including k.c.)

*Important question: can one incorporate resummation of the single logs as well in the nonlinear case?*

# Impact factors at NLL

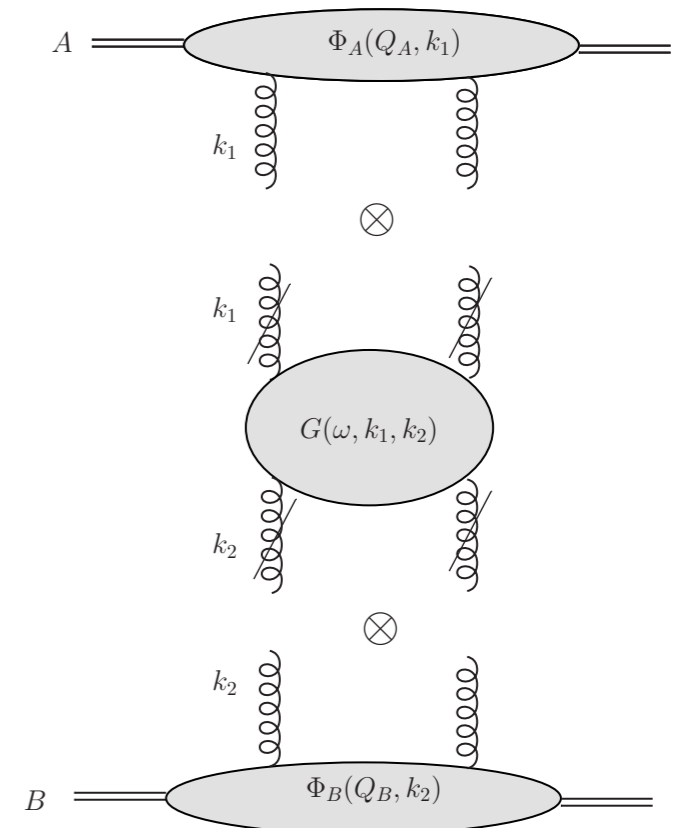
For the complete NLL calculation of the cross section one needs both the NLL calculation of the evolution and the impact factors.

Example of NLL corrections to the photon-gluon impact factor

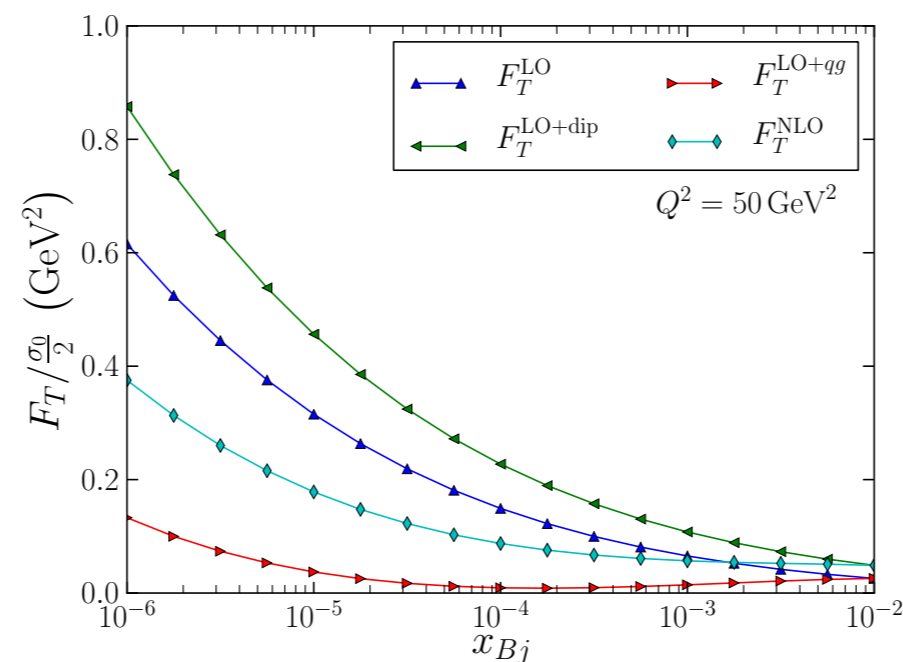


Complete NLL calculation

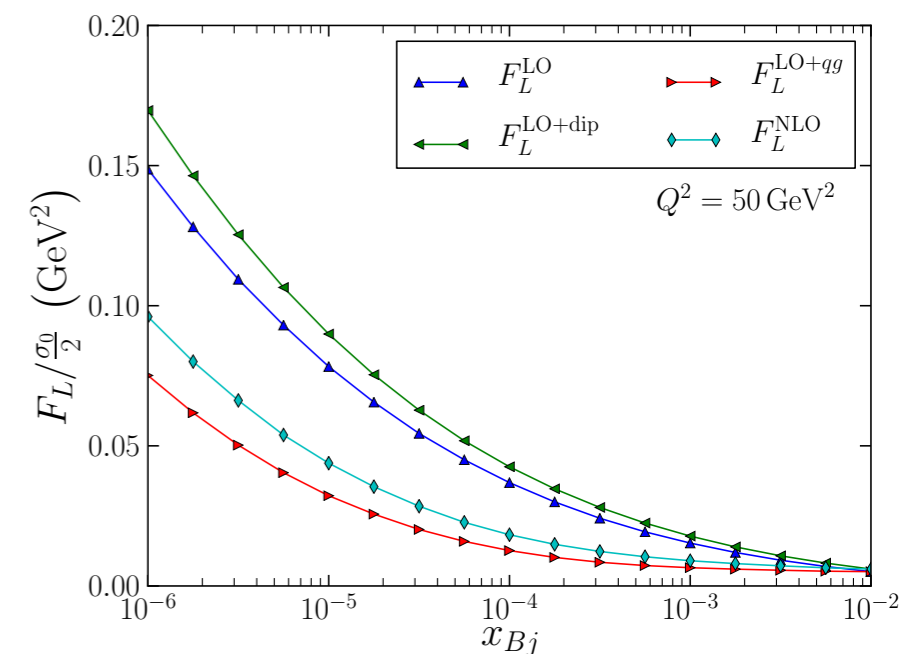
*Beuf; Balitsky, Chirilli*



Numerical studies demonstrated the importance of the NLL correction to the impact factor



*Ducloe, Hanninen, Lappi, Zhu*



# BFKL at NNLL ?

NNLL BFKL correction obtained in N=4 SYM.

*Gromov, Levkovich-Maslyuk, Sizov;  
Velizhanin*

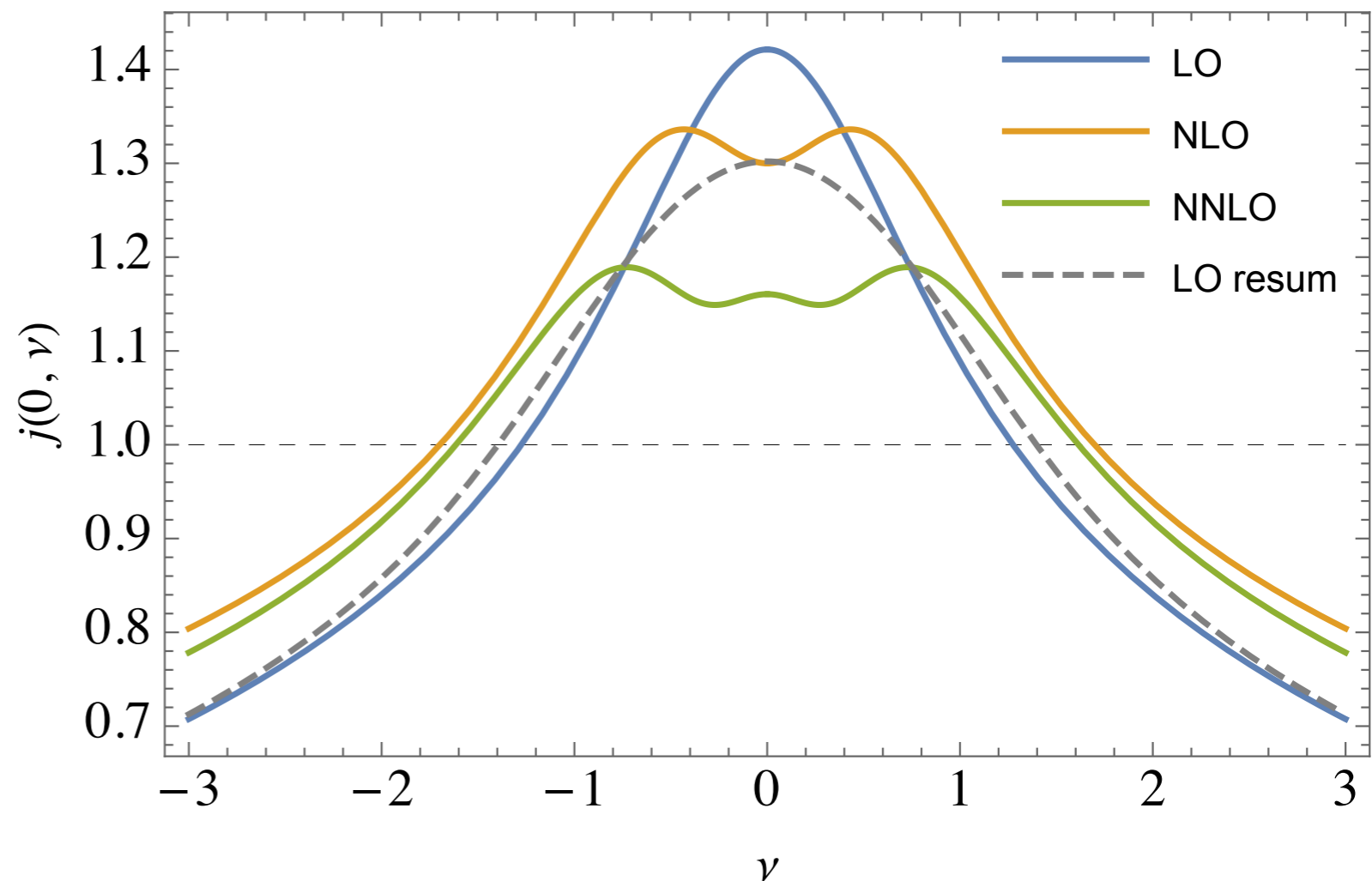
One of the methods relies on the equivalence between the forward scattering and the jet physics of wide-angle soft gluon radiation.

