The ambiguity of the PDFs' scale energy $\tilde{Q}^2$ in the Z and H production via ep-DIS

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Z-production via the DIS

We discuss Z-production via the process

$$e + p \rightarrow e + Z + X. \qquad (1)$$

In contrast to the simple DIS ($e + p \rightarrow eX$) where the choice of $\tilde{Q}^2$ is unambiguouos. In process (1) it is not clear, since Q^2 is different to Q'^2, and the momentum transfer square can be both of these quantities, at the quark level, depending on the reaction mechanisms.

Our aim in this work is to show that the total cross section rates, of process (1), depend strongly on the prescription adopted for making the convolution of the Parton Distribution Functions (PDFs) with the square amplitude!

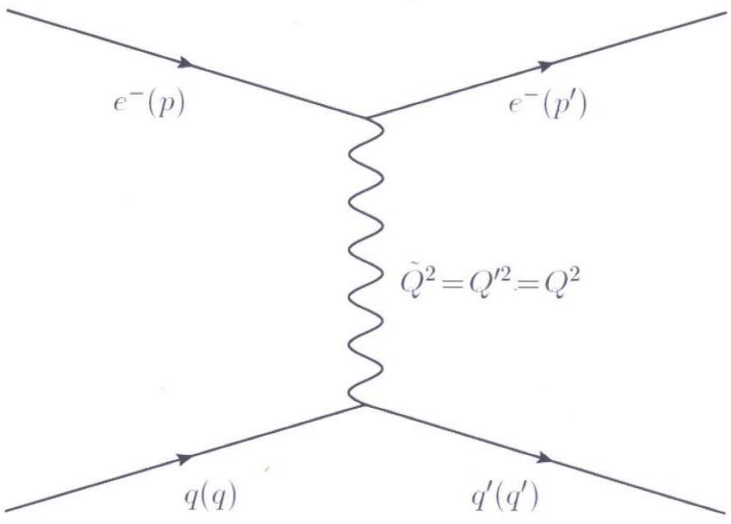
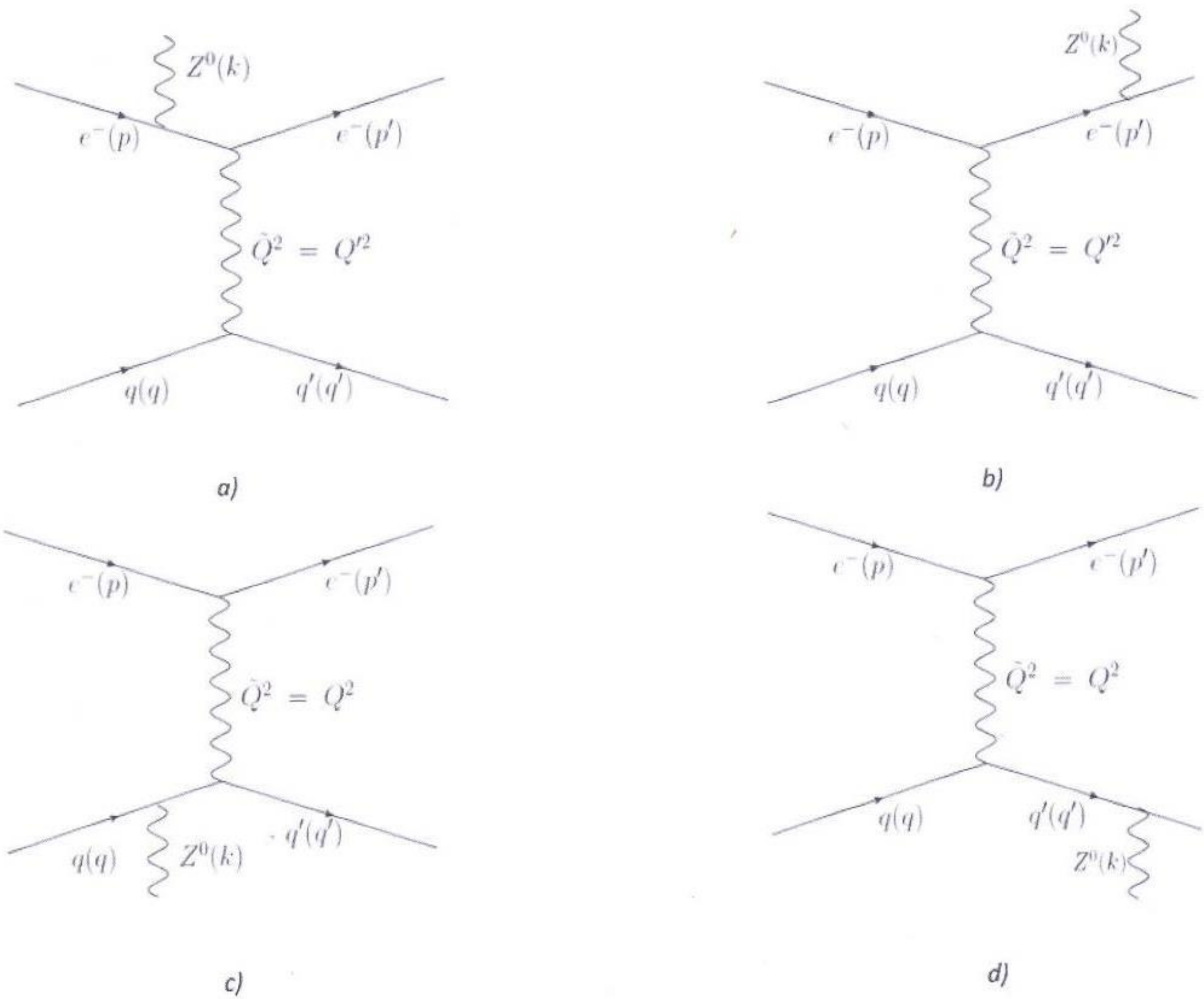
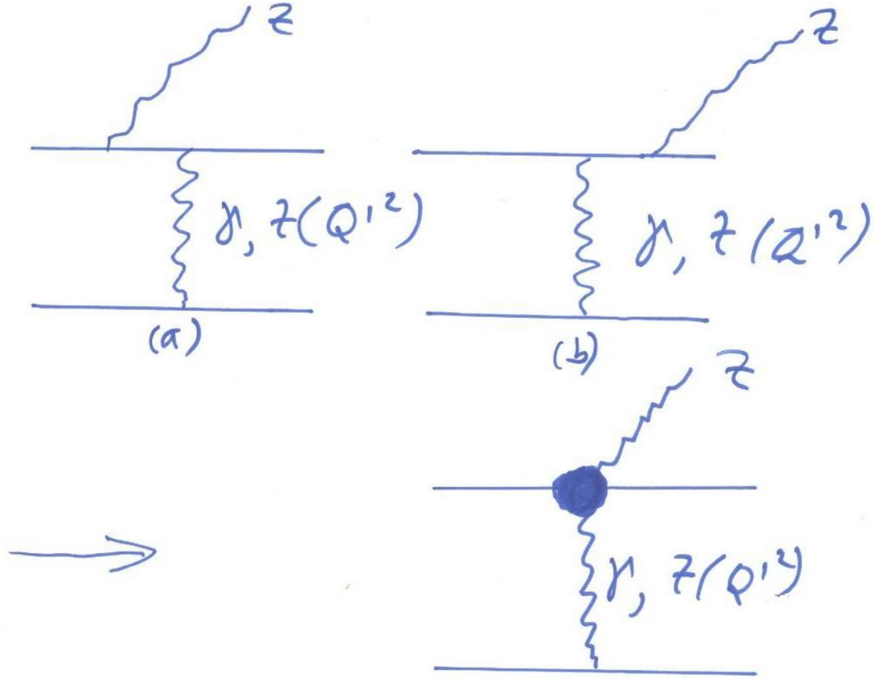


