Recent progress in NNLO QCD calculations

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Outline

- Introduction
- An extension of the subtraction method to NNLO

  - Higgs production

  - W and Z production

  - The W asymmetry
- Summary

Introduction

Until few years ago the standard for QCD theoretical predictions was essentially limited to NLO (plus possibly the all-order resummation of some logarithmically enhanced terms)

Recent years have seen an impressive amount of new results on NLO calculations which is witnessed by the number of talks presented here on the subject !

This effort was motivated by the fact that NLO is the first order where reliable predictions can be obtained

Introduction

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This effort was motivated by the fact that NLO is the first order where reliable predictions can be obtained

➡ **NNLO is thus the first order at which a reliable estimate of the error can be given**

Does it mean that NNLO calculations are essential for every process ?

Well, we can say that NNLO predictions are desirable at least in the following cases:

- For those processes whose NLO corrections are comparable to the LO contributions

  ➡ e.g. Higgs production at hadron colliders

- For those benchmark processes measured with high experimental accuracy

  - α_S measurements from e^+e^- event shape variables

  ➡ - W, Z hadroproduction

  - heavy quark hadroproduction

  ....

- For some important background processes

  ➡ e.g. WW for Higgs boson searches

(Fully) inclusive processes

In the case of one-scale quantities double real, real virtual and double virtual contributions can be analytically computed and the singularities explicitly cancelled

- DIS structure functions — E. Zijlstra, W. Van Neerven (1992)
- Single hadron production — P.J.Rijken, W.L.Van Neerven (1997); A.Mitov, S.Moch (2006)
- DY lepton pair production — R.Hamberg, W.Van Neerven, T.Matsuura (1991)
- Higgs boson production — R.Harlander, W.B. Kilgore (2002); C. Anastasiou, K. Melnikov (2002); V. Ravindran, J. Smith, W.L.Van Neerven (2003)

....

+

Vector boson rapidity distribution

→ modelling the phase space constraint with an effective "propagator" — C.Anastasiou, K.Melnikov, L.Dixon,F.Petriello (2003)

But real experiments have finite acceptances !

What about more exclusive processes?

Many of the ingredients for NNLO corrections available since long time

Example: $e^+e^- \to 3$ jets

- Tree amplitude for $e^+e^- \to 5$ partons K. Hagiwara, D. Zeppenfeld (1989)
  F.A.Berends, W.Giele, H.Kuijf (1989)
- One-loop amplitude for $e^+e^- \to 4$ partons
  N. Glover, D. Miller (1996)
  Z.Bern, L.Dixon, D.Kosower,S.Weinzierl (1996,1997)
  J. Campbell, N. Glover, D. Miller (1997)
- Two-loop amplitude for $e^+e^- \to 3$ partons
  L.W. Garland et al. (2002)

Example: Drell-Yan

T.Matsuura, W.Van Neerven (1988)

R.Hamberg, W.Van Neerven, T.Matsuura (1991)

➡ Amplitudes known since almost 20 years !

Despite this fact until recently the computation of the corresponding NNLO corrections could not be performed

The IR singularity structure of the three contributions has now been understood

S. Catani (1998); J.Campbell, N. Glover (1998)
S. Catani, MG (1999); Z.Bern, V. Del Duca, W. Kilgore, C. Schmidt (1999), D. Kosower, P. Uwer (1999), S. Catani, MG (2000)
G.Sterman, M. Tejeda-Yeomans (2002)

However the organization of the calculation into finite pieces that can be integrated numerically is still a formidable task

Two main strategies have been followed:

- **Sector decomposition**
- **Subtraction method**

Sector decomposition

K. Hepp (1966)
T. Binoth, G.Heinrich (2000,2004)
C.Anastasiou, K.Melnikov, F.Petriello (2004)

Sector decomposition as implemented by Anastasiou and collaborators works by dividing the integration region into sectors each containing a single singularity that can be made explicit by expansion into distributions

➡ This leads to a fully automated procedure by which the coefficients of the poles as well as finite terms can be computed numerically

The method has been successfully applied to a number of important fully exclusive NNLO computations

- Higgs and vector boson production in hadron collisions

C.Anastasiou, K.Melnikov, F.Petriello (2004)
K.Melnikov, F.Petriello (2004)

- NNLO QED computation of muon decay

C.Anastasiou, K.Melnikov, F.Petriello (2005)

- Semileptonic decay $b \longrightarrow c\, l\, \bar{\nu}_l$

K.Melnikov (2008)

Subtraction method

R.K. Ellis, D.A.Ross, A.E.Terrano (1981)
S.Frixione, Z.Kunszt, A. Signer (1995)
S.Catani, M. Seymour (1996)

$$d\sigma = \int_{n+1} r d\Phi_{n+1} + \int_n v d\Phi_n$$

$$d\sigma = \int_{n+1} \left(r d\Phi_{n+1} - \tilde{r} d\tilde{\Phi}_{n+1} \right) + \int_{n+1} \tilde{r} d\tilde{\Phi}_{n+1} + \int_n v d\Phi_n$$

Add and subtract a (local) counterterm with the same singularity structure of the real contribution that can be integrated analytically over the phase space of the unresolved parton

How to extend this procedure to NNLO in a general way ?

This absolutely non trivial issue has attracted quite an amount of work

Goal ➡ Formulate a general scheme that can be possibly applied to any process

D. Kosower (1998,2003,2005)
S. Weinzierl (2003)
S. Frixione, MG (2004)
A. & T. Gehrmann, N. Glover (2005)
G. Somogyi, Z. Trocsanyi, V. Del Duca (2005, 2007)
P.Bolzoni, S.Moch, G.Somogyi, Z.Trocsanyi (2009)
P.Bolzoni, G.Somogyi (2010); M.Czakon (2010)

At present the only approach that has been proven to work is the antenna subtraction method by A. & T. Gehrmann and Glover

Counterterms constructed from antennae extracted from physical matrix elements

It led to the completion of the NNLO calculation of $e^+e^- \to 3$ jets

Impressive achievement of a five years project !

A. & T. Gehrmann, N. Glover, G. Heinrich (2007)

➡ Important impact on α_S measurement

Cross checked with a fully independent implementation

S.Weinzierl (2008)

Now the method is being applied to hadron collisions

R. Boughezal, A.Gehrmann, M.Ritzmann (2010)
T.Gehrmann et al. (2010)
N.Glover, J.Pires (2010)

Is there an alternative that works at least in some (relatively simple) cases ?

A shortcut

S. Catani, MG (2007)

Let us consider a specific, though important class of processes: the production of colourless high-mass systems F in hadron collisions (F may consist of lepton pairs, vector bosons, Higgs bosons......)

At LO it starts with $c\bar{c} \longrightarrow F$

Strategy: start from NLO calculation of F+jet(s) and observe that as soon as the transverse momentum of the F $q_T \neq 0$ one can write:

$$d\sigma^{F}_{(N)NLO}|_{q_T \neq 0} = d\sigma^{F+\text{jets}}_{(N)LO}$$

Define a counterterm to deal with singular behaviour at $q_T \to 0$

But.....
the singular behaviour of $d\sigma^{F+\text{jets}}_{(N)LO}$ is well known from the resummation program of large logarithmic contributions at small transverse momenta

G. Parisi, R. Petronzio (1979)
J. Collins, D.E. Soper, G. Sterman (1985)
S. Catani, D. de Florian, MG (2000)

→ choose $d\sigma^{CT} \sim d\sigma^{(LO)} \otimes \Sigma^F(q_T/Q)$

where $\Sigma^F(q_T/Q) \sim \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \sum_{k=1}^{2n} \Sigma^{F(n;k)} \frac{Q^2}{q_T^2} \ln^{k-1} \frac{Q^2}{q_T^2}$

Then the calculation can be extended to include the $q_T = 0$ contribution:

$$d\sigma^F_{(N)NLO} = \mathcal{H}^F_{(N)NLO} \otimes d\sigma^F_{LO} + \left[d\sigma^{F+\text{jets}}_{(N)LO} - d\sigma^{CT}_{(N)LO}\right]$$

where I have subtracted the truncation of the counterterm at (N)LO and added a contribution at $q_T = 0$ to restore the correct normalization

The function $\mathcal{H}^F$ can be computed in QCD perturbation theory

$$\mathcal{H}^F = 1 + \left(\frac{\alpha_S}{\pi}\right)\mathcal{H}^{F(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{F(2)} + \ldots\ldots$$

Note that:

- It is a subtraction method for which the counterterm $d\sigma^{CT}$ regularizes the singular behaviour of the *sum* of the *double-real* and *real-virtual* contribution

- The form of the counterterm is arbitrary: only its $q_T \to 0$ limit is fixed

- Once a form of the counterterm is chosen, the hard function $\mathcal{H}^F$ is uniquely identified ➡ **we choose the form used in our resummation work**

  G. Bozzi, S. Catani, D. de Florian, MG (2005)

- At NLO (NNLO) the physical information of the *one-loop* (*two-loop*) contribution is contained in the coefficient $\mathcal{H}^{F(1)}$ ($\mathcal{H}^{F(2)}$)

- Due to the simplicity of the LO process, jets appear only in $d\sigma^{F+\text{jets}}_{(N)LO}$

  ➡ cuts on the jets can be effectively accounted for through a (N)LO calculation

For a generic $pp \to F + X$ process:

- At NLO we need a LO calculation of $d\sigma^{F+\mathrm{jet(s)}}$ plus the knowledge of $d\sigma_{LO}^{CT}$ and $\mathcal{H}^{F(1)}$
  - the counterterm $d\sigma_{LO}^{CT}$ requires the resummation coefficients $A^{(1)}, B^{(1)}$ and the one loop anomalous dimensions
  - the general form of $\mathcal{H}^{F(1)}$ is known

D. de Florian, MG (2000)
G. Bozzi, S. Catani, D. de Florian, MG (2005)

- At NNLO we need a NLO calculation of $d\sigma^{F+\mathrm{jet(s)}}$ plus the knowledge of $d\sigma_{NLO}^{CT}$ and $\mathcal{H}^{F(2)}$
  - the counterterm $d\sigma_{NLO}^{CT}$ depends also on the resummation coefficients $A^{(2)}, B^{(2)}$ and on the two loop anomalous dimensions
  - we have computed the coefficient $\mathcal{H}^{F(2)}$ for Higgs and vector boson production

→ **We can complete the corresponding NNLO calculations !**

The function $\mathcal{H}^H$ can be computed in QCD perturbation theory as follows

$$\mathcal{H}^H = 1 + \left(\frac{\alpha_S}{\pi}\right)\mathcal{H}^{H(1)} + \left(\frac{\alpha_S}{\pi}\right)^2\mathcal{H}^{H(2)} + \ldots\ldots$$

S. Catani, MG (to appear)

consider integral of q_T distribution up to an arbitrary small Q_0

$$\int_0^{Q_0^2} dq_T^2 \frac{d\hat{\sigma}_{H\,ab}}{dq_T^2}(q_T, M, \hat{s} = M^2/z) \equiv z\sigma_H^{(0)}\hat{R}^H_{gg\leftarrow ab}(z, M/Q_0)$$

