Pouvoir prédictif des modèles effectifs de QCD
Le cas du point critique chiral

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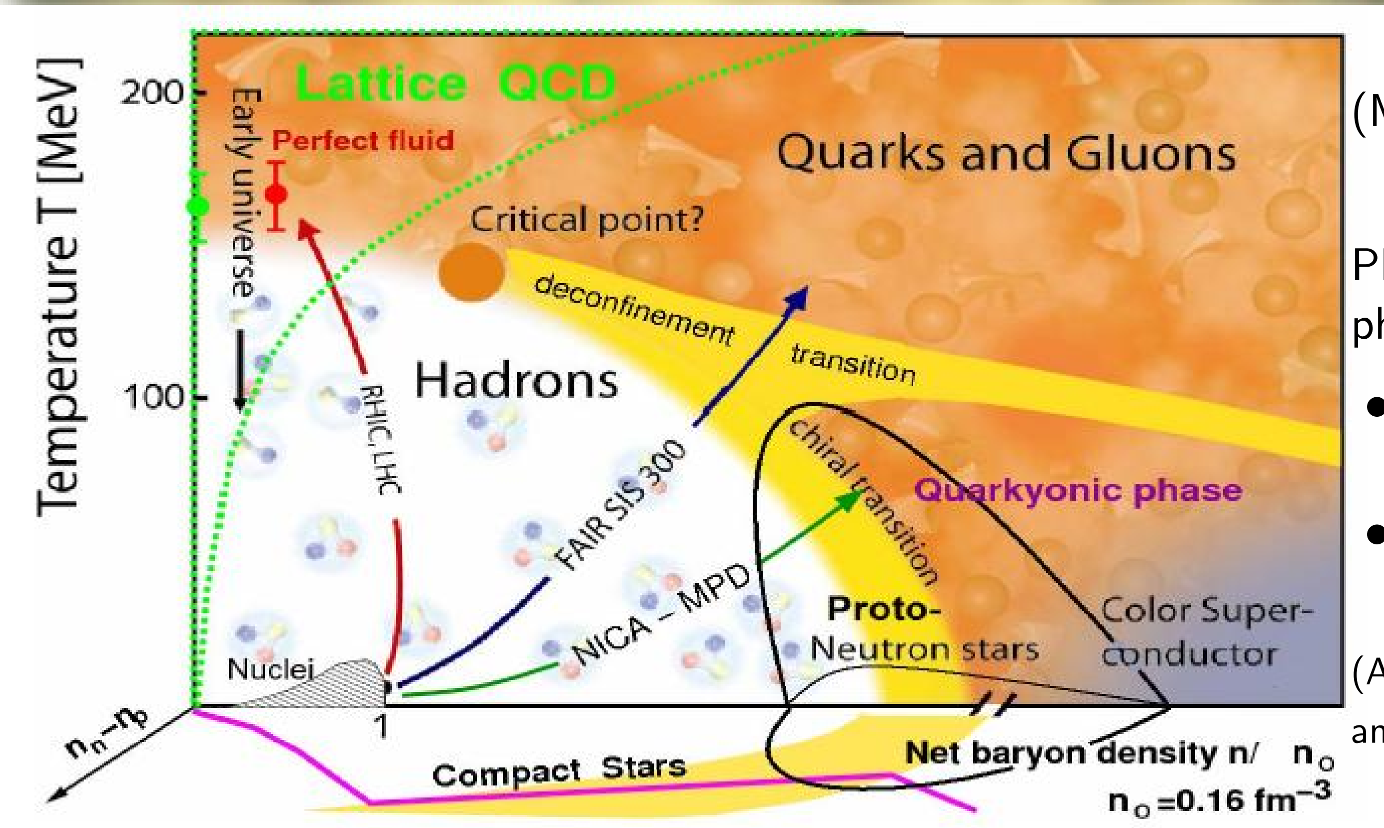
9-12 Octobre 2017

Schematic QCD phase diagram in the T, density plane



(Massimo Di Toro)

Phases arise mainly from two phenomena:

- **deconfinement** (related to color) ;
- **chiral symmetry** (related to the quark sector).

(Also quark condensation may mix color and flavor indices)

One to rule them all:

$$\boxed{\mathcal{L}_{QCD} = \bar{q}\,(\,i\gamma^{\mu} D_{\mu} - \hat{m}\,)\,q - \frac{1}{4} F^{a}_{\mu\nu}\, F^{\mu\nu}_{a} \;;\; D_{\mu} = \partial_{\mu} - ig\lambda^{a}\, A^{a}_{\mu}}$$

CEP (chiral Critical End Point): separation between first order and crossover chiral transition lines.

Motivation: Hot and Dense QCD

Study of the hot and dense phases of QCD with a quark effective model, at equilibrium, with a focus on the chiral transition.

Dense phase: Very few experimental and theoretical knowledge ; Different critical properties (first order) than zero density ; Compact (neutron) star phenomenology.