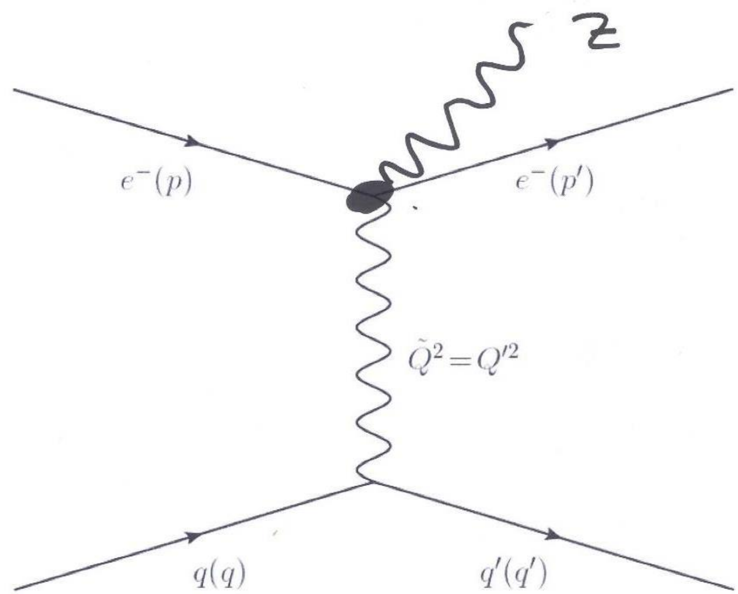
Fig.1 Feynman diagram which contribute at lowest order in α, to the Deep Inelastic Process e + p→ e + X, at the quark level.

Fig.2 Feynman diagrams which contribute at the lowest order in α to the Z-prouction e + p → e + Z + X, at the quark level. Z boson emitted from the initial (a) and final (b) electron, the initial(c) and final (d) quark.

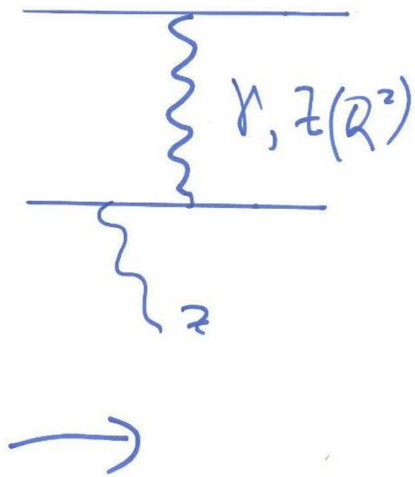




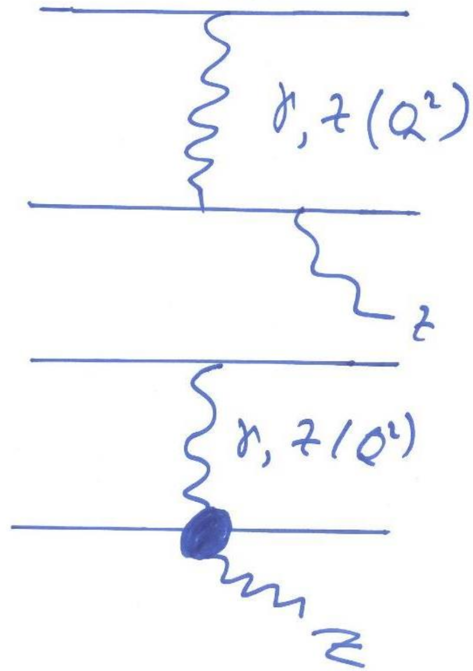
Z
γ, Z(Q'²)
(a)
Z
γ, Z(Q'²)
(b)
Z
γ, Z(Q'²)