Up to $\mathcal{O}(\alpha_S^2)$ the coefficients of the logarithmic expansion in $l_0 = \ln M_H^2/Q_0^2$ are all known

D. de Florian, MG (2000)

$$\hat{R}^{(1)}_{gg\leftarrow ab}(z, M/Q_0) = l_0^2\,\Sigma^{H(1;2)}_{gg\leftarrow ab}(z) + l_0\,\Sigma^{H(1;1)}_{gg\leftarrow ab}(z) + \mathcal{H}^{H(1)}_{gg\leftarrow ab}(z) + \mathcal{O}(Q_0^2/M^2)$$

$$\hat{R}^{(2)}_{gg\leftarrow ab}(z, M/Q_0) = l_0^4\,\Sigma^{H(2;4)}_{gg\leftarrow ab}(z) + l_0^3\,\Sigma^{H(2;3)}_{gg\leftarrow ab}(z) + l_0^2\,\Sigma^{H(2;2)}_{gg\leftarrow ab}(z)$$
$$+ l_0\left(\Sigma^{H(2;1)}_{ggab}(z) - 16\zeta_3\Sigma^{H(2;4)}_{ggab}(z)\right) + \left(\mathcal{H}^{H(2)}_{ggab}(z) - 4\zeta_3\,\Sigma^{H(2;3)}_{ggab}(z)\right) + \mathcal{O}(Q_0^2/M^2)$$

The only missing one is $\mathcal{H}^{H(2)}_{gg\leftarrow ab}(z)$

Total cross section — q_T distribution

→ solve this equation to obtain the $\mathcal{H}^{H(2)}_{gg\leftarrow ab}(z)$

$$\int_0^{Q_0^2} dq_T^2 \frac{d\hat{\sigma}_{H\,ab}}{dq_T^2}(q_T, M; z) = \hat{\sigma}^H_{ab}(z) - \int_{Q_0^2}^{\infty} dq_T^2 \frac{d\hat{\sigma}_{H\,ab}}{dq_T^2}(q_T, M; z)$$

HNNLO

http://theory.fi.infn.it/grazzini/codes.html

HNNLO is a numerical program to compute Higgs boson production through gluon fusion in pp or $p\bar{p}$ collisions at LO, NLO, NNLO

- $H \to \gamma\gamma \quad (\texttt{higgsdec} = 1)$
- $H \to WW \to l\nu l\nu \quad (\texttt{higgsdec} = 2)$
- $H \to ZZ \to 4l$
  - $H \to e^+e^-\mu^+\mu^- \quad (\texttt{higgsdec} = 31)$
  - $H \to e^+e^-e^+e^- \quad (\texttt{higgsdec} = 32)$

→ includes appropriate interference contribution

The user can choose the cuts and plot the required distributions by modifying the `cuts.f` and `plotter.f` subroutines

Results: $gg \to H \to WW \to l\nu l\nu$

MG (2007)

Use *preselection cuts* as in Davatz. et al (2003)

see also C.Anastasiou, G. Dissertori, F. Stockli (2007)

$p_T^l > 20$ GeV

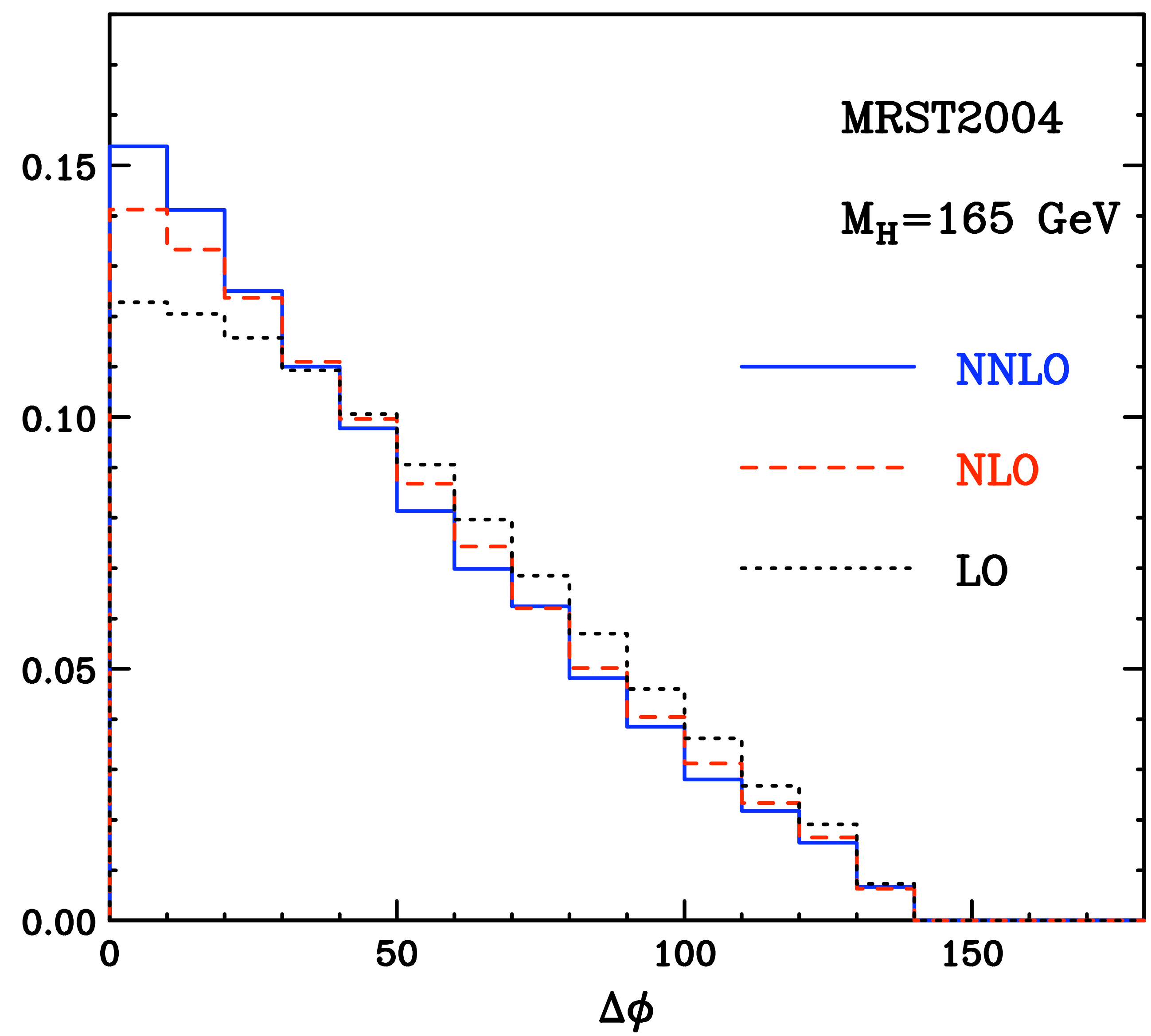
$|y_l| < 2$

$p_T^{\rm miss} > 20$ GeV

$\Delta\phi < 135^o$

$m_{ll} < 80$ GeV

normalized $\Delta\phi$ **distribution**



The distributions appears to be steeper when going from LO to NLO and from NLO to NNLO

Use now *selection cuts* as in Davatz. et al (2003)

$p_T^{\min} > 25$ GeV $m_{ll} < 35$ GeV $\Delta\phi < 45^o$

35 GeV $< p_T^{\max} <$ 50 GeV $|y_l| < 2$ $p_T^{\text{miss}} > 20$ GeV

Results for

$p_T^{\text{veto}} = 30$ GeV

σ (fb)	LO	NLO	NNLO
$\mu_F = \mu_R = M_H/2$	17.36 ± 0.02	18.11 ± 0.08	15.70 ± 0.32
$\mu_F = \mu_R = M_H$	14.39 ± 0.02	17.07 ± 0.06	15.99 ± 0.23
$\mu_F = \mu_R = 2M_H$	12.00 ± 0.02	15.94 ± 0.05	15.68 ± 0.20

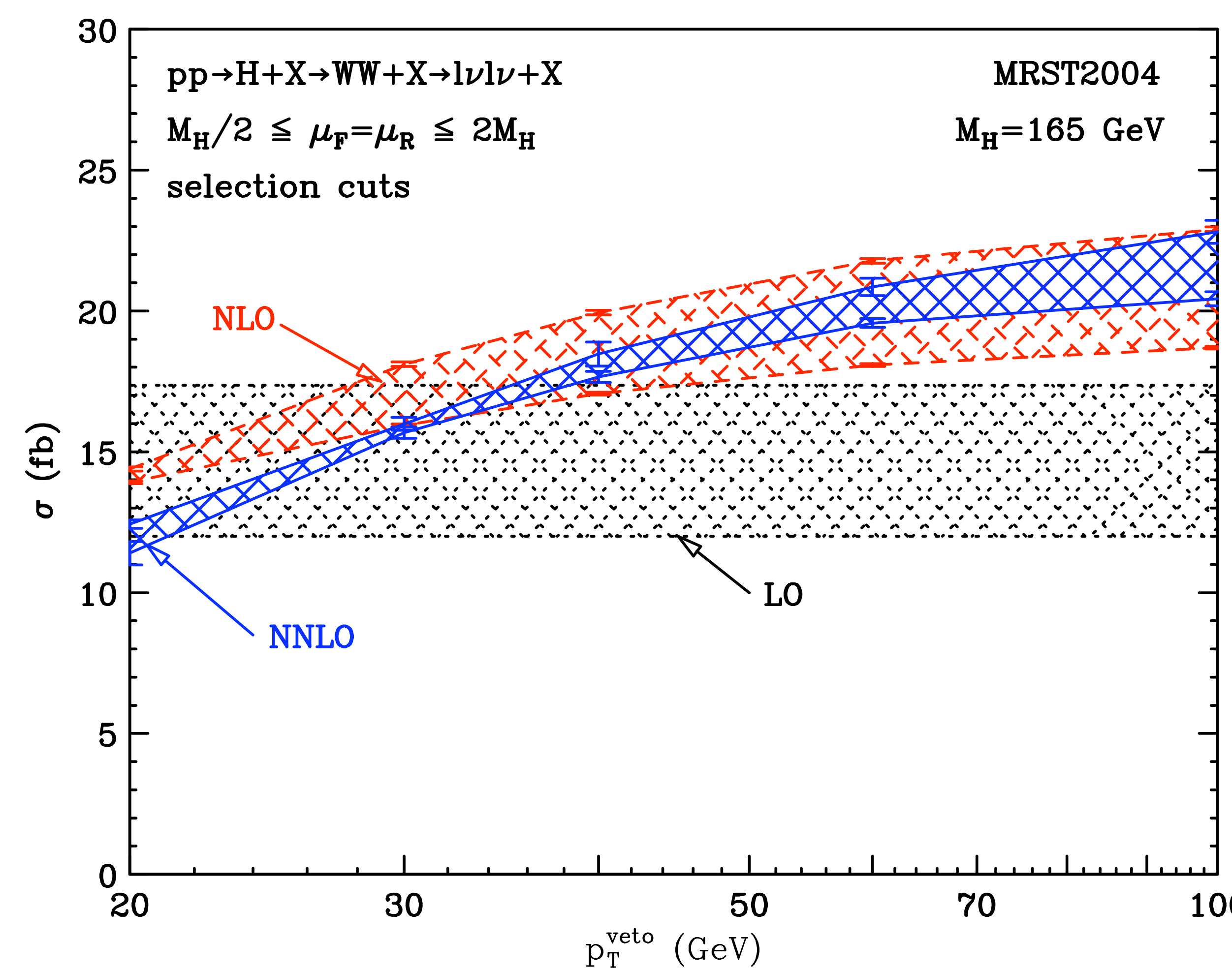
➡ **Impact of higher order corrections strongly reduced by selection cuts**

