All-Order Multi-jet Predictions with High Energy Jets

Jeppe R. Andersen in collaboration with J.M. Smillie

HO10 July 6, 2010

Overview

- We need (hard) perturbative at higher orders than at previous colliders in order to reliably describe the complicated final states
- Build framework (HEJ: High Energy Jets)
- Applications to H+jets, W+jets, pure jets

At colliders until now, **two separate** effects have ensured the suppression of hard radiative corrections:

- increasing powers of the coupling
- fast decrease of pdfs as the light-cone momentum fractions x are increased

$$\sigma_{ab\to 1\cdots N} = \int \prod_{i=1}^{N} \left(\frac{\mathrm{d}^{2} \mathbf{p}_{\perp i} \, \mathrm{d} y_{i}}{(2\pi)^{3} \, 2} \right) (2\pi)^{4} \delta^{2} \left(\sum_{i=1}^{N} \mathbf{p}_{\perp i} \right) \frac{1}{\hat{s}^{2}} \times x_{1} f_{1}(x_{1}, Q_{1}^{2}) x_{2} f_{2}(x_{2}, Q_{2}^{2}) \left| \mathcal{M}_{ab\to 1\cdots N} \right|^{2}$$

$$x_1 = \frac{1}{\sqrt{s}} \sum_{1}^{N} p_{i\perp} \exp(y_i)$$
 $x_2 = \frac{1}{\sqrt{s}} \sum_{1}^{N} p_{i\perp} \exp(-y_i)$

The Perturbative Description. The New Challenge by the LHC

If/when two jets are created far apart in rapidity, then the phase space suppression for additional central jets is exponentially small

$$x_1 = \frac{1}{\sqrt{s}} \sum_{1}^{N} p_{i\perp} \exp(y_i)$$
 $x_2 = \frac{1}{\sqrt{s}} \sum_{1}^{N} p_{i\perp} \exp(-y_i)$

This situation can arise as a result of

- event selection cuts (*H*+dijets, decay of TeV-scale resonances)
- 2 naturally because of dominance by qg initial state (W/Z+3,4,...) jets)

Why Hjj, The Problem, The Solution

Why study Higgs Boson production in Association with Dijets?

The distribution in the **azimuthal angle** between the **two** jets in *Hjj* allows for a **clean extraction** of CP properties

The Problem

... in a region of phase space where the **perturbative corrections** are large.

How do we deal with events with three or more jets?

The Solution

By constructing an azimuthal observable, which takes into account the **information from all the jets** of the event!

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Which Scalar?

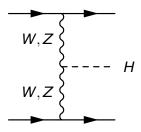
Once/if a scalar is discovered, it is important to determine whether this is the **Higgs Boson** of the **Standard Model** (or something else).

Measure the **strength** and **Lorentz structure** of the Higgs boson couplings:

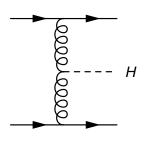
 production mechanism (independent of the Higgs decay channel)

or

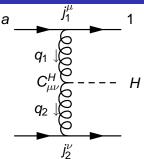
 detailed study of the Higgs boson decay products (independent on the production mechanism)



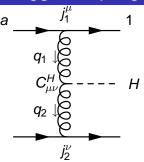
Considerations for Weak Boson Fusion



...and gluon fusion (Higgs coupling to gluons through top loop)



$$\mathcal{M} \propto rac{j_1^{\mu} \ C_{\mu
u}^{H} \ j_2^{
u}}{t_1 \ t_2}, \qquad j_1^{\mu} = \overline{\psi}_1 \gamma^{\mu} \psi_a \ C_H^{\mu
u} = a_2 \ (q_1 q_2 g^{\mu
u} \ - \ q_1^{
u} q_2^{\mu}) \ + a_3 \ arepsilon^{\mu
u
ho\sigma} \ q_{1
ho} \ q_{2\sigma}.$$



