Separatrix Folding with Multipoles

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Outline

- Some Historical Perspective
- Obligatory math stuff (take short nap here)
- Separatrix pictures
- ‘Step’ Size and beam loss on the septa
- Concluding remarks

Also note, recent related work:
“Phase Space Folding Studies for Beam Loss Reduction During Resonant Slow Extraction at the CERN SPS”
L.S. Stoel, M. Benedikt, K. Cornelis, M.A. Fraser, B. Goddard, V. Kain, F.M. Velotti
CERN, Geneva, Switzerland
IPAC 2017

Some Historical Perspective

Let's talk, for a moment, about this:

Y. Kobayashi and H. Takahashi,
"Improvement of the Emittance in the Resonant Beam Ejection",
Proc. VIth Int. Conf. on High Energy Accel., Massachusetts,
pp.347-351 (1967)

Improvement of the Emittance in the Resonant Beam Ejection

Y. Kobayashi and H. Takahashi

Institute for Nuclear Study, University of Tokyo

The resonant beam ejection from an AG Synchrotron, by the use of a quadrupole magnet and sextupole magnets, was proposed by Hereward[1] and performed successfully with the CERN PS.[2] A similar method by a current strip was first achieved at CEA.[3] Although many studies have been made on this subject, most of them were made numerically or experimentally. In this paper is given an analytical approach to this subject based on an approximate Hamiltonian.

A radial emittance in the resonant ejection is fairly small, generally. However, according to the predictions of the theory, it can be reduced further. We have investigated numerically motions of particles in a phase space, and found that the reduction of the emittance was really possible.

I. Hamiltonian

Suppose an arrangement shown in Fig. 1, where Q is a quadrupole magnet, K is a kick magnet and M's are non-linear multi-pole magnets. We consider the radial motion in a phase space (x, y), where $y = \frac{\beta}{\sqrt{1+\alpha^2}}\frac{dx}{ds}$, β and α are betatron functions at Q, x is a radial displacement from an equilibrium orbit, and s is a distance along the orbit. Near a m-th resonance, the transfer matrix for one period, that is for m revolutions, from the center of the quadrupole magnet, which will be called the point Q, can be written as

$$\Gamma = \begin{pmatrix} \cos\varepsilon - \alpha\sin\varepsilon & \sqrt{1+\alpha^2}\sin\varepsilon \\ -\sqrt{1+\alpha^2}\sin\varepsilon & \cos\varepsilon + \alpha\sin\varepsilon \end{pmatrix}, \qquad (1)$$

DISCUSSION (condensed and reworded)

M.Q. Barton (BNL): Does Figure 8 represent a numerical or theoretical value?

Kobayashi: This is the result of numerical calculation.

Barton: The non linearity that gave this orbit shift does not cause the separatrices extended to close. Did you have any trouble with the separatrices turning and coming back?

Kobayashi: When the non-linear field is very strong, there is some complicated behavior. In this case the non-linear field is not so strong, so even if the disc diameter converges, it would be larger than the aperture of the donut.

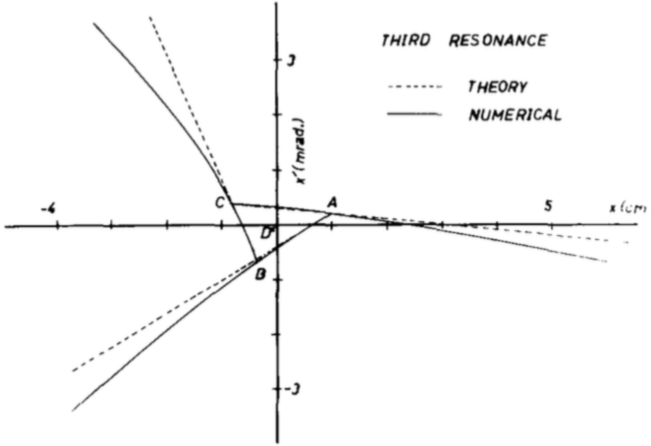


Fig. 3 A separatrix for a third resonance at K_1, where A, B and C are unstable fixed points and D is a stable fixed point. $g_1 = -0.2102$ and $g_2 = g_3 = 0$

