NLO QCD corrections to $pp \to t\bar{t}b\bar{b}$ at the LHC

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based on

A. Bredenstein, A. Denner, S. Dittmaier and S. P.

JHEP **0808** (2008) 108 [arXiv:0807.1248]
PRL **103** (2009) 012002 [arXiv:0905.0110]
JHEP **1003** (2010) 021 [arXiv:1001.4006]

HO10 Theory Institute, CERN, June 29, 2010

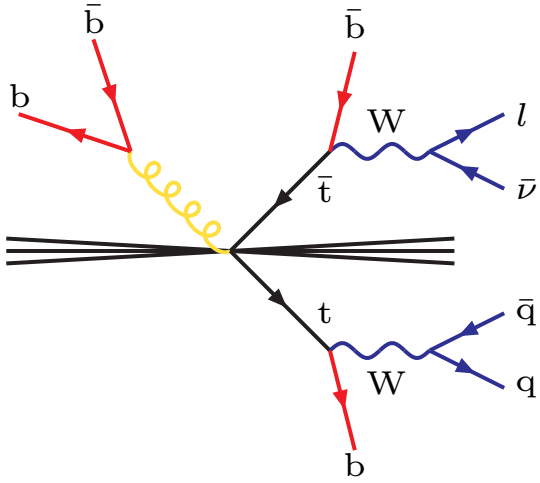
Outline of the talk

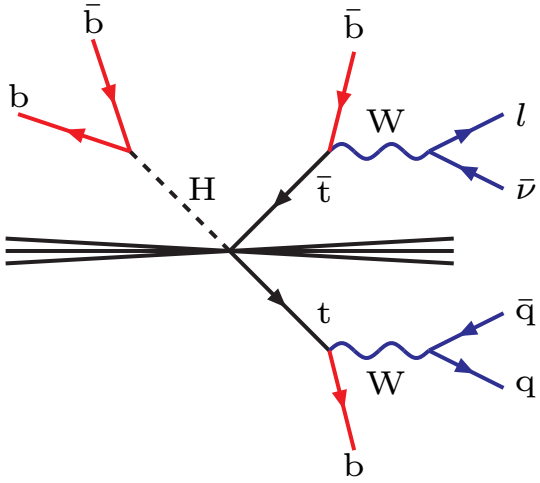
1. **Importance of** $\mathrm{t\bar{t}b\bar{b}}$ **as background to** $\mathrm{t\bar{t}H(H \to b\bar{b})}$
2. **One-loop amplitudes**
3. **NLO predictions for the LHC**

First $2 \to 4$ calculations of the '05/'07 Les Houches priority list

- **Two calculations for $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$ with permille agreement**
  - arXiv:0905.0110 by Bredenstein, Denner, Dittmaier and S. P.
    Feynman diagrams and tensor integrals
  - arXiv:0907.4723 by Bevilacqua, Czakon, Papadopoulos, Pittau and Worek
    OPP reduction and HELAC
- **Two calculations for $\mathrm{pp} \to Vjjj$**
  - arXiv:0906.1445 by Ellis, Melnikov and Zanderighi
    D-dimensional unitarity (leading colour)
  - arXiv:0907.1984 ($Wjjj$) and arXiv:1004.1659 ($Zjjj$) by Berger, Bern, Dixon, Febres Cordero, Forde, Gleisberg, Ita, Kosower and Maitre
    generalized unitarity (full colour)
- **$q\bar{q}$-channel contribution to $\mathrm{pp} \to \mathrm{b\bar{b}b\bar{b}}$**
  - arXiv:0910.4379 by Binoth, Greiner, Guffanti, Reuter, Guillet and Reiter
    Feynman diagrams and tensor integrals (GOLEM)
- **First result for $\mathrm{pp} \to \mathrm{t\bar{t}}jj$**
  - arXiv:1002.4009 by Bevilacqua, Czakon, Papadopoulos and Worek
    OPP reduction and HELAC

$t\bar{t}b\bar{b}$ as irreducible background to $t\bar{t}H(H \to b\bar{b})$



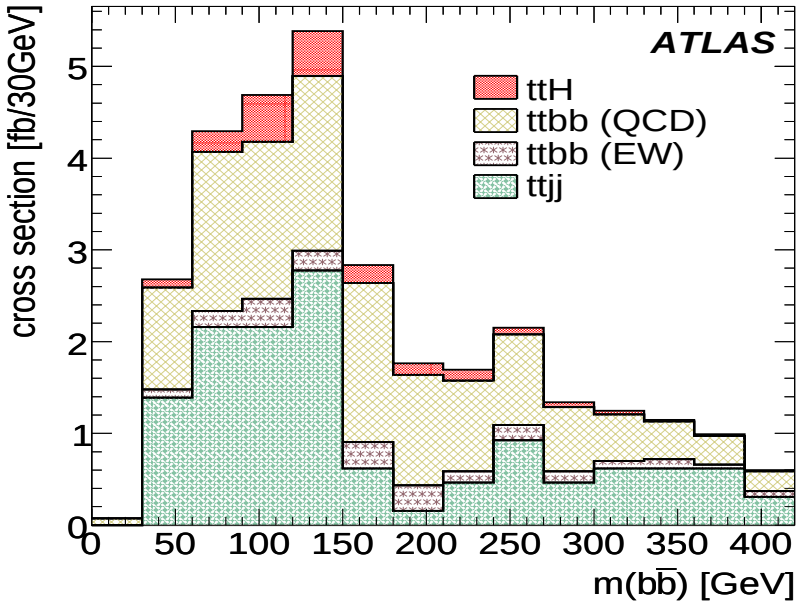


Importance of $t\bar{t}H(H \to b\bar{b})$

- exploit $H \to b\bar{b}$ for $M_H \sim 120\,\text{GeV}$
- direct measurement of Top-Yukawa coupling

ATLAS/CMS analysis ($30\ \text{fb}^{-1}$ @ $14\,\text{TeV}$)

- jet combinatorics dilutes Higgs resonance
- very low $S/B \simeq 1/10$
- to exploit $S/\sqrt{B} \simeq 2\sigma$ we need $\Delta B/B < 10\%$!



