

# *Perturbative QCD and the LHC physics*

*Kirill Melnikov*

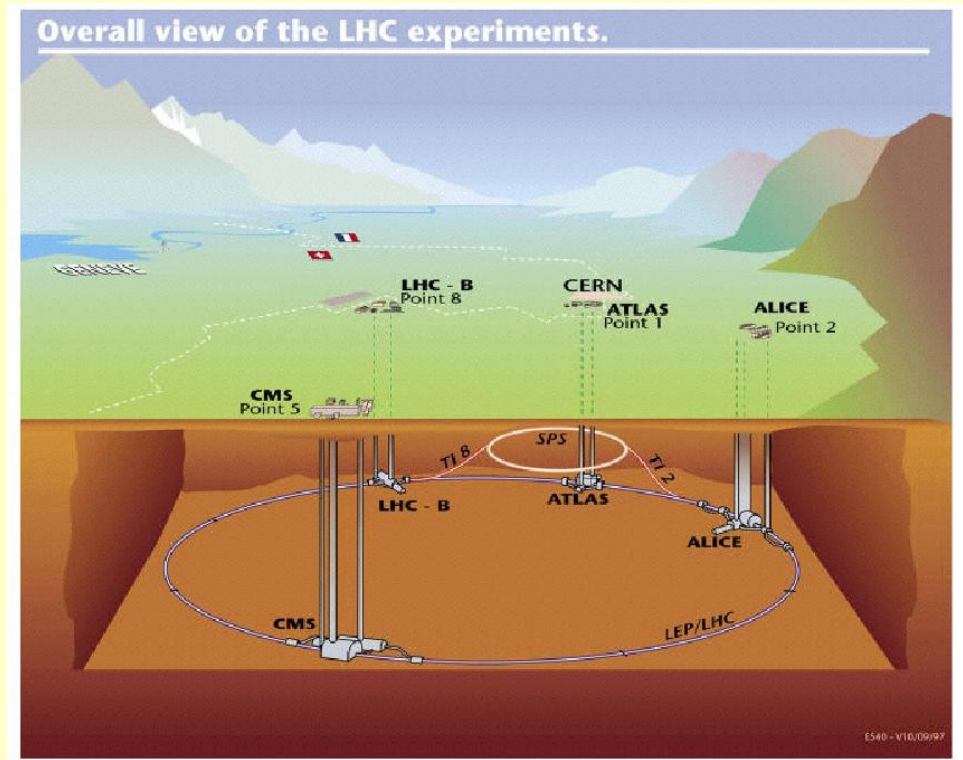
*Johns Hopkins University*

*CERN TH colloquium*

*July 7th, 2010*

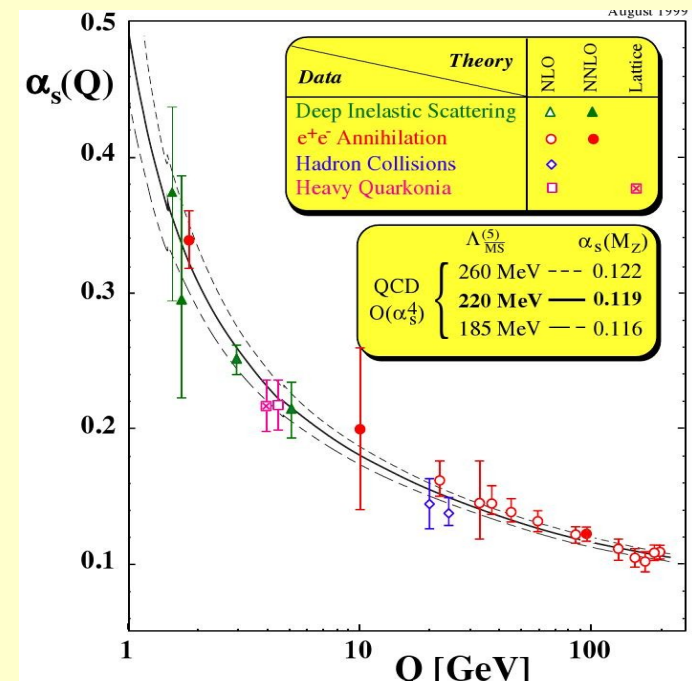
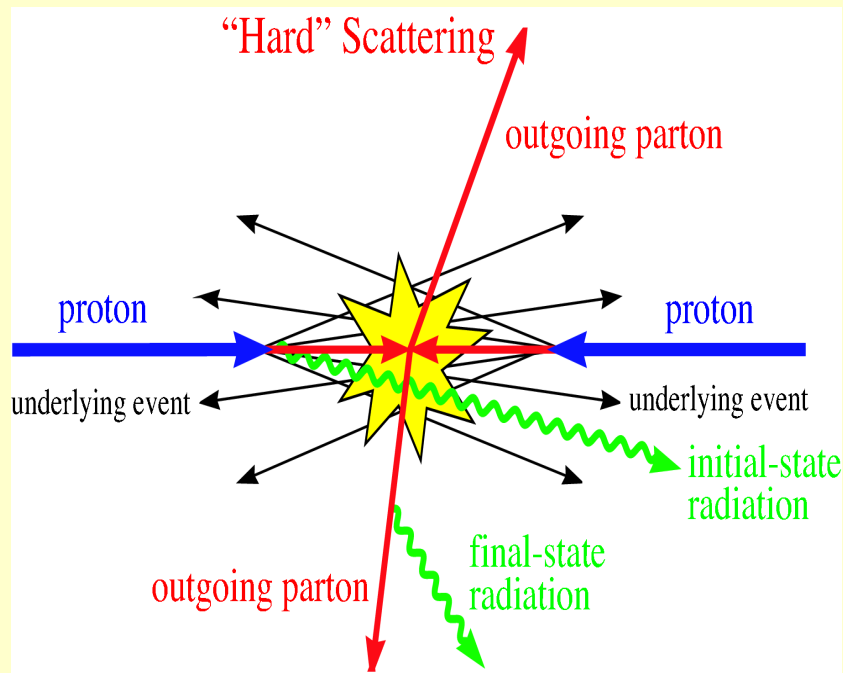
# The LHC

- Next decade of high-energy physics will be decade of the LHC, the proton proton collider here at CERN



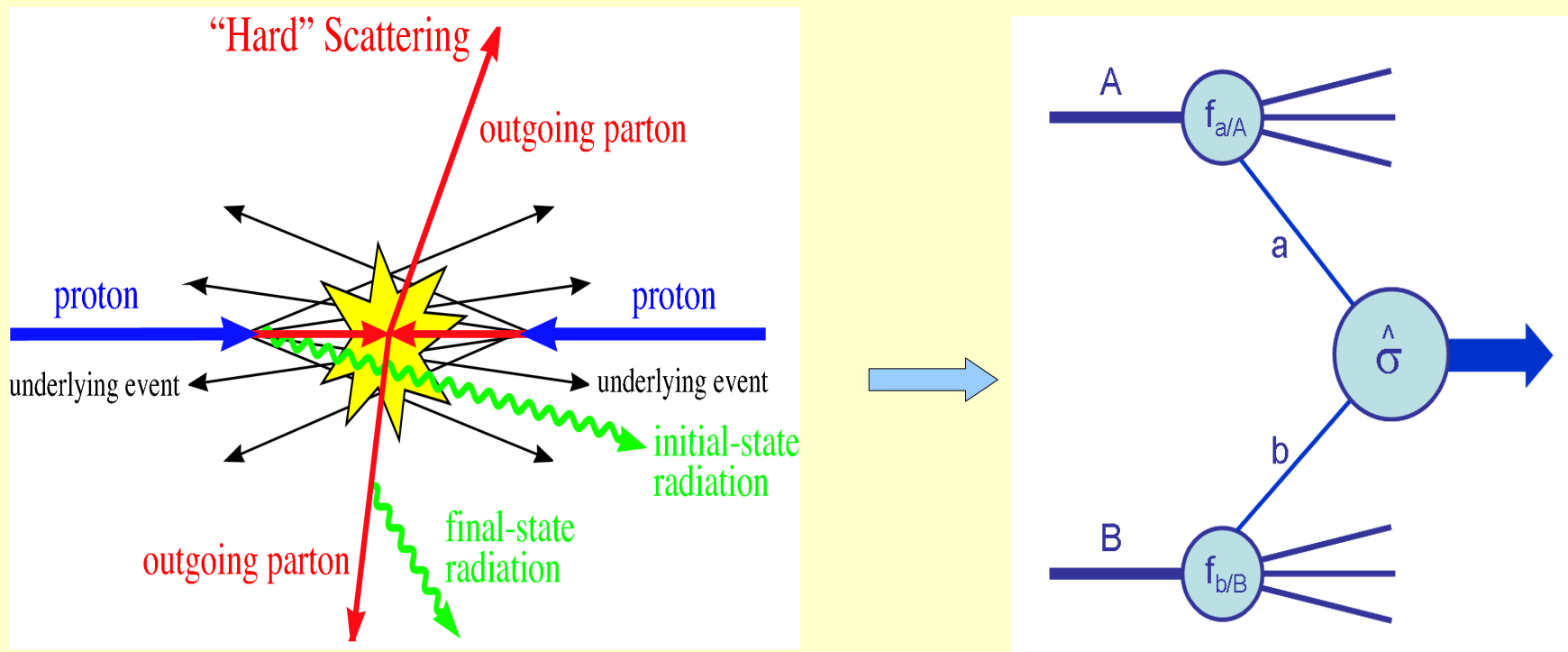
# Colliding hadrons

- Description of hadron collisions requires QCD. To search for “heavy BSM physics”, need description of hard QCD processes where protons desintegrate.
- These processes are responsible for a tiny fraction of proton proton scattering cross-section



# Perturbative QCD

- These rare processes are “double-deep-inelastic” and the formalism to describe them is similar



$$\langle \mathcal{O} \rangle = \sum_{i,j} \int dx_1 dx_2 f_i(x_1) f_j(x_2) d\sigma_{ij \rightarrow p} \mathcal{F}_{p \rightarrow \mathcal{O}} + \mathcal{O}(1/Q)$$

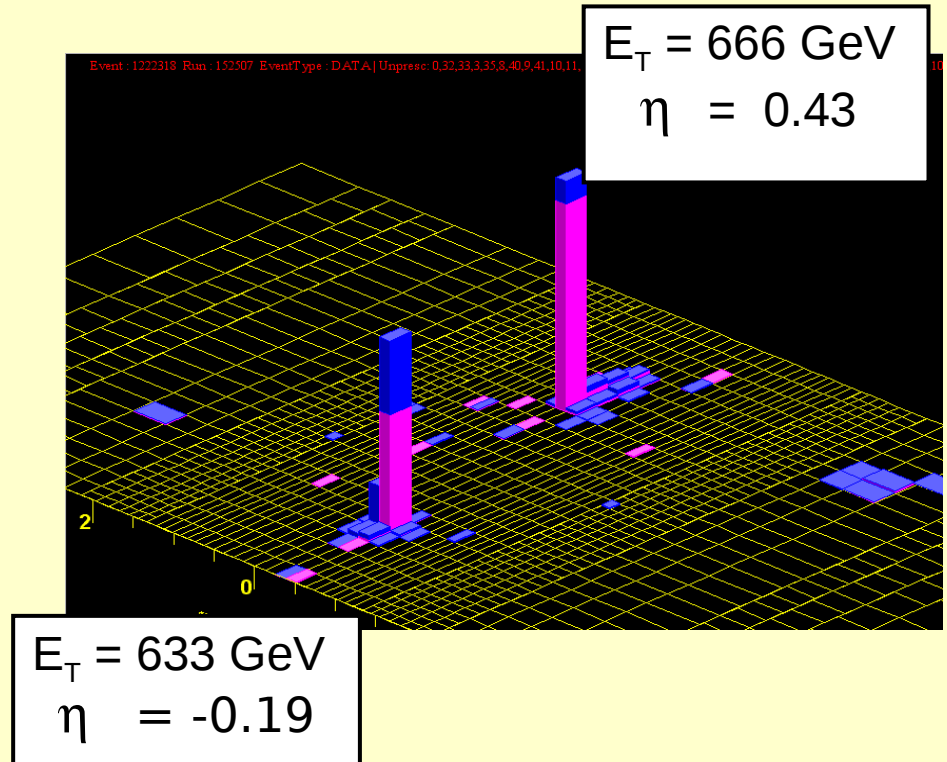
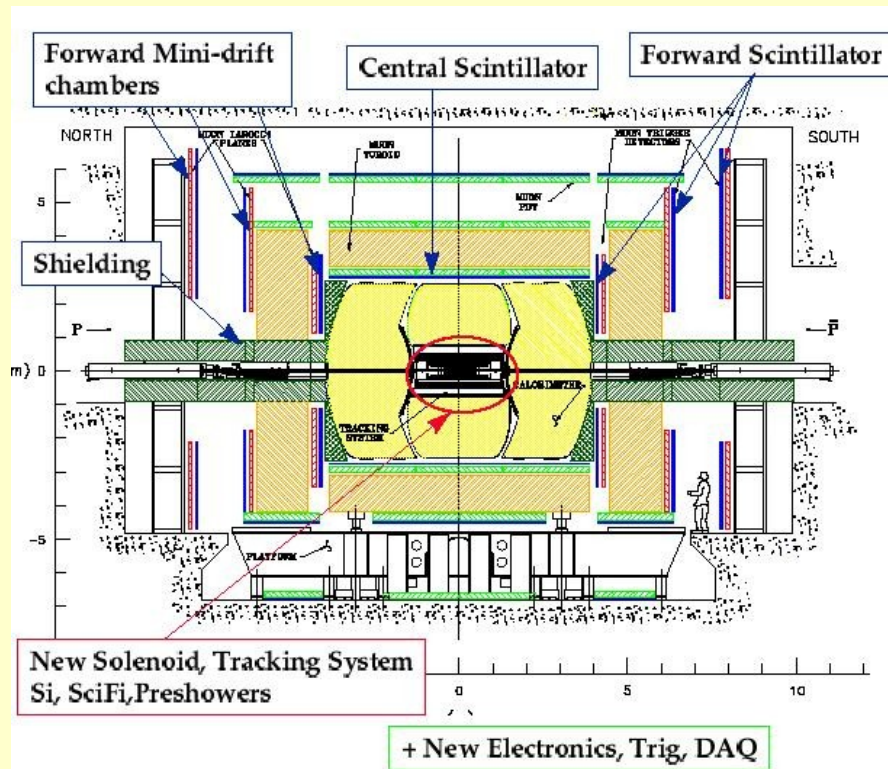
# The pQCD menu

- Perturbative QCD is an expansion in the strong coupling constant that leads to a large variety of approximations
  - Leading order matrix elements
  - Next-to-leading order matrix elements
  - Next-to-next-to-leading order matrix elements
  - Resummations
  - Parton showers
  - Parton showers merged with leading order matrix elements
  - Parton showers matched to next-to-leading order matrix elements



What would you like for your next project?

