Simulating NNLO QCD corrections for processes with giant K factors

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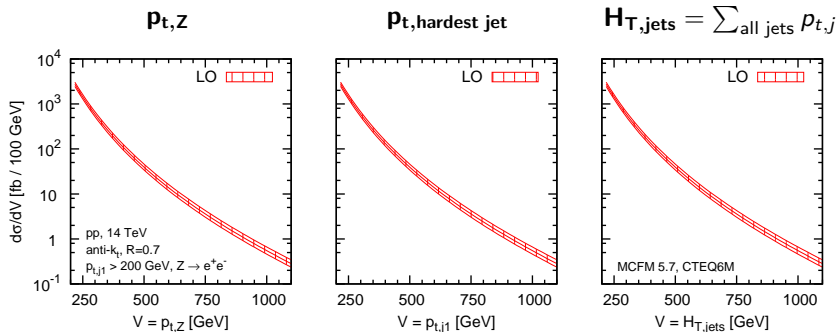
in collaboration with Gavin Salam and Mathieu Rubin [1]

CERN TH Institute, Perturbative higher-order effects at work at the LHC, June 21 – July 9, 2010
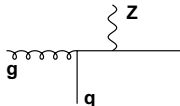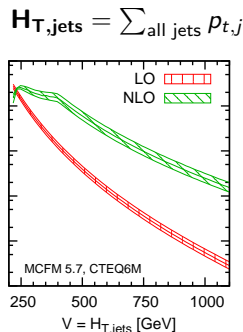
[1] M.Rubin, G.P.Salam and SS, arXiv:1006.2144 [hep-ph]
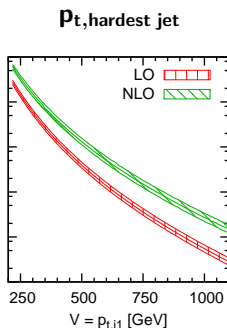
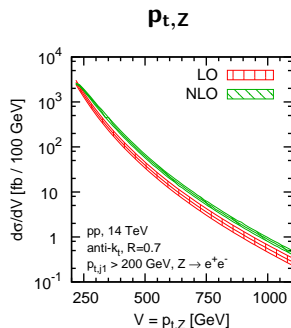# The problem of giant K factors

▶ Z+j at the LHC
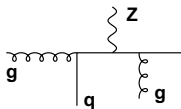

LO:

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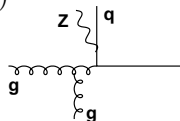
- Z+j at the LHC

NLO:

What do we have and what is missing?

▶ The large K factor for the Z+jet comes from the new "dijet type" topologies that appear at NLO
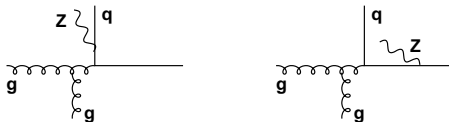
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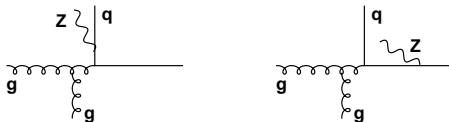▶ need for subleading contributions for Z+jet, in this case NNLO
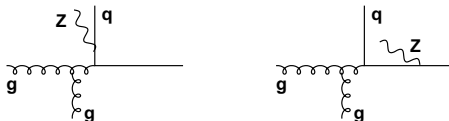
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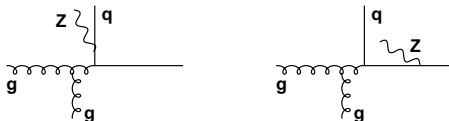▶ The large K factor for the Z+jet comes from the new "dijet type" topologies that appear at NLO

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▶ **2-loop part**
  ▶ we need it to cancel IR and collinear divergences from Z+2j at NLO result
  ▶ it will have the topology of Z+j at LO so it will not contribute much to the cross sections with giant K-factor

The basic idea

How to cancel the infrared and collinear singularities?

