The Weak Gravity Conjecture and Scalar Fields

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Are there constraints on quantum field theories which arise from requiring that they have an ultraviolet completion that includes quantum gravity?

No Global Symmetries

Consider a theory with a global symmetry and couple it to gravity

Black hole masses are not bound by their global symmetry charge

After Hawking radiation find an infinite number of states with sub-Planckian mass: $m_{q=1}$, $m_{q=2}$, $m_{q=3}$, … $< M_p$

Remnants dominate any amplitude, and renormalize the gravitational strength

The Weak Gravity Conjecture

[Arkani-Hamed, Motl, Nicolis, Vafa '06]

For a U(1) gauge symmetry, with gauge coupling g, charged black holes with charge Q and mass M satisfy an extremality bound

$$M \geq g\, Q\, M_p$$

Still obtain $\frac{1}{g}$ remnant states below M_p

If there exists a particle with charge q and mass m such that

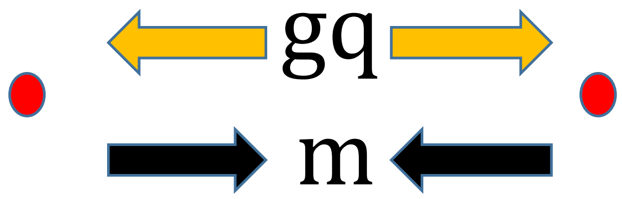
$$\mathrm{m} \leq g\, q\, M_p$$

Then none of the remnants are stable and will decay

Stable gravitationally bound states

Can also consider building up 'remnants' through bound states

Consider the particle with the largest charge-to-mass ratio



If $\mathrm{m} > g\ q\ M_p$ then can form a tower of stable gravitationally bound states which act as remnants

Introducing Scalar Fields

No remnants, or no bound states, are general principles but the WGC is their application to Einstein-Maxwell theory

Would like to consider the most general bosonic theory

$$\frac{R}{2} - g_{ij}(t)\,\partial_\mu t^i \partial^\mu t^j + \mathcal{I}_{IJ}(t)\,\mathcal{F}^I_{\mu\nu}\mathcal{F}^{J,\mu\nu} + \mathcal{R}_{IJ}(t)\,\mathcal{F}^I_{\mu\nu}\left(\star\mathcal{F}\right)^{J,\mu\nu}$$

N=2 Extremal Black Holes

Look at extremal black hole solutions to N=2 supergravity

$$\frac{R}{2} - g_{ij}\partial_\mu z^i \partial^\mu \overline{z}^j + \mathcal{I}_{IJ}\mathcal{F}^I_{\mu\nu}\mathcal{F}^{J,\mu\nu} + \mathcal{R}_{IJ}\mathcal{F}^I_{\mu\nu}\left(\star\mathcal{F}\right)^{J,\mu\nu}$$

$$z^i = b^i + it^i \qquad i = 1, \ldots, n_V \qquad I = 0, \ldots, n_V$$

The black hole ADM mass is given in terms of its charges

$$M_{\text{ADM}} = |Z|_\infty \qquad Z = e^{\frac{K}{2}}\left(q_I X^I - p^I F_I\right)$$

The WGC is based on the property of Reissner-Nordstrom black holes

$$g^2 q^2 = M_{\rm ADM}^2$$

There is a similar property of N=2 extremal black holes

$$\mathcal{Q}^2 = M_{\rm ADM}^2 + 4 g^{ij} \partial_i M_{\rm ADM} \overline{\partial}_j M_{\rm ADM}$$

$$\mathcal{Q}^2 \equiv -\frac{1}{2} \mathcal{Q}^T \mathcal{M} \mathcal{Q} \qquad \mathcal{Q} \equiv \begin{pmatrix} p^I \\ q_I \end{pmatrix} \qquad \mathcal{M} \equiv \begin{pmatrix} \mathcal{I} + \mathcal{R}\mathcal{I}^{-1}\mathcal{R} & -\mathcal{R}\mathcal{I}^{-1} \\ -\mathcal{I}^{-1}\mathcal{R} & \mathcal{I}^{-1} \end{pmatrix}$$