# The Tevatron legacy



$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4}G_{\mu\nu}^a G^{a,\mu\nu} + \sum_{i=1}^{n_f} \bar{\psi}_i \left( i\hat{D} - m_i \right) \psi_i$$

The Tevatron experiments confirmed that we know how to connect events registered in a detector and the QCD Lagrangian. Perturbation theory works.

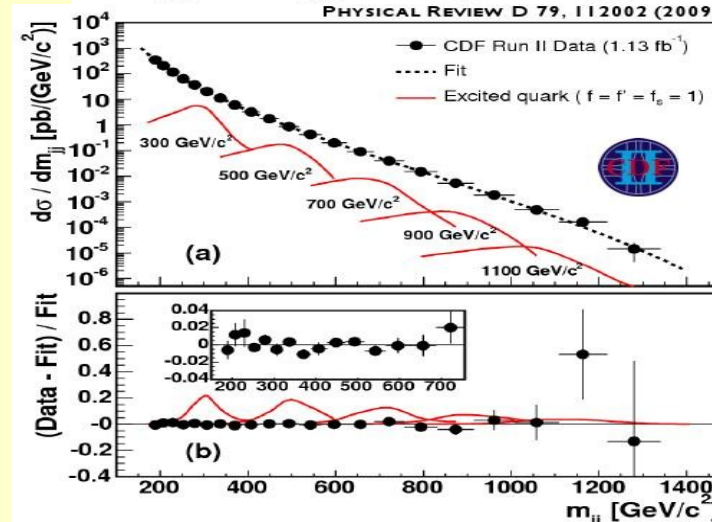
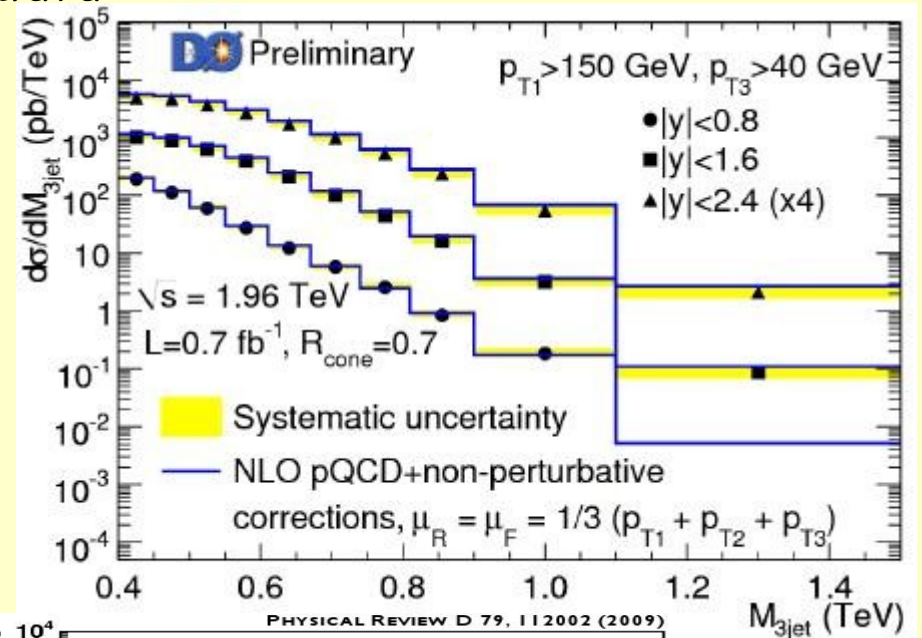
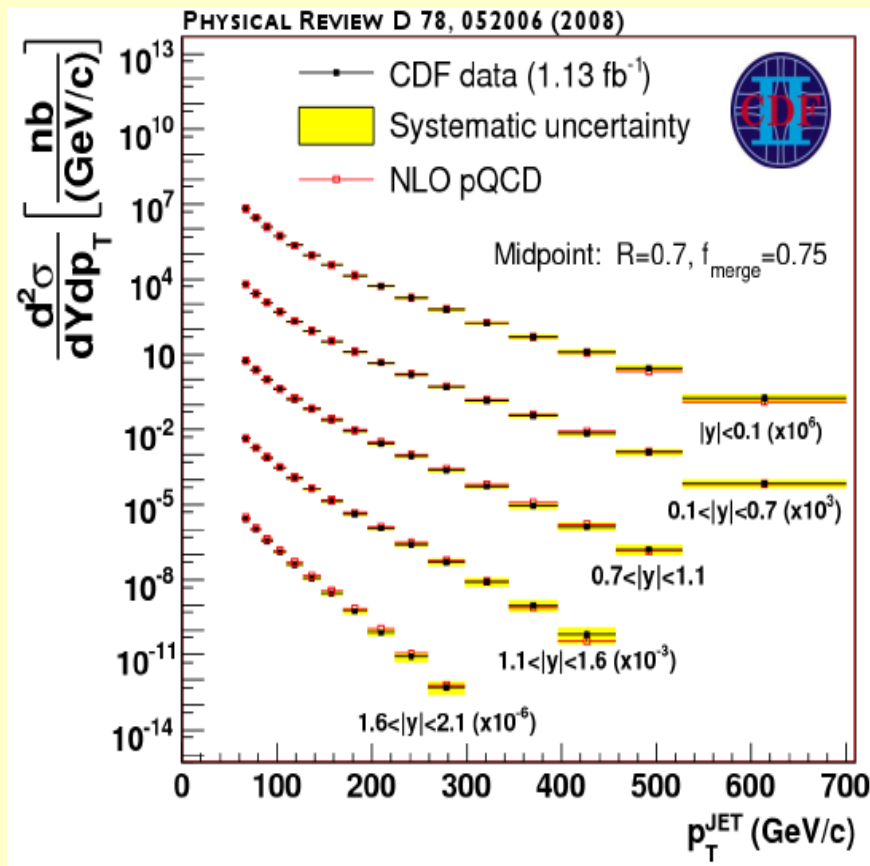


# What have we learned

- Tevatron allowed us to
  - verify the quality of various approximations
  - confirm that 20 GeV is indeed the “large momentum transfer”
- A clear message from the Tevatron is that all approximations work, most of the time. However,
  - LO/parton showers work if we choose input parameters carefully
  - NLO out-of-the-box works even if we choose input parameters carelessly (most of the times)
  - Matrix elements merged with parton showers (CKKW, MLM) work reasonably well for shapes;
  - CKKW tells us which scales are good; this has important effect on shapes of various distributions

# Perturbative QCD at the Tevatron

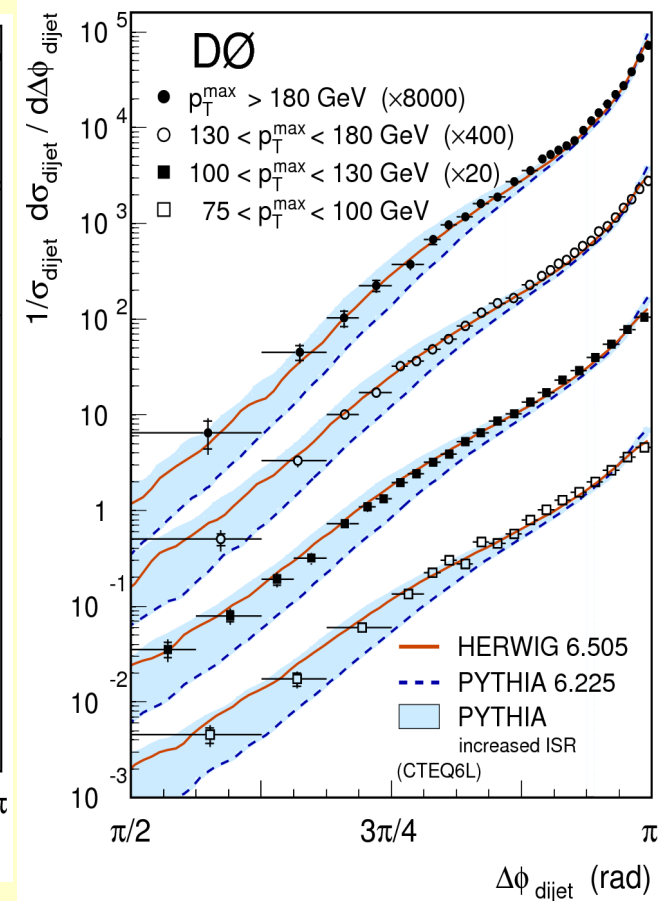
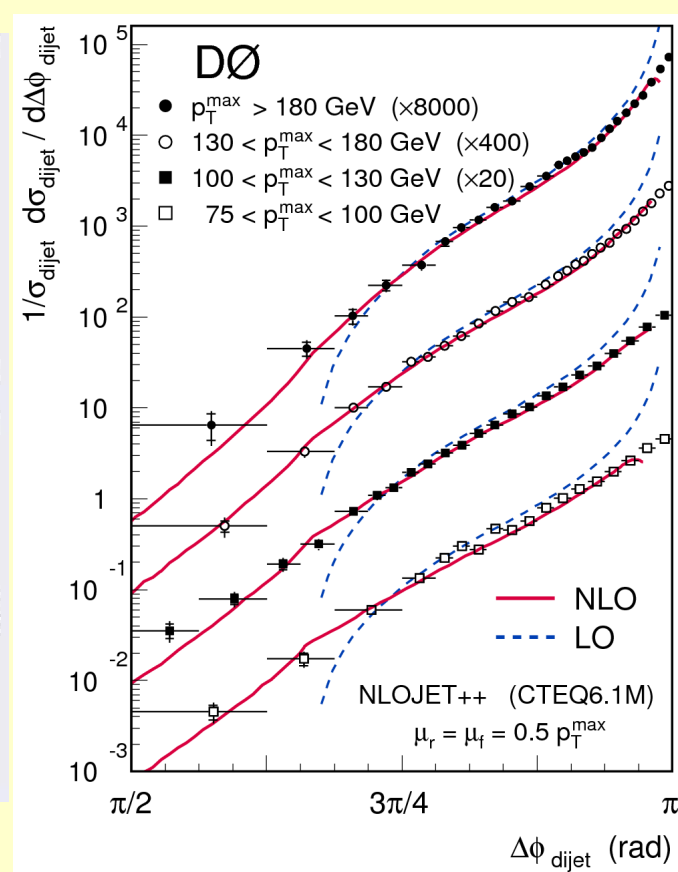
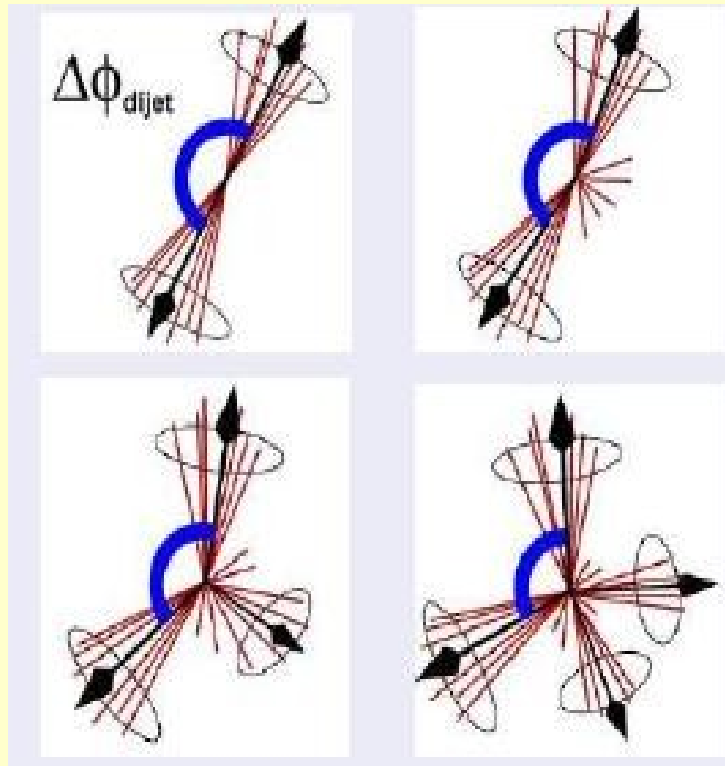
- Detailed studies of jet properties at the Tevatron show good agreement between pQCD and data