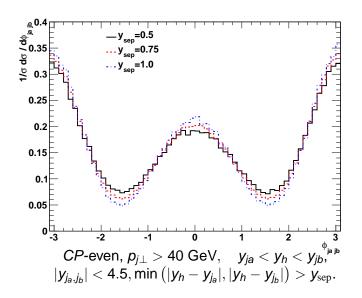
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u
ho\sigma} \ q_{1
ho} \ q_{2\sigma}.$$

Take e.g. the term $\varepsilon^{\mu\nu\rho\sigma}$ $q_{1\rho}$ $q_{2\sigma}$: for $|p_{1,z}|\gg |p_{1,x,y}|$ and for small energy loss (i.e. $p_{a,e}\sim p_{1,e}$):

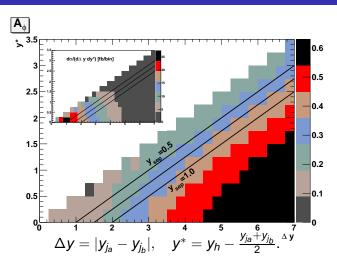
$$\left[j_1^0 \ j_2^3 - j_1^3 \ j_2^0\right] \left({f q}_{1\perp} imes {f q}_{2\perp}
ight).$$

In this limit, the azimuthal dependence of the propagators is also suppressed: $|\mathcal{M}|^2$:sin²(ϕ) (CP-odd), cos²(ϕ) (CP-even).

Azimuthal distribution

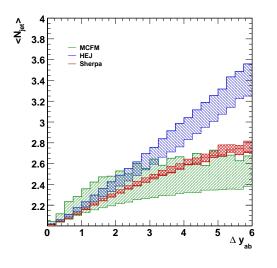


Signature and Cross Section



Rapidity separation between the jets and the Higgs Boson enhance the azimuthal correlation.

Increasing Rapidity Span → Increasing Number of Jets



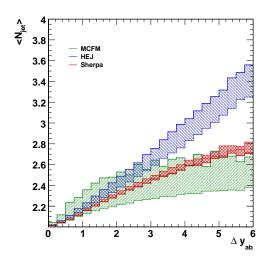
All models show a clear increase in the number of hard jets as the rapidity span increases.

How to extract the *CP*-structure of the Higgs boson coupling from events with **three or more** jets?

2 hardest jets?

J.R. Andersen, J. Campbell, S. Höche, arXiv:1003.

Increasing Rapidity Span → Increasing Number of Jets



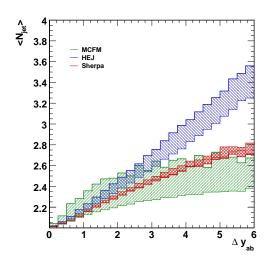
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2 hard jets furthest apart in rapidity?

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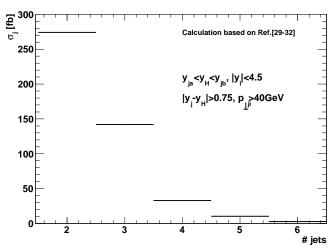
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Significant washing out of the azimuthal correlation observed at tree-level *hij*

J.R. Andersen, J. Campbell, S. Höche, arXiv:1003.

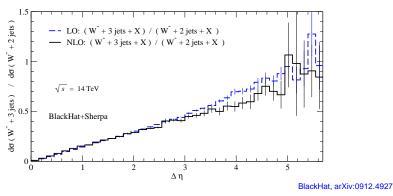
Many Jets!



Calculation based on all-order approximant to the *n*-particle matrix element, which reproduces the exact result in the limit of large invariant mass between all particles.

JRA&C.D. White, JRA&J.M. Smillie

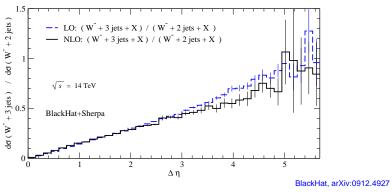
W+Multiple Jets @ NLO



The inclusive 3-jet rate is large compared to the inclusive 2-jet rate, even for normal rapidity spans obviously, the inclusive 3-jet rate "ought to" be smaller than the inclusive 2-jet rate.

