Processing one loop virtual corrections with SAMURAI

Francesco Tramontano
CERN Theory Group

work done in collaboration with
P. Mastrolia, G. Ossola and T. Reiter

OVERVIEW

- Introduction
- Methods
- Running SAMURAI
- Examples
- Conclusions and outlook

Introduction

- LHC successfully started collisions at 7 TeV on March 30th 2010

- The need of Next to Leading Order (NLO) multi-particle scattering predictions is more pressing

- New ideas in the field of loop corrections are giving the possibility to perform the automatic generation of NLO predictions for multi-leg processes

State of the art

Analytic calculations:

- W/Z/γ + 2jets Bern et al (1998)
- H + 2jets (eff. coupling) Badger, Berger, Campbell, Del Duca, Dixon, Ellis, Glover, Mastrolia, Risager, Sofianatos, Williams (2006-2009)

Numerical calculations:

- EW corr. e+e- > 4 fermions Denner and Dittmaier (2005)
- pp > W + 3jets Ellis et al, Berger et al (2009)
- pp > Z + 3jets Berger et al (2009)
- pp > ttbb Bredenstein et al, Bevilacqua et al (2009)
- pp > tt +2jets Czakon et al (2010)
- pp > 4b Binoth et al (2010)

We combined some of the recent techniques into a new computer program we called SAMURAI

Basic features of SAMURAI:

Scattering AMplitudes from Unitarity based Reduction Algorithm at Integrand level

- Is a fortran90 library for the calculation of the one loop corrections downloadable at the URL: www.cern.ch/samurai
- Main purpose was to provide a flexible and easy to use tool for the evaluation of the virtual corrections
- It works with any number/kind of legs
- Can process integrands written either as numerator of Feynman diagrams or as product of tree level amplitudes
- Rational terms are produced together with the cut-constructible one

And further:

- SAMURAI can be compiled in 2x or 4x precision
- It has a modular structure that allows for quick local updates
- It could also be useful to perform fast numerical check of analytic results
- Details and examples of applications can be found in: arXiv:1006.0710

Methods

SAMURAI: a numerical implementation of the OPP/D-dimensional generalized unitarity cuts technique

$$\mathcal{A}_n = \int d^d\bar{q}\ A(\bar{q},\epsilon)\ ,$$

$$A(\bar{q},\epsilon) = \frac{\mathcal{N}(\bar{q},\epsilon)}{\bar{D}_0\bar{D}_1\cdots\bar{D}_{n-1}}\ , \qquad \mathcal{N}(\bar{q},\epsilon) = N_0(\bar{q}) + \epsilon N_1(\bar{q}) + \epsilon^2 N_2(\bar{q}).$$

$$\bar{D}_i = (\bar{q}+p_i)^2 - m_i^2 = (q+p_i)^2 - m_i^2 - \mu^2,$$

$$\bar{\not{q}} = \not{q} + \not{\mu} \qquad \bar{q}^2 = q^2 - \mu^2$$

- OPP polynomials (n-ple cut, n=1,2,3,4) extended to the framework of D-dim unitarity [Ellis, Giele, Kunszt, Melnikov]
- 5-ple cut residue depending only on mu2 [Melnikov, Schultze]
- Integrand sampling with DFT for 3-ple and 2-ple cuts [Mastrolia, Ossola, Papdopoulos, Pittau]

OPP integrand decomposition: 4-dim

The power of the OPP method is the fact that for each phase space point the only requirement for the reduction is the knowledge of the numerical value of the numerator function N for a finite set of values of the loop momentum variable, solutions of the multiple cut conditions

- Any amplitude can be expressed as a linear combination of scalar integrals: boxes, triangles, bubbles, tadpoles plus rational terms

$$\int A = \sum_{i_0<i_1<i_2<i_3}^{m-1} d(i_0 i_1 i_2 i_3) D_0(i_0 i_1 i_2 i_3) + \sum_{i_0<i_1<i_2}^{m-1} c(i_0 i_1 i_2) C_0(i_0 i_1 i_2)$$
$$+ \sum_{i_0<i_1}^{m-1} b(i_0 i_1) B_0(i_0 i_1) + \sum_{i_0}^{m-1} a(i_0) A_0(i_0) + \text{rational terms}$$

- At integrand level the structure is enriched by terms that integrate to zero

$$N(q) = \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d(i_0 i_1 i_2 i_3) + \tilde{d}(q; i_0 i_1 i_2 i_3)\right] \prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} D_i + \sum_{i_0<i_1<i_2}^{m-1} \left[c(i_0 i_1 i_2) + \tilde{c}(q; i_0 i_1 i_2)\right] \prod_{i\neq i_0,i_1,i_2}^{m-1} D_i$$
$$+ \sum_{i_0<i_1}^{m-1} \left[b(i_0 i_1) + \tilde{b}(q; i_0 i_1)\right] \prod_{i\neq i_0,i_1}^{m-1} D_i + \sum_{i_0}^{m-1} \left[a(i_0) + \tilde{a}(q; i_0)\right] \prod_{i\neq i_0}^{m-1} D_i$$

Extension to D-dim

- Once fixed a parametrization for the loop momentum in terms of a linear combination of known four-vectors (p_0, e_i) the vanishing term are polynomials of x_i and mu2

$$\bar{\not{q}} = \not{q} + \not{\mu} \qquad \bar{q}^2 = q^2 - \mu^2 \qquad q = -p_0 + x_1 e_1 + x_2 e_2 + x_3 e_3 + x_4 e_4$$

- The problem is to fit the coefficients of the Δ-polynomials

$$N(\bar{q}) = \sum_{i<<m}^{n-1} \Delta_{ijk\ell m}(\bar{q}) \prod_{h\neq i,j,k,\ell,m}^{n-1} \bar{D}_h + \sum_{i<<\ell}^{n-1} \Delta_{ijk\ell}(\bar{q}) \prod_{h\neq i,j,k,\ell}^{n-1} \bar{D}_h +$$
$$+ \sum_{i<<k}^{n-1} \Delta_{ijk}(\bar{q}) \prod_{h\neq i,j,k}^{n-1} \bar{D}_h + \sum_{i<j}^{n-1} \Delta_{ij}(\bar{q}) \prod_{h\neq i,j}^{n-1} \bar{D}_h + \sum_{i}^{n-1} \Delta_{i}(\bar{q}) \prod_{h\neq i}^{n-1} \bar{D}_h$$