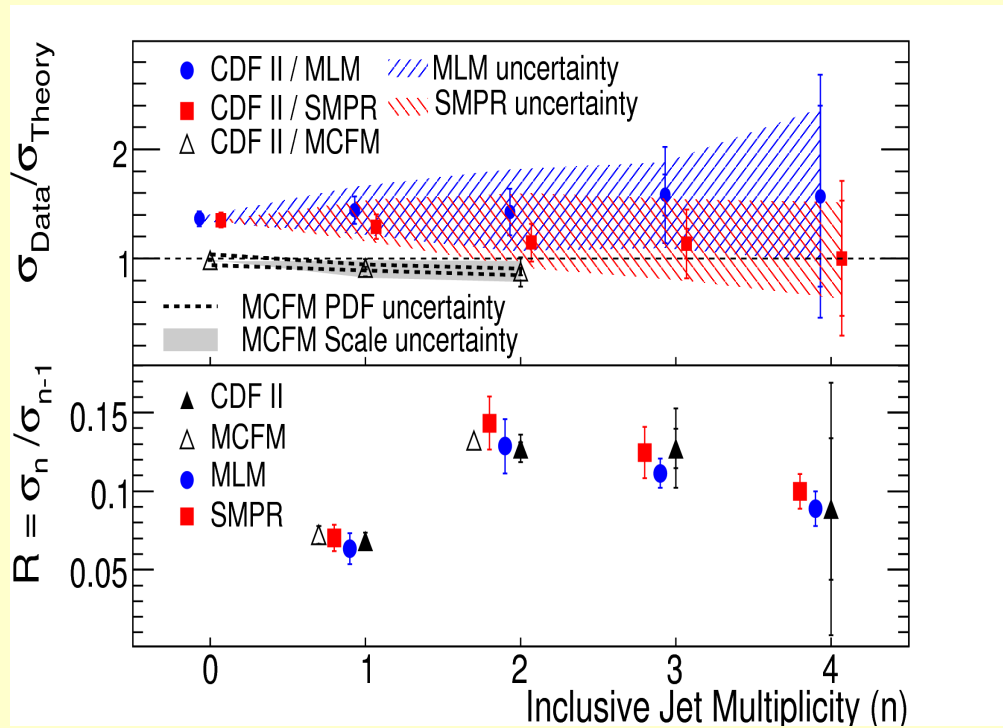
# Perturbative QCD at the Tevatron

- Jet angular correlations allow us to trace how things work when additional jets are being created in the hard process



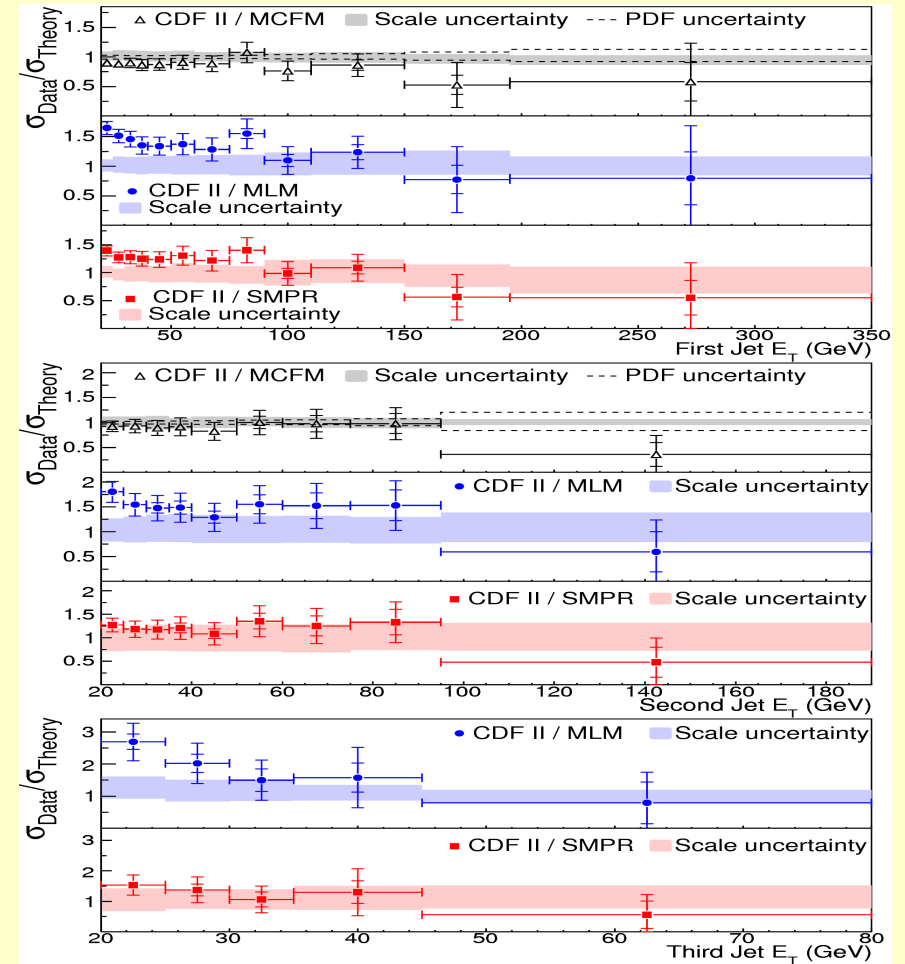
# Perturbative QCD at the Tevatron

- Production of  $W$  bosons in association with jets



number of jets	CDF	LO	NLO
1	$53.5 \pm 5.6$	$41.40(0.02)^{+7.59}_{-5.94}$	$57.83(0.12)^{+4.36}_{-4.00}$
2	$6.8 \pm 1.1$	$6.159(0.004)^{+2.41}_{-1.38}$	$7.62(0.04)^{+0.62}_{-0.86}$
3	$0.84 \pm 0.24$	$0.796(0.001)^{+0.488}_{-0.276}$	$0.882(0.005)^{+0.057}_{-0.138}$

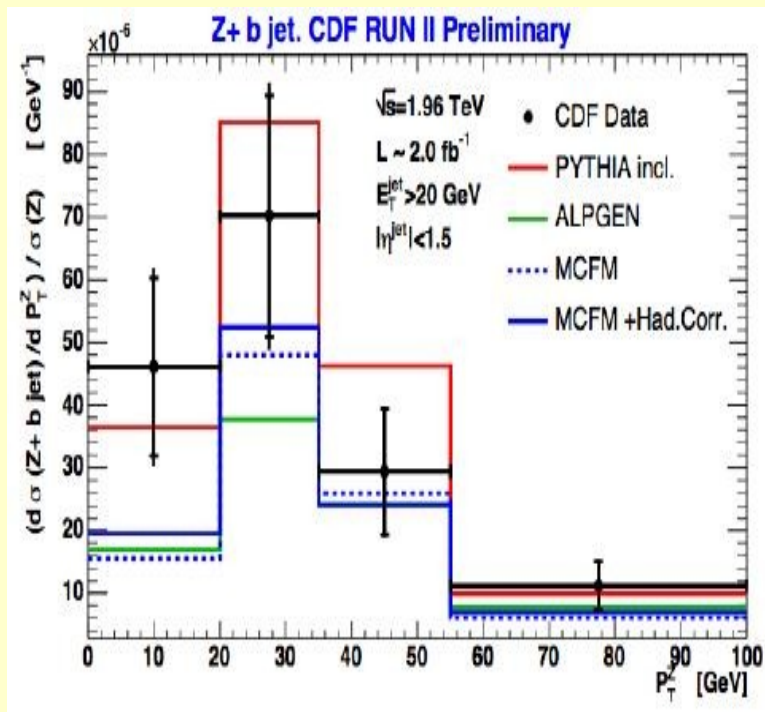
Blackhat/Sherpa collaborations



Interesting issues related to the consistency of theoretical and experimental jet algorithms

# Perturbative QCD at the Tevatron

- But sometimes the situation is very puzzling – for example when  $b$ -quarks are involved



CDF	$2.74 \pm 0.27$ (stat) $\pm 0.42$ (syst) pb
ALPGEN	0.78 pb
PYTHIA	1.10 pb
MCFM (combined 4F+5F)	$1.22 \pm 0.14$ (scale) pb

There is not much of a difference between  $Z+b$  and  $W+b$  production theoretically, so it is hard to understand why one works reasonably and the other fails badly

# pQCD: from the Tevatron to the LHC

- NLO QCD works very well for hard collisions at the Tevatron; similar predictions for the LHC background processes will be very useful.
- Because first NLO QED computations appeared nearly 50 years ago, it looked like an easy task but it turned out to be highly non-trivial.

• First computations of processes from the wishlist with 4 final state particles had to wait until 2009 when  $pp \rightarrow W+3j$  and  $pp \rightarrow tt$   $bb$  were computed through NLO in pQCD.

## An experimenter's wishlist April 2001

■ Hadron collider cross-sections one would like to know at NLO

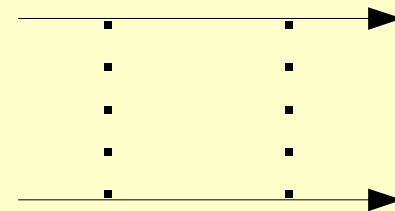
Run II Monte Carlo Workshop, April 2001

Single boson	Diboson	Triboson	Heavy flavour
$W + \leq 5j$	$WW + \leq 5j$	$WWW + \leq 3j$	$t\bar{t} + \leq 3j$
$W + b\bar{b} + \leq 3j$	$WW + b\bar{b} + \leq 3j$	$WWW + b\bar{b} + \leq 3j$	$t\bar{t} + \gamma + \leq 2j$
$W + c\bar{c} + \leq 3j$	$WW + c\bar{c} + \leq 3j$	$WWW + \gamma\gamma + \leq 3j$	$t\bar{t} + W + \leq 2j$
$Z + \leq 5j$	$ZZ + \leq 5j$	$Z\gamma\gamma + \leq 3j$	$t\bar{t} + Z + \leq 2j$
$Z + b\bar{b} + \leq 3j$	$ZZ + b\bar{b} + \leq 3j$	$WZZ + \leq 3j$	$t\bar{t} + H + \leq 2j$
$Z + c\bar{c} + \leq 3j$	$ZZ + c\bar{c} + \leq 3j$	$ZZZ + \leq 3j$	$t\bar{b} + \leq 2j$
$\gamma + \leq 5j$	$\gamma\gamma + \leq 5j$		$b\bar{b} + \leq 3j$
$\gamma + b\bar{b} + \leq 3j$	$\gamma\gamma + b\bar{b} + \leq 3j$		
$\gamma + c\bar{c} + \leq 3j$	$\gamma\gamma + c\bar{c} + \leq 3j$		
	$WZ + \leq 5j$		
	$WZ + b\bar{b} + \leq 3j$		
	$WZ + c\bar{c} + \leq 3j$		
	$W\gamma + \leq 3j$		
	$Z\gamma + \leq 3j$		

# A new way of computing one-loop graphs

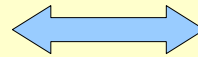
- What do we need to know to compute one-loop amplitudes in arbitrary renormalizable quantum field theory?
  - Traditional (and still viable) answer; Feynman diagrams;
  - New answer: **on-shell leading order amplitudes** for complex (!) on-shell momenta in two space-times with integer dimensionality higher than 4.
- Higher-dimensional space-times address the issue of divergences
- Complex momenta make the Coulomb field “propagate”

$$D_{00} = \frac{i}{\vec{q}^2}, \quad D_{ij} = \frac{-i}{q^2} \left( \delta_{ij} - \frac{q_i q_j}{\vec{q}^2} \right).$$

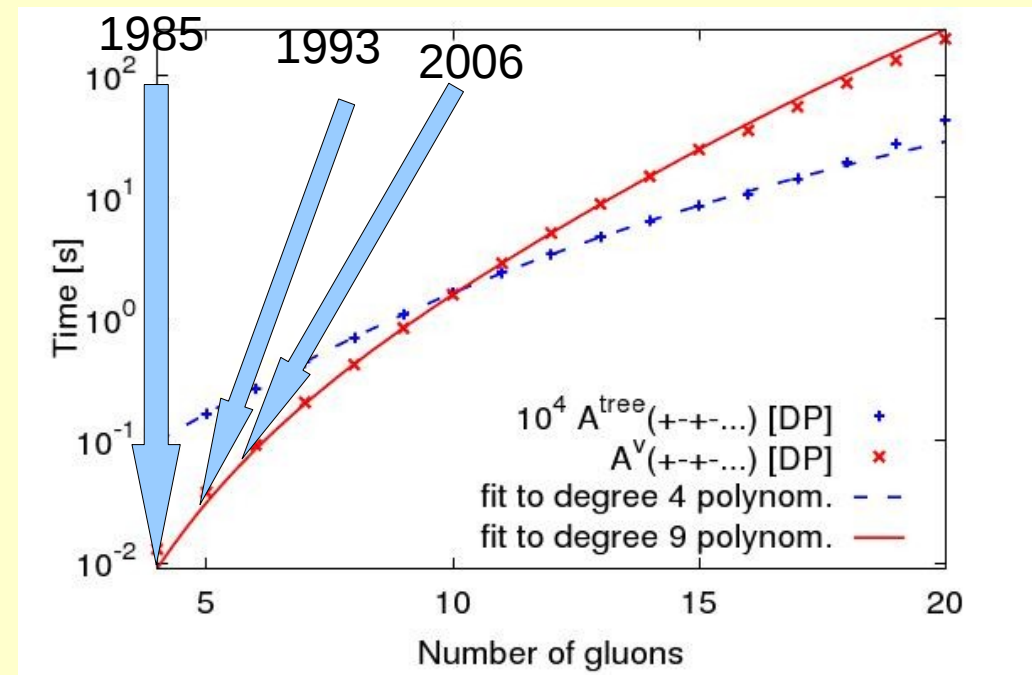
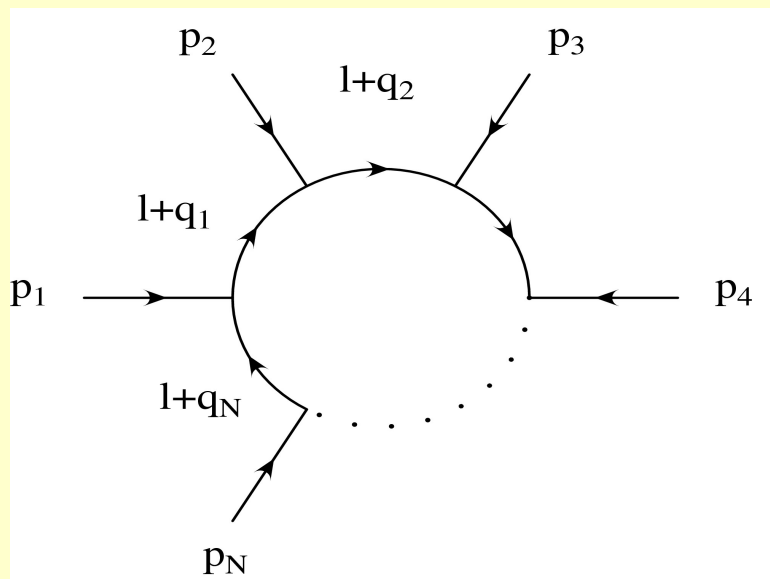


# The power of unitarity: $N$ -gluon amplitudes

$$20! \approx 2.4 \times 10^{18}$$



100 years of calculating



*Giele, Zanderighi*

$N$ -gluon amplitudes can be calculated for arbitrary  $N$ . Explicit numerical results available for  $N$  through 20. Factorial growth in the number of Feynman diagrams makes this computation impossible with traditional methods.



