

Perturbative higher-order effects at work at the LHC - HO10

C E R N T H E O R Y I N S T I T U T E

June 21 - July 9

Topics will include:

- * Progress in, and prospects for, attacking the yet uncalculated processes considered priorities at the LHC, and revisiting the NLO priority list formulated at Les Houches in order to target the experimental accuracies.
- * The role of NLO and NNLO QCD calculations for Higgs boson searches and coupling measurements.
- * Integrating automated NLO calculations into parton showers.
- * The role of NLO electroweak corrections and merging of QCD and electroweak corrections for benchmark processes such as W and Z production.
- * The role of (semi-)analytical resummations at the LHC, and their comparisons with those provided by Monte Carlos.
- * Key signatures of new physics at the LHC and challenging discovery modes in exotic models.
- * Processes, such as top-quark pair and di-boson production, for which NNLO QCD is desirable.
- * The status of parton distribution function errors and prospects for their improvement with LHC data.

New techniques for NLO calculations

Zoltan Kunszt, ETH, Zurich

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"QCD applications and predictions"

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- Lattice QCD successfully describes the properties of low lying hadrons.
- QCD Improved Parton Model (PQCD) successfully describes hard scattering processes

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The validity of PQCD assumptions must be tested with data

**More than 30 years of studies with e+e- colliders, lepton-hadron and hadron-hadron
colliders have convinced us that QCD and the PQCD framework are reliable.**

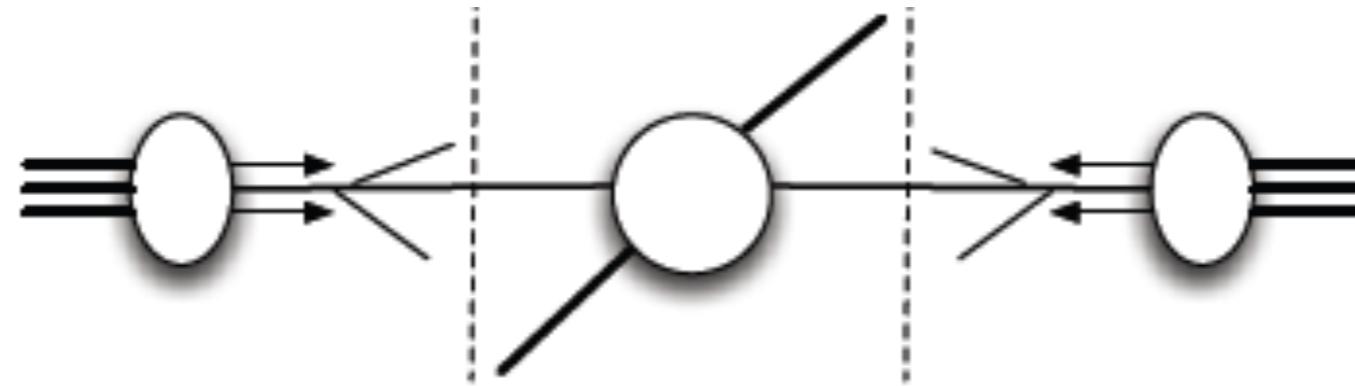
LHC: QCD plays a dominant role in all kind of interactions

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Hard scattering processes: beams of proton ~ beam of partons

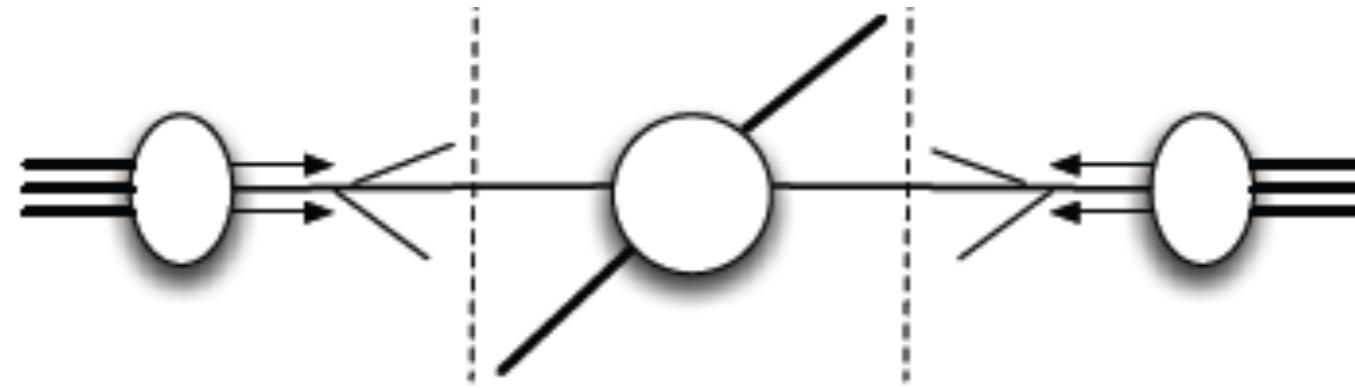
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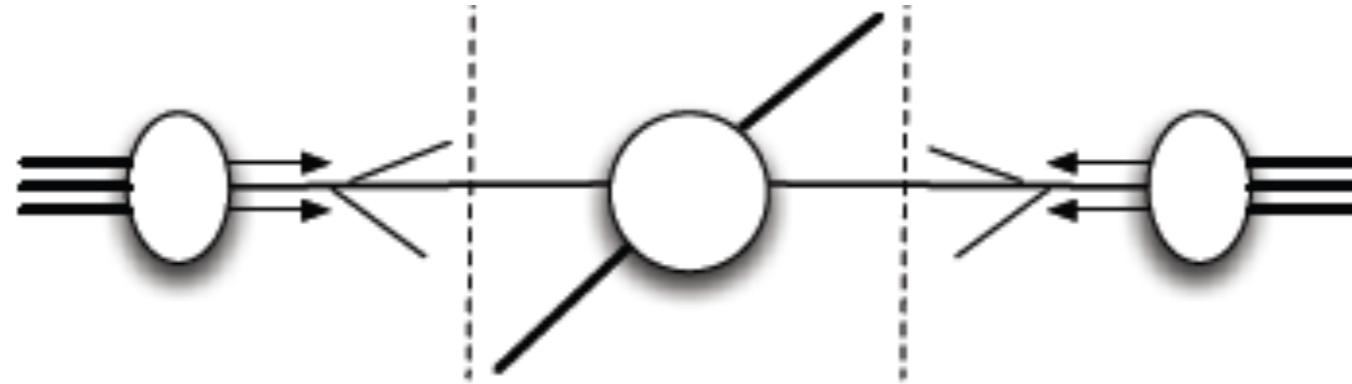
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Quantitative description in QCD with perturbation theory

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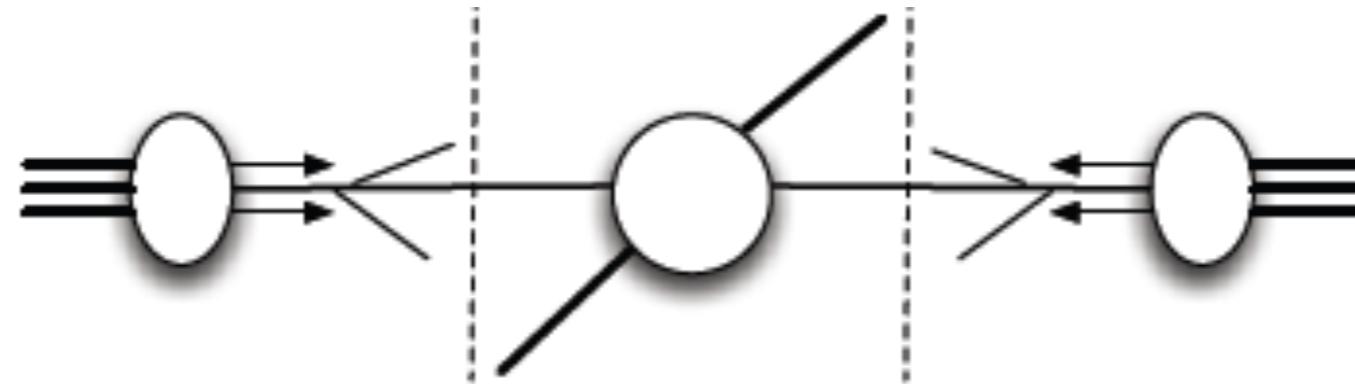


Quantitative description in QCD with perturbation theory

$$\frac{d\sigma_{\text{pp} \rightarrow \text{hadrons}}}{dX} = \sum_{a,b} \int dx_1 dx_2 f_a(x_1, \mu_F) f_b(x_2, \mu_F) \times \frac{d\hat{\sigma}_{ab \rightarrow \text{partons}}(\alpha_s(\mu_R), \mu_R, \mu_F)}{dX} + \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}^n}{Q^n}\right)$$

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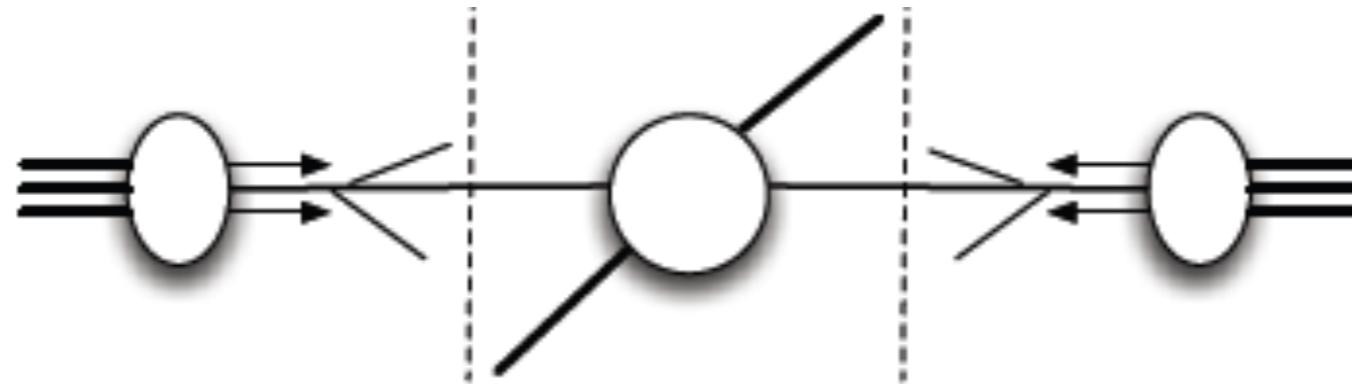
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parton number densities

A blue arrow points from the text "parton number densities" to the term $\int dx_1 dx_2 f_a(x_1, \mu_F) f_b(x_2, \mu_F)$ in the equation.

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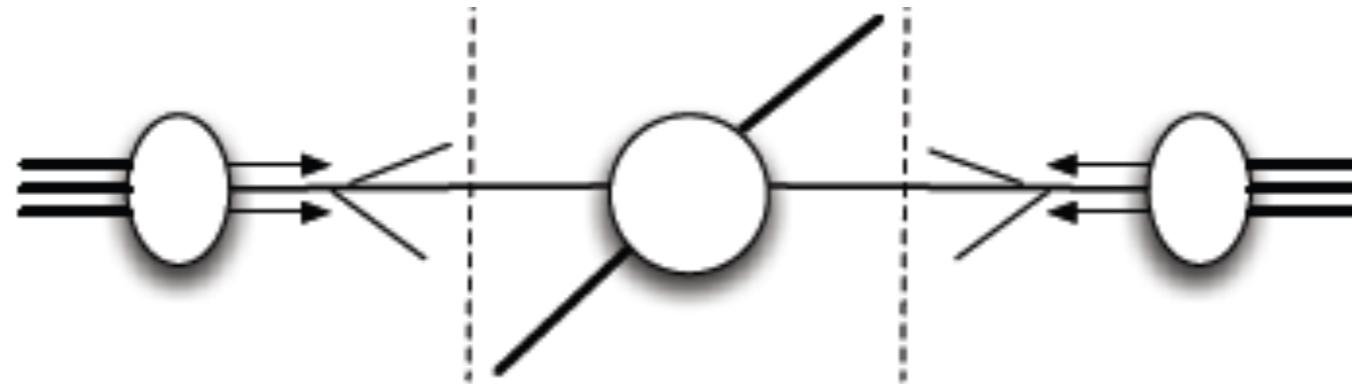
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FACTORIZATION

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Fully automated tree level cross section calculations for SM and BSM

ALPGEN, MADGRAPH, HELAC-PHEGAS , SHERPA, ComHep, COMIX, ...

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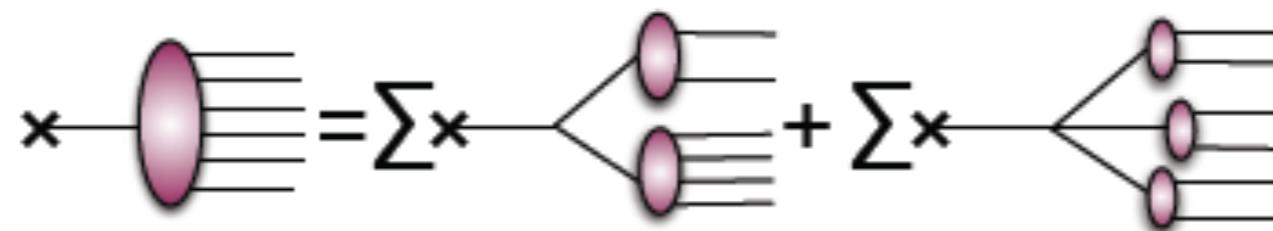
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Feynman diagrams or recursion relations (Berends-Giele)



Factorial vs. polynomial (exponential) growths of the evaluation time with the number of the external legs

**LO order matrix elements with shower MC is not enough?
Uncertainties from long distance effects ?**

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Calculation of the NLO corrections are justified

- if the ambiguities in the long distance part of the process are well controlled: jet definitions, parton number densities, fragmentation effects effects, threshold effects, power corrections
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Improved jet definition and recombination algorithms (M. Cacciari, G. Salam, G. Soyez)

- Fast recombination technique: FastJET
- Cone algorithms: IR-safe to all order SISCone
- Cone-formed jets: anti k_T algorithm

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Improved treatment of threshold effects and soft resummation

- Higgs-production, $e^+ + e^-$ annihilation (Catani et.al)
- Recent progress for top pair production (NLO+NNLO)
Moch, Uwer; Kidonakis: dominant contribution
Czakon, Mitov, Sterman: full contribution
Ahrens, et.al.: SCET

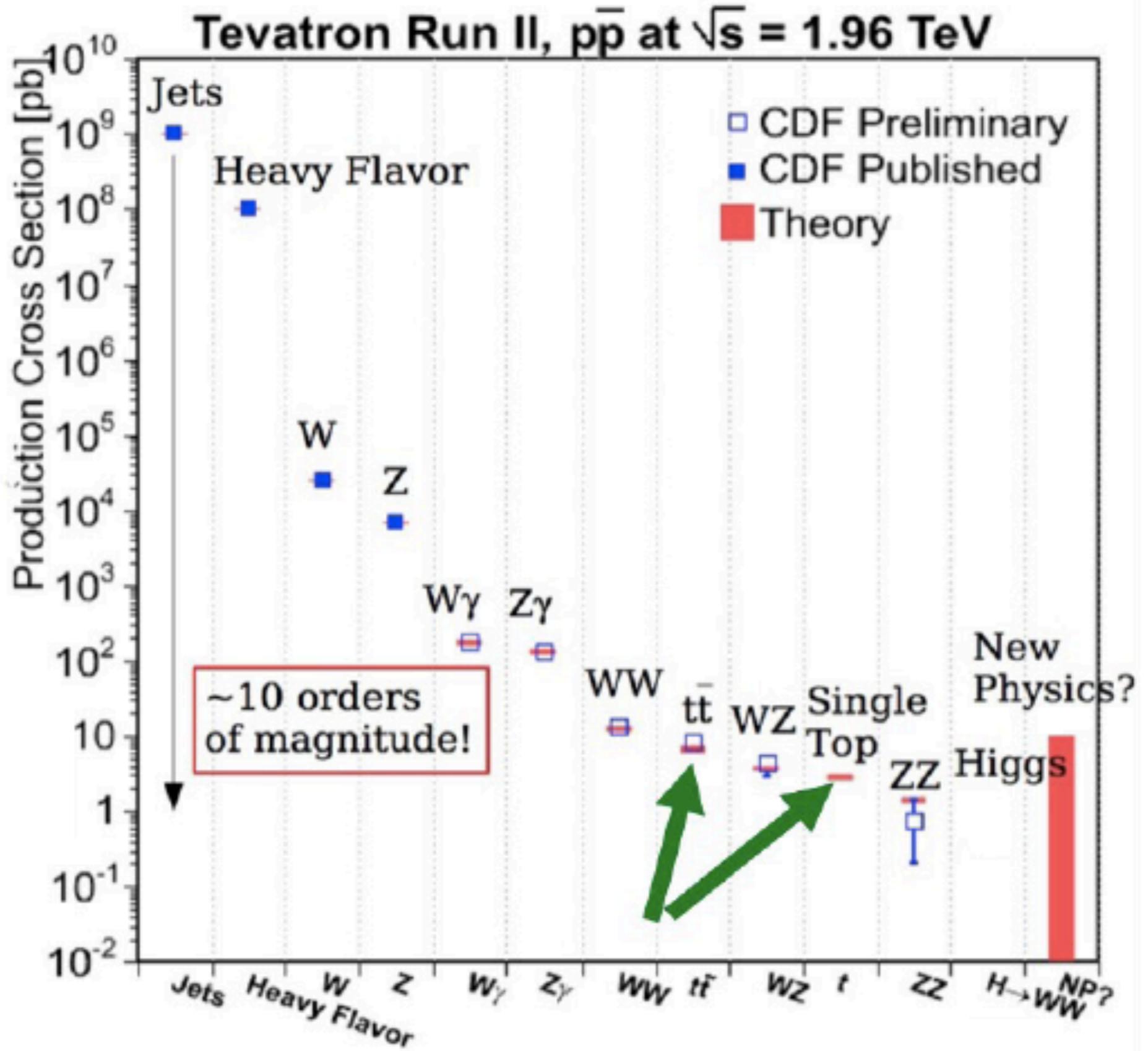
Evidence for the need of NLO from Tevatron

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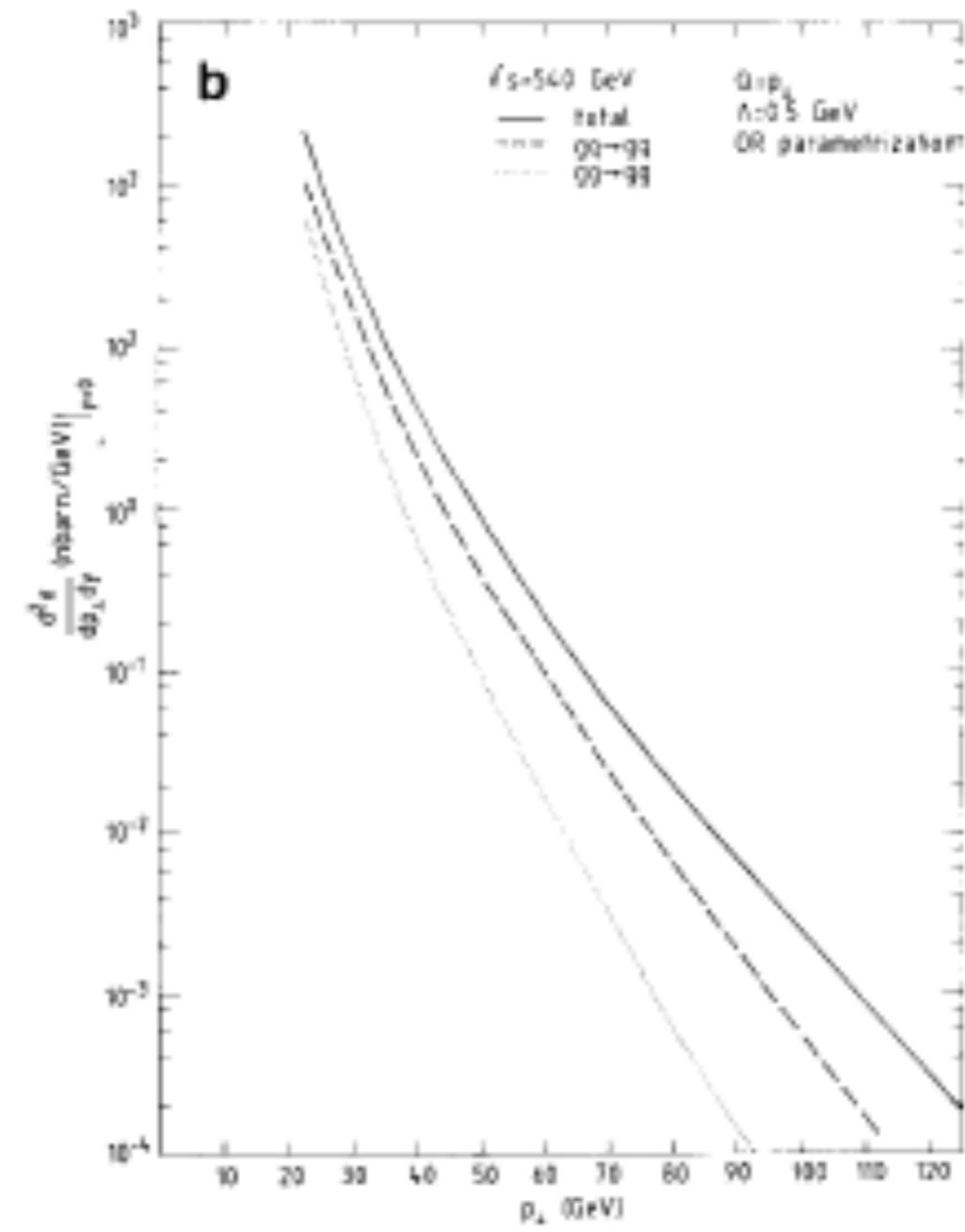
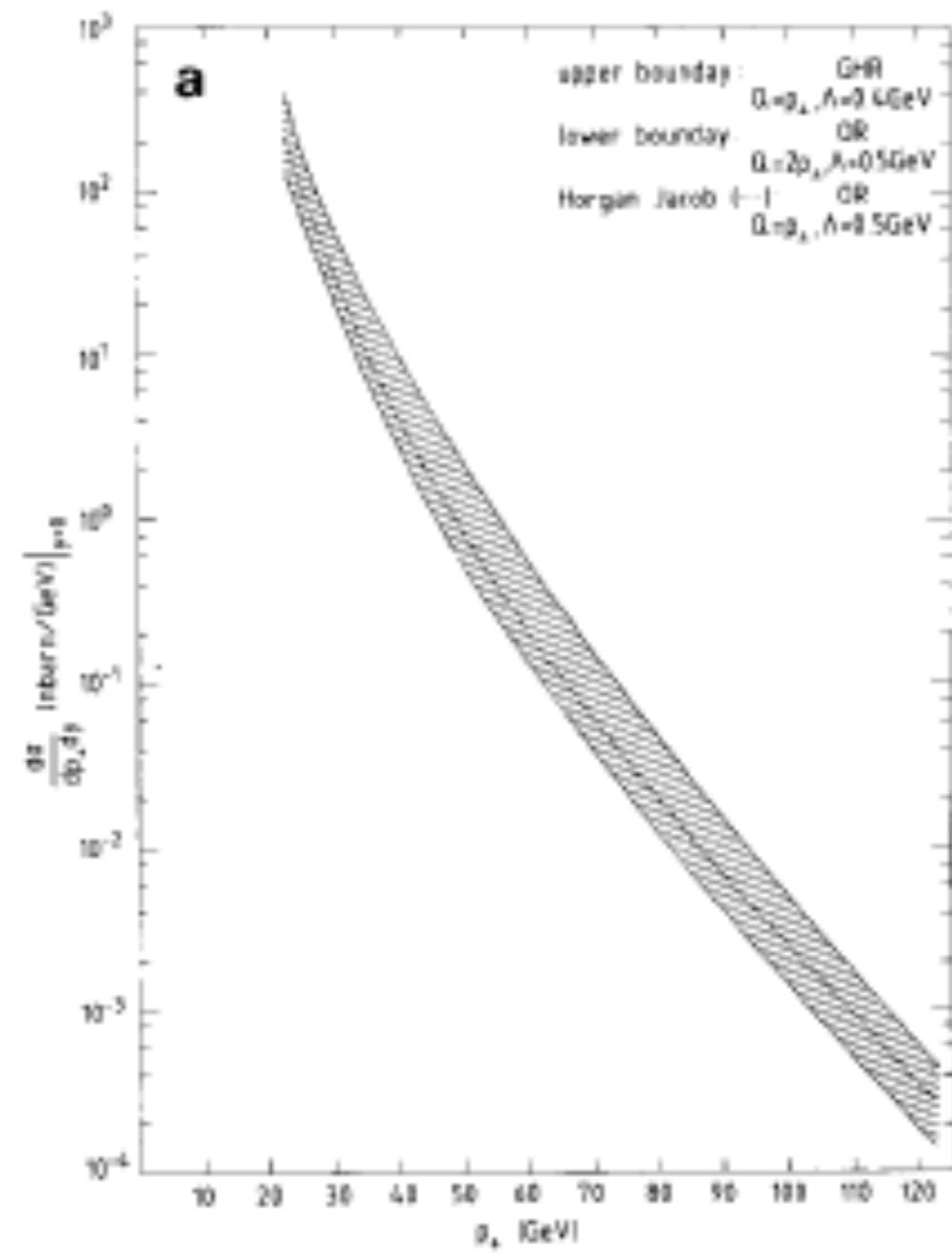
Jet production at NLO

