
Higher order threshold effects for heavy coloured particles at hadron colliders

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(Based on M.Beneke, P.Falgari, CS, arXiv:0907.1443 [hep-ph], NPB828:69 (2010);
arXiv:1001.4621 [hep-ph], arXiv:1001.4627 [hep-ph], (RADCOR09) and arXiv:1007.nnnn
M.Beneke, M.Czakon, P.Falgari, A.Mitov, CS arXiv:0911.5166 [hep-ph], PLB B690:483 (2010))

Introduction

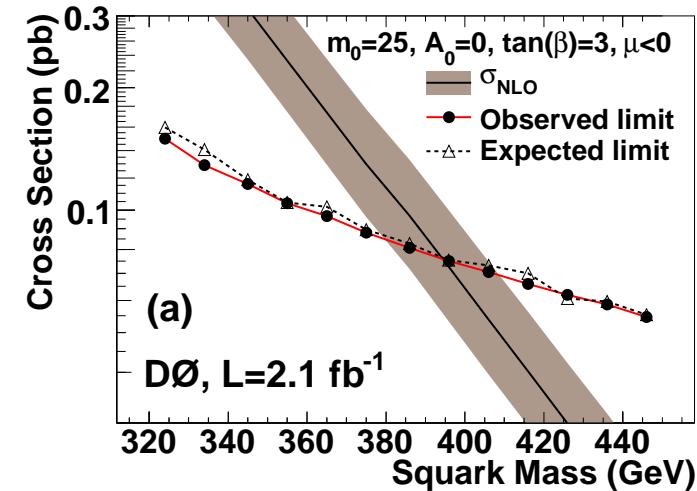
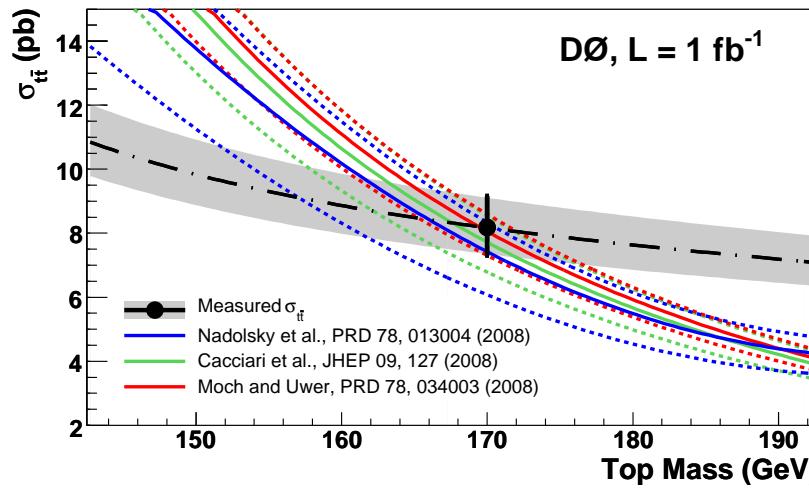
Pair production of heavy coloured particles at Tevatron/LHC

$$N(K_1)N'(K_2) \rightarrow H(p_1)H'(p_2) + X$$

- N, N' : $pp, p\bar{p}$; HH' : **top-quark, squark, gluino...** pairs

Precise knowledge of total cross sections:

- **top-quarks**: sensitivity on mass, constraining gluon PDFs
- **new particles**: Exclusion bounds, model discrimination,...



Introduction

NLO corrections enhanced for $\beta = \sqrt{1 - \frac{(M_H + M_{H'})^2}{\hat{s}}} \rightarrow 0$

$$\hat{\sigma}_{pp' \rightarrow HH'}^{(1)} = \hat{\sigma}_{pp' \rightarrow HH'}^{(0)} \alpha_s \left[\underbrace{a \log^2(8\beta^2) + b \log(8\beta^2)}_{\text{"threshold logarithms"} } + \underbrace{c \frac{1}{\beta}}_{\text{"Coulomb singularity"} } + \dots \right]$$

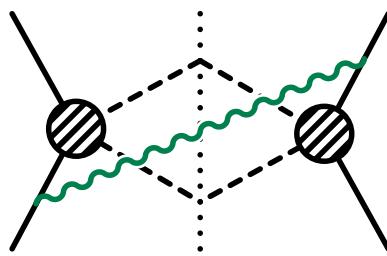
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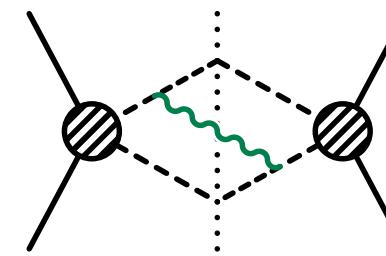
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Soft corrections:

(Resummation in Mellin space: Sterman 87; Catani, Trentadue 89, Kidonakis, Sterman 97, Bonciani et.al. 98, ...)



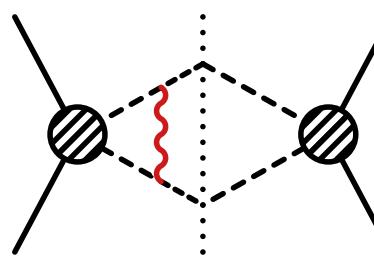
$$\Rightarrow \alpha_s \log^2(8\beta^2)$$



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Coulomb gluon corrections

(Fadin, Khoze 87; Peskin, Strassler 90, NRQCD, ...)