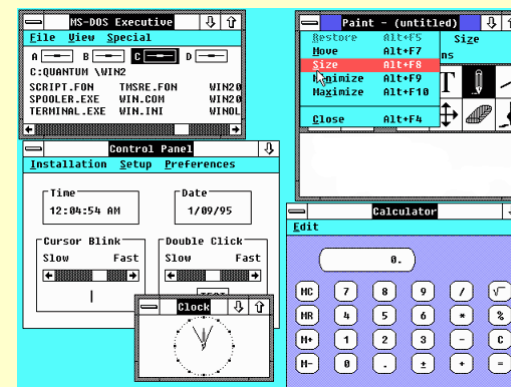
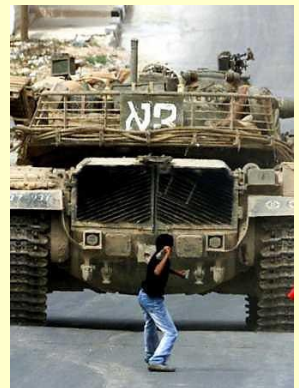
# The recent progress

- $pp \rightarrow ttbb$  *Bredenstein, Denner, Dittmaier, Pozzorini  
Bevilacqua, Czakon, Papadopoulos, Worek*
- $pp \rightarrow tt+2\text{jets}$  *Bevilacqua, Czakon, Papadopoulos, Worek*
- $pp \rightarrow W+3\text{ jets}$  *Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre  
RKEllis, G.Zanderighi, KM.*
- $pp \rightarrow Z +3\text{ jets}$  *Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre*
- Many important  $2 \rightarrow 3$  processes such as  $pp \rightarrow VV + \text{jet}$ ,  $pp \rightarrow H + 2j$ ,  
 $pp \rightarrow VVV$ ,  $pp \rightarrow V + bb$ ,  $pp \rightarrow tt + \text{jet}$  became known/refined in recent  
years  
*Kallweit, Uwer, Campbell, Binoth, Karg, Kauer, Sanguinetti, Ciccolini, Badger, Glover,  
Mastrolia, Williams, Lazopoulos, Petriello, Camparano, Hankele, Zeppenfeld, Ossola,  
Pittau, Wackerath, Reina, Weinzierl, Schulze*

The recent progress could not have happened without the OPP procedure,  
due to Ossola, Pittau, Papadopoulos

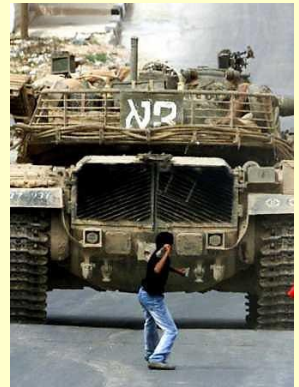
# December 1987

- NASA awards contracts to build the space station Freedom
- First intifada in Gaza Strip and West bank
- Japanese rock band BOOWY announces their breakup
- Cosmonaut Yuri Romanenko of USSR returns to Earth after 326 days in space
- Microsoft releases Windows 2



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- Microsoft releases Windows 2
- S. Dawson, K. Ellis and P. Nason complete computation of NLO QCD corrections to heavy quark pair production

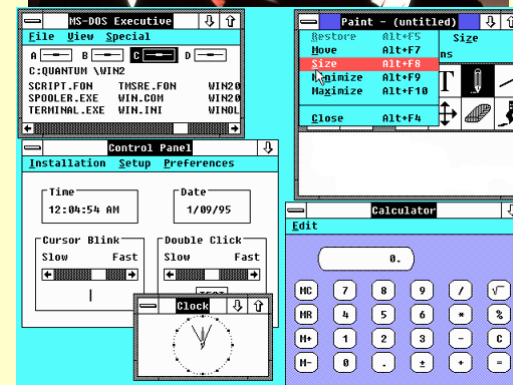


The total cross section for the production of heavy quarks  
in hadronic collisions

**P. Nason and S. Dawson**  
Brookhaven National Laboratory  
Upton, LI, New York 11973

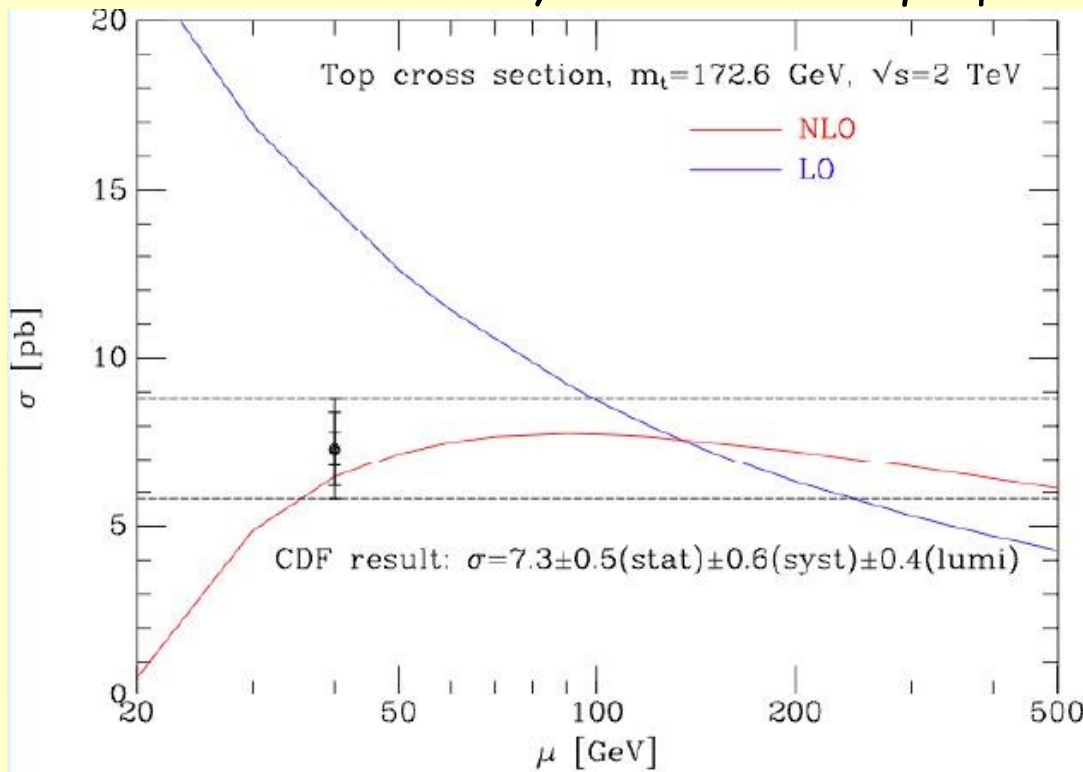
**R. K. Ellis**  
Fermi National Accelerator Laboratory  
P. O. Box 500, Batavia, Illinois 60510

December 23, 1987

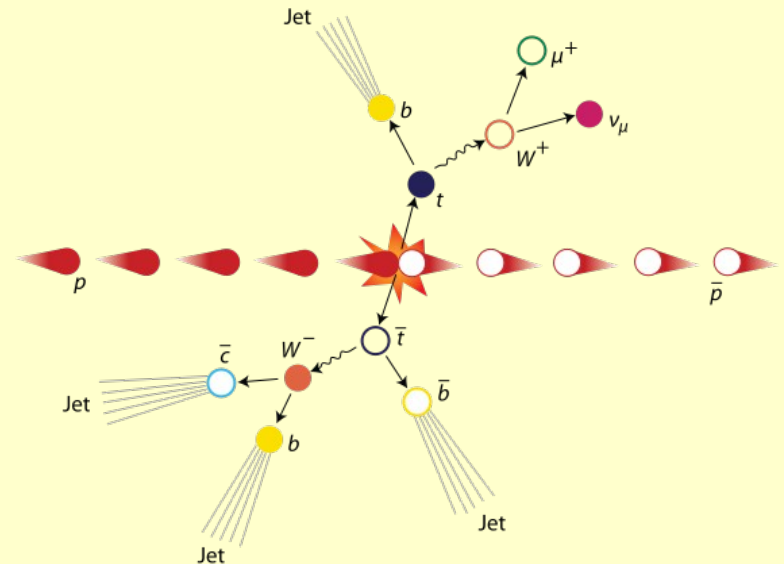


# Top quarks are not stable

- It is hard to believe but we still have their NLO QCD result! It is peculiar perseverance because back in 1990, it was sensible to treat heavy quarks as stable.
- But we know by now that top quarks decay !



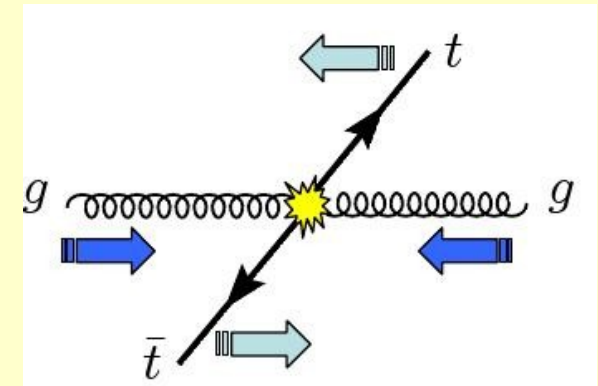
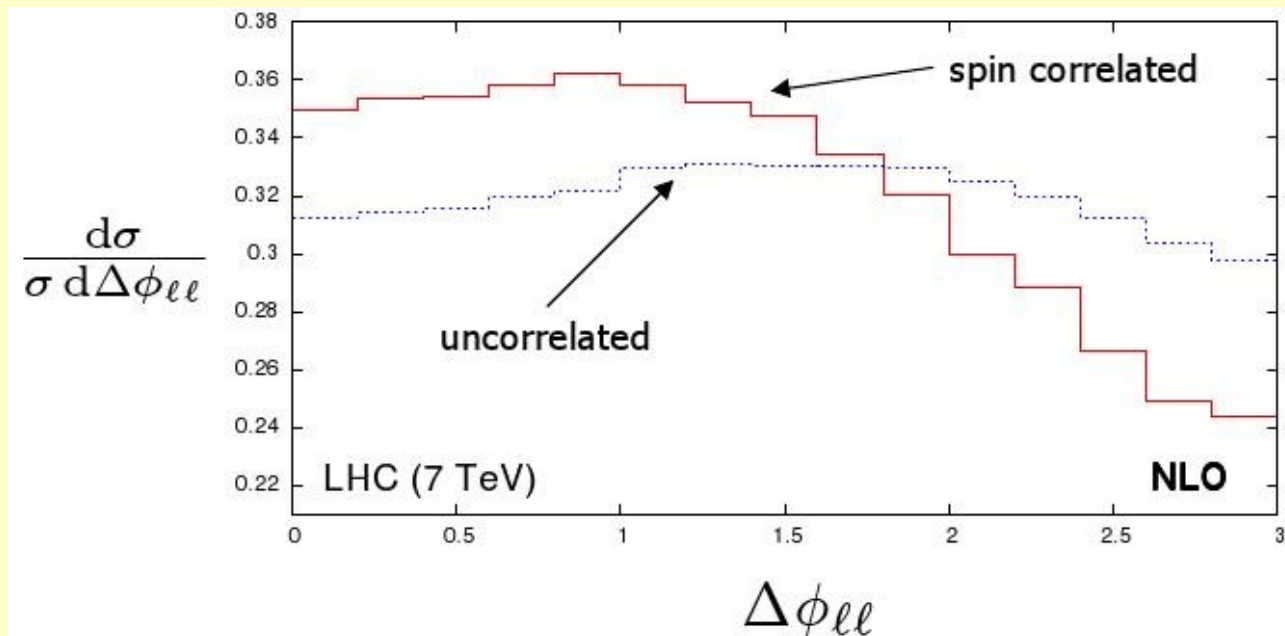
RKELLIS, MCFM



