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# Fermion mass hierarchy from nonuniversal abelian extensions of the Standard Model

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# Agenda



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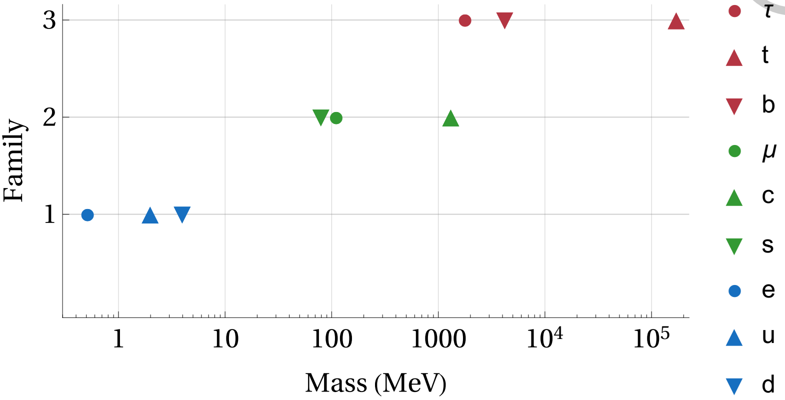
The fermion masses reported in [1] are

| Family | Particle | Mass |
|---|---|---|
| 1 | $u$ | $2.2^{+0.6}_{-0.4}$ MeV |
| | $d$ | $4.7^{+0.5}_{-0.4}$ MeV |
| | $e$ | 0.511 MeV |
| 2 | $c$ | $1.27 \pm 0.03$ GeV |
| | $s$ | $96^{+8}_{-4}$ MeV |
| | $\mu$ | 105.7 MeV |
| 3 | $t$ | $173.21 \pm 0.71$ GeV |
| | $b$ | $4.18^{+0.04}_{-0.03}$ GeV |
| | $\tau$ | 1.776 GeV |

# Fermion mass hierarchy (FMH)
## Introduction






| | | | |
|---|---|---|---|
| *u*, *d*, *e* | Units of MeV | $\mu$, *s* | Hundreds of MeV |
| *c*, *b*, $\tau$ | Units of GeV | *t* | Hundreds of GeV |

# Chiral anomaly equations
## Abelian Extensions



The FMH might be addressed by nonuniversal abelian extensions. However, the new model may not cancel chiral anomalies at each family but when the three are taken into account. It is possible new exotic quarks and leptons should be added to ensure cancelation[2, 3].

$$
\begin{aligned}
\left[\mathrm{SU}(3)_C\right]^2 \mathrm{U}(1)_X \rightarrow \quad A_C = &\quad \sum_Q X_{Q_L} - \sum_Q X_{Q_R} \\
\left[\mathrm{SU}(2)_L\right]^2 \mathrm{U}(1)_X \rightarrow \quad A_L = &\quad \sum_\ell X_{\ell_L} + 3\sum_Q X_{Q_L} \\
\left[\mathrm{U}(1)_Y\right]^2 \mathrm{U}(1)_X \rightarrow \quad A_{Y^2} = &\quad \sum_{\ell,Q}\left[Y_{\ell_L}^2 X_{\ell_L} + 3Y_{Q_L}^2 X_{Q_L}\right] - \sum_{\ell,Q}\left[Y_{\ell_R}^2 X_{L_R} + 3Y_{Q_R}^2 X_{Q_R}\right] \\
\mathrm{U}(1)_Y\left[\mathrm{U}(1)_X\right]^2 \rightarrow \quad A_Y = &\quad \sum_{\ell,Q}\left[Y_{\ell_L} X_{\ell_L}^2 + 3Y_{Q_L} X_{Q_L}^2\right] - \sum_{\ell,Q}\left[Y_{\ell_R} X_{\ell_R}^2 + 3Y_{Q_R} X_{Q_R}^2\right] \\
\left[\mathrm{U}(1)_X\right]^3 \rightarrow \quad A_X = &\quad \sum_{\ell,Q}\left[X_{\ell_L}^3 + 3X_{Q_L}^3\right] - \sum_{\ell,Q}\left[X_{\ell_R}^3 + 3X_{Q_R}^3\right] \\
[\mathrm{Grav}]^2 \mathrm{U}(1)_X \rightarrow \quad A_{\mathrm{G}} = &\quad \sum_{\ell,Q}\left[X_{\ell_L} + 3X_{Q_L}\right] - \sum_{\ell,Q}\left[X_{\ell_R} + 3X_{Q_R}\right]
\end{aligned}
$$