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Volume 132B, number 4,5,6

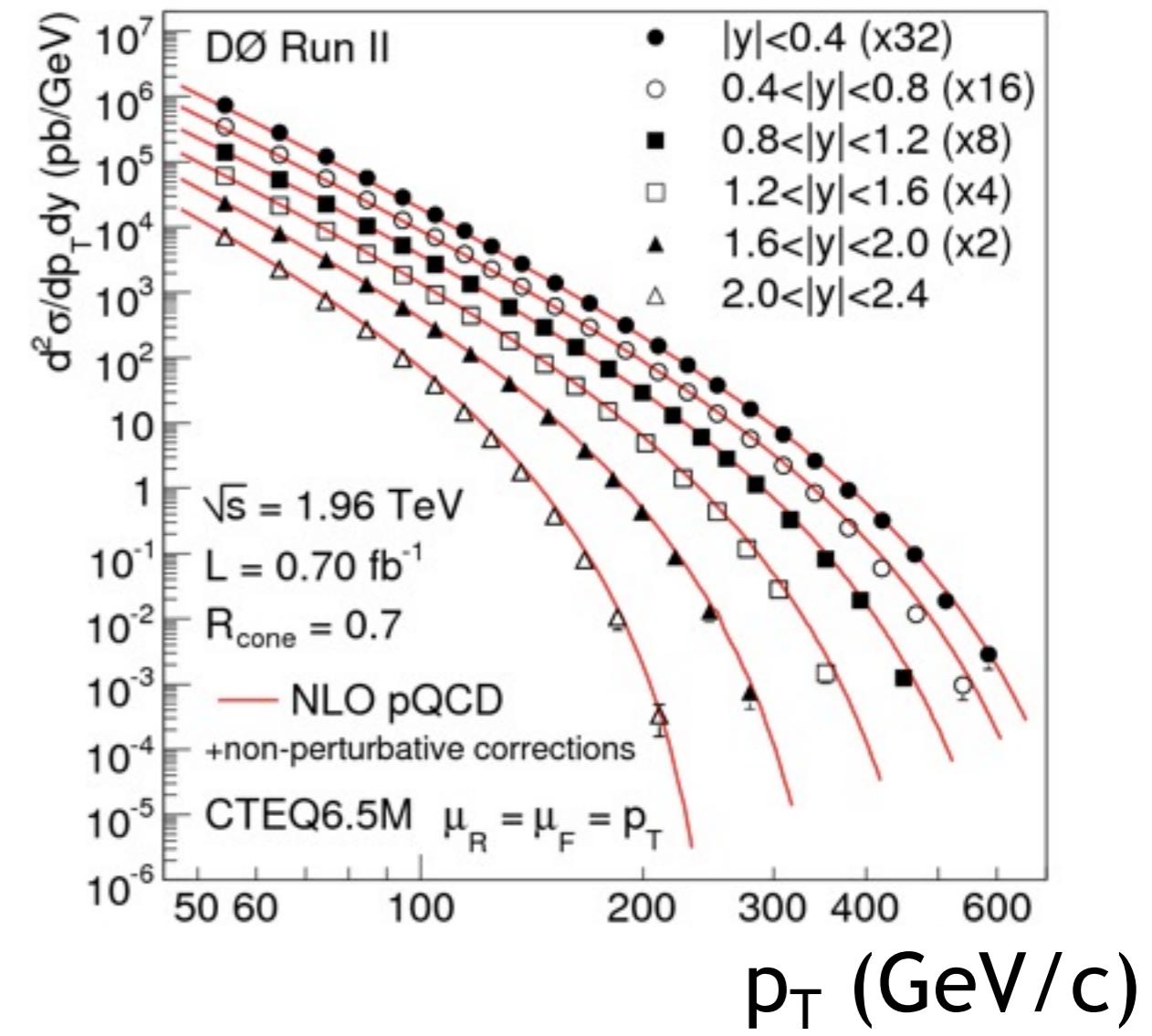
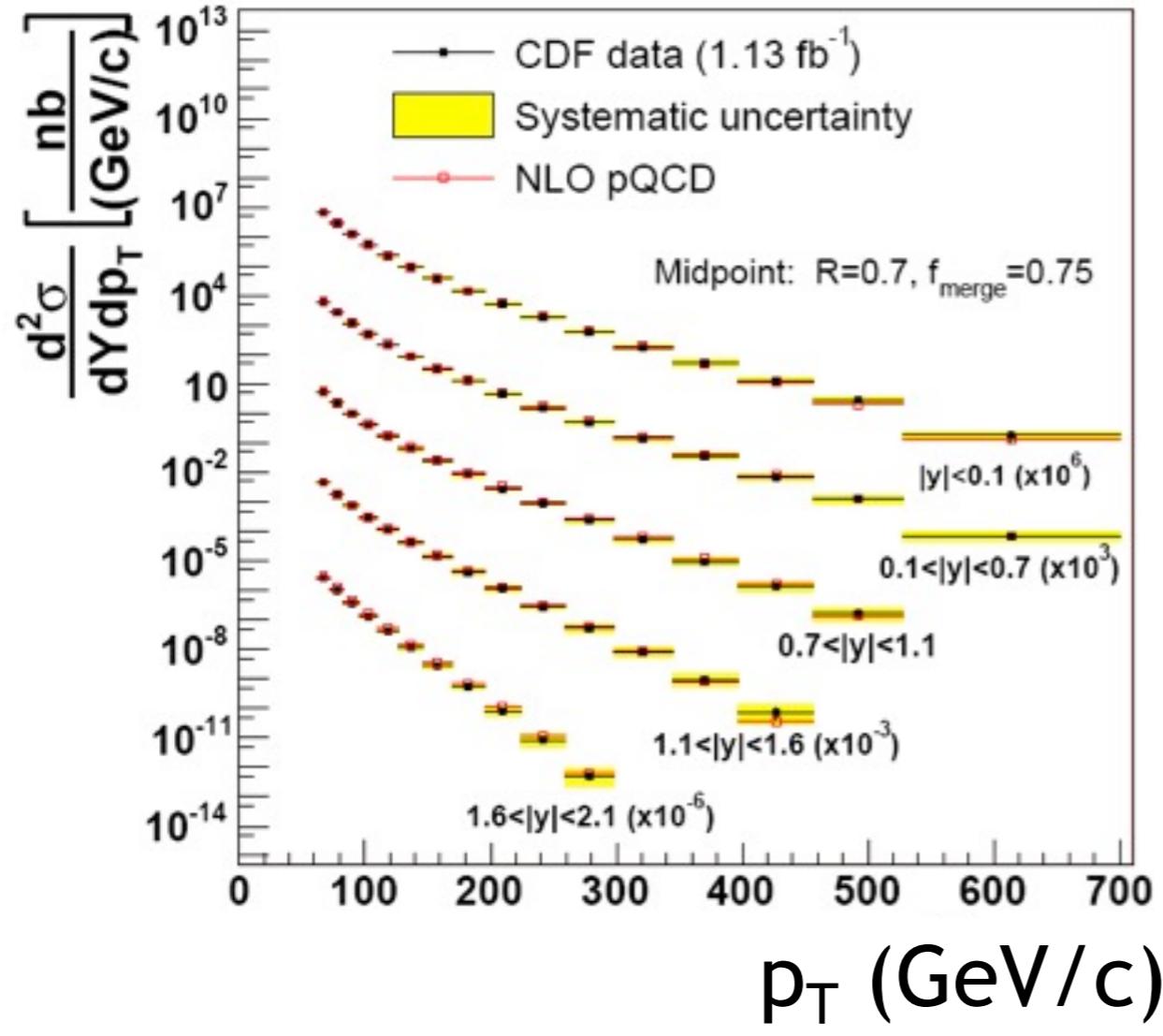
PHYSICS LETTERS

1 December 1983

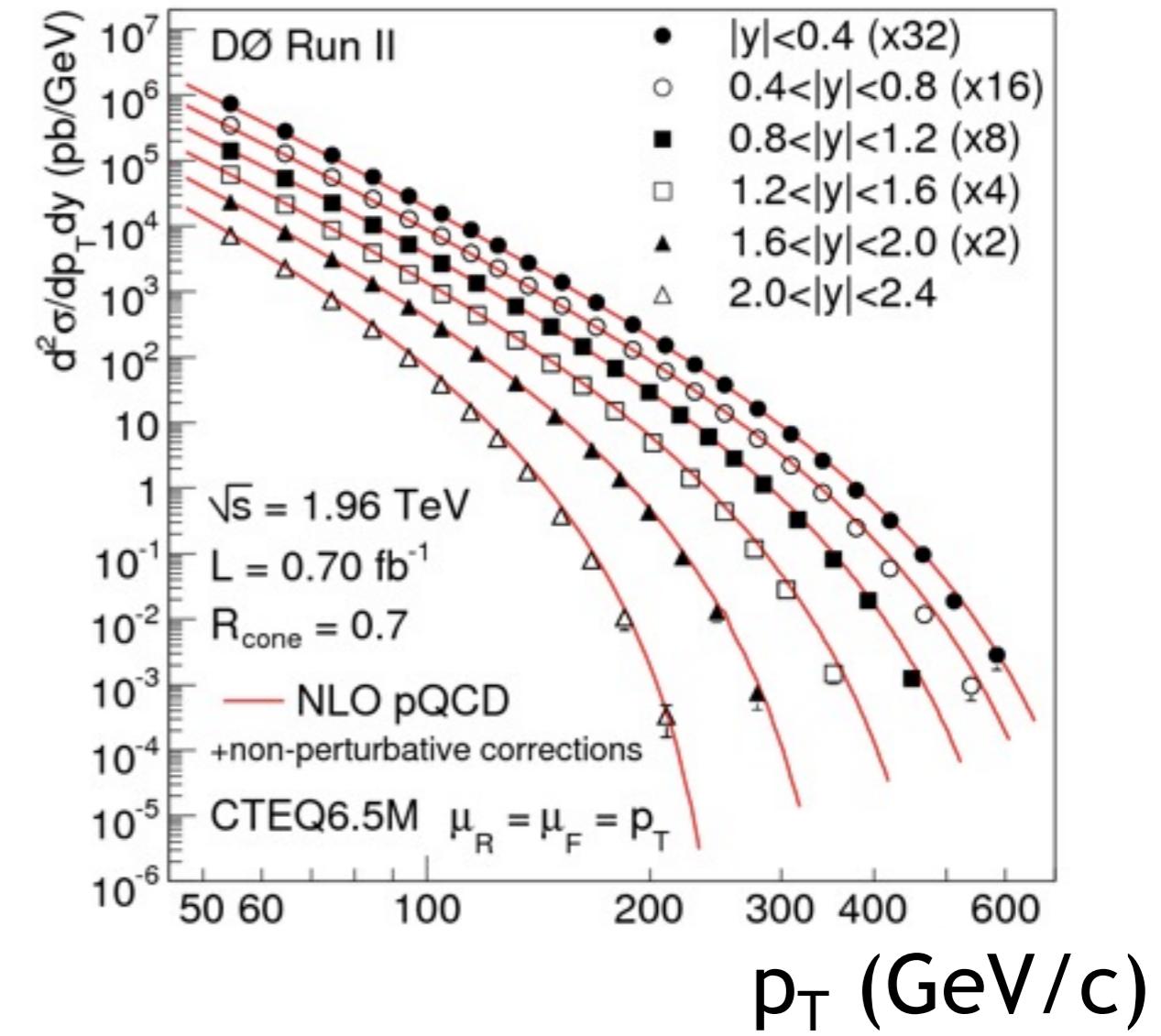
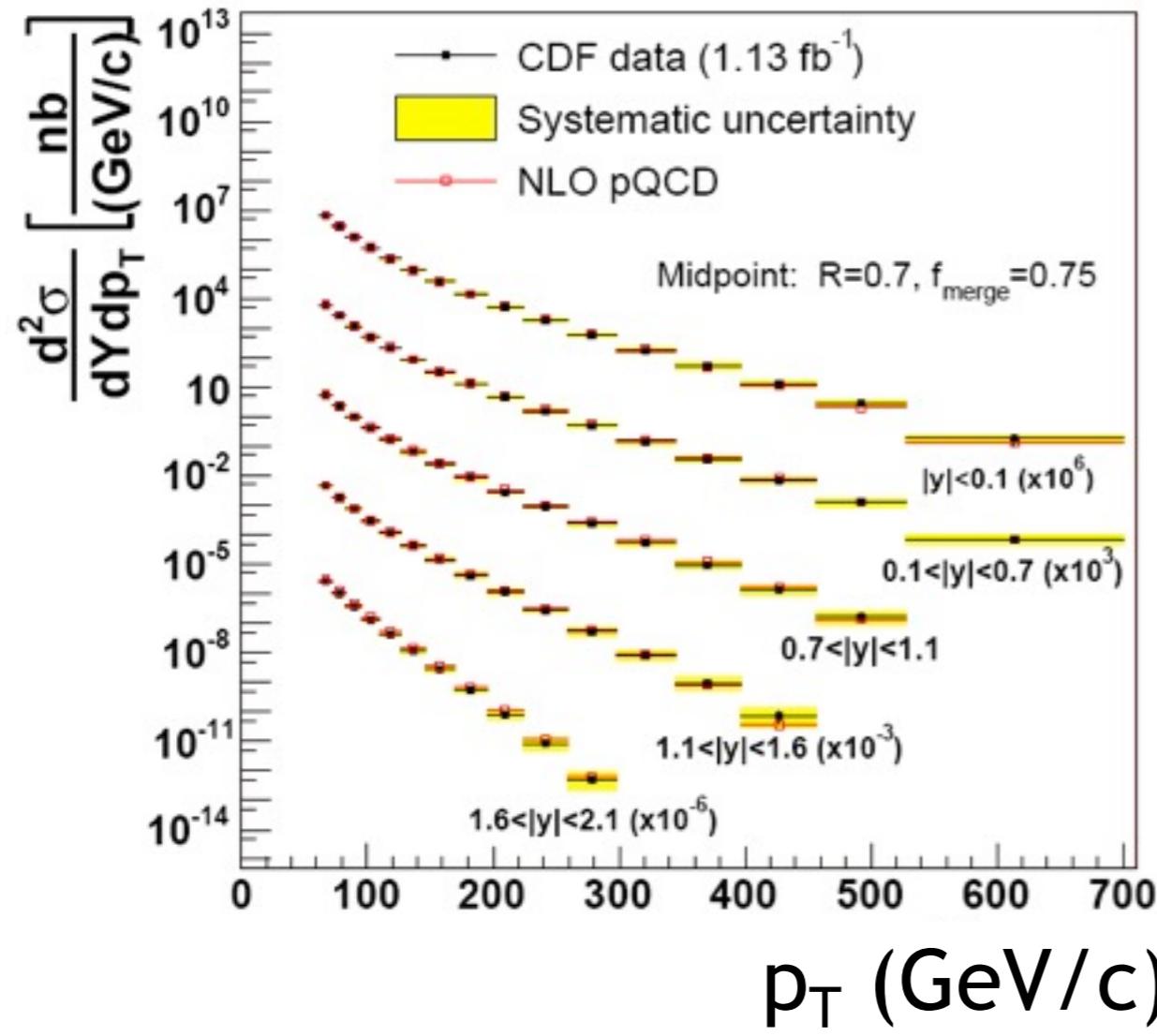


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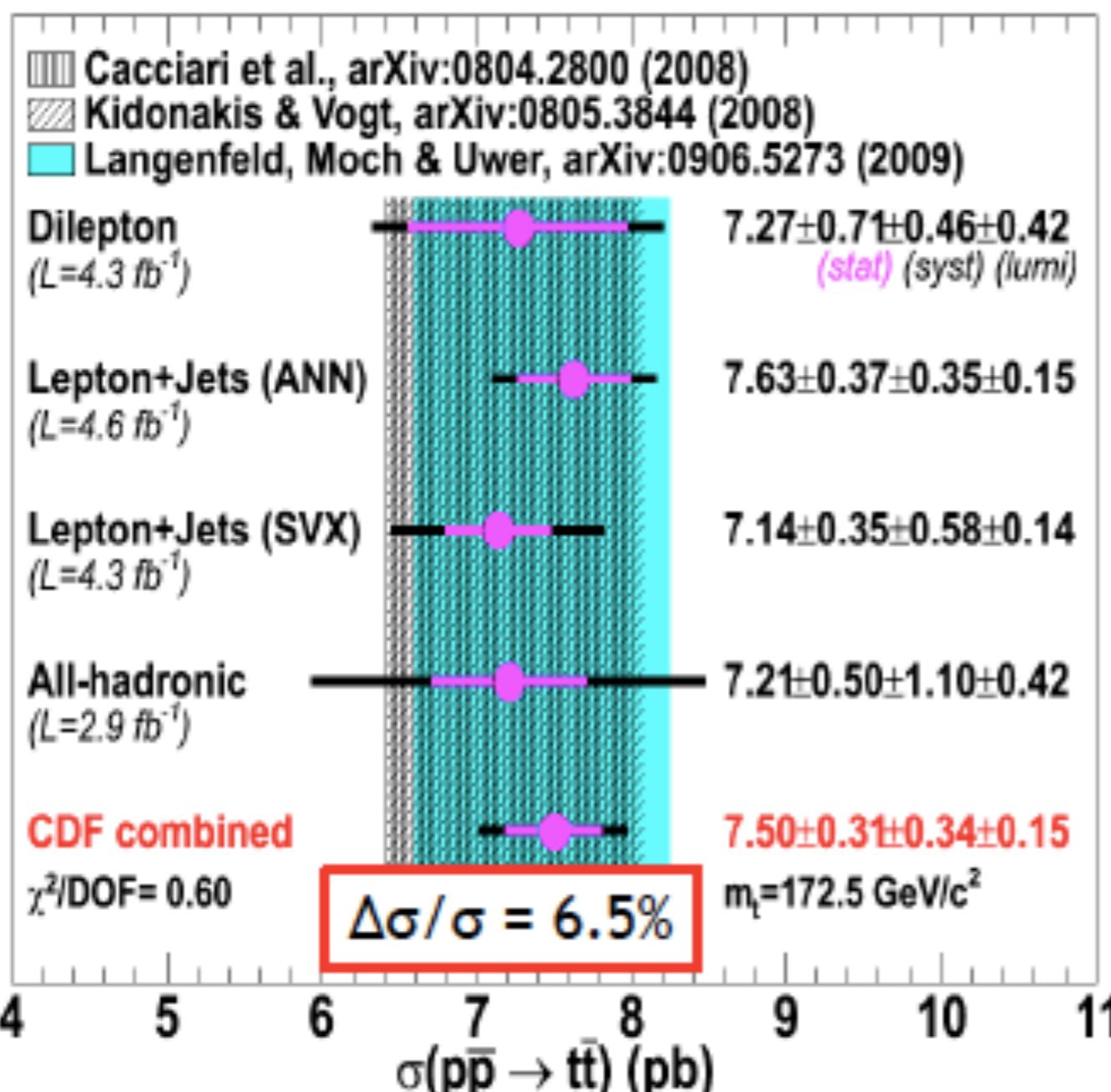


- Measurements span over 8 order of magnitude in $d\sigma^2/dp_T dy$
- Highest $p_T^{\text{jet}} > 600 \text{ GeV}/c$

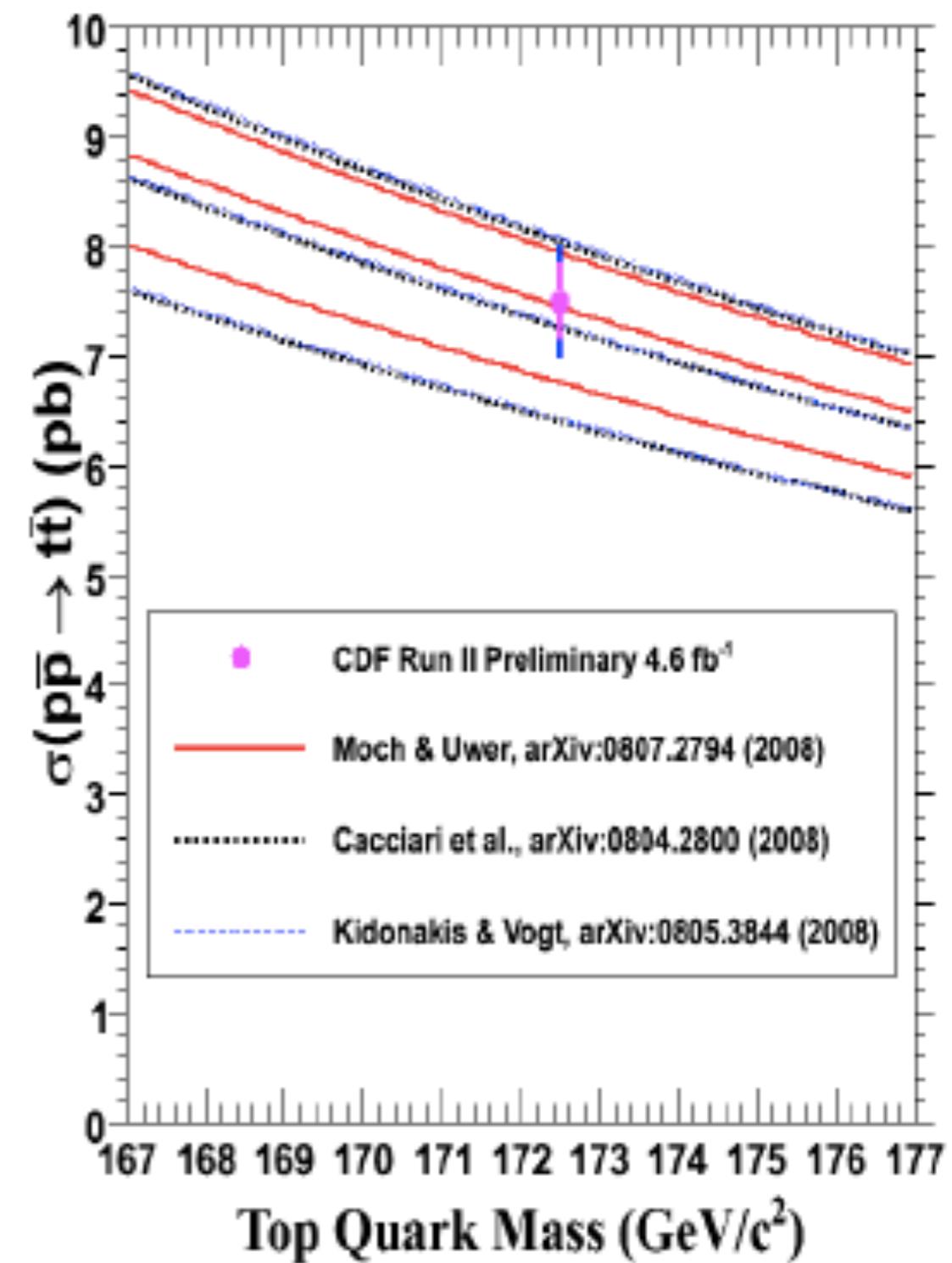
Top production at NLO

Good agreement:

- among different channels
- with theoretical prediction



*(not yet updated with the latest results)



Gauge boson pair production ...