# Top quarks are not stable

- Top quark decay products are observed in experiment
- Kinematics of top quark decay products is affected by top quark spin correlations – *a unique feature of top quark pair production!*
- Until 2009, *no NLO QCD calculation included spin correlations, corrections to top decay and allowed arbitrary cuts on hadronic and leptonic final states.*

$$pp \rightarrow t\bar{t} \rightarrow bl^+\nu_l + \bar{b}l\bar{\nu}_l$$



$$m_{l+l-} < 100 \text{ GeV}$$

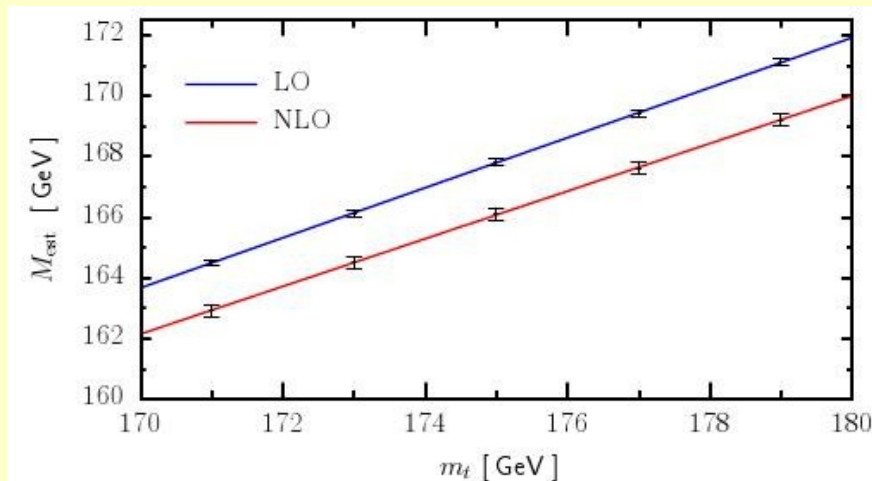
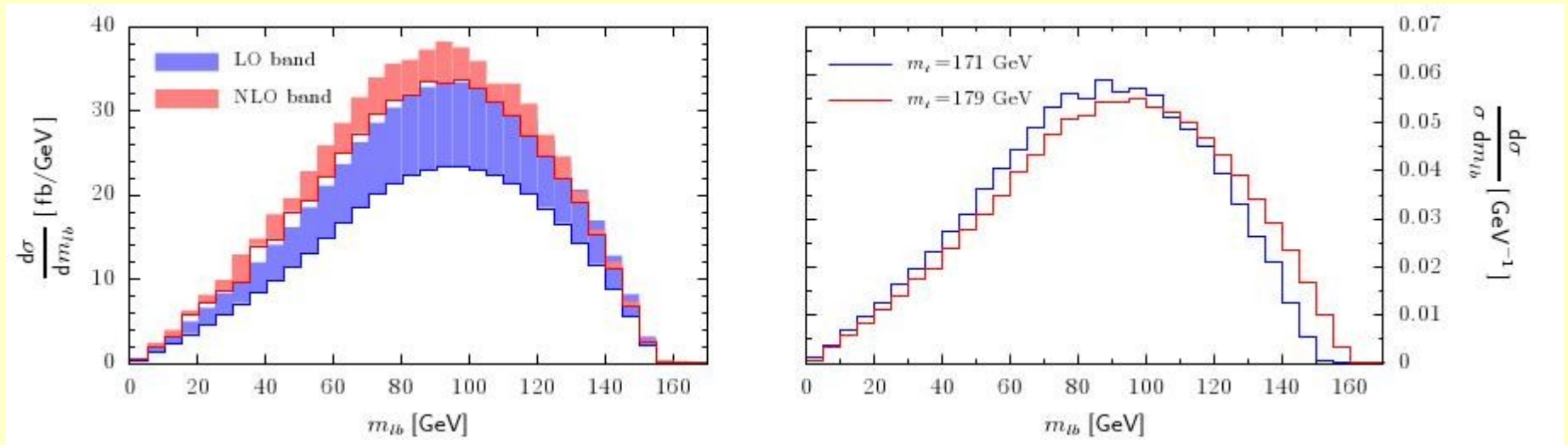
$$p_{\perp, l} < 50 \text{ GeV}$$

Schulze, KM.



# Top quarks are not stable

- Kinematics of top quark decay products is correlated with the top quark mass. *Correlations always evaluated with PYTHIA!*



*Biswas, Schulze, K.M.*

$$M_{\text{est}}^2 = m_W^2 + \frac{2\langle m_{lb}^2 \rangle}{1 - \langle \cos \theta_{lb} \rangle}$$

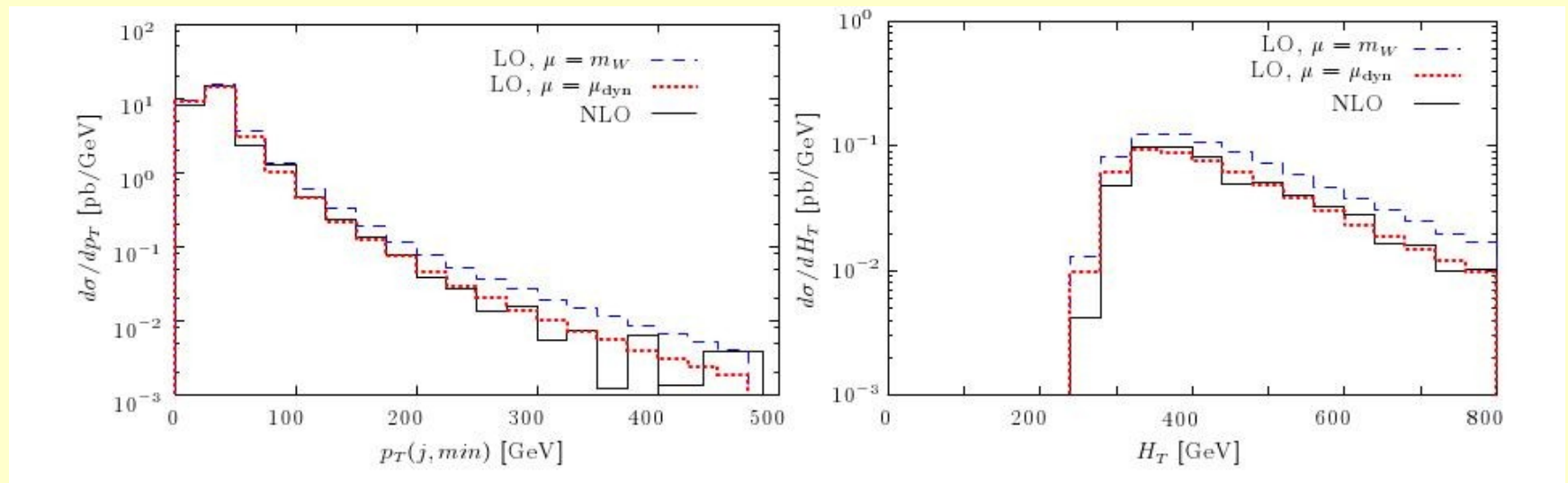
$$M_{\text{est}}^{\text{LO}} = 0.8262m_t + 23.22 \text{ GeV}$$

$$M_{\text{est}}^{\text{NLO}} = 0.7850m_t + 28.70 \text{ GeV}$$



# NLO calculations: choices of scales

- Bauer and Lange showed that the choice of the renormalization scale of the strong coupling constant leads to important effects for kinematic distributions.
- Properly chosen, dynamical renormalization/factorization scales in LO reproduce *shapes* of NLO computations



$$\mu_{dyn} = \sqrt{\left(\frac{m_{JJ}}{2}\right)^2 + m_W^2}$$

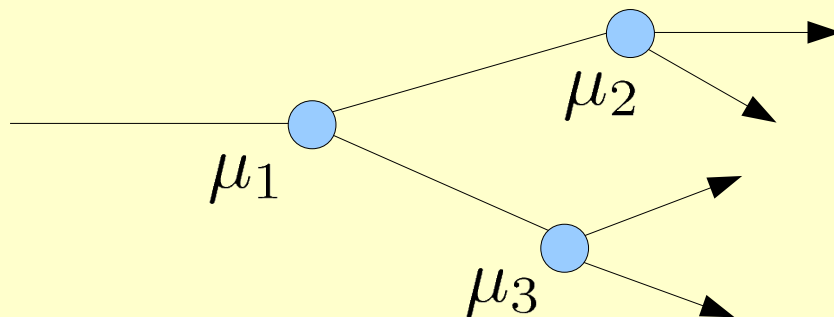
Bauer, Lange

# Learning from the parton shower

- The Bauer-Lange analysis works well because it respects a well-known feature of QCD partons branchings

$$\text{Prob}(a \rightarrow bc) \sim \alpha_s(p_\perp)$$

- *The CKKW/MLM procedure respects this choice and, in fact, does more careful scale adjustment.* The scales are chosen on an event-by-event basis by identifying most probable “history” of an event
  - iteratively cluster particles that are closest according to some measure (usually,  $k_\perp$  algorithm is used).
  - for each node, choose the relative momentum of the daughters as the scale for the strong coupling constant – this is the parton shower choice.

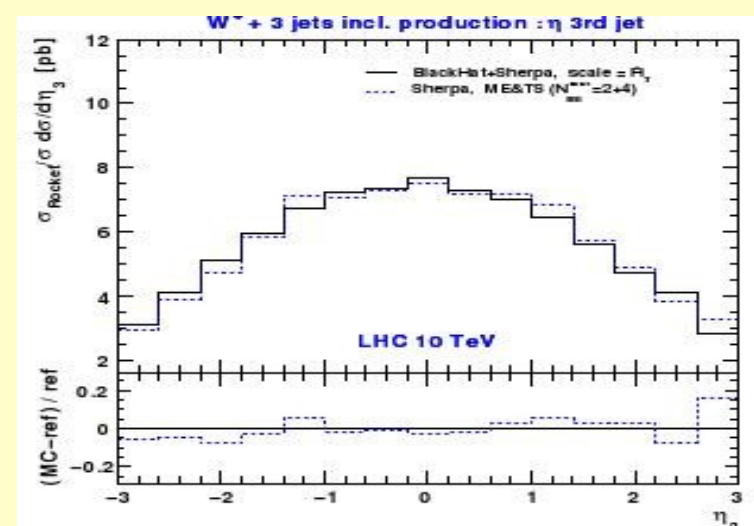
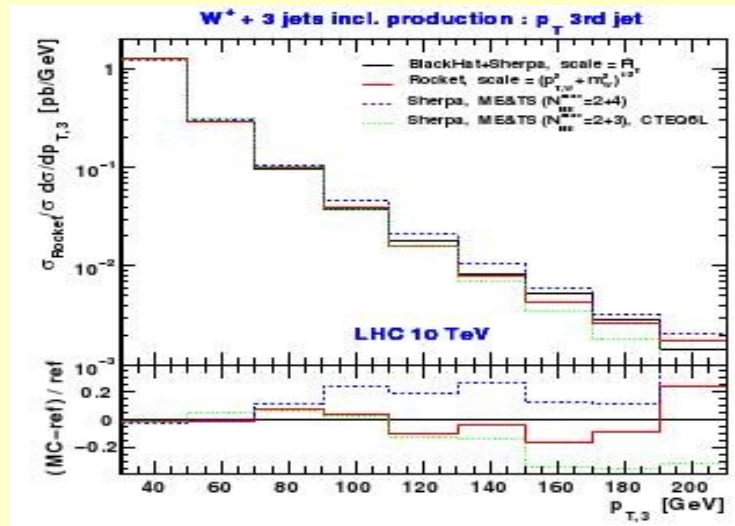
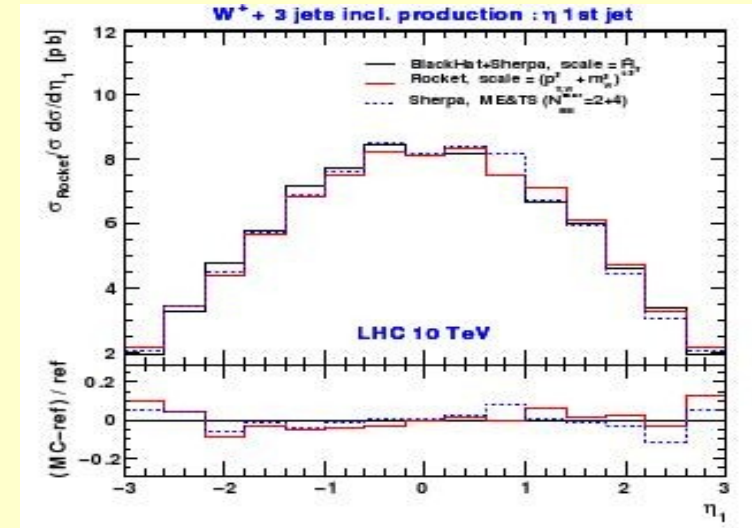
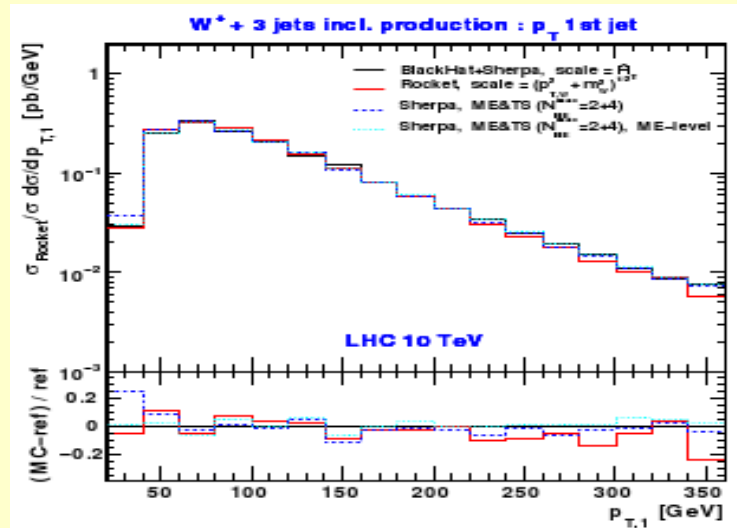