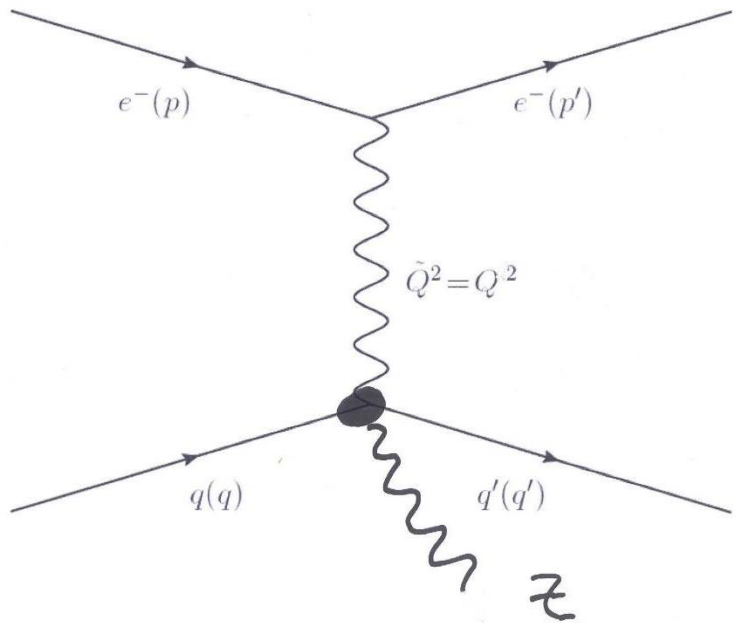

Z
$e^-(p)$
$e^-(p')$
$\tilde{Q}^2 = Q'^2$
$q(q)$
$q'(q')$

(c)



(d)





$e^-(p)$
$e^-(p')$
$\tilde{Q}^2 = Q^2$
$q(q)$
$q'(q')$
Z

Kinematics

$$e(p) + P(P_P) \to e(p') + Z(k) + X(P_X)$$

As usual the following invariants are defined:

$$\begin{aligned}
s &= (p + P_P)^2, \\
Q^2 &= -(p - p')^2, \\
\nu &= P_P(p - p'), \\
s' &= (p + P_P - k)^2, \\
Q'^2 &= -(p - p' - k)^2, \\
\nu' &= P_P(p - p' - k),
\end{aligned}$$

and the dimensionless variables:

$$x = \frac{Q^2}{2\nu}, \quad y = \frac{2\nu}{s}, \quad \tau = \frac{s'}{s}, \quad x' = \frac{Q'^2}{2\nu'}, \quad y' = \frac{2\nu'}{s}.$$

The differential cross section $d\sigma^{eP}$ for process is calculated in the Parton Model from the cross section $d\sigma^{eq}$ of the quark subprocess

$$e(p) + q(q) \to e(p') + q(q') + Z(k)$$

and the PDFs. As usual we define for the parton process the following invariant variables:

$$\begin{aligned}
\hat{s} &= (p + q)^2 &&= x's, \\
\hat{Q}^2 &= -(p - p')^2 &&= Q^2, \\
\hat{\nu} &= q(p - p') &&= x'\nu, \\
\hat{s}' &= (p + q - k)^2 &&= s' - (1 - x')s + 2(1 - x')(\nu - \nu'), \\
\hat{Q}'^2 &= -(p - p' - k)^2 &&= Q'^2, \\
\hat{\nu}' &= q(p - p' - k) &&= x'\nu'.
\end{aligned}$$

The variables $\hat{s}, ..., \hat{\nu}'$ are not independent. For massless quarks we have $\hat{Q}'^2 = 2\hat{\nu}'$ and consequently $x' = Q'^2/2\nu'$.

Formulae

The quark cross section is obtained from the invariant matrix element $\mathcal{M}(eq \to eqZ)$:

$$d\sigma_{tot}^{eq} = \frac{2(2\pi^{-5})}{\hat{s}} \frac{1}{4} \mid \mathcal{M}_{tot}^{eq} \mid^2 d\Gamma_3$$

The 3-particle phase space $d\Gamma_3$ can be expressed with help of the different sets of variables

$$\begin{aligned} d\Gamma_3 &= \frac{d^3p'}{2\,E'} \frac{d^3q'}{2\,E'_q} \frac{d^3k}{2\,E_k} \\ &= \frac{\pi}{8\hat{s}} \frac{d\hat{Q}^2\; d\hat{\nu}\, d\hat{s}\, d\hat{Q}'^2\, d\hat{\nu}'}{\sqrt{-\Delta_4\,(p,\ q,\ p',\ k)}} \delta(\hat{Q}'^2 - 2\hat{\nu}') \\ &= \frac{\pi}{8s} \frac{dQ^2\; d\nu\; ds'\; dQ'^2\; d\nu'}{\sqrt{-\Delta_4\,(p,\ P_P,\ p',\ k)}} \delta(Q'^2 - 2x'\nu') \\ &= \frac{\pi s^3}{32}\, y\, \frac{dx\; dy\; dy'\; d\tau}{\sqrt{-\Delta_4\,(p,\ P_P,\ p',\ k)}} \end{aligned}$$

with $\Delta_4(p, q, p', k)$ as the Jacobi determinant.

$$\mathcal{M}_{tot}^{eq} = \mathcal{M}_l^{eq} + \mathcal{M}_q^{eq}$$

The final step in the evaluation of $d\sigma^{eP}$ consists now in setting the product of the parton cross sections $d\sigma^{eq}$ and the parton distribution functions $f_q(x', \tilde{Q}^2)$. In contrast to deep inelastic eP scattering the choice of $\tilde{Q}^2$ is not unambiguous in this case since the momentum transfer square to the proton depends on the reaction mechanism (in other words, whether the boson is emitted at the lepton or at the quark line). In order to make clear how strong depend the cross section rates on the choice of the scale $\tilde{Q}^2$, we calculate here with the following simple prescriptions:

PARTON MODEL

$$d\sigma^{eP} = \sum_q \Big[\int dx' f_q(x', \tilde{Q}^2) \cdot d\sigma^{eq}_{lepton} + \int dx' f_q(x', \tilde{Q}^2) \cdot d\sigma^{eq}_{quark} + \int dx' f_q(x', \tilde{Q}^2) \cdot d\sigma^{eq}_{inter} \Big]$$

Prescription A

$$d\sigma^{eP} = \sum_q \Big[\int dx' f_q(x', M_Z^2) \cdot d\sigma^{eq}_{lepton} + \int dx' f_q(x', M_Z^2) \cdot d\sigma^{eq}_{quark} + \int dx' f_q(x', M_Z^2) \cdot d\sigma^{eq}_{inter} \Big]$$

Prescription B

$$d\sigma^{eP} = \sum_q \Big[\int dx' f_q(x',\ Q^2) \cdot d\sigma^{eq}_{lepton} + \int dx' f_q(x',\ Q^2) \cdot d\sigma^{eq}_{quark} + \int dx' f_q(x',\ Q^2) \cdot d\sigma^{eq}_{inter} \Big]$$

Prescription C

$$d\sigma^{eP} = \sum_q \Big[\int dx' f_q(x',\ Q'^2) \cdot d\sigma^{eq}_{lepton} + \int dx' f_q(x',\ Q'^2) \cdot d\sigma^{eq}_{quark} + \int dx' f_q(x',\ Q'^2) \cdot d\sigma^{eq}_{inter} \Big]$$

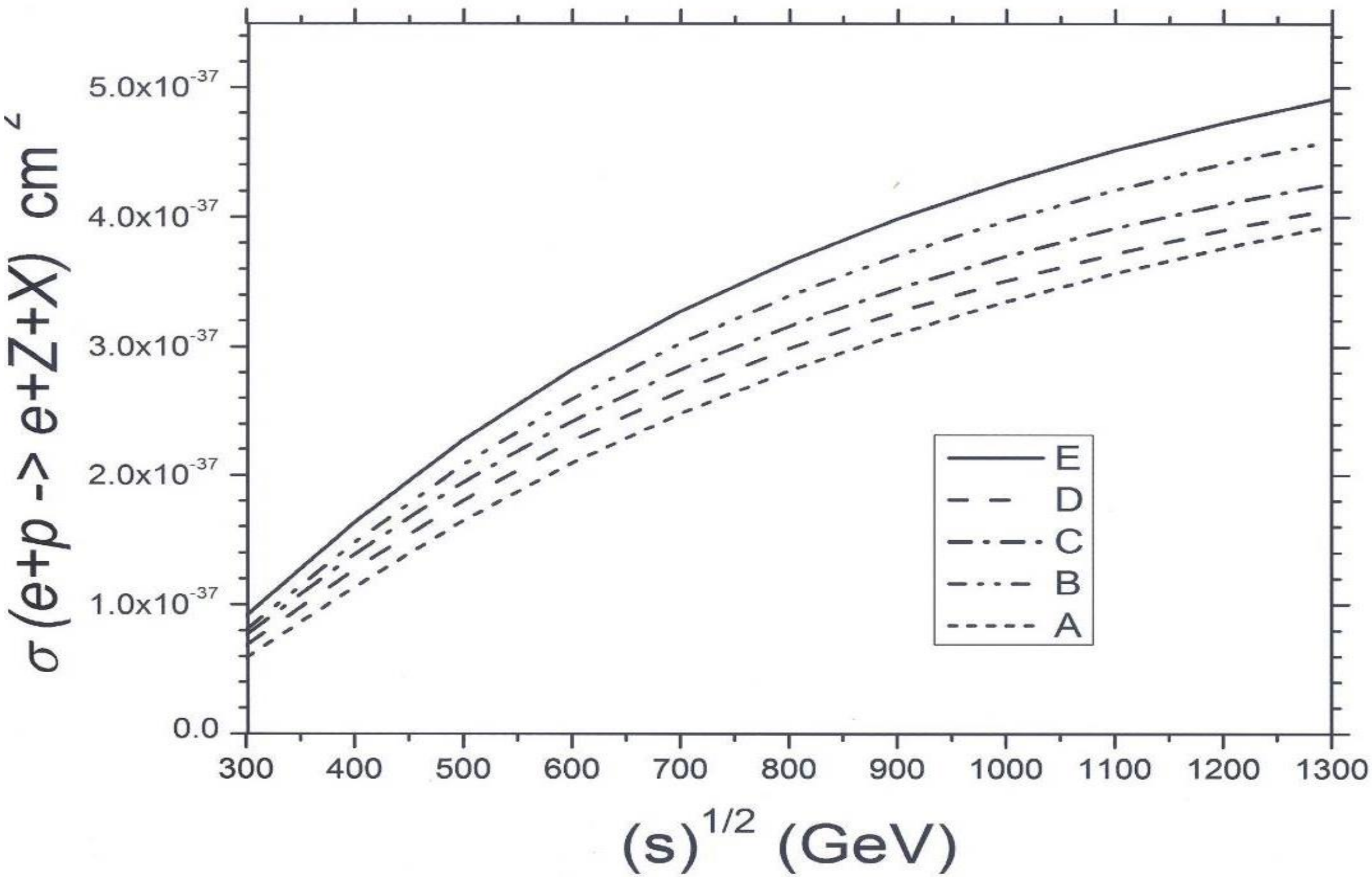
Prescription D

$$d\sigma^{eP} = \sum_q \Big[\int dx' f_q(x', (Q^2+Q'^2)/2) \cdot d\sigma^{eq}_{lepton} + \int dx' f_q(x', (Q^2+Q'^2)/2) \cdot d\sigma^{eq}_{quark} + \int dx' f_q(x', (Q^2+Q'^2)/2) \cdot d\sigma^{eq}_{inter} \Big]$$