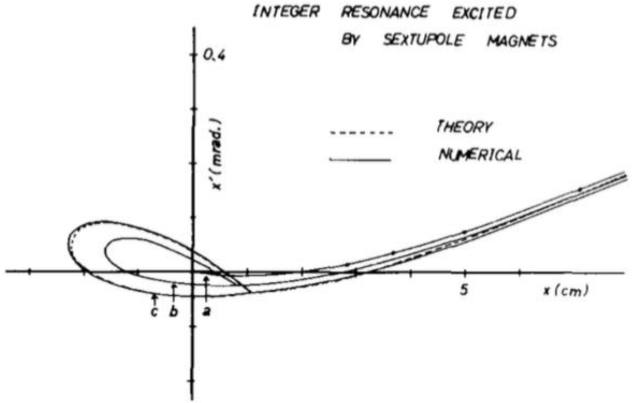


Fig. 2 Separatrices for an integer resonance excited by sextupole magnets. The positions of magnets are indicated in Fig. 4 and separatrices were calculated at K_1. Parameters of sextupoles are $g_1 = g_3 = -0.02102$ and $g_2 = 0.02102$. Phase space area of the separatrices are; a --- 0, b --- 0.117 cm·mrad and c --- 0.280 cm·mrad.

Note: 'g' is an their notation for multipole strength.

$$\begin{pmatrix} x_\nu \\ y_\nu \end{pmatrix} = \Gamma \begin{pmatrix} x_{\nu-1} \\ y_{\nu-1} \end{pmatrix} + \begin{pmatrix} \xi_\nu \\ \eta_\nu \end{pmatrix},$$

$$\begin{pmatrix} \xi_\nu \\ \eta_\nu \end{pmatrix} = \Gamma \sum_{i=1}^{n} \Gamma_i^{-1} \begin{pmatrix} 0 \\ \dfrac{\beta}{\sqrt{1+\alpha^2}} \cdot \psi_i \end{pmatrix},$$

$$\psi_i = g_i x_i^{\ell}$$

(b) The emittance can be appreciably reduced by the use of ten-pole magnets. Examples are shown in Fig. 5 and Fig. 6. In this case, the emittance can be reduced by a factor of three compared with that in an integer resonance, for the same initial amplitude. A disadvantage of the use of ten-pole magnets is that the rate of amplitude blow up changes somewhat rapidly with the amplitude, compared with the case of sextupole magnets. This disadvantage may be removed if the field distribution at a large displacement be shaped so as to show gx^2-like distribution.

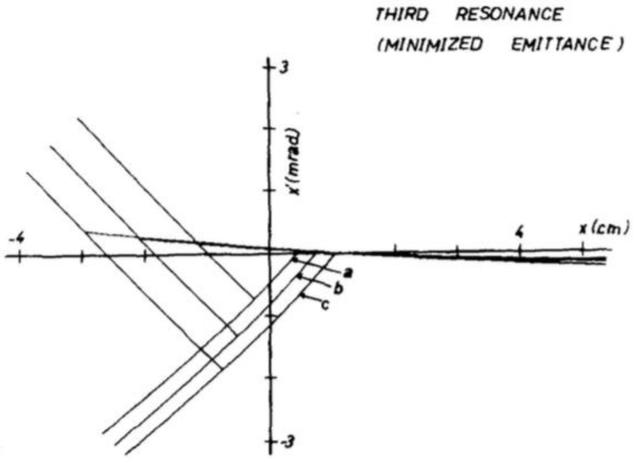


Fig. 7 Separatrices for a third resonance at K_2. g_1 = - 0.?051, g_2 = 0 and g_3 = 0.06. Phase space area; a --- 0.56 cm·mrad, b --- 1.63 cm·mrad. and c --- 3.85 cm·mrad..

(2) Third Resonance

The emittance in a third resonance ejection depends largely on arrangements of an ejection system, and usually it is substantially larger than that in an integer resonance. In the third resonance, however, a size of the separatrix and a displacement of the equilibrium orbit can be determined independently, and in principle it is always possible to realize a condition that an out-going separatrix, a extension of a side of separatrix, passes the origin of the phase space. When such a condition is realized at a point where the beam leaves the orbit, the emittance of the ejected beam will be reduced substantially. An example is shown in Fig. 7. As illustrated here, the emittance can be reduced to the same order of magnitude with that in an integer resonance. Since, in such cases, a displacement of the equilibrium orbit is large, the theory loses its accuracy and behaviors of the orbit

Slow Extraction and Higher Order Multipoles

A perturbation from a sextupole

$$\Delta X = 0 \qquad \Delta Y = 0$$

$$\Delta X' = S(X^2 - \frac{\beta_y}{\beta_x}Y^2) \qquad \Delta Y' = -2S\frac{\beta_y}{\beta_x}XY$$

From a ‘thin’ duodecapole

$$\Delta X = 0$$

$$\Delta X' = \mathcal{D}(X^5 + 5X\left(\frac{\beta_y}{\beta_x}\right)^2 Y^4 - 10\frac{\beta_y}{\beta_x}X^3Y^2)$$