*Caron-Huot, Herranen*

BFKL eigenvalue

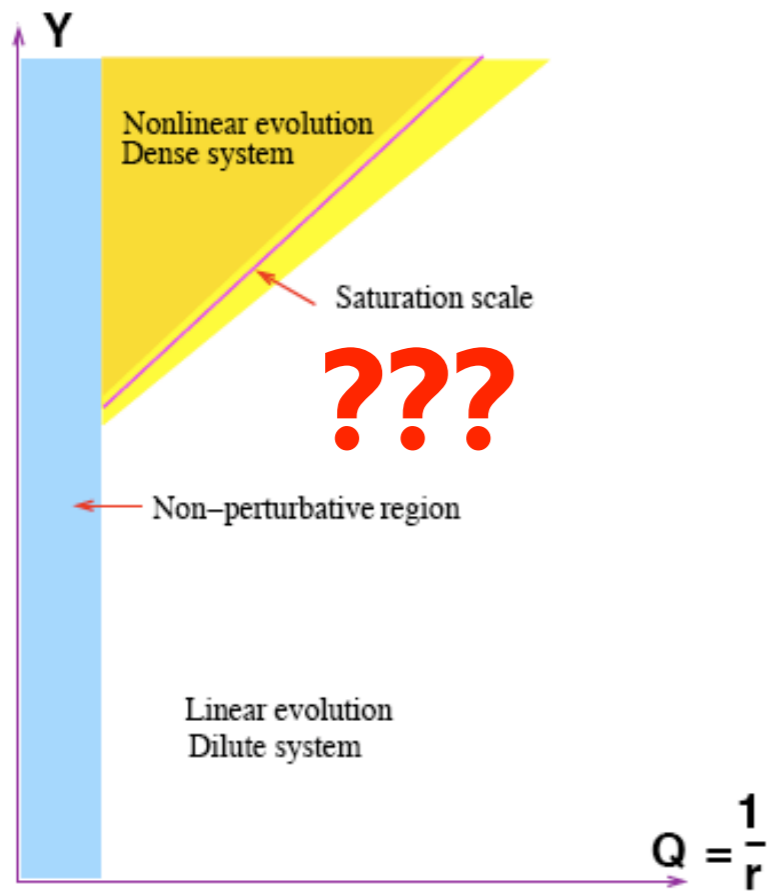
$$j(0, \nu) = 1 + \int \frac{dk'^2}{k'^2} \left( \frac{k'^2}{k^2} \right)^{\frac{1}{2} + \frac{i\nu}{2}} \mathcal{K}(k', k)$$

How the resummation methods developed so far compare with the exact NNLL?



# Impact parameter and low $x$ physics

# What about spatial distribution in small $x$ evolution?

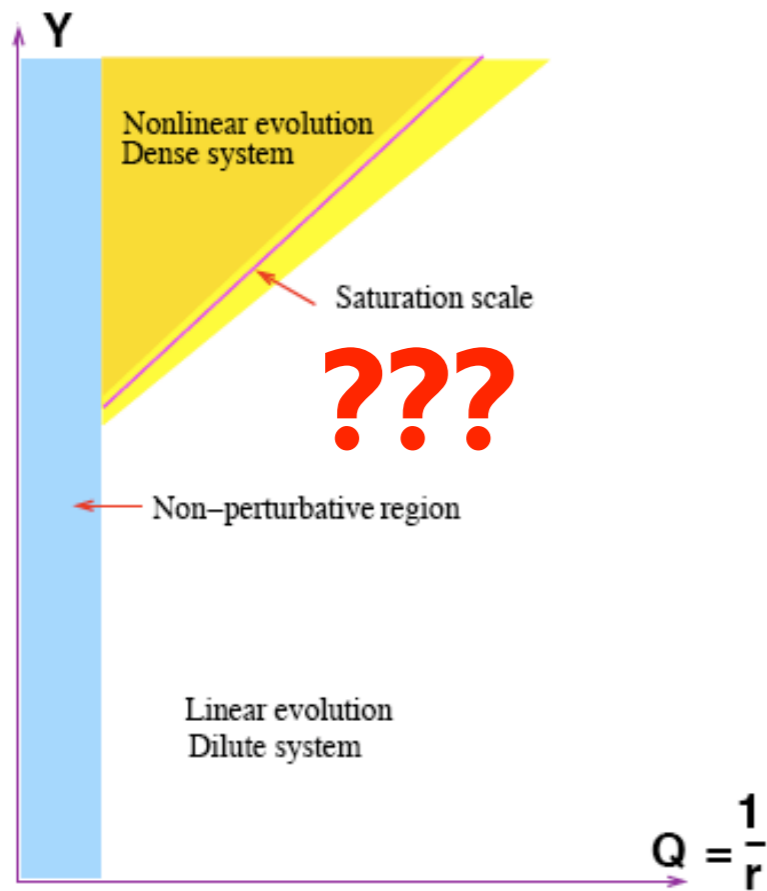


Usual approximation:

$$\mathcal{U}(Y; \mathbf{x}_0, \mathbf{x}_1) = \mathcal{U}(Y; |\mathbf{x}_0 - \mathbf{x}_1|)$$

- The target has infinite size.
- Local approximation suggests that the system becomes more perturbative as the energy grows.
- But this cannot be true everywhere (IR in QCD)

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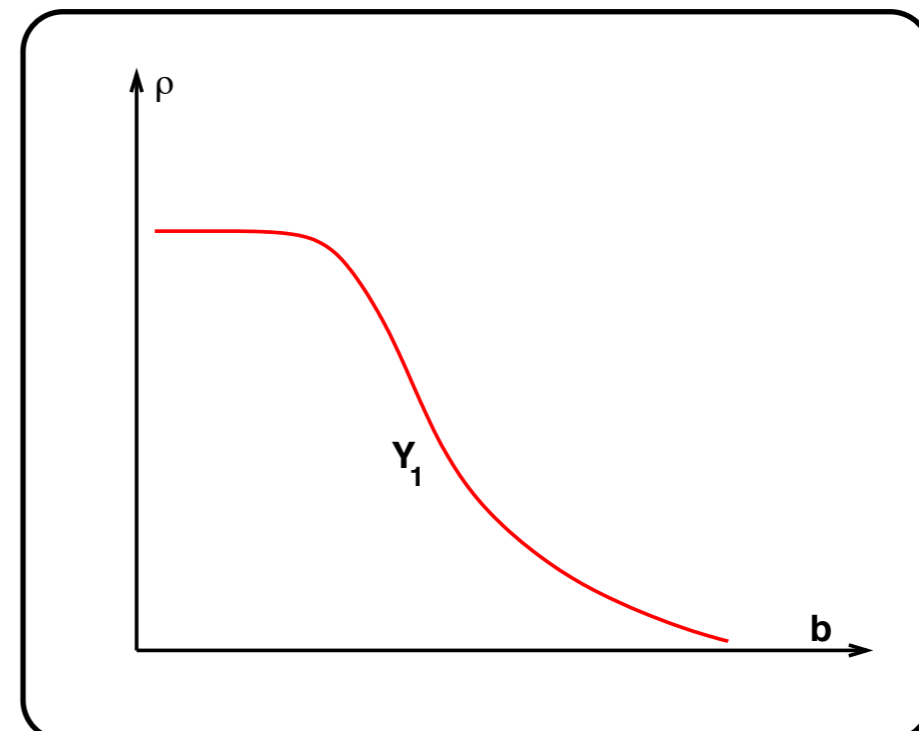
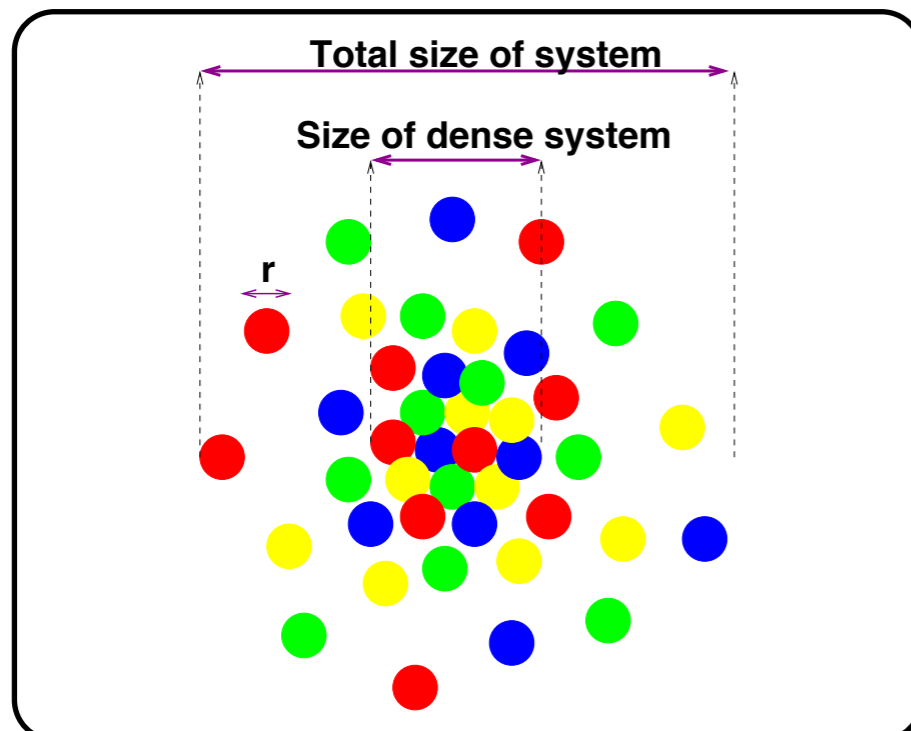