The NNLO band overlaps with the NLO one for $p_T^{\text{veto}} \gtrsim 30$ GeV

The bands do not overlap for $p_T^{\text{veto}} \lesssim 30$ GeV

NNLO efficiencies found in good agreement with MC@NLO

Anastasiou et al. (2008)



NEW: DYNNLO

http://theory.fi.infn.it/grazzini/dy.html

DYNNLO is a parton level MC program to compute vector boson production in pp or ppbar collisions up to NNLO in QCD perturbation theory

- $W^+ \to l^+\nu$ (nproc=1)
- $W^- \to l^-\nu$ (nproc=2)
- $Z \to l^+l^-$ (nproc=3)

The user can choose the cuts and plot the required distributions by modifying the cuts.f **and** plotter.f **subroutines**

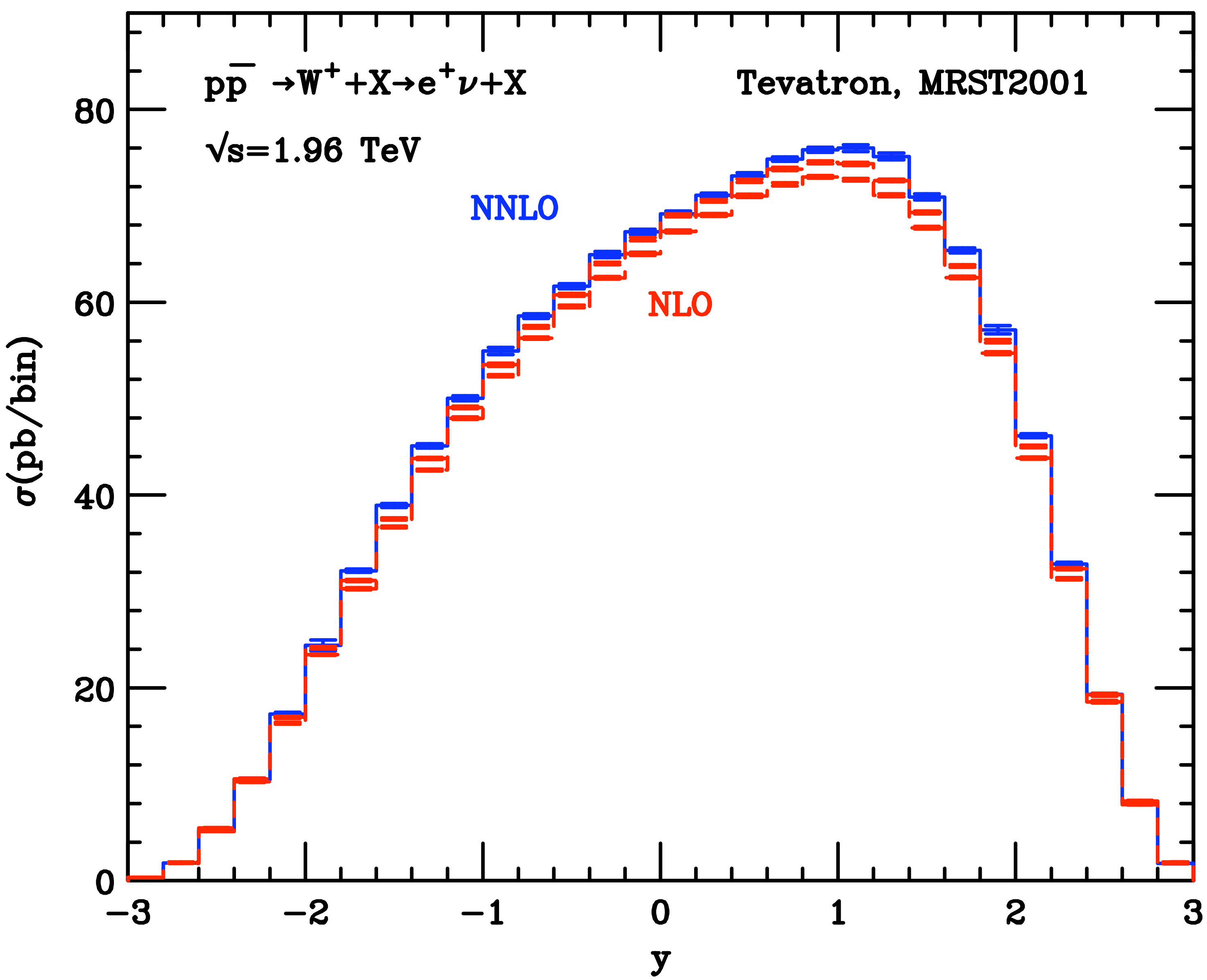
DYNNLO works exactly in the same way as HNNLO for Higgs production

Rapidity distribution of the vector boson

When no cuts are applied our numerical program provides the first independent check of the vector boson rapidity distribution up to NNLO

C.Anastasiou et al. (2003)

Tuned comparison for on shell W production at the Tevatron:



In this plot I compare the NNLO result with the NLO band (obtained by varying $\mu_F=\mu_R$ between 0.5 m_W and 2m_W) and with the result by Anastasiou et al.

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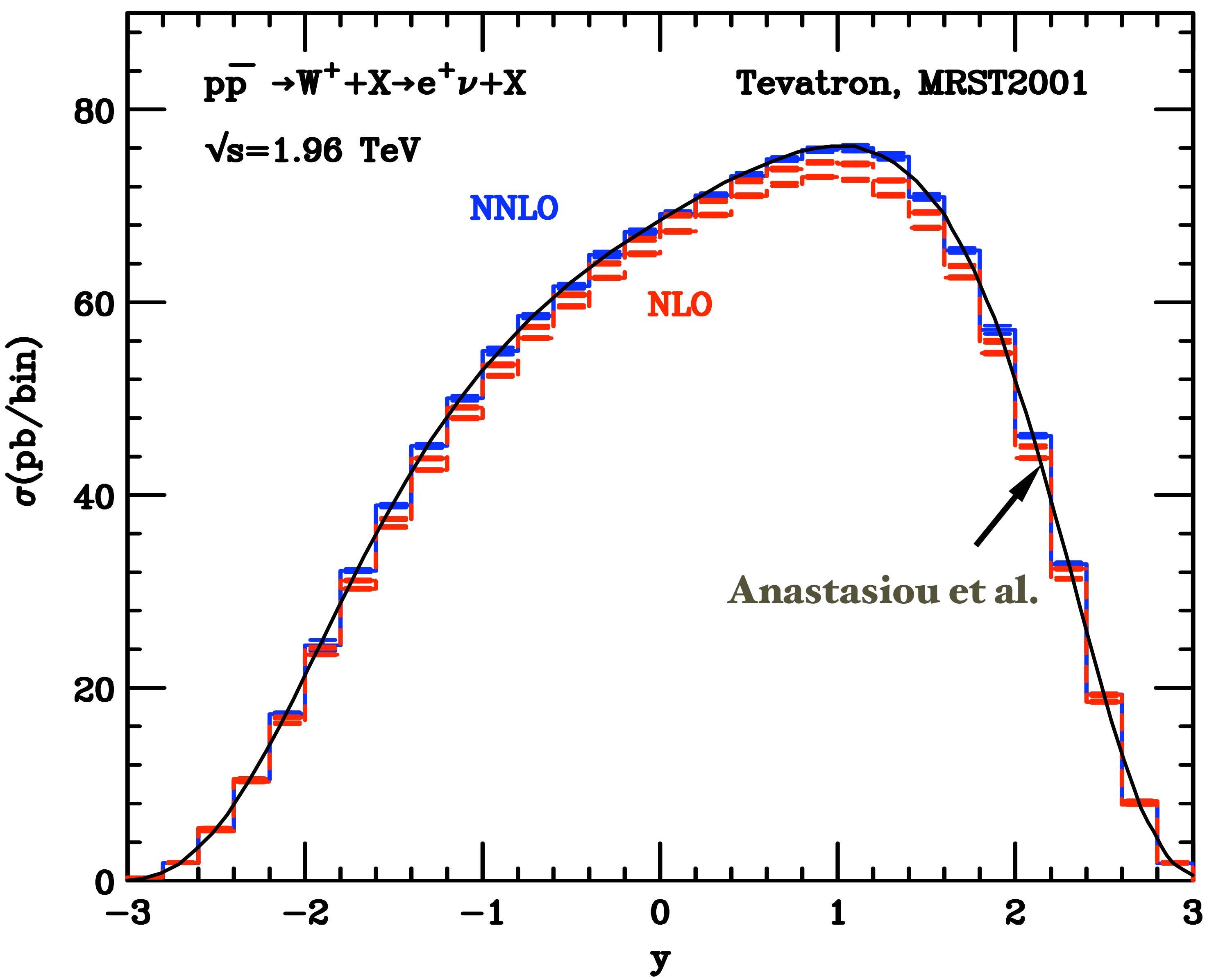