For example the 3-ple cut residue (function of the unfrozen components) reads:

$$\Delta_{ijk}(\bar{q}) = c_{3,0}^{(ijk)} \boxed{+ c_{3,7}^{(ijk)} \mu^2} - \Big((c_{3,1}^{(ijk)} \boxed{+ c_{3,8}^{(ijk)} \mu^2}) x_4 + (c_{3,4}^{(ijk)} \boxed{+ c_{3,9}^{(ijk)} \mu^2}) x_3 \Big) (e_1 \cdot e_2) +$$
$$+ \Big(c_{3,2}^{(ijk)} x_4^2 + c_{3,5}^{(ijk)} x_3^2 \Big) (e_1 \cdot e_2)^2 - \Big(c_{3,3}^{(ijk)} x_4^3 + c_{3,6}^{(ijk)} x_3^3 \Big) (e_1 \cdot e_2)^3 \, .$$

5-ple cut residue

$$\Delta_{ijk\ell m}(\bar{q}) = c_{5,0}^{(ijk\ell m)} + c_{5,1}^{(ijk\ell m)}\ \mu^2 + c_{5,2}^{(ijk\ell m)}\ \mu^4$$

$$\int \frac{d^4p}{(2\pi)^4} \int \frac{d^{-2\epsilon}\mu}{(2\pi)^{-2\epsilon}}\ (\mu^2)^r f(p^\alpha, \mu^2) \ = -\epsilon(1-\epsilon)(2-\epsilon)\cdots(r-1-\epsilon)\ (4\pi)^r \int \frac{d^{4+2r-2\epsilon}P}{(2\pi)^{4+2r-2\epsilon}}\ f(p^\alpha, \mu^2)$$

$$I_n^{D=6-2\epsilon} = \frac{1}{(n-5+2\epsilon)c_0}\left[2I_n^{D=4-2\epsilon} - \sum_{i=1}^{n} c_i I_{n-1}^{(i),D=4-2\epsilon}\right]$$

$$I_n^{D=8-2\epsilon} = \frac{1}{(n-7+2\epsilon)c_0}\left[2I_n^{D=6-2\epsilon} - \sum_{i=1}^{n} c_i I_{n-1}^{(i),D=6-2\epsilon}\right]$$

Linear dependence

Best choice:

$$\Delta_{ijk\ell m}(\bar{q}) = +c_{5,1}^{(ijk\ell m)}\ \mu^2$$

- ✓ avoid scalar pentagon decomposition
- ✓ avoid pentagon subtraction for tadpoles
- ✓ numerically more stable

Numerical Sampling

$$P(x) = \sum_{\ell=0}^{n} c_\ell \, x^\ell$$

- ❑ Cut-5: completely frozen
- ❑ Cut-4: mu2 sampling
- ❑ Cut-3,2: mu2 sampling + DFT:
- ❑ Cut-1: trivial

1. generate the set of discrete values P_k $(k = 0, ..., n)$,

$$P_k = P(x_k) = \sum_{\ell=0}^{n} c_\ell \, \rho^\ell \, e^{-2\pi i \frac{k}{(n+1)} \ell}$$

by sampling $P(x)$ at the points

$$x_k = \rho \, e^{-2\pi i \frac{k}{(n+1)}}$$

2. using the orthogonality relation

$$\sum_{n=0}^{N-1} e^{2\pi i \frac{k}{N} \, n} \, e^{-2\pi i \frac{k'}{N} \, n} = N \, \delta_{kk'}$$

each coefficient c_ℓ finally reads,

$$c_\ell = \frac{\rho^{-\ell}}{n+1} \sum_{k=0}^{n} P_k \, e^{2\pi i \frac{k}{(n+1)} \ell}$$

- straightforward extension to multi-variate DFT projection
- Sampling on different circles for stable solutions
- number of the integrand samplings = number of the unknowns
- dynamical mu2-sampling

Amplitudes & Master Integrals

$$\mathcal{A}_n = \sum_{i<j<k<\ell}^{n-1} \left\{ c_{4,0}^{(ijk\ell)} I_{ijk\ell}^{(d)} + \frac{(d-2)(d-4)}{4} c_{4,4}^{(ijk\ell)} I_{ijk\ell}^{(d+4)} \right\}$$
$$+ \sum_{i<j<k}^{n-1} \left\{ c_{3,0}^{(ijk)} I_{ijk}^{(d)} - \frac{(d-4)}{2} c_{3,7}^{(ijk)} I_{ijk}^{(d+2)} \right\}$$
$$+ \sum_{i<j}^{n-1} \left\{ c_{2,0}^{(ij)} I_{ij}^{(d)} + c_{2,1}^{(ij)} J_{ij}^{(d)} + c_{2,2}^{(ij)} K_{ij}^{(d)} - \frac{(d-4)}{2} c_{2,9}^{(ij)} I_{ij}^{(d+2)} \right\}$$
$$+ \sum_{i}^{n-1} c_{1,0}^{(i)} I_i^{(d)}$$

$$\int d^d\bar{q} \frac{\bar{q}\cdot e_2}{\bar{D}_i \bar{D}_j} = J_{ij}^{(d)}$$
$$\int d^d\bar{q} \frac{(\bar{q}\cdot e_2)^2}{\bar{D}_i \bar{D}_j} = K_{ij}^{(d)}$$

The sources of rational terms are the integrals with mu2 powers in the numerator

They are generated by the reduction algorithm, but could also be present ab initio in the numerator function as a consequence of the algebraic manipulations

$$\int d^d\bar{q} \frac{\mu^2}{\bar{D}_i \bar{D}_j} = -\frac{(d-4)}{2} I_{ij}^{(d+2)}$$
$$\int d^d\bar{q} \frac{\mu^4}{\bar{D}_i \bar{D}_j \bar{D}_k \bar{D}_\ell} = \frac{(d-2)(d-4)}{4} I_{ijk\ell}^{(d+4)}$$
$$\int d^d\bar{q} \frac{\mu^2}{\bar{D}_i \bar{D}_j \bar{D}_k} = -\frac{(d-4)}{2} I_{ijk}^{(d+2)}$$