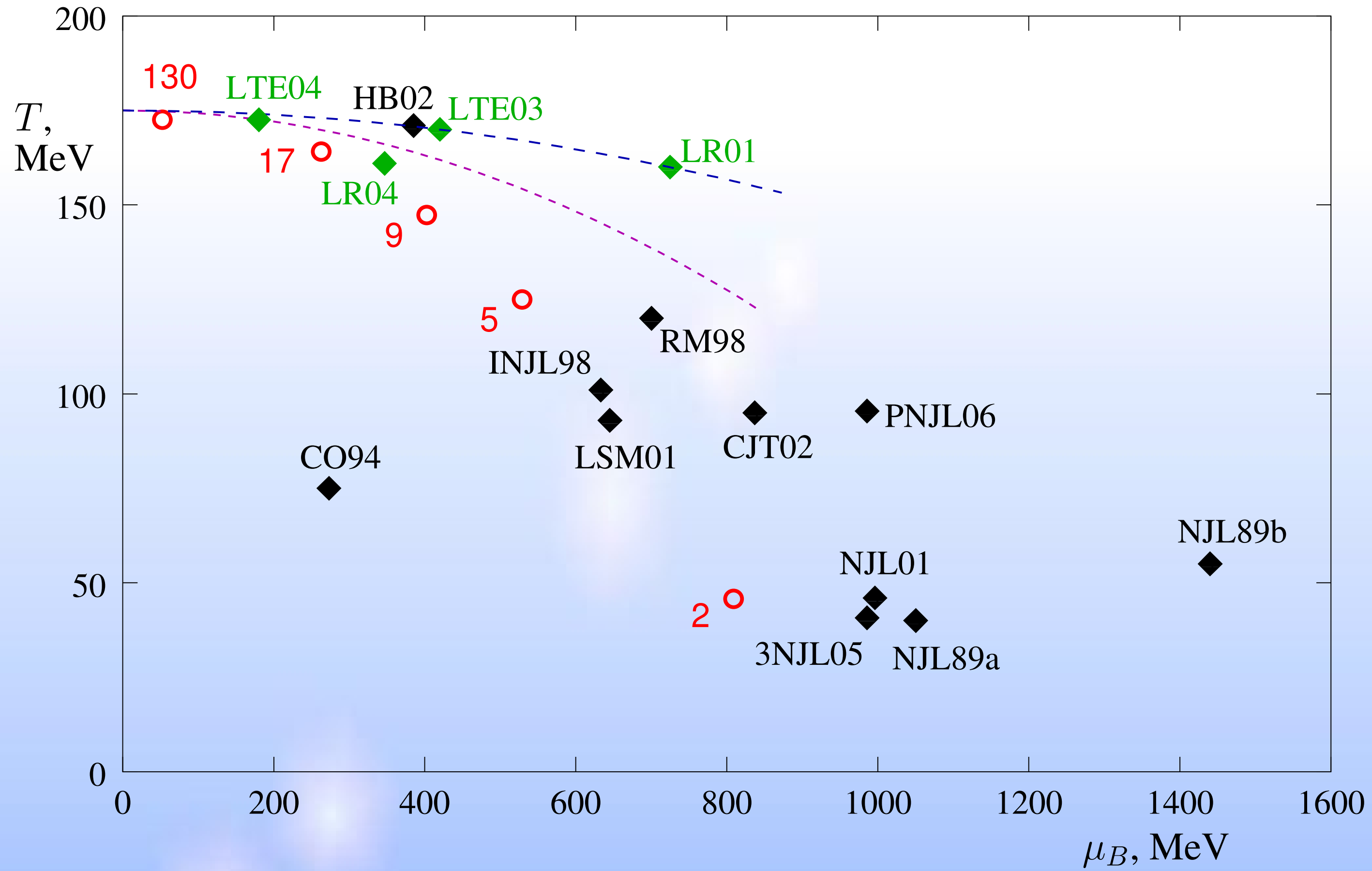
Equilibrium properties: First step before understanding out of equilibrium ; can be an input for transport code based on local thermal equilibrium ; quark matter at equilibrium may exists in core of compact stars.

Chiral physics: Chiral symmetry governs important properties of hadronic physics (e.g. the lightness of the pion $\leftrightarrow$ nucleon-nucleon interaction, the rho meson, etc.) in the low mass region.

Effective models

- Non-perturbative finite density properties inaccessibles to QCD or even Lattice QCD:
  Basically, model provides an extrapolation (based on some QCD ingredients, not a glorified polynomial) from known inputs to some predictions.
- calculation of phases and critical properties ; mesonic fluctuations description ; provide microscopic predictions (cross sections, viscosity, etc) ; microscopic mechanism related to QCD (chiral symmetry breaking, statistical confinement effect, etc.) ;