ATLAS CSC note, CERN-OPEN-2008-020

Fat Jets can bring $t\bar{t}H(H \to b\bar{b})$ back to life! [Plehn/Salam/Spannowsky arXiv:0910.5472]

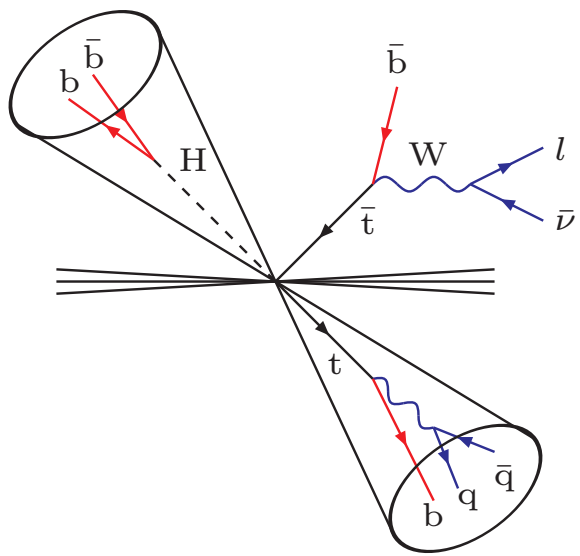
Selecting Higgs & Top Fat Jets ($p_T > 200\,\text{GeV}, R = 1.5$)

Higgs-jet: two b-tags & $|m_{b\bar{b}} - M_H| < 10\,\text{GeV}$

Top-jet: $m_{jjj} \sim m_t$ & $m_{jj} \sim M_W$

2nd Top: 1 lepton + 3rd b-tag (optional)

removes combinatorial ambiguity



ATLAS vs Fat-Jet Analysis ($M_H = 120\,\text{GeV}$ @ $14\,\text{TeV}$ & $30\ \text{fb}^{-1}$)

# of events	$t\bar{t}H$	$t\bar{t}b\bar{b} + t\bar{t}jj$	$t\bar{t}jj/t\bar{t}b\bar{b}$	$S/\sqrt{B}$	S/B
ATLAS	38	309	0.5–1.0	2.2	12%
Fat Jets (2b)	30	135	0.6	2.6	22%
Fat Jets (3b)	14	41	0.06	2.2	36%

sizable S/B increase and possible $t\bar{t}jj$ suppression!

Huge LO uncertainty in $t\bar{t}b\bar{b}$

$$\frac{\Delta\sigma_{t\bar{t}b\bar{b}}}{\sigma_{t\bar{t}b\bar{b}}} \simeq \frac{\Delta\alpha_S^4(\mu)}{\alpha_S^4(\mu)} \simeq 80\%$$

Scale in LO simulations (ATLAS)

$$\mu_{QCD} = \mathbf{E_{thr}/2}$$

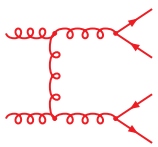
NLO for $pp \to t\bar{t}H$, $t\bar{t}j$, $t\bar{t}Z$ ($K \simeq 1$) and $t\bar{t}b\bar{b}$ ($K \simeq 2$) completely different!

Process	μ_{QCD}	K-factor	Reference
$pp \to t\bar{t}H$	$m_t + M_H/2$	1.2	Beenakker/Dittmaier/Krämer/Plümper/Spira/Zerwas (2001) Dawson/Reina/Wackeroth/Orr/Jackson (2001) Peng/Wen-Gan/Hong-Shen/Ren-You/Yi (2005)
$pp \to t\bar{t}j$ ($p_{T,jet} > 20$–50 GeV)	m_t	1.0–1.15	Dittmaier/Uwer/Weinzierl (2007) Melnikov/Schulze (2010)
$pp \to t\bar{t}Z$	$m_t + M_Z/2$	1.35	Lazopoulus/McElmurry/Melnikov/Petriello (2007)
$pp \to t\bar{t}b\bar{b}$ ($p_{T,jet} > 20$ GeV)	$m_t + m_{b\bar{b}}/2$	1.8	Bredenstein/Denner/Dittmaier/S. P. (2009) Bevilacqua/Czakon/Papadopoulos/Pittau/Worek (2009)

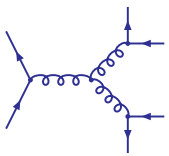
(2) One-loop amplitudes

Ingredients of $pp \to t\bar{t}b\bar{b}$ at NLO

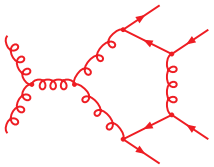
Partonic channels



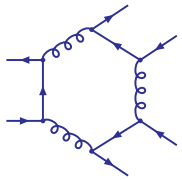
36 trees



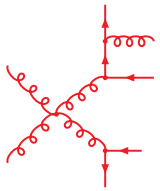
7 trees



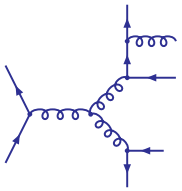
1003 loops



188 loops



341 NLO trees



64 NLO trees

64 NLO trees

92% 3% 5%

Full calculation twice and independently

Diagram generation

- `FeynArts 1.0 / 3.2`

Algebraic reduction

- `FormCalc 5.2` & `MATHEMATICA`

Tensor integrals & numerics

- `Fortran77` $\Rightarrow$ 100MB executable

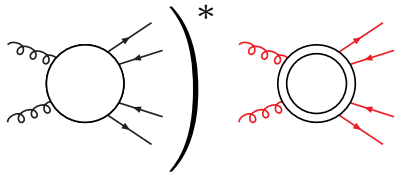
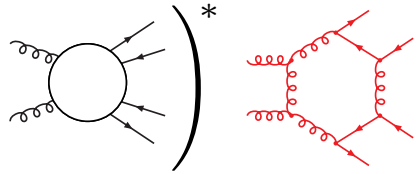
Real emission & IR Subtraction

- `Madgraph` & Spinors & Off-shell recursions
- Dipoles [Catani/Dittmaier/Seymour/Trócsányi '02] & `MadDipole` [Frederix/Gehrmann/Greiner '08]

PS integration

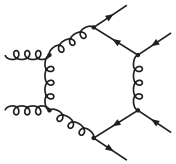
- adaptive MC with 1400 channels

Diagrammatic approach with systematic *reduction of factorial complexity*

$\sum_{\text{col,pol}}$  $=$ $\sum_{\text{col,pol}}$  $+\ \mathcal{O}(1000)$ more diagrams

Colour sums at zero cost

thanks to *colour factorisation*

 $\times\ f^{a_1 bd} f^{a_2 cd} \left(T^b T^e\right)_{i_3 i_4} \left(T^c T^e\right)_{i_5 i_6}$

$$\sum a_{i_1 \ldots j_P}\ \epsilon_{\mu_1} \epsilon_{\mu_2} \left[\bar{v}_3\, \gamma_{\mu_3} \ldots\ u_4\right] \left[\bar{v}_5\ \ldots \gamma_{\mu_k}\, u_6\right] \{g \ldots p\}^{\mu_1 \ldots \nu_P}_{i_1 \ldots j_P} \int \mathrm{d}^D q\, \frac{q_{\nu_1} \ldots q_{\nu_P}}{N_0 \ldots N_{N-1}}$$

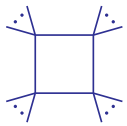
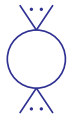
Covariant decomposition of tensor integrals

decouples *helicity structures* from topologies

$$\sum T^{(N)}_{j_1 \ldots j_P} \{g \ldots p\}^{\nu_1 \ldots \nu_P}_{j_1 \ldots j_P}$$