Running SAMURAI

calls:

```
call initsamurai(imeth,isca,verbosity,itest)
call InitDenominators(nleg,Pi,msq,v0,m0,v1,m1,...,vlast,mlast)
call samurai(xnum,tot,totr,Pi,msq,nleg,rank,istop,scale2,ok)
call exitsamurai
```

A dedicated module (kinematic) is also available in the release that contains useful functions to evaluate:

- ✓ Polarization vectors for massless vectors
- ✓ Scalar and spinor products with both real and complex four vectors as arguments

```
call initsamurai(imeth,isca,verbosity,itest)
```

- ✓ imeth = 'diag' for an integrand given as numerator of a Feynman diagram
  'tree' for an integrand given as the product of tree level amplitudes

- ✓ isca = 1, scalar integrals evaluated with the QCDLoop package (Ellis and Zanderighi)
  2, scalar integrals evaluated with the AVH-OLO package (van Hameren)

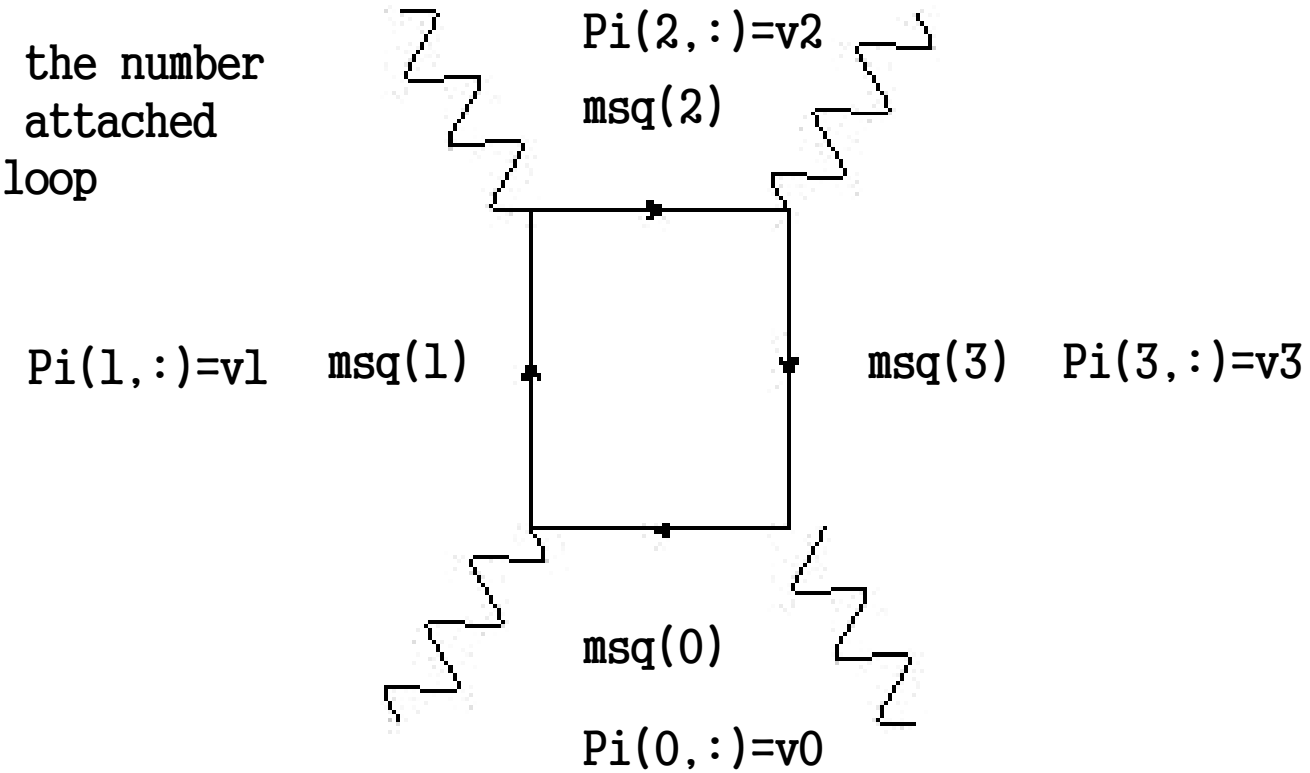
- ✓ verbosity = 0, nothing is printed by the reduction
  1, the coefficients are printed out
  2, also the value of the MI are printed out
  3, also the results of the tests are printed out

- ✓ itest = 0, none test
  1, global n=n test is performed (not avail. for imeth= 'tree')
  2, local n=n test is performed
  3, power test is performed (not avail. for imeth= 'tree')
  new - based on the mismatch of the polynomial degree of the given integrand and the reconstructed one

Optionally, to fill the denominators

```
call InitDenominators(nleg,Pi,msq,v0,m0,v1,m1,...,vlast,mlast)
```

nleg is the number of legs attached to the loop




```
Denominator(j) = [ q + Pi(j,:) ]^2 - mu2 - msq(j)
```

```
call samurai(xnum,tot,totr,Pi,msq,nleg,rank,istop,scale2,ok)
```

- ✓ xnum [i]= the name of the function to reduce with arguments xnum(cut, q, mu2) for imeth=tree the cut play a selective role to use the relative tree product
- ✓ tot [o] = contains the result of the reduction convoluted with the MI
- ✓ totr [o]= contains the rational part only
- ✓ rank [i] = the rank of the numerator, useful to speed up the reduction
- ✓ istop [i] = when stop the reduction, i.e. after pentuple cut (5) quadruple (4)…
- ✓ scale2 [i] = the value of the renormalization scale (square)
- ✓ ok [o] = a logical variable giving the result of the test if they are evaluated

About the precision

- ✓ Gram Determinant -> induce large cancellations between contributions from the MI that carry such a factor (the tests coded in SAMURAI detect the associated instabilities)

- ✓ Big cancellations between diagrams -> on-shell methods seems to be the best option

- ✓ If running with big internal masses -> big cancellations between cut-constructible and rational term