The basic idea
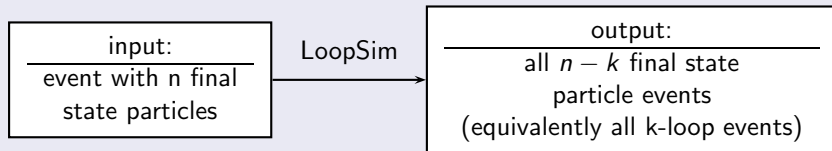
How to cancel the infrared and collinear singularities?

- use unitarity to simulate the divergent part of 2-loop diagrams

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LoopSim procedure

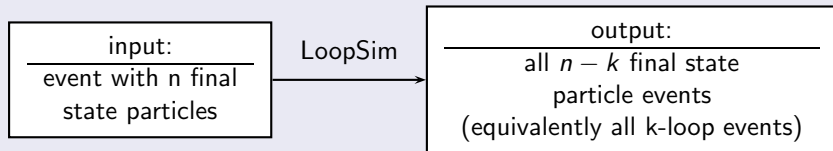| input:	LoopSim	output:
event with n final state particles	→	all $n - k$ final state particle events (equivalently all k-loop events)

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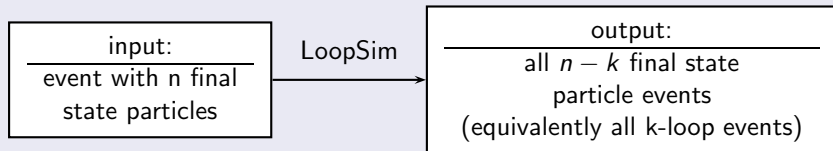
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▶ notation:

$\bar{n}$**LO** – simulated 1-loop
$\bar{n}\bar{n}$**LO** – simulated 2-loop and simulated 1-loop
$\bar{n}$**NLO** – simulated 2-loop and exact 1-loop

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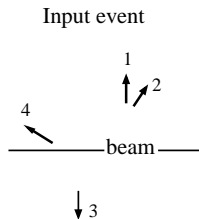- ▶ use unitarity to simulate the divergent part of 2-loop diagrams
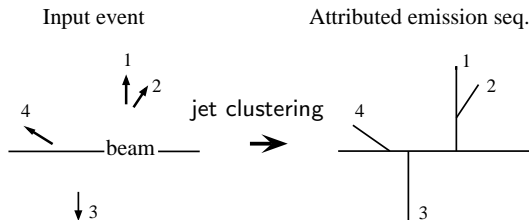
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- ▶ this will work very for well for the processes with large K factors e.g.

$$\sigma_{\bar{n}\text{NLO}} = \sigma_{\text{NNLO}} \left(1 + \mathcal{O}\left(\frac{\alpha_s^2}{K_{\text{NNLO}}} \right) \right), \quad K_{\text{NNLO}} \gtrsim K_{\text{NLO}} \gg 1$$

Input event

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jet clustering

► jet clustering $ij \rightarrow k$ is reinterpreted as the splitting $k \rightarrow ij$

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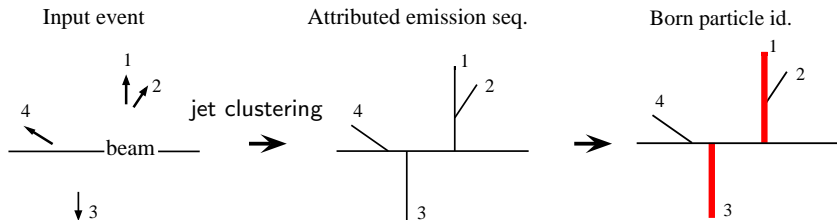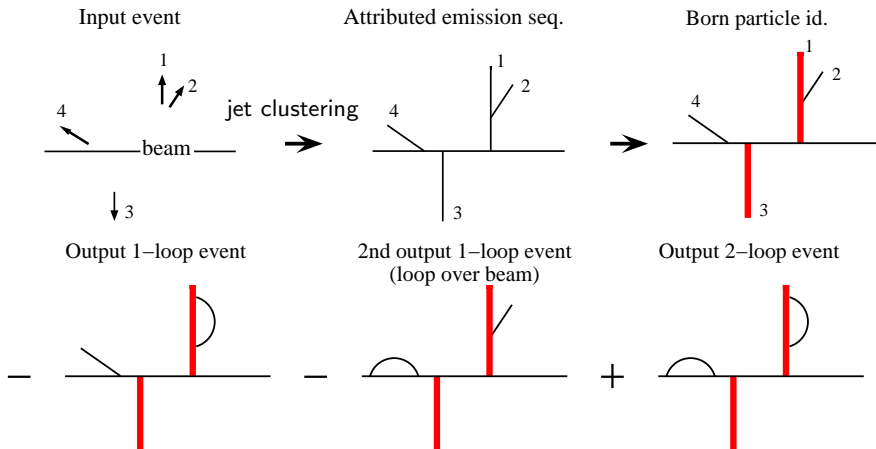
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- beware: the loops above are just a shortcut notation!