Kobayashi Hamiltonian (normal 3rd integer)

$$H = \frac{\epsilon}{2}(X^2 + X'^2) + \frac{S}{4}(3XX'^2 - X^3)$$

And, when we add a duodecapole (simple 3rd integer)

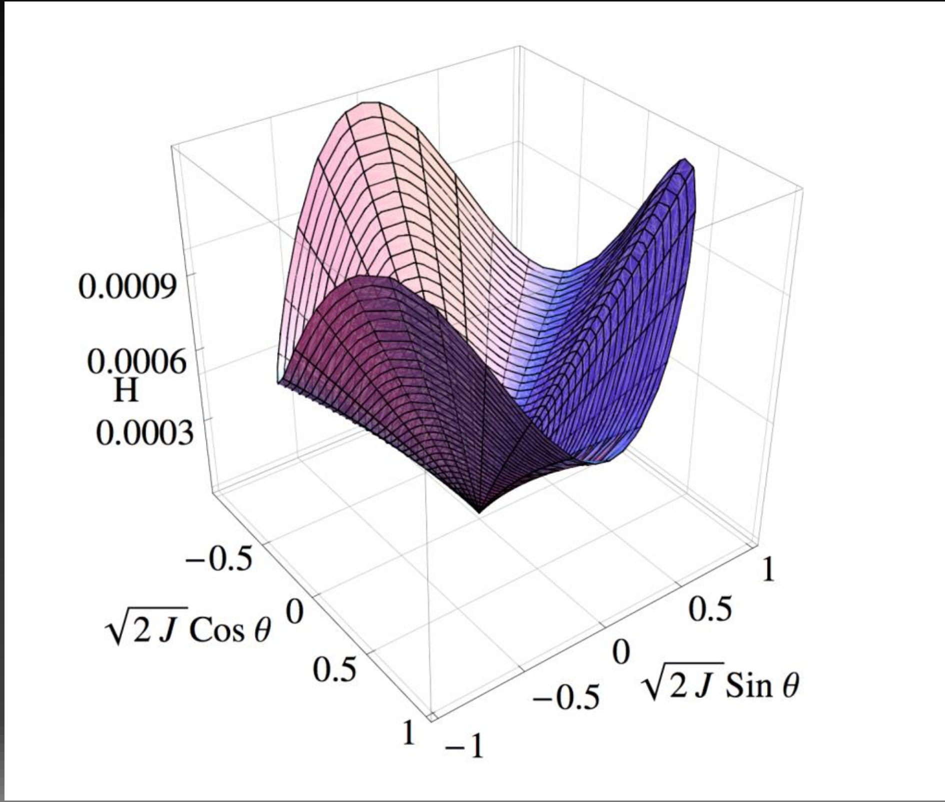
$$H = \frac{\epsilon}{2}(X^2 + X'^2) + \frac{S}{4}(3XX'^2 - X^3) + \frac{1}{64}\mathcal{D}(11X^6 + 15X^4X'^2 + 45X^2X'^4 + 9X'^6)$$