Quadruple precision solves these issues, but is time consuming

For numerical studies and checks SAMURAI compiles also in quad

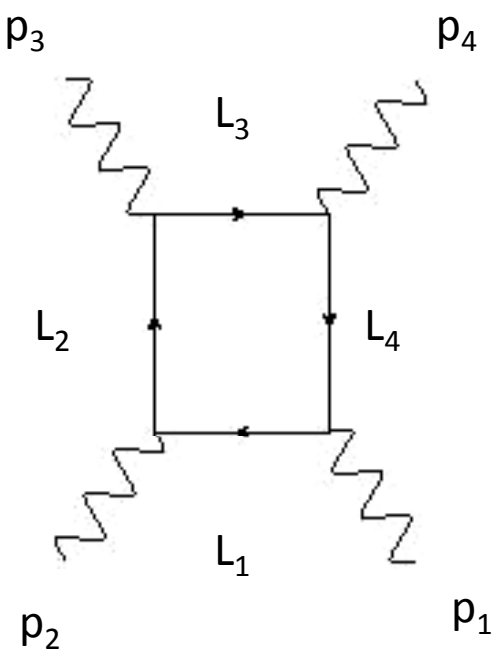
Examples

Note:

- Are chosen to address some typical technical issues that one encounter performing one loop virtual calculation
- Should show the flexibility of the framework
- Are part of the release and could also be used as templates for other calculations

4-photons

- imeth= 'diag'
- nleg = 4, rank = 4
- 6 permutations, only 3 relevant



$$\bar{L}_1 = \bar{q}\ ,\ \bar{L}_2 = \bar{q} + p_2\ ,\ \bar{L}_3 = \bar{q} + p_{23}\ ,\ \bar{L}_4 = \bar{q} + p_{234}$$

$$N(\bar{q}) = -\mathrm{Tr}\Big[(\bar{\not{L}}_1 + m)\,\not{e}_2\,(\bar{\not{L}}_2 + m)\,\not{e}_3\,(\bar{\not{L}}_3 + m)\,\not{e}_4\,(\bar{\not{L}}_4 + m)\,\not{e}_1\Big]$$

$$\begin{aligned} N(q,\mu^2) = & -(m^4 - \mu^2 m^2 + \mu^4)\,\mathrm{Tr}[\not{e}_2\,\not{e}_3\,\not{e}_4\,\not{e}_1] \\ & - (m^2 - \mu^2)\Big(\mathrm{Tr}[\not{e}_2\,\not{e}_3\,\not{e}_4\,\not{L}_4\,\not{e}_1\,\not{L}_1] + \mathrm{Tr}[\not{e}_2\,\not{e}_3\,\not{L}_3\,\not{e}_4\,\not{e}_1\,\not{L}_1] \\ & + \mathrm{Tr}[\not{e}_2\,\not{e}_3\,\not{L}_3\,\not{e}_4\,\not{L}_4\,\not{e}_1] + \mathrm{Tr}[\not{e}_2\,\not{L}_2\,\not{e}_3\,\not{e}_4\,\not{e}_1\,\not{L}_1] \\ & + \mathrm{Tr}[\not{e}_2\,\not{L}_2\,\not{e}_3\,\not{e}_4\,\not{L}_4\,\not{e}_1] + \mathrm{Tr}[\not{e}_2\,\not{L}_2\,\not{e}_3\,\not{L}_3\,\not{e}_4\,\not{e}_1]\Big) \\ & - \mathrm{Tr}[\not{L}_1\,\not{e}_2\,\not{L}_2\,\not{e}_3\,\not{L}_3\,\not{e}_4\,\not{L}_4\,\not{e}_1]\ , \end{aligned}$$

$$\mathtt{Pi} = (0, p_2, p_2 + p_3, p_2 + p_3 + p_4),\ \mathtt{msq} = (m^2, m^2, m^2, m^2)$$

Denominators:

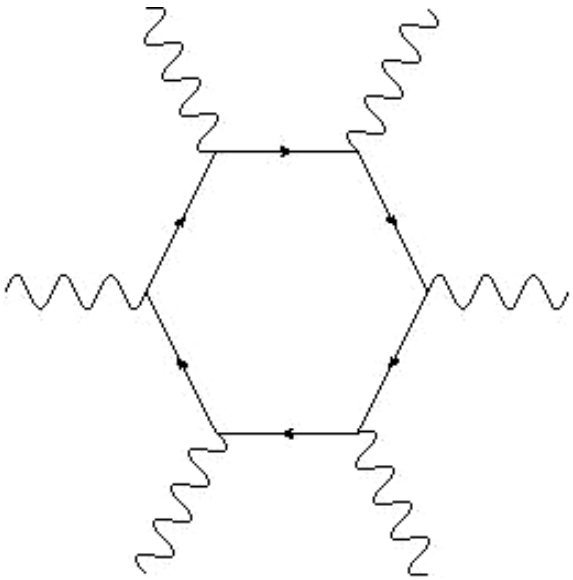
$$(\bar{L}_1^2 - m^2)\ (\bar{L}_2^2 - m^2)\ (\bar{L}_3^2 - m^2)\ (\bar{L}_4^2 - m^2)$$

- mu2 terms give zero contribution
- mu2 $q^\alpha q^\beta$ cancel in the sum
- mu2^2 gives rise to the correct rational part

Results numerically checked vs. Gounaris et al (1999)

6-photons

- imeth = 'diag'
- nleg = 6, rank = 6
- 120 permutations, only 60 relevant



$$N(q,\mu^2) = N(q) = -\mathrm{Tr}\Big[\not{L}_1 \not{\epsilon}_2 \not{L}_2 \not{\epsilon}_3 \not{L}_3 \not{\epsilon}_4 \not{L}_4 \not{\epsilon}_5 \not{L}_5 \not{\epsilon}_6 \not{L}_6 \not{\epsilon}_1\Big].$$