Including exact loops

$E_{n,l}$ –   input event with n final state particles and l loops

U_l^b –   operator producing event with b Born particles and l loops

$U_\forall^b$ –   operator generating all necessary loop diagrams at given order

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$$U_\forall^b(E_{n,0}) \quad + \quad U_\forall^b(E_{n-1,1})$$

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▶ inclusion of exact loops helps reducing scale uncertainties

▶ straightforward generalization to arbitrary number of exact loops

Validation

Drell-Yan at NNLO: spectrum of harder lepton
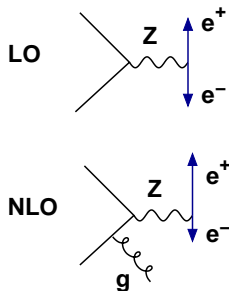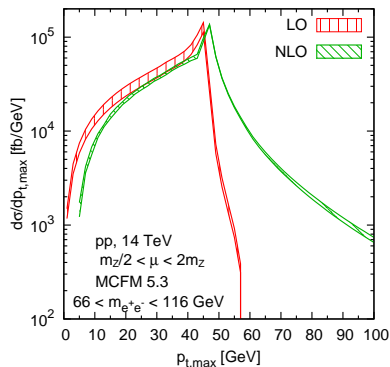
- ▶ giant K factor due to a boost caused by initial state radiation

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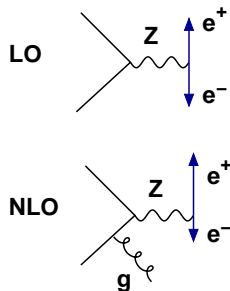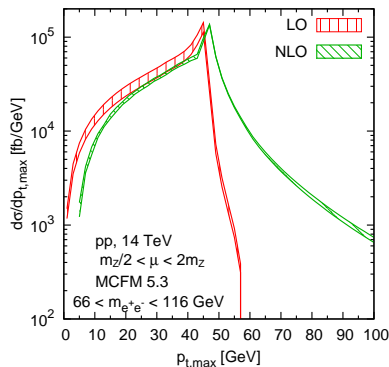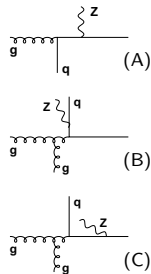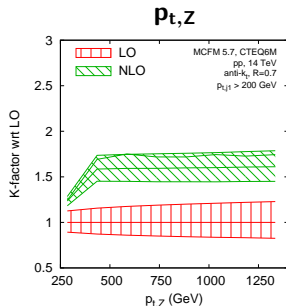