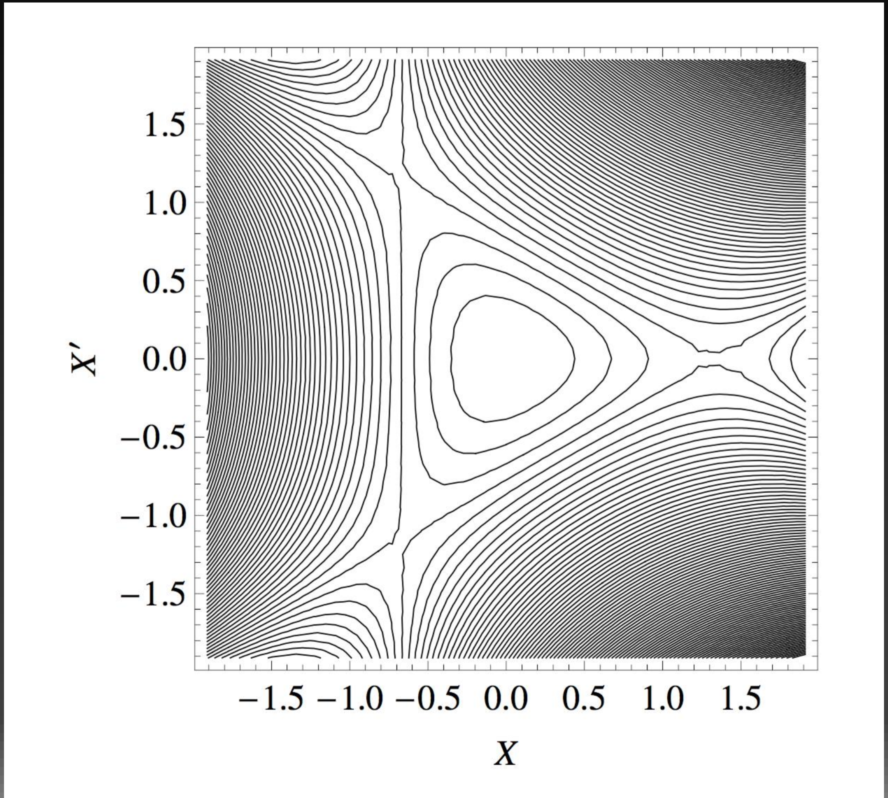
And to make life a little easier,

$$\mathcal{H} = -\left(\frac{J^3\mathcal{D}(10 + cos(6\theta))}{8}\right) - \frac{J^{3/2}Scos(3\theta)}{\sqrt{2}} + J\epsilon$$

What does it look like?

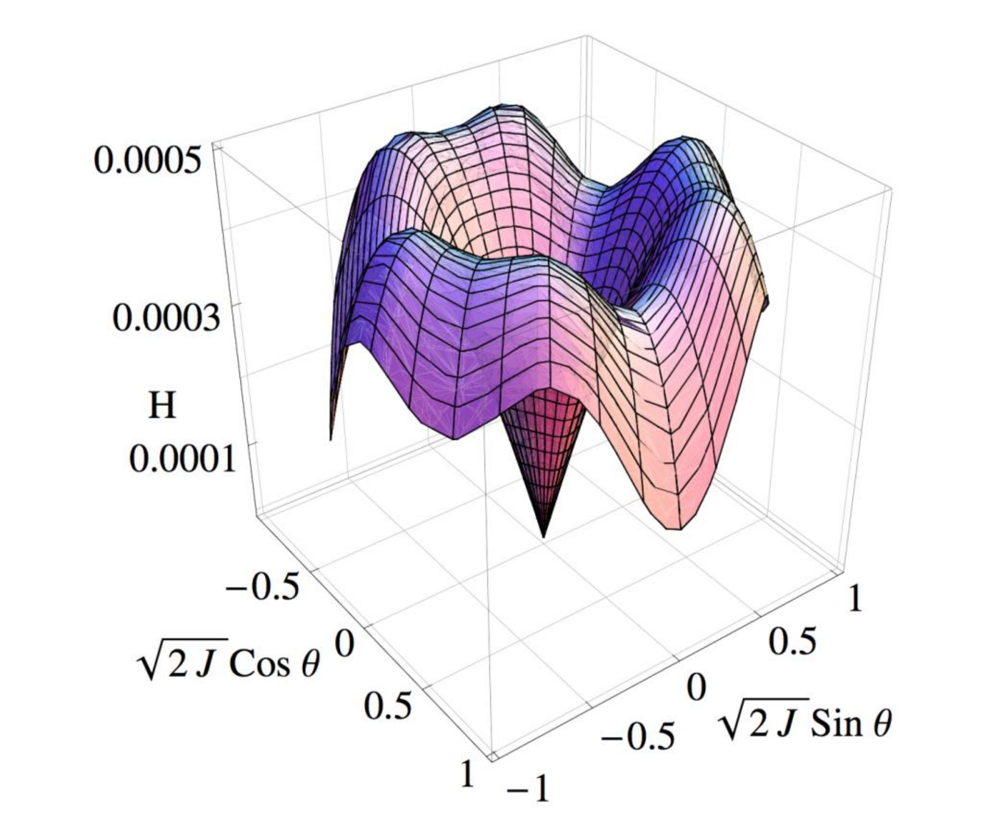
ϵ=0.001, S=0.0005, and $\mathcal{D}$=0.0