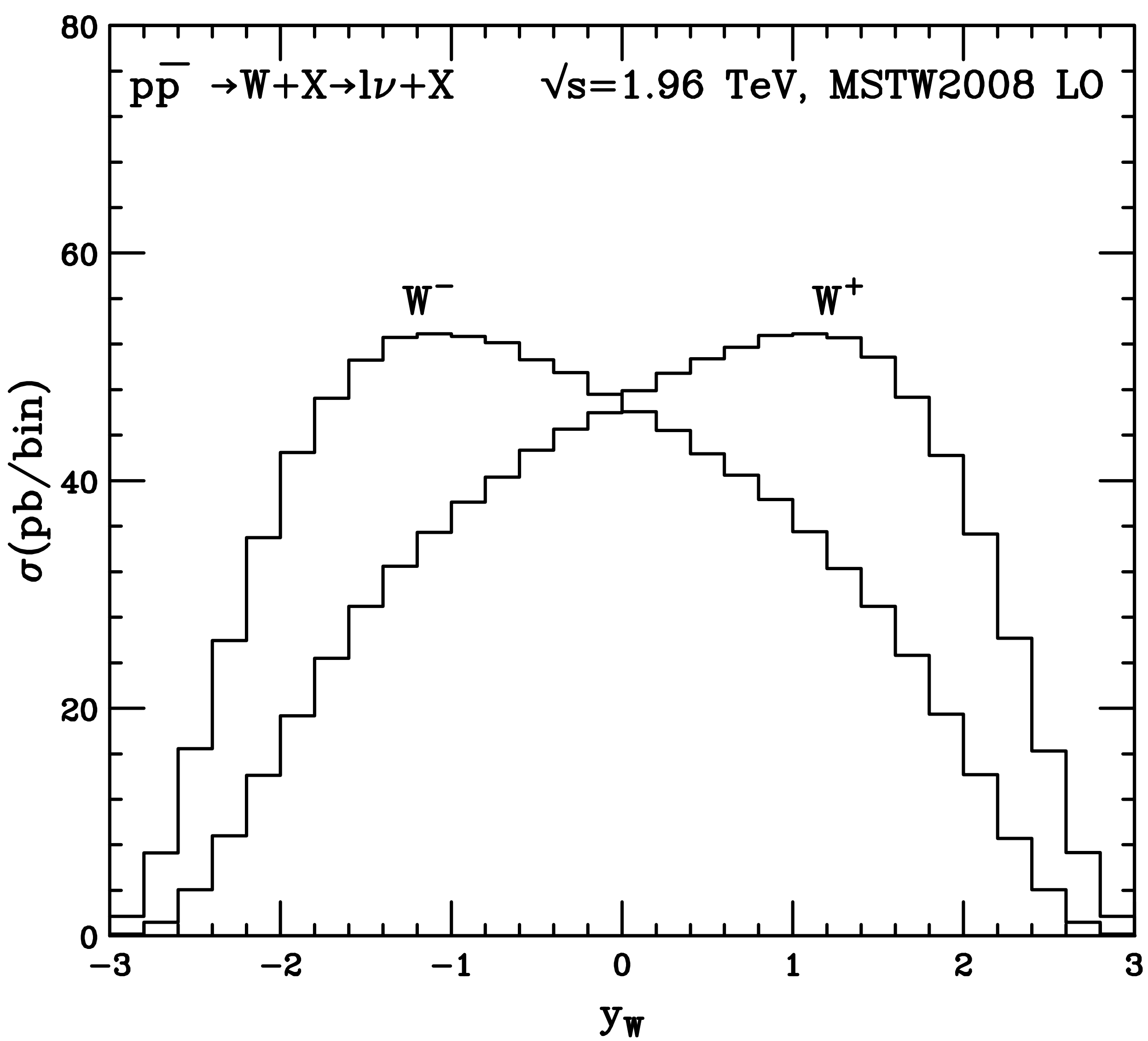
In this plot I compare the NNLO result with the NLO band (obtained by varying $\mu_F=\mu_R$ between 0.5 m_W and $2m_W$) and with the result by Anastasiou et al.

The agreement is good

NEW: W charge asymmetry

S. Catani, G.Ferrera, MG (2010)

An important observable in W hadroproduction is the asymmetry in the rapidity distributions of the W bosons



$$A(y_W) = \frac{\frac{d\sigma(W^+)}{dy_W} - \frac{d\sigma(W^-)}{dy_W}}{\frac{d\sigma(W^+)}{dy_W} + \frac{d\sigma(W^-)}{dy_W}}$$

In $p\bar{p}$ collisions the W^+ and W^- are produced with equal rates but W^+ (W^-) is produced mainly in the proton (antiproton) direction

These asymmetries are mainly due to the fact that, on average, the u quark carries more proton momentum fraction than the d quark

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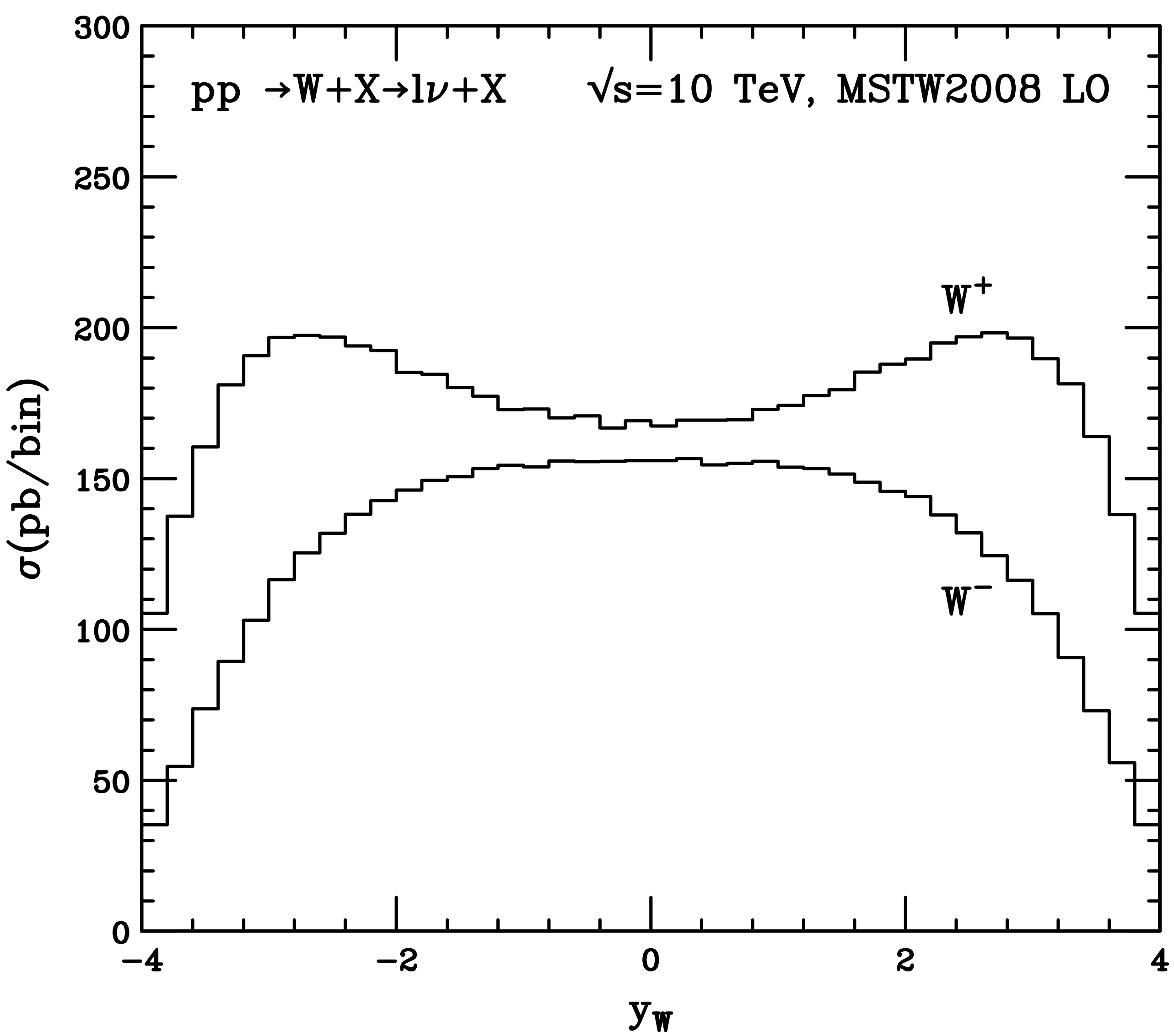

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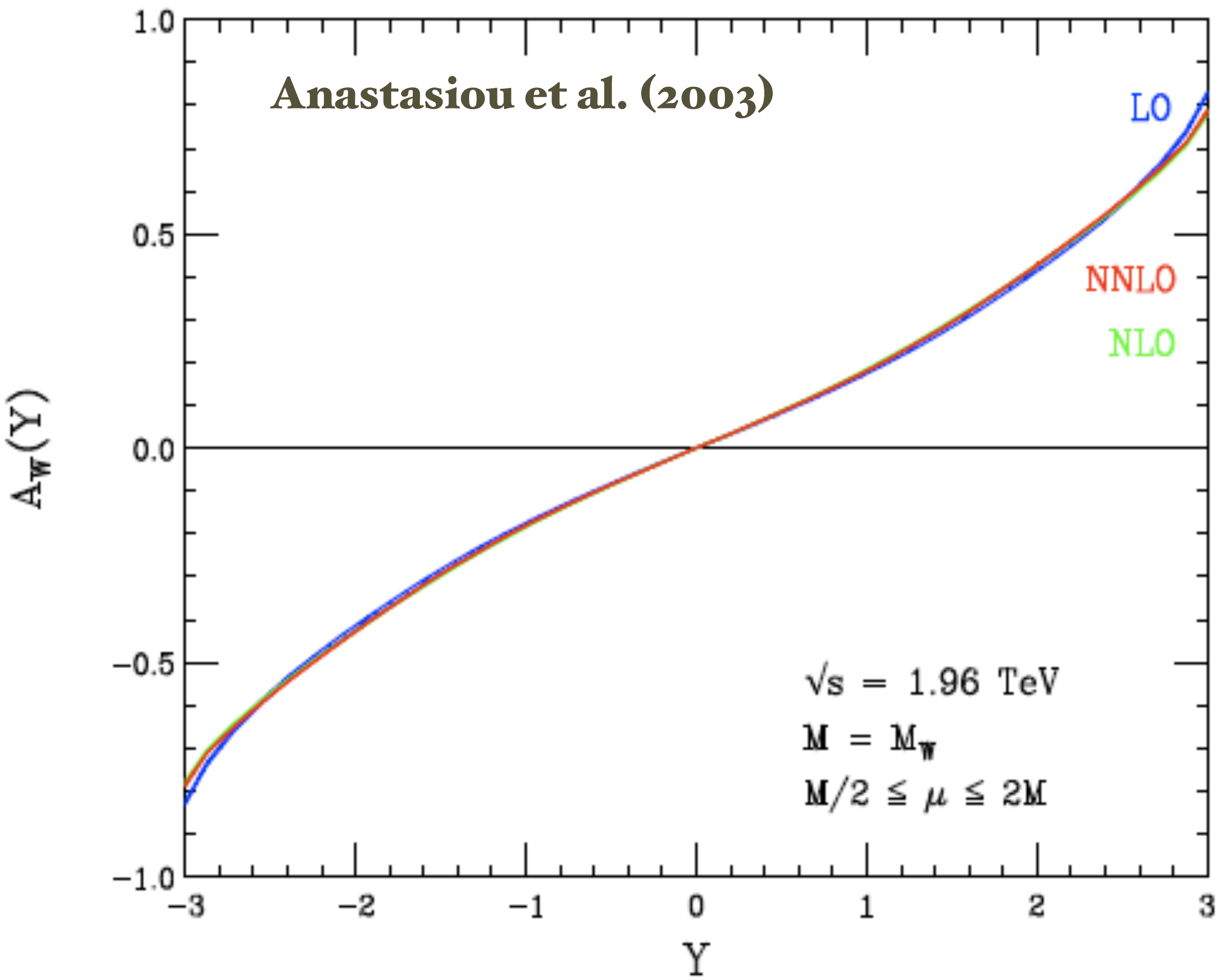
In pp collisions the W^+ and W^- are produced with different rates but W^+ and W^- rapidity distributions are forward-backward symmetric W^- distribution is central, whereas W^+ is produced at larger rapidities