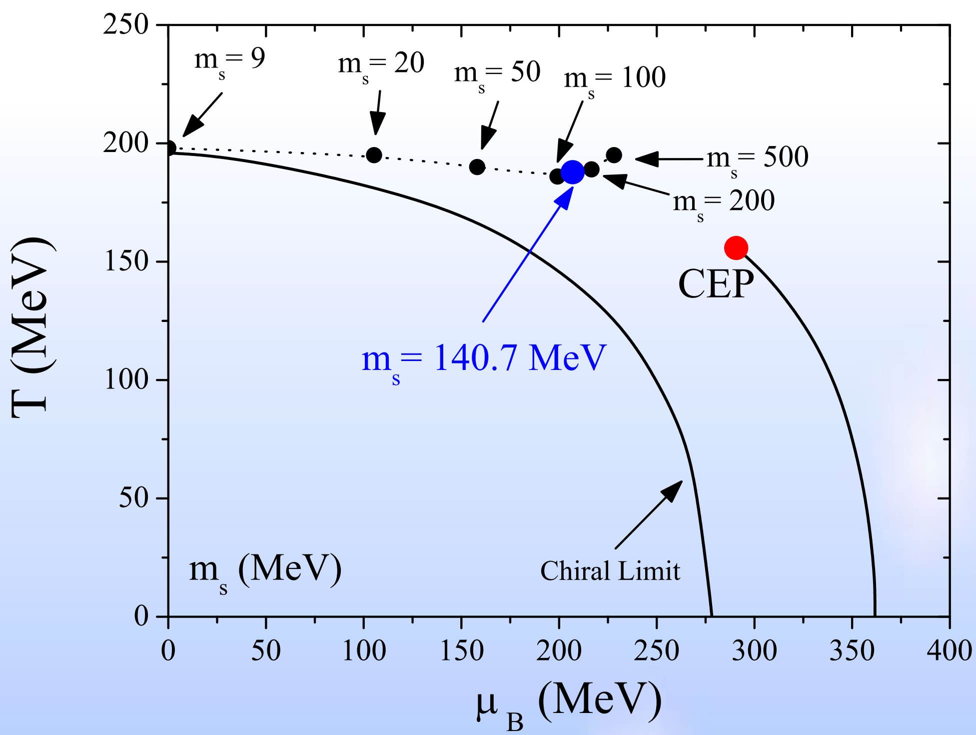
Predictive power of models, the case of the chiral CEP

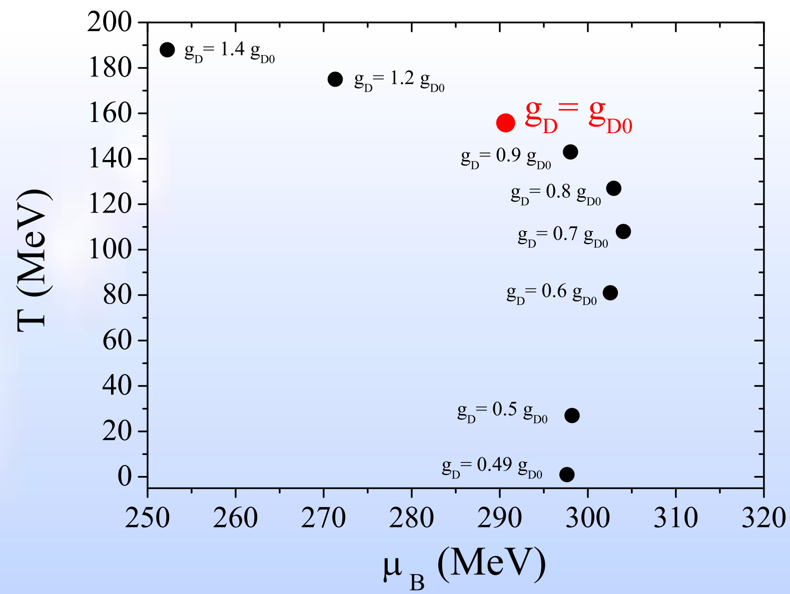


Comparison of predictions for the location of the CEP in the (T, μ_B) plane (the baryonic chemical potential $\mu_B = 3\mu_{quark}$). Black points are model predictions, green one are LQCD predictions and red one are freeze-out points measured in HIC. From Stephanov, PoS LAT **2006** (2006) 024.

Different effects, same output on the predictions

Evolution with strange quark sector properties (mass and t'Hooft flavor mixing)



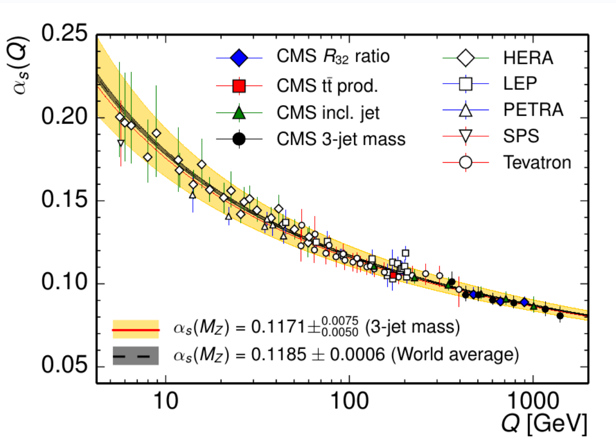


TCP (tricritical point between 1st and 2nd order transition) in the chiral limit for the light sector only $(m_u = m_d = 0)$

TCP position becomes very sensitive to m_s around the physical value of m_s ; also sensitive to the variation of the t'Hooft interaction and in the "same direction".

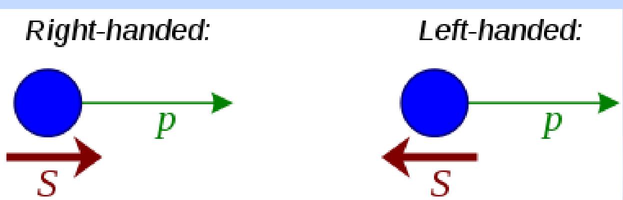
Some methods to study QCD and symmetries

$$\boxed{\mathcal{L}_{QCD} = \bar{q}\,(\,i\gamma^{\mu} D_{\mu} - \hat{m}\,)\,q - \tfrac{1}{4} F^{a}_{\mu\nu}\, F^{\mu\nu}_{a} \;;\; D_{\mu} = \partial_{\mu} - ig\lambda^{a} A^{a}_{\mu}}$$



- small α_S perturbative QCD, hard thermal loop.
- Lattice QCD (vaccuum or finite temperature and small densities)
- Schwinger-Dyson approach, etc.
- **Effective models, based on symmetries** (in the spirit of Landau) for the whole $(T,\,\mu)$ plane.