$$|\mathcal{M}|^2 \sim \prod_{i=1}^{N_{\text{part}}} \alpha_s(\mu_i)$$

Catani, Krauss, Kuhn, Webber

# Scale setting and W+3 jets at NLO

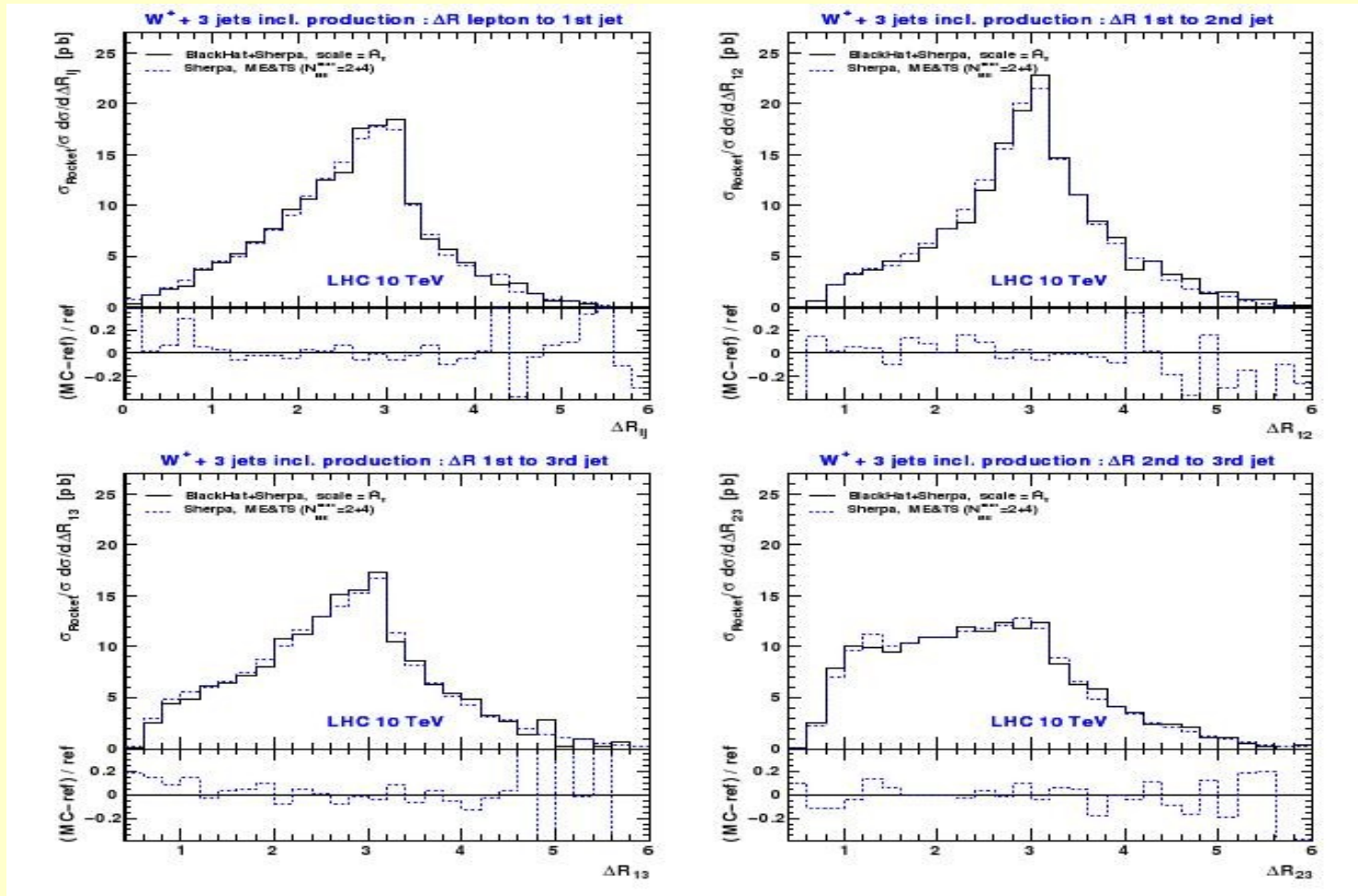
- CKKW/MLM procedure does a very good jobs in describing shapes.



Blackhat/Rocket/Sherpa comparison S. Hoche, J. Huston, D. Maitre, J. Winter, G. Zanderighi

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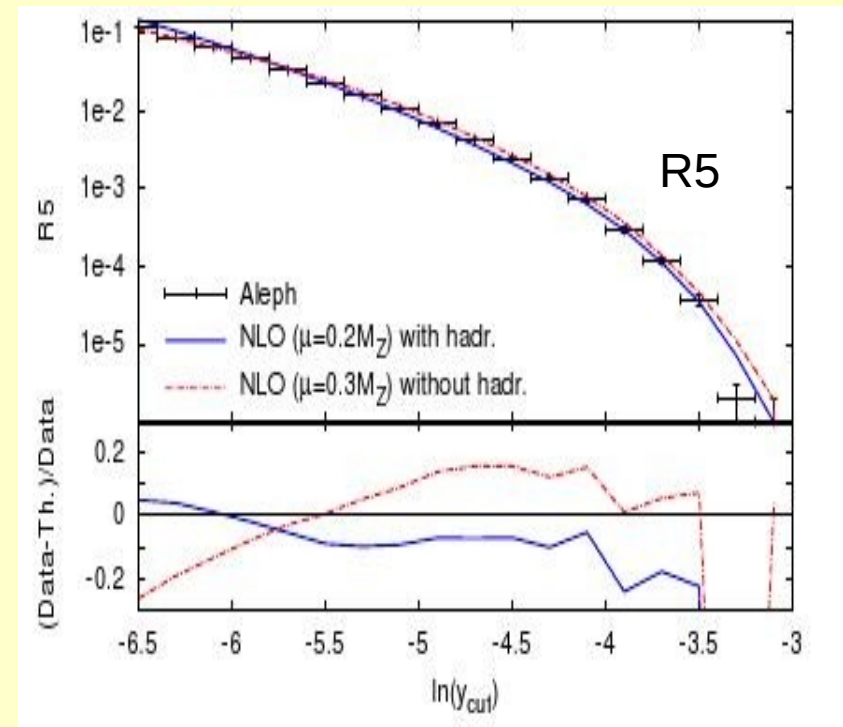
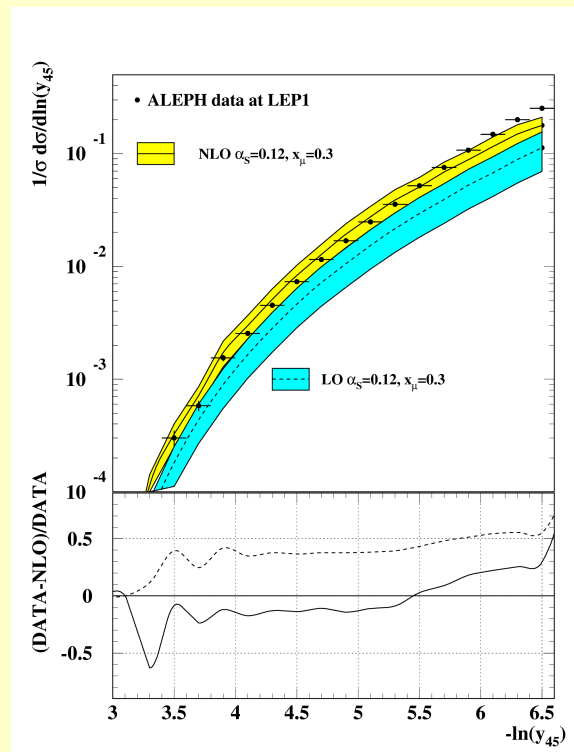
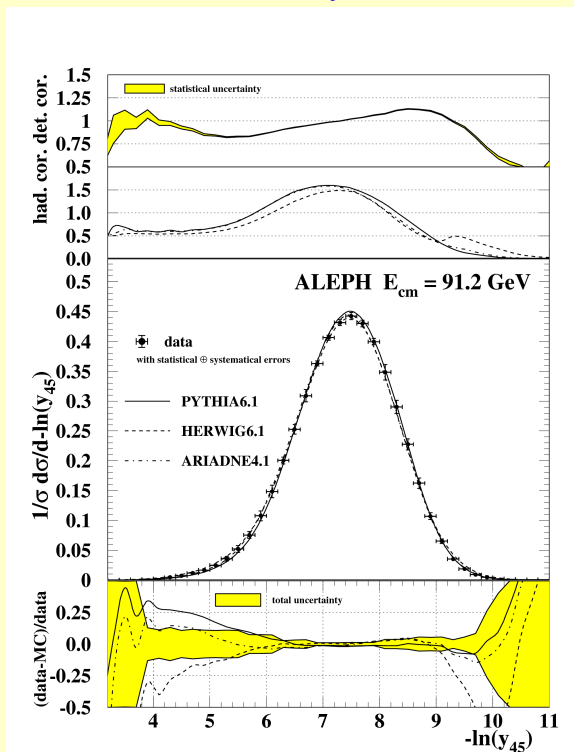


Blackhat/Sherpa comparison

S. Hoche, J. Huston, D. Maitre, J. Winter, G. Zanderighi

# “Perturbative” hadronization

- LHC will be a jetty place – will we know what we are doing?  
Look at the highest exclusive jet multiplicity studied at LEP –  
*five jets !*



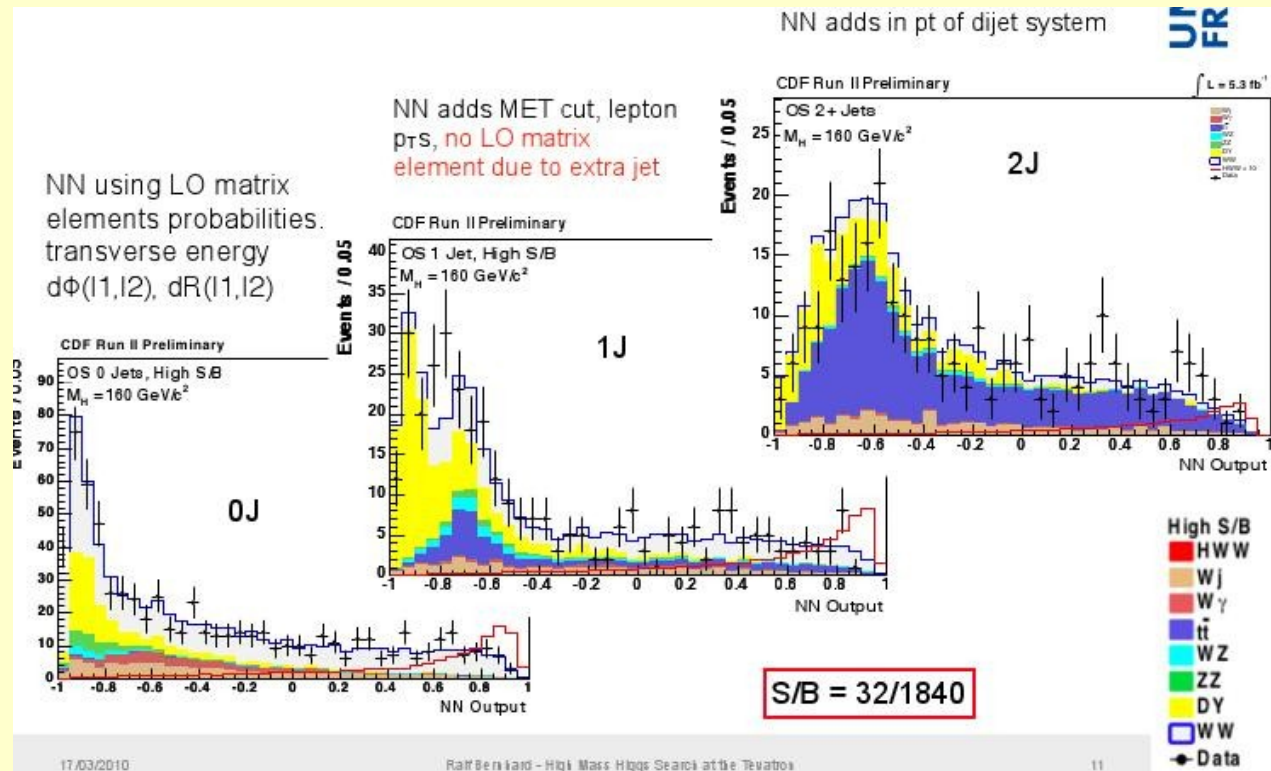
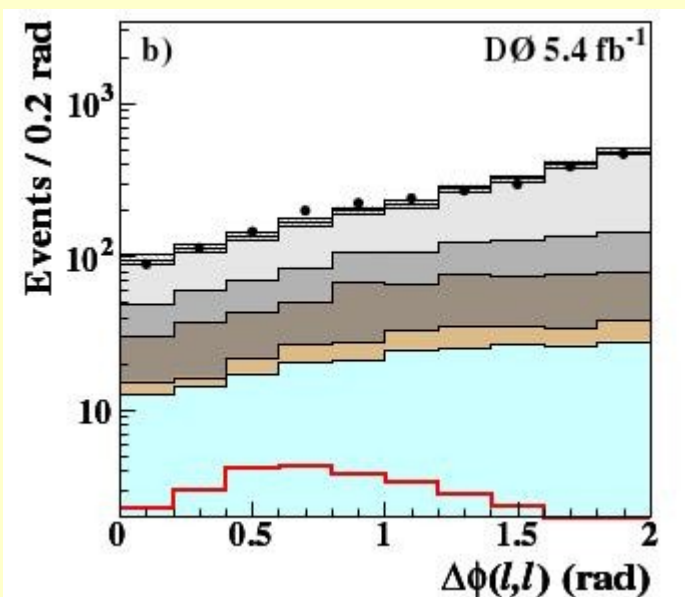
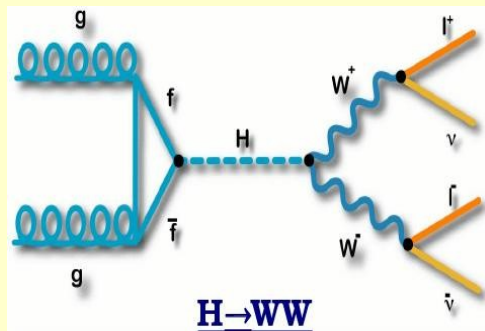
Frederix, Frixione, Zanderighi, K.M., Stenzel

Hadronization corrections and perturbative corrections become entangled for high jet multiplicity



# NLO QCD and “correct variables”

- The high-mass Higgs search at the Tevatron *is a neural net festival*.  
Can one calculate the neural net variable distributions at NLO?