These asymmetries are mainly due to the fact that, on average, the u quark carries more proton momentum fraction than the d quark

W asymmetry



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In ppbar collisions:

W^+ and W^- rapidity distributions symmetric → $A(0) = 0$

W^- cross section vanishes faster at the kinematical boundary → $A(y_{W\max}) = 1$

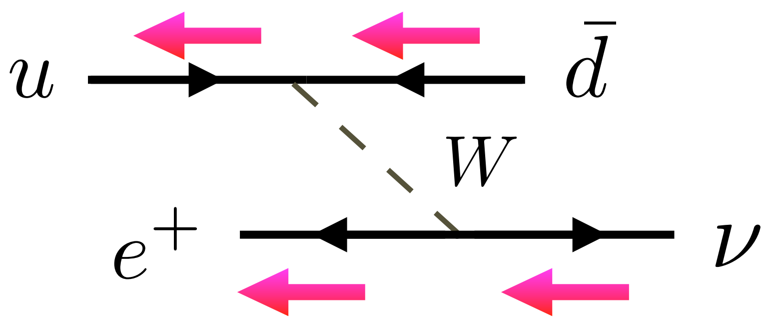
Results extremely stable against radiative corrections

But W bosons are identified through their leptonic decay $W \to l\nu$

The longitudinal component of the neutrino momentum is not measured → What is typically measured is the lepton asymmetry

Charged lepton rapidity distributions

W production and decay mechanisms are correlated



Angular momentum conservation: the charged lepton is mainly produced in the direction of the down quark

Scattering angle in the W rest frame

$$\frac{1}{\hat{\sigma}^{(0)}_{U\overline{D}}} \frac{d\hat{\sigma}^{(0)}_{U\overline{D}}}{d\cos\theta^*_{lD}} = \frac{1}{\hat{\sigma}^{(0)}_{D\overline{U}}} \frac{d\hat{\sigma}^{(0)}_{D\overline{U}}}{d\cos\theta^*_{lD}} = \frac{3}{8}(1+\cos\theta^*_{lD})^2$$

In the case of ppbar collisions the W boson tends to follow the colliding up quark

→ The dynamical correlation produced by the V-A interaction acts in the opposite direction and the rapidity distribution of the positive (negative) charged lepton is shifted backward (forward) with respect to the distribution of the parent W

Charged lepton rapidity distributions

The relative weight of the two competitive effects depends on kinematics and in particular on the lepton E_T

The lepton and W rapidities are related by $y_l = y_W + \frac{1}{2}\ln\frac{1+\cos\theta^*}{1-\cos\theta^*}$

where ϑ^* is the lepton scattering angle in the W rest frame

At LO and in the NWA E_T is bounded by $M_W/2$

The scattering angle ϑ^* is related to the lepton E_T by $1-\cos^2\theta^* = 4E_T^2/M_W^2$

→ Increasing the lepton E_T the lepton rapidity is close to the W rapidity thus minimizing the effect of EW correlation

The Tevatron data

The first measurement of the lepton charge asymmetry was done by CDF at the Tevatron Run I and dates back to 1992

The measurements that I consider here are:

- CDF, electrons, hep-ex/0501023 Used in the MSTW2008 fit

  $E_T^\nu > 25$ GeV $|\eta_e| < 2.45$

  Two E_T bins: 25 GeV < E_T < 35 GeV and 35 GeV < E_T < 45 GeV

  Isolation: total transverse energy in a cone of radius R=0.4 must be smaller than 10% of electron E_T

- DØ, electrons, arXiv:0807.3367

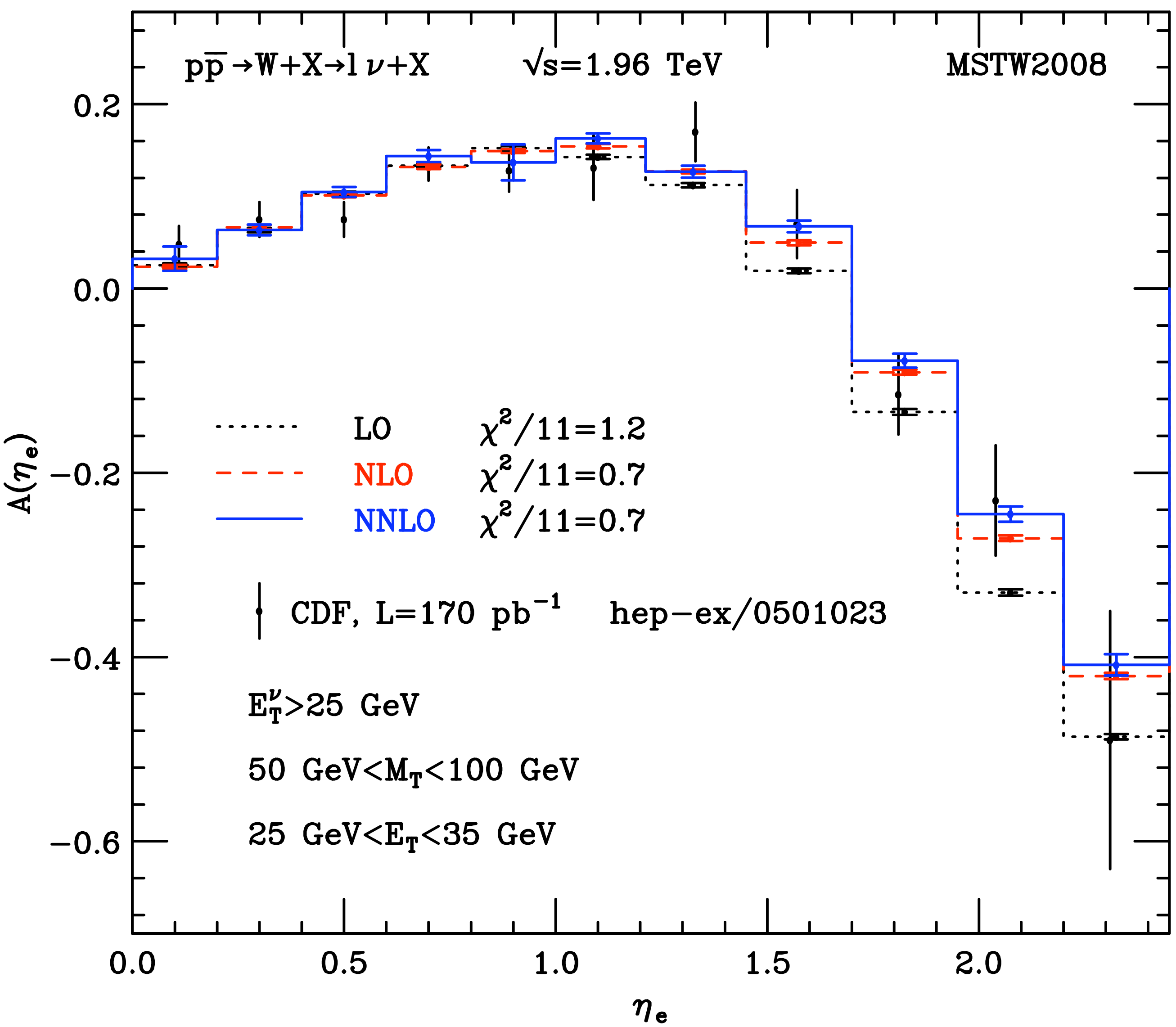
  $E_T^\nu > 25$ GeV $M_T > 50$ GeV $|\eta_e| < 3.2$

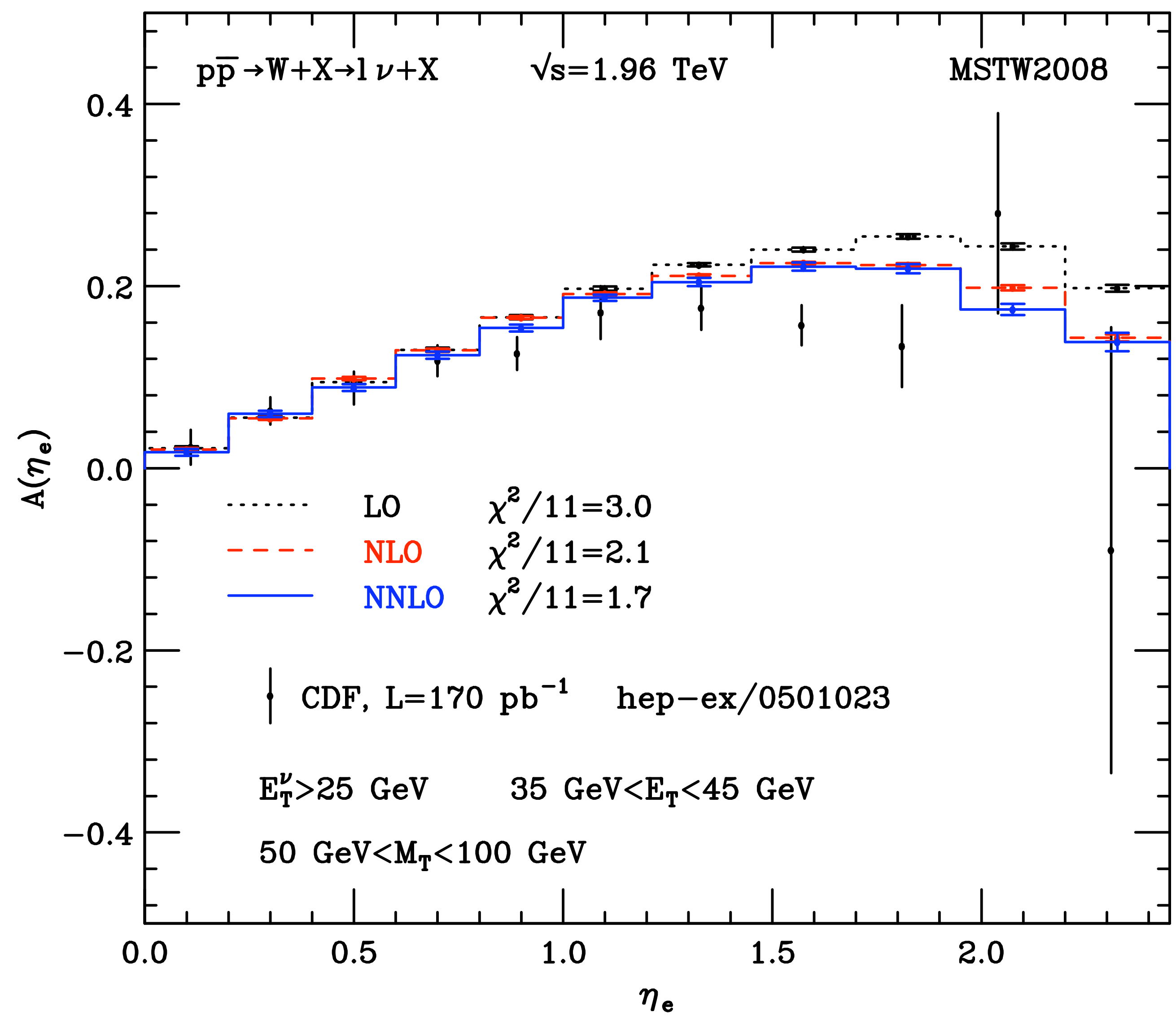
  Two E_T regions: $E_T > 25$ GeV and $E_T > 35$ GeV