Numerical tensor-intergral reduction

avoids *gigantic expressions* and *instabilities*

$\sum d_i$  $+\ c_j$  $+\ b_k$  $+\ a_l$ 

Reduction of $gg \to t\bar{t}b\bar{b}$ helicity structures to *common* and *minimal set* of SMEs

SMEs (compact & diagram-indep. spinor chains) **strongly boost helicity sums**

$$\{g\ldots p\}_m^{\mu_1\ldots\mu_k}\ \epsilon_{\mu_1}\epsilon_{\mu_2}\,[\bar{v}_3\,\gamma_{\mu_3}\ldots\,u_4]\,[\bar{v}_5\,\ldots\gamma_{\mu_k}\,u_6] = \sum_{n=1}^{N_{\rm SME}} K_{mn}\mathcal{S}_n$$

(A) **Process-independent reduction**

$\epsilon_i p_{1,2} = 0$, Dirac eq. & algebra, $\sum p_i = 0$, standard ordering, and

$$\gamma^{\mu_1}\gamma^{\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\gamma^{\mu_5} = g^{\mu_1\mu_2}\gamma^{\mu_3}\gamma^{\mu_4}\gamma^{\mu_5} + \ldots + g^{\mu_1\mu_2}g^{\mu_3\mu_4}\gamma^{\mu_5}$$

$\Rightarrow$ $\mathbf{N_{SME} = 970}$

(B) **Process-dependent optimisation**

sophisticated algorithm based on $u_j \Rightarrow [\omega_+ + \omega_-]\,u_j$ and

$$\left(\gamma^\mu\gamma^\alpha\gamma^\beta\,\omega_\pm\right)\otimes\left(\gamma_\mu\,\omega_\mp\right) = \left(\gamma^\mu\,\omega_\pm\right)\otimes\left(\gamma^\alpha\gamma^\beta\gamma_\mu\,\omega_\mp\right) \quad \ldots$$

$\Rightarrow$ $\mathbf{N_{SME} = 502}$

Surprise: A & B yield same CPU efficiency

High speed w.o. process-dependent manipulations $\Rightarrow$ automation!

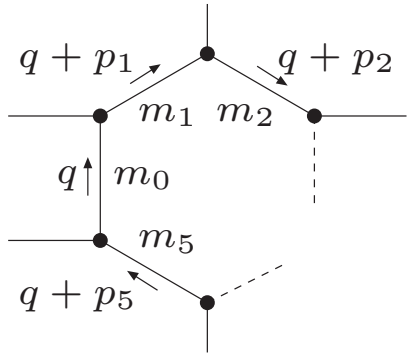
Reduction of tensor integrals – *collection of* $e^+e^- \to 4f$ *methods* [Denner/Dittmaier '05]

(A) Space-time 4-dim ($N \geq 5$ prop.)

simultaneous prop. & rank reduction

Melrose '65; Denner/Dittmaier '02&'05; Binoth et. al. '05

$$\begin{vmatrix} q^\mu & 2qp_1 & \dots & 2qp_5 \\ p_1^\mu & 2p_1p_1 & \dots & 2p_1p_5 \\ \vdots & \vdots & \ddots & \vdots \\ p_4^\mu & 2p_4p_1 & \dots & 2p_4p_5 \\ 0 & f_1 & \dots & f_5 \end{vmatrix} = \mathcal{O}(D-4)$$



(B) Lorentz invariance ($N \leq 4$ prop.)

reduction of rank (P)

Passarino/Veltman '79; Denner '93

$$2(D+P-N-1)\, T^{(P)}_{00i_3\dots i_P} = \sum_{k=1}^{N-1} f_k\, T^{(P-1)}_{ki_3\dots i_P} + 2m_0^2\, T^{(P-2)}_{i_3\dots i_P} + \text{lower-point}$$

$$\sum_{n=1}^{N-1} Z_{mn}\, T^{(P)}_{ni_2\dots i_P} = -2\sum_{r=2}^{P} \delta_{mi_r}\, T^{(P)}_{00i_2\dots\hat{i}_r\dots i_P} - f_m\, T^{(P-1)}_{i_2\dots i_P} + \text{lower-point}$$

inversion of Gram matrix $Z_{mn} = 2p_mp_n$ **unstable** when $\det(Z) \to 0$

$$\tilde{X}_{0j} T^{(P)}_{i_1\ldots i_P} = \det(Z)\, T^{(P+1)}_{j i_1\ldots i_P} + 2\sum_{n=1}^{N-1} \tilde{Z}_{jn} \sum_{r=1}^{P} \delta_{n i_r} T^{(P+1)}_{00 i_1\ldots \hat{i}_r \ldots i_P} + \text{lower-point}$$

$$2\tilde{Z}_{kl} T^{(P+1)}_{00 i_2\ldots i_P} = \Big\{ -\det(Z)\, T^{(P+1)}_{k l i_2\ldots i_P} + 2 m_0 \tilde{Z}_{kl} T^{(P-1)}_{i_2\ldots i_P} + \sum_{n,m=1}^{N-1} \Big[f_n f_m T^{(P-1)}_{i_2\ldots i_P} + 2\sum_{r=2}^{P} (f_n \delta_{m i_r} + f_m \delta_{n i_r})$$

$$\times T^{(P)}_{00 i_2\ldots \hat{i}_r \ldots i_P} + 4 \sum_{\substack{r,s=2 \\ r\neq s}}^{P} \delta_{n i_r} \delta_{m i_s} T^{(P+1)}_{0000 i_2\ldots \hat{i}_r \ldots \hat{i}_s \ldots i_P} \Big] \tilde{\tilde{Z}}_{(kn)(lm)} + \text{lower-point} \Big\} (D+1+P-N+\sum_{r=2}^{P} \bar{\delta}_{i_r 0})^{-1}$$

(C) Treatment of $\det(Z)$ instabilities

Denner/Dittmaier '05

- **Iterative expansion** with K adapted to $\det(Z)$ and target precision

$$T^{(P)} = T^{(P)}_K + \mathcal{O}\left[\det(Z)^{K+1}\right] \qquad \text{requires} \quad T^{(P+K)}_0$$

- **Various alternative methods**
  further expansions ($\tilde{Z}_{kl}$ or $\tilde{X}_{0j} = -\sum_k \tilde{Z}_{jk} f_k$ small); modified set of MIs ($T_0 \to T_{00\ldots00}$); solutions of PV-identities with $\det(Z) \to \det(Y)$

General, flexible & robuts solution of instability problems

CPU efficiency

(3GHz Intel Xeon & `pgf77`)

- The **tensor reduction is strongly boosted by a chache system** that recycles tensor integrals from diagrams that share *common subtopologies*

all loop integrals for $gg \to t\bar{t}b\bar{b}$	CPU time/point
A_0, B_0, C_0, D_0	$t_{\rm MIs} \simeq 10$ ms
$T_N^{(P)}$ with $N \leq 6$ and $P \leq 4$	$t_{\rm red} \simeq 30$ ms