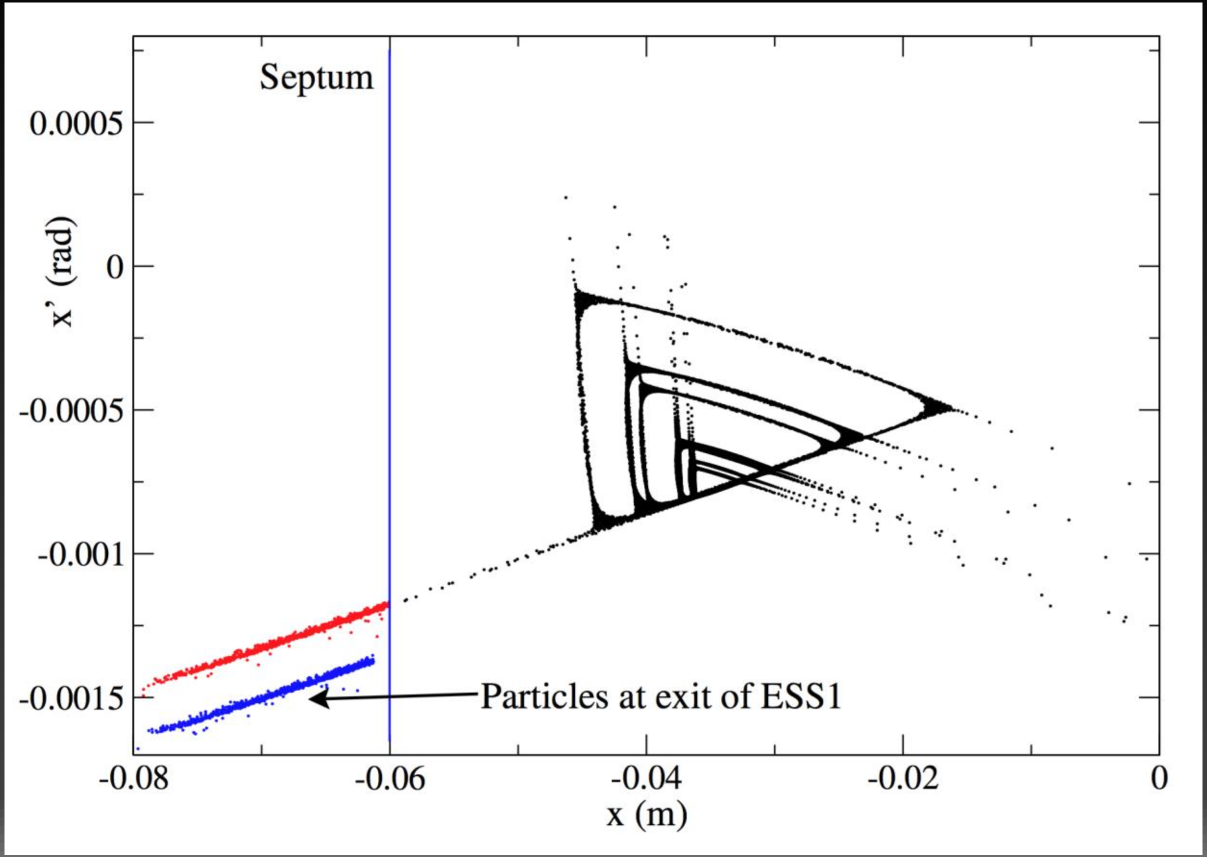


What does it look like?

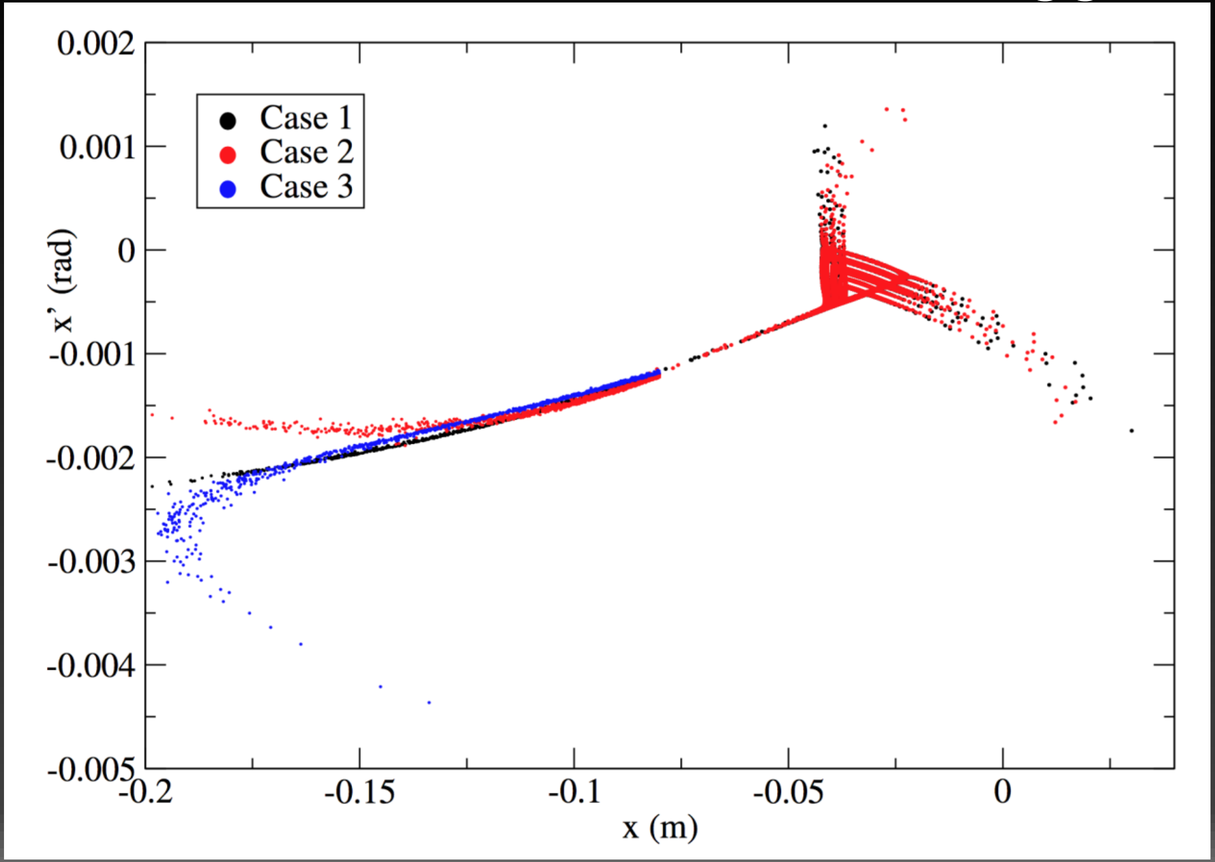
ϵ=0.001, S=0.0001, and $\mathcal{D}$=0.00055