# Suppresion square texture (SST)



The simplest example of the SST comprises two fermions $f$ and $\mathcal{F}$ coupled by two Higgs scalars $\phi_{1,2}$ with the vacuum hierarchy (VH) $v_1 < v_2$. The Yukawa Lagrangian is

$$-\mathcal{L}_Y = Ae^{ia}\overline{f_L}\phi_1\left(s_\alpha f_R + c_\alpha \mathcal{F}_R\right) + Be^{ib}\overline{\mathcal{F}_L}\phi_2\left(s_\beta f_R + c_\beta \mathcal{F}_R\right) + \mathrm{h.c.},$$

where the Yukawa coupling constants are parametrized in polar coordinates, i.e., the coupling constant among $f_L$, $\mathcal{F}_R$ and $\phi_1$ is $Ae^{ia}c_\alpha$. The corresponding mass matrix after evaluating at the VEVs is

$$M_{\mathrm{SST}} = \begin{pmatrix} Ae^{ia}v_1 \sin\alpha & Ae^{ia}v_1 \cos\alpha \\ Be^{ib}v_2 \sin\beta & Be^{ib}v_2 \cos\beta \end{pmatrix}.$$

# Suppresion square texture (SST)



The mass eigenvalues as well as the mixing angle of the left-handed fermions are obtained by diagonalizing the matrix $MM^{\dagger}$. The eigenvalues and the mixing angle of the left-handed fermions are

$$m_f^2 \approx A^2 v_1^2 \sin^2(\alpha - \beta),$$
$$m_F^2 \approx B^2 v_2^2 + A^2 v_1^2 \cos^2(\alpha - \beta),$$

The suppression in the first eigenvalue of the matrix through the sine of the difference between $\alpha$ and $\beta$, and the enhancement of the second eigenvalue because of the addition of the complementary function of the first eigenvalue make the SST an appealing structure to address FMH.

| Doublets | $X^{\pm}$ | Singlets | $X^{\pm}$ |
|---|---|---|---|
| $\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{h_1+v_1+i\eta_1}{\sqrt{2}} \end{pmatrix}$ | $+2/3^{+}$ | $\chi = \frac{\xi_\chi + v_\chi + i\zeta_\chi}{\sqrt{2}}$ | $+1/3^{+}$ |
| $\Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{h_2+v_2+i\eta_2}{\sqrt{2}} \end{pmatrix}$ | $+2/3^{-}$ | $\psi = \frac{\xi_\psi + v_\psi}{\sqrt{2}}$ | $0^{-}$ |
| $\Phi_3 = \begin{pmatrix} \phi_3^+ \\ \frac{h_3+v_3+i\eta_3}{\sqrt{2}} \end{pmatrix}$ | $+1/3^{+}$ | $\sigma$ | $+1/3^{-}$ |

The scalar sector is composed by three Higgs doublets with

$$v_1^2 + v_2^2 + v_3^2 = (246\,\mathrm{GeV})^2\,,$$

and two Higgs singlets $\chi$ and $\psi$ which break the new abelian symmetry $\mathrm{U}(1)_X$ and give masses to non-SM fermions. Additionaly, the VH assumed to obtain the FMH is[4]

| | | | |
|---|---|---|---|
| $v_\chi$ | Units of TeV | $v_1$ | Hundreds of GeV |
| $v_\psi$ | Hundreds of GeV | $v_2$ | Units of GeV |
| $\mu_\mathcal{N}$ | Units of keV | $v_3$ | Hundreds of MeV |