12



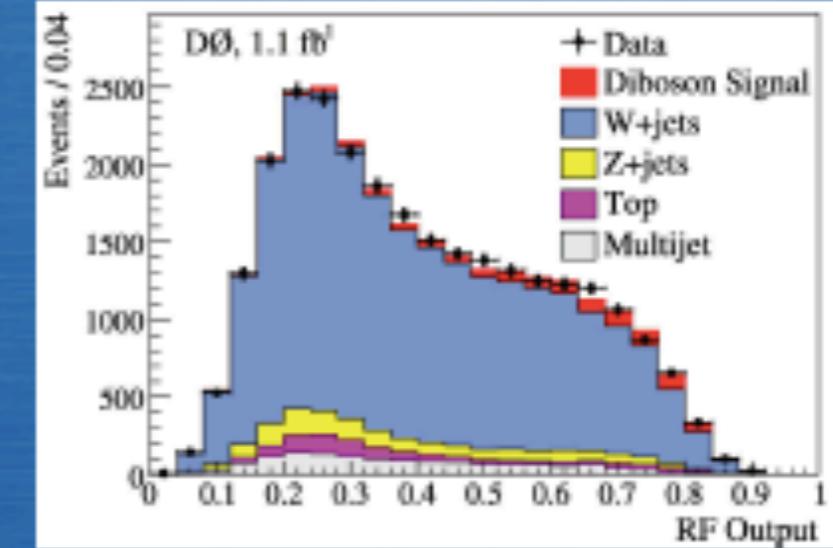
$$WW + WZ \rightarrow l\nu + jj$$

- Similar to $WH \rightarrow l\nu + jj$
- Same Analysis Technique
 - Validate multivariate technique to extract small signal in large background

In 1.1 fb^{-1} :

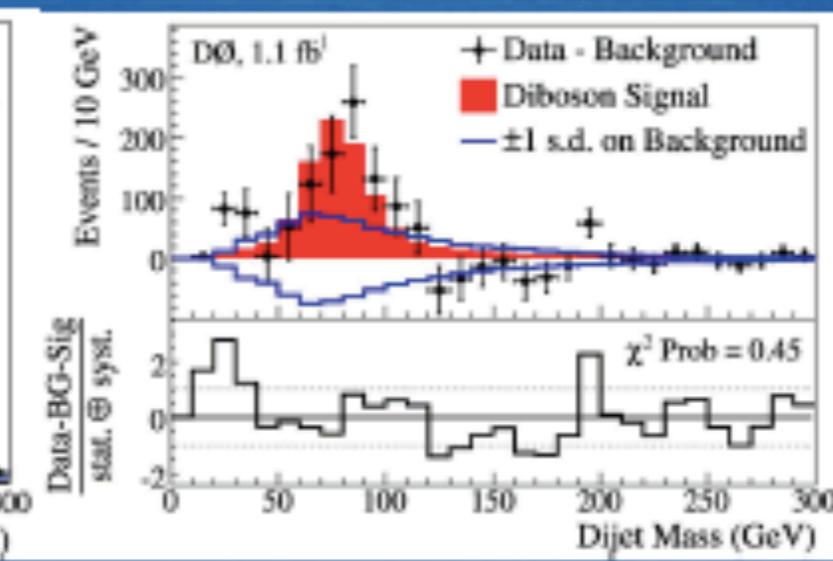
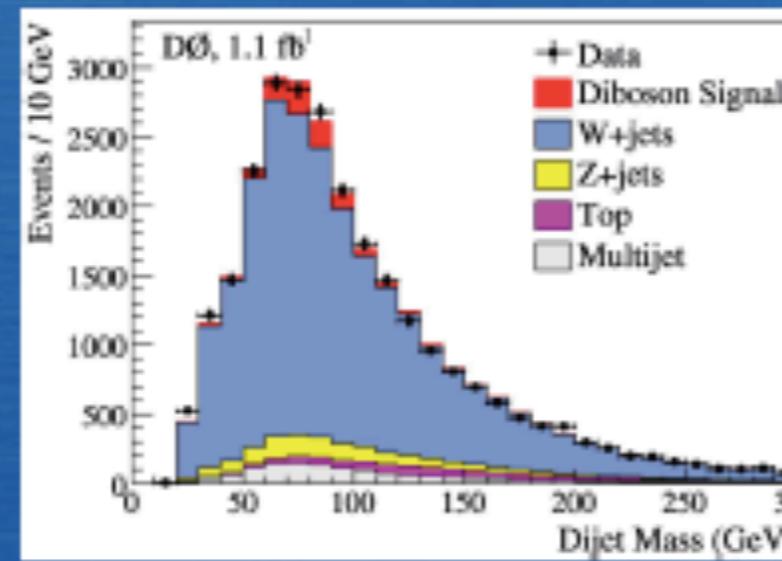
$$\sigma(pp \rightarrow WW + WZ) = \\ 20.2 \pm 2.5(\text{stat}) \pm 3.6(\text{syst}) \pm 1.2(\text{lum}) \text{ pb}$$

NLO Theory = $16.1 \pm 0.9 \text{ pb}$



Signal Significance 4.4σ
First Evidence

PRL 102, 161801 (2009)



Cross section calculations at NLO

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$$d\sigma_n^{(1)} \approx |M_n^{(0)}|^2 d\Phi_{n-2} + 2\text{Re}(M_n^{(0)\dagger} M_n^{(1)}) d\Phi_{n-2} + |M_{n+1}^{(0)}|^2 d\Phi_{n-1}$$

Cross section calculations at NLO

tree amplitude squared

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tree amplitude squared
virtual contributions
subtracted tree amplitude squared

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MOST PROMISING: algorithms based on d-dimensional unitarity
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Cross section calculations at NLO

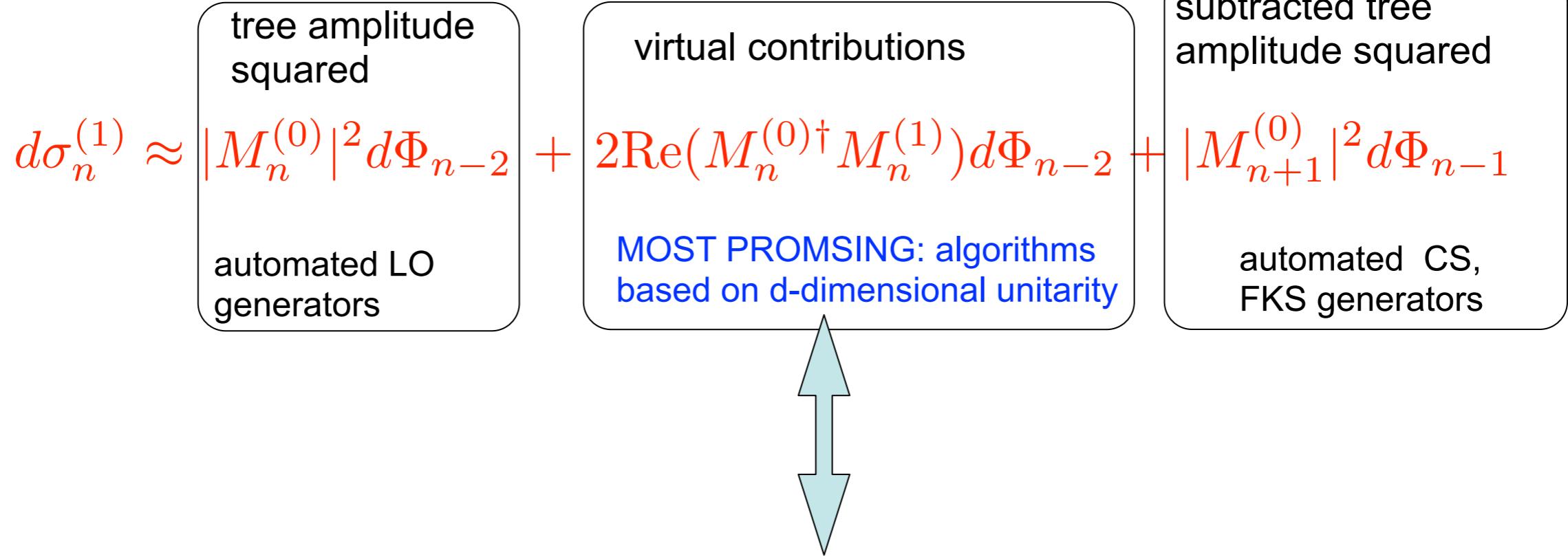
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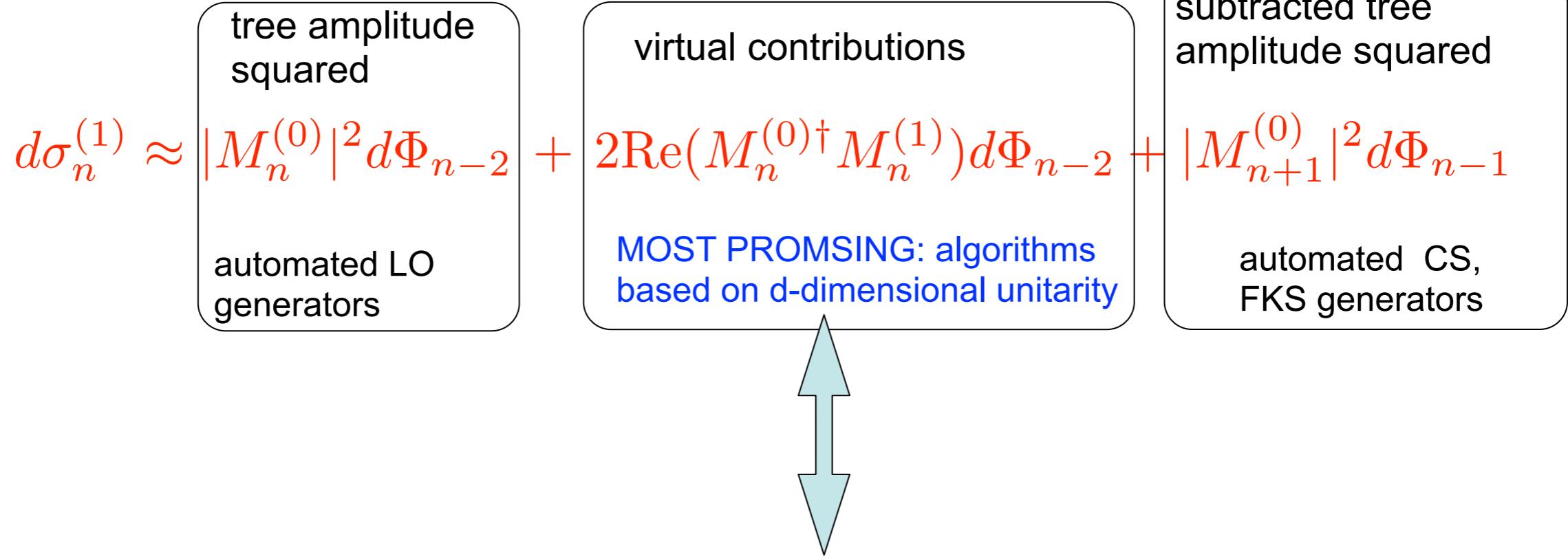
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3. Number of cuts grows with the number of the external legs (n) much slower than the number of the Feynman diagrams

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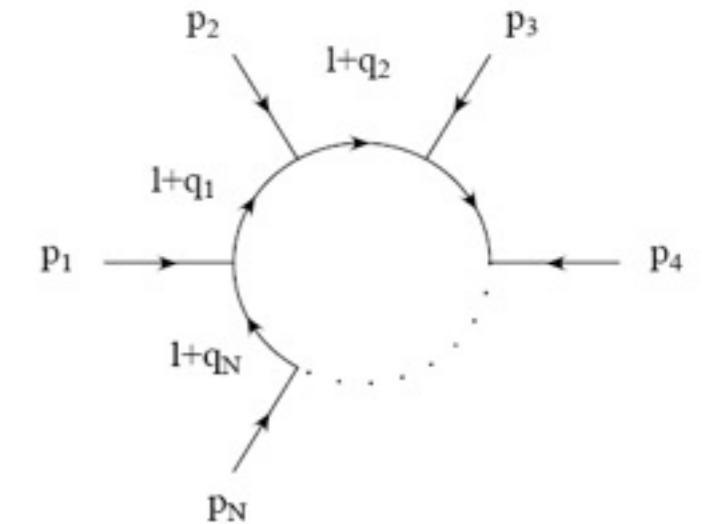
Automatic calculation up to $n=8\dots 10\dots$ appears to be feasible

Color summing(unordering)
leads to exponential law

Scaling of the computer time with the number of the external legs

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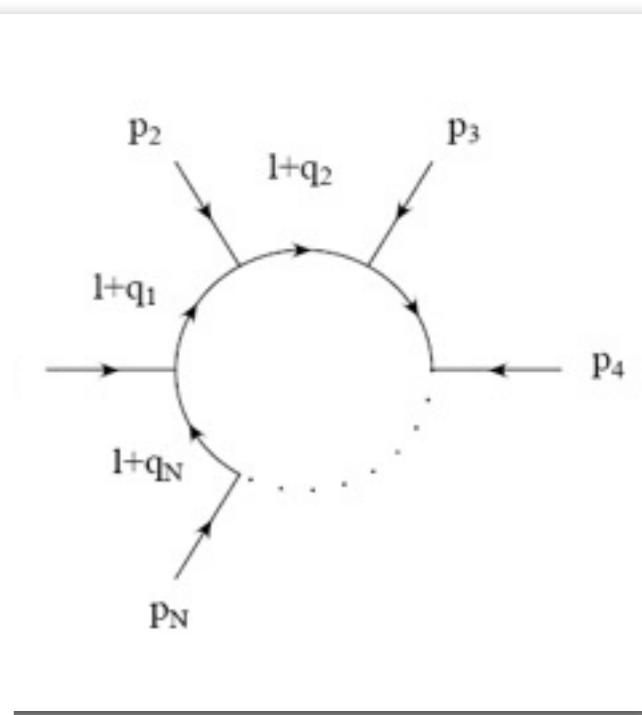
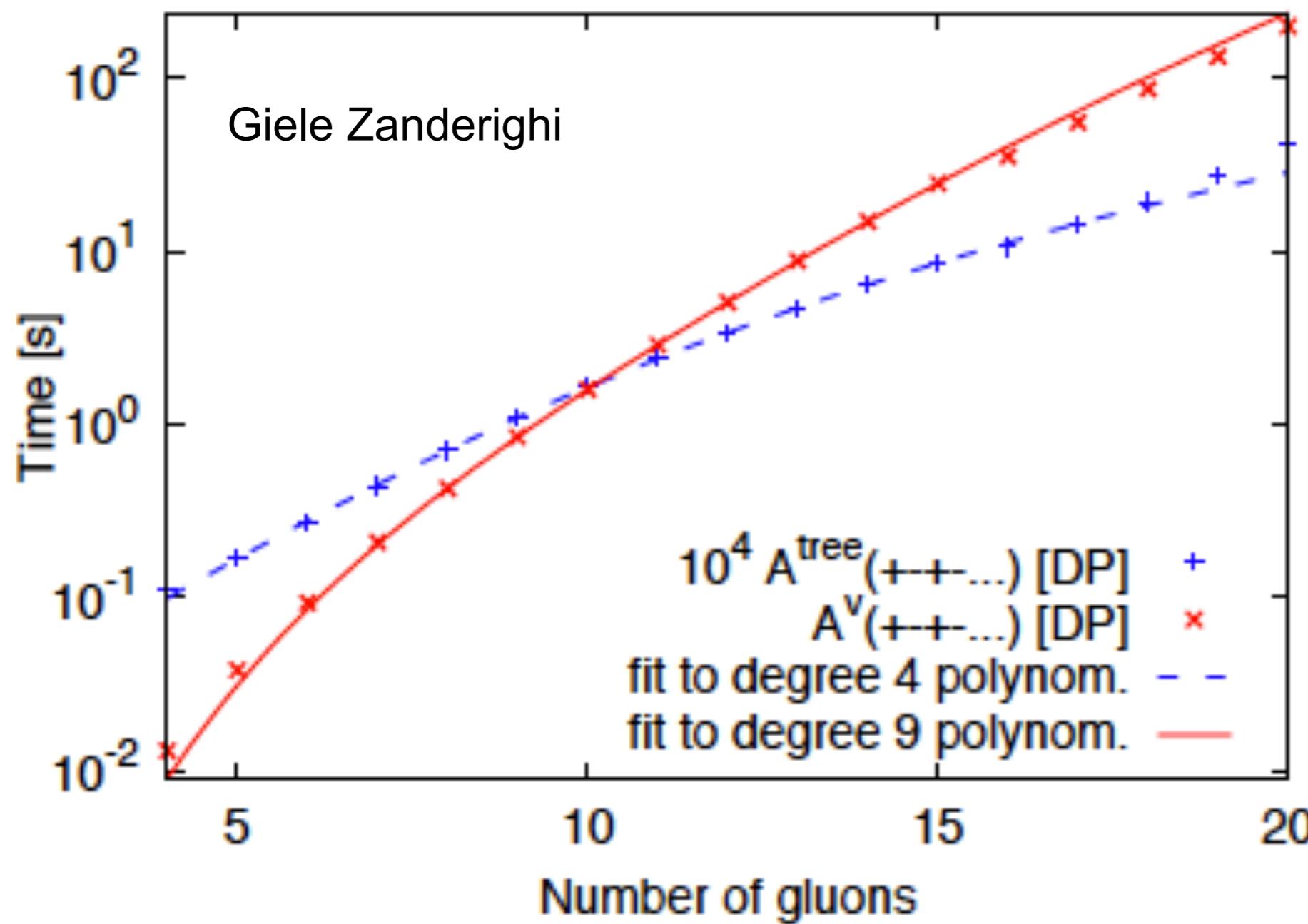


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Scaling of the computer time with the number of the external legs

CPU time: N^9 , 7^N , $(N!!)^2$

(Color) ordered amplitudes with n-gluons



Color summing(unordering)
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Vast number of results, very impressive progress

Results in MCFM

Standard Model processes involving
vector boson+jets, top quarks, Higgs

NLOJET++ (Nagy, Trocsanyi)

Results in MCFM

Standard Model processes involving
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W/Z; diboson, Wbb, Zbb, W/Z+1 jet, W/Z+2 jets, Wc, Zb, Zb + jet
H, H+1 jet, H+ 2jets (gluon fusion); H via WBF; Hb, WH,ZH
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MCFM: <http://mcfm.fnal.gov> (v5.8, April,2010)

J.M. Campbell, R.K. Ellis (main authors); Frederix, Maltoni, Tramontano

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Latest addition: top production with spin correlation
Higgs + 2 jet production

NLOJET++ (Nagy, Trocsanyi)

Results for physical cross-sections

- * NLO W+3jes cross section, leading color

Ellis, Melnikov, Zanderighi **hep-ph/0901.4101**, BlackHat+Sherpa(Gleisberg) **arXiv:0902.1835**, |

- * NLO W+3jets cross section, full result

BlackHat+Sherpa **arXiv:0907.1984**

- * NLO t+tbar, NLO t+tbar + 1 jet cross section with decay and full spin correlation

Melnikov, Schultze , **arXiv:0907.3090**

- * NLO Z/γ +3jets at the Tevatron **arXiv:1004.1659**

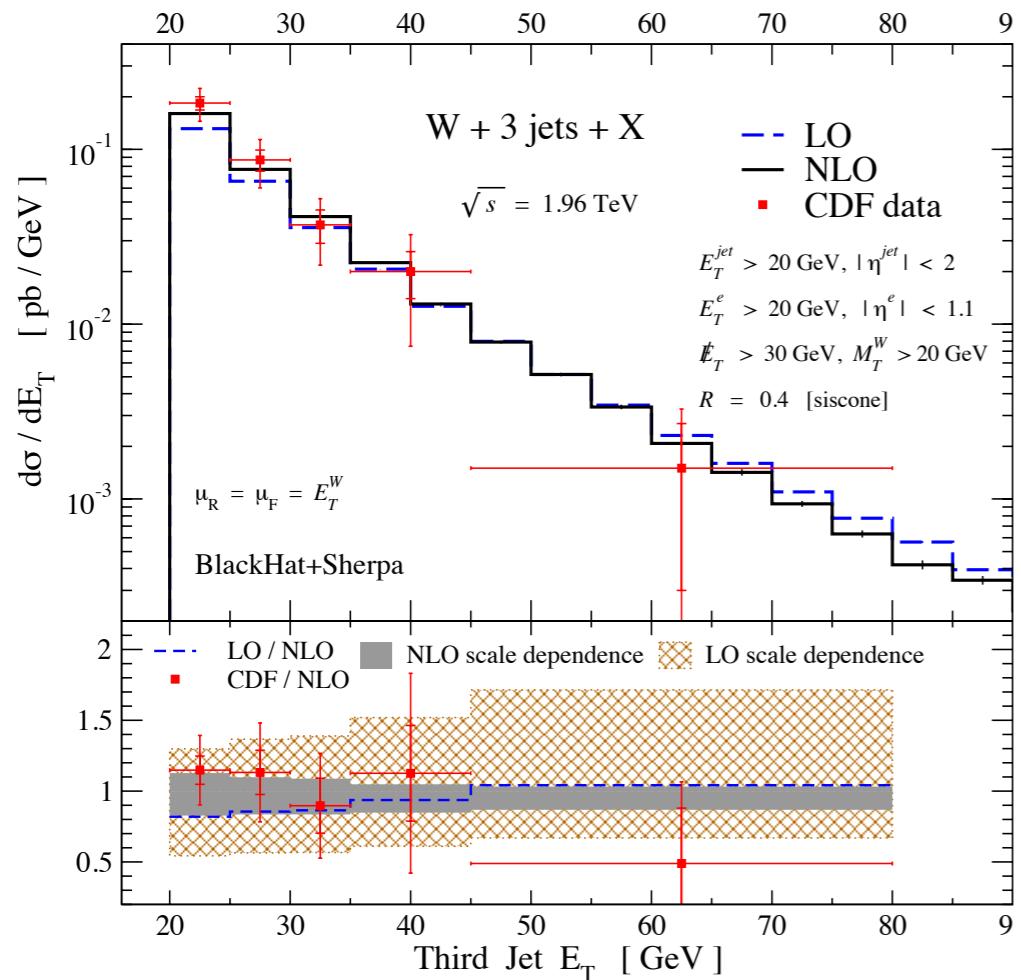
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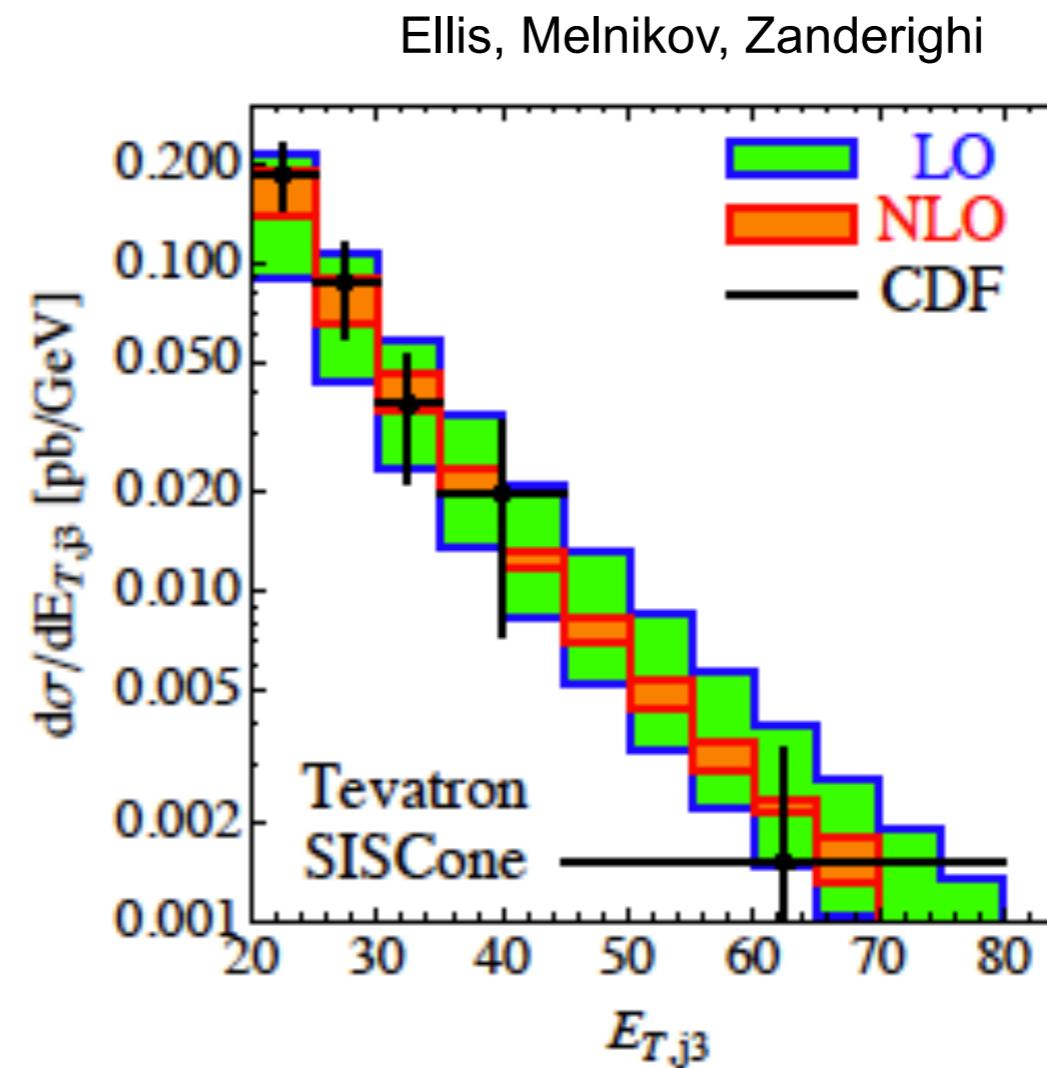
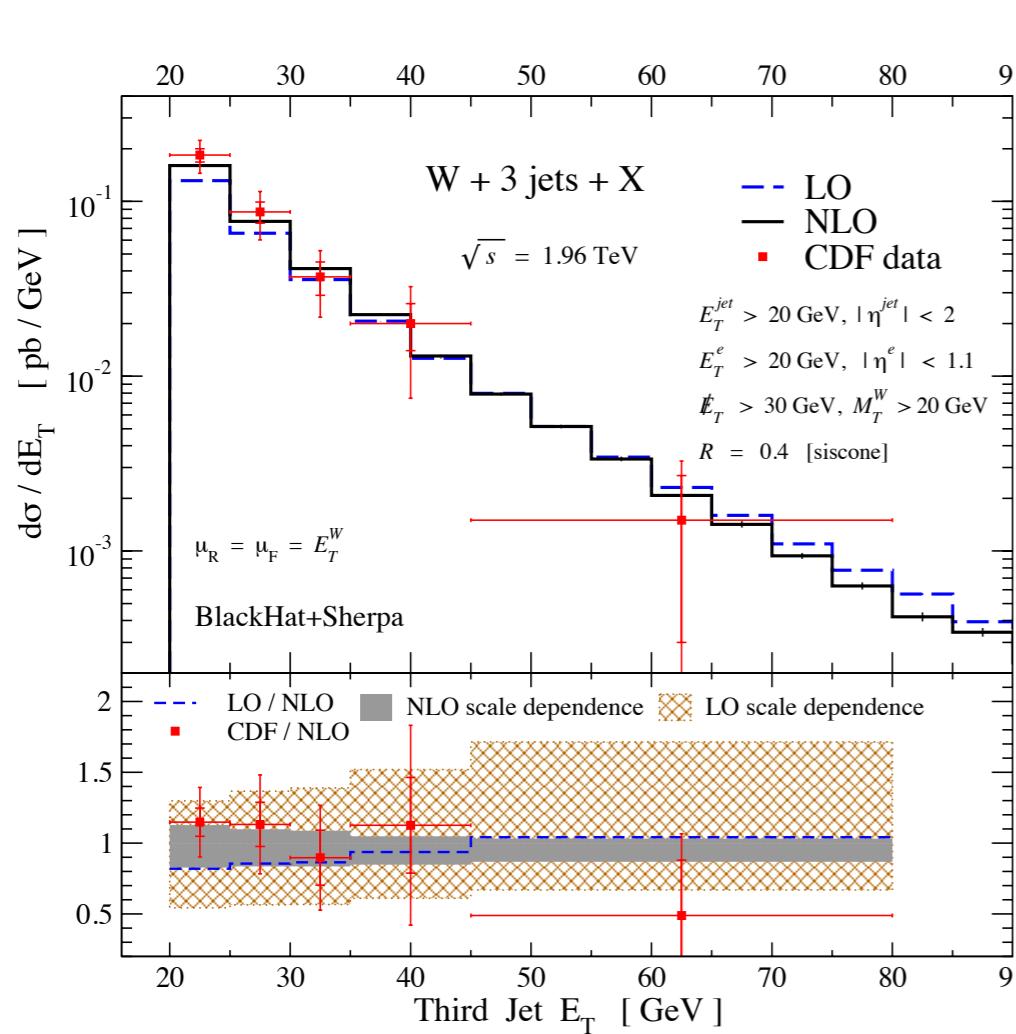
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- * NLO W+3jets cross section, full result
BlackHat+Sherpa [arXiv:0907.1984](#)
- * NLO t+tbar, NLO t+tbar + 1 jet cross section with decay and full spin correlation
Melnikov, Schultze , [arXiv:0907.3090](#)
- * NLO Z/ γ +3jets at the Tevatron [arXiv:1004.1659](#)
Berger,Bern,Dixon,Febres Cordero, Forde, Gleisberg, Ita, Kosover, Maitre
- * NLO t+tbar+b+bar
A. Bredenstein, A. Denner, S. Dittmaier and S. Pozzorini, [arXiv:0807.1248](#); [0905.0110](#); [1001.4006](#) ;
G. Bevilacqua, M. Czakon, C. G. Papadopoulos, R. Pittau and M. Worek, [arXiv:0907.4723](#) ;
T. Binoth, N. Greiner, A. Guffanti, J. P. Guillet, T. Reiter and J. Reuter, [arXiv:0910.4379](#) ;
recently combined with D-dim. unitarity
- * NLO t+tbar+ 2 jets
G. Bevilacqua, M. Czakon, C. G. Papadopoulos and M. Worek, [arXiv: 1002.4009](#)