The large contribution from real radiative corrections to W+dijets is not revealed by the inclusive K-factor (actually less than one)

W+Multiple Jets @ NLO

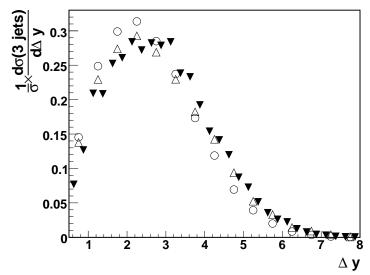


All calculational methods and processes will agree on the opening of phase space as Δy increases

The mechanism for emission **differ** between processes (WBF vs. GF) and calculational methods (full NLO, shower, ...). Can be **tested against data!**

$1/\sigma d\sigma/d\Delta y$

JRA, M. Campanelli, J. Campbell, V. Ciulli, J. Huston, P. Lenzi, R. Mackeprang, arXiv:1003.1241



 $\Delta y \approx 2-3$ (where σ_{3j}/σ_{2j} is already very large) is not "tail of distribution"!

HEJ (High Energy Jets)

Goal (inspired by the great Fadin & Lipatov)

Sufficiently **simple** model for hard radiative corrections that the all-order sum can be evaluated explicitly (completely exclusive)

but...

Sufficiently accurate that the description is relevant

Resummation and Matching

Consider the **perturbative expansion** of an observable

$$R = r_0 + r_1 \alpha_s + r_2 \alpha^2 + r_3 \alpha^3 + r_4 \alpha^4 + \cdots$$

Fixed order pert. QCD will calculate a fixed number of terms in this expansion. r_n may contain **logarithms** so that $\alpha_s \ln(\cdots)$ is large.

$$R = r_0 + \left(r_1^{LL} \ln(\cdots) + r_1^{NLL}\right) \alpha_s + \left(r_2^{LL} \ln^2(\cdots) + r_2^{NLL} \ln(\cdots) + r_2^{SL}\right) \alpha_s^2 + \cdots$$

$$= r_0 + \sum_n r_n^{LL} (\alpha_s \ln(\cdots))^n + \sum_n r_n^{NLL} \alpha_s (\alpha_s \ln(\cdots))^n + \text{sub-leading terms}$$

Need simplifying assumptions to get to all orders - useful **iff the terms** really do describe **the dominant part** of the **full pert. series**. **Matching** combines **best of both worlds**:

$$R = r_0 + r_1 \alpha_s + r_2 \alpha^2 + \left(r_3^{LL} \ln^3(\cdots) + r_3^{NLL} \ln^2(\cdots) + r_3^{SL} \right) \alpha^3 + \cdots$$

It is **well known** that QCD matrix elements **factorise** in certain kinematical limits:

Collinear limit enters all parton shower (and much else) resummation.

Like all good limits, this approximation is applied **outside its strict** region of validity.

Will discuss the **less well-studied factorisation** of scattering amplitudes in a different kinematic limit, better suited for describing perturbative corrections from **hard parton emission**

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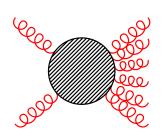
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The Possibility for Predictions of *n*-jet Rates

The Power of Reggeisation



High Energy Limit

$$|\hat{t}|$$
 fixed, $\hat{s} o \infty$

$$\mathbf{K_b}, \mathbf{y_b}$$

 $\mathbf{k_4}, \mathbf{y}$

 \mathbf{k}_{3}

 \mathbf{k}_2, y_2

 \mathbf{k}_1, y_1

$$(g_{0,1})(y_0-y_{0,1})$$
 $\Gamma_{B'B}$

 $\mathcal{A}_{2\to 2+n}^{R} = \frac{\Gamma_{A'A}}{q_{n}^{2}} \left(\prod_{i=1}^{n} e^{\omega(q_{i})(y_{i-1}-y_{i})} \frac{V^{J_{i}}(q_{i}, q_{i+1})}{q_{i}^{2} q_{i+1}^{2}} \right) e^{\omega(q_{n+1})(y_{n}-y_{n+1})} \frac{\Gamma_{B'B}}{q_{n+1}^{2}}$