$$\Rightarrow \alpha_s \frac{1}{\beta}$$

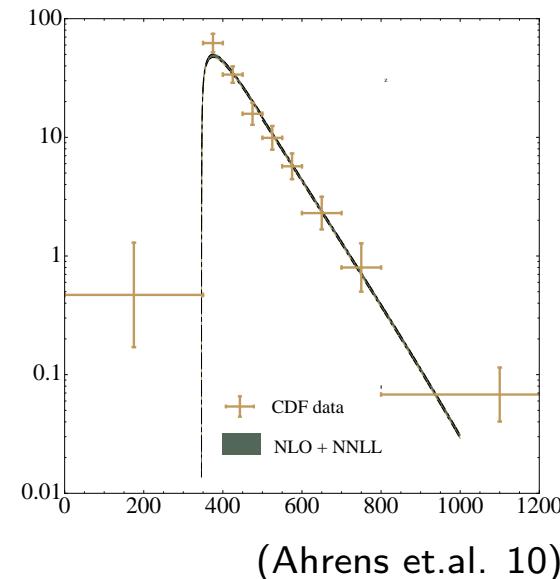
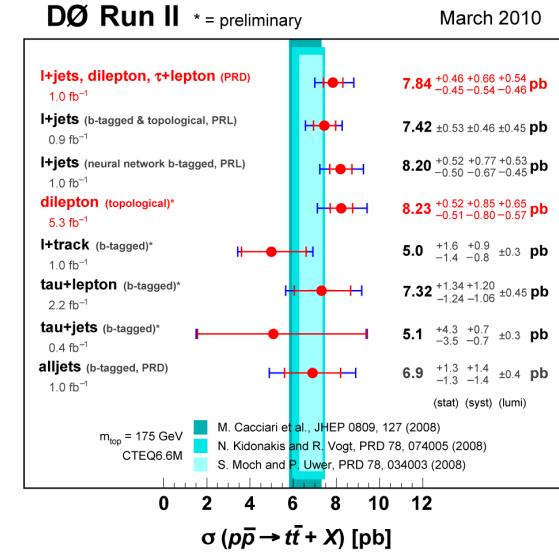
Threshold resummation

Resummation of threshold logs:

- Estimate of higher order corrections
- Accelerated convergence of perturbative series
- Reduced scale dependence

Recent applications

- total top quark cross section
(Moch, Uwer; Cacciari et.al.; Kidonakis Vogt 08)
- $t\bar{t}$ invariant mass distribution
(Ahrens, Ferroglio, Neubert, Pecjak, Yang 09/10)
- squark, gluino production
(Kulesza, Moytka 08/09; Langenfeld, Moch 09;
Beenakker et.al. 09/10)
- octet scalars (Idilbi, Kim, Mehen 09/10)



Threshold resummation

Factorization at partonic threshold $\hat{s} \sim M_{HH'}^2$: (Kidonakis/Sterman 97)

$$\frac{d\hat{\sigma}}{dM_{HH'}^2} \sim \Delta_1 \Delta_2 H_{ij} S_{ij}$$

- Δ_i : resum collinear logs; RGE for soft function S_{ij}
⇒ angle-dependent soft anomalous dimension matrix $\Gamma_{s,ij}$

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Resummation for total cross section: (Bonciani et.al. 98)

$$\hat{\sigma}^N(M^2, \mu) / \hat{\sigma}^{(0)N}(M^2, \mu) = g^0(M^2, \mu) \exp(G^{N+1}(M^2, \mu^2))$$

$$G^N(M^2, \mu^2) = \int_0^1 dz \frac{z^{N-1} - 1}{1-z} \left[\int_\mu^{4M^2(1-z)^2} \frac{dq^2}{q^2} (2A(\alpha_s(q^2)) + D_{HH'}(\alpha_s(4M^2(1-z)^2))) \right]$$

Relation between two approaches (e.g. Czakon,Mitov,Sterman 09)

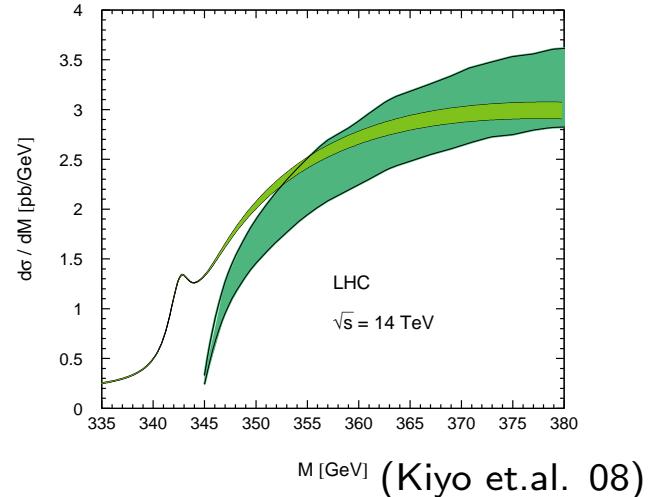
$$D_{HH'} \Leftrightarrow \Gamma_s \quad \text{at threshold} \quad M_{HH'} = m_H + m_{H'}$$

\Rightarrow no angular dependence, can be diagonalized once and for all

Threshold resummation

Combination of Coulomb- and soft effects?

- First Coulomb \otimes soft resummation
(Bonciani et.al. 98)
 - resummed Coulomb \otimes soft
(Hagiwara et.al. 08, Kiyo et.al. 09)
- \Rightarrow Soft \otimes Coulomb factorization?



Not trivial:

- Heavy particles **nonrelativistic** near threshold:

$$E \sim m\beta^2, \quad |\vec{p}| \sim m\beta$$

- soft gluon momenta of same order: $q_s \sim m\beta^2 \sim E$

\Rightarrow heavy particles “feel” soft radiation
violates eikonal approximation

(see also Mitov, Sterman, Sung 10)

Effective theory for

(Beneke, Falgari, CS 09/10)

- collinear $p, p' +$ soft gluons \Rightarrow SCET
- non-relativistic HH' + soft gluons \Rightarrow NRQCD

Factorization into hard, soft and Coulomb functions

$$\hat{\sigma}_{pp' \rightarrow HH'}|_{\hat{s} \rightarrow 4M^2} = H_{ij} \otimes W_{ij} \otimes J$$

- Momentum space threshold resummation (Becher, Neubert 06)
- simultaneous summation of Coulomb corrections in J

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Application to top quark, squark and gluino production

- Diagonal colour bases for $3 \otimes 3, 3 \otimes \bar{3}, 3 \otimes 8, 8 \otimes 8$
- NLL results for squark-antisquark production
- $\mathcal{O}(\alpha_s^4)$ threshold expansion of $t\bar{t}$ cross section