Tracking Simulations: J-PARC SE



With a little exaggeration

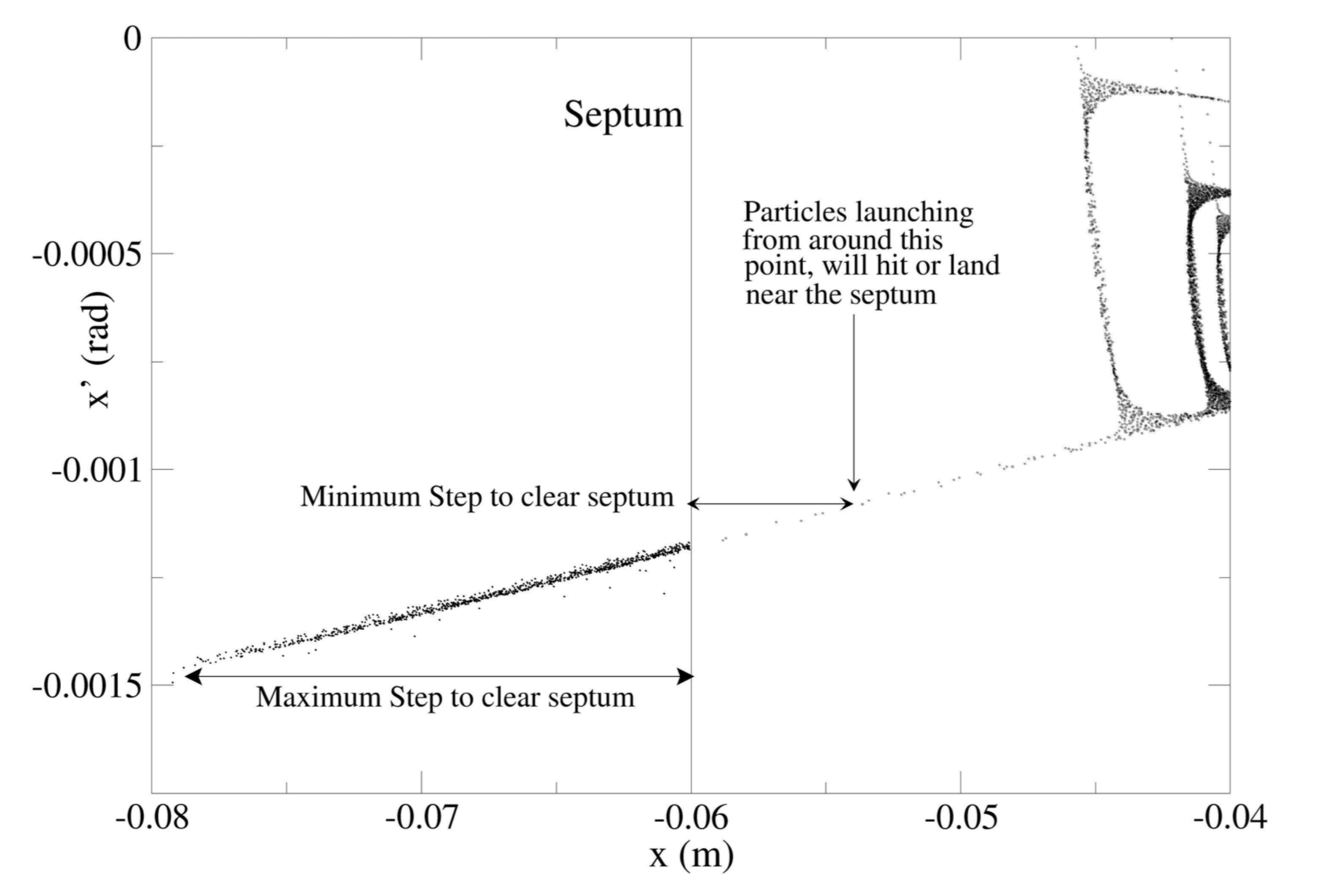