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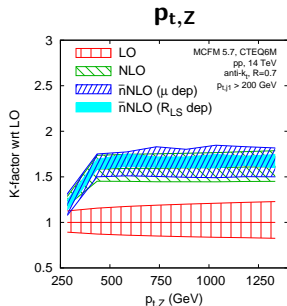
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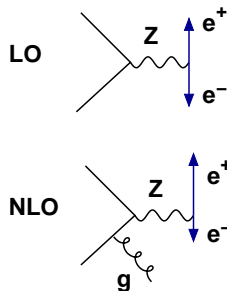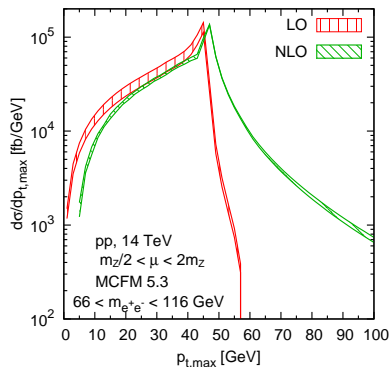
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- ► three regions of $p_{t,max}$: $\quad \lesssim \frac{1}{2}M_Z \quad [\frac{1}{2}M_Z, 58\,\text{GeV}] \quad > 58\,\text{GeV}$

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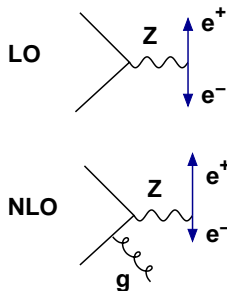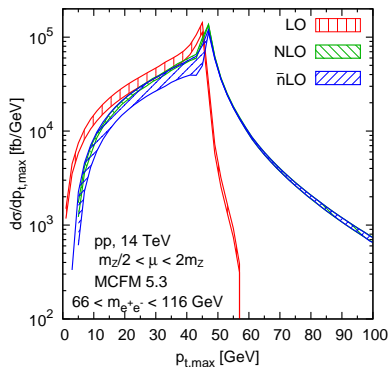
pp, 14 TeV
$m_Z/2 < \mu < 2m_Z$
MCFM 5.3
$66 < m_{e^+e^-} < 116$ GeV

LO

NLO

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	$\bar{n}$LO vs NLO	very good (not guaranteed)	excellent (expected)	perfect (expected)

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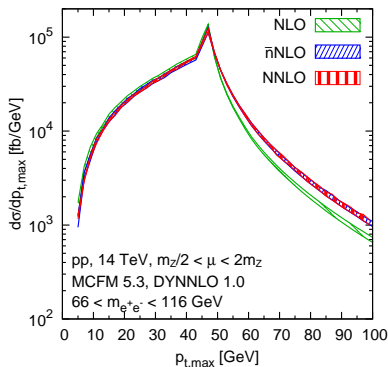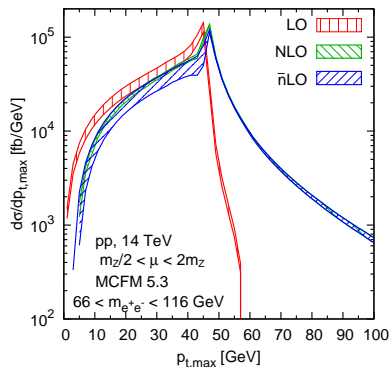
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$\bar{n}$NLO predictions for LHC

$p_{t,z}$

$p_{t,z}$

MCFM 5.7, CTEQ6M
pp, 14 TeV
anti-k_t, R=0.7
$p_{t,j} > 200$ GeV

- LO
- NLO
- $\bar{n}$NLO (μ dep)
- $\bar{n}$NLO (R_{LS} dep)

▶ $p_{t,Z}$: no correction; topology (A) dominant at high $p_{t,Z}$
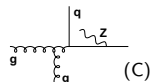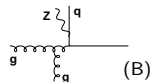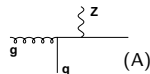(extra loops w.r.t. NLO do not change much)