The General Weak Gravity Conjecture

Inspired by this we conjecture that, for the most general bosonic theory, there must exist a particle with

$$\mathcal{Q}^2 \geq m^2 + g^{ij}\mu_i\mu_j \qquad \mu_i = \partial_{t^i} m$$

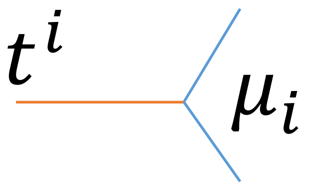
Black Hole Motivation: Evidence that $\mathcal{Q}^2 = M_{\mathrm{ADM}}^2 + 4g^{ij}\partial_i M_{\mathrm{ADM}}\bar{\partial}_j M_{\mathrm{ADM}}$ is tied to extremality rather than supersymmetry

There are known extremal black hole solutions, not N=2, which can be formulated in terms of a 'fake superpotential'

$$V_{\mathrm{BH}} = \mathcal{Q}^2 = \mathcal{W}^2 + 4g^{ij}\partial_i\mathcal{W}\bar{\partial}_j\mathcal{W}$$

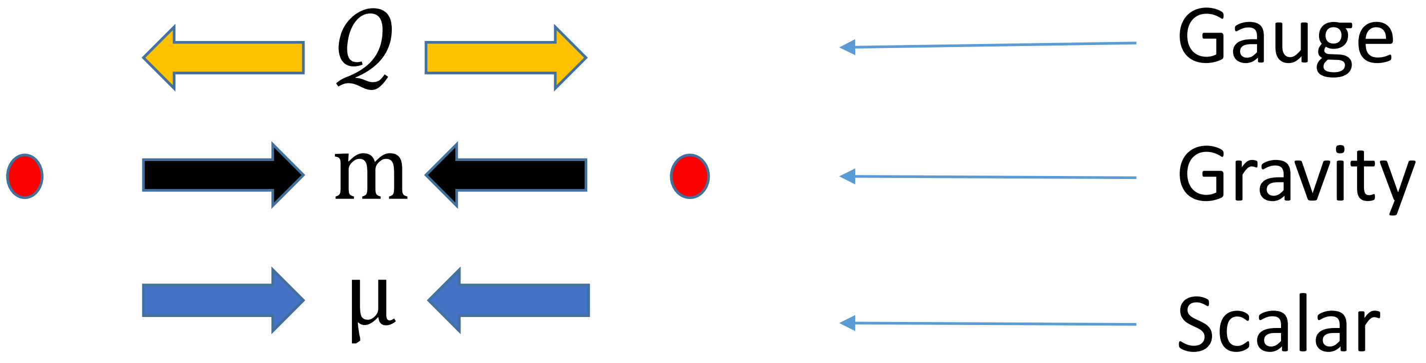
No Stable Gravitationally Bound States

The μ_i are the scalar couplings to the WGC states



Can interpret the bound as a statement about forces

$$\mathcal{Q}^2 \geq m^2 + g^{ij}\mu_i\mu_j$$



This is general, all the motivations for the WGC in terms of the absence of remnants, or stable gravitationally bound states, follow

Distances in Field Space

Consider the proper distance in field space along a linear combination of the scalar fields

$$\rho = \sum_i h_i t^i \qquad\qquad \Delta\phi \equiv \int_\gamma \sqrt{g_{ij}\frac{\partial t^i}{\partial\rho}\frac{\partial t^j}{\partial\rho}}d\rho = \int_{\rho_i}^{\rho_f} \left(h_i g^{ij} h_j\right)^{-\frac{1}{2}} d\rho$$

For moduli fields of a Calabi-Yau, in the large volume regime we have that

$$\mathcal{I}^{IJ} = -\frac{6}{\kappa}\begin{pmatrix} 1 & b^i \\ b^i & \frac{1}{4}g^{ij} + b^i b^j \end{pmatrix}$$

The N=2 identity, which leads to the formulation of the WGC

$$\mathcal{Q}^2 = |Z|^2 + g^{ij} D_i Z \overline{D}_j \overline{Z}$$

Gives for flux choices $q_I = (0, h_i) \quad p^I = 0$

$$h_i g^{ij} h_j = \frac{4\kappa}{3}\left(|Z|^2 + g^{ij} D_i Z \overline{D}_j \overline{Z}\right) = |\rho|^2 + \frac{4\kappa}{3} g^{ij} D_i Z \overline{D}_j \overline{Z}$$

Which means we can write

$$\Delta\phi = \int_{\rho_i}^{\rho_f} \frac{1}{\left(|\rho|^2 + F(\rho)^2\right)^{\frac{1}{2}}} d\rho \leq 1 - \frac{\rho_i}{\rho_c} + \ln\left(\frac{\rho_f}{\rho_c}\right)$$

$$|\rho_c| = F(\rho_c)$$

A similar calculation gives the same result for any N=1 Calabi-Yau orientifold compactifications:

The field distance grows at best logarithmically with moduli at super-Planckian distances

This implies that, since the moduli control infinite towers of states, we have infinite tower of states with mass behaving as

$$m\left(\phi+\Delta\phi\right)\leq m\left(\phi\right)e^{-\alpha\frac{\Delta\phi}{M_p}} \qquad \Delta\phi\geq M_p$$

The Refined Swampland Conjecture

For any scalar field there is an infinite tower of states with mass

$$m\left(\phi+\Delta\phi\right) \leq m\left(\phi\right)e^{-\alpha\frac{\Delta\phi}{M_p}} \qquad \Delta\phi \geq M_p$$

Conjecture applies to all fields, not just strict moduli

Strong version of an earlier Swampland Conjecture for infinite distances in moduli space [Ooguri, Vafa '06]

Monodromy Axions

Monodromy axions have their periodic symmetry spontaneously broken

$$L = f^2(\partial a)^2 - m^2 a^2$$

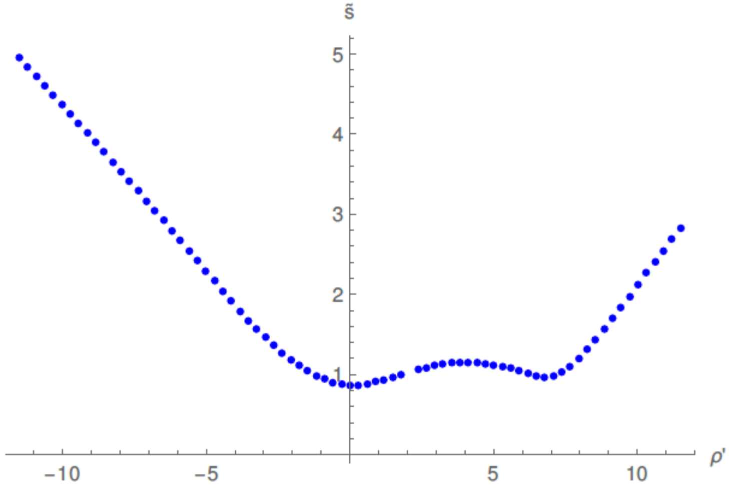
De-compactify the axion field space allowing $\Delta a \to \infty$

The axion decay constant f is independent of the axion a

Can test in string theory in compactifications of type IIA string theory on a Calabi-Yau in presence of fluxes

As the axion develops a large vev, the gravitational backreaction of its potential m^2a^2 causes moduli fields to track the axion $t = a$

This modifies its own field space metric $f(t) \rightarrow f(a)$, leading to logarithmic normalisation L = $\left(\frac{\partial a}{a}\right)^2$



Induces a power-law dependence of the mass of a tower of states on a

Find that the SC behavior emerges at $\Delta\phi > M_p$, for all fluxes

Supported by further studies so far [Valenzuela '16; Blumenhagen et al. '17]