R. Bernhard, Talk at QCD Moriond 2010

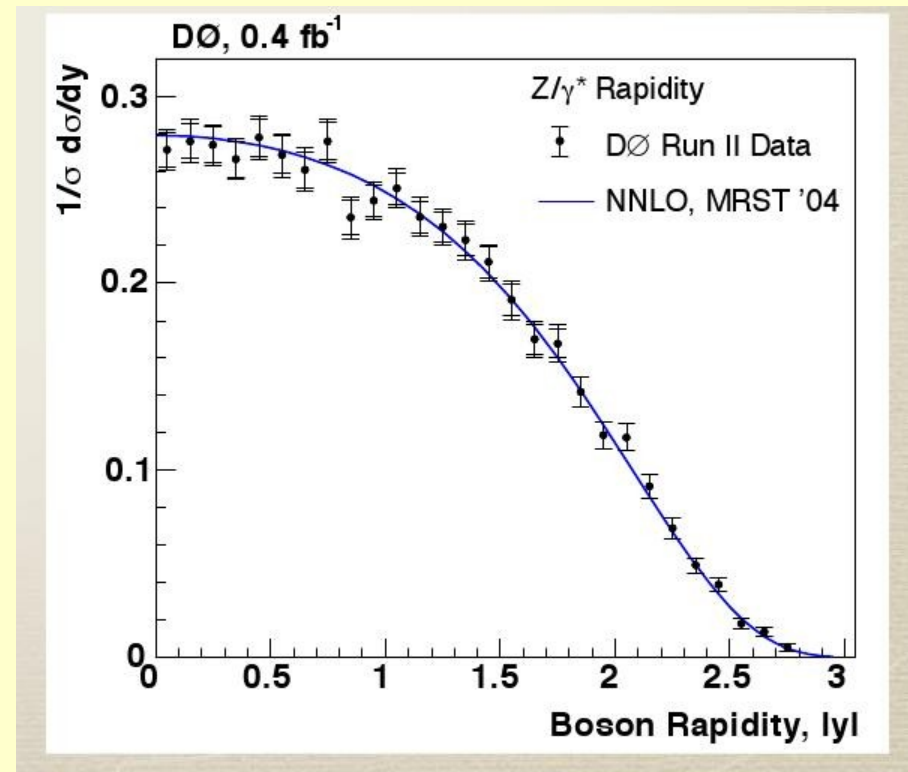
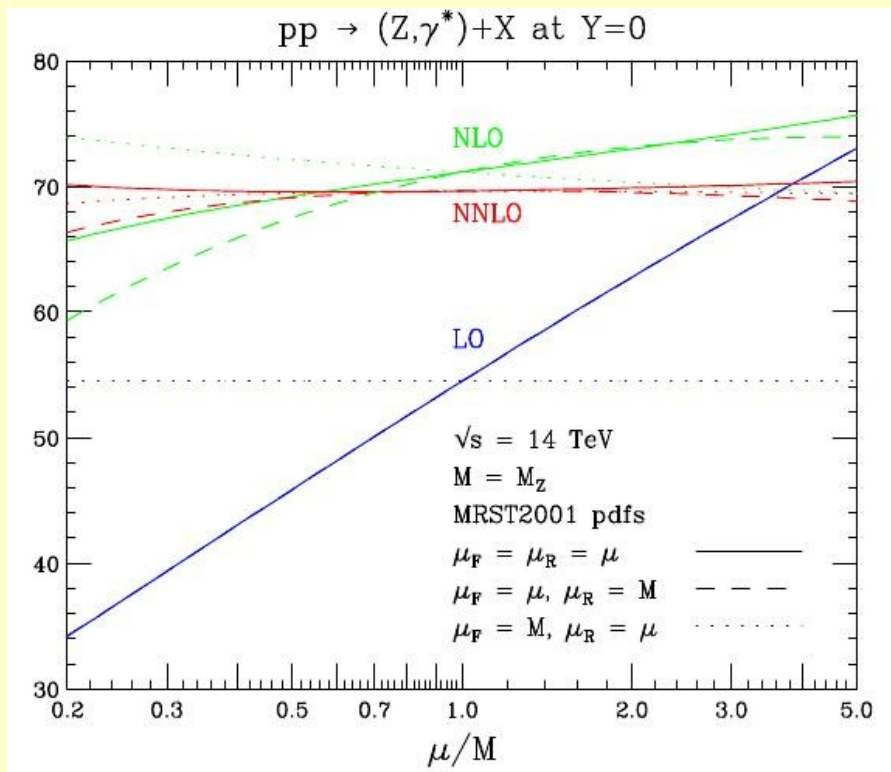


# Beyond the NLO: NNLO

- There are attempts to extend fixed order perturbative computations beyond the NLO.
- Useful for extraordinary clean or important hard processes
- NNLO calculations are in their infancy, and our abilities are very limited. Realistic results for collider processes are available for
  - $e^+e^- \rightarrow 3j \rightarrow$  the value of the strong coupling constant  
*Gehrmann-De Ridder, Gehrmann, Glover, Heinrich, Weinzierl*
  - $pp \rightarrow W, Z, H$ ; the W-mass, parton distributions, Higgs boson exclusion limits at the Tevatron  
*Anastasiou, Petriello, K.M., ; Catani, Grazzini, Cieri, De Florian,*

# NNLO: the ultimate goal

- The rapidity distribution of the Z-boson is known through NNLO and shows all the benefits of going to that high an order in pQCD

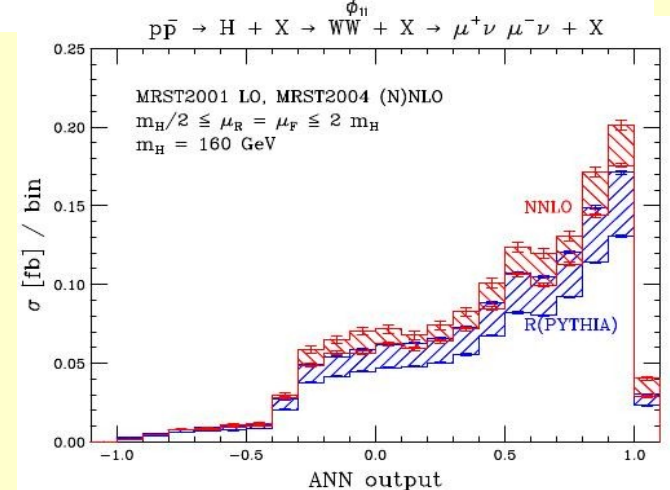
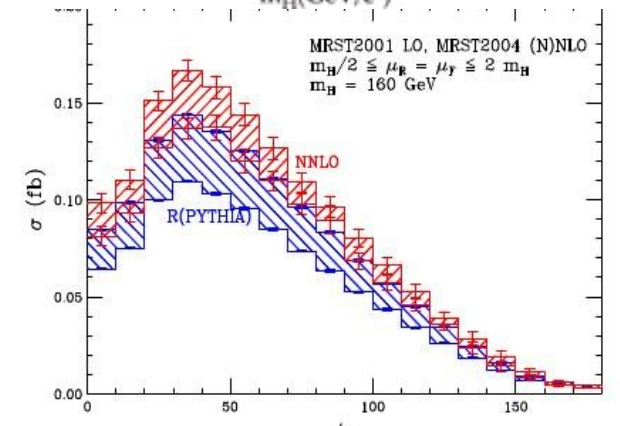
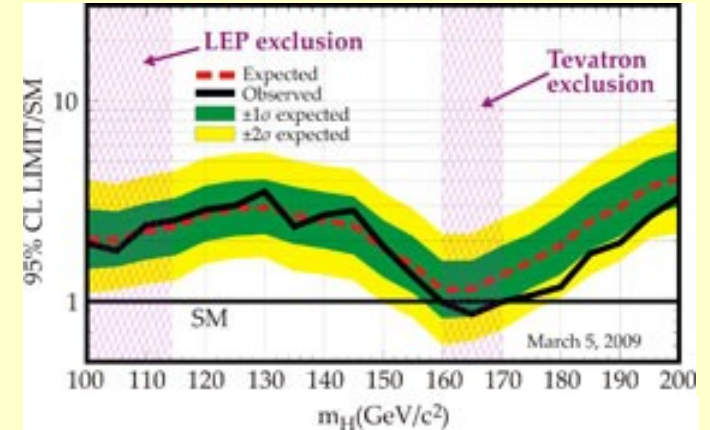


# Excluding the SM Higgs in $H \rightarrow W^+W^- \rightarrow l^+l^-\nu\bar{\nu}$

- CDF and D0 exclude the existence of the SM Higgs boson with the mass around 160 GeV.
- Good understanding of the Higgs signal is imperative. *But – done with PYTHIA!*
- Comparison of the NNLO computation with PYTHIA predictions shows *that PYTHIA acceptances are lower.*

$\sigma_{\text{acc}}/\sigma_{\text{incl}}$	Trigger	+ Jet-Veto	+ Isolation	All Cuts
NNLO ( $\mu = m_H/2$ )	44.7%	39.4% (88.1%)	36.8% (93.4%)	27.8% (75.5%)
NNLO ( $\mu = 2 m_H$ )	44.9%	41.8% (93.1%)	40.7% (97.4%)	31.0% (76.2%)
MC@NLO ( $\mu = m_H/2$ )	44.4%	38.1% (85.8%)	35.3% (92.5%)	26.5% (75.2%)
MC@NLO ( $\mu = 2 m_H$ )	44.8%	38.8% (86.7%)	35.9% (92.5%)	27.0% (75.2%)
HERWIG	46.7%	40.8% (87.4%)	37.8% (92.7%)	28.6% (75.7%)
PYTHIA	46.6%	37.9% (81.3%)	32.2% (85.0%)	24.4% (75.8%)

Anastasiou, Dissertori, Grazzini, Stoeckli, Webber



# NNLO: constraining BSM

- One can use limits on Higgs boson production cross-section to constrain physics beyond the Standard Model

- supersymmetry

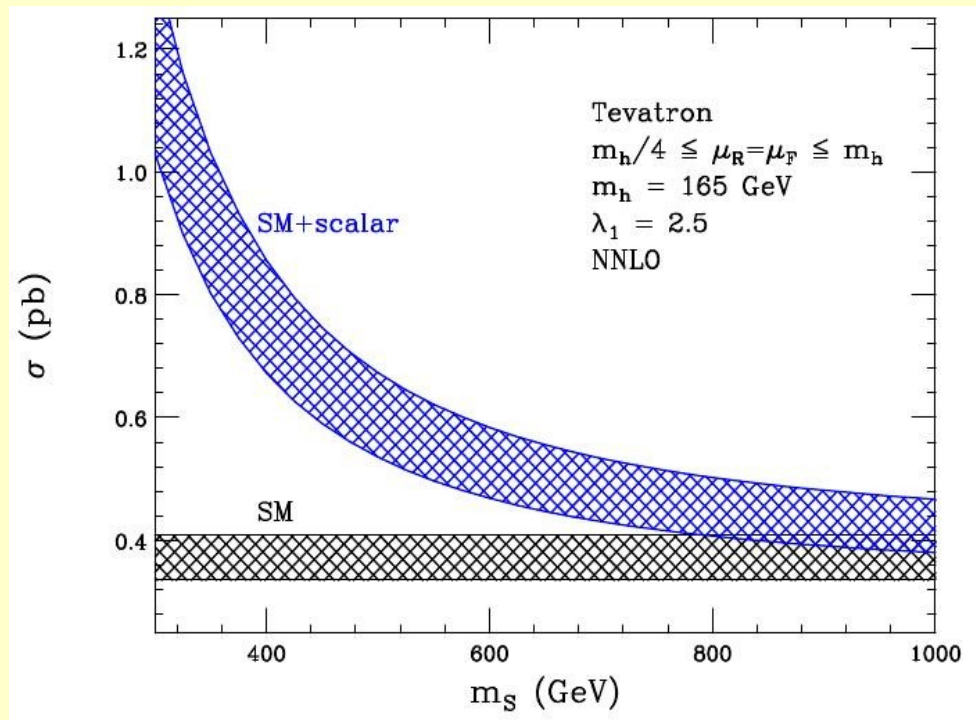
*Anastasiou, Beerli, Daleo, Muhlleitner, Rzehak, Spira*