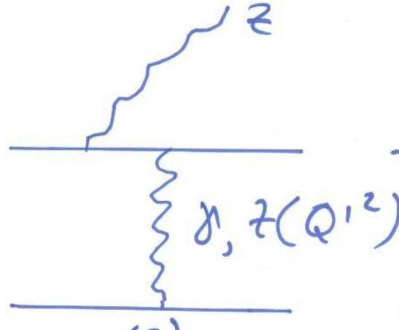
Prescription E

$$d\sigma^{eP} = \sum_q \Big[\int dx' f_q(x',\ Q'^2) \cdot d\sigma^{eq}_{lep} + \int dx' f_q(x',\ Q^2) \cdot d\sigma^{eq}_{quark} + \int dx' \sqrt{f_q(x',\ Q^2)} \sqrt{f_q(x',\ Q'^2)} \cdot d\sigma^{eq}_{inter} \Big]$$

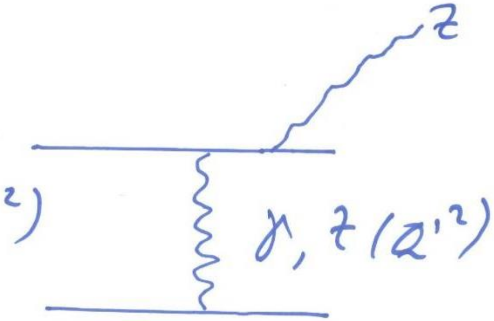
$$f(x',Q'^2)\ |M_a + M_b|^2 +$$
$$f(x',Q^2)\ |M_c + M_d|^2 +$$
$$[f(x',Q'^2)]^{1/2}\ [f(x',Q^2)]^{1/2}$$
$$\mathrm{Re}\{[M_a + M_b]\ [M_c + M_d]^*\}$$



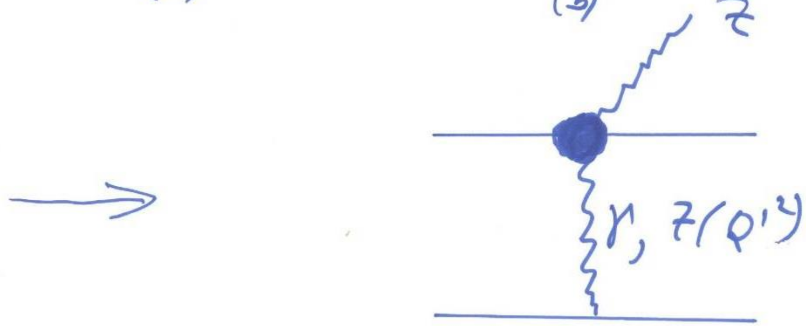
σ (e+p -> e+Z+X) cm^2
5.0x10^{-37}
4.0x10^{-37}
3.0x10^{-37}
2.0x10^{-37}
1.0x10^{-37}
0.0
300
400
500
600
700
800
900
1000
1100
1200
1300
(s)$^{1/2}$ (GeV)
E
D
C
B
A

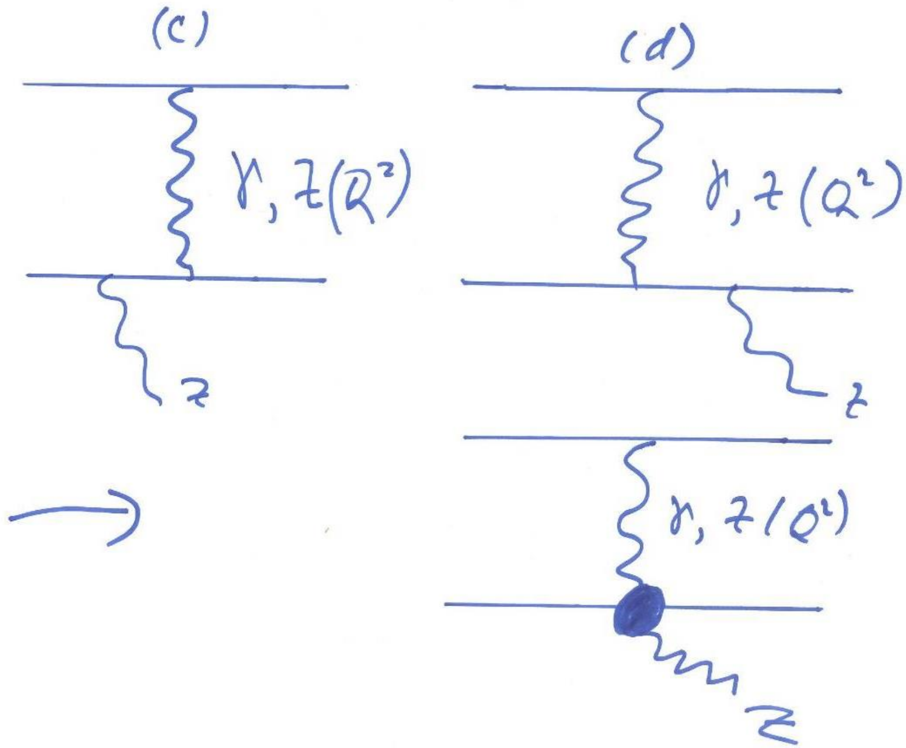


(a)



(b)




(c)
(d)
$\gamma, Z(Q^2)$
Z
$\gamma, Z(Q^2)$
Z
$\gamma, Z(Q^2)$
Z

We take an integrated Luminosity of 0.1 **(ab)** $^{-1}$**/year**

$\sqrt{s}$ (GeV)	σ_A	σ_B	σ_C	σ_D	σ_E
300	0.587	0.682	0.763	0.803	0.909
1300	3.934	4.064	4.257	4.590	4.912

Table 1. Cross section rates in $10^{-37} cm^2$ for Z production for $\sqrt{s} = 300$ GeV (HERA) and 1300 GeV (LHeC), for the different prescriptions that we have taken for making the convolution of the PDFs with the amplitude of the quark processes.