Black Holes Scalar Hair

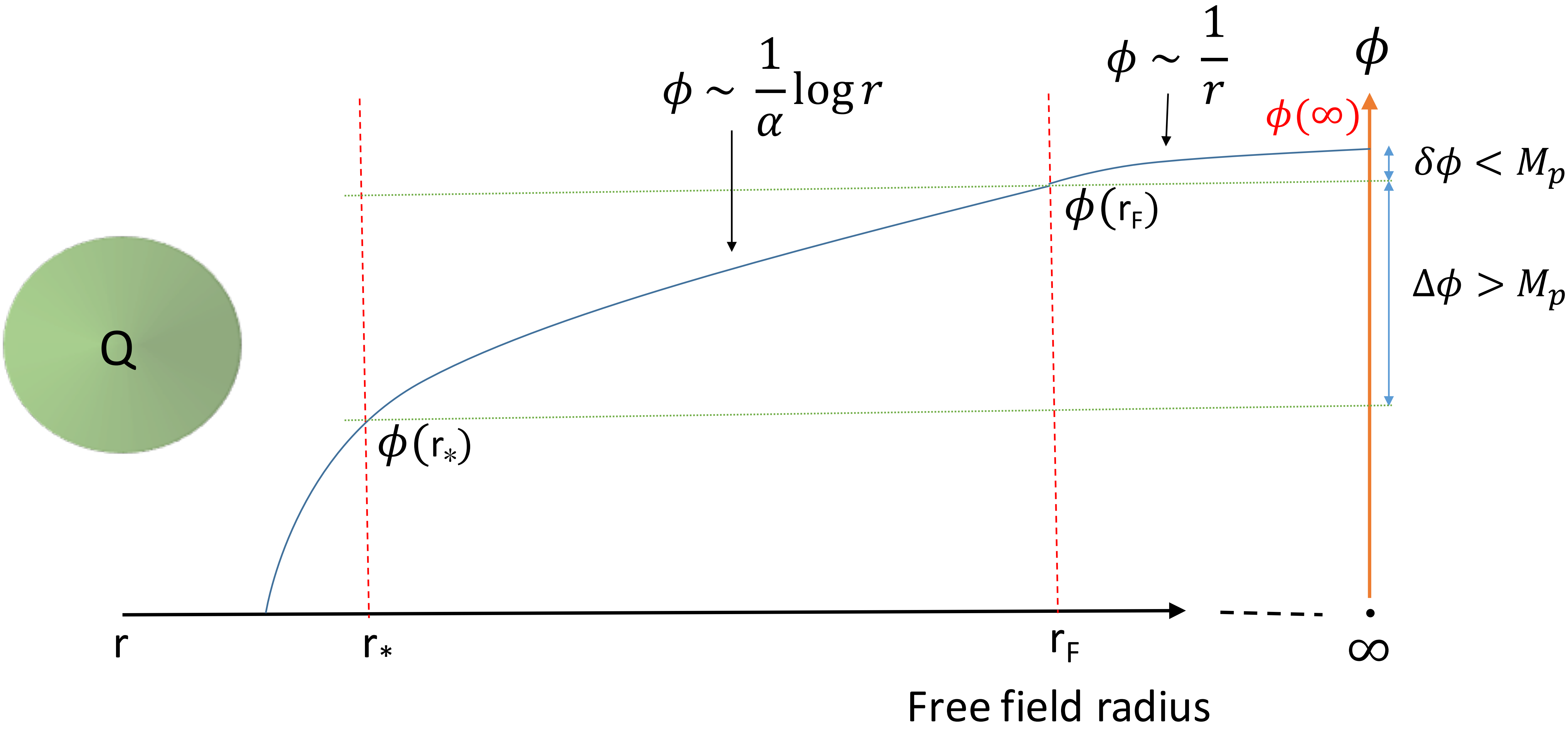
Consider a theory with gravity, gauge field, and scalar field

$$S = \frac{1}{2}\int \sqrt{g} d^4x \left[R - 2\left(\partial\phi\right)^2 - \frac{1}{2g\left(\phi\right)^2} F^2 \right]$$

A charged black hole will induce a spatial gradient $\phi(r)$

There is a ‘speed limit’ on how fast a scalar field can spatially vary due to the gravitational backreaction of its kinetic term

Keeping gravity in the Newtonian regime:

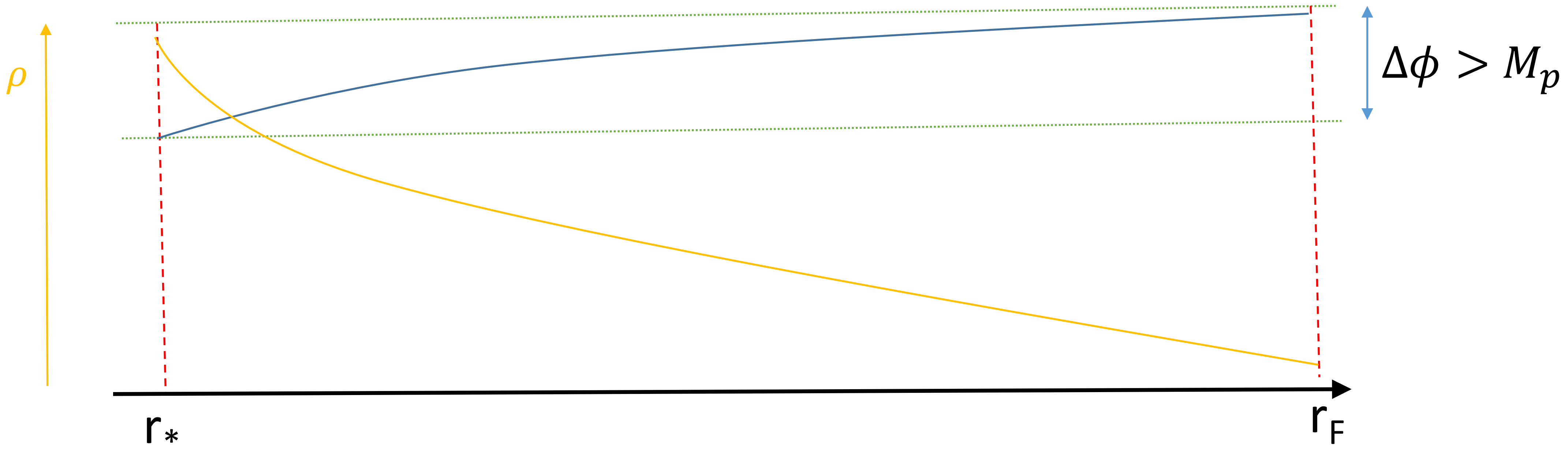


For logarithmic spatial running of the scalar field we have

$$\rho(r)^{\frac{1}{2}} > \partial\phi = \frac{1}{\alpha r}$$

Therefore the energy density is exponentially decreasing

$$\frac{\rho(r_F)^{\frac{1}{2}}}{\rho(r_*)^{\frac{1}{2}}} = \frac{r_*}{r_F} \leq e^{-\alpha\Delta\phi}$$



Can show that at r_F the gauge coupling can not be much higher than the energy density

The (Lattice) Weak Gravity Conjecture $m \leq g$ therefore implies that a tower of WGC states is not much higher than the energy density

For the black hole solution to exist within the effective theory the mass of the states must remain above the energy density in the solution

It must therefore depend exponentially on $\Delta\phi$

Gravity as the Weakest Force

In N=2, the scalar force between charged states which have vanishing gauge interaction is repulsive and cancels gravity

$$|Z|\left|Z'\right| + \mathrm{Re}\left(4g^{ij}\partial_i\left|Z'\right|\overline{\partial}_j\left|Z\right|\right) = \mathcal{Q}\mathcal{Q}'\mathrm{Re}\left(\frac{Z\overline{Z}'}{\left|Z\overline{Z}'\right|}\right) - \frac{1}{2}\left(q_I p'^I - q'_I p^I\right)\mathrm{Im}\left(\frac{Z\overline{Z}'}{\left|Z\overline{Z}'\right|}\right)$$

Avoiding gravitationally bound states between WGC particles which have no gauge interaction (eg. Electron and Monopole), requires a scalar field acting stronger than gravity

$$-\,g^{ij}\left(\partial_{t^i} m\right)\left(\partial_{t^j} m'\right) \geq mm'$$

Also in N=2 the scalar force acts stronger than gravity between the same states

Generally this is $$g^{ij}\left(\partial_{t^i} m\right)\left(\partial_{t^j} m\right) > m^2$$

This is the statement that Gravity is the Weakest Force

However, the existence and meaning of a tower of states is much more subtle and complicated than before, so the evidence is weaker

Consider imposing that gravity is the weakest force for a single scalar field

$$|\partial_t m| > m$$

For large $t \gg 1$, This implies* $m = e^{-\alpha t}$ $|\alpha| > 1$

*Only generically, there are interesting ways around this, which are utilised by axions for example

So the Refined Swampland Conjecture is nothing but the statement that gravity is the weakest force, applied to the scalar field!

Summary

Introduced conjectures regarding scalar fields

WGC: $\mathcal{Q}^2 \geq m^2 + g^{ij}\mu_i\mu_j$ $\mu_i = \partial_{t^i} m$

RSC: $m\left(\phi + \Delta\phi\right) \leq m\left(\phi\right) e^{-\alpha\frac{\Delta\phi}{M_p}}$ $\Delta\phi \geq M_p$

Evidence from black hole physics, quantum gravity expectations, and string theory