NB: for us, typical scale $\Lambda_{QCD} \simeq 200$ MeV $\simeq 1$ fm.



Chirality:
special relativity distinguish left (L) and right (R) fermion ; obviously boosting a massive particle changes its chirality.

QCD almost invariant (exact if $\hat{m}_{uds} = 0$; in practice, OK because small compared to Λ_{QCD}) by the $U_R(N_f = 3) \times U_L(N_f = 3)$ symmetry and (at finite temperature) Z_{N_c} (center of $SU(N_c)$).

But experimentally ...

Experimental facts, chiral symmetry breaking and modelisation

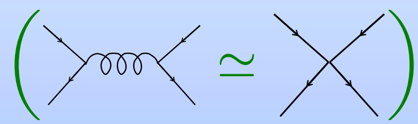
Only hadrons in vacuum: **Quark confinement** in the non-perturbative regime and **asymptotic freedom** (color deconfinement) at higher energy, related to breaking of Z_{N_c} at finite temperature

No Wigner realization of the chiral symmetry in vacuum: **Spontaneous chiral symmetry breaking of $SU_R(3) \times SU_L(3)$ to $SU(3)_V$**
$\Rightarrow$ octet of (almost) **Goldstone** bosons, the off-scale light pseudoscalar octet.

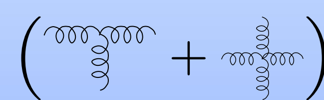
η' not of the Goldstone type: **Adler–Jackiw–Bell $U_A(1)$ anomaly** breaks $U_R(N_f) \times U_L(N_f)$ to $U_V(1) \times SU_R(N_f) \times SU_L(N_f)$ ('t Hooft picture: interaction with instantons change chirality).

$\Rightarrow$ **the PNJL chiral model** ($q = (q_u, q_d, q_s)$ are the light quark fields) :

$$\mathcal{L}_{PNJL} = \bar{q}(i\gamma_\mu D^\mu - \hat{m})q + \frac{1}{2} g_S \sum_{a=0}^{8} [(\bar{q}\lambda^a q)^2 + (\bar{q}\, i\gamma_5 \lambda^a q)^2]$$