EFT approach to heavy pair production

LO NRQCD Lagrangian for particles H, H' in representations R, R' :

$$\begin{aligned}\mathcal{L}_{\text{PNRQCD}} = & \psi^\dagger \left(iD_s^0 + \frac{\vec{\partial}^2}{2m_H} + \frac{i\Gamma_H}{2} \right) \psi + \psi'^\dagger \left(iD_s^0 + \frac{\vec{\partial}^2}{2m_{H'}} + \frac{i\Gamma_{H'}}{2} \right) \psi' \\ & + \int d^3\vec{r} \left[\psi^\dagger \mathbf{T}^{(R)a} \psi \right] (\vec{r}) \left(\frac{\alpha_s}{r} \right) \left[\psi'^\dagger \mathbf{T}^{(R')^a} \psi' \right] (0),\end{aligned}$$

with $D_s^0 = \partial^0 - ig_s A_s^0(x_0, \vec{0})$.

Decoupling for heavy particle fields:

$$\begin{aligned}\psi(x) = & S_v^{(R)}(x_0) \psi^{(0)\dagger}(x), \quad S_v^{(R)}(x) = \overline{\mathcal{P}} \exp \left[-ig_s \int_0^\infty ds v \cdot \mathbf{A}^a(x + vs) \mathbf{T}^{(R)a} \right] \\ \Rightarrow D_s^0 \psi = & S_v \partial^0 \psi^0\end{aligned}$$

same $v = (1, \vec{0})$ for both heavy particles at threshold

Works at **leading order** in PNRQCD (\Rightarrow Higher orders see below)

Analogous redefinition in SCET (Bauer, Pirjol, Stewart 01)

Apply soft-gluon decoupling to amplitude:

$$\mathcal{A}_{pp' \rightarrow HH'X} \Rightarrow \sum_i C^{(i)} \langle HH' | \psi^{(0)\dagger} \psi'^{(0)\dagger} | 0 \rangle \langle 0 | \phi_c^{(0)} \phi_{\bar{c}}^{(0)} | pp' \rangle \langle X | S_n S_{\bar{n}} c^{(i)} S_v^\dagger S_{\bar{v}}^\dagger | 0 \rangle$$

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Inserting into formula for σ , summing over complete set of $|X\rangle \dots$

$$\hat{\sigma}_{pp'}(\hat{s}, \mu) = \sum_{i,i'} H_{ii'}(M, \mu) \int d\omega \sum_{R_\alpha} J_{R_\alpha}(\sqrt{\hat{s}} - 2M - \frac{\omega}{2}) W_{ii'}^{R_\alpha}(\omega, \mu)$$

Irreducible representations $R \otimes R' = \sum_{R_\alpha} R_\alpha$ e.g. $3 \otimes \bar{3} = 1 \oplus 8$.

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Potential function:

$$J_{R_\alpha}(E) = \int d^4z e^{iEz^0} \langle 0 | [\psi^{(0)} \psi'^{(0)}](z^0) P^{R_\alpha} [\psi^{(0)\dagger} \psi'^{(0)\dagger}](0) | 0 \rangle = 2\text{Im} G_C^{R_\alpha}(0, 0, E)$$

Soft function:

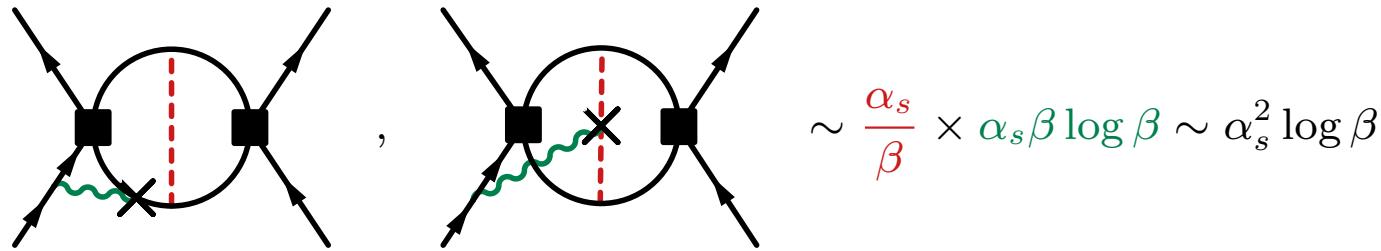
$$W_{ii'}^{R_\alpha}(\omega) = \int \frac{dz_0}{4\pi} e^{i\omega z_0/2} \langle 0 | \overline{T}[S_v S_v c^{(i')*} S_{\bar{n}}^\dagger S_n^\dagger](0) P^{R_\alpha} T[S_n S_{\bar{n}} c^{(i)} S_v^\dagger S_v^\dagger](x_0) | 0 \rangle$$

Subleading PNRQCD and SCET interactions:

$$\psi^\dagger \vec{x} \cdot \vec{E}_{us}(x_0, 0) \psi'^\dagger, \quad \bar{\xi} \left(x_\perp^\mu n_-^\nu W_c g F_{\mu\nu}^{us} W_c^\dagger \right) \frac{\not{n}_+}{2} \xi \dots$$

Soft gluons not decoupled by field redefinitions.

Possibly relevant at NNLL in soft \otimes potential corrections :



Related to three-parton colour correlations in IR singularities of amplitudes (Ferroglia et.al. 09)

σ_{tot} : effects vanish at NNLL!