The general WGC has a non-trivial infrared limit $m \longrightarrow 0$

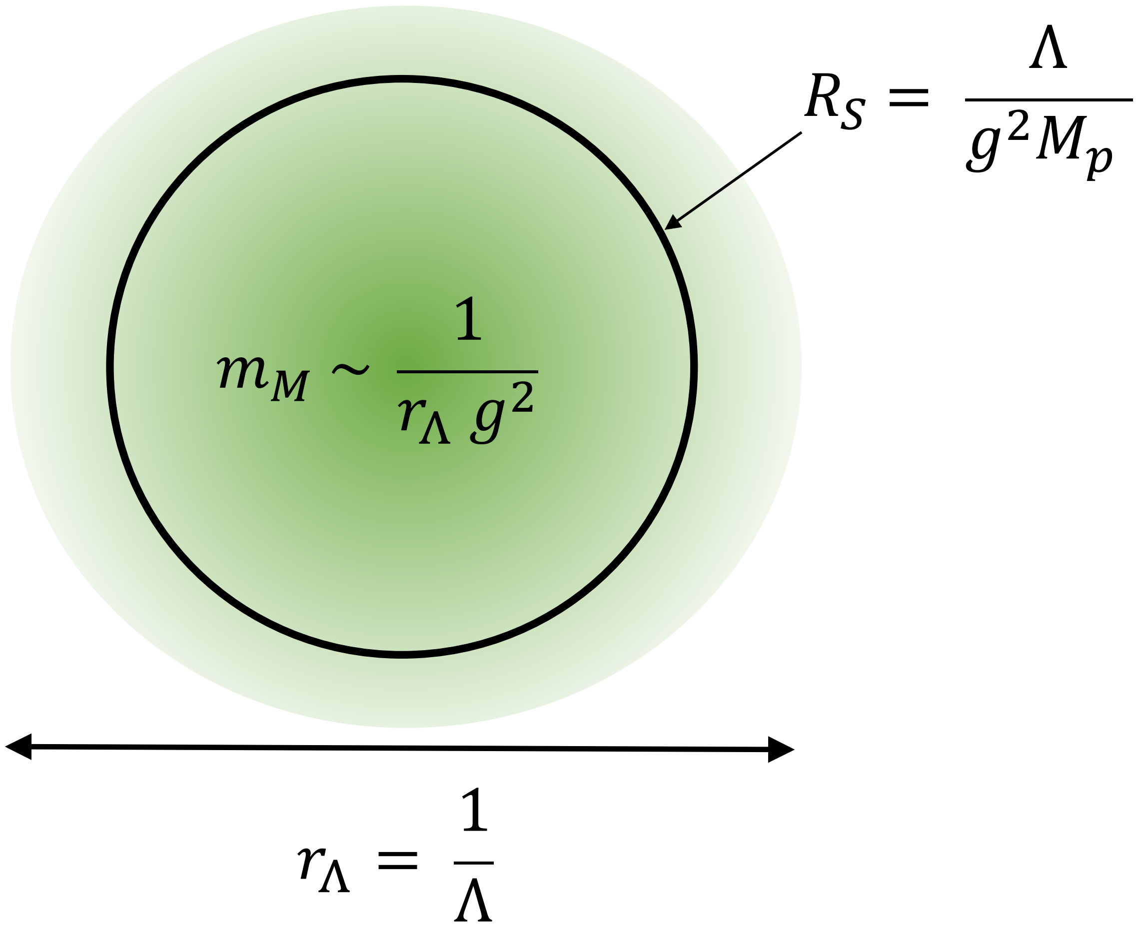
The RSC is a universal bound on primordial tensor modes

Thank You

Evidence for gM_p as a QG cut-off

[Arkani-Hamed, Motl, Nicolis, Vafa ’06]

Consider the magnetic dual of the WGC $\frac{q_M}{g} M_p \geq m_M$, apply to monopole:



- Apply magnetic WGC
- Require unit-charged monopole to not be a classical Black Hole

$$\Lambda < gM_p$$

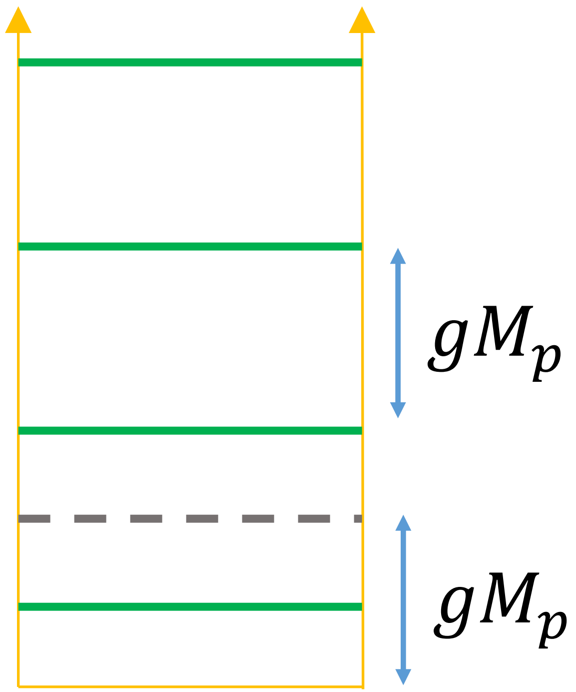
$$\Rightarrow gM_p^2 > \rho(r_\Lambda)^{\frac{1}{2}}$$