Bernicot et al (2007,2008)

$$\frac{s}{\alpha^3} A(-,-,+,+,+,+) = 11075.04009210435 \ ,$$

$$\frac{s}{\alpha^3} A(+,-,-,+,+,-) = 7814.762085902767 \ ,$$

SAMURAI with istop=2

$$\frac{s}{\alpha^3} A(-,-,+,+,+,+) = \underline{11075.040}174990 \ ,$$

$$\frac{s}{\alpha^3} A(+,-,-,+,+,-) = \underline{7814.7623}429908 \ .$$

SAMURAI with istop=3, subtracting totr

$$\frac{s}{\alpha^3} A(-,-,+,+,+,+) = \underline{11075.040092102} \ ,$$

$$\frac{s}{\alpha^3} A(+,-,-,+,+,-) = \underline{7814.7620859}084 \ .$$

PS point as in Nagy and Soper (2006)

$$\vec{p}_3 = (33.5, 15.9, 25.0)$$

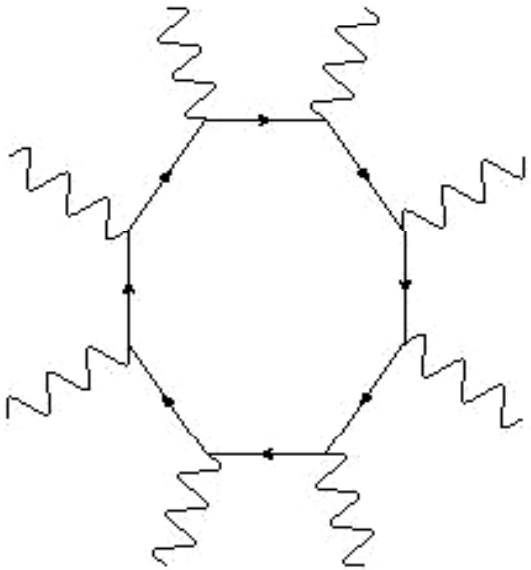
$$\vec{p}_4 = (-12.5, 15.3, 0.3)$$

$$\vec{p}_5 = (-10.0, -18.0, -3.3)$$

$$\vec{p}_6 = (-11.0, -13.2, -22.0)$$

Results numerically checked vs. Bernicot et al (2007,2008)

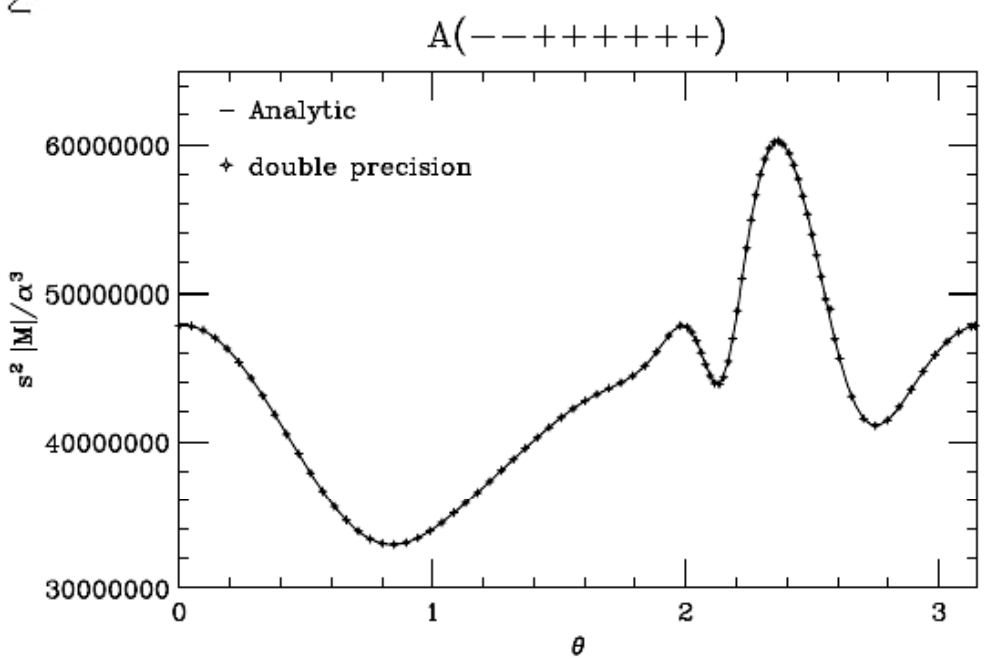
8-photons



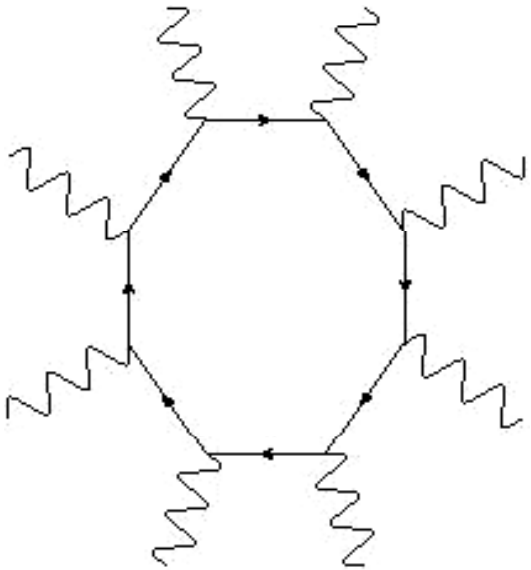
- imeth = 'diag'
- nleg = 8, rank = 8
- 5040 permutations, only 2520 relevant
- sampling set as in Gong et al (2008)

$$N(q,\mu^2) = -\mathrm{Tr}\left[\not{L}_0 \not{\epsilon}_1 \not{L}_1 \not{\epsilon}_2 \not{L}_2 \not{\epsilon}_3 \not{L}_3 \not{\epsilon}_4 \not{L}_4 \not{\epsilon}_5 \not{L}_5 \not{\epsilon}_6 \not{L}_6 \not{\epsilon}_7 \not{L}_7 \not{\epsilon}_8\right]$$

MHV result numerically checked vs. Mahlon (1993)