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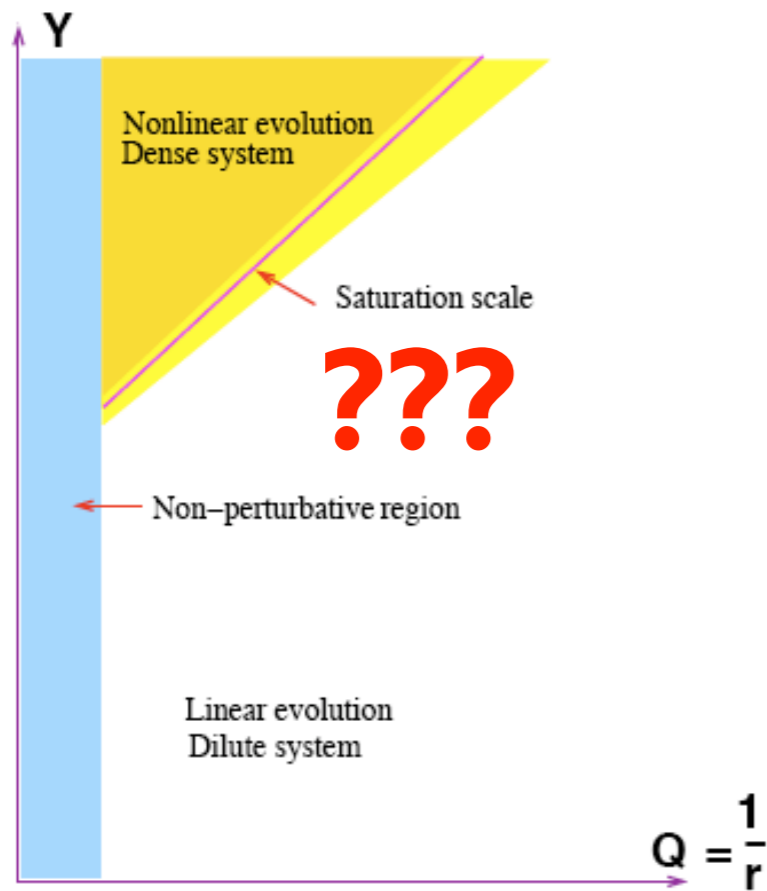
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## Impact parameter profile





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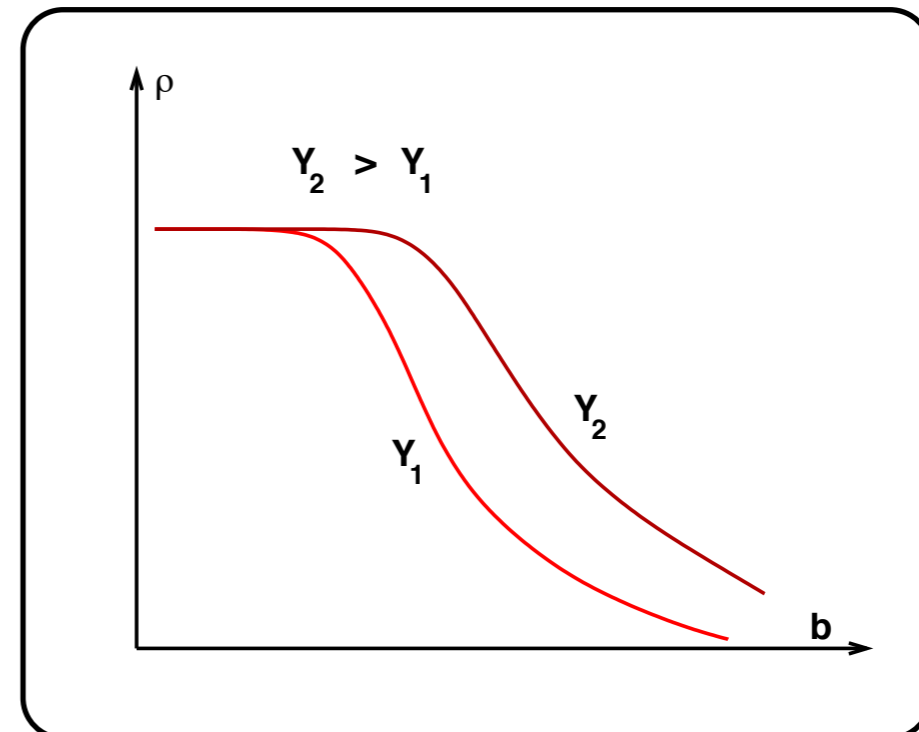
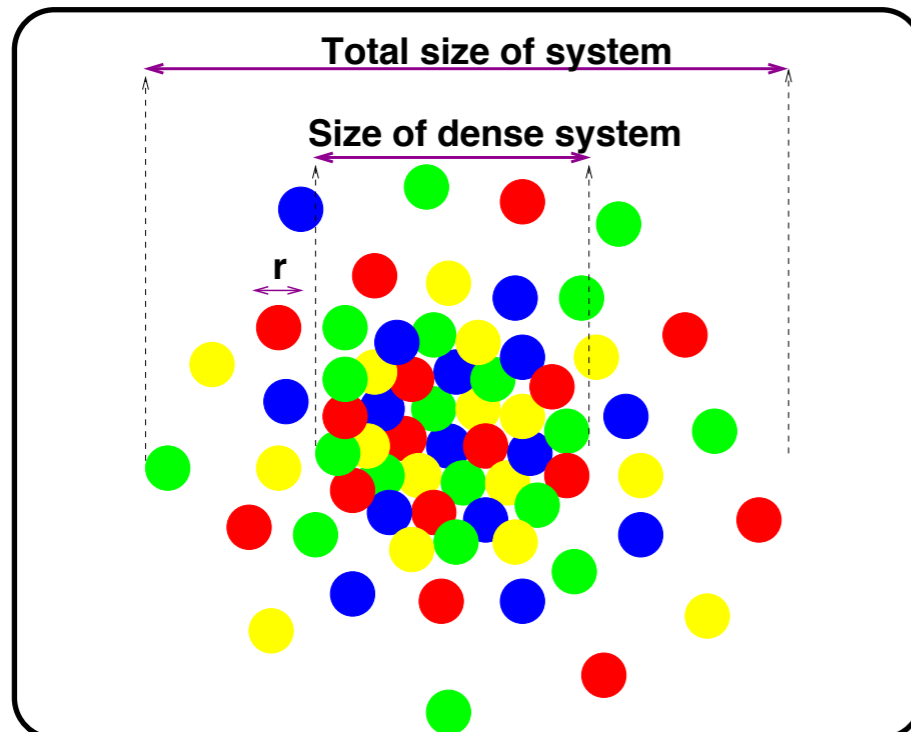


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## Impact parameter profile



# Impact parameter representation

Why do we care about impact parameter?

Impact parameter profile can provide the information how close the amplitudes are to the unitarity limit. Important to address the issue of correlations and in the double parton scattering context.

Impact parameter representation for total, elastic and inelastic pp cross section

$$\sigma_{tot}(s) = 2 \int d^2\mathbf{b} \operatorname{Re} \Gamma(s, b),$$

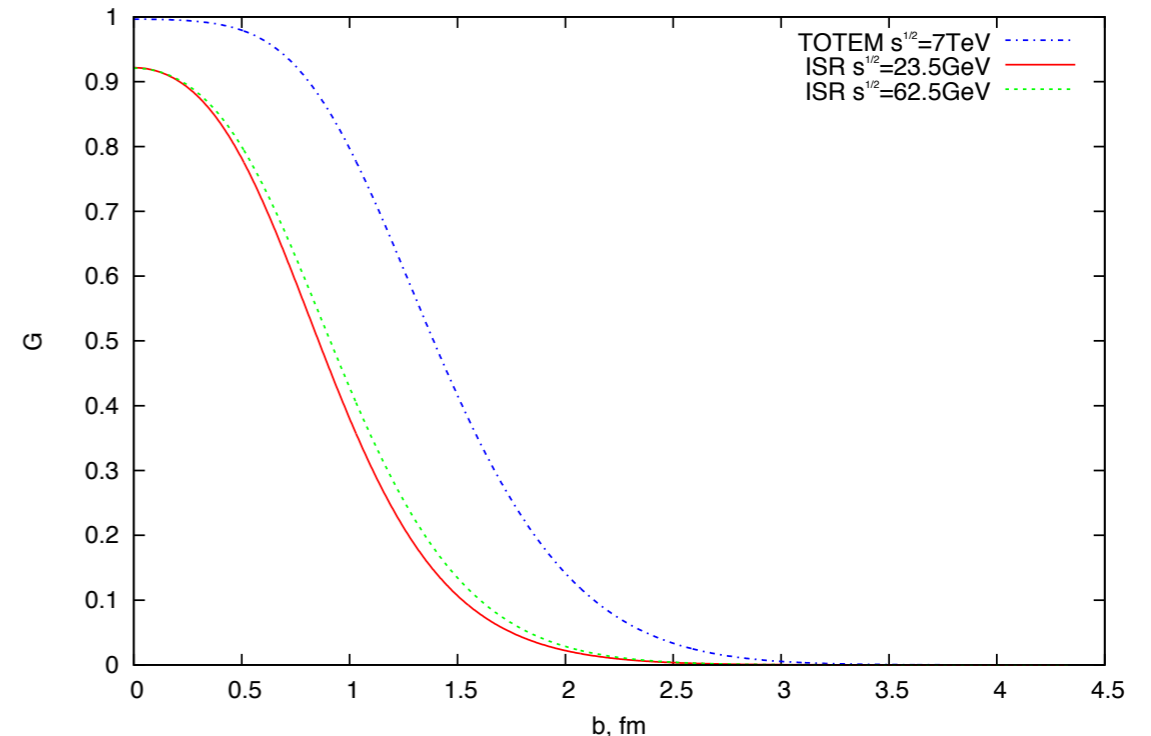
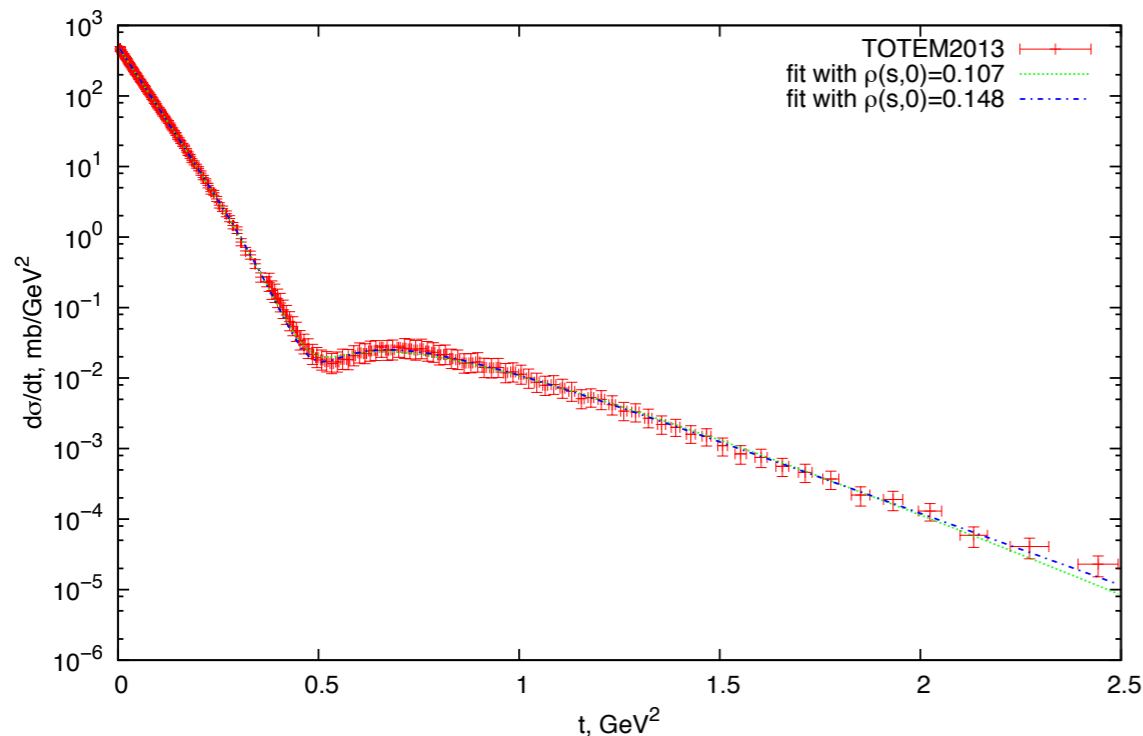
$$\sigma_{el}(s) = \int d^2\mathbf{b} |\Gamma(s, b)|^2,$$