(Beneke, Czakon, Falgari, Mitov, CS 09)

- no collinear/potential correction $\sim \beta$ for $k_\perp = 0$
- no potential/soft corrections due to rotational invariance
(no heavy particle three-momentum available)

Physical picture:

(Bonciani et.al. 98)

soft radiation off total colour charge of HH' system

Construction of colour basis

(Beneke, Falgari, CS 09)

adapted to initial/final state representations:

$$r \otimes r' = \sum_{\alpha} r_{\alpha}, \quad R \otimes R' = \sum_{R_{\alpha}} R_{\alpha}$$

Basis tensors from Clebsch Gordan coefficients:

$$c_{\{a\}}^{(i)} = \frac{1}{\sqrt{\dim(r_{\alpha})}} C_{\alpha a_1 a_2}^{r_{\alpha}} C_{\alpha a_3 a_4}^{R_{\beta}*} \quad r_{\alpha} \sim R_{\beta}$$

- obtain soft function for **single final-state particle** in R_{α}
- diagonalizes soft function to all orders
- Bases for all squark/gluino production processes $\tilde{q}\bar{\tilde{q}}$, $\tilde{q}\tilde{q}$, $\tilde{g}\tilde{g}$, $\tilde{q}\tilde{g}$
(one loop: Kidonakis/Sterman 97; Kulesza/Moytka 08, Beenakker et.al. 09)
- Example: basis for $gg \rightarrow \tilde{g}\tilde{g}$ corresponds to
 $(1, 1), (8_S, 8_S), (8_A, 8_A), (8_S, 8_A), (8_A, 8_S), (10, 10), (\bar{10}, \bar{10}), (27, 27)$

Evolution equations from reduction to $2 \rightarrow 1$ process

(adequate up to NNLL: scale independent potential function, no three parton correlations):

Hard function

(from Becher/Neubert 09)

$$\frac{d}{d \ln \mu} H_i(M, \mu) = \left(\gamma_{\text{cusp}}(C_r + C_{r'}) \ln \left(\frac{4M^2}{\mu^2} \right) + 2 \left(\underbrace{\gamma^r + \gamma^{r'}}_{\text{as for Drell-Yan/Higgs}} + \gamma_{H,s}^{R_\alpha} \right) \right) H_i^S(M, \mu).$$

Soft anomalous dimension:

(agrees with Czakon, Mitov, Sterman 09)

$$\gamma_{H,s}^{R_\alpha} = \frac{\alpha_s}{4\pi} (-2C_{R_\alpha}) + \left(\frac{\alpha_s}{4\pi} \right)^2 C_{R_\alpha} \left[-C_A \left(\frac{98}{9} - \frac{2\pi^2}{3} + 4\zeta_3 \right) + \frac{40}{18} n_f \right] + \mathcal{O}(\alpha_s^3).$$

(using Becher, Neubert 09; Korchemsky Radyushkin 92, Kidonakis 09)

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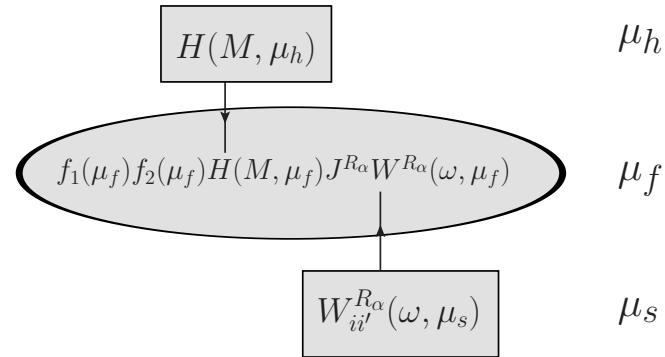
Soft function from scale independence of $\sigma = f_1 \otimes f_2 \otimes H \otimes W \otimes J$

$$\frac{d}{d \log \mu} W_i^{R_\alpha}(z^0, \mu) = \left(2\gamma_{\text{cusp}}(C_r + C_{r'}) \log \left(\frac{iz_0 \mu e^{\gamma_E}}{2} \right) - 2(\gamma_{H,s}^{R_\alpha} + \gamma_s^r + \gamma_s^{r'}) \right) W_i^{R_\alpha}(z^0, \mu)$$

(same form as for Drell-Yan: Korchemsky, Marchesini 92)

Resummation of threshold logs

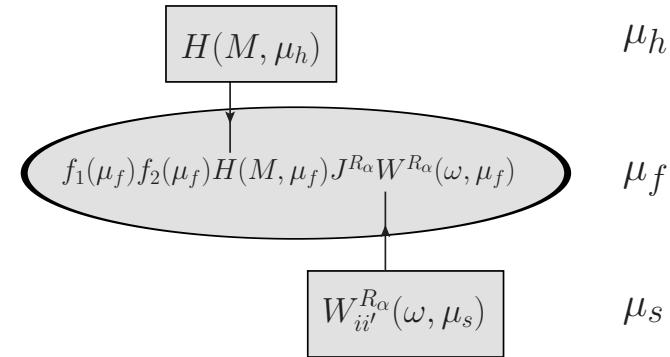
- Solution to RGE in momentum space (Becher/Neubert 06)
- evolve hard function from $\mu_h \sim Q \sim 2M$ to μ_f
- evolve soft function from μ_s to μ_f
- solution in Mellin-space corresponds to $\mu_s = Q/N, \mu_h = \mu_f$
 (Korchemsky/Marchesini 92, Manohar 03, Becher/Neubert/Xu 07)



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Coulomb resummation

NLL: use LO Coulomb-Green function

$$D_1 = -C_F, \quad D_8 = \frac{1}{2N_C}$$

$$J^{R_\alpha(0)}(E) \sim \text{Im} \left\{ \sqrt{-\frac{E}{m_{\tilde{q}}}} - D_{R_\alpha} \alpha_s(\mu_C) \left[\frac{1}{2} \ln \left(-\frac{4m_{\tilde{q}}E}{\mu^2} \right) + \psi \left(1 + \frac{\alpha_s(\mu_C) D_{R_\alpha}}{2\sqrt{-E/(m_{\tilde{q}})}} \right) \right] \right\}$$

Scale independent by itself \Rightarrow evaluate at μ_C

Combined NLL soft/Coulomb resummation

(Beneke, Falgari, CS)

- NLL solutions for H , $\textcolor{teal}{W}$, LO resummed $\textcolor{red}{J}$
- Simplified setup: equal squark masses, exclude stop
- Matching to NLO results (Beenakker et.al. 96, PROSPINO (Plehn et.al.))

$$\hat{\sigma}_{pp'}^{\text{match}}(\hat{s}, \mu_f) = [\hat{\sigma}_{pp'}^{\text{NLL}}(\hat{s}, \mu_f) - \hat{\sigma}_{pp'}^{\text{NLL}}(\hat{s}, \mu_f)|_{\text{NLO}}] + \hat{\sigma}_{pp'}^{\text{NLO}}(\hat{s}, \mu_f)$$