Result for physical cross-sections

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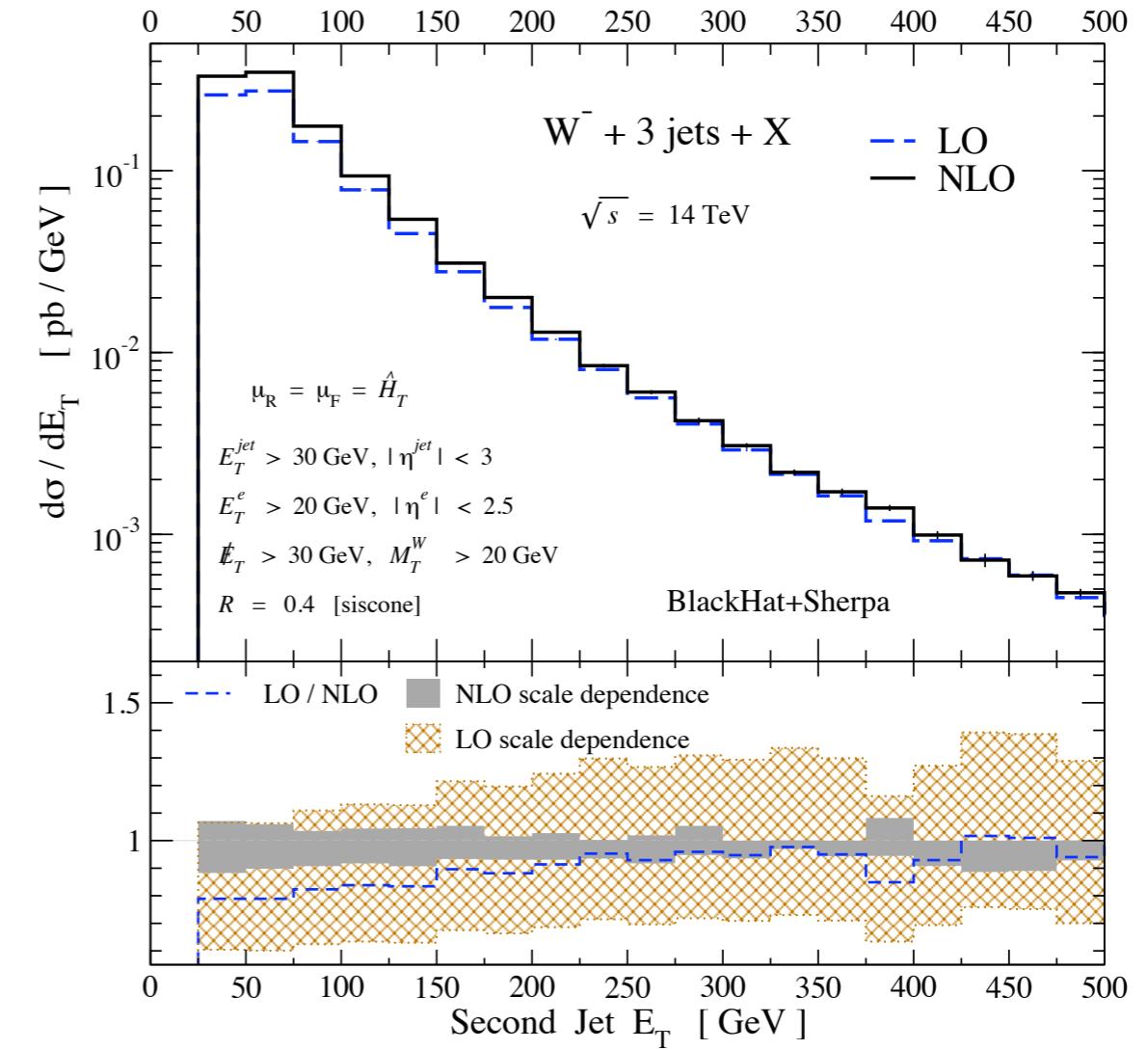
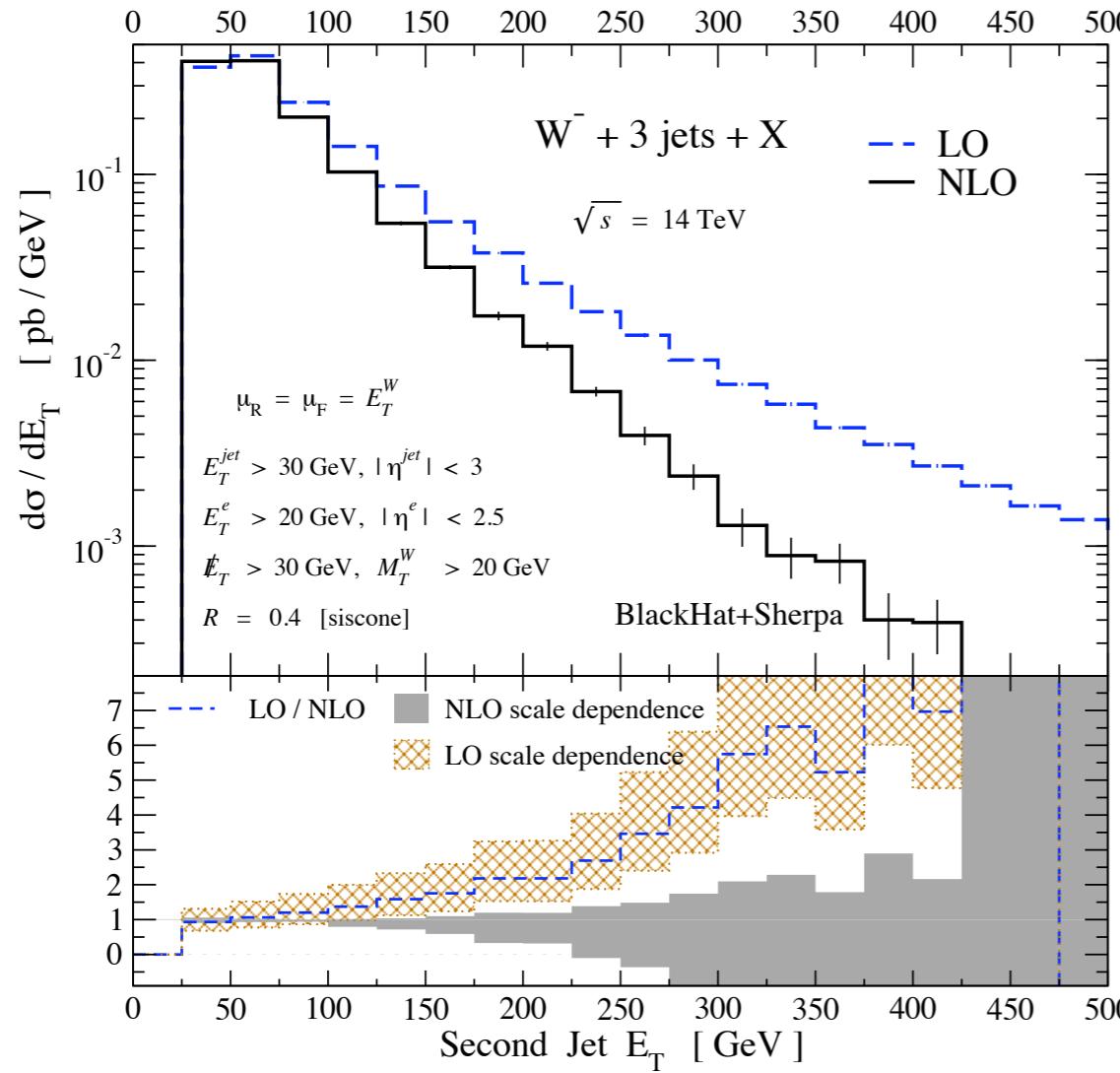
Result for physical cross-sections



Choosing the scale is an issue

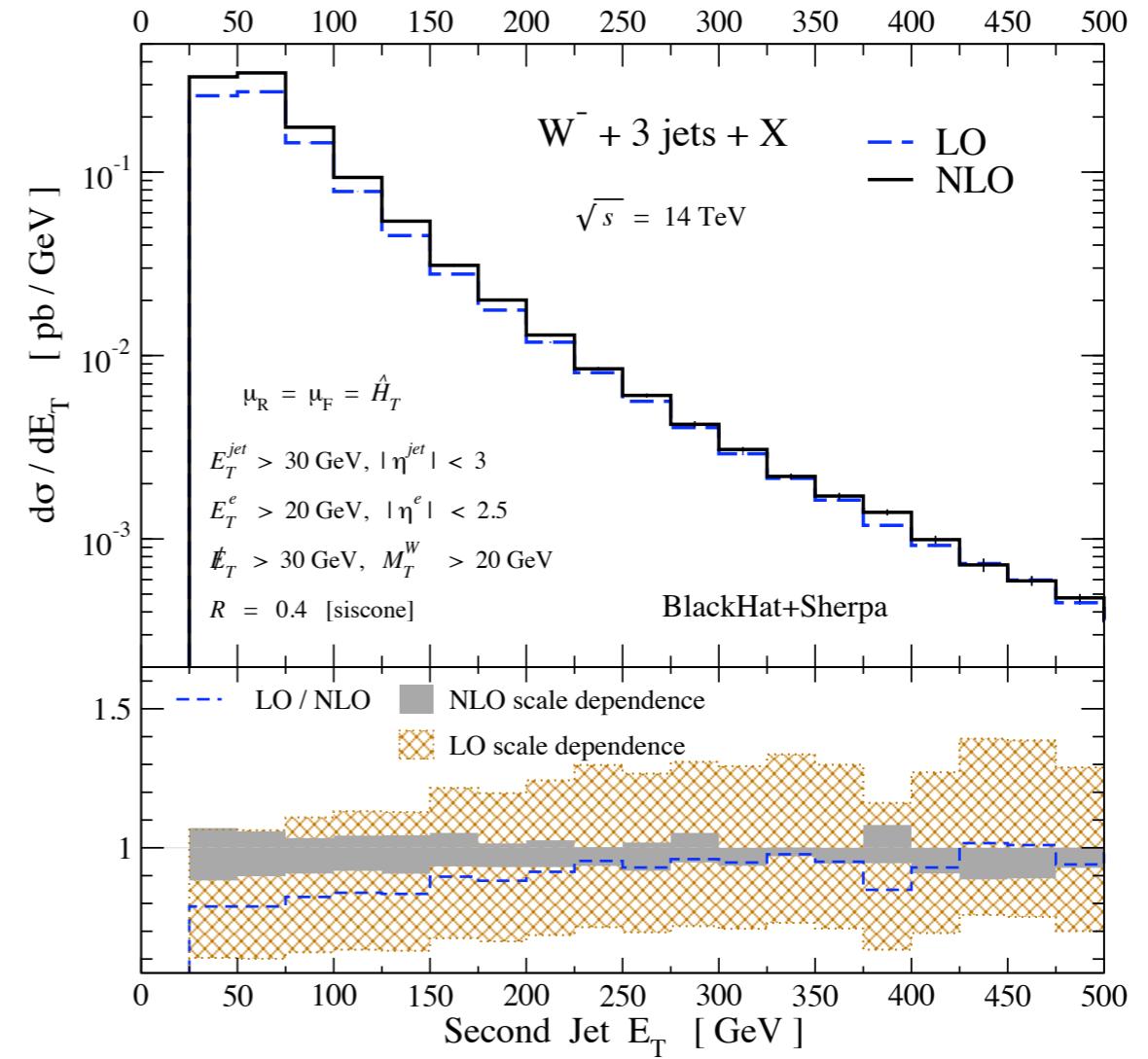
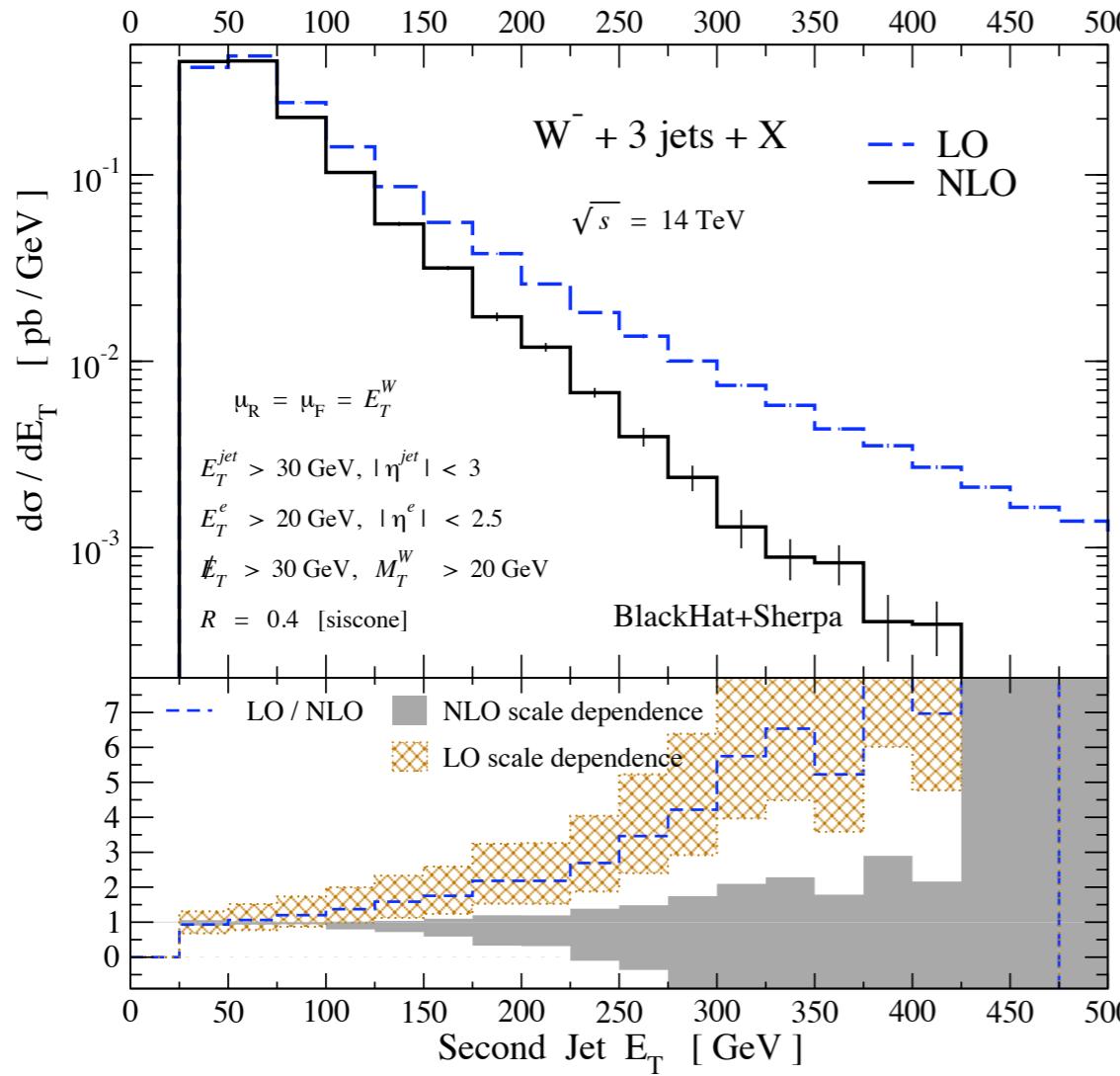
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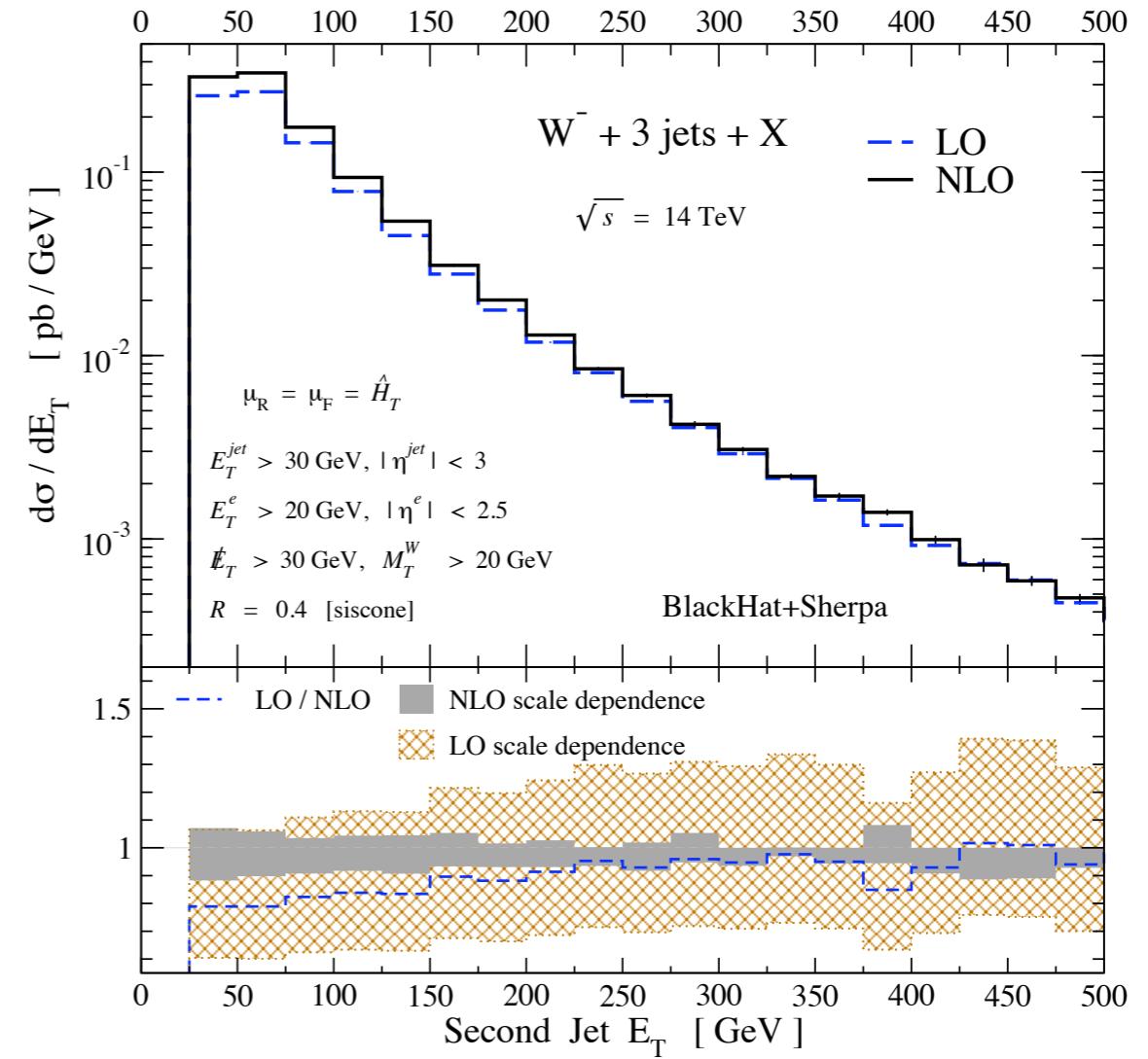
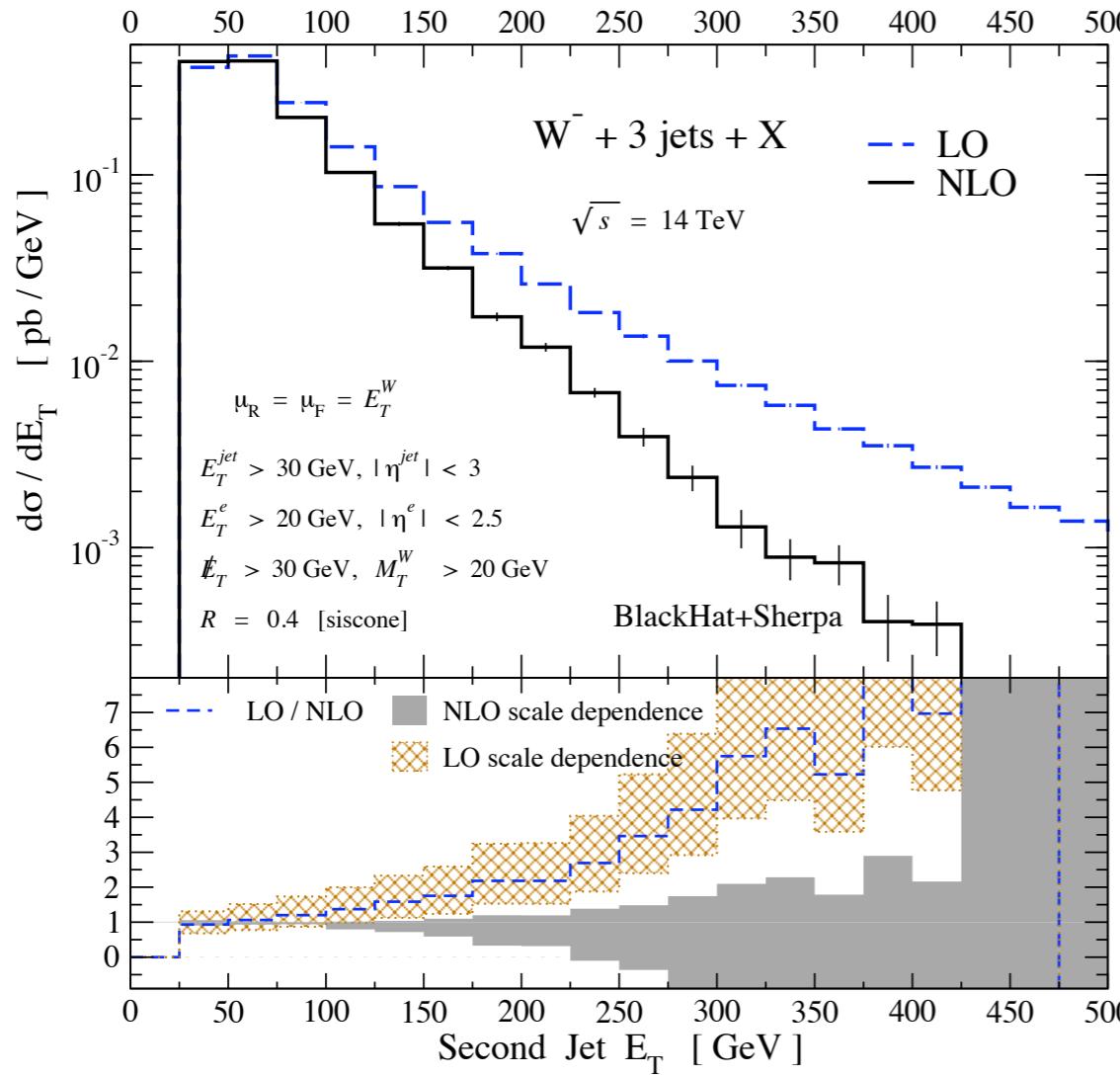
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Bauer and Lange('09): suggest using a dynamical scale better agreement with LO & NLO