$$\sigma_{inel}(s) = \int d^2\mathbf{b} \left( 2 \operatorname{Re} \Gamma(s, b) - |\Gamma(s, b)|^2 \right),$$

$$\Gamma(s, b) = \frac{1}{2is(2\pi)^2} \int d^2\mathbf{q} e^{i\mathbf{q}\cdot\mathbf{b}} A(s, t)$$

Unitarity limit:

$$\Gamma(s, b) \leq 1$$



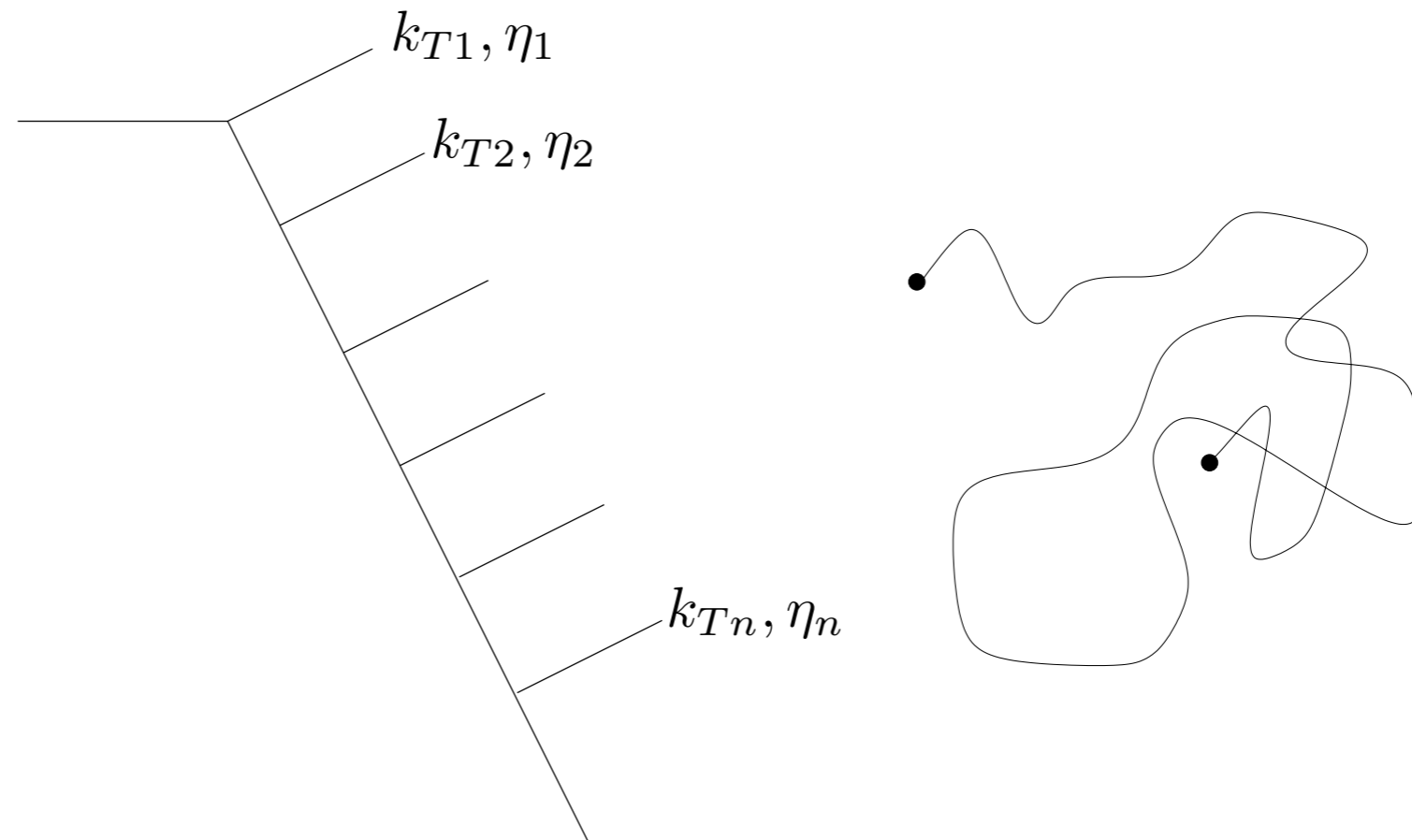
*Dremin, Nechitailo*

Impact parameter amplitude provides information about the unitarity limit.

# Gribov diffusion in parton model

Gribov

Emission of particles, with some transverse momenta leads to the diffusion in impact parameter space.



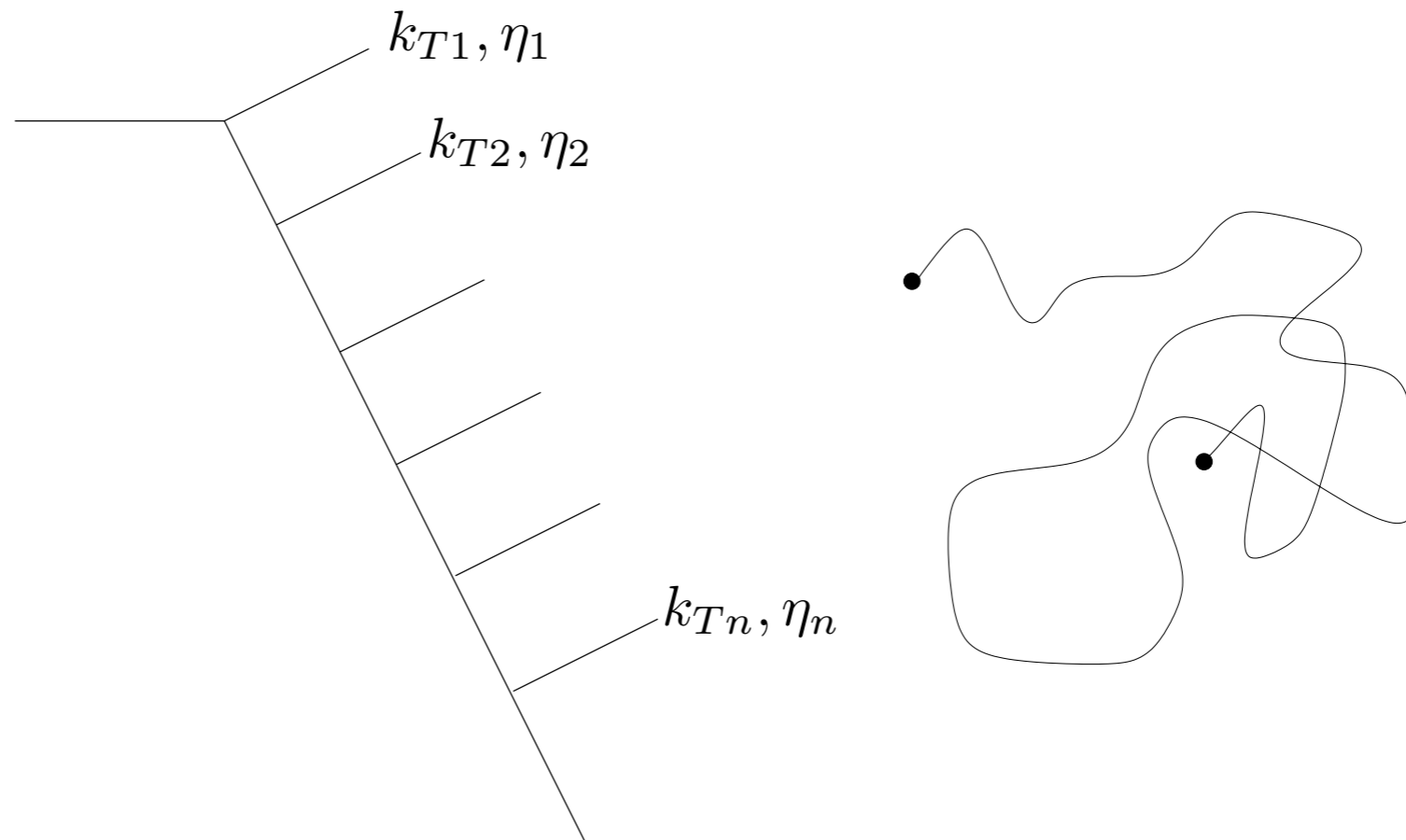
# Gribov diffusion in parton model

*Gribov*

Emission of particles, with some transverse momenta leads to the diffusion in impact parameter space.