$\sqrt{s}$ (GeV)	N_A	N_B	N_C	N_D	N_E
1300	3.934	4.064	4.257	4.590	4.912

Table 2. Number of Z bosons ($= N_X \times 10^4$) that will be produced through $eP \to eZX$ taking $\sqrt{s} = 1300$ GeV (LHeC), for the different prescriptions that we have adopted for making the convolution of the PDFs with the amplitude of the quark processes.

Conclusions

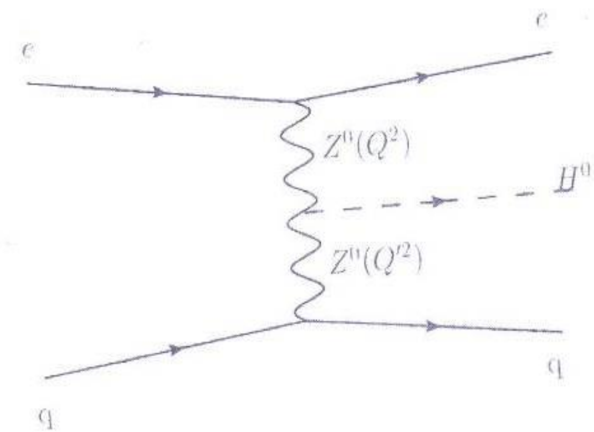
- In order to have precision in the calculations performed by using the Parton Model it is necessary to make a proposal to do the identification of the scale energy parameter $\tilde{Q}^2$.
- We provide a naive prescription to make the convolution of the PDFs and the square amplitude. The advantages of our proposal is that it is simple and practical.

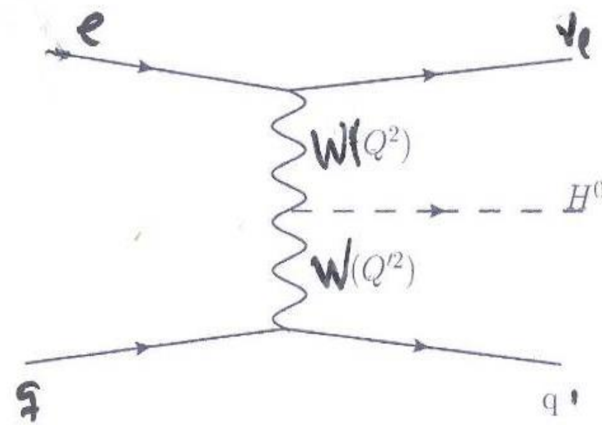
- Is simple because in order to apply it, it is necessary only to do a geometrical analysis of the diagrams which contribute to the cross section rates of the process under consideration.
- It is practical, because retains the idea that in the Parton Model the intensity of the collision of the electron and the proton is given for the square transfer momentum in each contributing diagram.