Choosing the scale is an issue

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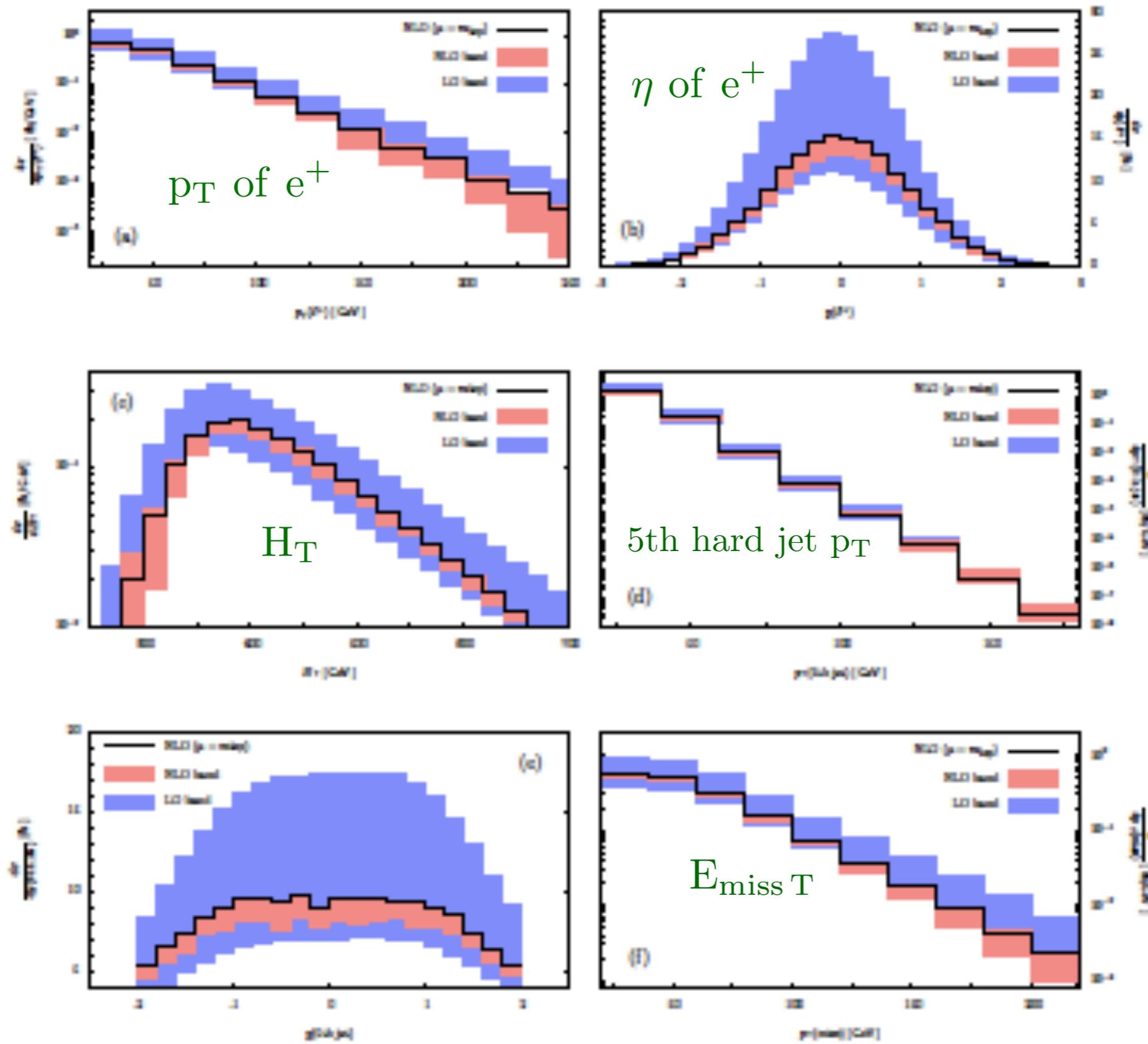
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Matrix element generator with CKKW matching uses local scales

t+tbar + jet production with top decay at Tevatron

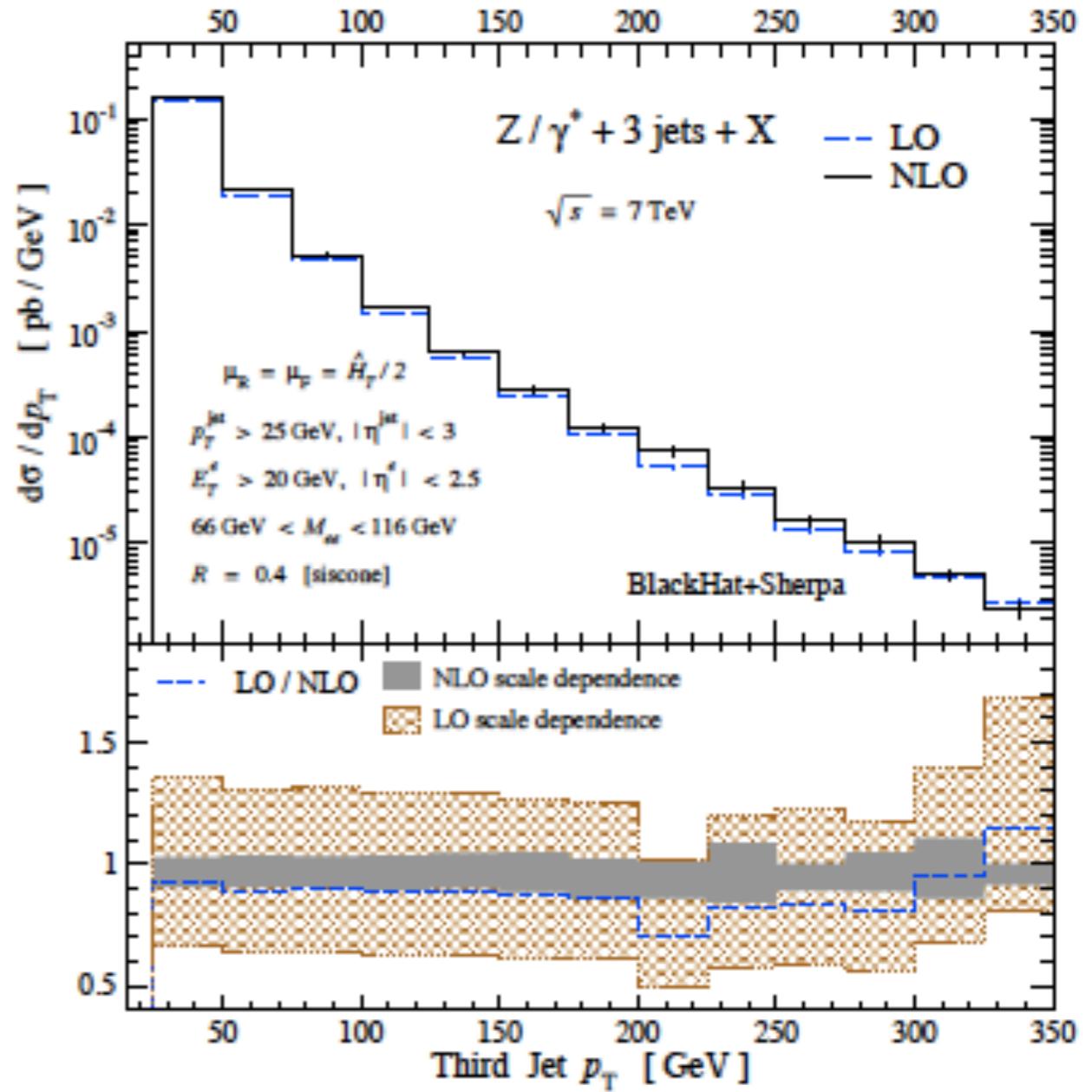
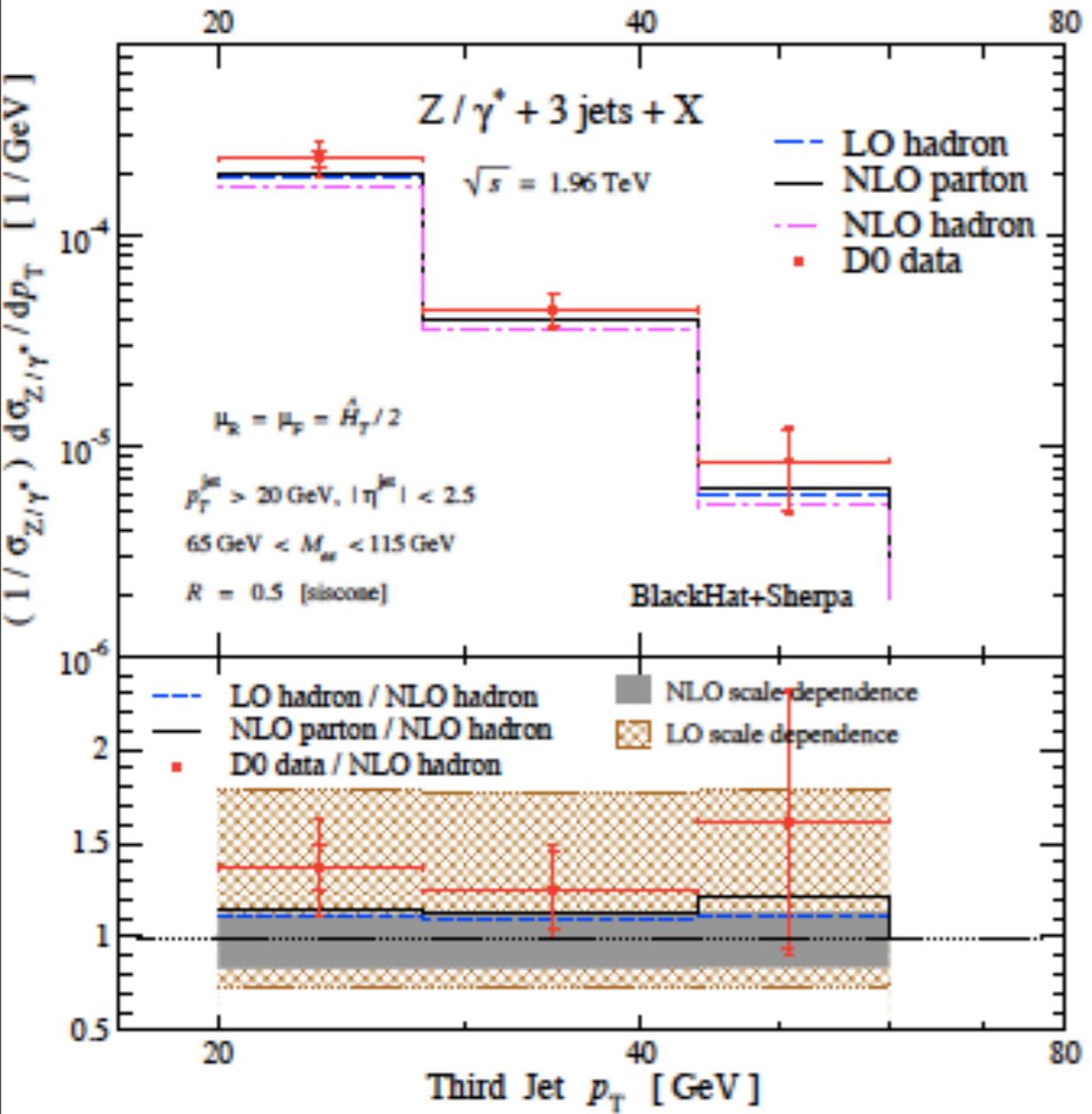
Melnikov, Schulze

$$p\bar{p} \rightarrow (t \rightarrow l^+ \nu b) + (\bar{t} \rightarrow j_1 j_2 \bar{b}) + j$$

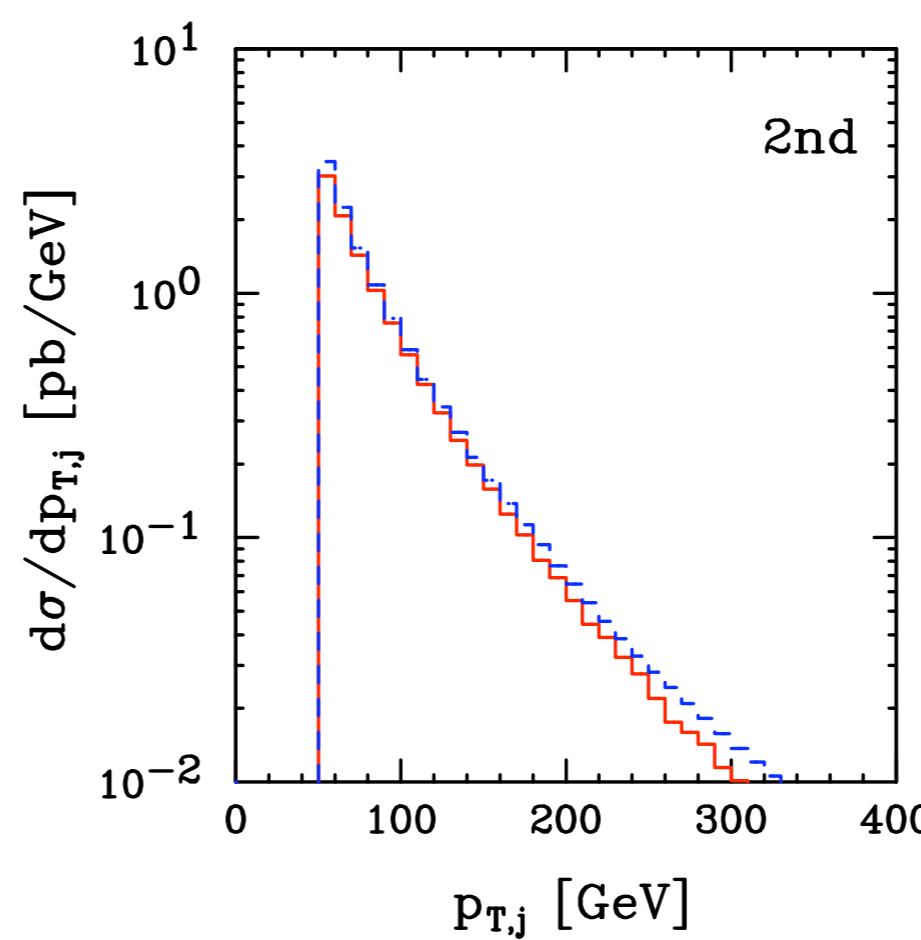
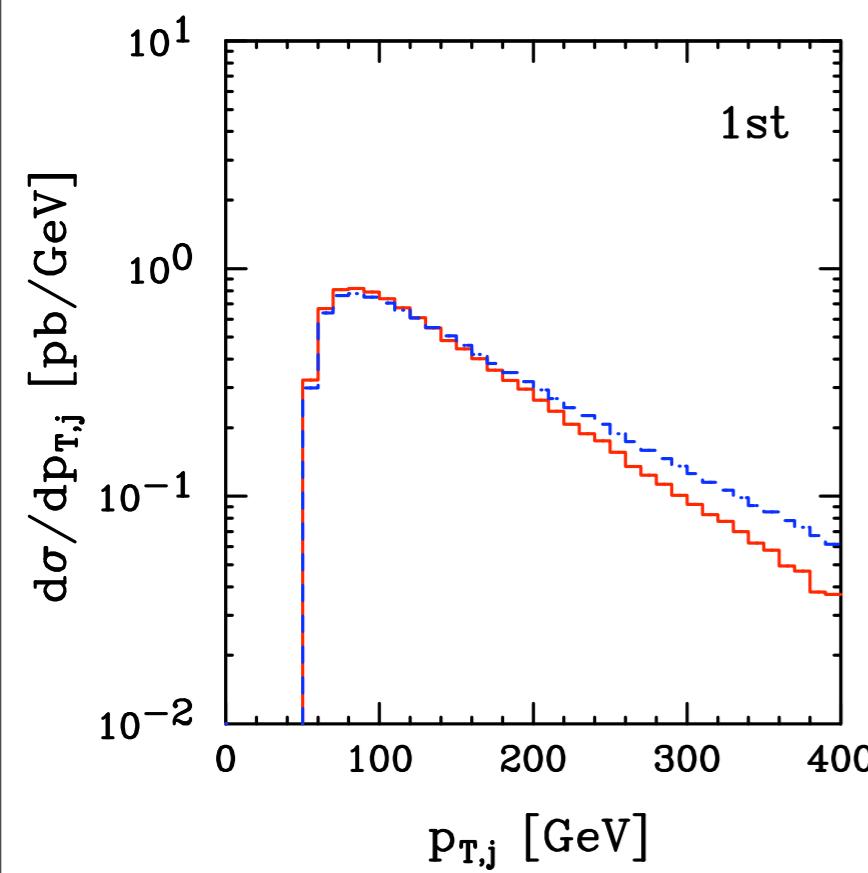


Z/gamma + 3 jets production

Black Hat



t+tbar+2jets

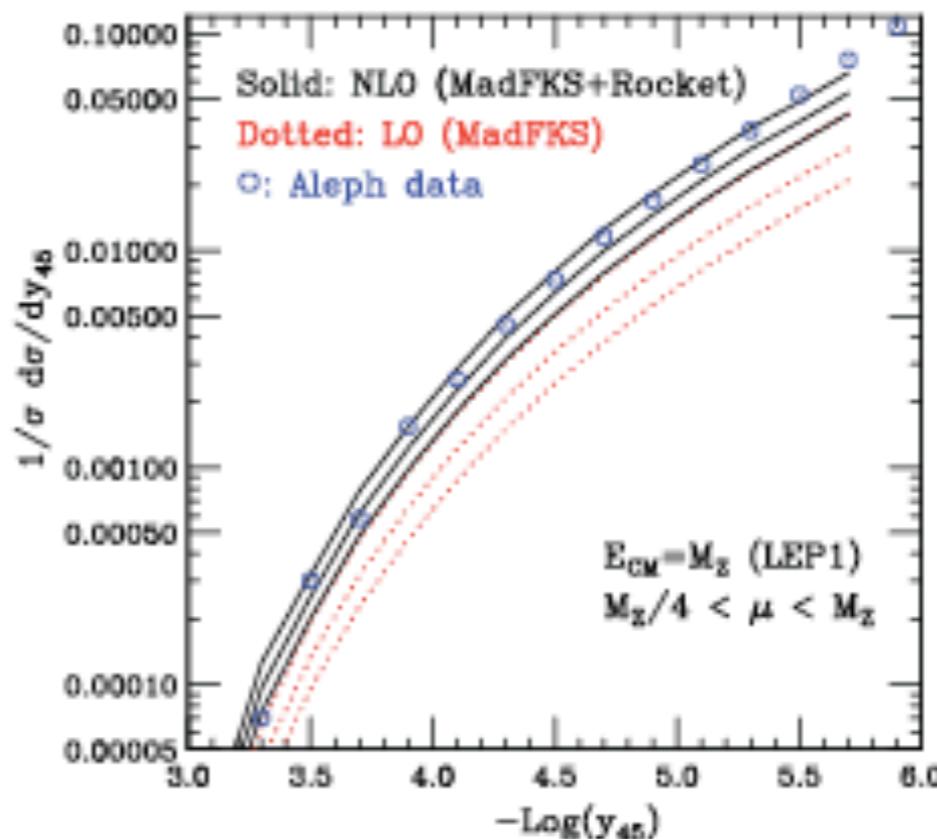


Bevilaqua, Czakon, Papadopoulos, Worek

NLO multileg: $e^+e^- \rightarrow 5$ jets preliminary

- ▶ one-loop amplitudes are crossing of hadron-collider V+3j
 - ▶ computed using Rocket package
- ▶ real radiation from MadGraph
 - ▶ subtraction using MadFKS
- ▶ improved scale dependence
- ▶ better agreement with data
- ▶ possibly new extraction of α_s

R. Frederix, S. Frixione, K. Melnikov, H. Stenzel, G. Zanderighi



T. Gehrmann's talk at DIS 2010



DIS 2010 Firenze

In preparation: W+4jets (Black Hat)
NLO 4jets ?

Subtracted real radiation codes based on LO generators

Subtracted real radiation codes based on LO generators

$$d\sigma_n^{(1)} \approx |M_n^{(0)}|^2 d\Phi_{n-2} + 2\text{Re}(M_n^{(0)\dagger} M_n^{(1)}) d\Phi_{n-2} + |M_{n+1}^{(0)}|^2 d\Phi_{n-1}$$

Subtracted real radiation codes based on LO generators

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- ❖ residue subtraction (Frixione, ZK, Signer)
 - MadFKS (R. Frederix, S. Frixione, F. Maltoni, T. Stelzer)
- ❖ dipole subtraction (Catani, Seymour, Dittmaier, Trocsanyi)
 - SHERPA (T. Gleisberg, F. Krauss)
 - MadDipole (R. Frederix, N. Greiner, TG)
 - TeVJet (M. Seymour, C. Tevlin)
 - AutoDipole (K. Hasegawa, S. Moch, P. Uwer)
 - Helac/Phegas (M. Czakon, C.G. Papadopoulos, M. Worek)
 - Black Hat
 - Rocket
- ❖ Treatment of color is an important issue

More powerful computers with GPU

Large scale grids to run the code, expensive infrastructure

Massively parallel GPGPU (General Purpose Graphics Processing Units) ?

Giele, Stavanger, Winter :

GPU NVIDIA Tesla chip is designed for numerical applications and the CUDA C compiler

A leading-order, leading-color parton-level event generator is developed for use on a multi-threaded GPU. Speed-up factors between 150 and 300 are obtained compared to an unoptimized CPU-based implementation of the event generator.

Hardware limitations:

GPU memory is independent from the CPU memory and divided into the off-chip global memory and the on-chip memory. This distinction is important as the off-chip memory is large (of the order of gigabytes) but slow to access by the threads. The on-chip memory is fast to access, but limited in size (of the order of tens of kilobytes).

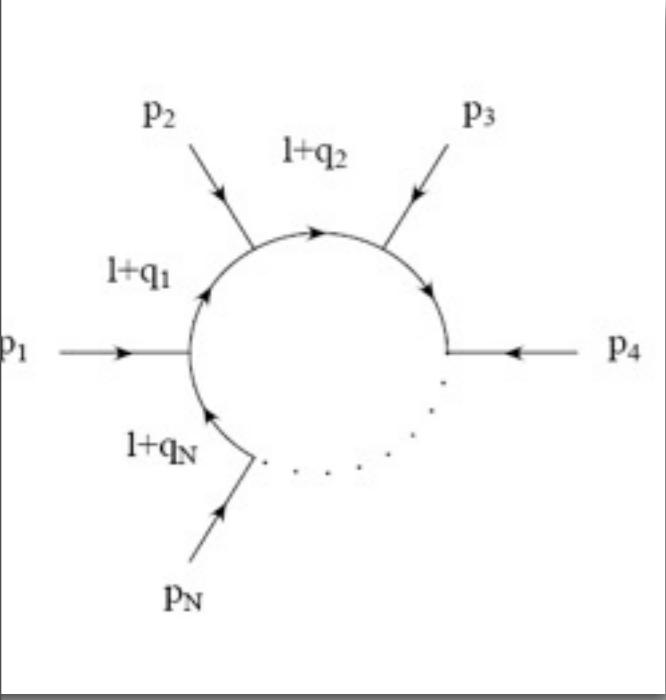
Hardware and software limitation may be removed in the near future?

Large on-chip memory (gigabytes), Fortran interface for programming

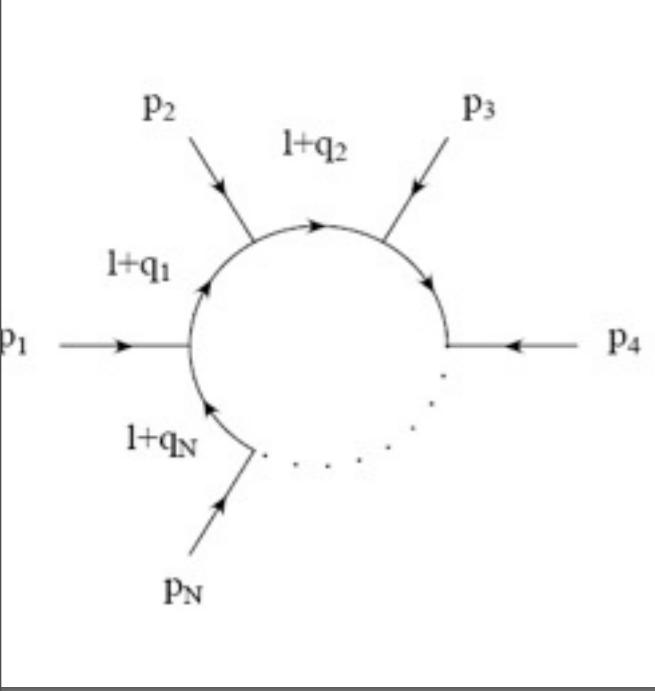
Some technical aspects of D-dimensional unitarity method

- ❖ Loop integration is a parametric integration
- ❖ Calculating the parameters in terms of tree amplitudes in numerically stable fashion
- ❖ Rational parts; D-dimensional unitarity
- ❖ Massive external particles, gauge invariance, renormalization
- ❖ Color decomposition
- ❖ Scalar integrals with complex particle mass values

Any one-loop N-point amplitude can be given in terms
of set of master integrals

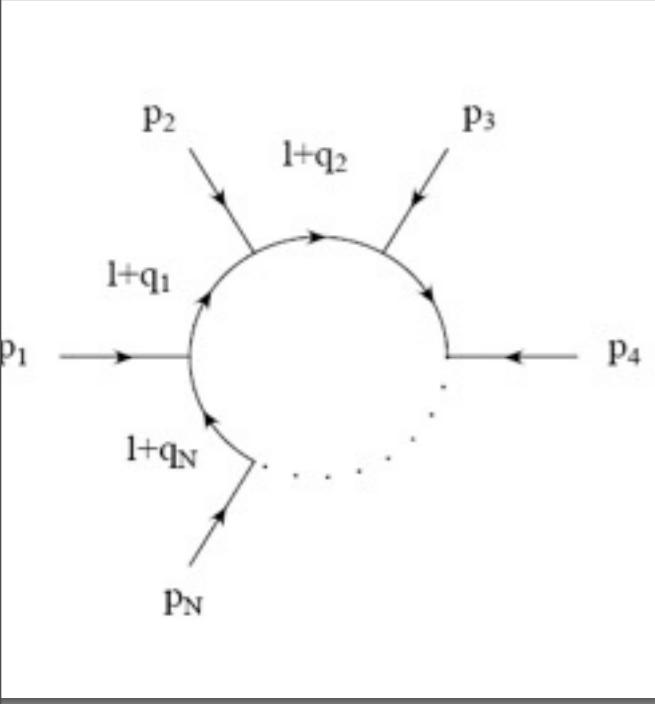


Any one-loop N-point amplitude can be given in terms of set of master integrals



$$\begin{aligned}\mathcal{A}_N(p_1, p_2, \dots, p_N) = & \sum_{1 \leq i_1 < i_2 < i_3 < i_4 \leq N} d_{i_1 i_2 i_3 i_4}(p_1, p_2, \dots, p_N) I_{i_1 i_2 i_3 i_4} \\ & + \sum_{1 \leq i_1 < i_2 < i_3 \leq N} c_{i_1 i_2 i_3}(p_1, p_2, \dots, p_N) I_{i_1 i_2 i_3} \\ & + \sum_{1 \leq i_1 < i_2 \leq N} b_{i_1 i_2}(p_1, p_2, \dots, p_N) I_{i_1 i_2} \\ & + \sum_{1 \leq i_1 \leq N} a_{i_1}(p_1, p_2, \dots, p_N) I_{i_1}\end{aligned}$$