- The **CPU cost of the one-loop amplitude** including colour/helicity sums (180 ms) is not far from the **absolute minimum** ($t_{\rm MIs} = 10$ ms)

- Virtual corrections require less CPU time (25%) than real emission + dipoles (75%)

- Integrating the **NLO cross section** with **statistical accuracy of** $\mathcal{O}(10^{-3})$ requires $\mathcal{O}(10^7)$ events obtained within **2–3 days on a single CPU**

(3) NLO predictions for $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$ at the LHC

Setup and input parameters

Parton masses

- $m_{\mathrm{t}} = 172.6\,\mathrm{GeV}$ and $m_{\mathrm{b}} = 0$ (massless approximation better than 3% at LO)

Recombination of collinear $\mathrm{b}\bar{\mathrm{b}}$, $\mathrm{b}g$, $\bar{\mathrm{b}}g$, with k_{T}-Jet-Algorithm hep-ex/0005012

- partons with $|\eta| < 5 \quad \Rightarrow \quad$ b-jets with $\sqrt{\Delta\phi^2 + \Delta y^2} > D = 0.4$

Require two b-jets ($y < 2.5$, $p_{\mathrm{T}} > 20\,\mathrm{GeV}$) **with**

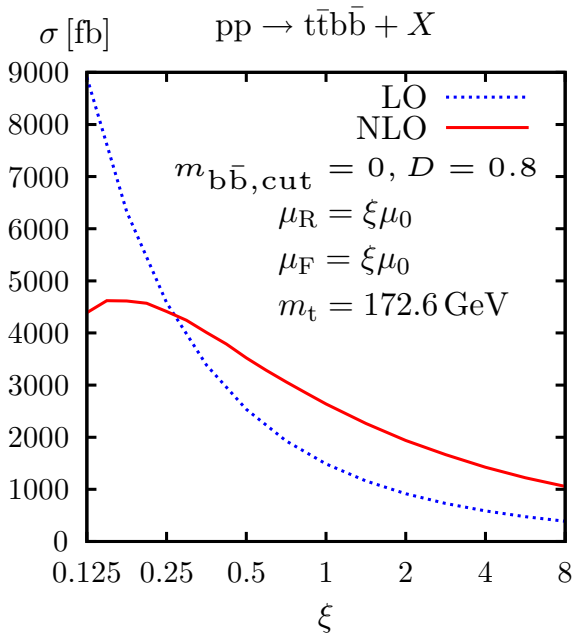
(1) ATLAS setup: $m_{\mathrm{b}\bar{\mathrm{b}}} > 100\,\mathrm{GeV}$

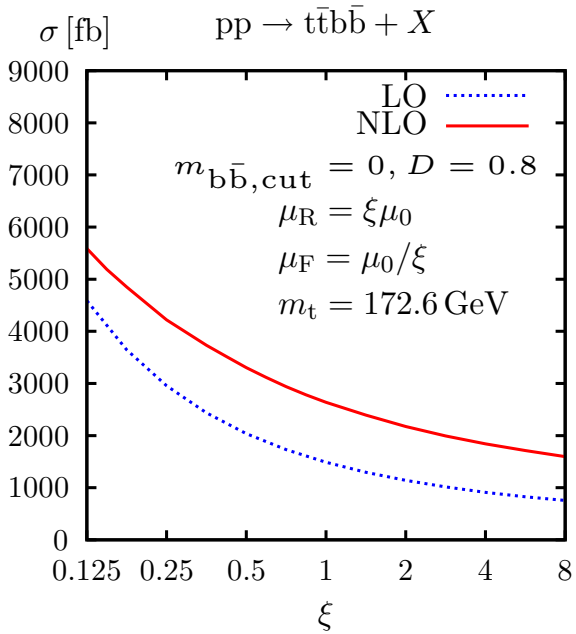
(2) FAT-JET setup: $p_{\mathrm{T},\mathrm{b}\bar{\mathrm{b}}} > 200\,\mathrm{GeV}$

CTEQ6M PDFs with factor-2 variations of $\mu_{\mathrm{R,F}}$ around central values

(A) ATLAS scale choice: $\mu_0 = E_{\mathrm{thr}}/2$

(B) NEW dynamical scale: $\mu_0^2 = m_{\mathrm{t}}\sqrt{p_{\mathrm{T},\mathrm{b}}\,p_{\mathrm{T},\bar{\mathrm{b}}}}$





Impact on $t\bar{t}H$ ATLAS studies: $m_{b\bar{b}} > 100\,\mathrm{GeV}$

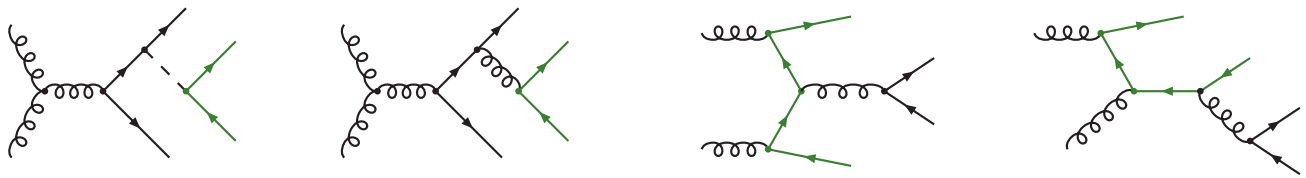
High sensitivity to scale choice

$$\frac{\Delta\sigma_{\mathrm{LO}}}{\sigma_{\mathrm{LO}}} \simeq \frac{\Delta\alpha_S^4(\mu)}{\alpha_S^4(\mu)} \quad \Rightarrow \quad 78\% \text{ uncertainty}$$

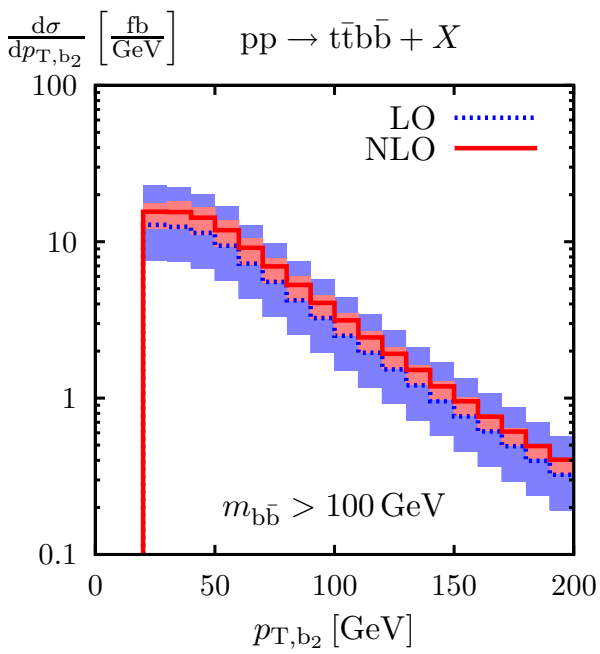
ATLAS scale choice $\mu_0 = E_{\mathrm{thr}}/2 = m_t + m_{b\bar{b}}/2$