8-photons

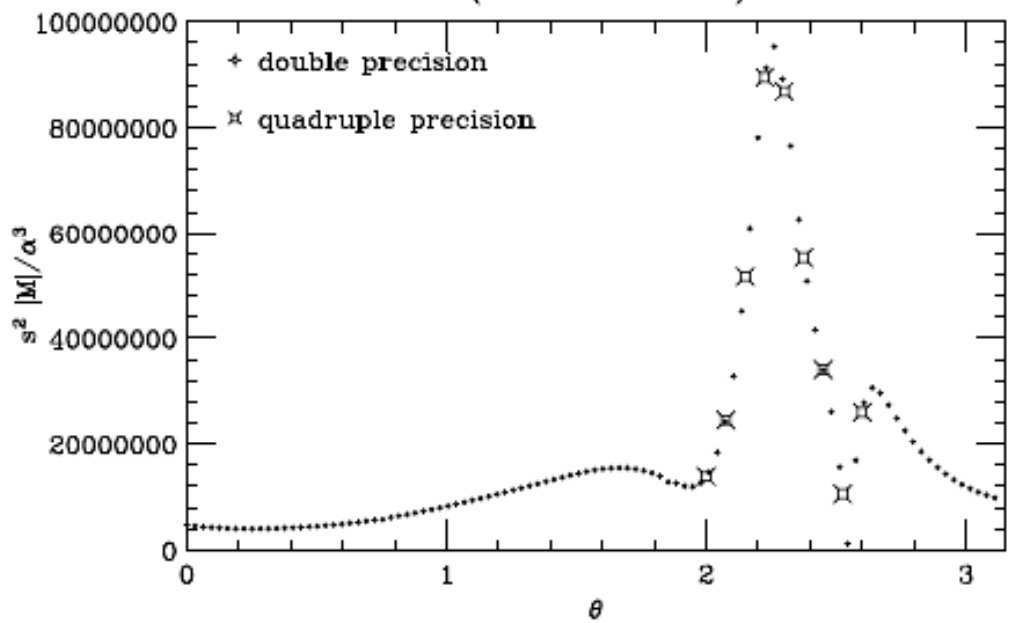


- imeth = 'diag'
- nleg = 8, rank = 8
- 5040 permutations, only 2520 relevant
- sampling set as in Gong et al (2008)

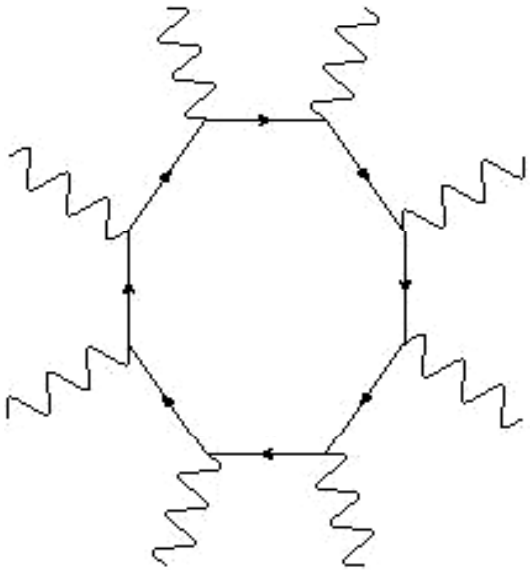
$$N(q,\mu^2) = -\mathrm{Tr}\left[\not{L}_0\not{\epsilon}_1\not{L}_1\not{\epsilon}_2\not{L}_2\not{\epsilon}_3\not{L}_3\not{\epsilon}_4\not{L}_4\not{\epsilon}_5\not{L}_5\not{\epsilon}_6\not{L}_6\not{\epsilon}_7\not{L}_7\not{\epsilon}_8\right]$$

NMHV result (new) numerically confirm the structure in Badger et al (2009)
The points in quadruple precision (x) have been calculated with istop=2, i.e. retaining all the cut constructible and rational pieces

$A(-++-++-+)$



8-photons

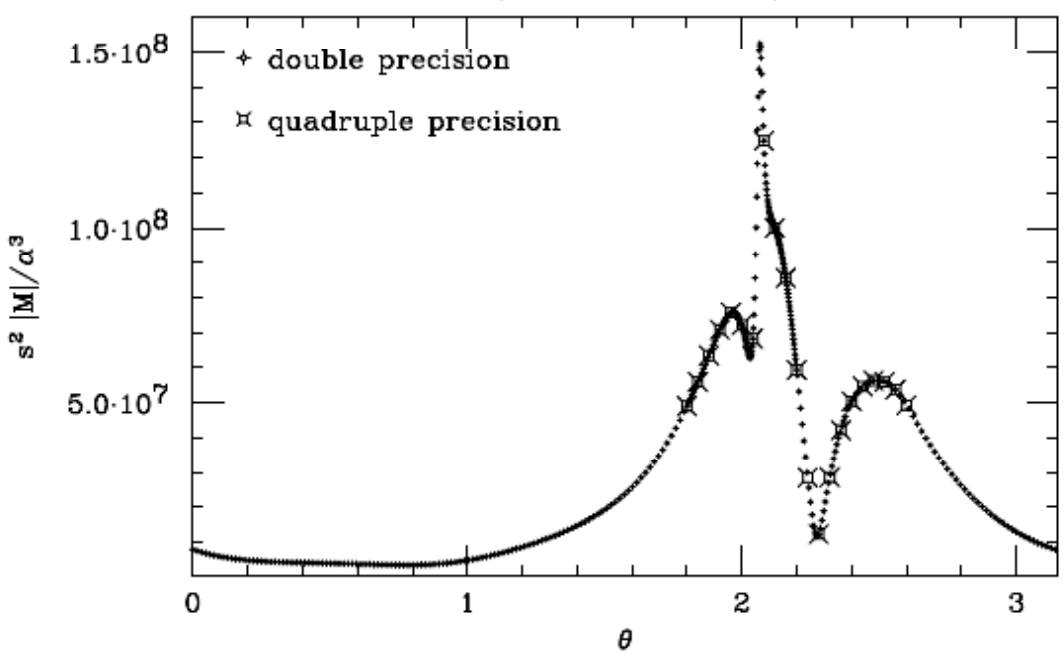


- imeth = 'diag'
- nleg = 8, rank = 8
- 5040 permutations, only 2520 relevant
- sampling set as in Gong et al (2008)