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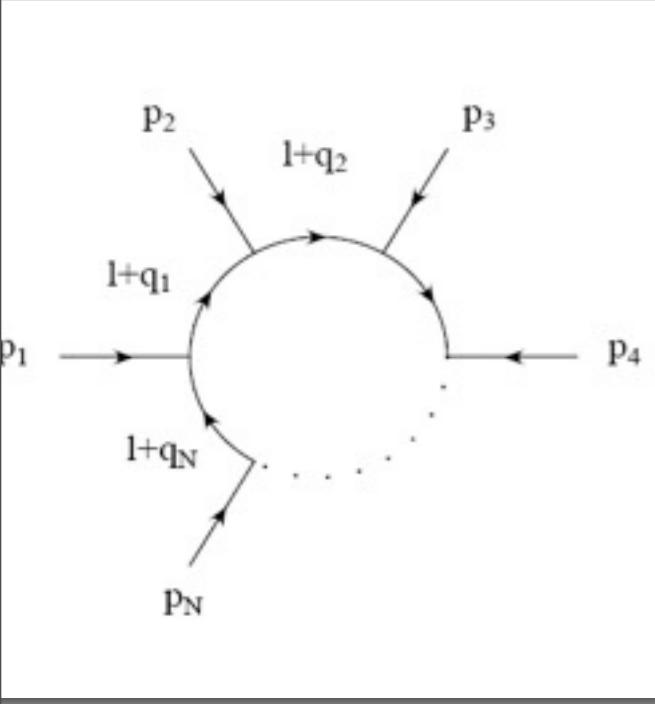
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$$I_{i_1 \dots i_M} = \int [dl] \frac{1}{d_{i_1} \cdots d_{i_M}}$$

$$d_{i_1} = (l + q_{i_1})^2 - m_{i_1}^2, \quad q_{i_1} = \sum_{j=1}^{i_1} p_j$$

e.g. Ellis, Zanderighi and others

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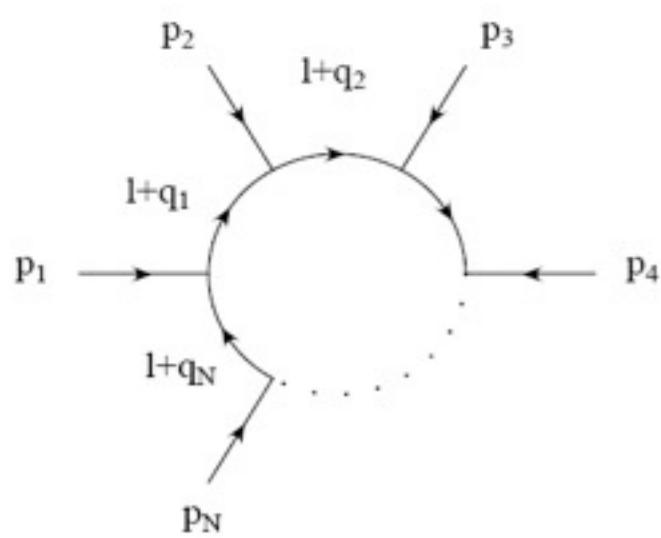
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$$\mathcal{A}_N(\{p_i\}) = \sum d_{i_1 i_2 i_3 i_4} \text{ (square loop)} + \sum c_{i_1 i_2 i_3} \text{ (triangle)} + \sum b_{i_1 i_2} \text{ (circle)} + \mathcal{R}$$

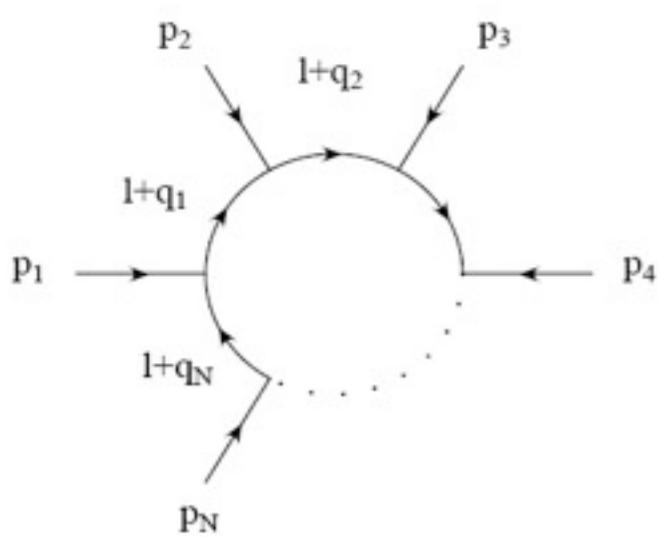
OPP method for the cut constructible part of the amplitude

parametric integral over the loop momentum



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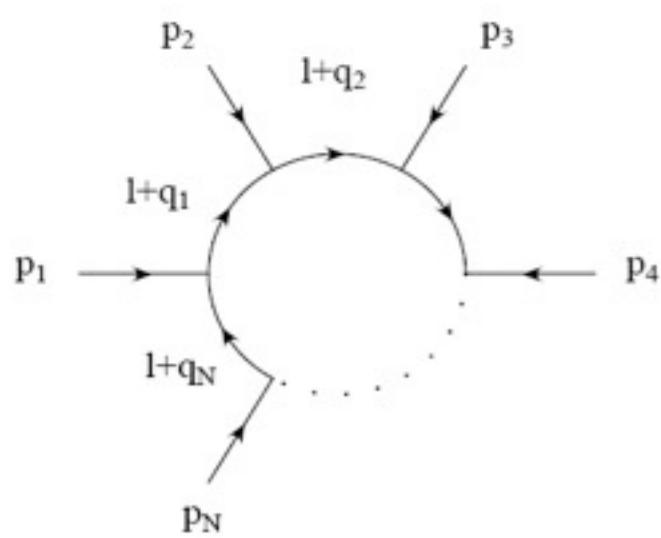
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“Amplitude integrand” for fully ordered external legs: many diagrams but maximal N different l -dependent scalar propagators.
This gives unique prescription of the integrand function as a function of modulo overall shift.

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parametric integral over the loop momentum

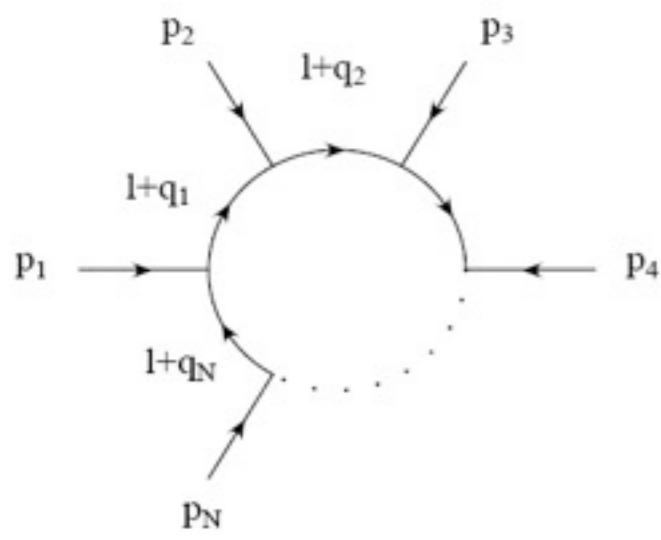


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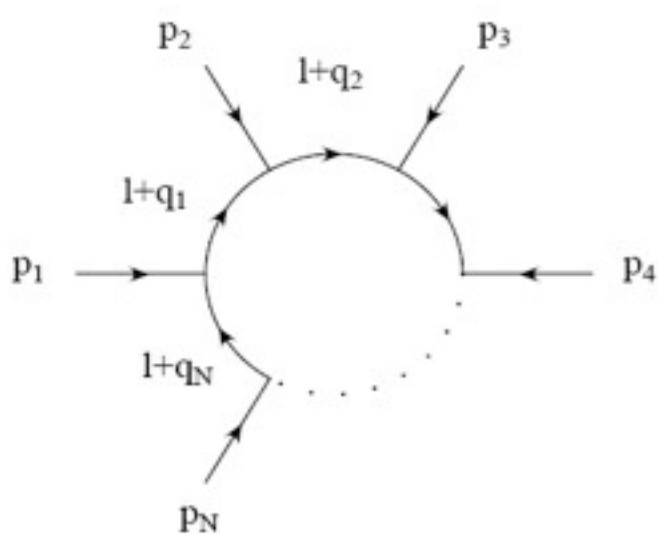
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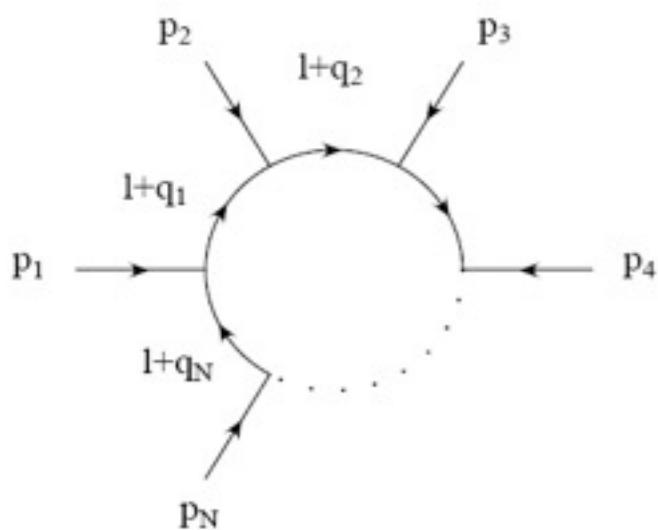
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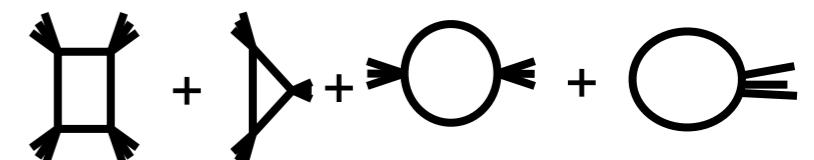
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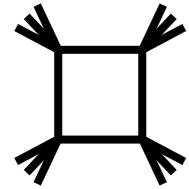
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$$\begin{aligned} \mathcal{A}_N(p_1, p_2, \dots, p_N; l) &= \frac{\mathcal{N}(p_1, p_2, \dots, p_N; l)}{d_1 d_2 \cdots d_N} = \\ &= \sum_{1 \leq i_1 < i_2 < i_3 < i_4 \leq N} \frac{\bar{d}_{i_1 i_2 i_3 i_4}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} + \sum_{1 \leq i_1 < i_2 < i_3 \leq N} \frac{\bar{c}_{i_1 i_2 i_3}(l)}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{1 \leq i_1 < i_2 \leq N} \frac{\bar{b}_{i_1 i_2}(l)}{d_{i_1} d_{i_2}} + \sum_{1 \leq i_1 \leq N} \frac{\bar{a}_{i_1}(l)}{d_{i_1}} \end{aligned}$$



Multi-pole structure in loop momentum, physical spaces and “trivial spaces” of different dimensions

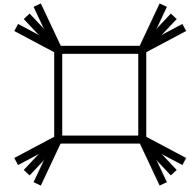
Multi-pole structure in loop momentum, physical spaces and “trivial spaces” of different dimensions



$$\frac{d_{[i_1|i_4]}(\{p_i\}; l)}{[(l + q_{i_1})^2 - m_{i_1}^2][(l + q_{i_1} + k_{i_1|i_2})^2 - m_{i_2}^2][(l + q_{i_1} + k_{i_2|i_3})^2 - m_{i_3}^2][(l + q_{i_1} + k_{i_3|i_4})^2 - m_{i_4}^2]}$$

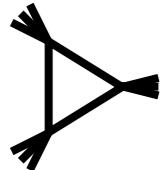
three independent outflowing momenta: $k_{i_1|i_2}$, $k_{i_2|i_3}$, $k_{i_3|i_4}$ $D_P = 3$, $D_T = D - D_P = D - 3$

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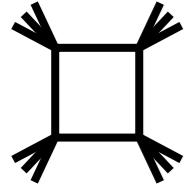
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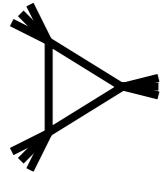
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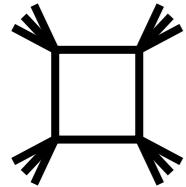
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The loop momenta is decomposed into “physical” and “transverse” component

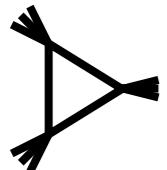
$$l^\nu = l_P^\nu + l_T^\nu , \quad l^2 = l_P^2 + l_T^2 = l_P^2 - \mu^2$$

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We need to choose some convenient basis vectors for the
“physical” and “trivial” spaces

Loop momentum decomposed in Vermaseren van Nerveen basic vectors: reduction at the integrand level

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Important is the split to trivial and physical space. Various convenient choices of the basis vectors in the two spaces.

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define: a set of dual momenta v_i , $v_i p_j = \delta_{ij}$

and : a set of orthogonal unit vectors n_i , $n_i p_j = 0$

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$$\delta_{\nu_1 \nu_2 \cdots \nu_R}^{\mu_1 \mu_2 \cdots \mu_R} = \begin{vmatrix} \delta_{\nu_1}^{\mu_1} & \delta_{\nu_2}^{\mu_1} & \cdots & \delta_{\nu_R}^{\mu_1} \\ \delta_{\nu_1}^{\mu_2} & \delta_{\nu_2}^{\mu_2} & \cdots & \delta_{\nu_R}^{\mu_2} \\ \vdots & \vdots & & \vdots \\ \delta_{\nu_1}^{\mu_R} & \delta_{\nu_2}^{\mu_R} & \cdots & \delta_{\nu_R}^{\mu_R} \end{vmatrix} ,$$

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$$v_i^\mu(k_1, \dots, k_{D_P}) \equiv \frac{\delta_{k_1 \dots k_{i-1} k_i k_{i+1} \dots k_{D_P}}^{k_1 \dots k_{i-1} \mu k_{i+1} \dots k_{D_P}}}{\Delta(k_1, \dots, k_{D_P})} , \quad \delta_{\nu_1 \nu_2 \dots \nu_R}^{\mu_1 \mu_2 \dots \mu_R} = \begin{vmatrix} \delta_{\nu_1}^{\mu_1} & \delta_{\nu_2}^{\mu_1} & \dots & \delta_{\nu_R}^{\mu_1} \\ \delta_{\nu_1}^{\mu_2} & \delta_{\nu_2}^{\mu_2} & \dots & \delta_{\nu_R}^{\mu_2} \\ \vdots & \vdots & \ddots & \vdots \\ \delta_{\nu_1}^{\mu_R} & \delta_{\nu_2}^{\mu_R} & \dots & \delta_{\nu_R}^{\mu_R} \end{vmatrix} ,$$

Gram-determinant



Loop momentum decomposed in Vermaseren van Nerveen basic vectors: reduction at the integrand level

Important is the split to trivial and physical space. Various convenient choices of the basis vectors in the two spaces.

define: a set of dual momenta v_i , $v_i p_j = \delta_{ij}$
 and : a set of orthogonal unit vectors n_i , $n_i p_j = 0$

$$l^\mu = V_R^\mu + \sum_{i=1}^{D_P} \frac{1}{2} (d_i - d_{i-1}) v_i^\mu + \sum_{i=1}^{D_T} \alpha_i n_i^\mu , \quad V_R^\mu = -\frac{1}{2} \sum_{i=1}^{D_P} ((q_i^2 - m_i^2) - (q_{i-1}^2 - m_{i-1}^2)) v_i^\mu$$

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Even at fixed external kinematics, for every cut we have different the split to trivial and physical space and choice of basis vectors.

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Box (rank four): $D_P = 3, D_T = 1, \quad l_T^2 = s_1^2 \sim 1 \quad l_T^2 = l^2 - l_P^2, \quad l_P^\mu = (l q_i) v_i^\mu$

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$$\mathcal{N}(l) = \mathcal{N}(\tilde{l}, \mu), \quad l^2 = \tilde{l}^2 - \mu^2$$

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- Choose two integer values $D_s = D_1$ and $D_s = D_2$ to reconstruct the full D_s dependence.
- Suitable for numerical implementation
- $D_s = 4 - 2\epsilon$ 't Hooft Veltman scheme, $D_s = 4$ FDHS
- for closed fermion loops

$$\mathcal{N}^{D_s}(l) = 2^{(D_s-4)/2} \mathcal{N}_0(l)$$

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The full ϵ -dependence comes from the integrals

$$\int \frac{d^D l}{(i\pi)^{D/2}} \frac{s_e^2}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} = -\frac{D-4}{2} I_{i_1 i_2 i_3 i_4}^{D+2},$$

$$\int \frac{d^D l}{(i\pi)^{D/2}} \frac{s_e^4}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} = \frac{(D-2)(D-4)}{4} I_{i_1 i_2 i_3 i_4}^{D+4},$$

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$$R_N = - \sum_{[i_1 | i_4]} \frac{d_{i_1 i_2 i_3 i_4}^{(4)}}{6} + \sum_{[i_1 | i_3]} \frac{c_{i_1 i_2 i_3}^{(7)}}{2} - \sum_{[i_1 | i_2]} \left(\frac{(q_{i_1} - q_{i_2})^2}{6} - \frac{m_{i_1}^2 + m_{i_2}^2}{2} \right) b_{i_1 i_2}^{(9)},$$

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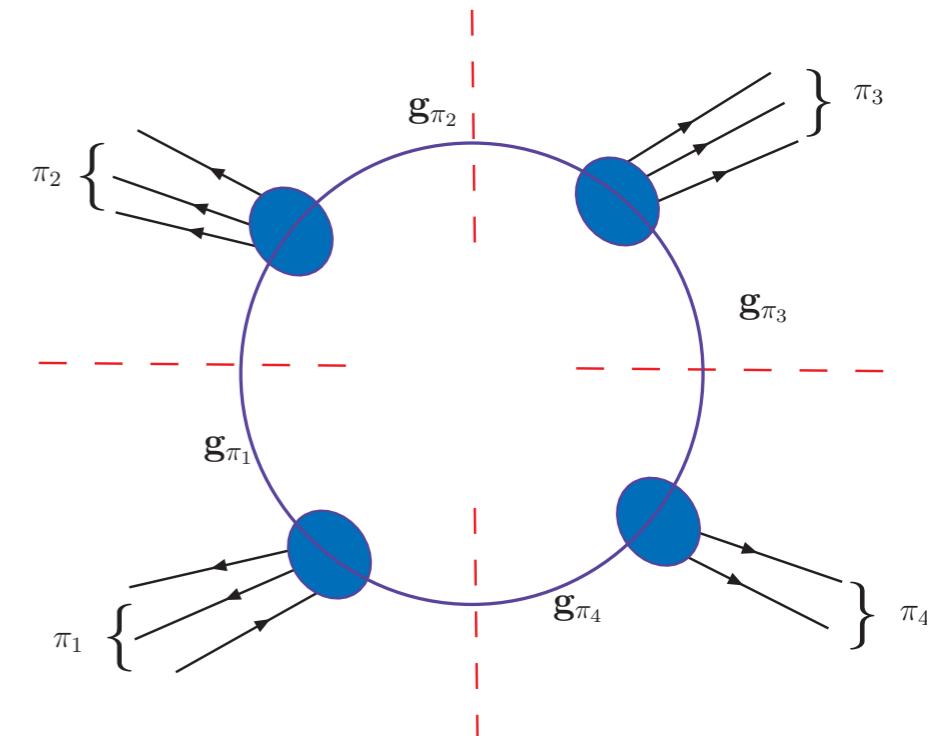
Pentagon contribution in 4 dimension is decomposed into box contributions

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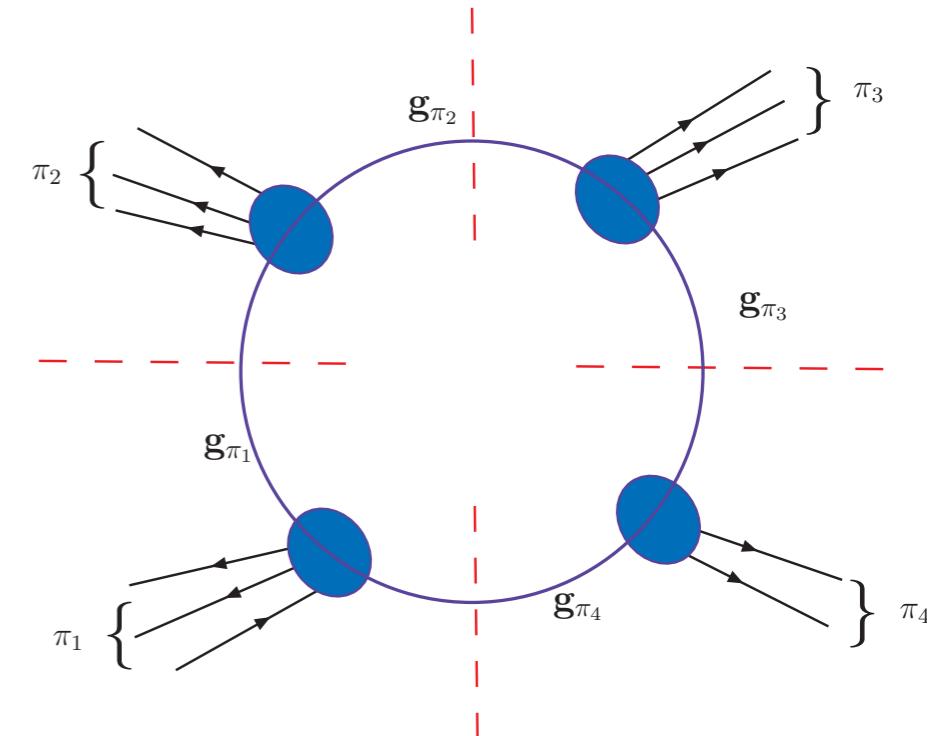
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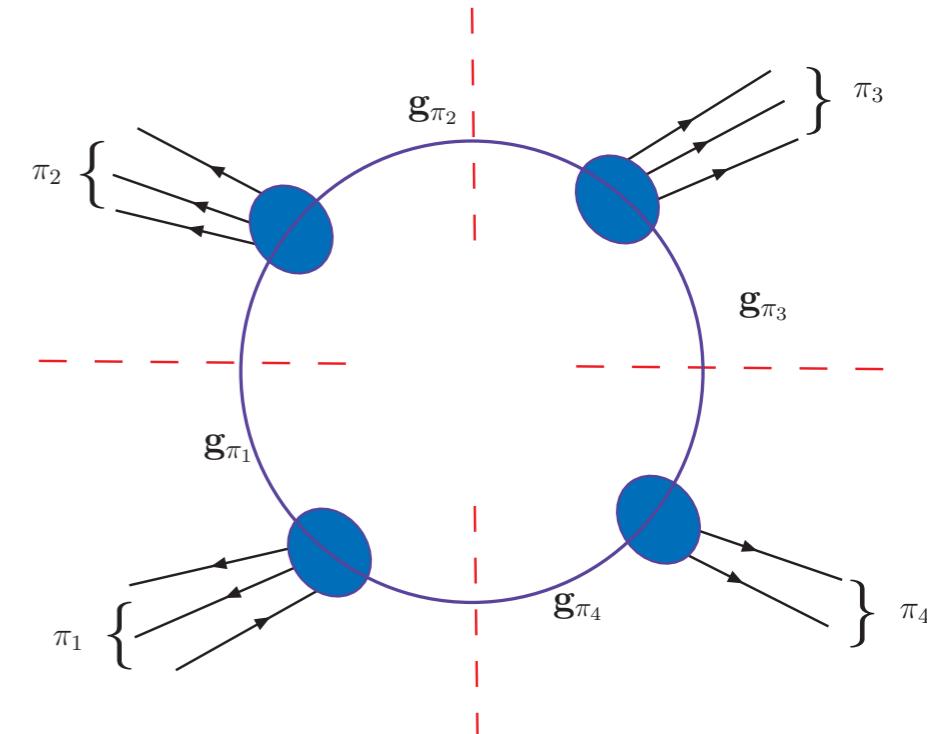