Lattice WGC: The state satisfying the WGC is the first in an infinite tower of states, of increasing mass and charge, all satisfying the WGC

[Heidenreich, Reece, Rudelius '15]

Evidence

- Appears to be the case in String Theory
- Black Holes charged under both KK U(1) and gauge U(1) violate the WGC unless there is such a tower
- Sharpening of Completeness Conjecture [Polchinski '03]
- Matches cut-off constraint for monopole to not be a Black Hole $\Lambda < gM_p$, $gM_p^2 > \rho(r_\Lambda)^{\frac{1}{2}}$



Gravitational effect of kinetic term

The Newtonian potential Φ sets the scale of strong gravity physics

$$ds^2 = -\left[1 + 2\Phi(r)\right] dt^2 + \left[1 - 2\Phi(r)\right] \left(dr^2 + r^2 d\Omega\right)$$

Consider an arbitrary power-law profile for a scalar field

$$\phi\left(r\right) = \frac{\beta}{\alpha}\left(\frac{r}{r_F}\right)^{\frac{1}{\beta}}$$

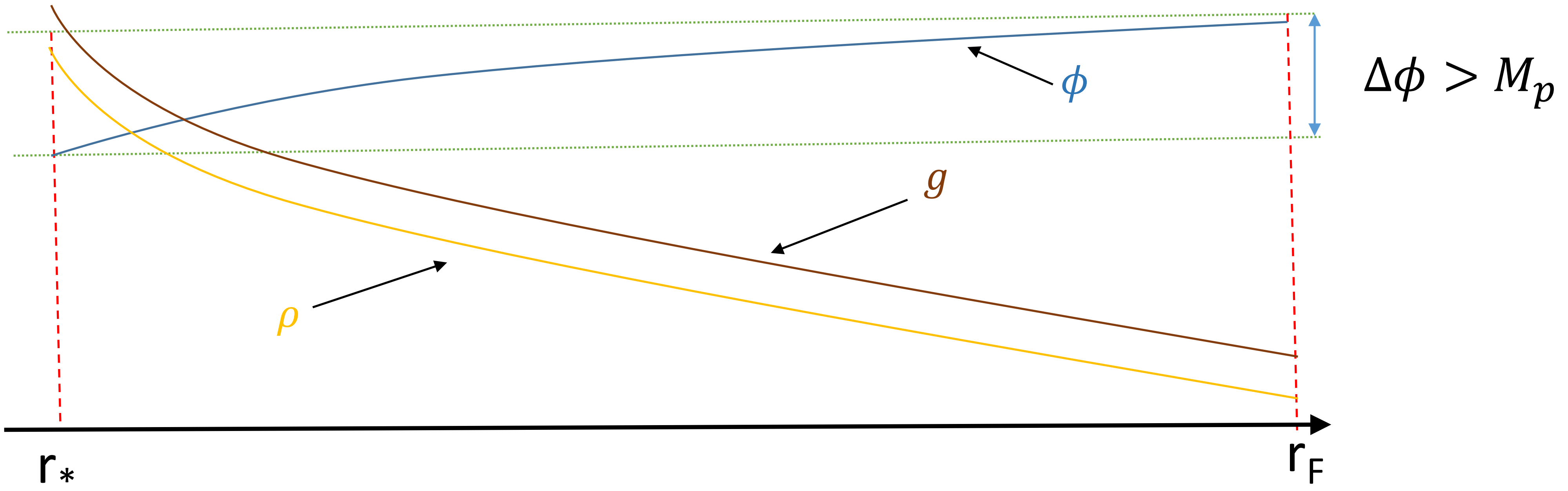
Find that for a variation from r_* to r_F have

$$\Delta\phi = \frac{\beta}{\alpha}\left(1 - \left(\frac{r_*}{r_F}\right)^{\frac{1}{\beta}}\right) \qquad \Phi > \frac{\Delta\phi^2}{\beta}\left(\frac{1 + \left(\frac{r_*}{r_F}\right)^{\frac{1}{\beta}}}{1 - \left(\frac{r_*}{r_F}\right)^{\frac{1}{\beta}}}\right) \qquad |\Phi| < 1 \;\Rightarrow \beta > (\Delta\phi)^2$$