Assumption:  
each emission leads to the change of impact parameter of the order of some scale

$$b \sim \frac{1}{\mu}$$



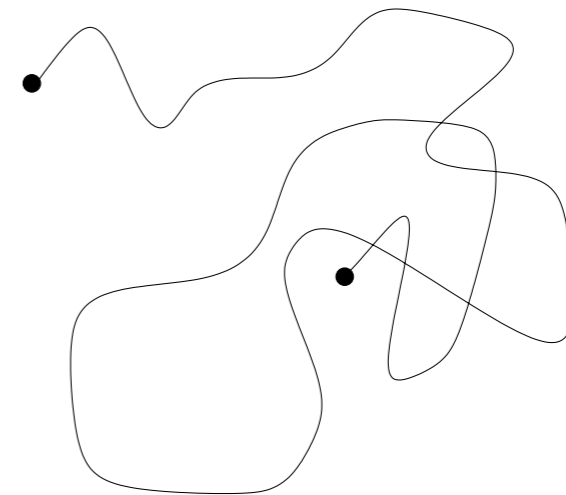
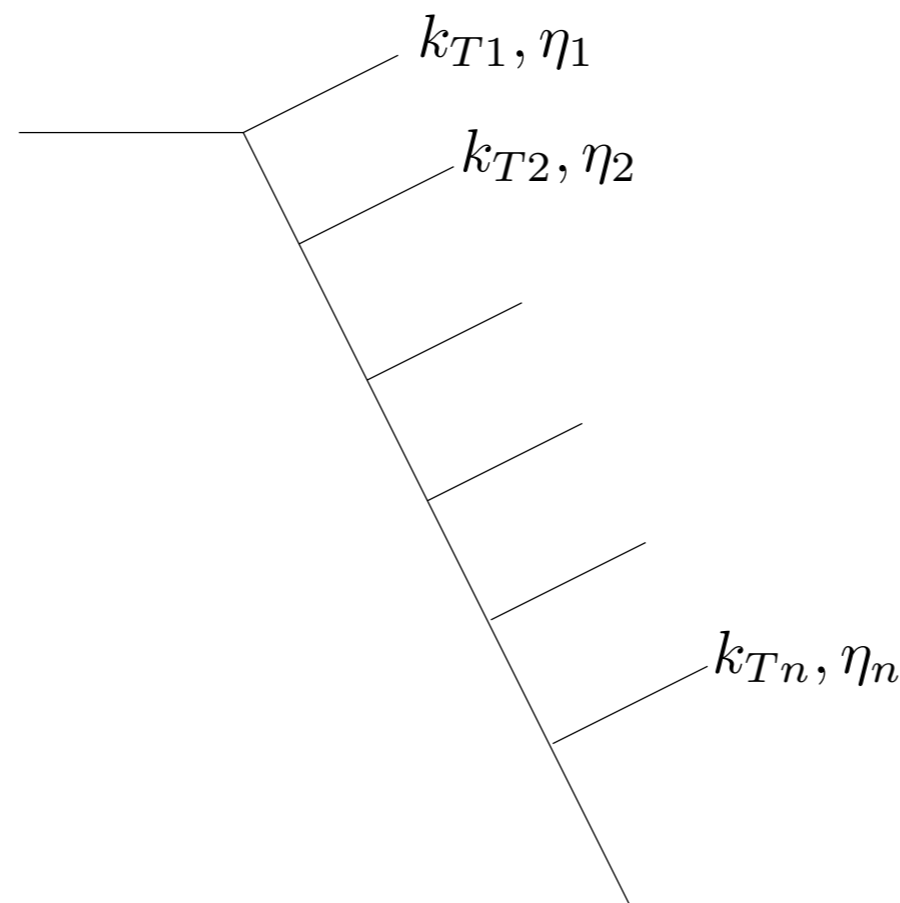
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$$\langle (\Delta b)^2 \rangle = c(\eta_1 - \eta_n)$$

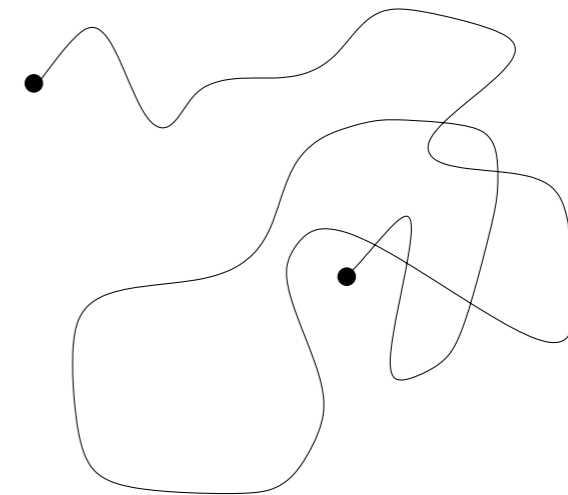
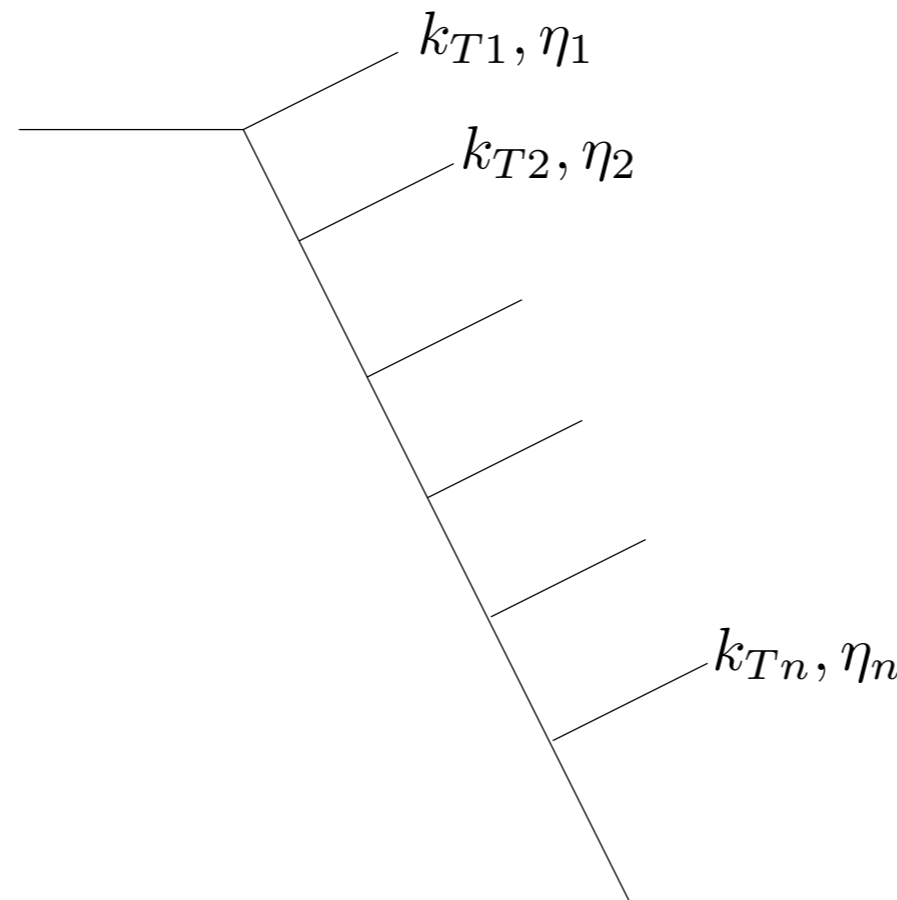
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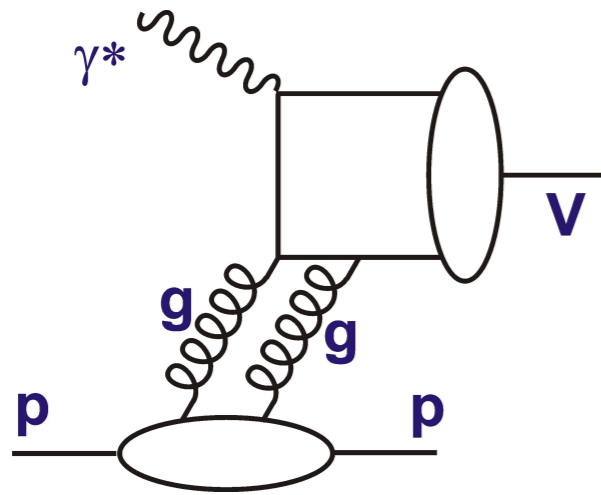