 $q_i^2 q_{i+1}^2$ f_{n+1}^2 f_{n+1}^2 LL: Fadin, Kuraev, Lipatov; NLL: Fadin, Fiore, Kozlov, Reznichenko

 $q_i = \mathbf{k_a} + \sum_{l=1}^{i-1} \mathbf{k_l}$ Maintain (at LL) terms of the form

$$\left(\alpha_{s} \ln \frac{\hat{\mathsf{s}}_{ij}}{|\hat{t}_{i}|}\right)$$

At LL only gluon production; at NLL also quark—anti-quark pairs produced. Approximation of any-jet rate possible.

to all orders in α_s .

Universal behaviour of scattering amplitudes in the HE limit:

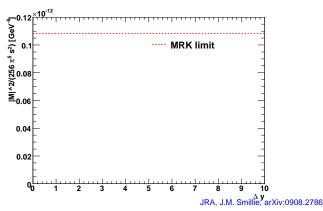
$$\forall i \in \{2, \dots, n-1\} : y_{i-1} \gg y_i \gg y_{i+1}$$

 $\forall i, j : |p_{i\perp}| \approx |p_{j\perp}|$

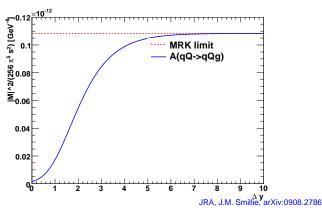
$$\begin{split} \left| \overline{\mathcal{M}}_{gg \to g \cdots g}^{MRK} \right|^2 &= \frac{4 \ s^2}{N_C^2 - 1} \ \frac{g^2 \ C_A}{|p_{1 \perp}|^2} \left(\prod_{i=2}^{n-1} \frac{4 \ g^2 C_A}{|p_{i \perp}|^2} \right) \frac{g^2 \ C_A}{|p_{n \perp}|^2}. \\ \left| \overline{\mathcal{M}}_{qg \to qg \cdots g}^{MRK} \right|^2 &= \frac{4 \ s^2}{N_C^2 - 1} \ \frac{g^2 \ C_F}{|p_{1 \perp}|^2} \left(\prod_{i=2}^{n-1} \frac{4 \ g^2 C_A}{|p_{i \perp}|^2} \right) \frac{g^2 \ C_A}{|p_{n \perp}|^2}, \\ \left| \overline{\mathcal{M}}_{qQ \to qg \cdots Q}^{MRK} \right|^2 &= \frac{4 \ s^2}{N_C^2 - 1} \ \frac{g^2 \ C_F}{|p_{1 \perp}|^2} \left(\prod_{i=2}^{n-1} \frac{4 \ g^2 C_A}{|p_{i \perp}|^2} \right) \frac{g^2 \ C_F}{|p_{n \perp}|^2}, \end{split}$$

Allow for analytic resummation (BFKL equation). However, how well does this actually approximate the amplitude?

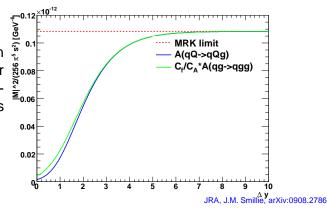
Study just a slice in phase space:



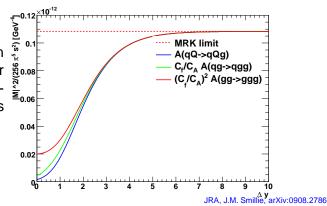
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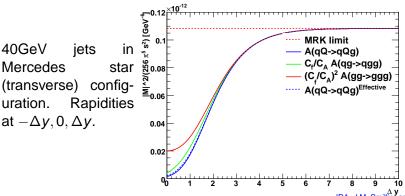
Study just a slice in phase space:



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- 1) Inspiration from Fadin&Lipatov: dominance by t-channel JRA, J.M. Smillie, arXiv:0908.2786
- 2) No kinematic approximations in the position of these poles (denominator)
- 3) Accurate definition of currents (coupling through *t*-channel exchange)
- 4) Gauge invariance. Not just asymptotically.