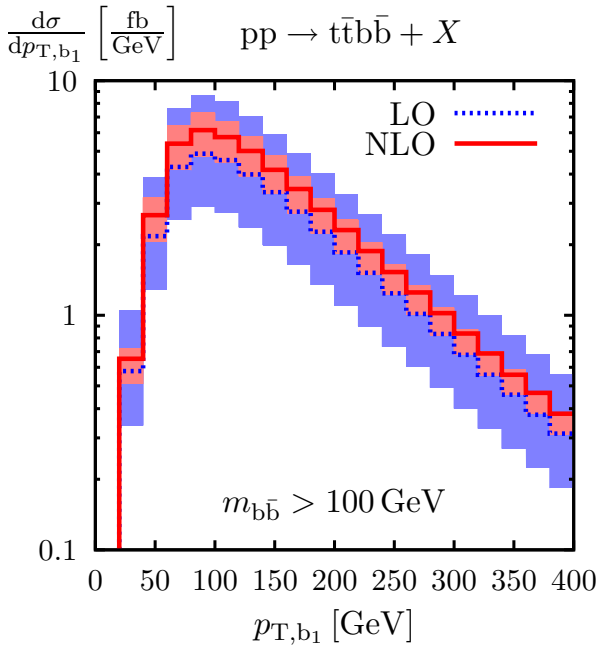
- $t\bar{t}H$ signal receives small K-factor (1.2)
- $t\bar{t}b\bar{b}$: large K-factor and scale dep. ($K = 1.67 \pm 34\%$)

QCD dynamics of $t\bar{t}H/t\bar{t}b\bar{b}$ completely different



- various channels involving $g \to b\bar{b}$ splittings
- $t\bar{t}b\bar{b}$ is a multi-channel multi-scale process





Stabilizing $\sigma_{t\bar{t}b\bar{b}}$ with an appropriate μ_{QCD}

Pragmatic approach: geometric average of widely separated scales observed in $t\bar{t}b\bar{b}$ distributions

$$\mu_0^2 = m_t \sqrt{p_{T,b} p_{T,\bar{b}}}$$

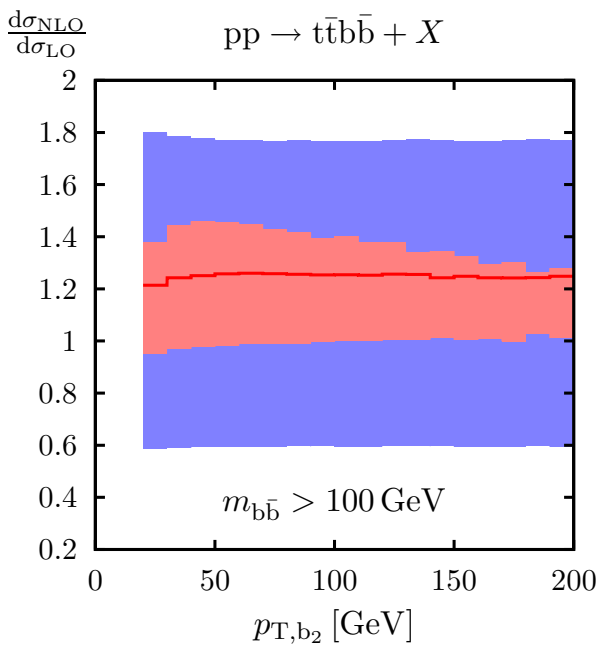
p_T-distributions of individual b-jets

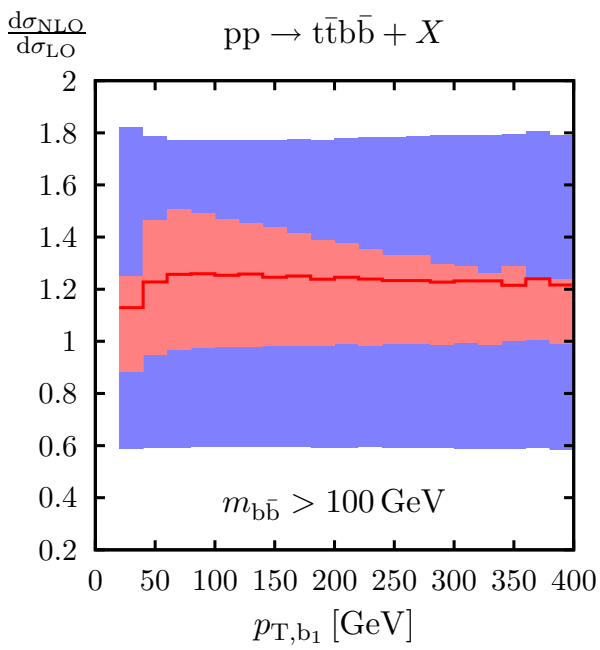
The two b-jets have typically

$$p_{T,b} \ll m_t$$

and rather different distributions

- softest b-jet (upper plot) tends to saturate the cut at 20 GeV
- hardest b-jet (lower plot) has $p_T \sim 100\,GeV$ and extends over wider p_T-range