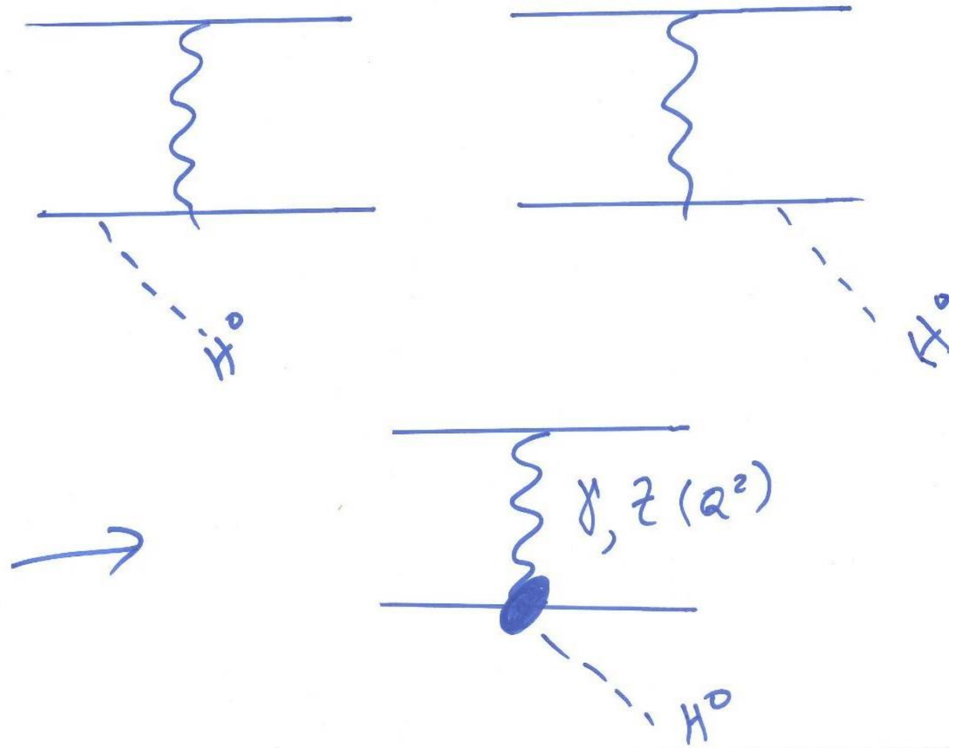
H-Production via ep-DIS

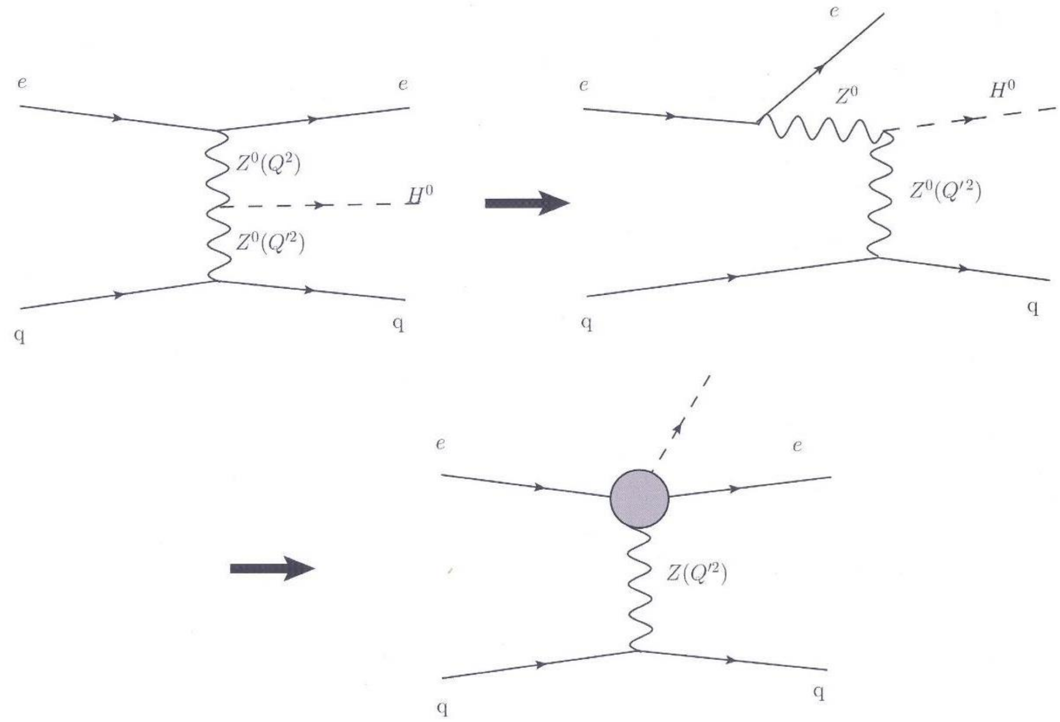
$$e + p \rightarrow e + H + X \qquad (2)$$

$$e + p \rightarrow \nu + H + X \qquad (3)$$

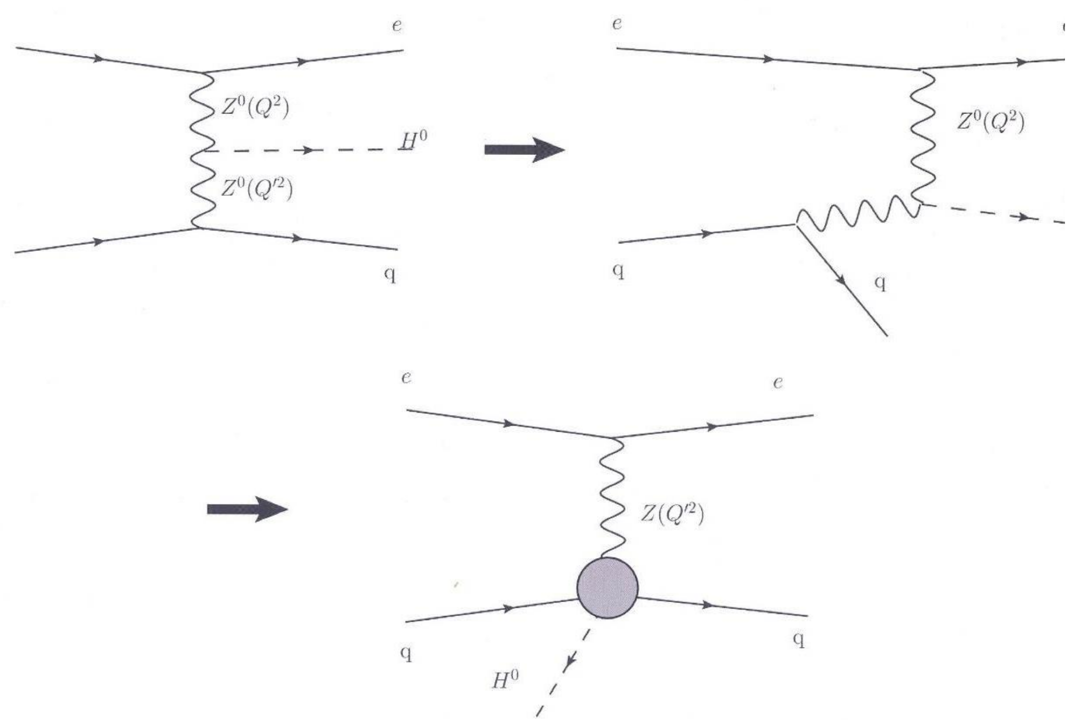

e
e
$Z^0(Q^2)$
H^0
$Z^0(Q'^2)$
q
q


e
ν_e
$W(Q^2)$
H^0
$W(Q'^2)$
q
q'