Scattering of qQ-Helicity States

Start by describing quark scattering. Simple matrix element for $q(a)Q(b) \rightarrow q(1)Q(2)$:

$$M_{q^-Q^-
ightarrow q^-Q^-} = \langle 1|\mu|a
angle rac{g^{\mu
u}}{t} \langle 2|
u|b
angle$$

t-channel factorised: Contraction of (local) currents across *t*-channel pole

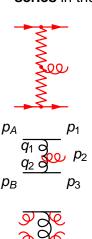
$$\begin{split} \left| \overline{\mathcal{M}}_{qQ \to qQ}^t \right|^2 &= \frac{1}{4 \left(N_C^2 - 1 \right)} \, \left\| \mathbb{S}_{qQ \to qQ} \right\|^2 \\ &\quad \cdot \, \left(g^2 \, \mathit{C}_{\!\mathit{F}} \, \frac{1}{t_1} \right) \\ &\quad \cdot \, \left(g^2 \, \mathit{C}_{\!\mathit{F}} \, \frac{1}{t_2} \right). \end{split}$$

Extend to $2 \rightarrow n \dots$

J.M.Smillie and JRA: arXiv:0908.2786

Building Blocks for an Amplitude

Identification of the **dominant contributions** to the **perturbative series** in the limit of well-separated particles



mit of well-separated particles
$$q = \frac{1}{q^2} \exp\left(\hat{\alpha}(q)\Delta y\right)$$

$$q_{i-1} = \frac{1}{q^2} \exp\left(\hat{\alpha}(q)\Delta y\right)$$

$$q_i = \frac{1}{q^2} \exp\left(\hat{\alpha}(q)\Delta y\right)$$

$$j^{\nu} = \frac{1}{q^2} \exp\left(\frac{1}{q^2} + \frac{1}{q^2} + \frac{1$$

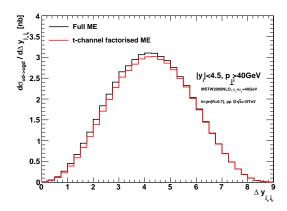
 $-\frac{p_B^\rho}{2}\left(\frac{q_2^2}{p_2 \cdot p_B} + \frac{p_2 \cdot p_A}{p_B \cdot p_A} + \frac{p_2 \cdot p_1}{p_A \cdot p_1}\right) - p_B \leftrightarrow p_3.$

Building Blocks for an Amplitude

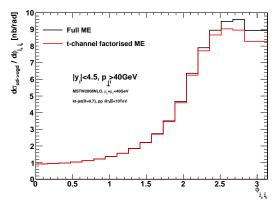
 $p_g \cdot V = 0$ can easily be checked (gauge invariance) The approximation for $qQ \rightarrow qgQ$ is given by

$$\begin{split} \left| \overline{\mathcal{M}}_{qQ \to qgQ}^t \right|^2 &= \frac{1}{4 \left(N_C^2 - 1 \right)} \, \left\| S_{qQ \to qQ} \right\|^2 \\ &\quad \cdot \, \left(g^2 \, \mathit{C}_{\!\mathit{F}} \, \frac{1}{t_1} \right) \cdot \, \left(g^2 \, \mathit{C}_{\!\mathit{F}} \, \frac{1}{t_2} \right) \\ &\quad \cdot \left(\frac{-g^2 \mathit{C}_A}{t_1 t_2} \, \mathit{V}^\mu(q_1, q_2) \mathit{V}_\mu(q_1, q_2) \right). \end{split}$$

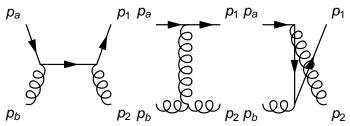
3 Jets @ 10 TeV



3 Jets @ 10 TeV



"What happens in $2 \rightarrow 2$ -processes with gluons? Surely the *t*-channel factorisation is spoiled!"