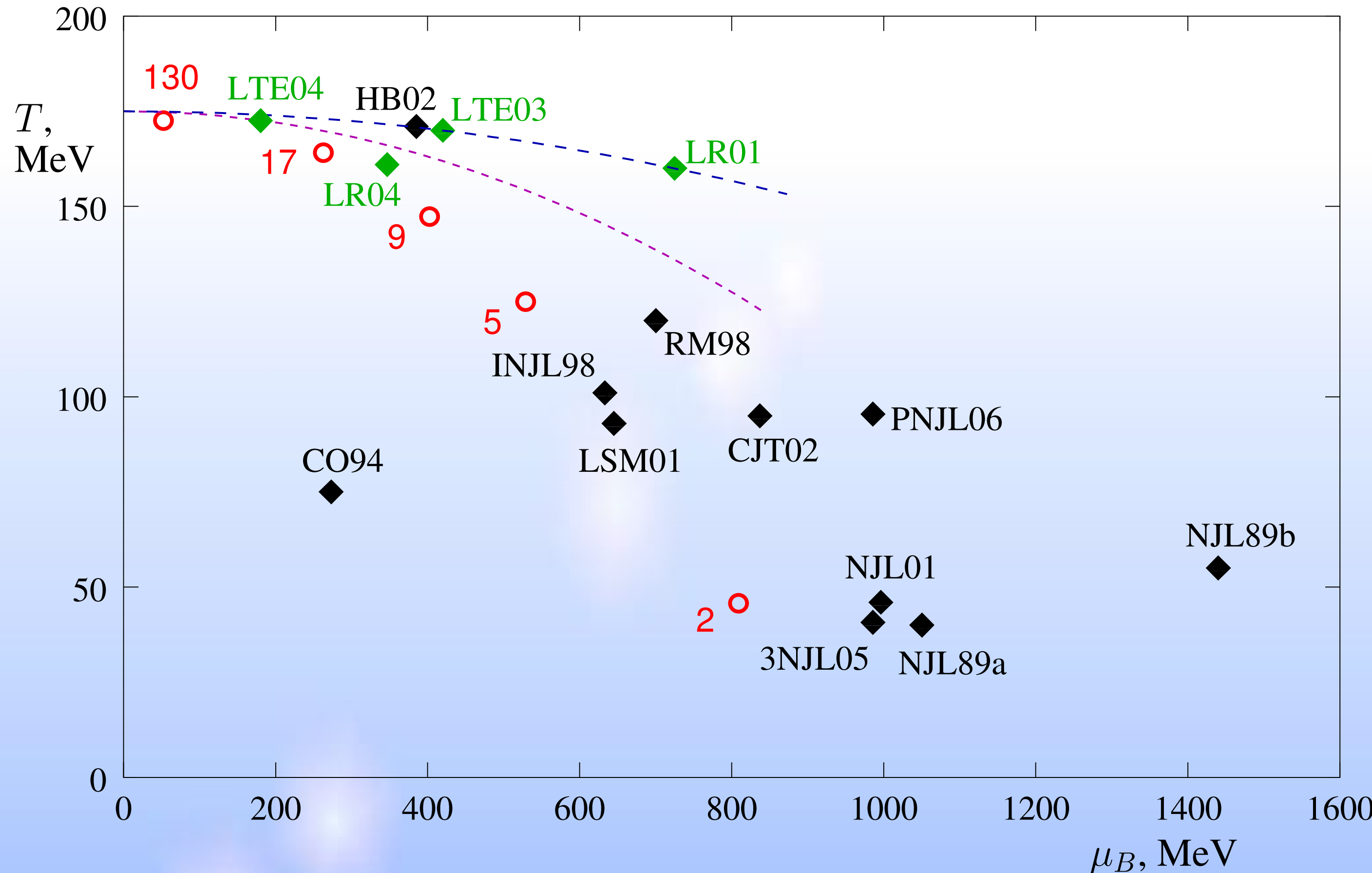


$$+ g_D\{\det[\bar{q}(1+\gamma_5)q] + \det[\bar{q}(1-\gamma_5)q]\} - \mathcal{U}(\Phi[A], \bar{\Phi}[A]; T)$$



Rem: baryonic mass generation and chiral symmetry:
Even with zero bare quark mass, if the "quark condensate" $\langle \bar{q}q \rangle \neq 0$, $(\bar{q}q)^2 \simeq 2\langle \bar{q}q \rangle \bar{q}q + \ldots \Rightarrow$ generation of a dynamical mass $-2g_S\langle \bar{q}q \rangle$ that breaks spontaneously the chiral symmetry.
$-g_S\langle \bar{q}q \rangle \simeq 330$ MeV $\simeq M_N/3$.
When $\langle \bar{q}q \rangle \to 0$ at finite temperature/density: chiral phase transition.

Predictive power of models, the case of the chiral CEP



What are the relevant microscopic mechanism ?

What can we do in the absence of data ?

Parametrisation of the NJL model

❖ "Toy model" two flavors NJL model with scalar interactions:

$$\mathcal{L}_{NJL} = \bar{q}(i\gamma^{\mu}\partial_{\mu} - m_0)q + G\left[(\bar{q}q)^2 + (\bar{q}i\gamma_5\boldsymbol{\tau}q)^2\right] \ . \tag{1}$$

We have one **"a priori"**: the quark scalar sector of the NJL model is relevant to study the chiral properties of QCD. Every conclusions gathered from this model has to be evaluated with respect to this hypothesis.

❖ Three dimension-full "free" parameters (loosely constrained by phenomenology):

- m_0 the quark mass around the u and d quark masses
- Λ the three-dimensional cutoff of the order of the (hadronic) Λ_{QCD}
- G the "Fermi" coupling constant, $G = g/\Lambda^2$, $g \in [1, 10]$.

❖ (At least) three phenomenological inputs: pion + condensate phenomenology, in vacuum

$$\begin{pmatrix} m_{\pi} \\ f_{\pi} \\ c = -\langle\bar{q}q\rangle^{1/3} \end{pmatrix} = \begin{pmatrix} 137 & \text{MeV} \\ 93.0 & \text{MeV} \\ 315 & \text{MeV} \end{pmatrix} \ . \tag{2}$$

The inverse problem

NB: here, model $\equiv$ Lagrangian + approximations + parametrisation procedure.

- **Direct problem** $\boxed{\Lambda, m_0, G \Rightarrow m_\sigma, m_\pi, f_\pi, c, \text{CEP}}$ (or other predictions)

- **Inverse problem** $\boxed{m_\pi, f_\pi, c \Rightarrow \Lambda, m_0, G}$

For example one can minimize a merit function as a χ^2.
Remark: the value of the χ^2 is important to quantify the quality of the fit but the shape of the function (very flat or very narrow) is also an important information concerning the robustness of the fit. We will indirectly get an access to this information.

- **Here, exact inverse problem** with Hartree + Ring + Quasi-Goldstone approximation.

Luckily, with physical values for the inputs $\Rightarrow$ unique solution.

Sensitivity and ill-posedness of a problem

Concern: parametrisation in vacuum $\Rightarrow$ prediction in medium (position of the CEP).

How good is this **extrapolation** ? Is the problem **ill-posed** (no solution, unique solution) ?

Previous studies:

How different physical sectors **qualitatively** affect the CEP position ?
To do this $\Rightarrow$ variation of the parameters (thus destroying the vacuum phenomenology).

Goals: Systematic study of the variations of the whole parameter space compatible with the "true" inputs of the model (m_π, f_π, c) and assessment of the sensitivity of the extrapolation with a **quantitative** criterium.

$\Rightarrow$ **introduction of a sensitivity coefficient with respect to the inputs.**

Rem: we cannot simply propagate the experimental uncertainties on the input since the effective model is not QCD. We necessarily have unknown systematic theoretical errors.