  Isolation: total transverse energy in a cone R=0.4 must be smaller than 15% of the electron energy

  Not used in the MSTW2008 fit: tension with DIS data

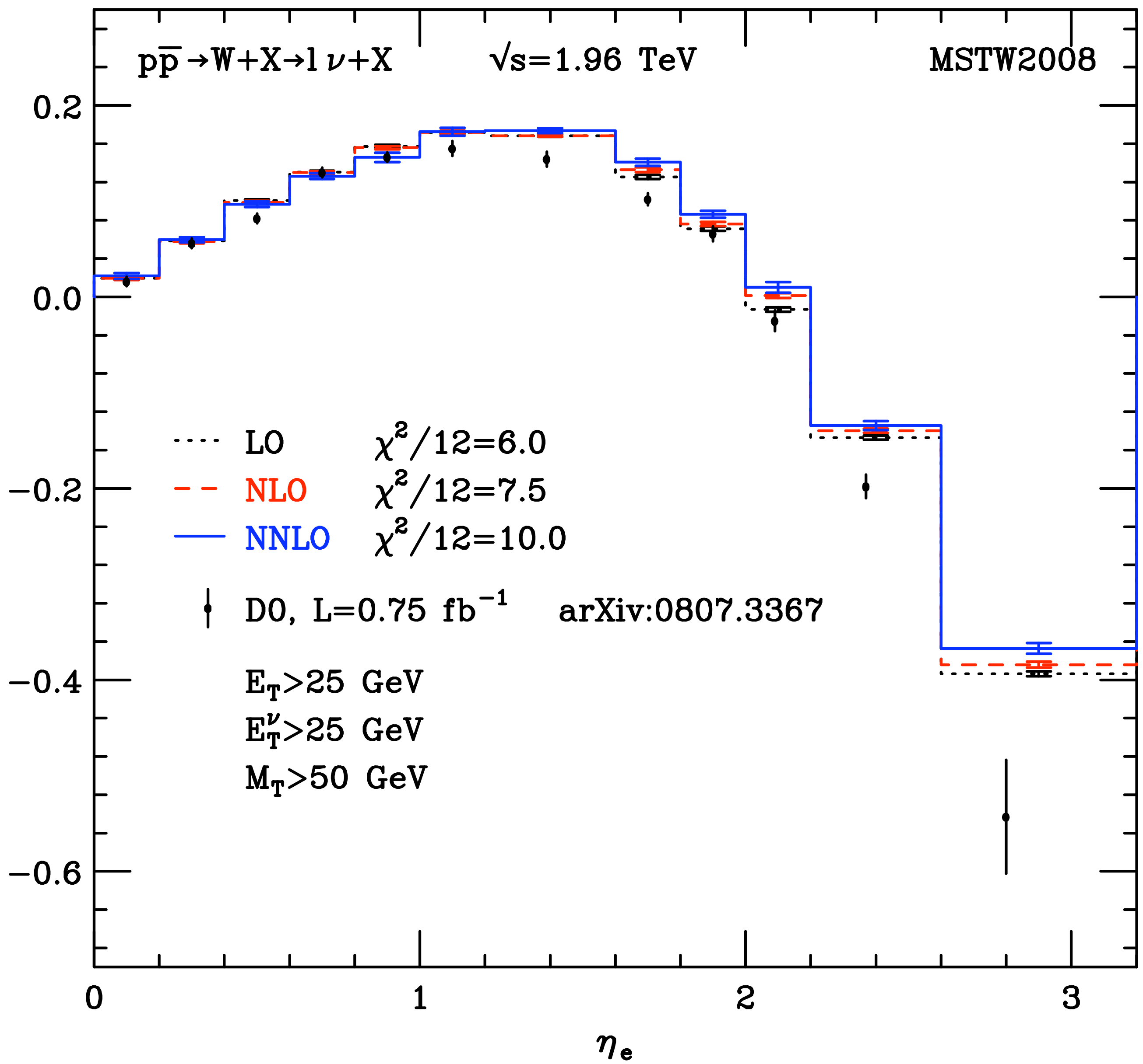
Lepton asymmetry and CDF data

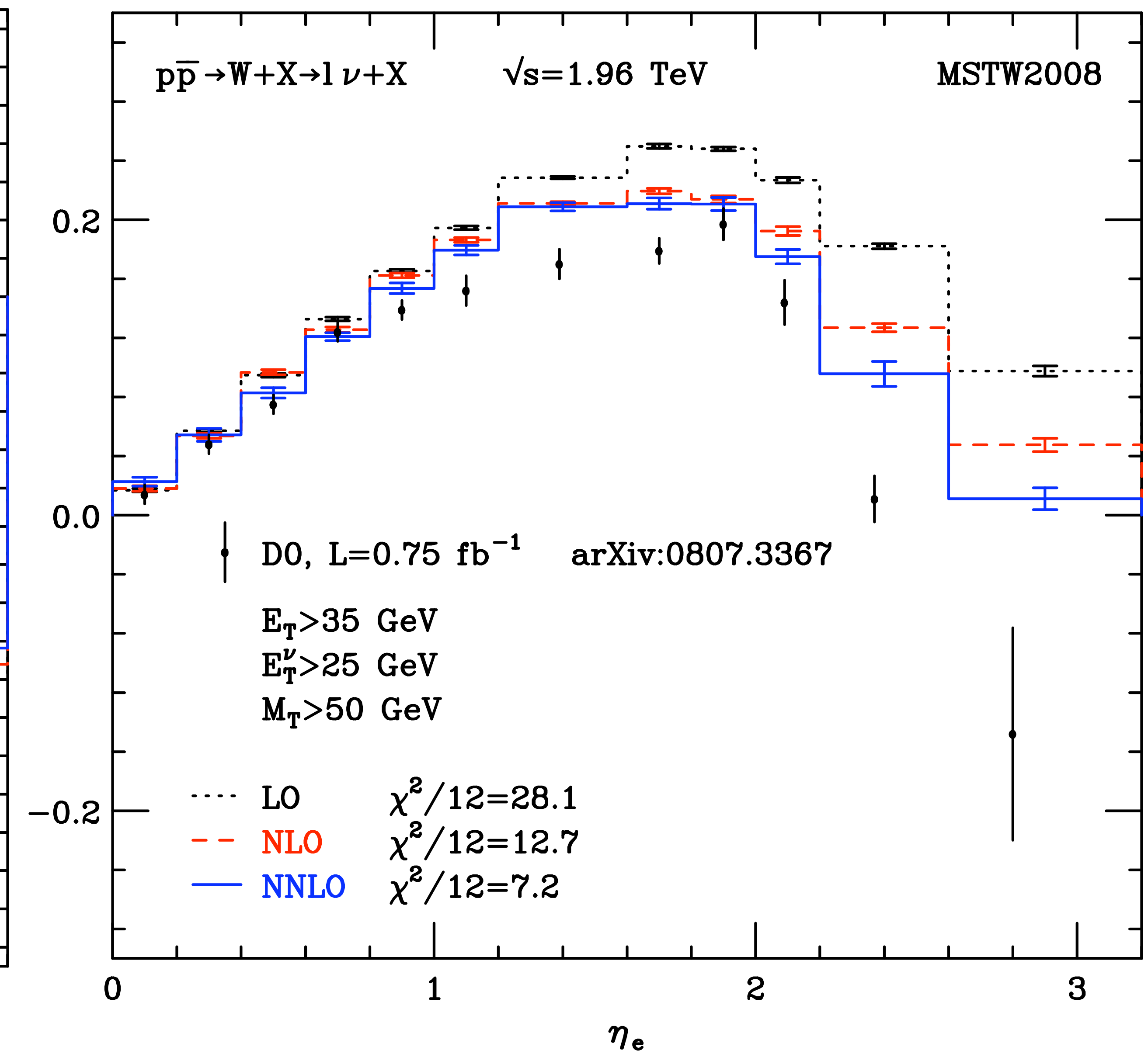




- Effect of V-A correlation evident: the asymmetry even becomes negative !
- The effect of the correlation is particularly evident in the lower E_T bin

Lepton asymmetry and new DØ data





- As expected, the agreement with the data is poor
- In the higher-E_T region the inclusion of NLO and NNLO corrections improves the situation but a substantial disagreement persists

Summary & Outlook

- Fully exclusive NNLO calculations are important in many cases

  - they provide a precise estimate of higher order corrections when cuts are applied

  - the corresponding acceptances can be compared with those obtained with standard MC event generators

- After some years of work the first fully exclusive NNLO computations have appeared, most notably