Direct calculation $(q^-g^- o q^-g^-)$:

$$M = \frac{g^2}{\hat{t}} \times \frac{\rho_{2\perp}^*}{|\rho_{2\perp}|} \left(t_{\text{ae}}^2 t_{\text{e1}}^b \sqrt{\frac{\rho_b^-}{\rho_2^-}} - t_{\text{ae}}^b t_{\text{e1}}^2 \sqrt{\frac{\rho_2^-}{\rho_b^-}} \right) \langle b | \sigma | 2 \rangle \ \times \langle 1 | \sigma | a \rangle. \label{eq:mass_model}$$

Complete t-channel factorisation!

J.M.Smillie and JRA

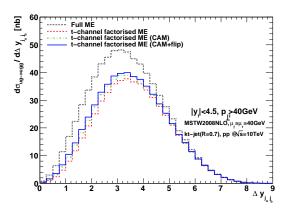
For the helicity choices where a qQ-channel exists, the t-channel current generated by a gluon in qg scattering is that of a quark, but with a colour factor

$$\frac{1}{2} \left(C_A - \frac{1}{C_A} \right) \left(\frac{\rho_b^-}{\rho_2^-} + \frac{\rho_2^-}{\rho_b^-} \right) + \frac{1}{C_A}$$

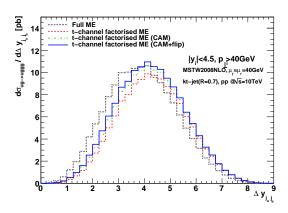
instead of C_F . Tends to C_A in MRK limit.

Similar results for e.g. $g^+g^- \rightarrow g^+g^-$. **Exact, complete** *t*-channel factorisation.

By using the formalism of **current-current scattering**, we get a better description of the t-channel pole than by using just the kinematic limit.



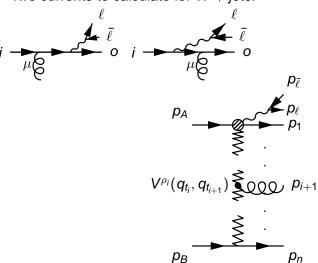
J.M.Smillie and JRA



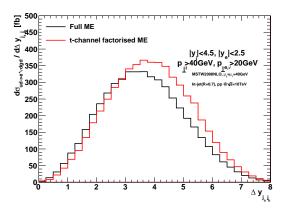
J.M.Smillie and JRA

W+Jets

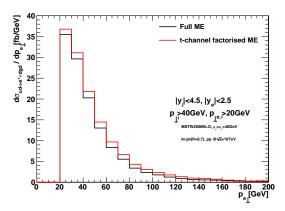
Two currents to calculate for W + jets:



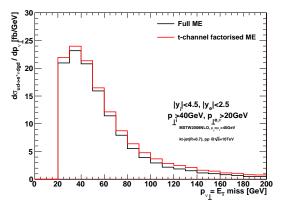
W+ 3 Jets @ LHC



W+ 3 Jets @ LHC



W+3 Jets @ LHC



Performing the Explicit Resummation

Soft divergence from real radiation:

$$\left|\mathcal{M}_{\mathrm{HE}}^{\rho_a\rho_b\to\rho_0\rho_1\rho_h\rho_2}\right|^2 \overset{\mathbf{p}_1^2\to0}{\longrightarrow} \left(\frac{4g_s^2C_A}{\mathbf{p}_1^2}\right) \left|\mathcal{M}_{\mathrm{HE}}^{\rho_a\rho_b\to\rho_0\rho_h\rho_2}\right|^2$$

Integrate over the soft part $\mathbf{p}_i^2 < \lambda^2$ of phase space in $D = 4 + 2\varepsilon$ dimensions

$$\begin{split} & \int_0^\lambda \frac{\mathrm{d}^{2+2\varepsilon} \mathbf{p} \ \mathrm{d}y_1}{(2\pi)^{2+2\varepsilon} \ 4\pi} \left(\frac{4g_s^2 C_A}{\mathbf{p}^2} \right) \mu^{-2\varepsilon} \\ & = \frac{4g_s^2 C_A}{(2\pi)^{2+2\varepsilon} 4\pi} \Delta y_{0h} \frac{\pi^{1+\varepsilon}}{\Gamma(1+\varepsilon)} \frac{1}{\varepsilon} (\lambda^2/\mu^2)^{\varepsilon} \end{split}$$