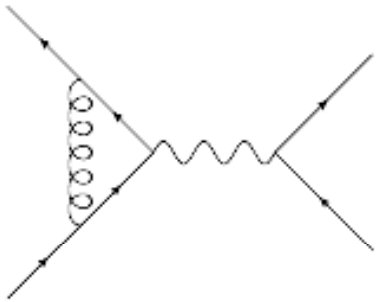
$$N(q,\mu^2) = -\mathrm{Tr}\left[\not{L}_0 \not{\epsilon}_1 \not{L}_1 \not{\epsilon}_2 \not{L}_2 \not{\epsilon}_3 \not{L}_3 \not{\epsilon}_4 \not{L}_4 \not{\epsilon}_5 \not{L}_5 \not{\epsilon}_6 \not{L}_6 \not{\epsilon}_7 \not{L}_7 \not{\epsilon}_8\right]$$

NNMHV result (new) numerically confirm the structure in Badger et al (2009)
The points in quadruple precision (x) have been calculated with istop=2, i.e. retaining all the cut constructible and rational pieces

$A(----++++)$



Drell-Yan



If one want to consider regularization schemes giving rise to O(ε) terms and reduce them, then one needs to process N_0 and N_1 below separately

$$\mathcal{N}(\bar{q},\epsilon) = N_0(\bar{q}) + \epsilon N_1(\bar{q}) + \epsilon^2 N_2(\bar{q}).$$

- imeth = 'diag'
- nleg = 3, rank = 2

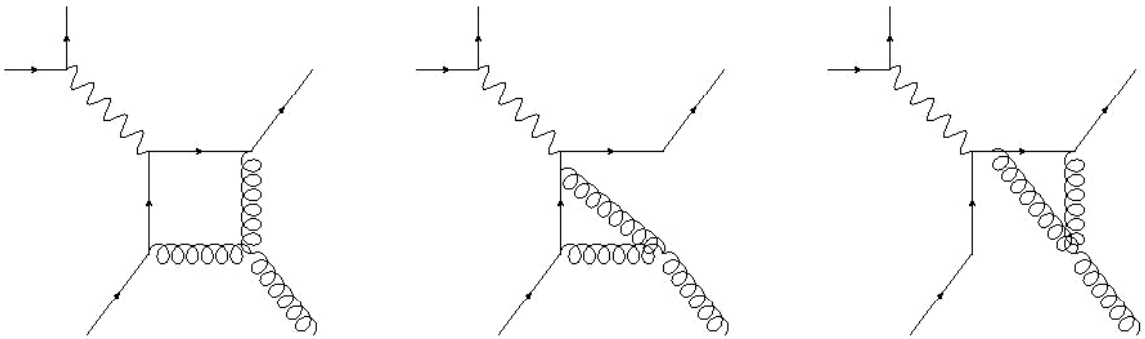
d=4 -> Dim Red
d=4-2 ε -> CDR

$$N(q,\mu^2) = C_F\, g_s^2\, e^2\, \bar{u}(p_{e^-})\, \gamma^\mu\, v(p_{e^+})\, \bar{v}(p_{\bar{u}}) \left[2\,(2-d)\, \bar{q}^\mu\, \not{\bar{q}} + \left[(d-2)\, \bar{q}^2 + 4\,(p_u \cdot \bar{q} - p_{\bar{u}} \cdot \bar{q} - p_u \cdot p_{\bar{u}}) \right] \gamma^\mu \right] u(p_u)$$

Denominators: $\bar{q}^2 \quad (\bar{q}+p_u)^2 \quad (\bar{q}+p_u+p_{e^-}+p_{e^+})^2$

- msq = { 0, 0, 0}
- Pi = { $\underline{0}$, p_u, $p_u + p_{e^-} + p_{e^+}$ }
- N_1 generate a rational term = - g_s^2 C_F LO

VB+lj: *leading color*

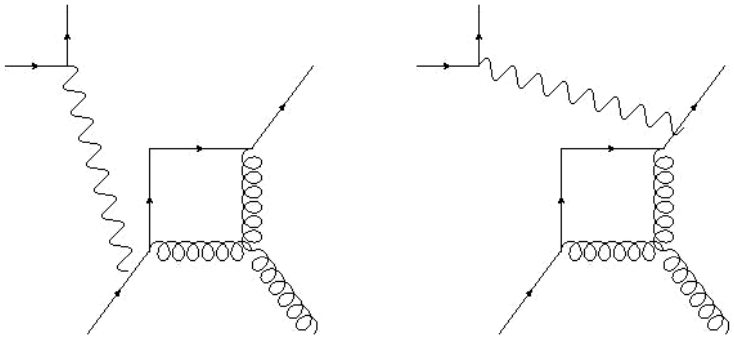


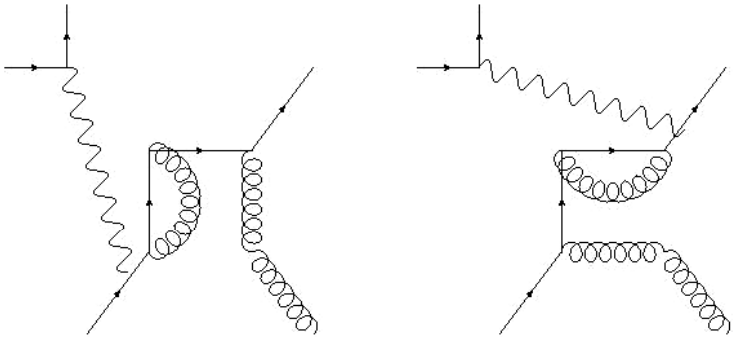
- imeth = 'diag'

- 1 Box nleg=4, rank=3
  4 Tri nleg=3, rank=2
  2 Bub nleg=2, rank=1

- Diagrams can be collected on a common box denominator

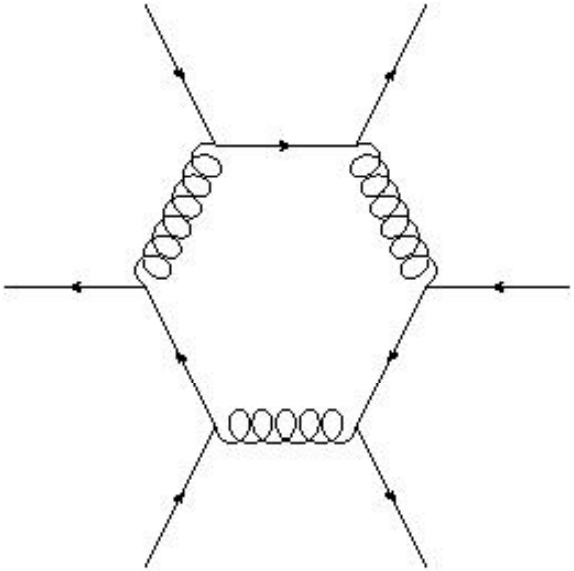
- Studing Left-handed current needs of a prescription for gamma5:
  adopting DR w/anticommuting gamma5
  we added $-N_c/2$ times the Tree Level amplitude