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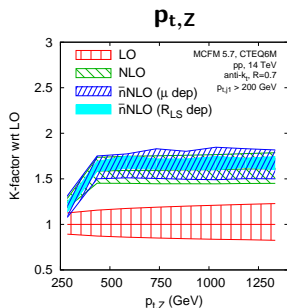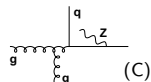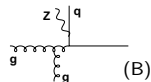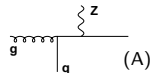
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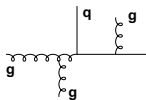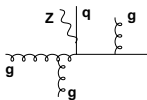
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- $H_{T,jets}$: significant correction; K factor ~ 2; given that its more like going from LO to NLO this may happen sometimes, especially for nontrivial observables like H_T; can we understand it here?
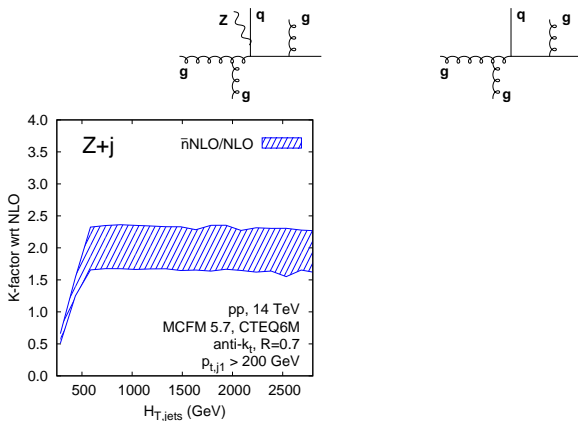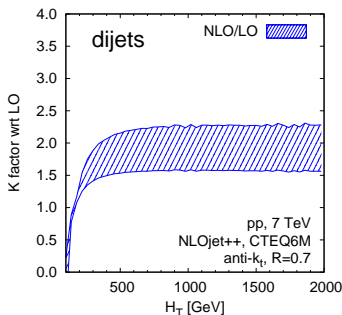
H_T type observables at $\bar{n}$NLO for Z+jet and for dijets

▶ Z+jet at NNLO like dijets at NLO
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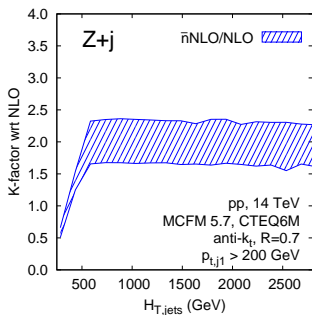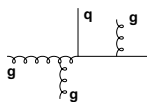
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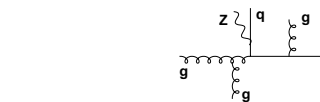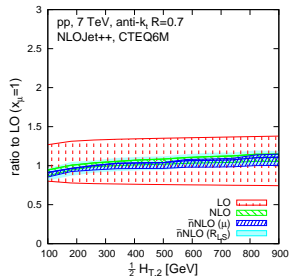
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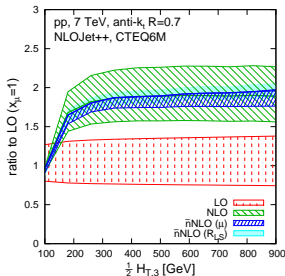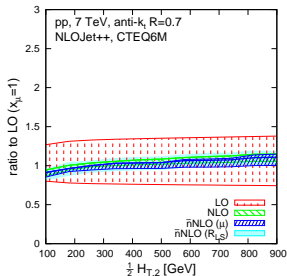 (same topology, Z only provides the enhancement $\mathcal{O}\big(\alpha_s \ln^2 p_{t,j1}/m_Z\big)$)

- ▶ H_T for dijets receives large contributions at NLO!
  - ▶ caused by appearance of the third jet from initial state radiation

Dijets at $\bar{n}$NLO

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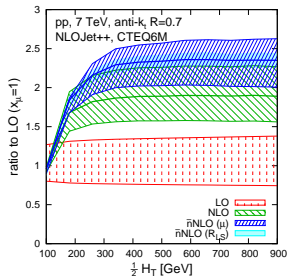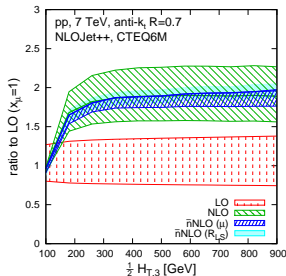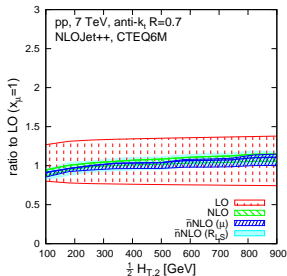
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- ▶ **$H_{T,3}$ converges, significant reduction of scale uncertainty:** the observable comes under control at $\bar{n}$NLO