Soft scale $\tilde{\mu}_s$ that minimizes hadronic $\Delta\sigma_{\text{soft}}^{\text{NLO}}$

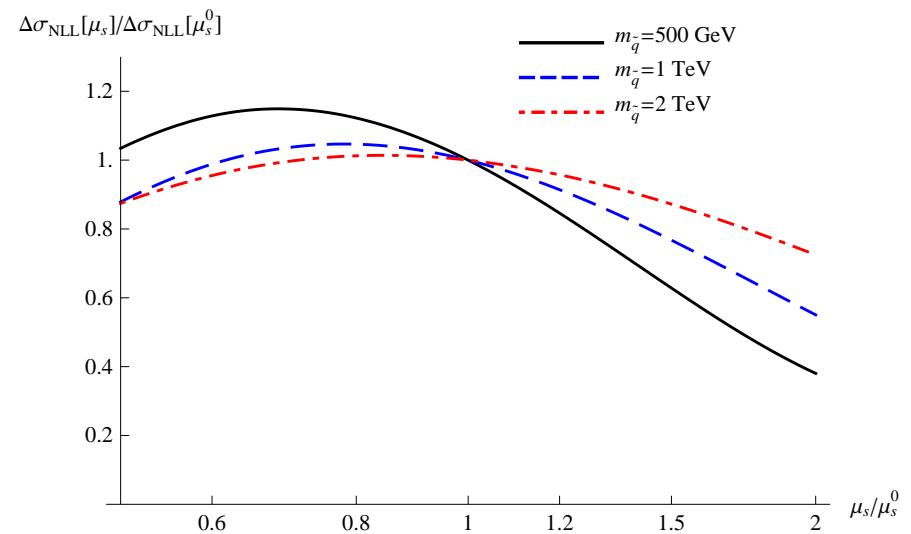
(Becher, Neubert, Xu 07)

$\tilde{\mu}_s/m_{\tilde{q}} \approx 0.5 \dots 0.2$
for $m_{\tilde{q}} = 0.5, \dots 2 \text{ TeV}$

Hard scale: $\tilde{\mu}_h = 2m_{\tilde{q}}$

Coulomb scale:

$$\mu_C = \max\{2m_{\tilde{q}}\beta, C_F m_{\tilde{q}} \alpha_s(\mu_C)\}$$



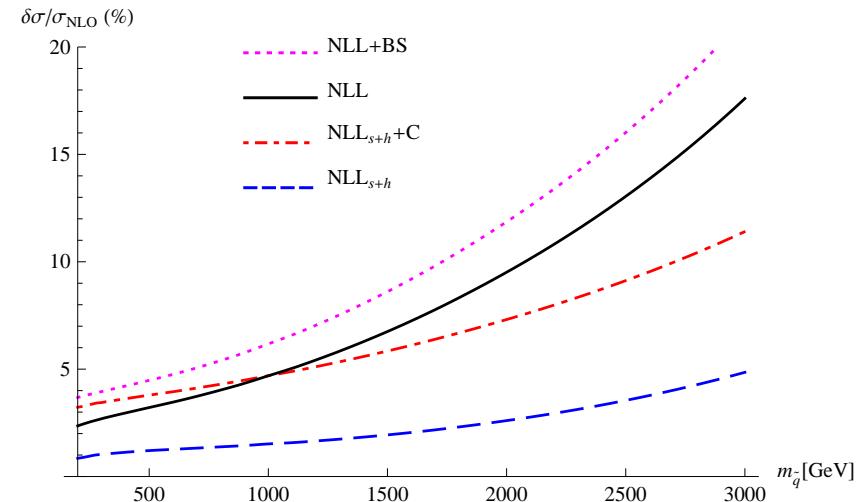
Impact of higher order corrections

NLL_{s+h}: resummation of H and W

C: Coulomb resummation

NLL: full Coulomb \otimes res. soft

BS Bound state effects



($\sqrt{s} = 14 \text{ TeV}$, $m_{\tilde{g}}/m_{\tilde{q}} = 1.25$ MSTW08NLO)

Comparison to Mellin-approach: (Kulesza, Motyka 08/09, Beenakker et.al. 09)

Good agreement for appropriate choice of scales ($\mu_h = \mu_f$: NLL_s):

$m_{\tilde{q}} [\text{GeV}]$	NLO [pb]	NLL _{Mellin} [pb]	NLL _s [pb]	NLL [pb]
500	1.6×10^1	1.61×10^1 (1.2%)	1.62×10^1 (1.3%)	1.65×10^1 (3%)
1000	2.89×10^{-1}	2.93×10^{-1} (1.7%)	2.94×10^{-1} (1.7%)	3.02×10^{-1} (4.4%)
2000	1.11×10^{-3}	1.14×10^{-3} (3.4%)	1.14×10^{-3} (3.1%)	1.21×10^{-3} (9%)

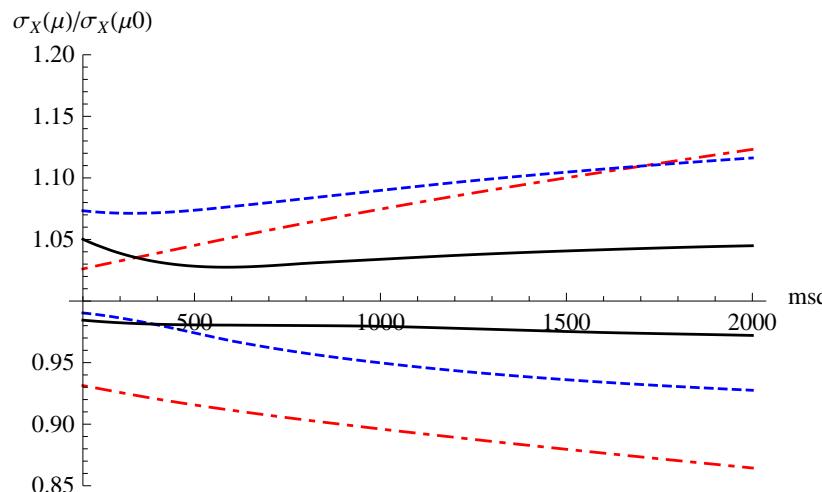
(LHC 14 TeV, $m_{\tilde{g}} = m_{\tilde{q}}$)