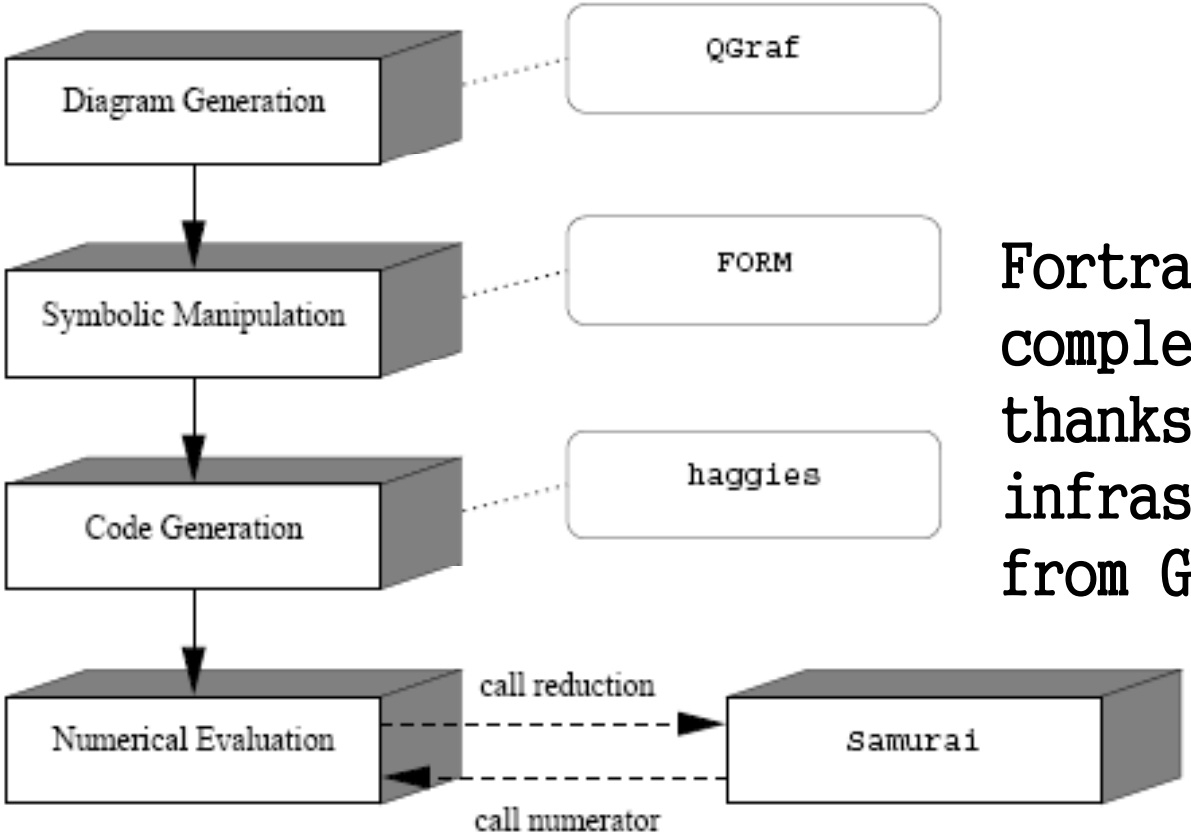




Results numerically checked vs. Bern et al (1997)

6-quarks amplitudes

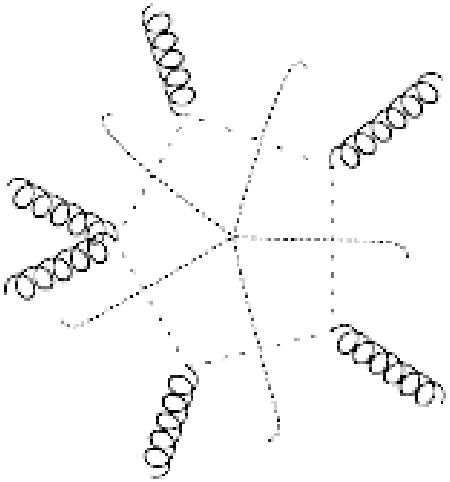




Fortran Code generation completely automated thanks to an infrastructure derived from Golem-2.0

… see Thomas Reiter talk

5 and 6-gluons all plus: *massive scalar loop*



- imeth= 'tree'
- nleg = 6, rank = 6

$$A_3^{\text{tree}}(1_s;2^+;3_s)=\frac{[2|1|r_2\rangle}{\langle 2\,r_2\rangle}\ ,$$

$$A_4^{\text{tree}}(1_s;2^+,3^+;4_s)=\frac{\mu^2\ [2\,3]}{\langle 2\,3\rangle(p_{12}^2-\mu^2)}\ ,$$

$$A_5^{\text{tree}}(1_s;2^+,3^+,4^+;5_s)=\frac{\mu^2\ [2|1\ (2+3)|4]}{\langle 2\,3\rangle\langle 3\,4\rangle(p_{12}^2-\mu^2)(p_{45}^2-\mu^2)}\ ,$$

$$N(q,\mu^2)=A_4(L_1;1^+,2^+;-L_2)\times A_3(L_2;3^+;-L_3)\times A_3(L_3;4^+;-L_4)$$
$$\times A_3(L_4;5^+;-L_5)\times A_3(L_5;6^+;-L_1)$$

For this helicity choice the result is purely rational

Results numerically checked vs. Badger's table

Conclusions

- We wrote a fortran90 library we called SAMURAI for the automatic evaluation of the NLO virtual correction to scattering processes, once the integrand is given in the form of Feynman diagrams or as products of tree level amplitudes

- We produced several examples to show its main features

- We tried to make things as effective and simple as possible to allow for interfaces with other tools

Outlook

- Improve on velocity and stability
  especially for degenerate kinematic configurations