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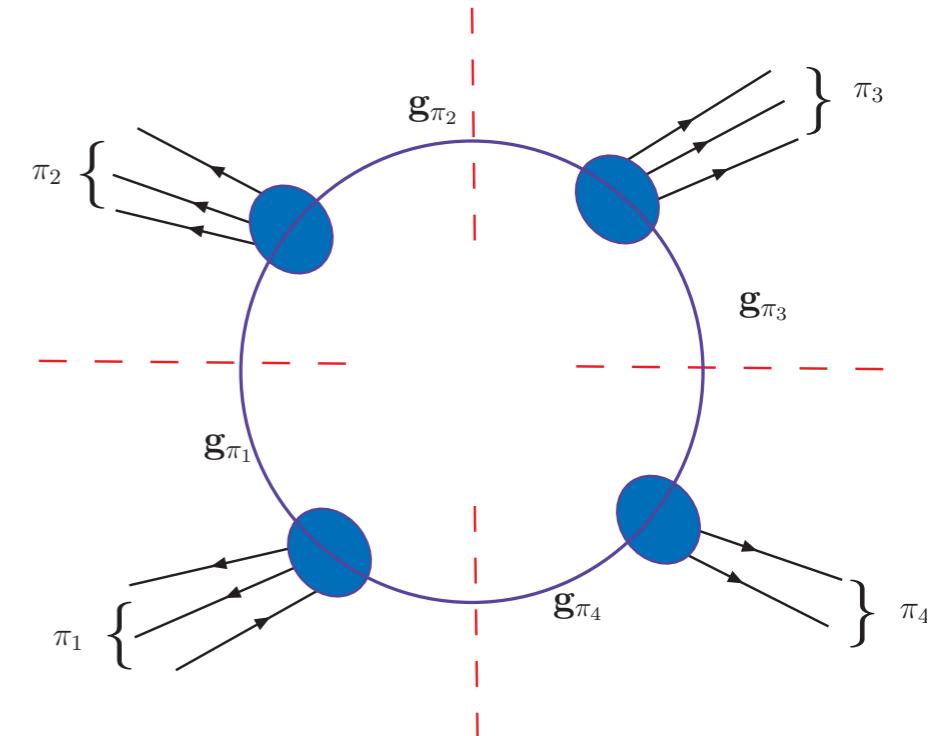
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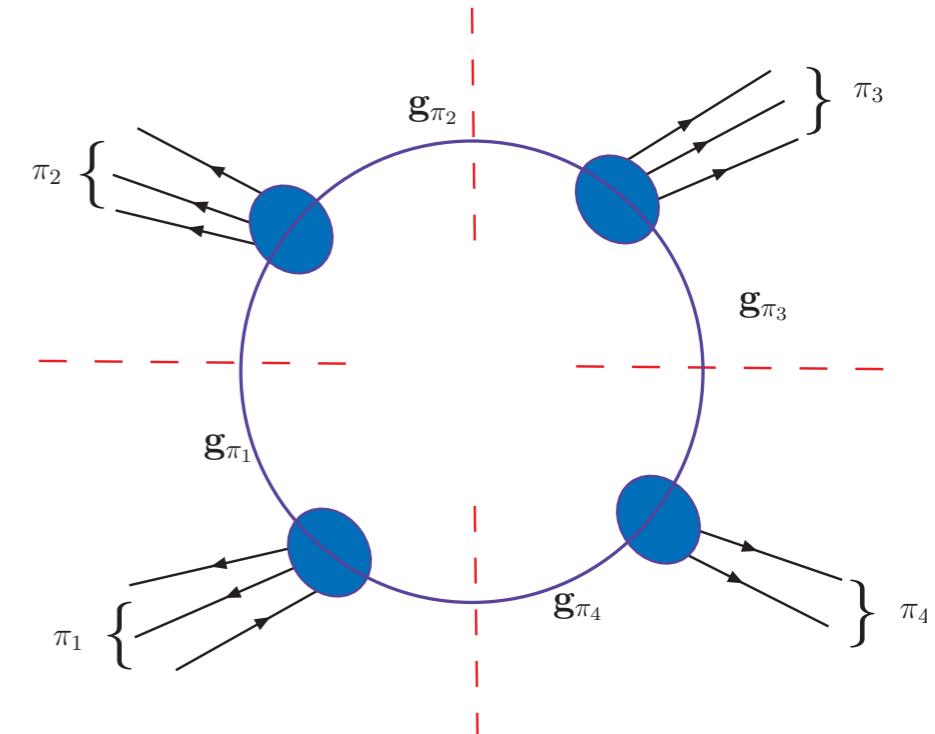
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- A) parameters are independent from the loop momenta: they can be calculated from the value of the residua of the amplitude in the loop momentum

Taking residues constraints the allowed values of the loop momentum. In D=4:

$$l^\mu = V_R^\mu + \sum_{i=1}^{D_P} \frac{1}{2} (d_i - d_{i-1}) v_i^\mu + \sum_{i=1}^{D_T} \alpha_i n_i^\mu ,$$

1. Quadrupole cut $d_i=d_j=d_k=d_l=0$ (two solutions)

$$l^\mu = V_4^\mu + \alpha_1 n_1^\mu$$

$$l_\pm^\mu = V_4^\mu \pm i \sqrt{V_4^2 - m_l^2} \times n_1^\mu$$

2. Triple cut, infinite number of solutions (on a circle circle)

$$l^\mu = V_3^\mu + \alpha_1 n_1^\mu + \alpha_2 n_2^\mu$$

$$l_{\alpha_1 \alpha_2}^\mu = V_3^\mu + \alpha_1 n_1^\mu + \alpha_2 n_2^\mu; \quad \alpha_1^2 + \alpha_2^2 = -(V_3^2 - m_k^2)$$

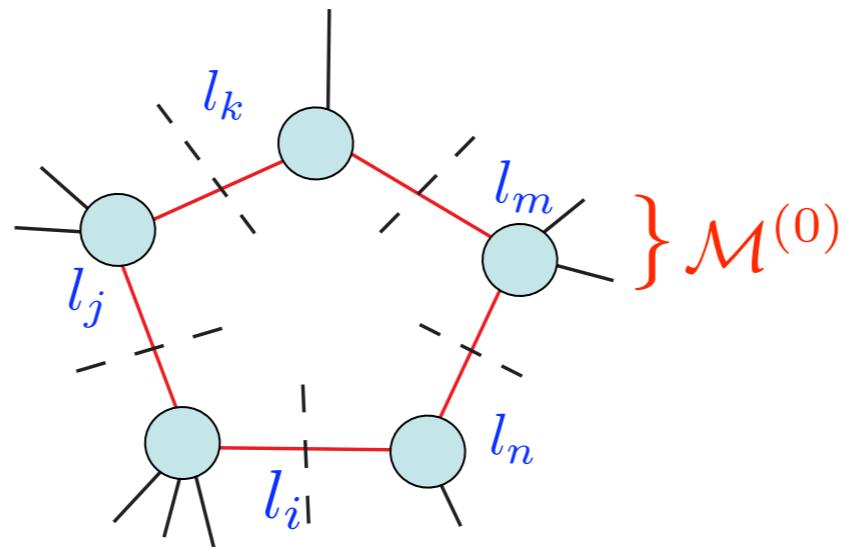
3. Double cut, infinite number of solutions (on a “sphere”)

$$l^\mu = V_2^\mu + \alpha_1 n_1^\mu + \alpha_2 n_2^\mu + \alpha_3 n_3^\mu$$

$$l_{\alpha_1 \alpha_2 \alpha_3}^\mu = V_2^\mu + \alpha_1 n_1^\mu + \alpha_2 n_2^\mu + \alpha_3 n_3^\mu; \quad \alpha_1^2 + \alpha_2^2 + \alpha_3^2 = -(V_2^2 - m_j^2) .$$

B) The residua factorize into the products of tree amplitudes

$$\text{Res}_{i_1 \dots i_M} \left[\mathcal{A}_N^{(D_s)}(\ell) \right] = \sum_{\{\lambda_1, \dots, \lambda_M\}=1}^{D_s-2} \left\{ \prod_{k=1}^M \mathcal{M}^{(0)} \left(\ell_{i_k}^{(\lambda_k)}; p_{i_k+1}, \dots, p_{i_{k+1}}; -\ell_{i_{k+1}}^{(\lambda_{k+1})} \right) \right\}$$



New issues for tree-level generator coming from D-dimensional generalized unitarity

- Two legs in 6 and 8 dimensions
- Two legs with complex four momentum
- Two legs having complex mass value
- Adding new physics vertices efficiently

Factorization into tree amplitudes can be replaced by efficient Feynman diagram calculation of the integrand function

Factorization into tree amplitudes can be replaced by efficient Feynman diagram calculation of the integrand function

Main issue: Efficient calculation of the coefficients of the OPP decomposition

One can work in four dimension

R_1 and R_2 part of the rational part

CutTool, Samurai (Mastrolia, et.al), GOLEM+Samurai

D-dimensional unitary algorithm for massive fermions (EGKM)

Application to ttbar and ttbar+jet production

- We have to choose even values for D_s
$$\mathcal{A}^{\text{FDH}} = 2\mathcal{A}_{(D,D_s=6)} - \mathcal{A}_{(D,D_s=8)}$$
 - Pentagon, box, triangle ,bubble and tadpole cuts
 - The treatment of bubble and tadpole cuts is more subtle:
 - i) light-like bubbles, tadpoles
 - ii) (1,n-1) partitioning of the n-legs has to be included
unitarity has difficulty with self-energy insertions on external legs
 - Particles of different flavors: more sophisticated bookkeeping
 - Color and “flavor ordered” primitive amplitudes
 - More master integrals (use QCDLoop, Ellis, Zanderighi)

External self energies

External self energies

The box coefficient of the integrand function has pole terms

$$\frac{1}{p_i^2 - m_i^2}$$

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$$\frac{\bar{c}_{j_1 j_2 j_3}^{(D_s)}(l)}{d_{j_1} d_{j_2} d_{j_3}}$$

$$\bar{b}_{i-1,i}^{D_s}(l) = \text{Res}_{i-1,i} \left(\frac{\mathcal{N}^{(D_s)}(l)}{d_1 \dots d_N} - \sum_{[j_1|j_5]} \frac{\bar{e}_{i_1 j_2 j_3 j_4 j_5}^{(D_s)}(l)}{d_{j_1} d_{j_2} d_{j_3} d_{j_4} d_{j_5}} - \sum_{[j_1|j_4]} \frac{\bar{d}_{j_1 j_2 j_3 j_4}^{(D_s)}(l)}{d_{j_1} d_{j_2} d_{j_3} d_{j_4}} - \sum_{[j_1|j_3]} \frac{\bar{c}_{j_1 j_2 j_3}^{(D_s)}(l)}{d_{j_1} d_{j_2} d_{j_3}} \right)$$

$$\bar{b}_i^{(D_s)}(l) = \frac{\tilde{b}_{i,i-1}(l)}{p_i^2 - m_i^2} + \hat{b}_{i-1,i}^{(D_s)}(l)$$

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We need to calculate both term using unitarity cut in numerically stable way.

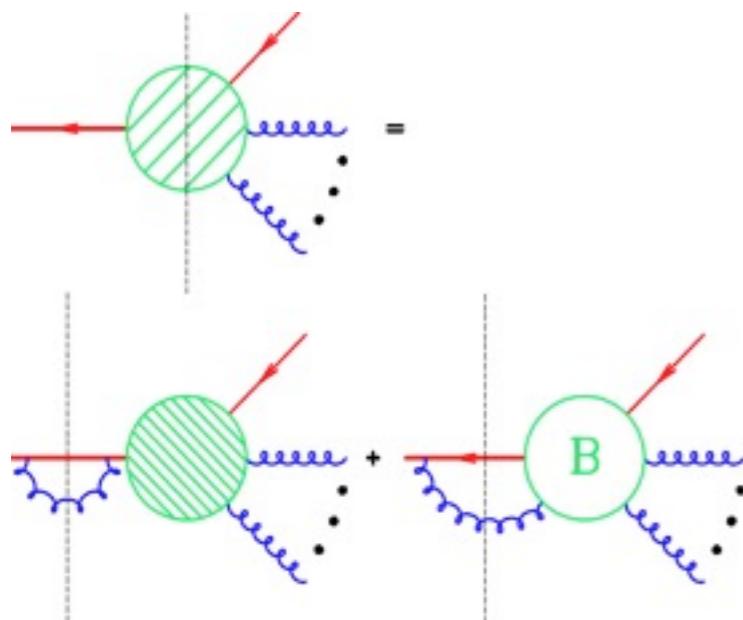
External self energies

The box coefficient of the integrand function has pole terms

$$\frac{1}{p_i^2 - m_i^2} \left(\bar{p}_i^{(D_s)}(l) - \sum_{[j_1|j_5]} \frac{\bar{e}_{i_1 j_2 j_3 j_4 j_5}^{(D_s)}(l)}{d_{j_1} d_{j_2} d_{j_3} d_{j_4} d_{j_5}} - \sum_{[j_1|j_4]} \frac{\bar{d}_{j_1 j_2 j_3 j_4}^{(D_s)}(l)}{d_{j_1} d_{j_2} d_{j_3} d_{j_4}} - \sum_{[j_1|j_3]} \frac{\bar{c}_{j_1 j_2 j_3}^{(D_s)}(l)}{d_{j_1} d_{j_2} d_{j_3}} \right)$$

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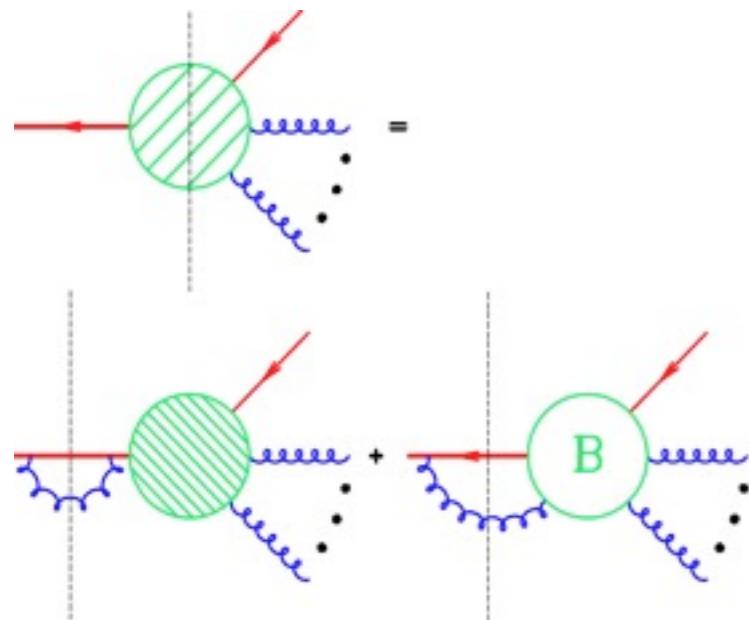
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$$\begin{aligned} \bar{b}_{i-1,i}^{D_s}(l) &= \text{Res}_{i-1,i} \left(\frac{\mathcal{N}^{(D_s)}(l)}{d_1 \dots d_N} - \sum_{[j_1|j_5]} \frac{\bar{e}_{i_1 j_2 j_3 j_4 j_5}^{(D_s)}(l)}{d_{j_1} d_{j_2} d_{j_3} d_{j_4} d_{j_5}} - \sum_{[j_1|j_4]} \frac{\bar{d}_{j_1 j_2 j_3 j_4}^{(D_s)}(l)}{d_{j_1} d_{j_2} d_{j_3} d_{j_4}} - \sum_{[j_1|j_3]} \frac{\bar{c}_{j_1 j_2 j_3}^{(D_s)}(l)}{d_{j_1} d_{j_2} d_{j_3}} \right) \\ \bar{b}_i^{(D_s)}(l) &= \frac{\tilde{b}_{i,i-1}(l)}{p_i^2 - m_i^2} + \hat{b}_{i-1,i}^{(D_s)}(l) \end{aligned}$$

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$$\text{Res} \left(\mathcal{A}^{(1)}(Q; 1 \dots; P) \right) =$$

$$\sum_{\lambda_1 \lambda_2} \mathcal{A}^{(0)}(Q; -l_1^{-\lambda_1}; -l_2^{-\lambda_2}) \mathcal{A}^{(0)}(l_2^{\lambda_2}; l_1^{\lambda_1}; 1 \dots n; P)$$

External self energies

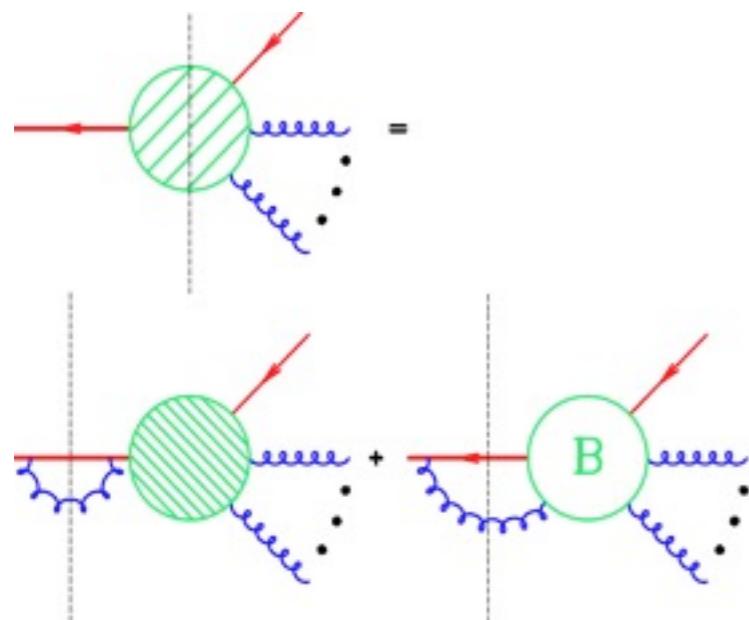
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External self energies

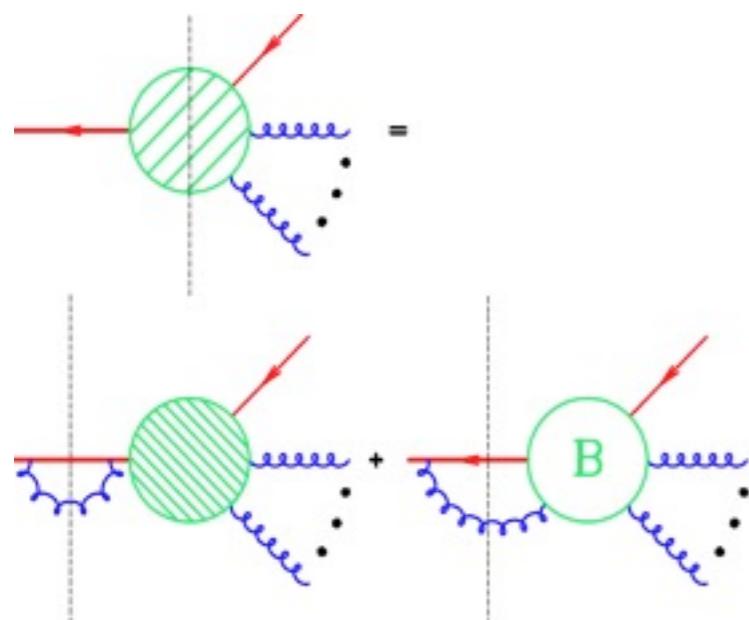
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$$\mathcal{A}^{[0]}(t^*, g^*, g_1, \dots, g_n, \bar{t}) = \frac{R(t^*, g^*, g_1, \dots, g_n, \bar{t})}{(p_{t^*} + p_{g^*})^2 - m_t^2} + B(t^*, g^*, g_1, \dots, g_n, \bar{t}).$$

Separation of pole terms is consistent with BG recursion relations

$$J(12 \cdots n; P) = \frac{1}{\cancel{P} + \cancel{K}_1 + \cdots + \cancel{K}_n + m} \sum_{k=1}^n J(1 \cdots k) J(K+1 \cdots n; P)$$

$$\mathcal{A}^{(0)}(Q; 12 \cdots n; P) = \bar{u}(Q) (\cancel{P} + \cancel{K}_1 + \cdots + \cancel{K}_n + m) J(12 \cdots n; P)$$

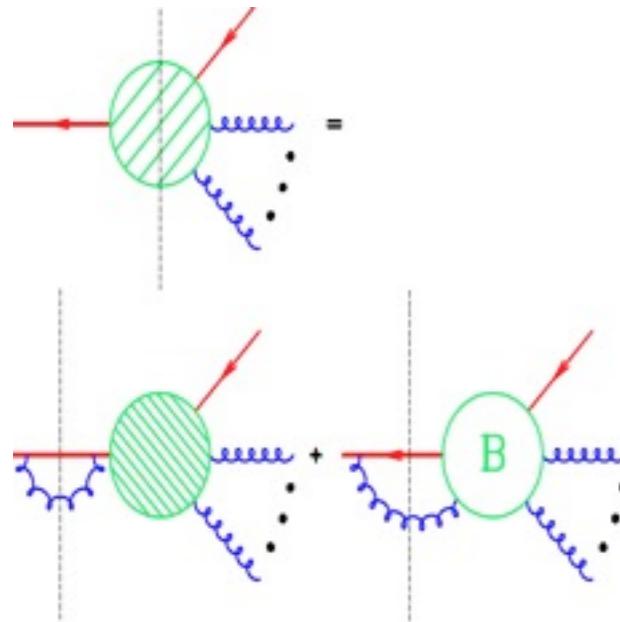
right hand side of the cut:

$$\mathcal{A}^{(0)}(l_2^{\lambda_2}; l_1^{\lambda_1}; 1 \cdots n; P) = \bar{u}(l_2^{\lambda_2}) \not{d}(l_1^{\lambda_1}) J(1 \cdots n; P) + \sum_{k=1, \dots, n} \bar{u}(l_2^{\lambda_2}) J(l_1^{\lambda_1} 1 \cdots k) J((k+1) \cdots n; P)$$

$$\mathcal{A}^{[0]}(t^*, g^*, g_1, \dots, g_n, \bar{t}) = \frac{R(t^*, g^*, g_1, \dots, g_n, \bar{t})}{(p_{t^*} + p_{g^*})^2 - m_t^2} + B(t^*, g^*, g_1, \dots, g_n, \bar{t}).$$

If sum over four polarization states for the cut gluon line the first term is the standard self energy correction in Feynman-gauge

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right hand side of the cut:

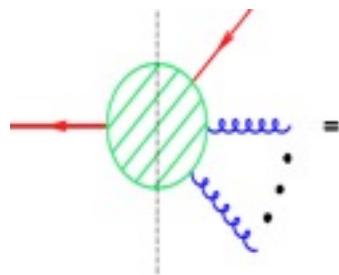
$$\mathcal{A}^{(0)}(l_2^{\lambda_2}; l_1^{\lambda_1}; 1 \cdots n; P) = \bar{u}(l_2^{\lambda_2}) \not{d}(l_1^{\lambda_1}) J(1 \cdots n; P) + \sum_{k=1, \dots, n} \bar{u}(l_2^{\lambda_2}) J(l_1^{\lambda_1} 1 \cdots k) J((k+1) \cdots n; P)$$

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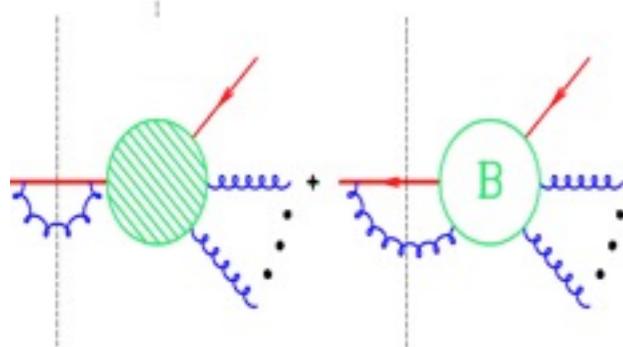
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Separation of pole terms is consistent with BG recursion relations

Tree amplitudes with spinorial recursion relations



$$J(12 \cdots n; P) = \frac{1}{\cancel{P} + \cancel{K}_1 + \cdots + \cancel{K}_n + m} \sum_{k=1}^n J(1 \cdots k) J(K+1 \cdots n; P)$$



$$\mathcal{A}^{(0)}(Q; 12 \cdots n; P) = \bar{u}(Q)(\cancel{P} + \cancel{K}_1 + \cdots + \cancel{K}_n + m) J(12 \cdots n; P)$$

right hand side of the cut:

$$\mathcal{A}^{(0)}(l_2^{\lambda_2}; l_1^{\lambda_1}; 1 \cdots n; P) = \bar{u}(l_2^{\lambda_2}) \not{e}(l_1^{\lambda_1}) J(1 \cdots n; P) + \sum_{k=1, \dots, n} \bar{u}(l_2^{\lambda_2}) J(l_1^{\lambda_1} 1 \cdots k) J((k+1) \cdots n; P)$$

$$\mathcal{A}^{[0]}(t^*, g^*, g_1, \dots, g_n, \bar{t}) = \frac{R(t^*, g^*, g_1, \dots, g_n, \bar{t})}{(p_{t^*} + p_{g^*})^2 - m_t^2} + B(t^*, g^*, g_1, \dots, g_n, \bar{t}).$$

If sum over four polarization states for the cut gluon line the first term is the standard self energy correction in Feynman-gauge

Comment on color treatment

Tree level:

BG recursion relations for colorless ordered amplitudes
different color basis (T-basis, F-basis, mixed basis, color-flow basis)

recursion relations for color dressed amplitude

One loop:

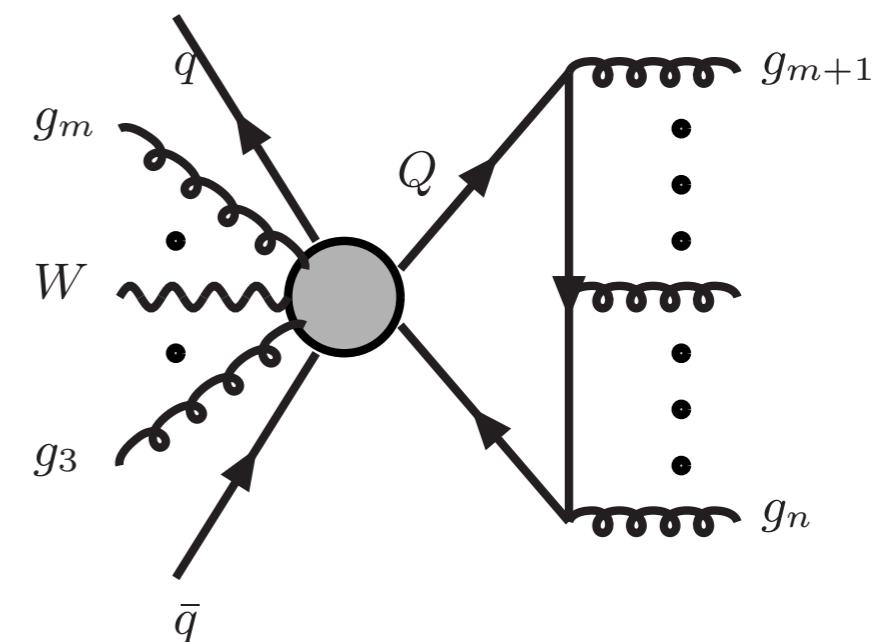
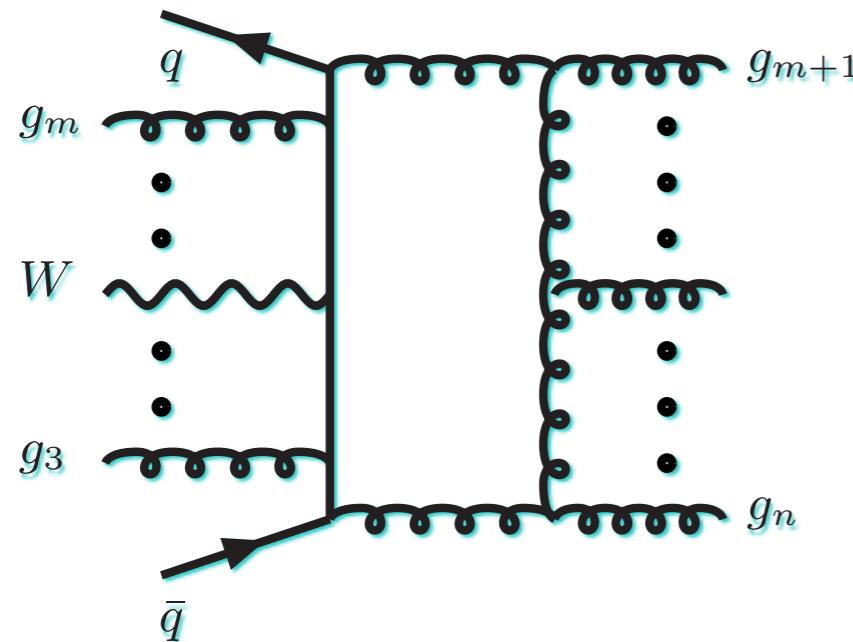
Decomposition to color and flavor ordered partial amplitudes
qbar+q gluon: mixed basis color decomposition