| Left-handed | $X^{\pm}$ | Right-handed | $X^{\pm}$ |
|---|---|---|---|
| **SM Quarks** | | | |
| $q_L^1 = \begin{pmatrix} u^1 \\ d^1 \end{pmatrix}_L$ | $0^+$ | $u_R^1$<br>$d_R^1$ | $+2/3^+$<br>$-2/3^+$ |
| $q_L^2 = \begin{pmatrix} u^2 \\ d^2 \end{pmatrix}_L$ | $+1/3^-$ | $u_R^2$<br>$d_R^2$ | $+2/3^-$<br>$-1/3^-$ |
| $q_L^3 = \begin{pmatrix} u^3 \\ d^3 \end{pmatrix}_L$ | $+1/3^+$ | $u_R^3$<br>$d_R^3$ | $+2/3^+$<br>$-1/3^-$ |
| **Non-SM quarks** | | | |
| $\mathcal{T}_L^1$ | $+1/3^-$ | $\mathcal{T}_R^1$ | $+2/3^-$ |
| $\mathcal{T}_L^2$ | $+1^-$ | $\mathcal{T}_R^2$ | $+4/3^-$ |
| $\mathcal{J}_L^1$ | $-1/3^+$ | $\mathcal{J}_R^1$ | $-2/3^+$ |
| $\mathcal{J}_L^2$ | $0^+$ | $\mathcal{J}_R^2$ | $+1/3^+$ |

The most general Yukawa Lagrangian invariant under $\mathrm{U}(1)_X \otimes \mathbb{Z}_2$ in the up-like quark sector is

$$\begin{aligned}
-\mathcal{L}_{Y,U} = & A_U e^{ia_U} \overline{q_L^1} \widetilde{\Phi}_3 \left( s_\alpha^U u_R^1 + c_\alpha^U u_R^3 \right) + \\
& + B_U e^{ib_U} \overline{q_L^1} \widetilde{\Phi}_2 \left( s_\beta^U u_R^2 + c_\beta^U \mathcal{T}_R^1 \right) + \\
& + C_U e^{ic_U} \overline{q_L^2} \widetilde{\Phi}_1 \left( s_\gamma^U u_R^2 + c_\gamma^U \mathcal{T}_R^1 \right) + \\
& + D_U e^{id_U} \overline{q_L^3} \widetilde{\Phi}_1 \left( s_\delta^U u_R^1 + c_\delta^U u_R^3 \right) + \\
& + E_U e^{ie_U} \overline{\mathcal{T}_L^1} \chi^* \left( s_\epsilon^U u_R^2 + c_\epsilon^U \mathcal{T}_R^1 \right) + \\
& + F_U e^{if_U} s_{\zeta 2}^U \overline{\mathcal{T}_L^2} \chi \left( s_{\zeta 1}^U u_R^2 + c_{\zeta 1}^U \mathcal{T}_R^1 \right) \\
& + F_U e^{if_U} c_{\zeta 2}^U \overline{\mathcal{T}_L^2} \chi^* \mathcal{T}_R^2 + \mathrm{h.c.}
\end{aligned} \tag{1}$$

$$\mathbb{M}_U = \frac{1}{\sqrt{2}} \begin{pmatrix} A_U e^{ia_U} s^U_\alpha v_3 & B_U e^{ib_U} s^U_\beta v_2 & A_U e^{ia_U} c^U_\alpha v_3 & B_U e^{ib_U} c^U_\beta v_2 & 0 \\ 0 & C_U e^{ic_U} s^U_\gamma v_1 & 0 & C_U e^{ic_U} c^U_\gamma v_1 & 0 \\ D_U e^{id_U} s^U_\delta v_3 & 0 & D_U e^{id_U} c^U_\delta v_3 & 0 & 0 \\ 0 & E_U e^{ie_U} s^U_\epsilon v_\chi & 0 & E_U e^{ie_U} c^U_\epsilon v_\chi & 0 \\ 0 & 0 & 0 & 0 & F_U e^{if_U} v_\chi \end{pmatrix}$$