  - Higgs and vector boson production in hadron collisions

  - $e^+e^- \to 3$ jets

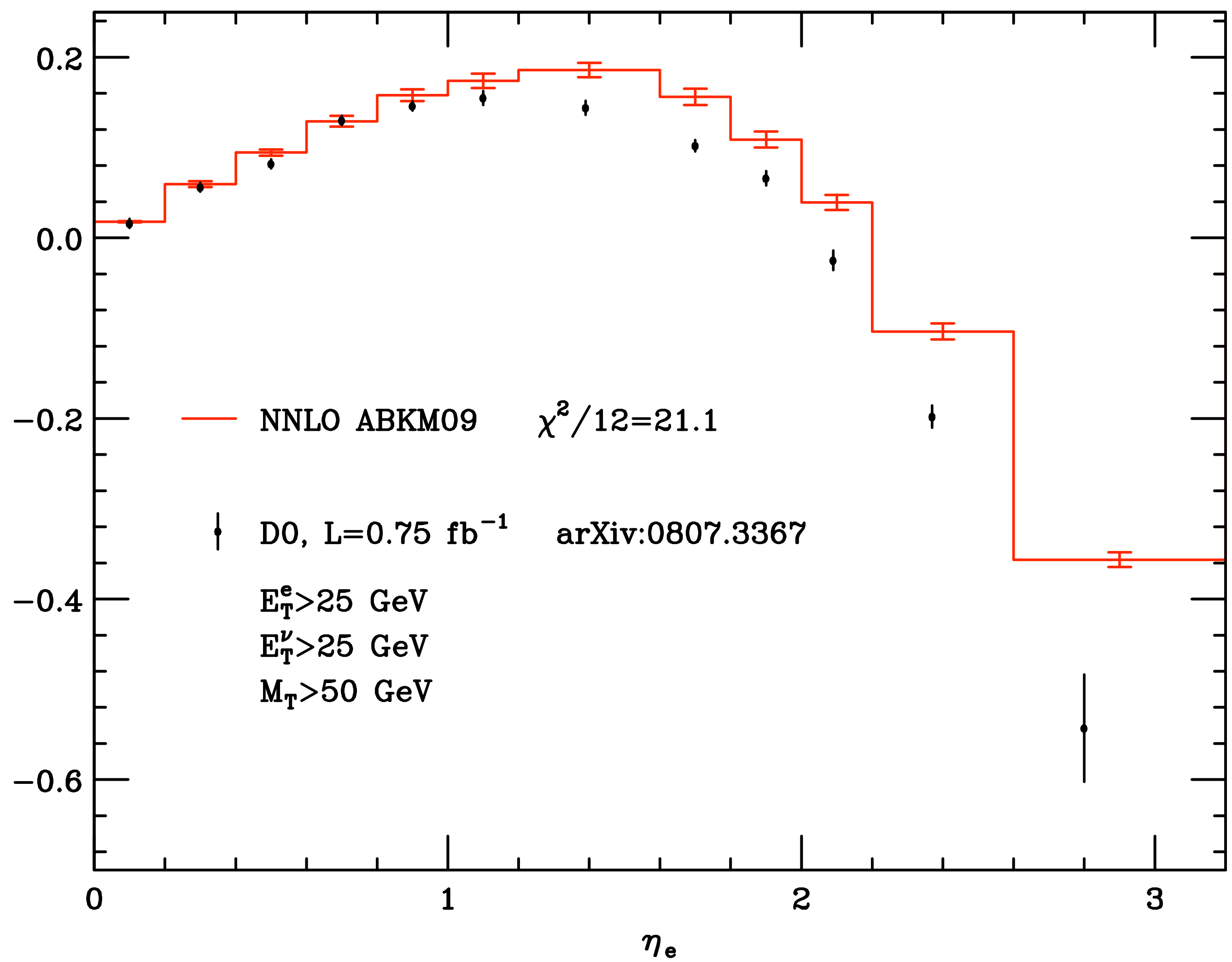
- A new powerful method, based on sector decomposition complements the more traditional approach of the subtraction method

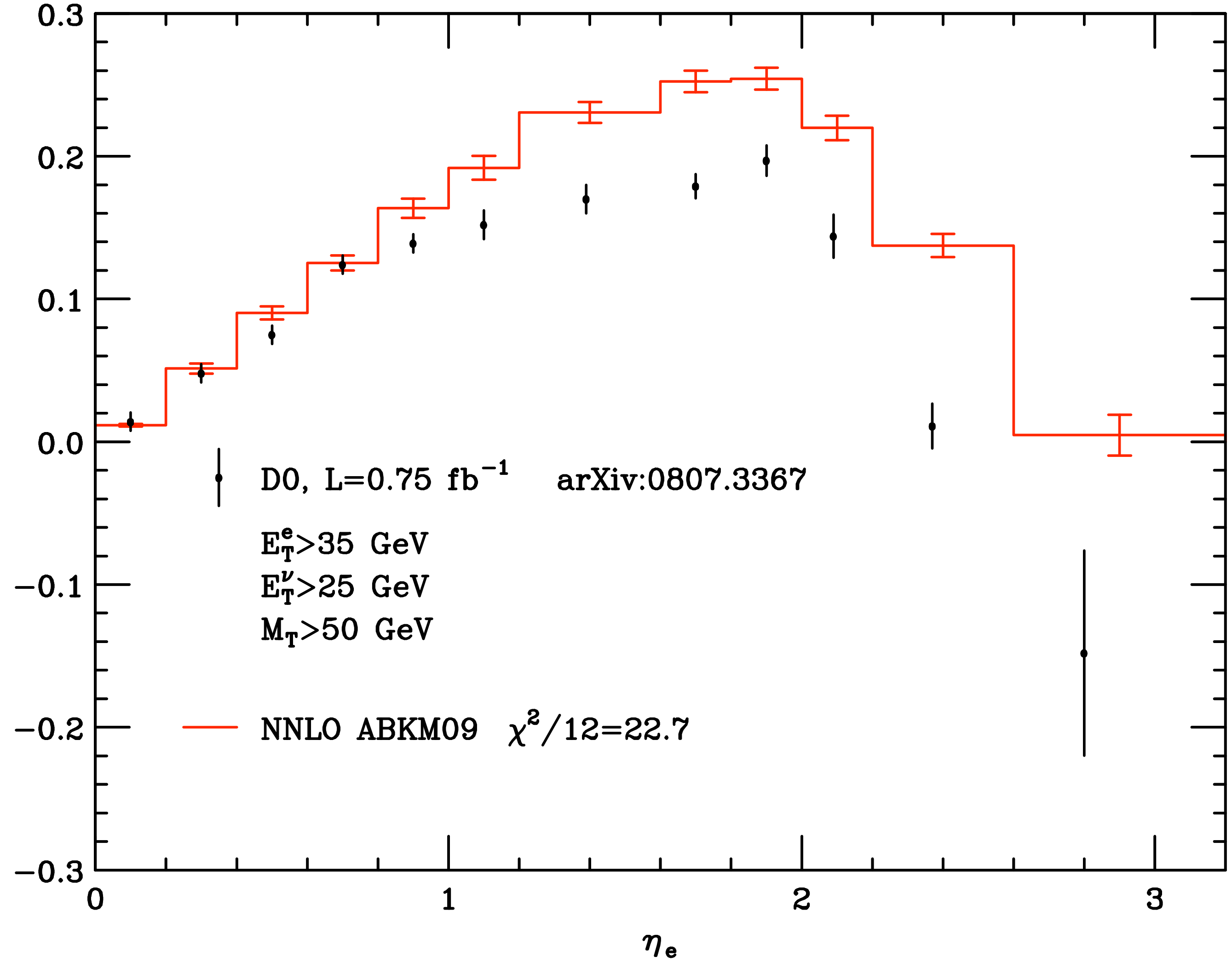
Summary & Outlook

- I have discussed an extension of the subtraction formalism that allowed us to complete the NNLO calculations for Higgs and vector boson production at hadron colliders
- The computations are implemented in the public codes HNNLO and DYNNLO
- Relatively simple standalone numerical programs that run on a single desktop computer
- The user can apply arbitrary cuts on the final state leptons (photons) and the associated jet activity, and obtain the desired distributions in the form of bin histograms
- I have presented selected numerical results for Higgs and vector boson production, and in particular for the W asymmetry