- ▶ **H_T does not converge:** again caused by the initial state radiation, this time a second emission which shifts the distribution of H_T to higher values and causes no effect for the $H_{T,3}$ distribution

Summary

- several cases of observables with giant NLO K factor exist
- those large corrections arise due to the appearance of new topologies at NLO
- we developed a method, called *LoopSim*, which allows one to obtain approximate NNLO corrections for such processes
- the method is based on unitarity and makes use of combining NLO results for different multiplicities
- we gave arguments why the method should produce meaningful results and we validated it against NNLO Drell-Yan and also NLO Z+j and NLO dijets
- we computed approximated NNLO corrections to Z+j and dijets at the LHC finding, depending on observable, either indication of convergence of the perturbative series or further corrections
- the latter has been understood and attributed to the initial state radiation

Outlook

- processes with W, multibosons, heavy quarks, . . .

BACKUP SLIDES

The LoopSim method: some more details

For a given input E_n event with n final state particles the weights of all diagrams generated by LoopSim sum up to zero

$$\sum_{\text{all diagrams}} w_n = \sum_{\ell=0}^{\upsilon} (-1)^\ell \binom{\upsilon}{\ell} = 0 \, , \qquad \ell - \text{number of loops}, \ \upsilon - \text{maximal } \ell$$

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The principle of the method looks rather simple. However, there is a number of issues that need to be addressed to fully specify the procedure and make it usable:

- ▶ infrared and collinear safety
- ▶ conservation of four-momentum
- ▶ choice of jet definition (algorithm, value of R)
- ▶ treatment of flavour (e.g. for processes with vector bosons)
  - ▶ Z boson can be emitted only from quarks and never emits itself
- ▶ extension to input events with exact loops; for example:

$$Z + j @ \bar{n} \text{NLO} = Z + j @ \text{NLO} + \text{LoopSim} \circ (Z + 2j @ \text{NLO}_{\text{only}})$$

Z+jet at NLO

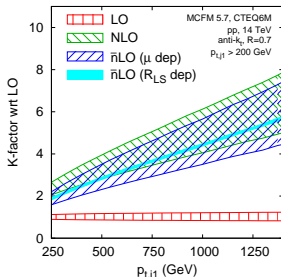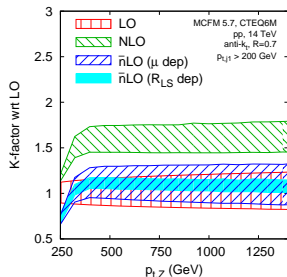- $Z + j @ \bar{n} \text{LO} = Z + j @ \text{LO} + \text{LoopSim} \circ (Z + 2j @ \text{LO})$

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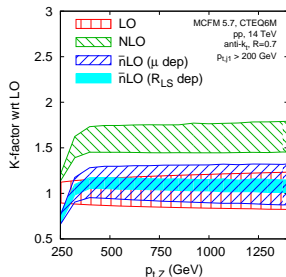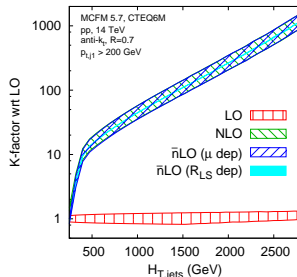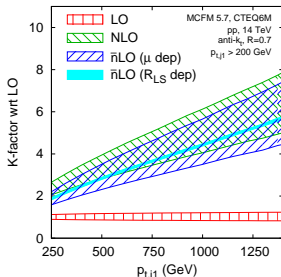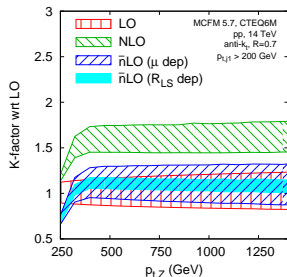