Scale uncertainty

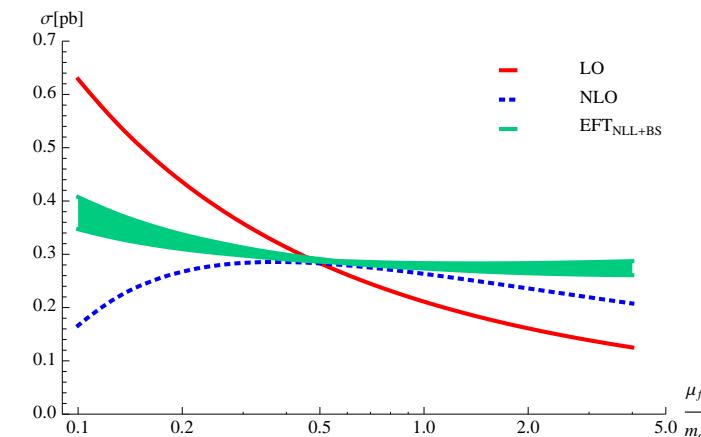
NLO $\frac{m_{\tilde{q}}}{2} < \mu_f < m_{\tilde{q}}$

NLL: vary all scales $\frac{\tilde{\mu}_i}{2} < \mu_i < 2\tilde{\mu}_i$, add in quadrature

⇒ significant reduction for combined resummation!



$(\sqrt{s} = 14 \text{ TeV}, \text{ MSTW08NLO}, m_{\tilde{q}}/m_{\tilde{q}} = 1.25)$



$(m_{\tilde{q}} = 1 \text{ TeV}, \mu_s^0/2 < \mu_s < 2\mu_s^0)$

Threshold expansion at $\mathcal{O}(\alpha_s^2)$

All threshold enhanced $\mathcal{O}(\alpha_s^2)$ terms (Beneke, Czakon, Falgari, Mitov, CS 09)

Pure soft corrections:

$$\Delta\sigma_s^{(2)} \sim \alpha_s^2 (c_{\text{LL}}^{(2)} \ln^4 \beta + c_{\text{NLL}}^{(2)} \ln^3 \beta + c_{\text{NNLL},2}^{(2)} \ln^2 \beta + \underbrace{c_{\text{NNLL},1}^{(2)} \ln \beta}_{\text{2-loop } \gamma_{H,s}})$$

Potential corrections: 2nd Coulomb, NLO potentials

$$\Delta\sigma_p^{(2)} \sim \alpha_s^2 \left(\frac{c_C{}^2}{\beta^2} + \frac{1}{\beta} (c_{C,0}^{(2)} + c_{C,1}^{(2)} \log \beta) + \underbrace{c_{n-C}^{(2)} \ln \beta}_{\text{spin-dependent}} \right)$$

(extracted from Beneke, Signer, Smirnov 99, Czarnecki/Melnikov 97/01)

mixed Coulomb/soft, hard corrections:

$$\Delta\sigma_{p \otimes \text{sh}}^{(2)} \sim \frac{\alpha_s}{\beta} \alpha_s (c_{\text{LL}}^{(1)} \ln \beta^2 + c_{\text{NLL}}^{(1)} \ln \beta + c + \underbrace{H^{(1)}}_{\text{process dependent}})$$

Numerical impact

of $\mathcal{O}(\alpha_s^2)$ correction on $t\bar{t}$ production at 14 TeV LHC:

$$\sigma_{\text{NLO}} = 842^{+97+30}_{-97-32} \text{ pb}$$

$$\sigma_{\text{NNLO}_{\text{app}}} = 895^{+29+31}_{-07-33} \text{ pb} \quad (\text{MSTW08NLO/NNLO})$$

Comparison to earlier NNLO_{app}

(Moch/Uwer+Langenfeld 08/09)

$$\gamma_{H,s} \quad \Delta\sigma \sim 6 \text{ pb} \quad (< 1\%)$$

$$(\alpha_s^2/\beta) \text{ terms :} \quad \Delta\sigma \sim 15 \text{ pb} \quad (\approx 2\%)$$

Resummation effects

(Beneke, Falgari, Klein, CS, preliminary):

NLL: very small impact on central value

improved scale dependence

NNLL: in progress...

EFT approach to $\log \beta$, β^{-n} resummation for σ_{tot}

- use SCET+NRQCD to factorize soft and Coulomb gluons
- $\log \beta$ resummation from momentum space solution to RGEs
- subleading soft interactions not relevant at NNLL

Colour structure of soft function

- **diagonal basis** to all orders for arbitrary colour
- two-loop soft anomalous dimension

Application to **squark-antisquark** production

- combined Soft and Coulomb resummation
- total corrections 4 – 10% for $m_{\tilde{q}} = 300$ GeV-2 TeV

Threshold expansion to $\mathcal{O}(\alpha_s^2)$ of $t\bar{t}$ cross section

NNLL resummation for $t\bar{t}$ in progress

(Beneke, Falgari, Klein, CS)

Why perform resummation?