Spatial distribution of partons

$$\langle (\Delta b)^2 \rangle = c(\eta_1 - \eta_n)$$

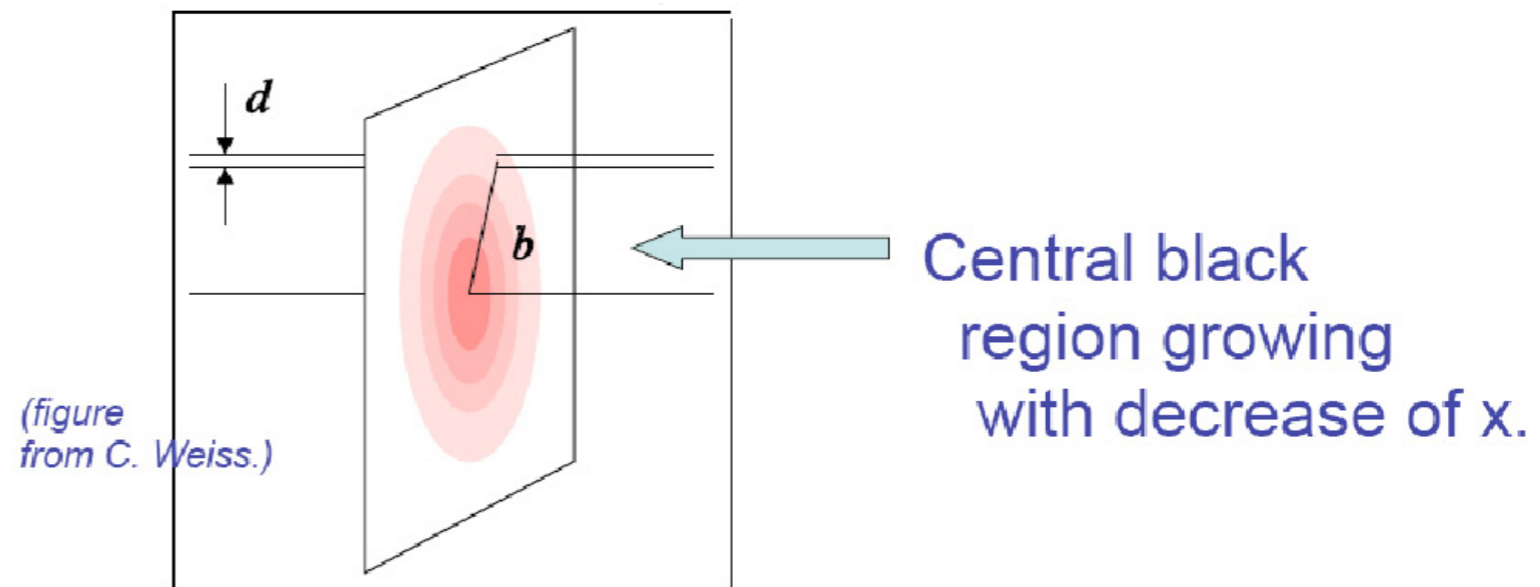
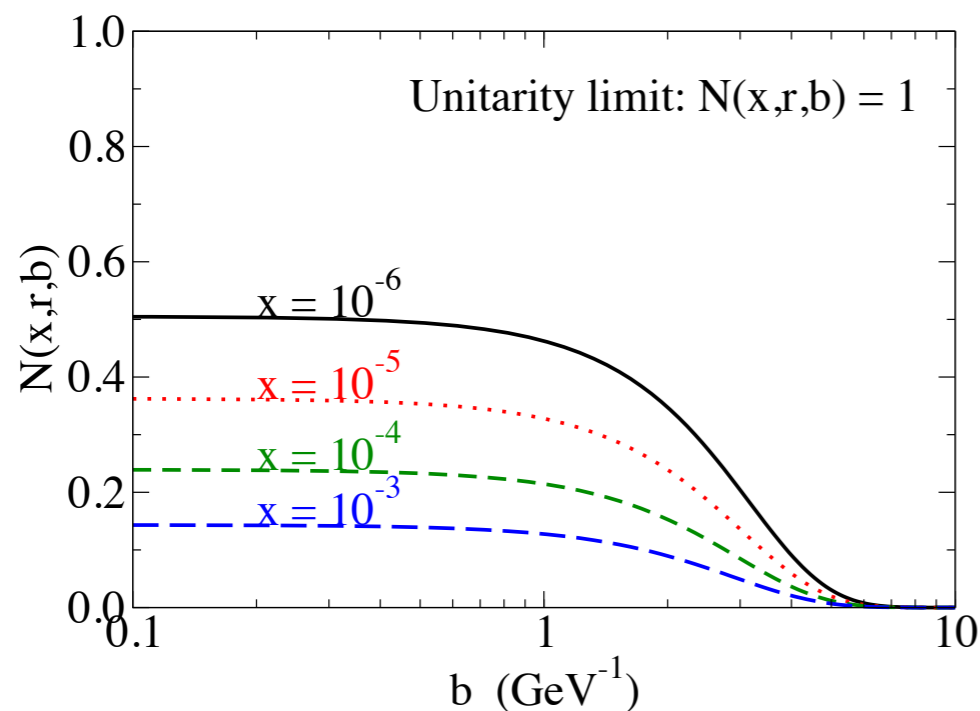
$$\phi(b, \eta) \sim \frac{1}{c(\eta_1 - \eta_n)} \exp\left(-\frac{b^2}{c(\eta_1 - \eta_n)}\right)$$

# Exclusive diffraction of vector mesons in DIS



- Exclusive diffractive production of VM is an excellent process for extracting the dipole amplitude and GPDs
- Suitable process for estimating the 'blackness' of the interaction.
- $t$ -dependence provides an information about the impact parameter profile of the amplitude.

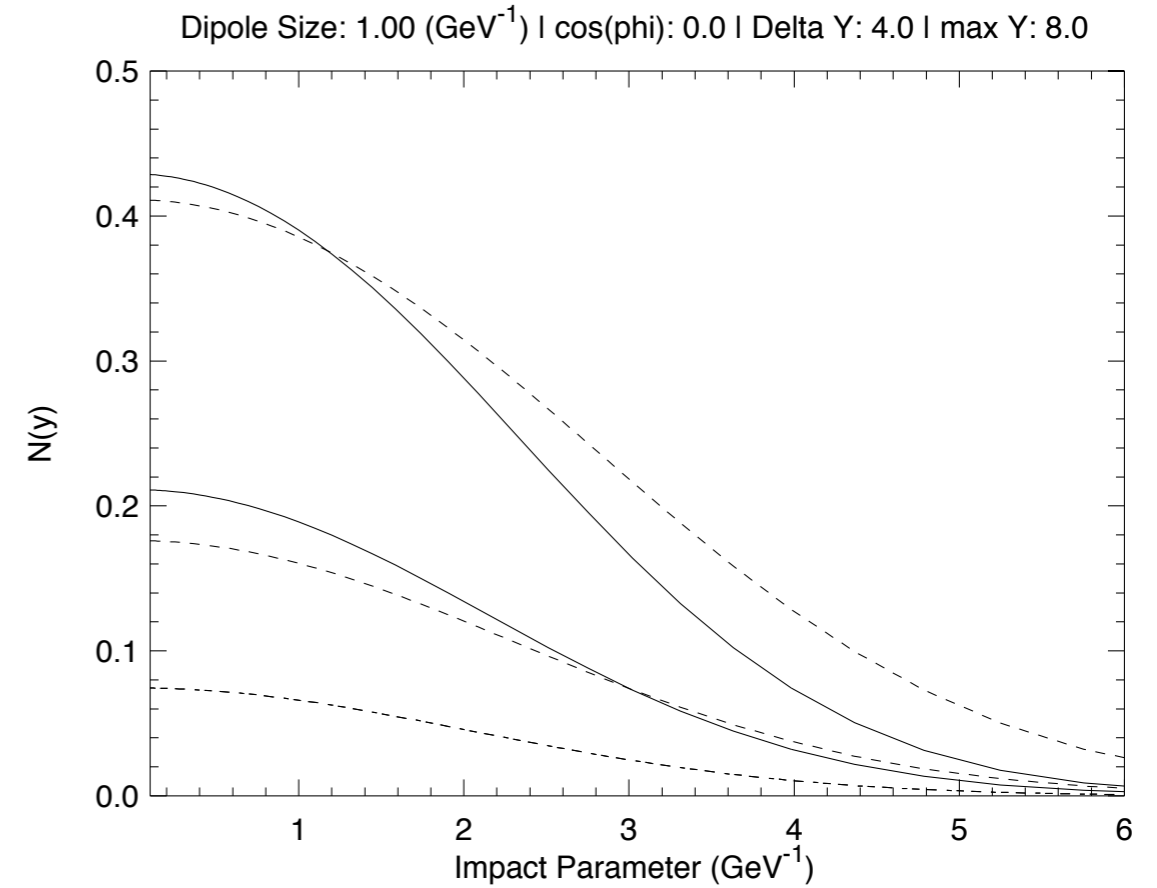
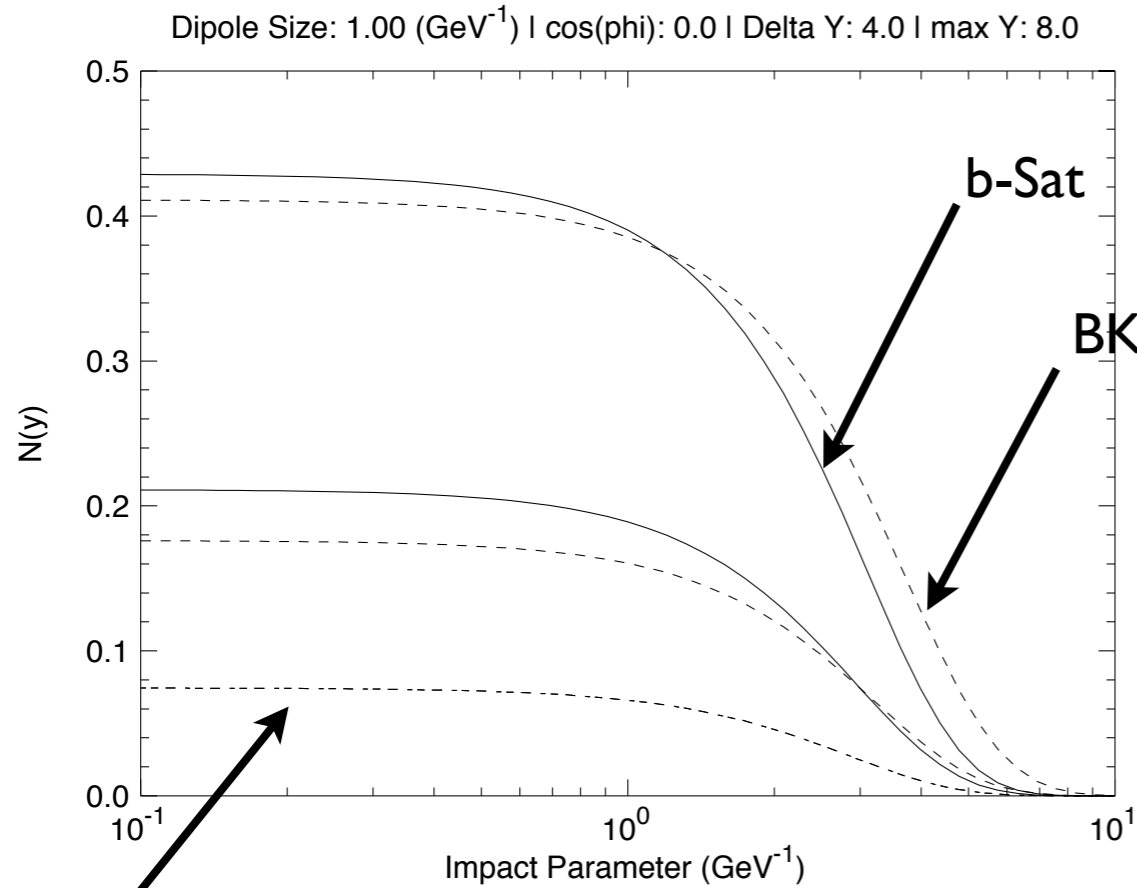
"b-Sat" dipole scattering amplitude with  $r = 1 \text{ GeV}^{-1}$



Large momentum transfer  $t$  probes small impact parameter where the density of interaction region is largest