▶ $p_{t,Z}$ (lack of large K-factor):
  ▶ finite loop contributions matter
  ▶ correctly reproduced dip towards $p_t = 200$ GeV

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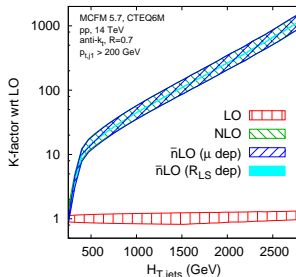
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  - very good agreement between $\bar{n}LO$ and NLO
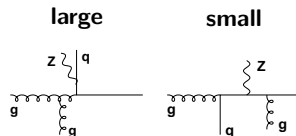
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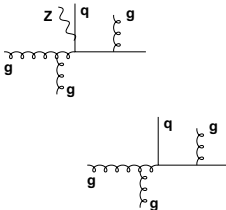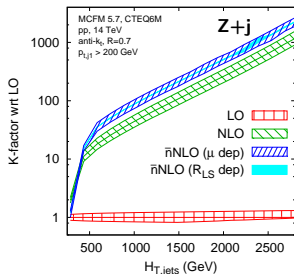► small R uncertainties – driven only by subleading diagrams

▶ Z+jet at NNLO like dijets at NLO
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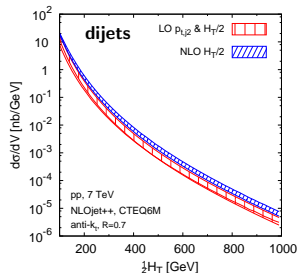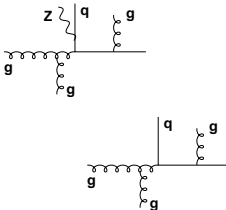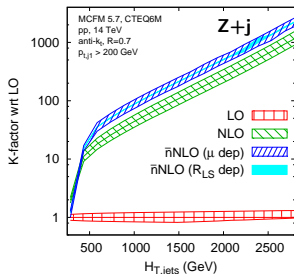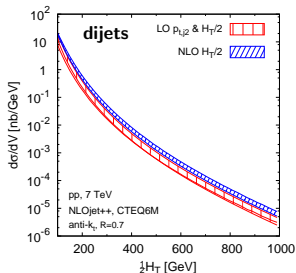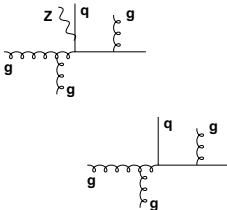- Z+jet at NNLO like dijets at NLO
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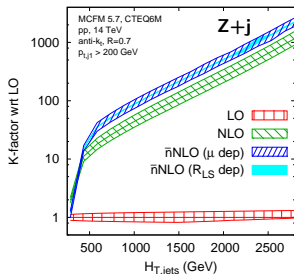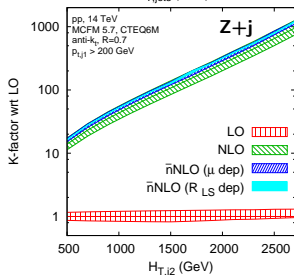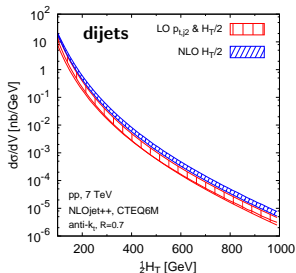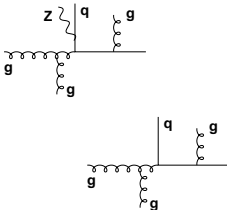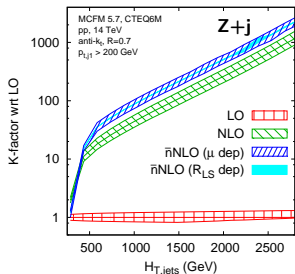
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▶ if the same is valid for $Z + j$ we should see only small correction for $H_{T,j2} = \sum_{i=1}^{2} p_{t,j_i}$
  ▶ and indeed it is small!