Pole in ε cancels with that from the virtual corrections

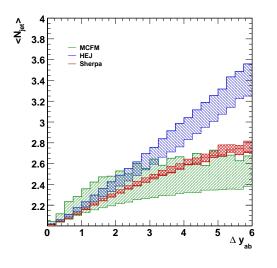
$$\hat{lpha}(t) = -rac{g_{s}^{2}C_{\!A}\Gamma(1-arepsilon)}{(4\pi)^{2+arepsilon}}rac{2}{arepsilon}\left(\mathbf{q}^{2}/\mu^{2}
ight)^{arepsilon}.$$

All-Orders and Regularisation

- Have prescription for $2 \to n$ matrix element, including virtual corrections: Lipatov Ansatz $1/t \to 1/t \exp(-\omega(t)\Delta y_{ij})$
- Organisation of cancellation of IR (soft) divergences is easy
- Can calculate the sum over the n-particle phase space explicitly $(n \sim 30)$ to get the all-order corrections (just as if one had provided all the $N^{30}LO$ matrix elements and a regularisation procedure)
- Match to n-jet tree-level where known

J.M. Smillie, JRA arXiv:0908.2786, arXiv: 0910:5113

Increasing Rapidity Span → Increasing Number of Jets



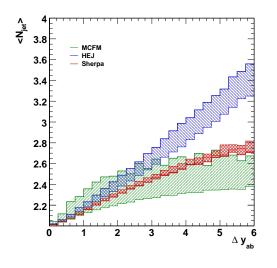
All models show a clear increase in the number of hard jets as the rapidity span increases.

How to extract the *CP*-structure of the Higgs boson coupling from events with **three or more** jets?

2 hardest jets?

J.R. Andersen, J. Campbell, S. Höche, arXiv:1003.

Increasing Rapidity Span → Increasing Number of Jets



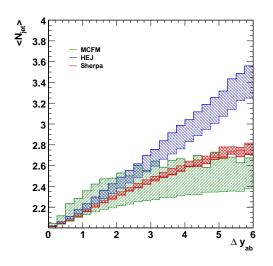
All models show a clear increase in the number of hard jets as the rapidity span increases.

How to extract the *CP*-structure of the Higgs boson coupling from events with **three or more** jets?

2 hard jets furthest apart in rapidity?

J.R. Andersen, J. Campbell, S. Höche, arXiv:1003.

Increasing Rapidity Span → Increasing Number of Jets



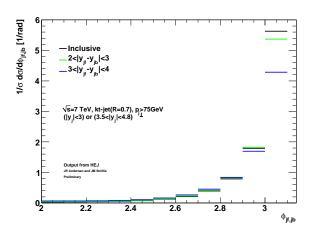
All models show a clear increase in the number of hard jets as the rapidity span increases.

How to extract the *CP*-structure of the Higgs boson coupling from events with **three or more** jets?

Significant washing out of the azimuthal correlation observed at tree-level *hjj*

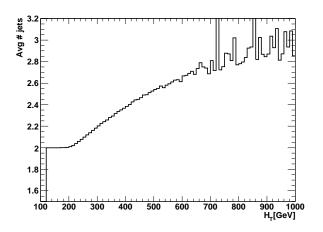
J.R. Andersen, J. Campbell, S. Höche, arXiv:1003.

Azimuthal correlations in dijets



Evolution should be measurable with dijets already in this years data

Dominance of Multi-Jets in Tails Too. W+dijets



Hard, radiate corrections are important for the description of the tails also in e.g. H_t .

Outlook and Conclusions

Conclusions

- Emerging framework for the study of processes with multiple hard jets
- For each number of particles n, the approximation to the matrix element (real and virtual) is sufficiently simple to allow for the all-order summation to be constructed as an explicit sum over n-particle final states (exclusive studies possible)
- Resummation based on approximation which really does capture the behaviour of the scattering processes at the LHC
- Matching will correct the approximation where the full matrix element can be evaluated