By taking advantage of the VH , the eigenvalues of $\mathbb{M}_U \mathbb{M}_U^\dagger$ are

$$m_u^2 \approx A_U^2 \sin^2\left(\alpha^U - \delta^U\right) \frac{v_3^2}{2},$$

$$m_c^2 \approx B_U^2 \sin^2\left(\beta^U - \epsilon^U\right) \frac{v_2^2}{2} + C_U^2 \sin^2\left(\gamma^U - \epsilon^U\right) \frac{v_1^2}{2},$$

$$m_t^2 \approx D_U^2 \frac{v_1^2}{2} + A_U^2 \cos^2\left(\alpha^U - \gamma^U\right) \frac{v_3^2}{2},$$

$$m_{T1}^2 \approx E_U^2 \frac{v_\chi^2}{2} + B_U^2 \cos^2\left(\beta^U - \epsilon^U\right) \frac{v_2^2}{2} + C_U^2 \cos^2\left(\gamma^U - \epsilon^U\right) \frac{v_1^2}{2},$$

$$m_{T2}^2 \approx F_U^2 \frac{v_\chi^2}{2}.$$

The most general Yukawa Lagrangian invariant under $\mathrm{U}(1)_X \otimes \mathbb{Z}_2$ in the down-like quark sector is

$$\begin{aligned}
-\mathcal{L}_{Y,D} = & A_D e^{ia_D}\overline{q_L^1}\Phi_3\left(s_\alpha^D d_R^1 + c_\alpha^D \mathcal{J}_R^1\right) + \\
& + B_D e^{ib_D}\overline{q_L^2}\Phi_3\left(s_\beta^D d_R^2 + c_\beta^D d_R^3\right) \\
& + C_D e^{ic_D}\overline{q_L^3}\Phi_2\left(s_\gamma^D d_R^2 + c_\gamma^D d_R^3\right) + \\
& + D_D e^{id_D}\overline{\mathcal{J}_L^1}\chi\left(s_\delta^D d_R^1 + c_\delta^D \mathcal{J}_R^1\right) \\
& + E_D e^{ie_D}\overline{\mathcal{J}_L^1}\psi\left(s_\epsilon^D d_R^2 + c_\epsilon^D d_R^3\right) + \\
& + F_D e^{if_D}\overline{\mathcal{J}_L^2}\chi^*\mathcal{J}_R^2 + \mathrm{h.c.}
\end{aligned} \tag{2}$$

$$\mathbb{M}_D = \frac{1}{\sqrt{2}} \begin{pmatrix} A_D e^{ia_D} s^D_\alpha v_3 & 0 & 0 & A_D e^{ia_D} c^D_\alpha v_3 & 0 \\ 0 & B_D e^{ib_D} s^D_\beta v_3 & B_D e^{ib_D} c^D_\beta v_3 & 0 & 0 \\ 0 & C_D e^{ic_D} s^D_\gamma v_2 & C_D e^{ic_D} c^D_\gamma v_2 & 0 & 0 \\ D_D e^{id_D} s^D_\delta v_\chi & E_D e^{ie_D} s^D_\epsilon v_\psi & E_D e^{ie_D} c^D_\epsilon v_\psi & D_D e^{id_D} c^D_\delta v_\chi & 0 \\ 0 & 0 & 0 & 0 & F_D e^{if_D} v_\chi \end{pmatrix}$$

By taking advantage of the VH , the eigenvalues of $\mathbb{M}_D \mathbb{M}_D^\dagger$ are

$$m_d^2 \approx A_D^2 \sin^2(\alpha^D - \delta^D) \frac{v_3^2}{2},$$

$$m_s^2 \approx B_D^2 \sin^2(\beta^D - \gamma^D) \frac{v_3^2}{2},$$

$$m_b^2 \approx C_D^2 \frac{v_2^2}{2} + B_D^2 \cos^2(\beta^D - \gamma^D) \frac{v_3^2}{2},$$

$$m_{J1}^2 \approx D_D^2 \frac{v_\chi^2}{2} + E_D^2 \frac{v_\psi^2}{2} + A_D^2 \cos^2\left(\alpha^U - \delta^U\right) \frac{v_3^2}{2},$$