Sensitivity definition

Infinitesimal sensitivity of a prediction based on the (statistical) propagation of an uncertainty:

Let X be a prediction depending on two inputs a and b.
Standard deviation of X (where $\sigma(a)$ and $\sigma(b)$ are deviation for the inputs with some distribution):

$$\sigma^2(X) = \left(\frac{\partial X}{\partial a}\right)^2 \sigma^2(a) + \left(\frac{\partial X}{\partial b}\right)^2 \sigma^2(b) \ . \tag{3}$$

Sensitivity:

$$\Sigma(X) = \lim_{\sigma \to 0} \frac{\sigma_{rel}(X)}{\sigma^I_{rel}} \tag{4}$$

where,

$$\sigma_{rel}(X) = \frac{\sigma(X)}{X} \tag{5}$$

$$\sigma^I_{rel} = \frac{1}{2}\left(\frac{\sigma(a)}{a} + \frac{\sigma(b)}{b}\right) , \tag{6}$$

and $\lim_{\sigma \to 0}$ means we take infinitesimal variations of the inputs.

In the NJL model

We choose vanishing *relative dispersion* of the inputs namely for $I = a$ or b, $\sigma(I)/I = p$ and $p \to 0$:

$$\Sigma(X) = \sqrt{\left(\frac{\partial X}{\partial m_\pi}\right)^2 \frac{m_\pi^2}{X^2} + \left(\frac{\partial X}{\partial f_\pi}\right)^2 \frac{f_\pi^2}{X^2} + \left(\frac{\partial X}{\partial c}\right)^2 \frac{c^2}{X^2}}\,. \tag{7}$$

We choose a uniform (no a priori) for the input distribution (and check that the results does not depend on this choice).

Sensitivity meaning

- Large (infinite) sensitivity $\Rightarrow$ the problem is ill-posed, the predictive power is low.
  Any small but finite errors in the inputs (experimental errors as for the condensate or theoretical systematic errors because of the approximations) will damaged the prediction.
- Small sensitivity $\Rightarrow$ the prediction is robust and can be trusted **if the model itself can be trusted.**

For our toy model and for the CEP prediction:

- $\Sigma \gg 1$: the extrapolation to the medium is not robust, the scalar sector does not bring enough constraint.
- $\Sigma \simeq 1$: the chiral properties describe by the scalar interaction are relevant for the given prediction.

Temperature and chemical potential CEP sensitivities

NB: parameter sensitivities small (around 3). At least the inverse problem is well-posed and one can use the model to compute predictions.

		Sensitivities	Values
Parameters	Λ	2.83	0.653 (GeV)
	m_0	4.11	0.0051 (GeV)
	$G\Lambda^2$	3.32	2.11
In-medium predictions	T_{CEP}	71.5	0.0299 (GeV)
	μ_{CEP}	1.05	0.327 (GeV)

Table 1: Sensitivities of the parameters and in-medium predictions considering infinitesimal changes of the inputs. The sensitivities of the parameters and of μ_{CEP} are close to 1. The sensitivity of the temperature coordinate of the CEP is very large. These values were computed numerically with a Monte-Carlo.

- For T_{CEP}, $\Sigma \simeq 70$: no consistent conclusion can be given within the model. Even the existence of the CEP may be questioned.
- For μ_{CEP}, $\Sigma \simeq 1$ (even lower than the parameters !): the chiral symmetry and scalar sector is part of the physics relevant for the chemical potential coordinate of the CEP. A model without this sector is probably meaningless to study chiral physics (and indeed: no Goldstone pion without it !).

Consequences of small but finite deviations of the inputs

Why finite variations are relevant ?

- Obvious for observables **not very well known experimentally** (as for the quark condensate).
- For the pion: experimental value very well known but **unknown systematic errors**.

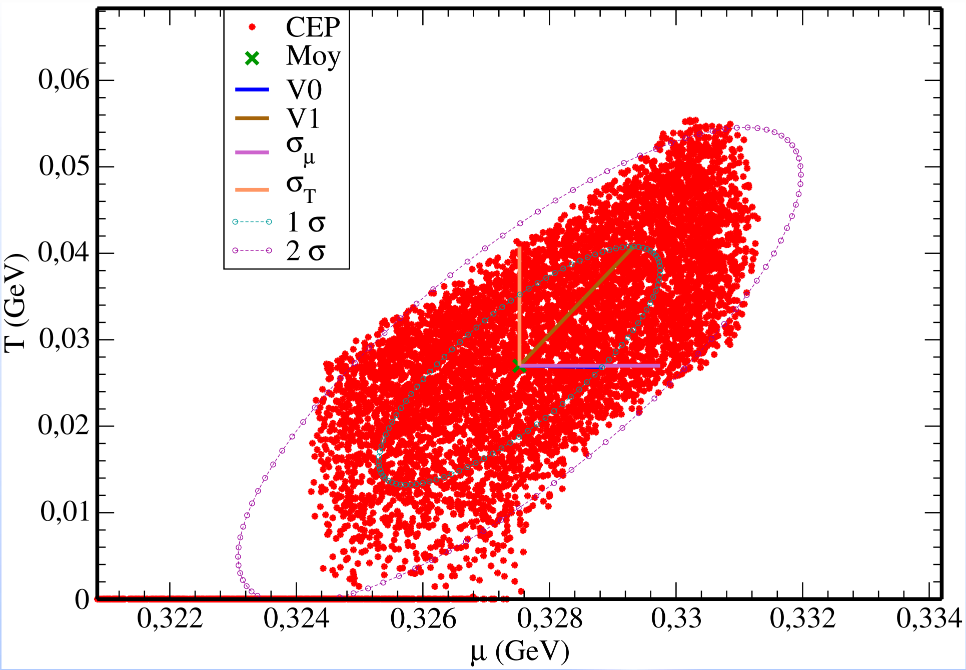
1. NJL is an effective (uncontroled) model of QCD. Only our "a priori" of the correctness of the model (symmetries for example) tells us we can use it.
2. Approximations within the NJL model:
   Quasi-Goldstone approximation $\Rightarrow$ 1% of variation without it.
   Next order in $1/N_c$ (meson loop approximation) $\Rightarrow$ 5%.