# Evolved solution for the dipole amplitude

Berger, AS



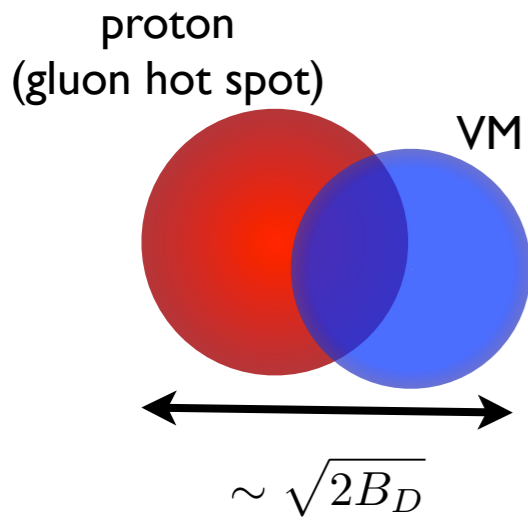
KMW (b-Sat model)  
initial condition

Profile in  $b$ : Solid line KMW, dashed lines BK with running coupling and cuts  
Small  $x$  evolution leads to the broader distribution in impact parameter  
Change of shape with decreasing  $x$

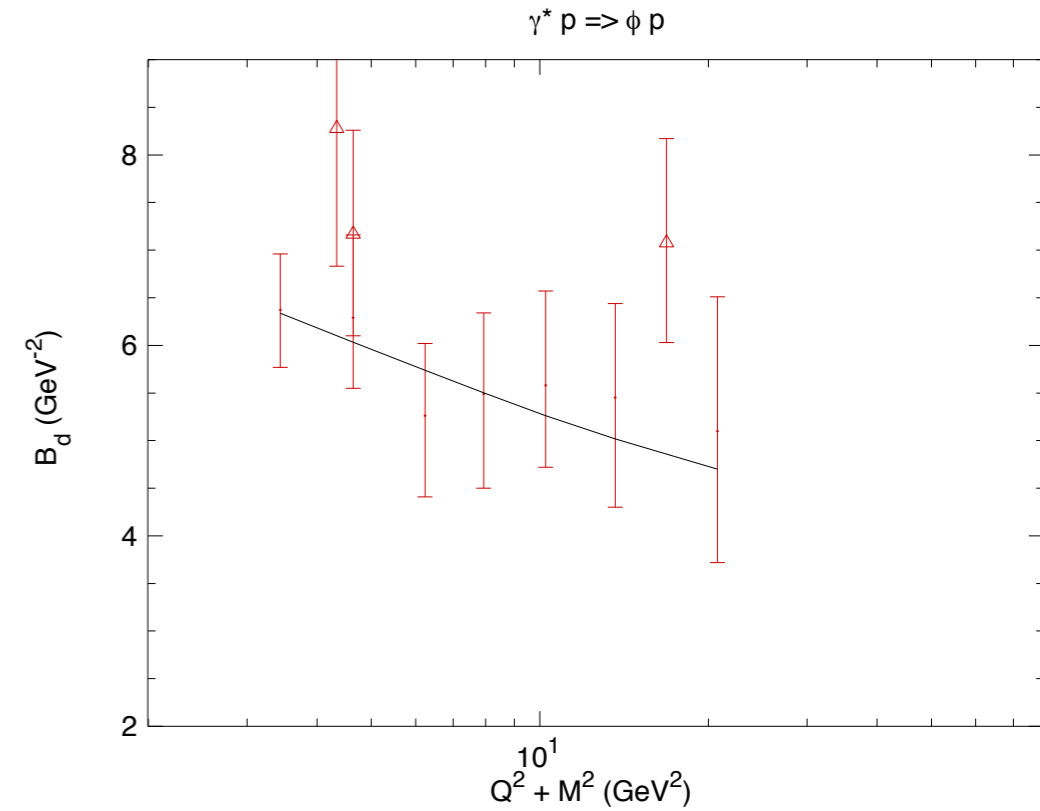
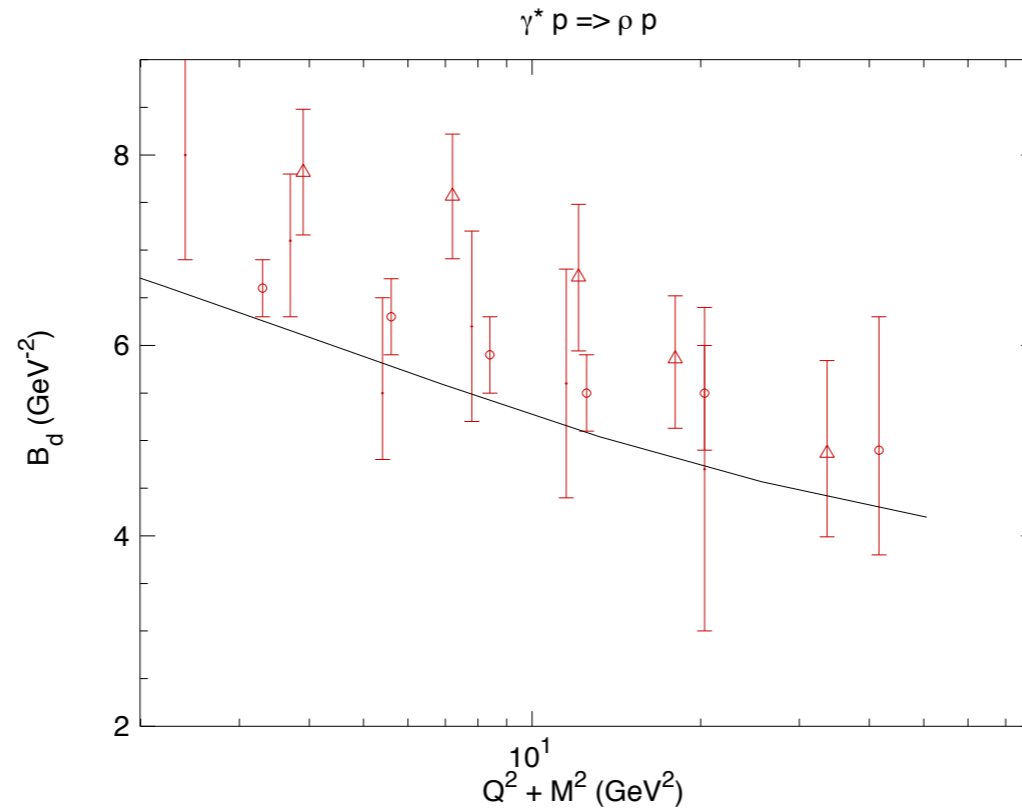


# Diffractive slope

HERA data

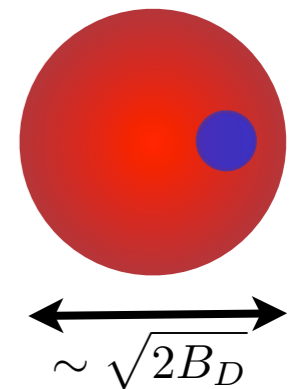
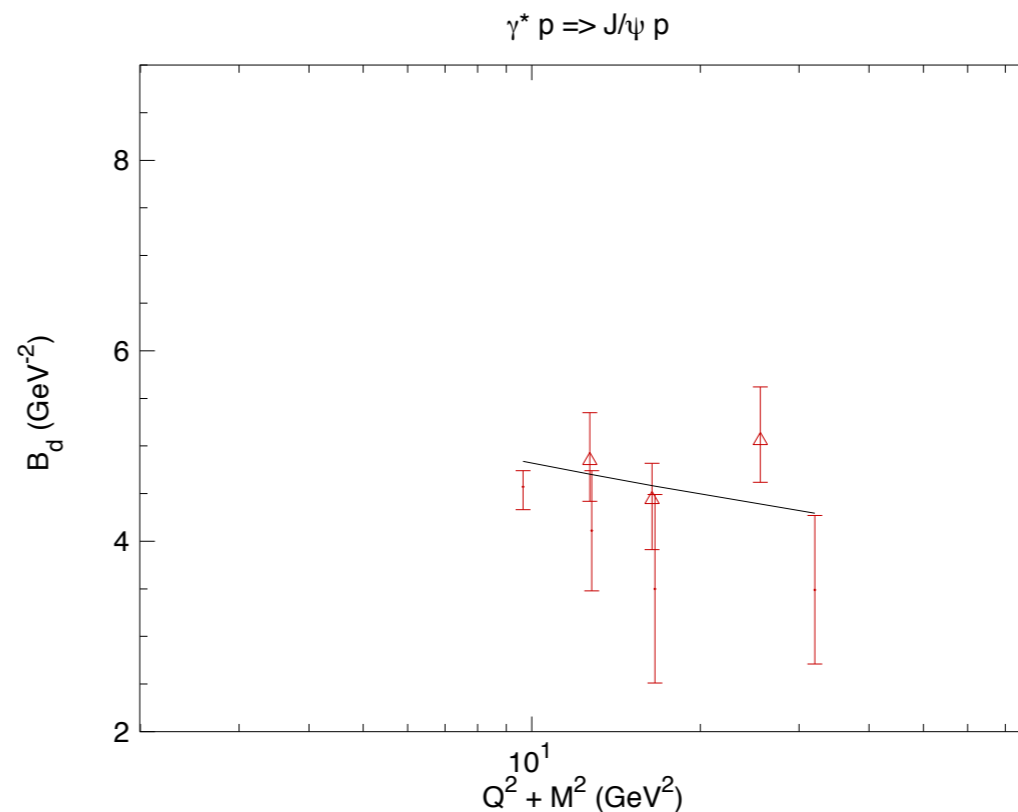