- color octet scalars

*Boughezal, Petriello*

- additional heavy fermions

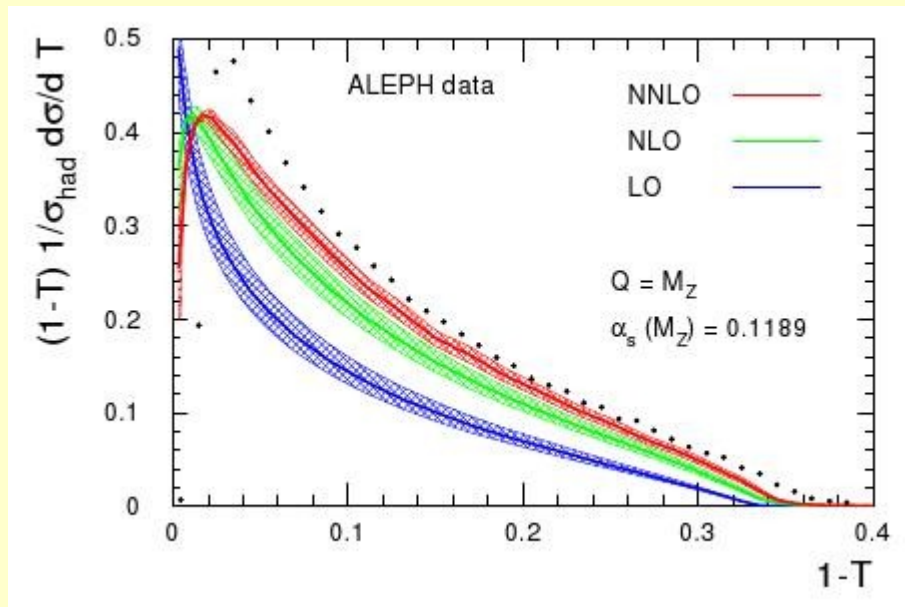
*Anastasiou, Boughezal, Furlan*



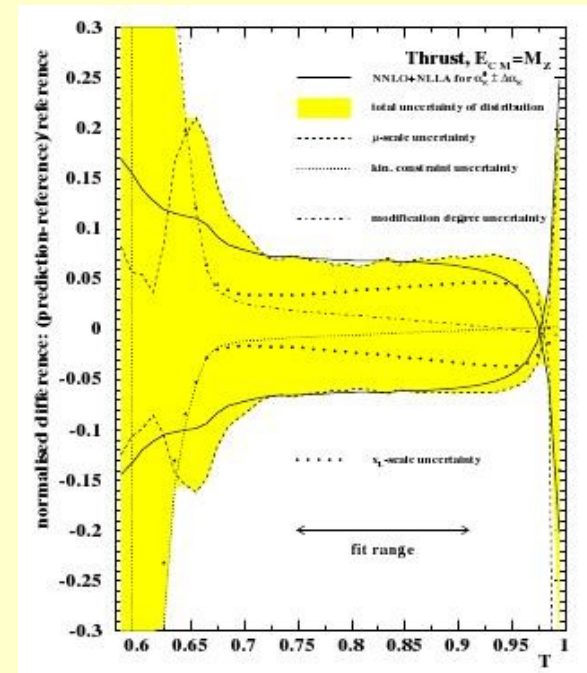
*Boughezal, Petriello*

# NNLO: the strong coupling constant

- Calculation of the NNLO QCD corrections to 3 jet production at LEP is one of the most heroic computations in high-energy physics
- Lead to active re-investigation of the value of the strong coupling constant



Gehrmann-De Ridder, Gehrmann, Glover, Heinrich



+ Dissertori, Luisoni

$$\alpha_s(M_Z) = 0.1224 \pm 0.0039$$



$$NNLO + SCET = \text{weak(er) coupling}$$

- Traditional determinations of the strong coupling constant from thrust distribution can be criticized on two occasions
  - resummations are limited to NLL
  - non-perturbative corrections are typically taken from parton showers
- SCET improves on both of these things –  $N^3LL$  resummations for thrust and self-consistent definition of non-perturbative soft function  
*Becher, Schwartz, Hoang, Stewart*
- Simultaneous fit for the strong coupling and the non-perturbative power corrections leads to a **smaller value** of  $\alpha_s(M_z)$

$$\alpha_s(M_z) = 0.1135 \pm 0.001$$

*Abbate, Fickinger, Hoang, Mateu, Stewart*

It seems that whenever the hadronization effects are fitted together with the strong coupling constant, the results for  $\alpha_s(M_z)$  are lower.



# NNLO for more difficult processes

- A NNLO computation to hadroproduction of  $N$  particles involves
  - Two-loop virtual corrections to  $2 \rightarrow N$  matrix element, integrated over  $N$ -particles phase-space;
  - One-loop virtual corrections to  $2 \rightarrow N+1$  matrix element, integrated over  $(N+1)$ -particles phase-space;
  - $2 \rightarrow N+2$  matrix element, integrated over  $(N+2)$ -particle phase-space.
- Each of these items lives in a different phase-space but, since they all diverge when integrated separately, they must be combined (and divergences extracted and canceled) before the integration.
- How this can be done efficiently is a matter of active research.

Note that a large number of two-loop amplitudes for  $2 \rightarrow 2$  scattering processes are known for almost ten years, already.

# NNLO: double real emission

- Two main lines of thought
  - subtraction techniques ( $e^+e^- \rightarrow 3j$ )
  - sector-decomposition for real emission ( $pp \rightarrow W,Z,H$ )
- The NLO analogs exist for both
  - subtraction techniques  $\rightarrow$  Catani-Seymour dipole formalism
  - sector-decomposition  $\rightarrow$  Frixione-Kunszt-Signer technique
- The FKS technique is the result of a simple observation
  - lets partition the phase-space for final state particles so that at any sector one definite particle can be soft or two definite particles can be collinear;
  - in all such sectors, optimal choices of variables and singularities of matrix elements are obvious.

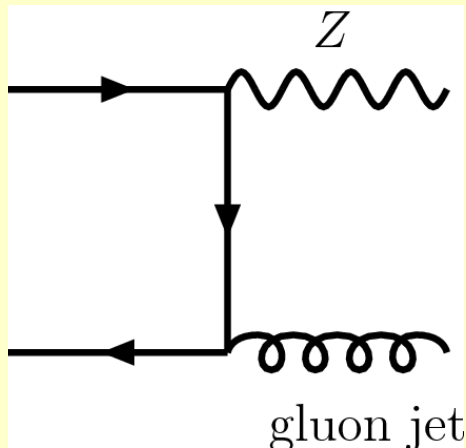
# NNLO: from sector decomposition to $CzFKS$

- Sector decomposition at NNLO attempts to construct global changes of variables, looking at various types of Feynman diagrams;
- This worked for  $pp \rightarrow W, Z, H$  ; would have trivially worked for  $pp \rightarrow t$  or  $pp \rightarrow tt$  but not for something more difficult
- However, it is clear that this limitation is not necessary and that partition of phase-space should exist such that for every sector one can clearly identify two or three partons that become unresolved
- It is harder to deal with such three unresolved partons; but since we have done  $pp \rightarrow W, Z, H$ , we know how to do this and what kind of sector decomposition needs to be employed.

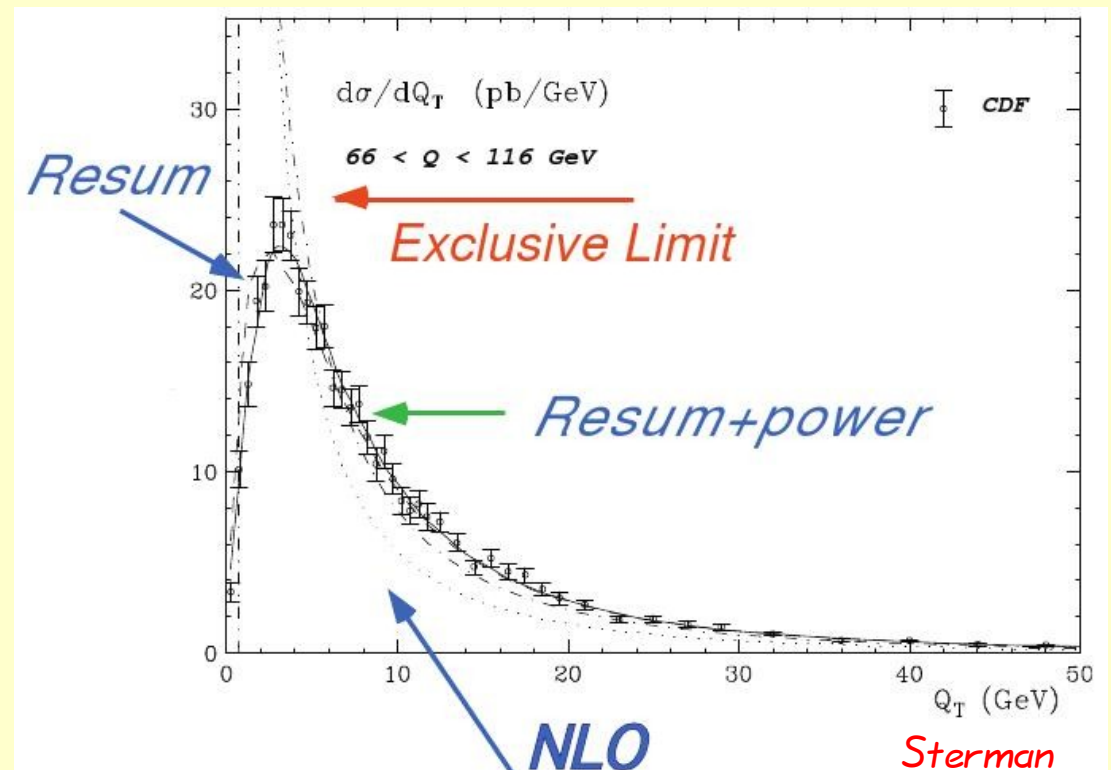
A combination of FKS ideas and sector decomposition is a very promising suggestion to develop generic NNLO “subtraction” technique

# Resummations

- One way to go beyond  $NLO/NNLO/\dots$  is to use resummed calculations. Classic example – transverse momentum distribution of  $Z$  or  $W$  boson



When the gluon “jet” is soft, we need to sum up multiple emissions to produce sensible results



# Resummations: EFT

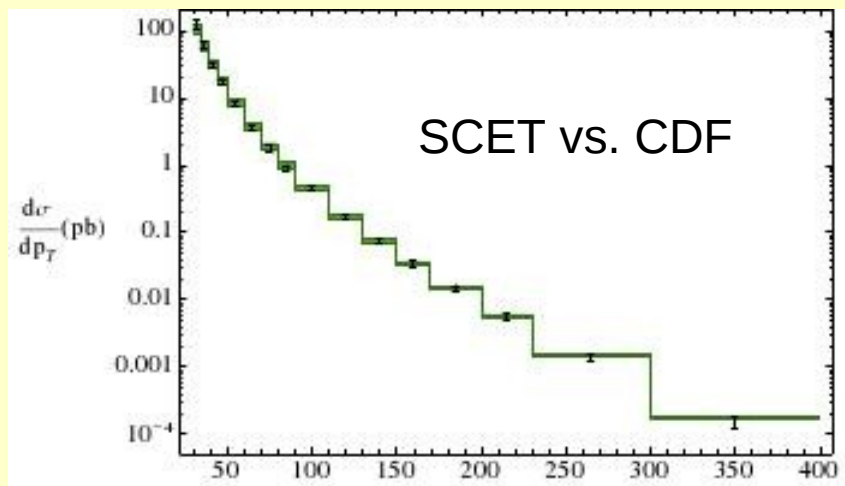
- General basis for resummations is the factorization formula

$$\mathcal{M} = H(\mu) \otimes J(\mu) \otimes S(\mu)$$

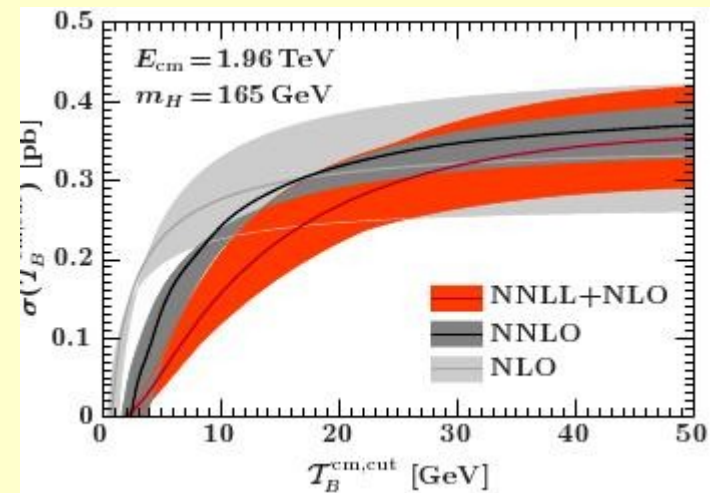
*Collins, Soper, Sterman*

- Effective field theory (SCET)  $\rightarrow$  factorization  $\rightarrow$  resummation

$$\mu \frac{dH}{d\mu} = \gamma \otimes H(\mu)$$



*Becher and Schwartz*



SCET vs. FEHIP

*Berger, Waalewijn, Marcatonini, Stewart, Tackmann*

# What is next?

- Further technical developments related to
  - one-loop computations ([MadGraph@NLO](#))
  - two-loop computations – complicated processes, fully differentially
- Realistic treatment of complicated background and signal processes (large number of jets, decays, spin-correlations, observables)
- Merging fixed order perturbative computations for differential jet production cross-sections with resummations, in a controlled fashion.
- Parton showers with quantum interferences
- Scale-setting prescriptions at NLO ([CKKW@NLO](#))



# Conclusion

- Discovering New Physics at the LHC is not possible without working theory of hadron collisions.
- “Practical” theory of hadron collisions is in good shape
- The theory of hadron collisions went through rapid development in the past ten years
  - new theoretical ideas
  - new computational techniques
  - better appreciation of what works and what does not
- Spectacular agreement with the Tevatron data over wide range of energies
- Every reason to believe that (practical) theory of hadron collisions -- as we have it now -- is up to the task that we face at the LHC