Case 1: normal, sext. Only

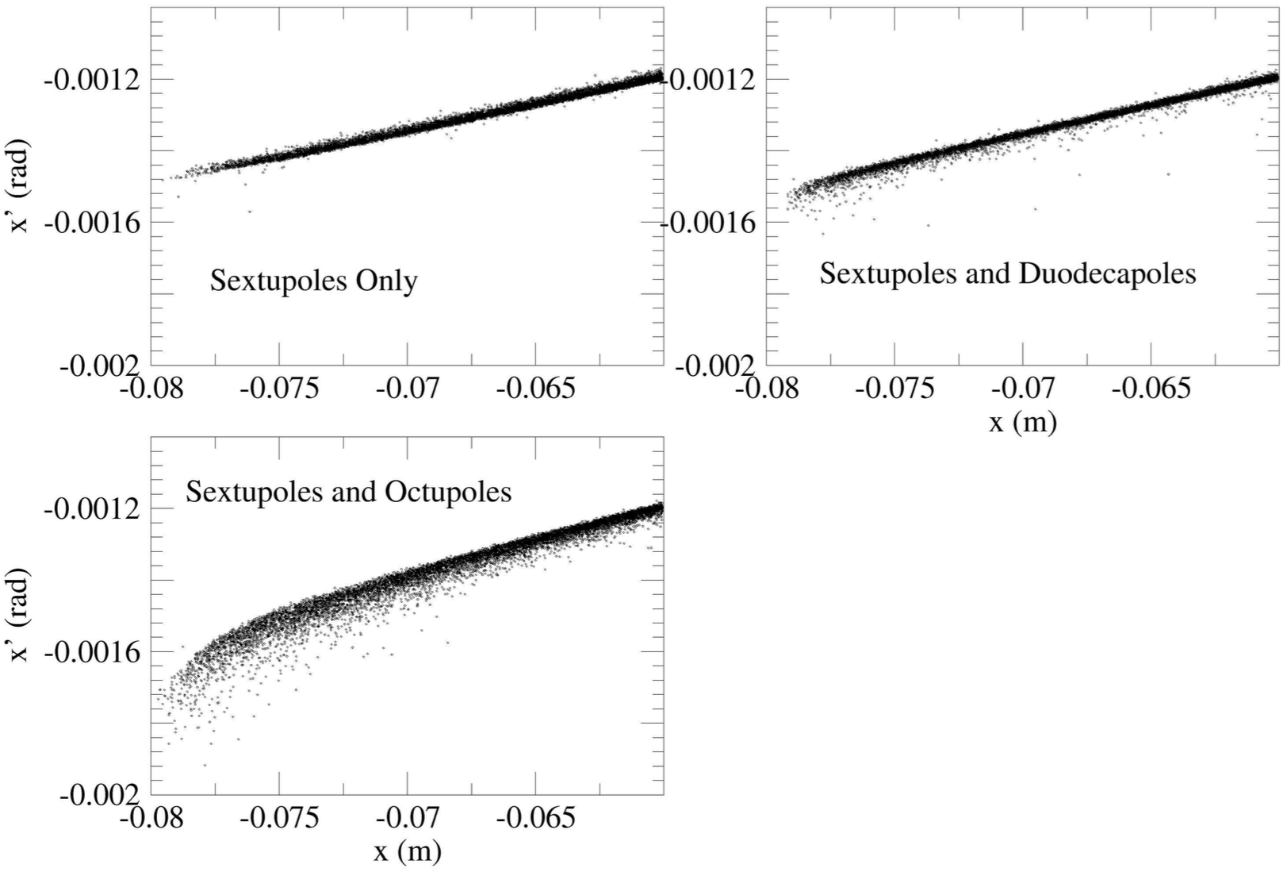
Case 2: with duodecapoles clockwise particle flow