Backup Slides

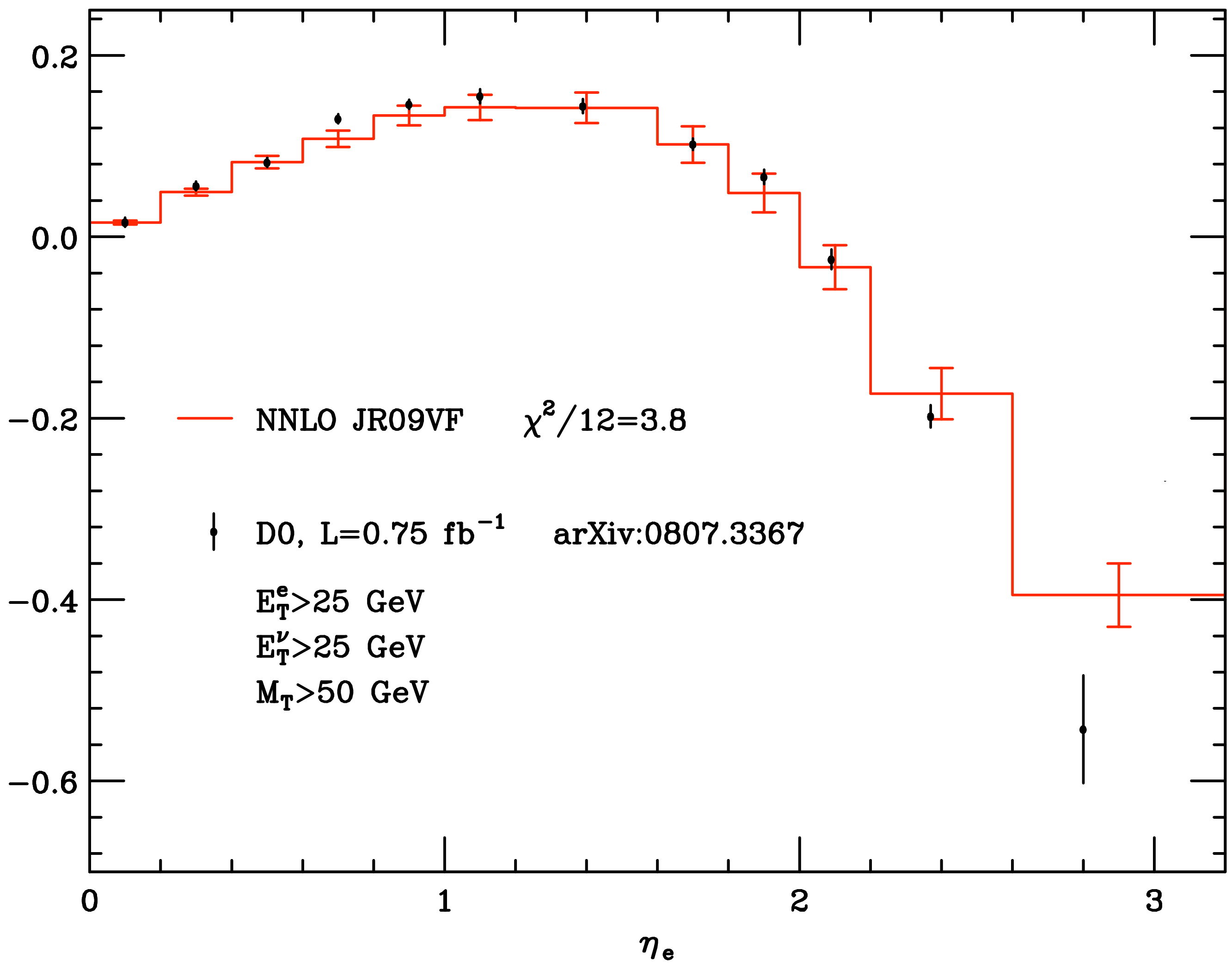
Lepton asymmetry and new DØ data

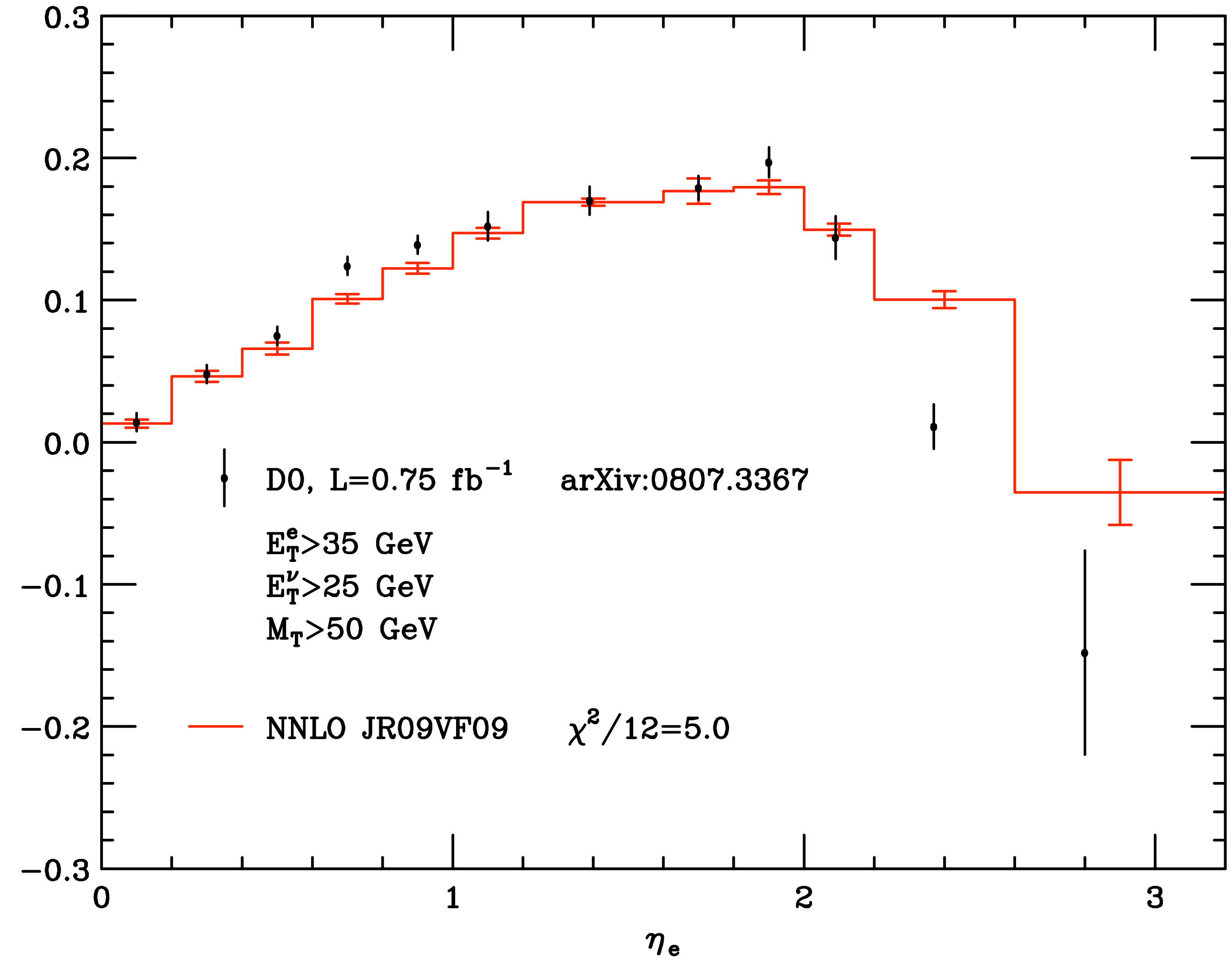




The agreement is even worse by using ABKM09 PDFs

Lepton asymmetry and new DØ data





The agreement is instead better by using JR09VF

Lepton asymmetry at the LHC

At the LHC the lepton charge asymmetry can be used to constrain PDFs at smaller values of x with respect to Tevatron energies

CMS has recently studied a possible measurement of the muon charge asymmetry

CMS-PAS-EWK-09-003

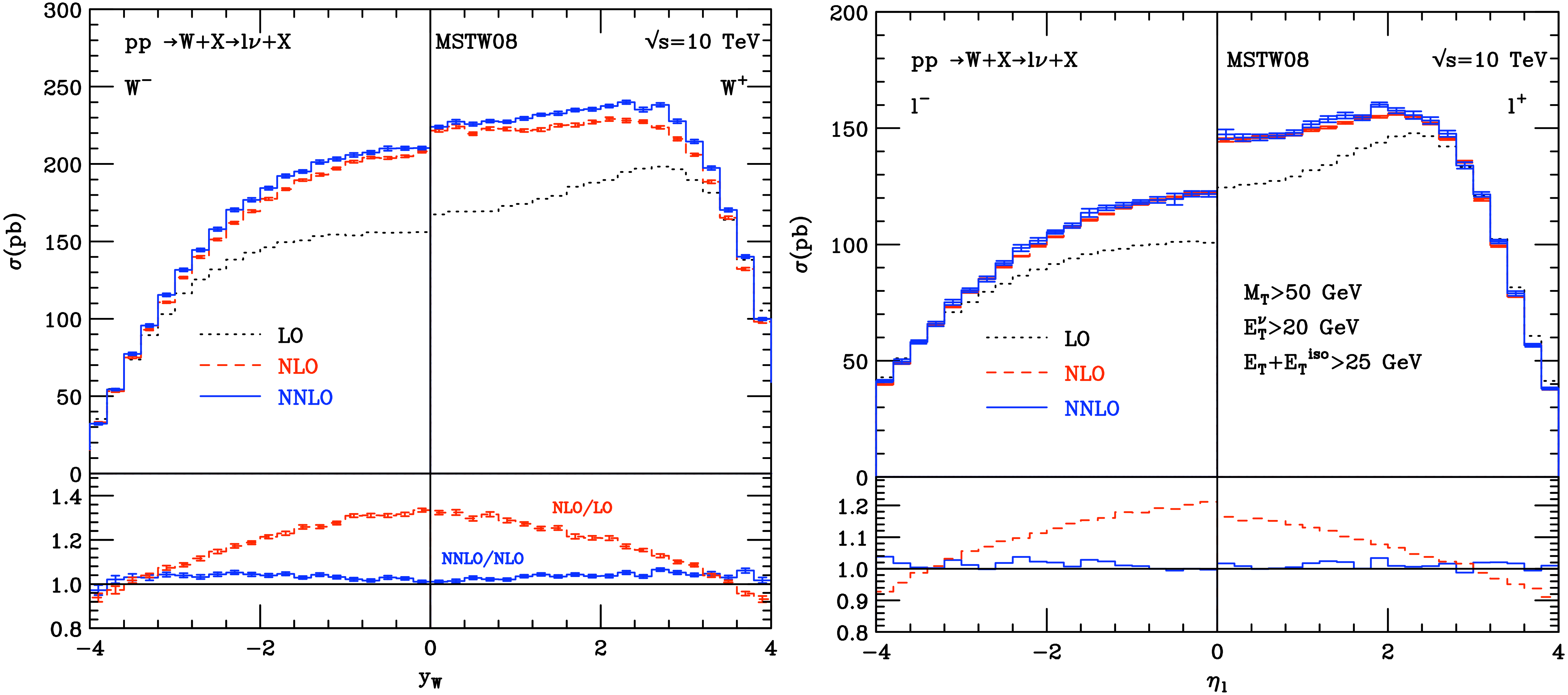
Cuts:

$E_T^\nu > 20$ GeV $M_T > 50$ GeV

$E_T + E_{Tiso} > 25$ GeV E_{Tiso} total transverse energy in a cone of radius R=0.3

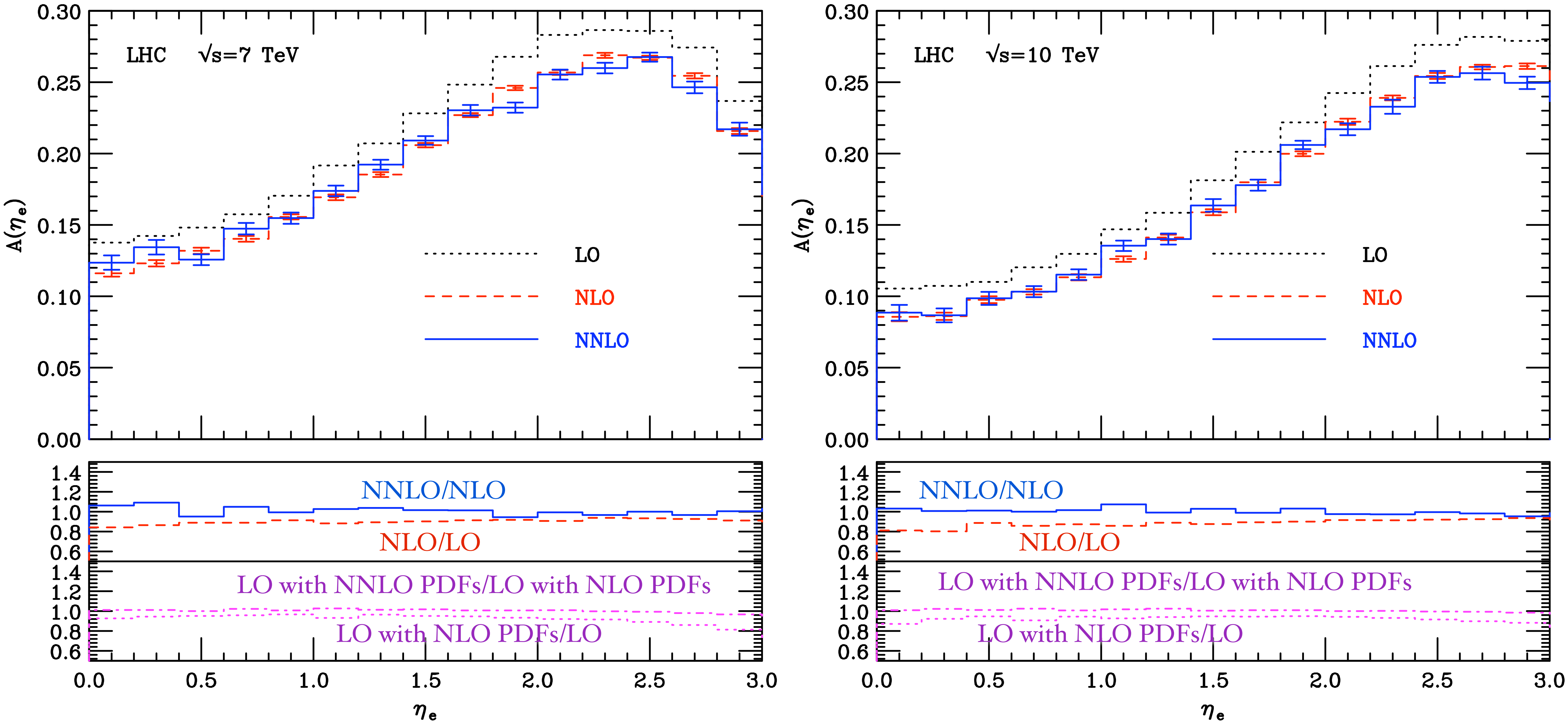
Isolation: $E_{Tiso} < z/(1-z)\, E_T$ where z=0.05

Rapidity distributions at the LHC



- Leptonic decay shifts the peak of the l^+ distribution towards smaller rapidities
- The opposite happens for the l^-

Lepton asymmetry at the LHC



From 7 to 10 TeV the asymmetry decreases
(at smaller x flavor asymmetries are less important)

NNLO contribution not particularly significant