$$m_{J2}^2 \approx F_D^2 \frac{v_\chi^2}{2}.$$

| Left-handed | $X^{\pm}$ | Right-handed | $X^{\pm}$ |
|---|---|---|---|
| **SM Leptons + RH neutrinos** | | | |
| $\ell_L^e = \begin{pmatrix} \nu^e \\ e^e \end{pmatrix}_L$ | $-{}^2/_3{}^+$ | $\nu_R^e$ | $+{}^1/_3{}^+$ |
| | | $e_R^e$ | $-{}^4/_3{}^+$ |
| $\ell_L^\mu = \begin{pmatrix} \nu^\mu \\ e^\mu \end{pmatrix}_L$ | $-{}^1/_3{}^-$ | $\nu_R^\mu$ | $0^-$ |
| | | $e_R^\mu$ | $-1^-$ |
| $\ell_L^\tau = \begin{pmatrix} \nu^\tau \\ e^\tau \end{pmatrix}_L$ | $-1^+$ | $\nu_R^\tau$ | $-{}^1/_3{}^-$ |
| | | $e_R^\tau$ | $-{}^4/_3{}^+$ |
| **Non-SM Leptons and Majorana** | | | |
| $\mathcal{E}_L^1$ | $+1^-$ | $\mathcal{E}_R^1$ | $+{}^4/_3{}^-$ |
| $\mathcal{E}_L^2$ | $-1^+$ | $\mathcal{E}_R^2$ | $-{}^4/_3{}^+$ |
| $\mathcal{E}_L^3$ | $+{}^5/_3{}^-$ | $\mathcal{E}_R^3$ | $+{}^4/_3{}^-$ |
| Majorana | | $\mathcal{N}_R^1$ | $0^+$ |
| | | $\mathcal{N}_R^2$ | $0^-$ |
| | | $\mathcal{N}_R^3$ | $0^+$ |

The most general Yukawa Lagrangian invariant under $\mathrm{U}(1)_X \otimes \mathbb{Z}_2$ in the neutral lepton sector is

$$
\begin{aligned}
-\mathcal{L}_{Y,N} = & B_{N1}\overline{\ell^2_L}\widetilde{\Phi}_2\nu^e_R + \boldsymbol{D}_N\overline{\nu^{e\,C}_R}\chi\boldsymbol{\mathcal{N}}_R + \\
& + A_{N1}\overline{\ell^1_L}\widetilde{\Phi}_2\nu^\mu_R + B_{N2}\overline{\ell^2_L}\widetilde{\Phi}_1\nu^\mu_R + \boldsymbol{E}'_N\overline{\nu^{\mu\,C}_R}\psi\boldsymbol{\mathcal{N}}_R + \\
& + A_{N2}\overline{\ell^1_L}\widetilde{\Phi}_1\nu^\tau_R + C_N\overline{\ell^3_L}\widetilde{\Phi}_3\nu^\tau_R + \boldsymbol{F}_N\overline{\nu^{\tau\,C}_R}\chi\boldsymbol{\mathcal{N}}_R + \\
& + \frac{\mu_{\mathcal{N}}}{2}\overline{\boldsymbol{\mathcal{N}}^C_R}\mathbb{G}_N\boldsymbol{\mathcal{N}}_R + \mathrm{h.c.}
\end{aligned} \tag{4}
$$

$$\mathbb{M}_N = \begin{pmatrix} 0 & \mathcal{M}_\nu^{\mathrm{T}} & 0 \\ \mathcal{M}_\nu & 0 & \mathcal{M}_\mathcal{N}^{\mathrm{T}} \\ 0 & \mathcal{M}_\mathcal{N} & M_\mathcal{N} \end{pmatrix},$$

with $\mathcal{M}_\nu$ as the Dirac mass matrix between $\nu_L$ and $\nu_R$

$$\mathcal{M}_\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & A_{N1} v_2 & A_{N2} v_1 \\ B_{N1} v_2 & B_{N2} v_1 & 0 \\ 0 & 0 & C_N v_3 \end{pmatrix},$$

$\mathcal{M}_N$ the Dirac mass matrix between $\nu_R^C$ and $\mathcal{N}_R$

$$\mathcal{M}_N = \frac{v_\chi}{\sqrt{2}} \begin{pmatrix} D_{N1} & D_{N2} & D_{N3} \\ E_{N1} & E_{N2} & E_{N3} \\ F_{N1} & F_{N2} & F_{N3} \end{pmatrix},$$

where $E_{N1} = \rho_\psi E'_{N1}$ with $\rho_\psi = v_\psi / v_\chi$, and $M_\mathcal{N} = \mathbb{G}_N \mu_\mathcal{N}$ is the Majorana mass of $\mathcal{N}_R$ with $\mu_\mathcal{N}$ the scale of the Majorana mass.