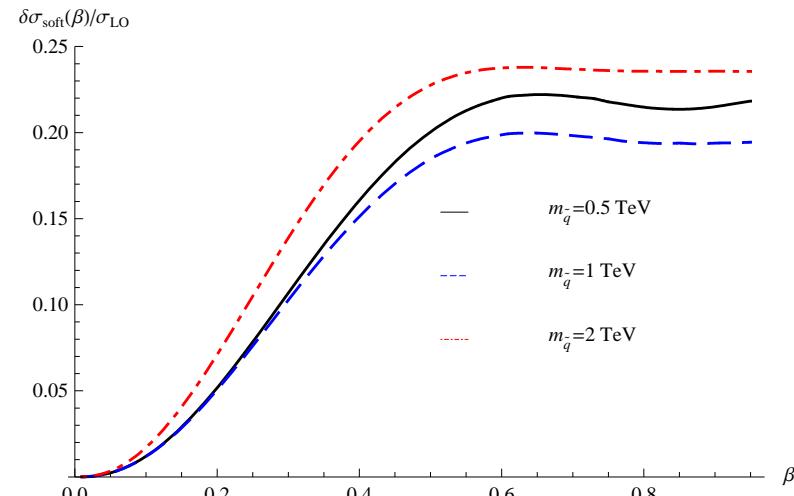
Resummation **required** for

$$\alpha_s \log(\beta) \sim 1$$

Example: $\tilde{q}\bar{\tilde{q}}$ -production
dominant contribution from

$$\beta > 0.2 \quad \Rightarrow |\alpha_s \log \beta| \lesssim 0.2$$

\Rightarrow resummation not **mandatory**



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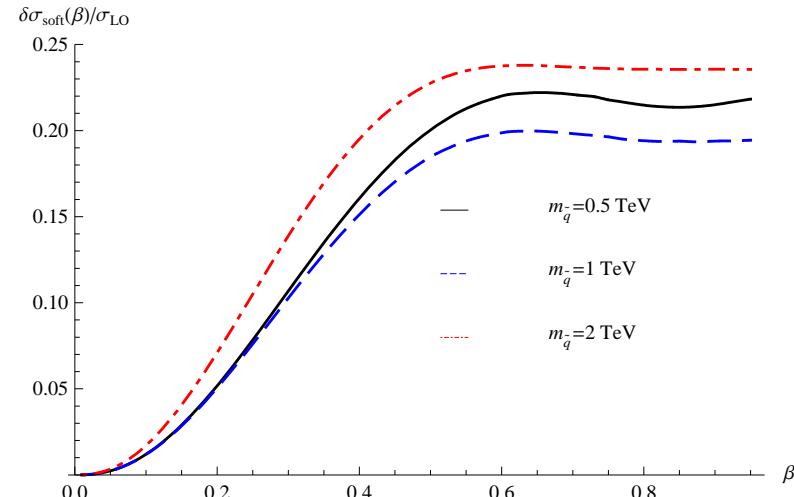
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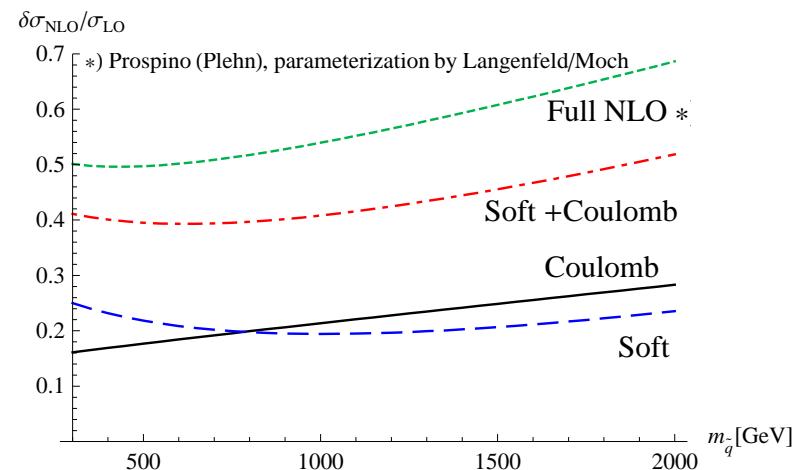
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Is it **useful**?

- threshold terms bulk of NLO also for $\beta > 0.1$
- predict higher order terms
- reduce scale dependence



Parametric representation of cross section near threshold:

$$\hat{\sigma}_{pp'} \propto \sigma^{(0)} \exp \left[\underbrace{\ln \beta g_0(\alpha_s \ln \beta)}_{\text{(LL)}} + \underbrace{g_1(\alpha_s \ln \beta)}_{\text{(NLL)}} + \underbrace{\alpha_s g_2(\alpha_s \ln \beta)}_{\text{(NNLL)}} + \dots \right]$$

$$\times \sum_{k=0} \left(\frac{\alpha_s}{\beta} \right)^k \times \left\{ 1 (\text{LL}, \text{NLL}); \alpha_s, \beta (\text{NNLL}); \dots \right\},$$

- Counting of soft logs usually defined by exponential only (Bonciani et.al. 98)
- Compared to $e^- e^+ \rightarrow t\bar{t}$ at threshold: $N^n \text{LL} \Leftrightarrow N^{n-1} \text{LL}|_{e^- e^+ \rightarrow t\bar{t}}$
- No contribution of odd powers in β to total σ

Fixed order expansion contains all terms of the form

$$\text{LL: } \alpha_s \left\{ \frac{1}{\beta}, \ln^2 \beta \right\}; \quad \alpha_s^2 \left\{ \frac{1}{\beta^2}, \frac{\ln^2 \beta}{\beta}, \ln^4 \beta \right\}; \dots,$$

$$\text{NLL: } \alpha_s \ln \beta; \quad \alpha_s^2 \left\{ \frac{\ln \beta}{\beta}, \ln^3 \beta \right\}; \dots$$

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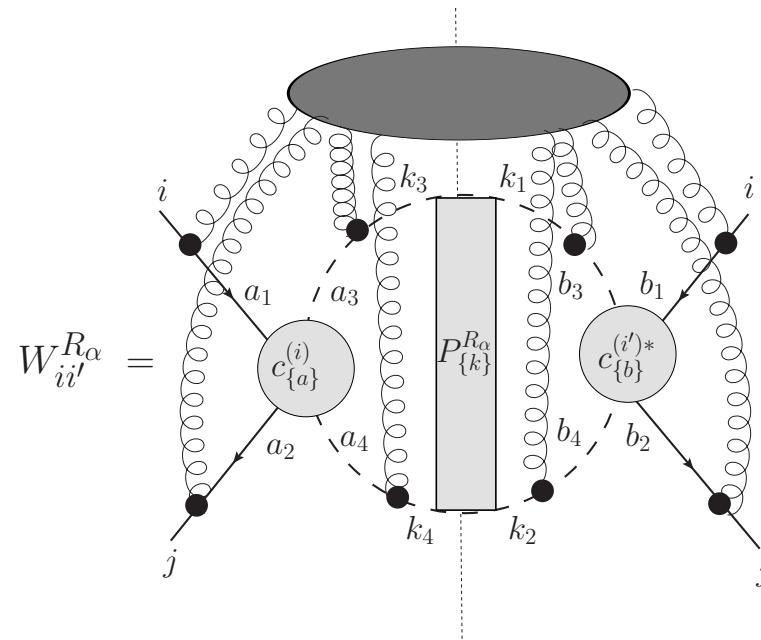
Colour structure