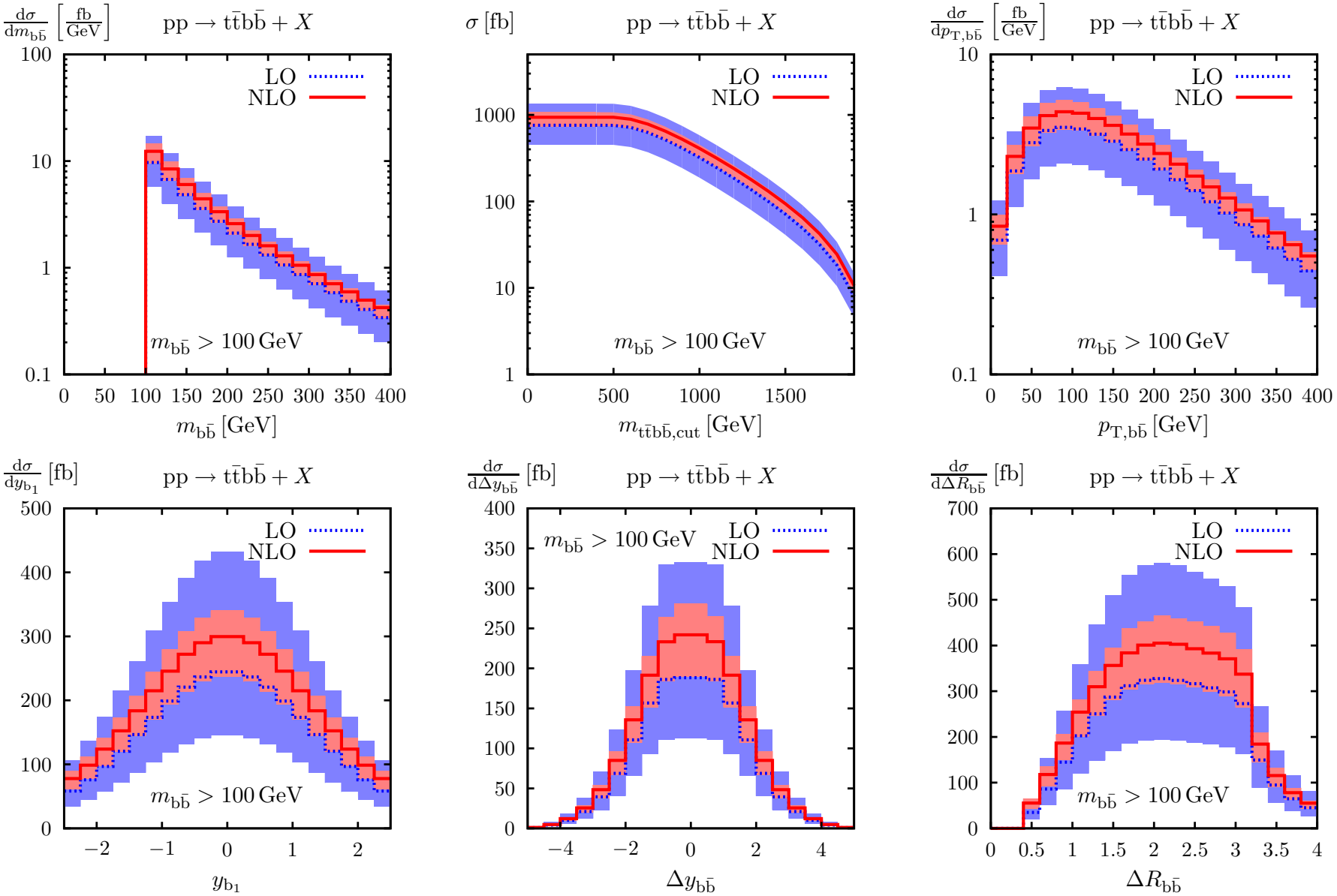
NLO/LO for b-jet p_T-distributions

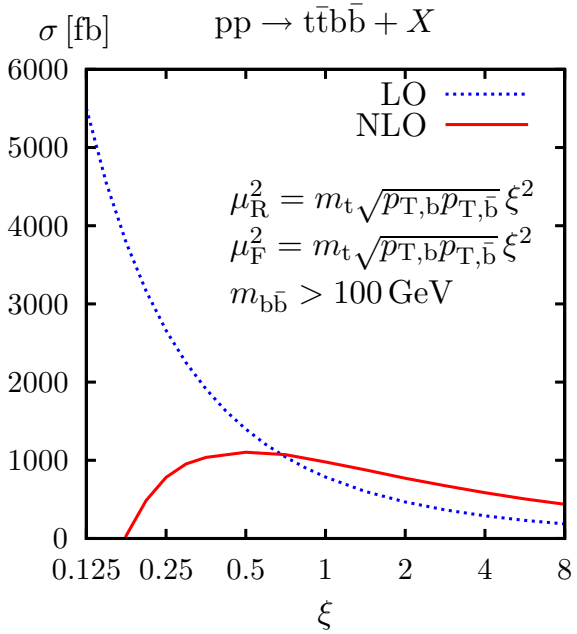
NLO band perfectly fits within LO band both for soft-b (upper plot) and hard-b (lower plot) distributions

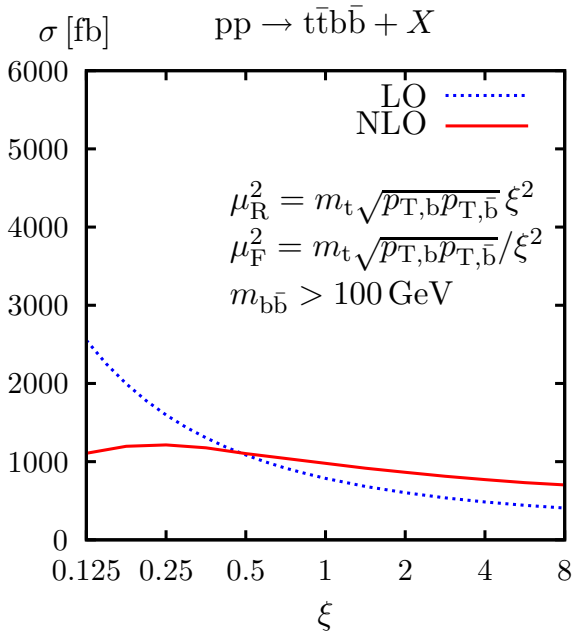
- much smaller NLO correction ($K \simeq 1.25$)
- smaller NLO uncertainty (20%)
- almost constant K-factor over wide p_T-range

New scale choice strongly improves convergence

In $m_{\mathrm{b\bar{b}}}$, $m_{\mathrm{t\bar{t}b\bar{b}}}$, $p_{\mathrm{T,b\bar{b}}}$, y_{b},$\Delta y_{\mathrm{b\bar{b}}}$, $\Delta R_{\mathrm{b\bar{b}}}$ distributions $\lesssim \mathcal{O}(10\%)$ shape distortions







LO and NLO scale dependence of $\sigma_{t\bar{t}b\bar{b}}$

Variations around new central scale

$$\mu_0^2 = m_t\sqrt{p_{T,b}p_{T,\bar{b}}}$$

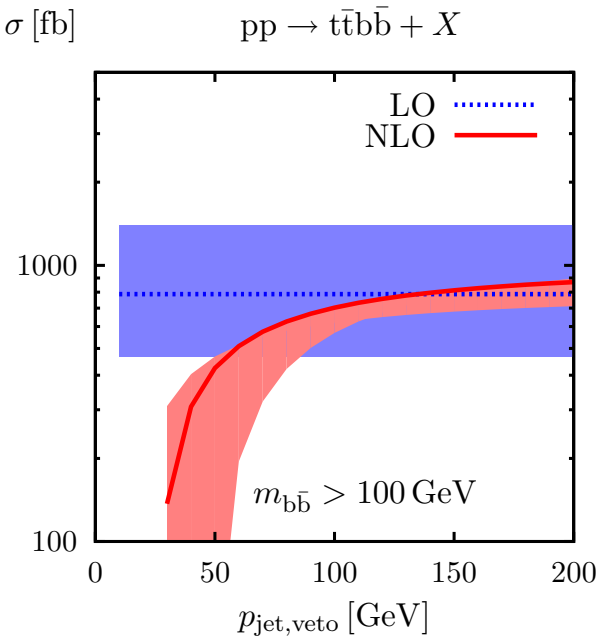
Good news for theory: improved convergence