$$\frac{d\sigma}{dt} \sim e^{-B_D |t|}$$

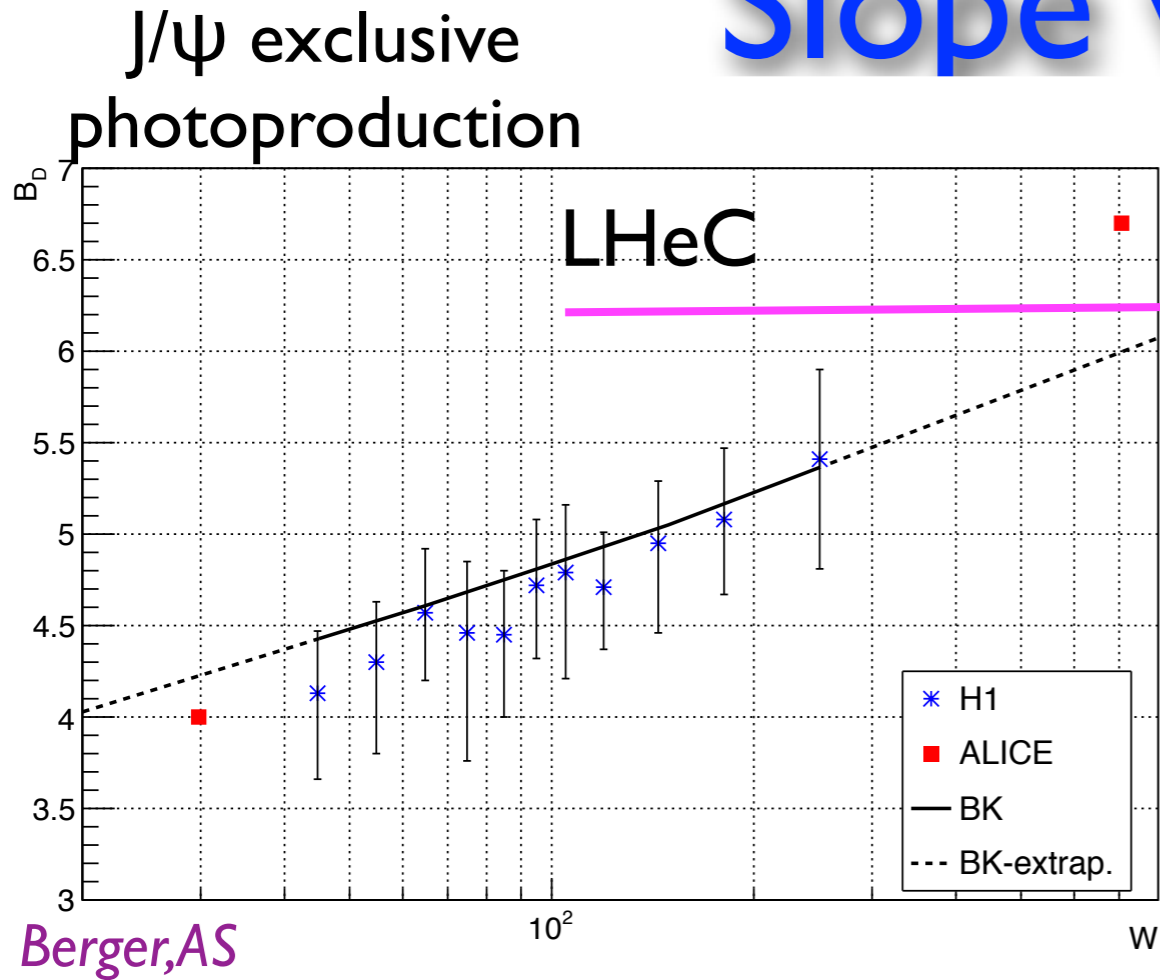


- The value of  $B_D$  is closely related to the transverse size of the interaction region which is a combination of the size of the VM and the size of the gluon hot-spot in the proton.
- In the case of the lighter mesons it is the first one which prevails.
- For heavier mesons, it is the larger size of the gluon distribution in the proton. Thus it does not depend on  $Q^2$  that much.

*Berger, AS*



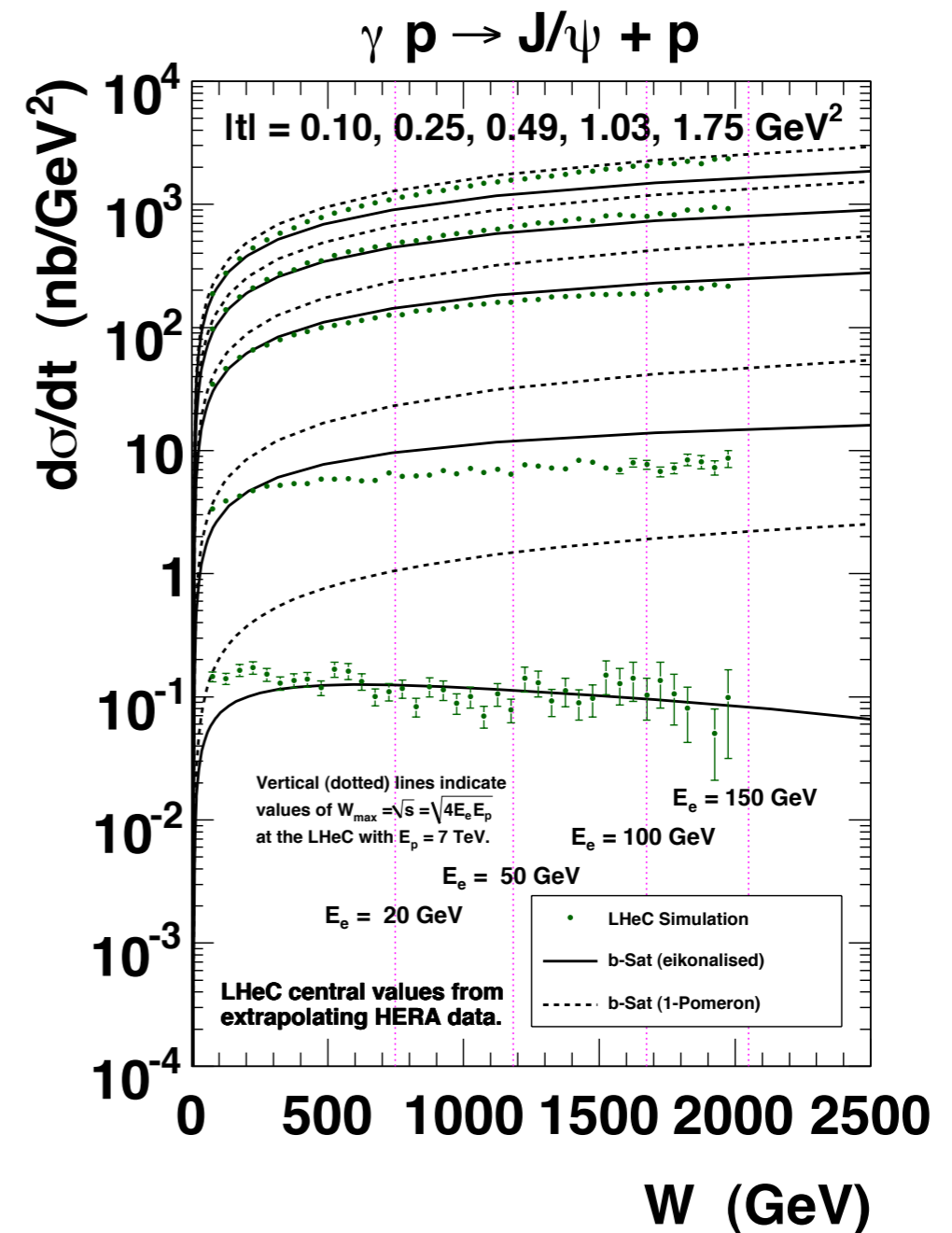
# Slope vs energy



ALICE (ultraperipheral collisions)

$$B_D(W = 29.8 \text{ GeV}) = 4 \text{ GeV}^{-2}$$

$$B_D(W = 706 \text{ GeV}) = 6.7 \text{ GeV}^{-2}$$

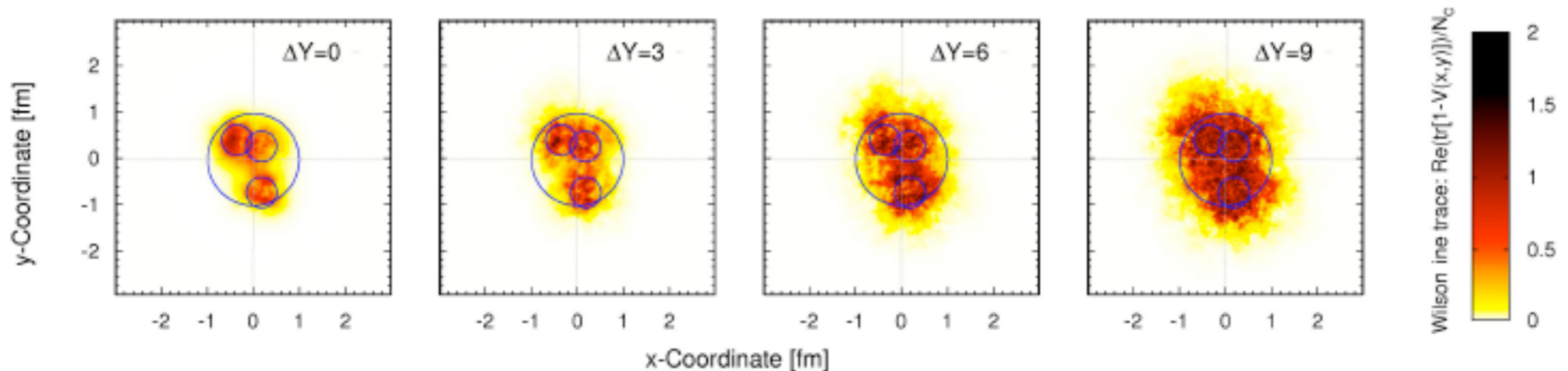


LHeC simulation

- Reasonable description of the diffractive slope from dynamical prediction based on BK evolution with cutoff
- LHeC through the measurements of the differential cross section in  $t, W, Q^2$  would provide detailed information about the shape of the proton and its variation with the energy.

# Shape of the proton

Schlichting, Schenke



Study of the evolution of the asymmetric shape of proton.  
Initial asymmetry is not washed out by evolution quickly.  
Important for ridge and  $v_n$  studies in pp/pA.

LHeC would provide the information about the shape of the proton, for example through the diffractive dijet production (constraints on Wigner function).

# Summary and outlook

- Great progress over the years in understanding and implementation of the NLL and resummation at low  $x$  at the level of linear BFKL.
- This knowledge has been already extended and applied into the non-linear evolution equations including saturation. Recent calculations of the NLL photon impact factor, still need to be applied to phenomenology and could be used to make predictions for LHeC.
- LHeC can provide with plethora of measurements to make the full tomography of the proton through the elastic diffractive vector meson production and other exclusive diffractive processes (dijet production).
- **Next steps:** in the 2012 CDR many predictions including saturation were presented: structure functions and VM production, mostly based on dipole models.
- Given recent progress in calculations at NLO and resummation in nonlinear case, predictions for LHeC with resummation and saturation could be made taking into account higher order terms.
- Diffraction is particularly sensitive to low  $x$  effects, diffractive pdfs for LHeC have been calculated (see talk by Paul Newman)
- VM production is an excellent process to study low  $x$  effects and proton shape. In particular energy dependence of the  $t$ -slope, for photo- and electroproduction should be studied in more detail at LHeC.