As $\Delta\phi \rightarrow \infty$ we have $\Delta\phi \rightarrow \frac{1}{\alpha}\log\left(\frac{r_F}{r_*}\right)$, converging rapidly for $\Delta\phi > 1$

The gauge coupling must track the energy density:

- The (Local) Weak Gravity Conjecture implies $g(r) > \rho(r)^{\frac{1}{2}}$
  (Black Holes describable in a semi-classical gravity regime outside horizon)
- At the free-field radius can show $g(r_F) < \rho^{\frac{1}{2}}(r_F)\left(-\alpha\,\partial_\phi(\ln g)\big|_{\phi(r_F)}\right)$



Find $g(\phi+\Delta\phi) \leq g(\phi)\,\Gamma(\phi,\Delta\phi)\,e^{-\alpha\Delta\phi}$ with $\Gamma(\phi,\Delta\phi)e^{-\alpha\Delta\phi} < 1$ for $\Delta\phi > 1$

Logarithmic spatial dependence at Strong Curvature

Extend the Newtonian analysis to an arbitrary spherically symmetric background

$$ds^2 = -e^{2U(r)}dt^2 + e^{-2U(r)}\left(dr^2 + f(r)\, r^2 d\Omega^2\right)$$

Re-parameterise $U = -\frac{\alpha}{1+\alpha^2}\ln\left(H_1^{\alpha} H_2^{\frac{1}{\alpha}}\right) + \frac{1}{2}\ln f \ , \quad \phi = \frac{\alpha}{1+\alpha^2}\ln\left(\frac{H_1}{H_2}\right)$

Can show that if H_1 and H_2 are Eigenfunctions of the Laplacian then for large spatial variation $\Delta\phi \gg 1$ have $\phi \simeq \frac{\alpha}{1+\alpha^2}\log r$

Imposing a relativistic version of the local WGC $\sqrt{R(r)} < g(r)\, M_p$

We have that $\sqrt{R(r)} \sim r^{-\frac{\alpha^2}{1+\alpha^2}}$ leads to the same exponential behaviour

Super-Planckian Field Variations in Cosmology

Interested in variations of scalar fields that are larger than the Planck mass

$$\Delta\phi > M_p$$

Arise often in scalar field cosmology and impact our understanding of contemporary observational cosmology

Lyth bound: $\frac{\Delta\phi}{M_P} \geq 0.25 \left(\frac{r}{0.01}\right)^{\frac{1}{2}}$

[BICEP3, Spider, CMBPol, …]: $r \sim 0.001$

Super-Planckian Field Variations in Cosmology: Inflation

If the Swampland Conjecture holds then there is a tower of states with mass

$$m = \beta M_p e^{-\alpha\Delta\phi} \quad \text{for} \quad \Delta\phi > \gamma M_p$$

This implies an exponential tension between a high energy scale cut-off and large field variations.

Primordial tensor modes in large field inflation requires both

Lyth bound: $\frac{\Delta\phi}{M_P} \geq 0.25 \left(\frac{r}{0.01}\right)^{\frac{1}{2}}$ Energy scale: $V^{\frac{1}{4}} \sim \left(\frac{r}{0.01}\right)^{\frac{1}{4}} 10^{16} GeV$

For $\beta = \gamma = 1$ we have that $\alpha = 2, 3, 4$ implies a bound on the tensor-to-scalar ratio of $r < 0.22, 0.11, 0.06$.

Super-Planckian Field Variations in Cosmology: Dark Energy

Power-law quintessence as a model of dark energy, $V \sim \frac{M^{4+p}}{\phi^p}$

Field mass: $m^2 \sim \frac{\partial^2 V}{\partial \phi^2} \sim \frac{\rho_\phi}{\phi^2}$

Hubble scale: $H^2 \sim \frac{\rho_\phi}{M_P^2}$

Onset of dark energy is at $m \sim H$ which implies $\phi \sim M_P$.

Super-Planckian fields are generic in quintessence models [Copeland, Sami, Tsujikawa '06]

Infrared gravity physics tied to Ultraviolet gravity physics !