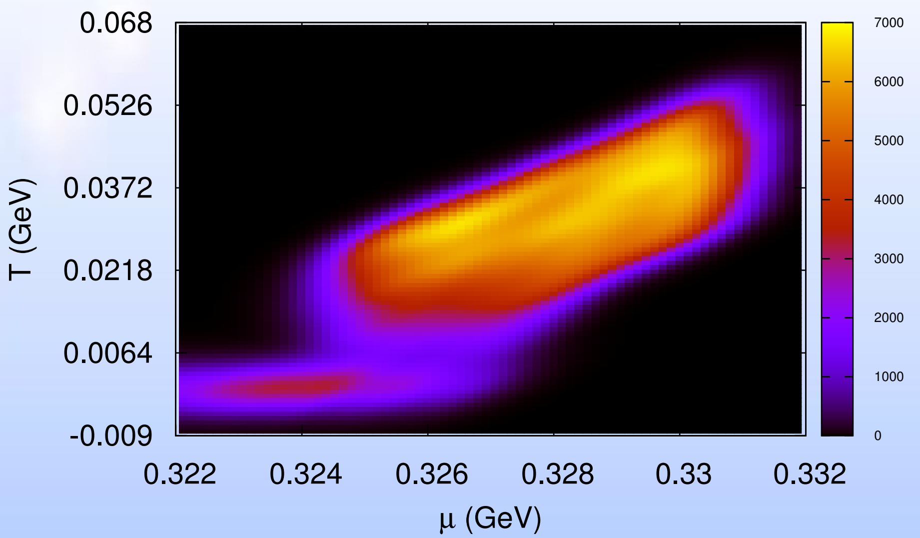
Hence it seems unreasonable to expect than the inverse problem as an accuracy better than 1%.

CEP unpredictability

We allow finite variation with relative dispersion $p = 1\%$ to compute the sensitivities:

$$\begin{array}{cccccc} m_\pi & \in & [& 135.6 & , & 138.4 &] \\ f_\pi & \in & [& 92.07 & , & 93.93 &] \\ \langle \bar{q}q \rangle^{1/3} & \in & [& -318.1 & , & -311.8 &] \end{array}$$





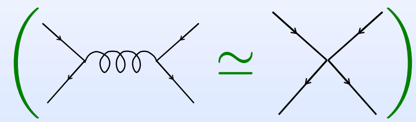
- Striking dispersion in the T direction.
- With only a 1% variation of the inputs, in 10% of the case there is no CEP !
  There is no mechanism in this model that ensures the CEP must exists.
- Very stable chemical potential prediction.

Sensitivity with finite temperature constraints

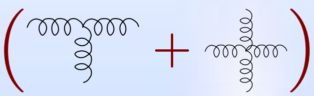
NJL use only zero temperature data ; PNJL use also as inputs a phenomenological potential $\mathcal{U}\left(\Phi[A],\bar{\Phi}[A];T\right)$ fitted on finite temperature LQCD **(pure gauge only)** results.

$\mathcal{U}\left(\Phi[A],\bar{\Phi}[A];T\right)$ can be thought as the thermal gluon pressure in which quark propagates, with a minimal coupling to gluons.

$$\mathcal{L}_{PNJL} = \bar{q}(i\gamma_\mu D^\mu - \hat{m})q + G[(\bar{q}q)^2 + (\bar{q}\,i\gamma_5\vec{\tau}\,q)^2]$$