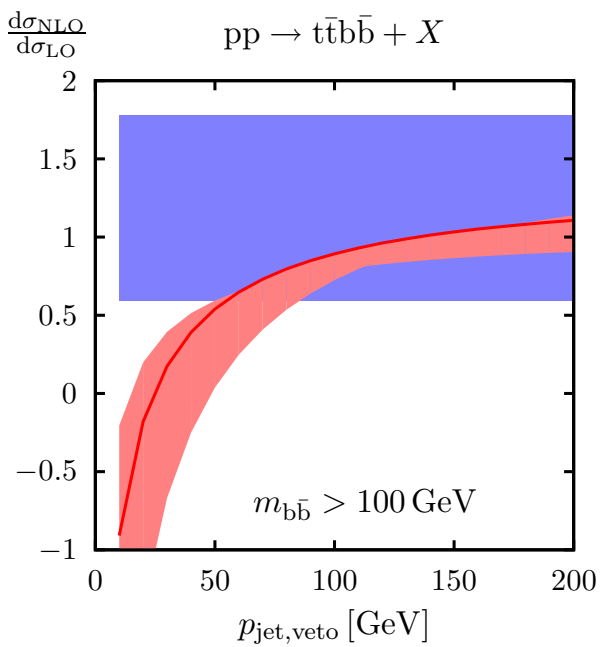
- small correction & uncertainty ($K = 1.25 \pm 21\%$)
- shape of NLO curves: μ_0 close to maximum

Bad news for experiment: $\sigma_{t\bar{t}b\bar{b}}$ enhanced by factor 2.2[a] wrt LO ATLAS simulations

$\sigma_{t\bar{t}b\bar{b}}$	LO	NLO	NLO/LO
$\mu_{R,F} = E_{thr}/2$	449 fb	751 fb	1.67
$\mu_{R,F}^2 = m_t\sqrt{p_{T,b}p_{T,\bar{b}}}$	786 fb	978 fb	1.24

[a] (Partially) taken into account in Fat-Jet analysis!





Jet-veto effects

How to reduce large $t\bar{t}b\bar{b}$ background?

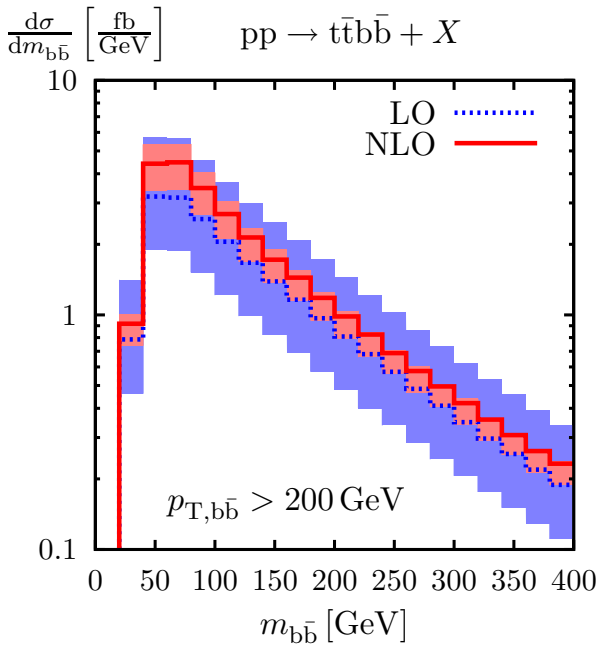
- $p_{\mathrm{jet,veto}} \sim 50\,\mathrm{GeV} \quad \Rightarrow \quad$ sizable suppression

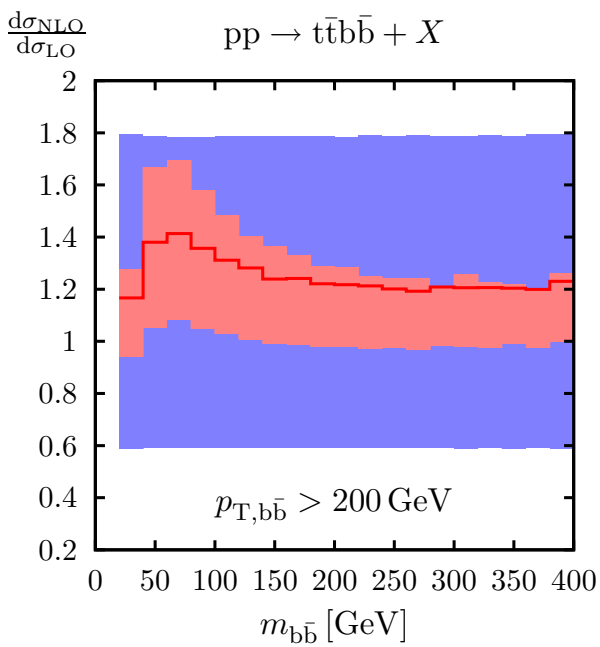
Perturbative instability

- small $p_{\mathrm{jet,veto}} \quad \Rightarrow \quad -\alpha_s^5 \ln^2(Q_0/p_{\mathrm{jet,veto}})$
- NLO uncertainty band enters $K < 0$ range when $p_{\mathrm{jet,veto}} \lesssim 50\,\mathrm{GeV}$

Safe jet veto: $p_{\mathrm{jet,veto}} \simeq 100\,\mathrm{GeV}$

- σ_{NLO} reduced by 30%
- optimal stability (19% uncertainty)





Fat-Jet kinematics: $p_{T,b\bar{b}} > 200\,\text{GeV}$

Dynamical scale keeps $\sigma_{t\bar{t}b\bar{b}}$ stable

$\sigma_{t\bar{t}b\bar{b}}$	LO	NLO	NLO/LO
$\mu^2_{R,F} = m_t\sqrt{p_{T,b}p_{T,\bar{b}}}$	452 fb	592 fb	1.31

- correction (31%) & uncertainty (22%) moderate

Some distributions significantly distorted

- distortion of $\mathcal{O}(20\%)$ in shape of $m_{b\bar{b}}$
- close to $m_{b\bar{b}} = 100\,\text{GeV}$ and tends to mimic $H \to b\bar{b}$ signal (both in shape and size)

Conclusions

NLO QCD calculation for $\mathrm{pp} \to \mathrm{t\bar{t}b\bar{b}}$ **at the LHC**

- mandatory to extract $\mathrm{t\bar{t}H(H \to b\bar{b})}$

QCD scale in ATLAS studies not adequate $\Rightarrow$ **new dynamical scale**

- stabilizes QCD predictions ($K \simeq 1.7 \Rightarrow 1.25$)
- doubles $\mathrm{t\bar{t}b\bar{b}}$ cross section wrt LO ATLAS studies

Technical test of diagrammatic tensor-reduction approach

- very high CPU efficiency with process-independent techniques
- very good perspectives to study other six-particle processes!