The inverse seesaw, together with the VH yield a massless neutrino, $\nu_L^1$, and two light neutrinos, $\nu_L^2$ and $\nu_L^3$,

$$m_{\nu 2}^2 \approx \frac{B_{N2}^2 G_{N2}}{E_{N2}^2}\frac{\mu_N v_1^2}{v_\chi^2} - \frac{2A_{N1}B_{N2}^3 E_{N1} G_{N2} G_{N4}}{G_{N2}(A_{N2}^2 E_{N2}^2 G_{N1} - B_{N2}^2 D_{N1}^2 G_{N2})}\frac{\mu_N v_1 v_2}{v_\chi^2},$$

$$m_{\nu 3}^2 \approx \frac{A_{N2}^2 G_{N1}}{D_{N1}^2}\frac{\mu_N v_1^2}{v_\chi^2} + \frac{2A_{N1}A_{N2}^2 B_{N2} G_{N2} G_{N1} G_{N4}}{E_{N1}(A_{N2}^2 E_{N2}^2 G_{N1} - B_{N2}^2 D_{N1}^2 G_{N2})}\frac{\mu_N v_1 v_2}{v_\chi^2},$$

and the masses of the exotic species

$$m_{N_R^1} = D_{1N}\frac{v_\chi}{\sqrt{2}} - \frac{G_{N1}\mu_N}{2}, \qquad m_{\tilde{N}_R^1} = D_{1N}\frac{v_\chi}{\sqrt{2}} + \frac{G_{N1}\mu_N}{2},$$

$$m_{N_R^2} = E_{2N}\frac{v_\chi}{\sqrt{2}} - \frac{G_{N2}\mu_N}{2}, \qquad m_{\tilde{N}_R^2} = E_{2N}\frac{v_\chi}{\sqrt{2}} + \frac{G_{N2}\mu_N}{2},$$

$$m_{N_R^3} = F_{3N}\frac{v_\chi}{\sqrt{2}} - \frac{G_{N3}\mu_N}{2}, \qquad m_{\tilde{N}_R^3} = F_{3N}\frac{v_\chi}{\sqrt{2}} + \frac{G_{N3}\mu_N}{2}.$$

The most general Yukawa Lagrangian invariant under $\mathrm{U}(1)_X \otimes \mathbb{Z}_2$ in the down-like quark sector is

$$\begin{aligned}
-\mathcal{L}_{Y,E} = & A_E e^{ia_E} \overline{\ell_L^1} \Phi_3 \left( s^E_{\alpha 1} s^E_{\alpha 2} e^e_R + c^E_{\alpha 1} s^E_{\alpha 2} e^\tau_R + c^E_{\alpha 2} \mathcal{E}^2_R \right) + \\
& + C_E e^{ic_E} \overline{\ell_L^3} \Phi_1 \left( s^E_{\gamma 1} s^E_{\gamma 2} e^e_R + c^E_{\gamma 1} s^E_{\gamma 2} e^\tau_R + c^E_{\gamma 2} \mathcal{E}^2_R \right) + \\
& + B_E e^{ib_E} \overline{\ell_L^2} \Phi_3 e^\mu_R + D_E e^{id_E} \overline{\mathcal{E}_L^1} \chi \left( s^E_\delta \mathcal{E}^1_R + c^E_\delta \mathcal{E}^3_R \right) + \\
& + E_E e^{ie_E} \overline{\mathcal{E}_L^2} \chi \left( s^E_{\epsilon 1} s^E_{\epsilon 2} e^e_R + c^E_{\epsilon 1} s^E_{\epsilon 2} e^\tau_R + c^E_{\epsilon 2} \mathcal{E}^2_R \right) + \\
& + F_E e^{if_E} \overline{\mathcal{E}_L^2} \psi e^\mu_R + G_E e^{ig_E} \overline{\mathcal{E}_L^3} \chi \left( s^E_\zeta \mathcal{E}^1_R + c^E_\zeta \mathcal{E}^3_R \right) + \mathrm{h.c.}
\end{aligned} \tag{5}$$