Case 3: with duodecapoles counter-clockwise flow

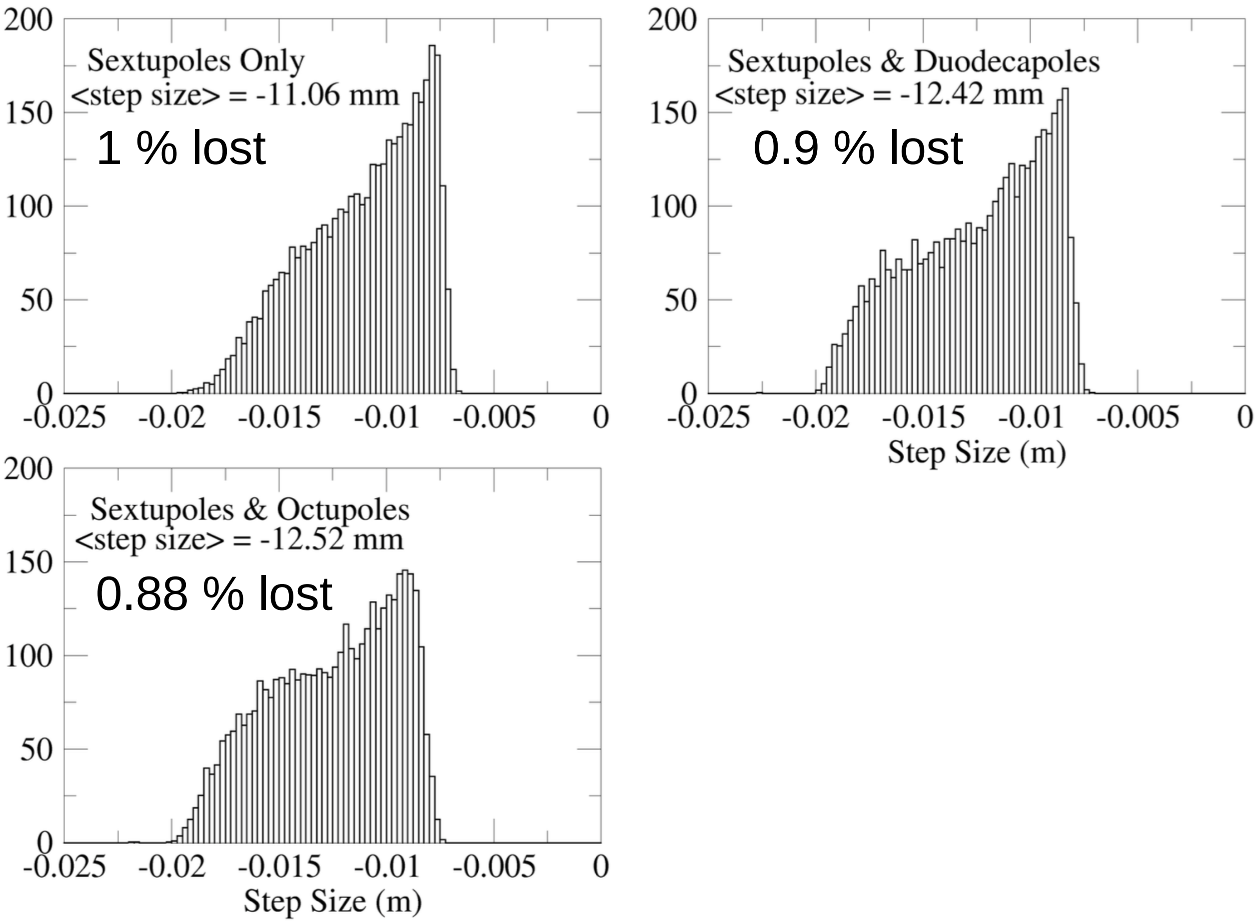
Step Size



Septum
Particles launching from around this point, will hit or land near the septum
Minimum Step to clear septum
Maximum Step to clear septum
0
-0.0005
-0.001
-0.0015
x' (rad)
-0.08
-0.07
-0.06
-0.05
-0.04
x (m)



Sextupoles Only
Sextupoles and Duodecapoles
Sextupoles and Octupoles
x' (rad)
x (m)
-0.0012
-0.0016
-0.002
-0.08
-0.075
-0.07
-0.065



The <step size> is just the simple average of the shown distribution.

However, the bigger the |<step size>| the fewer the number of particles that land near the septum.

Concluding Remarks

- The Goal is to reduce beam losses on the septa
- A consequence is possibly a ‘larger’ beam in the extraction channel
- Use of higher order multipoles can bend particles back in and allow for a larger ‘step’ size at the septa, and thus lower losses on the septa