BACKUP SLIDES

(d) Rational parts

$$K_{i_1\ldots i_P}(D) \quad \underbrace{T^{(N)}_{i_1\ldots i_P}}_{\frac{R_1}{(D-4)} + \frac{R_1}{(D-4)} + \frac{R_2}{(D-4)^2} + \text{finite part}} \quad \Rightarrow K'_{i_1\ldots i_P}(4)\,(R_1 + R_1) + \frac{1}{2}K''_{i_1\ldots i_P}(4)\,R_2 + \ldots$$

When tensor integrals are combined with their D-dimensional coefficients

- UV and IR poles require $(D-4)$ expansions (performed algebraically)
- this produces **rational terms** proportional to the pole residues

Rational terms of **IR origin**

- require the heaviest algebraic work but **cancel in any** *unrenormalized* **QCD amplitude** (proven in App. A of arXiv:0807.1248)
- can thus be neglected from the beginning

Rational terms of **UV origin**

- extracted automatically by means of a catalogue of UV residues R_1
- after the relevent $(D-4)$-expansions we can continue the calculation in $D = 4$

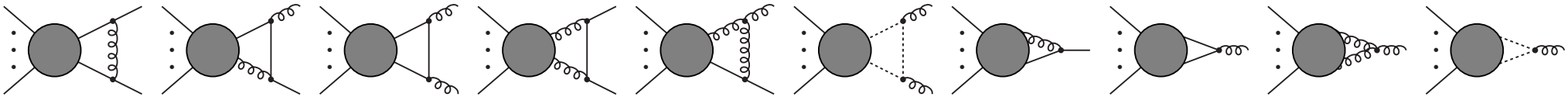
Cancellation of rational terms of IR origin: sketch of the proof (arXiv:0807.1248)

Rational terms originate from D-**dependent** $g^{\mu\nu}$-**contractions** of type $g_{\nu\lambda}\Gamma^{\nu\lambda}$

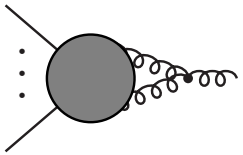
$$g_{\nu\lambda}\, g^{\nu\lambda} = D, \qquad g_{\nu\lambda}\, \gamma^{\nu} \not{p} \gamma^{\lambda} = (2-D)\not{p}, \ldots$$

(1) **The tensor-reduction is free from IR rational terms** since in the soft and collinear regions ($q^{\mu} \to x p^{\mu}$) the tensor integrals cannot produce $g^{\mu\nu}$

(2) **All possible diagrams involving IR-divergent integrals**



can be cast into a form where $g_{\nu\lambda}\Gamma^{\nu\lambda}$ **contractions cancel in IR regions**

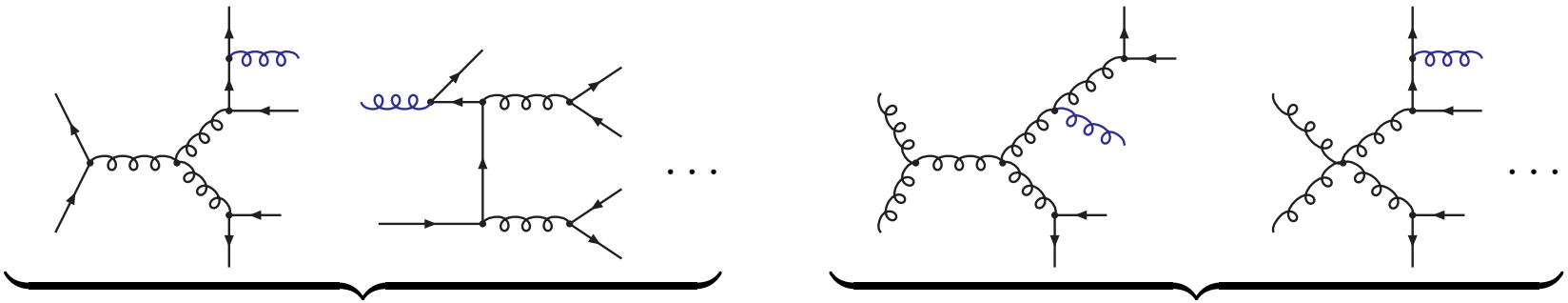


$$= \int \frac{\mathrm{d}^D q}{q^2 (q+p)^2} \underbrace{\epsilon^{\mu *}(p)\,(2q+p)_{\mu}}_{\to\, 0 \text{ in soft/coll. regions}} \; g_{\nu\lambda}\, \Gamma^{\nu\lambda}(q) + \ldots$$

(4.1) Matrix elements for real emission

All ingredients computed twice and independently

Two types of matrix elements



- Madgraph 4.1.33 [Alwall/Demin/deVisscher/Frederix/Herquet/Maltoni/Plehn/Rainwater/Stelzer'07] for all channels
- analytical calculation with Weyl–van der Waerden spinors [Dittmaier '98] for qq/qg channels
- in-house numerical algortihm based on off-shell recursions [Berends/Giele '88; Caravaglios/Moretti '95; Draggiotis/Kleiss/Papadopoulos '98] for gg channel

(4.2) Dipole subtraction of soft and collinear singularities

Catani/Seymour '96; Dittmaier '99; Catani/Dittmaier/Seymour/Trócsányi '02

$$\int \mathrm{d}\sigma_{2\to5} = \int \left[\mathrm{d}\sigma_{2\to5} - \sum_{\substack{i,j=1\\ i\neq j}}^{6} \mathrm{d}\sigma_{2\to5}^{\mathrm{dipole},ij} \right] + \sum_{\substack{i,j=1\\ i\neq j}}^{6} \mathcal{F}_{ij} \otimes \mathrm{d}\sigma_{2\to4}$$

- numerically stable/efficient but non-trivial: 30 qq/gg (10 qg) subtraction terms
- in-house dipoles checked against `MadDipole`[Frederix/Gehrmann/Greiner '08] (gg/qg) and PS slicing[Giele/Glover '92; Giele et al. '93; Keller/Laenen '98; Harris/Owens '01] (qq)
- initial-state collinear singularities cancelled by $\overline{\mathrm{MS}}$-redefinition of PDFs

Phase-space integration

- adaptive multi-channel Monte Carlo [Berends/Kleiss/Pittau '94; Kleiss/Pittau '94] as in `RACOONWW`[Denner/Dittmaier/Roth/Wackeroth'99]/`PROFECY4f`[Bredenstein/Denner/Dittmaier/Weber'06]
- $\mathcal{O}(1400)$ channels to map all peaks from propagators (300) and dipoles (1100)

	$\sigma/\sigma_{\rm LO}$	events (after cuts)	$(\Delta\sigma)_{\rm stat}/\sigma$	time	time/event
NLOtree (gg)	86%	5.3×10^6	0.4×10^{-3}	38min	0.4ms
virtual (gg)	-11%	0.26×10^6	0.6×10^{-3}	13h	180ms
real + dipoles (gg/qg)	49%	10×10^6	3×10^{-3}	40h	14ms
tot	124%		4×10^{-3}	53h	