$$\mathbb{M}_E = \frac{1}{\sqrt{2}} \begin{pmatrix} A_E s^E_{\alpha 1} s^E_{\alpha 2} v_3 & 0 & A_E c^E_{\alpha 1} s^E_{\alpha 2} v_3 & 0 & A_E c^E_{\alpha 2} v_3 & 0 \\ 0 & B_E v_3 & 0 & 0 & 0 & 0 \\ C_E s^E_{\gamma 1} s^E_{\gamma 2} v_1 & 0 & C_E c^E_{\gamma 1} s^E_{\gamma 2} v_1 & 0 & C_E c^E_{\gamma 2} v_1 & 0 \\ 0 & 0 & 0 & D_E s^E_{\delta} v_\chi & 0 & D_E c^E_{\delta} v_\chi \\ E_E s^E_{\epsilon 1} s^E_{\epsilon 2} v_\chi & F_E e^{i f_E} v_\psi & E_E c^E_{\epsilon 1} s^E_{\epsilon 2} v_\chi & 0 & E_E e^{i e_E} c^E_{\epsilon 2} v_\chi & 0 \\ 0 & 0 & 0 & G_E s^E_{\zeta} v_\chi & 0 & G_E c^E_{\zeta} v_\chi \end{pmatrix}$$

By taking advantage of the VH , the eigenvalues of $\mathbb{M}_E \mathbb{M}_E^\dagger$ are

$$m_e^2 \approx A_E^2 \sin^2\left(\alpha_1^E\right) \sin^2\left(\alpha_2^E - \gamma_2^E\right) \frac{v_3^2}{2}, \quad m_\mu^2 \approx B_U^2 \frac{v_3^2}{2},$$

$$m_\tau^2 \approx C_E^2 \sin^2\left(\gamma_2^E - \epsilon_2^E\right) \frac{v_1^2}{2}, \qquad m_{E1}^2 \approx D_E^2 \frac{v_\chi^2}{2},$$

$$m_{E2}^2 \approx E_E^2 \frac{v_\chi^2}{2} + C_E^2 \cos^2\left(\gamma_2^E - \epsilon_2^E\right) \frac{v_1^2}{2}, \quad m_{E3}^2 \approx G_E^2 \frac{v_\chi^2}{2}.$$

# Summary of fermion masses

## Conclusions



| Family | | Mass | | | Mass | |
|---|---|---|---|---|---|---|
| | | Up-like Quarks | | | Down-like Quarks | |
| 1 | $u$ | $A_U s_{\alpha U-\delta U}\frac{v_3}{\sqrt{2}}$ | 1 MeV | $d$ | $A_D s_{\alpha D-\delta D}\frac{v_3}{\sqrt{2}}$ | 1 MeV |
| 2 | $c$ | $C_U s_{\gamma U-\epsilon U}\frac{v_1}{\sqrt{2}}$ | 1 GeV | $s$ | $B_D s_{\beta D-\gamma D}\frac{v_3}{\sqrt{2}}$ | $10^2$ MeV |
| 3 | $t$ | $\frac{D_U v_1}{\sqrt{2}}$ | $10^2$ GeV | $b$ | $\frac{C_D v_2}{\sqrt{2}}$ | 1 GeV |
| | | Neutral Leptons | | | Charged Leptons | |
| 1 | $\nu_L^1$ | 0 | —— | $e$ | $A_E s_{\alpha E-\delta E}\frac{v_3}{\sqrt{2}}$ | 1 MeV |
| 2 | $\nu_L^2$ | $\frac{B_{N2}^2 G_{N2}}{E_{N2}^2}\frac{\mu_N v_1^2}{v_\chi^2}$ | 1 meV | $\mu$ | $\frac{B_E v_3}{\sqrt{2}}$ | $10^2$ MeV |
| 3 | $\nu_L^3$ | $\frac{A_{N2}^2 G_{N1}}{D_{N1}^2}\frac{\mu_N v_1^2}{v_\chi^2}$ | 1 meV | $\tau$ | $C_E s_{\gamma E-\epsilon E}\frac{v_1}{\sqrt{2}}$ | 1 GeV |

Table: Summary of fermion masses.

# Referencias



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Thank you for your attention and kindness!