$(s + s\bar{ }) + (t + t\bar{ }) + n_{\text{gluon}}$

of colorless amplitude is less than # of primitive amplitudes
use color-flow basis and dressed recursion relations

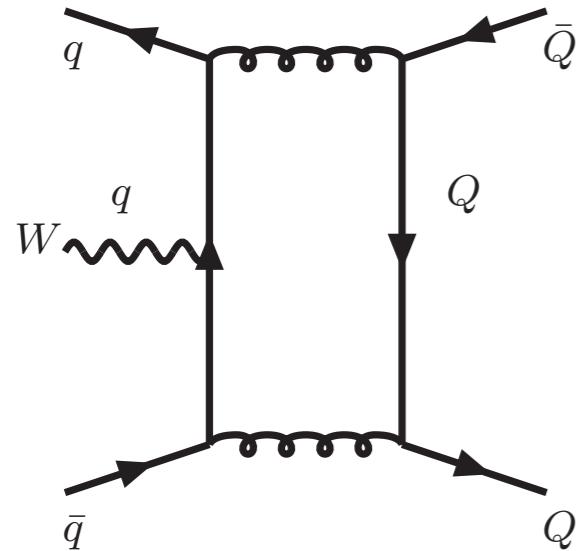
Primitive amplitudes W+3jets

Parent diagrams for primitive amplitudes

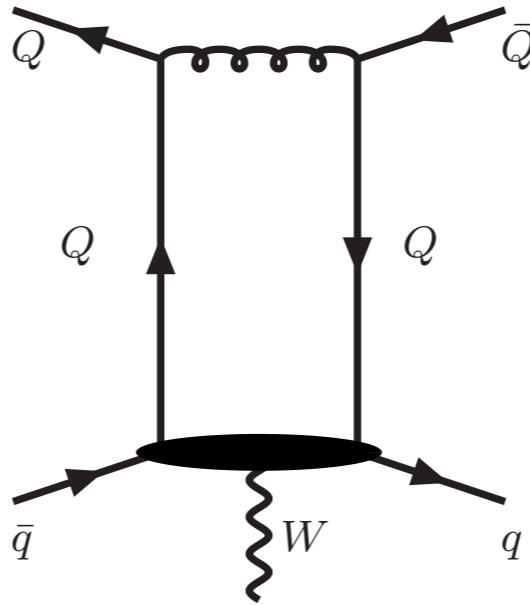


Parent diagrams for four quarks

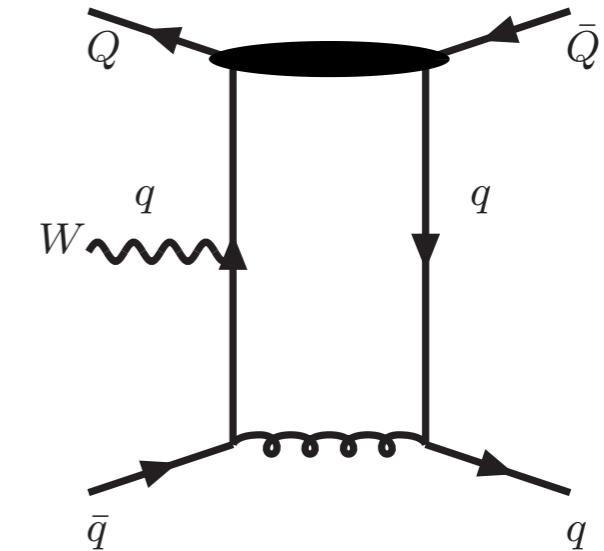
proliferation of ordered partial amplitudes



a)

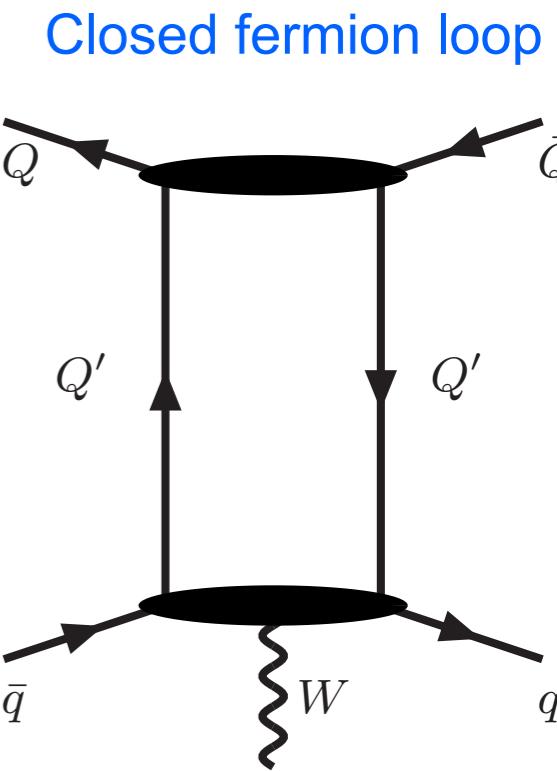


c)



b)

less than 5 propagators
in the loop



All use color ordered and primitive amplitudes

$$\mathcal{B}^{\text{1-loop}}(1_{\bar{q}}, 2_q, 3_{\bar{Q}}, 4_Q, 5_g) = g^5 \left[N_c (T^{a_5})_{i_4}^{\bar{i}_1} \delta_{i_2}^{\bar{i}_3} B_{7;1} + (T^{a_5})_{i_2}^{\bar{i}_1} \delta_{i_4}^{\bar{i}_3} B_{7;2} \right. \\ \left. + N_c (T^{a_5})_{i_2}^{\bar{i}_3} \delta_{i_4}^{\bar{i}_1} B_{7;3} + (T^{a_5})_{i_4}^{\bar{i}_3} \delta_{i_2}^{\bar{i}_1} B_{7;4} \right].$$

$$B_{7;1}^{[1],a} = \left(1 - \frac{1}{N_c^2} \right) A_L^{[1],a}(1_{\bar{q}}, 2_q, 3_{\bar{Q}}, 4_Q, 5_g) - \frac{1}{N_c^2} \left(-A_L^{[1],a}(1_{\bar{q}}, 5_g, 2_q, 3_{\bar{Q}}, 4_Q) \right. \\ \left. - A_L^{[1],a}(1_{\bar{q}}, 5_g, 2_q, 4_Q, 3_{\bar{Q}}) - A_L^{[1],a}(1_{\bar{q}}, 2_q, 5_g, 3_{\bar{Q}}, 4_Q) - A_L^{[1],a}(1_{\bar{q}}, 2_q, 5_g, 4_Q, 3_{\bar{Q}}) \right. \\ \left. - A_L^{[1],a}(1_{\bar{q}}, 2_q, 3_{\bar{Q}}, 5_g, 4_Q) - A_L^{[1],a}(1_{\bar{q}}, 2_q, 4_Q, 5_g, 3_{\bar{Q}}) + A_L^{[1],a}(1_{\bar{q}}, 2_q, 4_Q, 3_{\bar{Q}}, 5_g) \right), \quad (5.6)$$

$$B_{7;2}^{[1],a} = +A_L^{[1],a}(1_{\bar{q}}, 2_q, 5_g, 4_Q, 3_{\bar{Q}}) - A_L^{[1],a}(1_{\bar{q}}, 2_q, 4_Q, 5_g, 3_{\bar{Q}}) + A_L^{[1],a}(1_{\bar{q}}, 2_q, 4_Q, 3_{\bar{Q}}, 5_g) \\ - \frac{1}{N_c^2} \left(A_L^{[1],a}(1_{\bar{q}}, 5_g, 2_q, 3_{\bar{Q}}, 4_Q) + A_L^{[1],a}(1_{\bar{q}}, 5_g, 2_q, 4_Q, 3_{\bar{Q}}) \right), \quad (5.7)$$

$$B_{7;3}^{[1],a} = \left(1 - \frac{1}{N_c^2} \right) A_L^{[1],a}(1_{\bar{q}}, 2_q, 5_g, 3_{\bar{Q}}, 4_Q) - \frac{1}{N_c^2} \left(-A_L^{[1],a}(1_{\bar{q}}, 5_g, 2_q, 3_{\bar{Q}}, 4_Q) \right. \\ \left. - A_L^{[1],a}(1_{\bar{q}}, 5_g, 2_q, 4_Q, 3_{\bar{Q}}) - A_L^{[1],a}(1_{\bar{q}}, 2_q, 3_{\bar{Q}}, 5_g, 4_Q) - A_L^{[1],a}(1_{\bar{q}}, 2_q, 4_Q, 5_g, 3_{\bar{Q}}) \right. \\ \left. - A_L^{[1],a}(1_{\bar{q}}, 2_q, 3_{\bar{Q}}, 4_Q, 5_g) - A_L^{[1],a}(1_{\bar{q}}, 2_q, 4_Q, 3_{\bar{Q}}, 5_g) + A_L^{[1],a}(1_{\bar{q}}, 2_q, 5_g, 4_Q, 3_{\bar{Q}}) \right), \quad (5.8)$$

$$B_{7;4}^{[1],a} = -A_L^{[1],a}(1_{\bar{q}}, 5_g, 2_q, 4_Q, 3_{\bar{Q}}) - A_L^{[1],a}(1_{\bar{q}}, 2_q, 5_g, 4_Q, 3_{\bar{Q}}) - A_L^{[1],a}(1_{\bar{q}}, 2_q, 4_Q, 3_{\bar{Q}}, 5_g) \\ - \frac{1}{N_c^2} \left(A_L^{[1],a}(1_{\bar{q}}, 2_q, 4_Q, 5_g, 3_{\bar{Q}}) + A_L^{[1],a}(1_{\bar{q}}, 2_q, 3_{\bar{Q}}, 5_g, 4_Q) \right). \quad (5.9)$$

Dressed recursive technique for tree amplitudes

Draggiotis, Kleiss, Papadopoulos, Duhr, Maltoni, Comix:Gleisberg, Hoche

Monte-Carlo sampling over flavor, helicity, color, momentum quantum numbers of external sources

$$\mathbf{f}_i^{(r)} = \{f_i, h_{f_i}, C_{f_i}, K_i\}^{(r)} ,$$

$$d\sigma_{LO}(f_1 f_2 \rightarrow f_3 \cdots f_n) = \frac{W_S}{N_{\text{event}}} \times \sum_{r=1}^{N_{\text{event}}} dPS^{(r)}(K_1 K_2 \rightarrow K_3 \cdots K_n) \left| \mathcal{M}^{(0)} \left(\mathbf{f}_1^{(r)}, \mathbf{f}_2^{(r)}, \dots, \mathbf{f}_n^{(r)} \right) \right|^2$$

$$\mathcal{M}^{(0)}(\mathbf{f}_1, \dots, \mathbf{f}_n) = P^{-1} [J(\mathbf{f}_1, \dots, \mathbf{f}_{n-1}), J(\mathbf{f}_n)] .$$

Recursion relations are defined in terms of currents with n-1 on-shell external legs and one off-shell leg

$$P [J(\{\mathbf{f}\}_{\pi_1}), J(\{\mathbf{f}\}_{\pi_2})] = \sum_{\mathbf{g}_1 \mathbf{g}_2} J_{\mathbf{g}_1}(\{\mathbf{f}\}_{\pi_1}) P^{\mathbf{g}_1 \mathbf{g}_2}(K_{\pi_1}) J_{\mathbf{g}_2}(\{\mathbf{f}\}_{\pi_2})$$
$$\mathbf{g} = \{g, L_g, C_g, K_g\}$$

Instead of ordering we have partitioning

Generic method for any field theory: dressed recursive technique for one-loop amplitudes

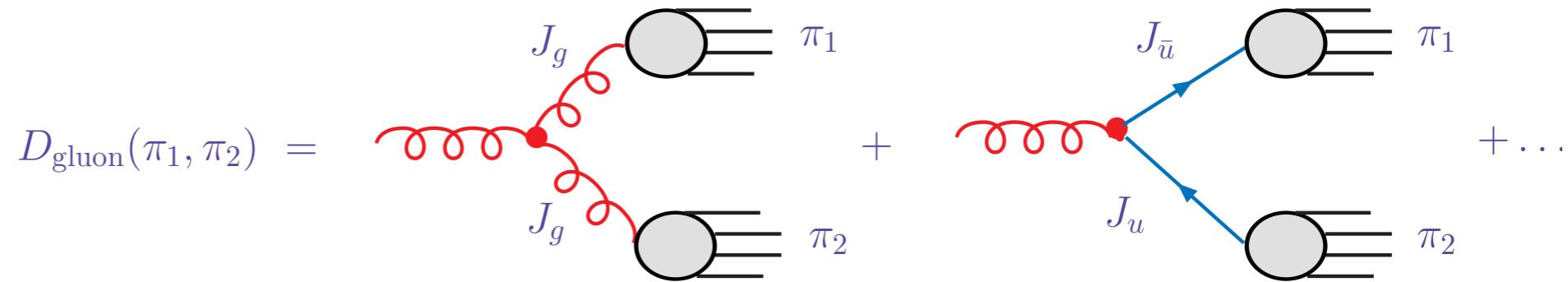
Giele, ZK, Winter

Monte-Carlo sampling over quantum numbers of external sources

$$d\sigma^{(V)}(f_1 f_2 \rightarrow f_3 \cdots f_n) = \frac{W_S}{N_{\text{event}}} \times \sum_{k=1}^{N_{\text{event}}} dPS^{(k)}(K_1 K_2 \rightarrow K_3 \cdots K_n) \\ 2\Re \left(\mathcal{M}^{(0)} \left(\mathbf{f}_1^{(k)}, \dots, \mathbf{f}_n^{(k)} \right)^\dagger \times \mathcal{M}^{(1)} \left(\mathbf{f}_1^{(k)}, \dots, \mathbf{f}_n^{(k)} \right) \right)$$

$$\mathcal{M}^{(1)} (\mathbf{f}_1, \dots, \mathbf{f}_n) = \int \frac{d^D \ell}{(2\pi)^D} \mathcal{A}^{(1)} (\mathbf{f}_1, \dots, \mathbf{f}_n \mid \ell)$$

Generic recursion with generic vertices for tree amplitudes



Vertices:

$$D_{\mathbf{g}_1 \dots \mathbf{g}_k}(Q_1, \dots, Q_k) = D_{g_1, \dots, g_k; C_{g_1} \dots, C_{g_k}}^{L_{g_1} \dots L_{g_k}}(Q_1, \dots, Q_k).$$

$$D_{\mathbf{g}} [J(\mathbf{f}(\pi_1)), \dots, J(\mathbf{f}(\pi_k))] = \sum_{\mathbf{g}_1 \dots \mathbf{g}_k} D_{\mathbf{g}\mathbf{g}_1 \dots \mathbf{g}_k}(-K_{\Pi_k}, K_{\pi_1}, \dots, K_{\pi_k}) \times J^{\mathbf{g}_1}(\mathbf{f}_{\pi_1}) \times \dots \times J^{\mathbf{g}_k}(\mathbf{f}_{\pi_k}),$$

$\Pi_k = \bigcup_{i=1}^k \pi_i$

BG recursion relations:

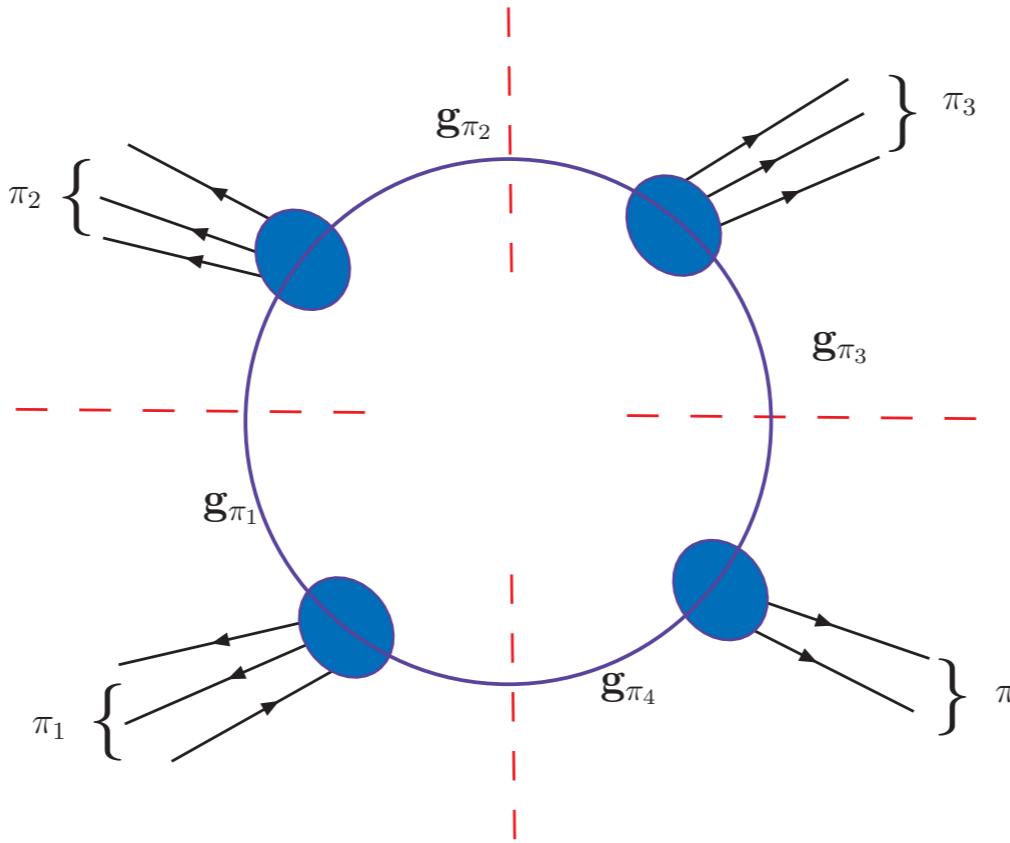
$$J_{\mathbf{g}}(\mathbf{f}_i) = \delta^{g f_i} \delta^{C_g C_{f_i}} S_{f_i L_g}^{h_{f_i} C_{f_i}}(K_i).$$

Stirling
number of 2nd kind

$$J_{\mathbf{g}}(\mathbf{f}_1, \dots, \mathbf{f}_n) = \sum_{k=2}^{V_{\max}-1} \sum_{P_{\pi_1 \dots \pi_k}(1, \dots, n)}^{\mathcal{S}_2(n, k)} P_{\mathbf{g}} \left[D[J(\mathbf{f}_{\pi_1}), \dots, J(\mathbf{f}_{\pi_k})] \right],$$

all possible partitions of n particle into k subsets is given by the Stirling number of second kind $S_2(n, k)$.

Generalized OPP parametrization



- **Sum over ordered cuts changes to sum over partitions including non-cyclic, non-reflective permutations of the external partons**

$$\mathcal{A}^{(1)}(\mathbf{f}_1, \dots, \mathbf{f}_n \mid \ell) = \sum_{k=1}^{C_{\max}} \sum_{RP_{\pi_1 \dots \pi_k}(1,2,\dots,n)}^{\max\left(1, \frac{1}{2}(k-1)!\right) \mathcal{S}_2(n,k)} \sum_{g_{\Pi_1}, \dots, g_{\Pi_k}} \frac{\mathcal{P}_k \left(\vec{C}_{g_{\Pi_1} \dots g_{\Pi_k}} \mid \ell \right)}{d_{g_{\Pi_1}}(\ell) d_{g_{\Pi_2}}(\ell) \dots d_{g_{\Pi_k}}(\ell)},$$

$$\Pi_k = \bigcup_{i=1}^k \pi_i$$

- sum over the propagator flavors $g_{\Pi_1}, \dots, g_{\Pi_k}$ is required as these are not uniquely defined for unordered amplitudes

Sum over ordered cuts changes to sum over partitions including non-cyclic, non-reflective permutations of the external partons

Number of the cuts is higher.

$$\mathcal{A}^{(1)}(\mathbf{f}_1, \dots, \mathbf{f}_n \mid \ell) = \sum_{k=1}^{C_{\max}} \sum_{RP_{\pi_1 \dots \pi_k}(1,2,\dots,n)}^{\max\left(1, \frac{1}{2}(k-1)!\right) S_2(n,k)} \sum_{g_{\Pi_1}, \dots, g_{\Pi_k}} \frac{\mathcal{P}_k \left(\vec{C}_{g_{\Pi_1} \dots g_{\Pi_k}} \mid \ell \right)}{d_{g_{\Pi_1}}(\ell) d_{g_{\Pi_2}}(\ell) \dots d_{g_{\Pi_k}}(\ell)},$$

Unordered cuts.									
#	pentagon cuts	box cuts	triangle cuts	bubble cuts	sum \equiv total	sum _n	sum _{n-1}	ordr total/unordr total	
n								orderings	(ab) _k (cd) _k
4	0	3	6	3	12			2.750	1.833 2.750
5	12	30	25	10	77	6.42		4.052	2.026 2.364
6	180	195	90	25	490	6.36		6.857	2.743 2.514
7	1,680	1,050	301	56	3,087	6.30		13.06	4.354 1.451
8	12,600	5,103	966	119	18,788	6.09		28.17	8.048 1.610
9	83,412	23,310	3,025	246	109,993	5.85		68.18	17.05 2.557
10	510,300	102,315	9,330	501	622,446	5.66		182.8	40.61 2.708
11	2,960,760	437,250	28,501	1,012	3,427,523	5.51		535.7	107.1
12	16,552,800	1,834,503	86,526	2,035	18,475,864	5.39		1699	308.9

$$\frac{(n-1)!}{2} \rightarrow S_2(n, 5) \approx 5^n$$

Calculating electroweak corrections with generalized D-dimensional unitarity?

Two “new” features:

- Unordered amplitudes
- Finite width effects:

Denner, Dittmaier: Scalar one-loop 4-point integrals
with complex internal masses

Conclusions

- ❖ The results of the last several years obtained in the field of PQCD framework for hard scattering processes is very impressive and has a fundamentally improves the analysis of the collider data.
- ❖ Construction of computer codes for automated numerical evaluation of NLO QCD correction for multi-leg processes is under way.
- ❖ Different competing proposals are under construction and study. A trade-off between speed and flexibility is an issue. With improved computer power (with GPU's) flexibility will be more and more important
- ❖ Results for W/Z+3jet, W/Z+4jets, ttbar+1jet, ttbar+2jets,... cross-sections. Many new interesting features emerged:
scale choice, non-universal K-factors, spin correlation effects.

Many interesting projects for the future:

- Fully automated NLO packages
- Interaction with SCET experts
- Interaction with Shower MC experts (MC@NLO, POWHEG)
- Interaction with the experimentalist colleagues
- Electroweak corrections with unitarity cut method
- NLO QCD corrections to new physics models
- Unitarity cut method for two loop applications
- Improved and more generalized recursive methods
- Improved subtraction methods and phase space integration methods
-

OPP and/or D-dimensional unitarity is used by

Ossola, Papadopoulos Pitrau, Ellis, Giele, Kunszt, Melnikov, Zanderighi, Lazopoulos, Winter, Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower, Maitre, Mastrolia, Tramontano, Greiner, Guffanti, Guillet, Reiter, Reuter, Czakon, Bevilacqua, Worek, Draggiotis, Garzelli,....

Codes:

Helac-Phegas, Cut-Tools, Helac-1L,
Black Hat, Rocket, Melnikov-Schultze, Lazopoulos, Winter, (with Giele),
Winter (with Giele ZK), Giele,(with Stavenga, Winter),
Samurai (Mastrolia, Ossola, Reiter, Tramontano,...

Perturbative higher-order effects at work at the LHC - HO10

C E R N T H E O R Y I N S T I T U T E

June 21 - July 9

Topics will include:

- * Progress in, and prospects for, attacking the yet uncalculated processes considered priorities at the LHC, and revisiting the NLO priority list formulated at Les Houches in order to target the experimental accuracies.
- * The role of NLO and NNLO QCD calculations for Higgs boson searches and coupling measurements.
- * Integrating automated NLO calculations into parton showers.
- * The role of NLO electroweak corrections and merging of QCD and electroweak corrections for benchmark processes such as W and Z production.
- * The role of (semi-)analytical resummations at the LHC, and their comparisons with those provided by Monte Carlos.
- * Key signatures of new physics at the LHC and challenging discovery modes in exotic models.
- * Processes, such as top-quark pair and di-boson production, for which NNLO QCD is desirable.
- * The status of parton distribution function errors and prospects for their improvement with LHC data.