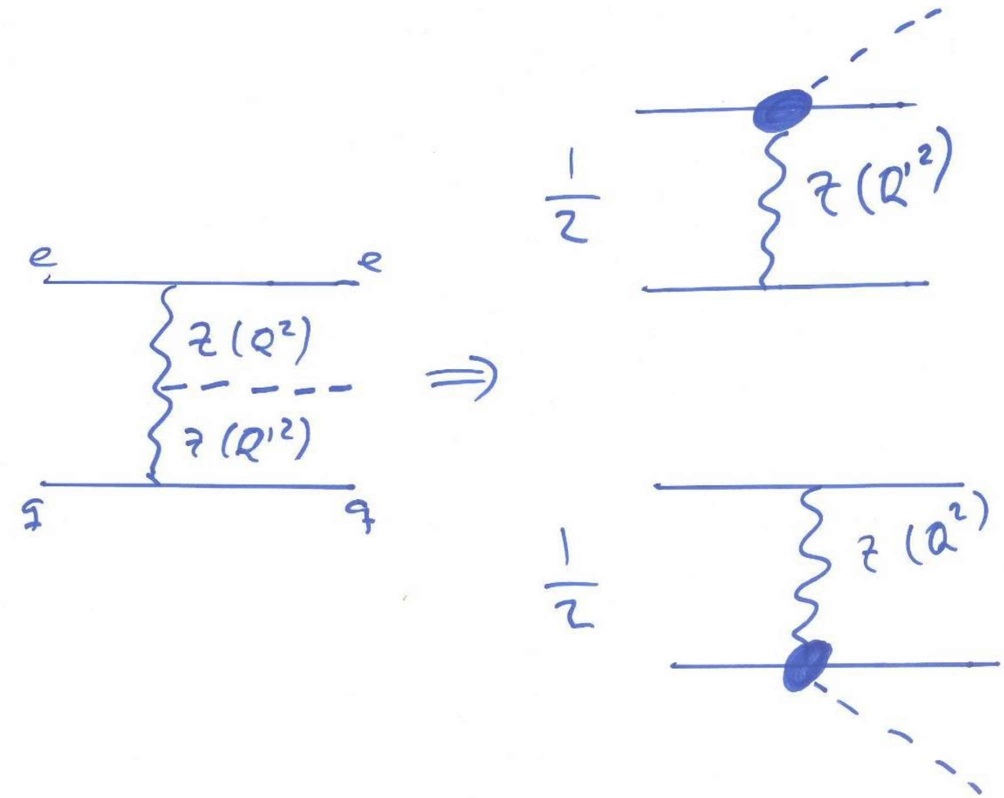

H^0
H^0
$\gamma, Z\ (Q^2)$
H^0



(a)



(b)


e
e
Z (Q²)
Z (Q'²)
q
q
1/2
Z (Q'²)
1/2
Z (Q²)

$\tilde{Q}^2$	M_H^2	Q^2	Q'^2	M_Z^2
$\sigma(eP \to eH^0X)(10^{-37}cm^2)$	2.966	3.328	3.325	3.146
$\sigma(eP \to \nu H^0X)(10^{-37}cm^2)$	9.978	11.518	11.529	11.063

Table 1. Cross section rates for H^0 production through the process $eP \to eH^0X$ for $\sqrt{s} =$ 300 GeV (HERA) and 1300 GeV (LHeC), for the different prescriptions that we have taken for making the convolution of the PDFs with the amplitude of the quark processes (see Eqs. (17)-(21)).

$\tilde{Q}^2$	M_H^2	Q^2	Q'^2	M_B^2
$\sigma(eP \to eH^0X)$	2.966×10^4	3.328×10^4	3.325×10^4	3.146×10^4
$\sigma(eP \to \nu H^0X)$	9.978×10^4	11.518×10^4	11.529×10^4	11.063×10^4

Table 2. Number of H^0's that will be produced through $eP \to lH^0X$ taking $\sqrt{s} = 1300$ GeV (LHeC) and assuming an integrated luminosity of ab^{-1}, for the different prescriptions that we have taken for making the convolution of the PDFs with the amplitude of the quark processes (see Eqs. (17)-(21)).

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$\tilde{Q}^2$	M_H^2	Q^2	Q'^2	M_B^2
M_H^2	—	12.20%	12.10%	6.07%
Q^2	-10.88%	—	-0.09%	-5.47%
Q'^2	-10.80%	0.09%	—	-5.38%
M_B^2	-5.72%	5.78%	5.69%	—

Table 3. In this table we show the difference in term of percentage for the number of events using several choices of $\widetilde{Q}^2$ in the $e + p \to e + H^0 + X$ channel. Here percentages were calculated using the values of the first row respect the first column. We can see that choosing $\widetilde{Q}^2 = Q'^2$ or $\widetilde{Q}^2 = Q^2$ is almost equivalent since the difference between number of events with this choices are less than 0.1%.

$\tilde{Q}^2$	M_H^2	Q^2	Q'^2	M_B^2
M_H^2	—	15.43%	15.54%	10.87%
Q^2	-13.37%	—	0.09%	-3.95%
Q'^2	-13.45%	-0.09%	—	-4.04%
M_B^2	9.81%	4.11%	4.21%	—

Table 4. In this table we show the difference in term of percentage for the number of events using several choices of $\widetilde{Q}^2$ in the $e + p \to \nu + H^0 + X$ channel. Here percentages were calculated using the values of the first row respect the first column. We can see that choosing $\widetilde{Q}^2 = Q'^2$ or $\widetilde{Q}^2 = Q^2$ is almost equivalent since the difference between number of events with this choices are less than 0.1%.

Conclusion

- In order to have precision in the calculations performed by using the Parton Model it is necessary to make a proposal to do the identification of the scale energy parameter $\tilde{Q}^2$.

- In this case our proposal is.

$$\{f(x',Q'^2)\}|M_{lep}|^2 + \{f(x',Q^2)\}|M_{had}|^2$$

$$+ \{[f(x',Q'^2)]^{1/2}\ [f(x',Q^2)]^{1/2}\}$$

$$2Re[\{M_{had}][M_{lep}]^*\}$$

$$\mathbf{M_{lep} = M_{had} = (1/2)\ M}$$

$$=\{f(x',Q'^2)|^2 + f(x',Q^2) +$$

$$[f(x',Q'^2)]^{1/2}\ [f(x',Q^2)]^{1/2}\}|M|^2/4$$

$$=\{[f(x',Q'^2)]^{1/2}+[f(x',Q^2)]^{1/2}\}^2\,(1/4)\,|M|^2$$

$$M_{lep}=M_{had}=(1/2)\,M$$