$$-\mathcal{U}\left(\Phi[A],\bar{\Phi}[A];T\right)$$

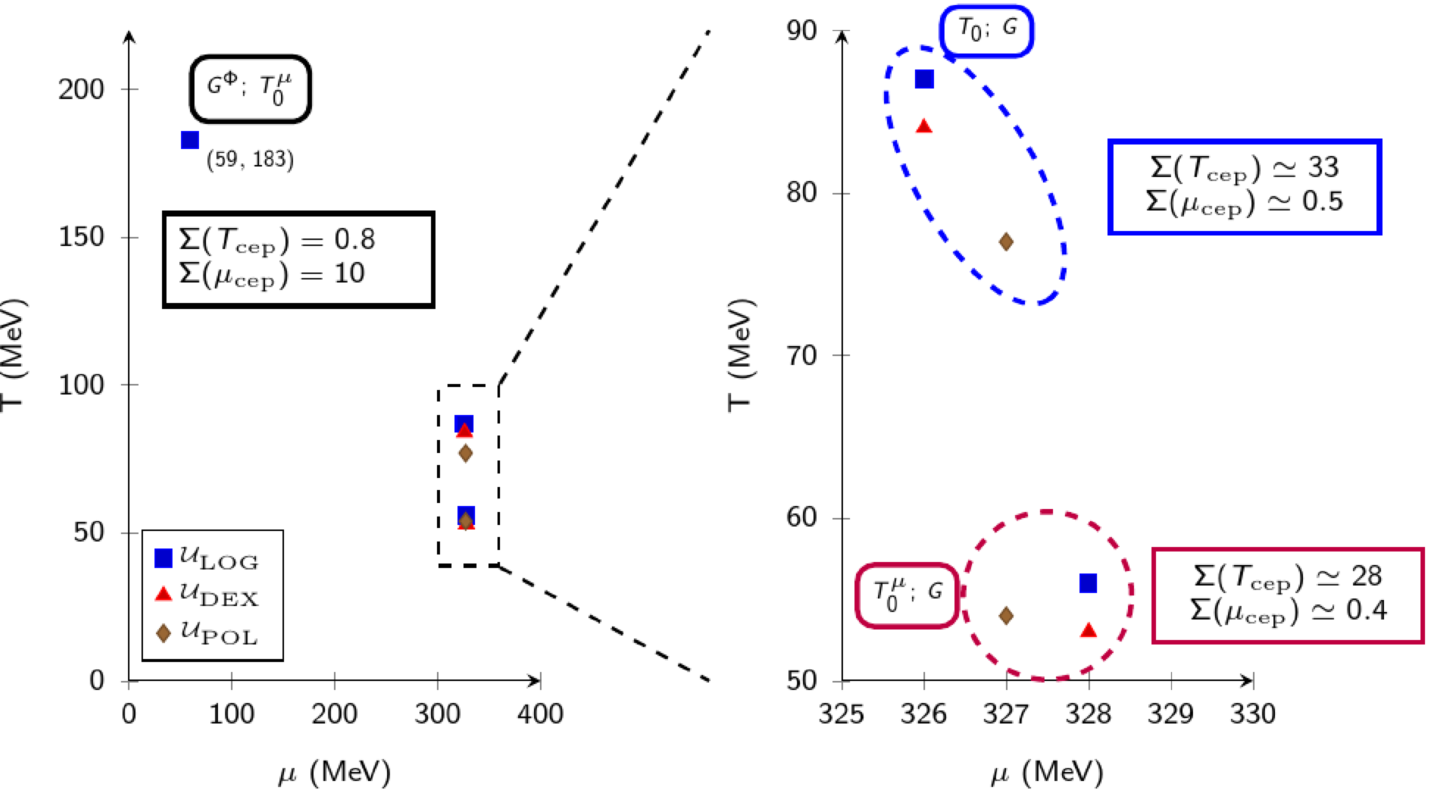


But it exists differents parametrisation based on the same LQCD data ($\mathcal{U}_{LOG}$, $\mathcal{U}_{DEX}$, $\mathcal{U}_{POL}$, etc) ;

Also different ways to take into account quark back reaction on the gluon ($G \to G(\Phi)$) ;

Or the effect of the number of flavor on deconfinement temperature ($T_0(\mu)$).

Partial sensitivity analysis (quark inputs variations only)



→ G^{Φ} et T_0 : Pas de CEP $\forall\ \mathcal{U}$
→ G et T_0 ou T_0^{μ} : CEP instable vs. inputs mais stable vs. $\mathcal{U}$
→ G^{Φ} et T_0^{μ} : CEP stable vs. inputs mais instable vs. $\mathcal{U}$

$\Sigma(T_{\rm CEP})_{\rm PNJL} < \Sigma(T_{\rm CEP})_{\rm NJL}$:
Contraintes LQCD $T \neq 0$ et $\mu = 0$

→ More constraints helps.

→ $G(\Phi)$ surprisingly stable but:

- the form of $G(\Phi)$ (or even its relevance) is not well established, there is no good inputs to fix its coefficient. The a priori one can have on $G(\Phi)$ is not very good (in particular, no microscopic explanation).
- CEP is unstable vs. the potential choice (CEP do not exist in some cases !).

Predictive power of effective models

- The parametrization (inverse problem) is a question that need to be addressed.
- Other problems as for example the coupling constant is supposed to be constant even in medium $\Rightarrow$ no universal admitted temperature and chemical potential dependence of the model parameters.
- Constrained in medium are needeed:
  To improve sensitivity and predictive power ;
  To have an idea of the parameter dependance with medium.
- They may be obtained:
  In future HIC experiment at lower energy reaching higher densities (but difficult to relate our thermodynamical calculations with real time evolution of the fireball) ;
  Compact star phenomenology (but one cannot probe the inside, essentially one can only have global information as mass and radius ; still very constraining for equation of states).

Conclusions

The construction of the phase diagram of QCD is in progress with the help of effective models (and other approaches) using experimental and LQCD inputs but the foundations has to be more firm $\Rightarrow$ more control is needed on the predictive power of models to get a better understanding of the non perturbative aspects of QCD:

Success of effective models:

- Understanding of the chiral dynamics
- Importance of the Polyakov loop
- Effective models needed to extrapolate and also understand microscopically or validate other approaches (e.g. LQCD at finite density and radius of the Taylor expansion).
- Experimental constraints needed, in particular in medium.

Perspective for the assessment of the predictive power:

- Sensitivity with respect to all the parameters of the model.
- Cross analysis of the sensitivity in different effective model (e.g. with PNJL and PQM) with the same inputs $\rightarrow$ determine if there is stable feature accross "effective model space" (hopefully a feature shared with QCD).
- Can the CEP and the compact star radius constrain one another (whoever is measured first) ?
  $\rightarrow$ calculation of the mutual information of the observable.
  If the mutual information is large we know that any knowledge of one (even with uncertainties) may constrained the other.