Diagonalization of soft function:

$$W_{ii'}^{R_\alpha}(z_0) = \langle 0 | \bar{\mathbf{T}}[S_v^4 S_v^3 c^{i'*} S_{\bar{n}}^{1\dagger} S_n^{2\dagger}](0) P^{R_\alpha} \mathbf{T}[S_n^1 S_{\bar{n}}^2 c^i S_v^{3\dagger} S_v^{4\dagger}](x_0) | 0 \rangle$$

Basis tensors and projectors from Clebsch-Gordan coefficients:

$$P^{R_\alpha} = C^{R_\alpha*} C^{R_\alpha}, \quad C^{(i)} = C^{r_\alpha} C^{R_\beta*} \quad (P_i = (r_\alpha, R_\beta), r_\alpha \sim R_\beta)$$



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Combine Wilson lines: $C^{R_\beta} S_v^3 S_v^4 = S_v^{R_\beta} C^{R_\beta}$

(Works only since both heavy particles have same velocity!)

⇒ reduce to soft function for **single final-state particle** in R_α :

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⇒ soft function automatically block diagonal, e.g. $8 \otimes 8 \rightarrow 3 \otimes \bar{3}$

$P_i, P_{i'} = (1, 1)$: single element $(8_S, 8), (8_A, 8)$: 2×2 matrix

no $8_S/8_A$ interference (Bose symm.) ⇒ $W_{ii'}^{R_\alpha}$ diagonal!

Soft-collinear Lagrangian

(Bauer et.al. 2000, Beneke et.al. 2002)

for quarks with momentum $p \sim n$, $n^2 = 0$: $(n \cdot \bar{n} = 2, n \cdot p_\perp = 0)$

$$\mathcal{L}_c = \bar{\xi}_c \left(i n \cdot D + i \not{D}_{\perp c} \frac{1}{i \bar{n} \cdot \not{D}_c} i \not{D}_{\perp c} \right) \frac{\not{n}}{2} \xi_c$$

Coupling to soft and collinear gluons:

$$i \not{D}_c = i \partial + g \not{A}_c \quad i D = i \not{D}_c + g \not{A}_s$$

Field redefinition

(Bauer, Pirjol, Stewart 01)

$$\xi_c(x) = S_n(x_-) \xi_c^{(0)}(x) \quad S_n(x) = \mathsf{P} \exp \left[i g_s \int_{-\infty}^0 dt n \cdot A_s^a(x+nt) T^a \right]$$

Wilson line satisfies

$$n \cdot \partial S_n(x_-) = i g_s n \cdot A_s^a(x_-) T^a S_n(x_-)$$

⇒ soft gluons decouple in Lagrangian:

$$\bar{\xi}_n (i n \cdot \not{D}_s) \xi_n = \bar{\xi}_n^{(0)} (i n \cdot \partial) \xi_n^{(0)}$$

Bound state effects

Nonvanishing imaginary part of singlet Coulomb Green function below threshold:

$$J_1^{(0)\text{bound}}(E) = 2 \sum_{n=1}^{\infty} \delta(E + E_n) \left(\frac{m_{\tilde{q}} \alpha_s(\mu_C) C_F}{2n} \right)^3 \quad E < 0$$

with the bound state energies

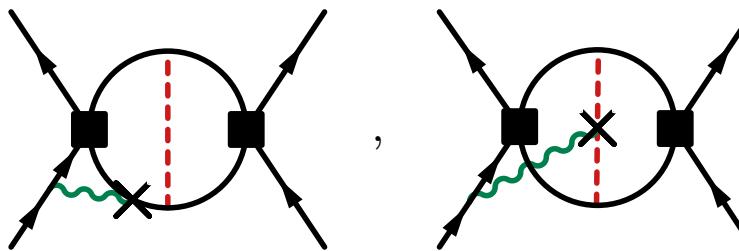
$$E_n = \frac{m \alpha_s^2 C_F^2}{4n^2}$$

Correction to cross section:

$$\Delta\sigma_{\text{bound}} = \sum_{pp'} \sum_n \mathcal{L}_{pp'}(z_n, \mu_f) \hat{\sigma}_{pp'(1)}^{(0)}(sz_n) \frac{\pi}{m_{\tilde{q}}^2} \sqrt{\frac{z_n}{s}} \left(\frac{m_{\tilde{q}} \alpha_s(\mu_C) C_F}{n} \right)^3$$

with $z_n = (2m_{\tilde{q}} + E_n)^2/s$.

Subleading soft \otimes potential corrections:



Related to three-parton colour correlations in IR singularities of amplitudes (Ferroglia et.al. 09)

Expansion of Hg_sH vertex: $(p = Mv + \mathbf{r}, (r^0, \vec{r}) \sim (\lambda, \sqrt{\lambda}), q \sim \lambda)$:

$$\frac{q}{p} + \frac{q}{\cancel{r}} : \quad \frac{v^\mu}{r^0 + q^0 - \frac{\vec{r}^2}{2M}} + \left[\frac{\vec{r}/M}{r^0 + q^0 - \frac{\vec{r}^2}{2M}} - v^\mu \frac{\vec{r} \cdot \vec{q}}{M} \left(\frac{1}{r^0 + q^0 - \frac{\vec{r}^2}{2M}} \right)^2 \right] + \mathcal{O}(\beta^2)$$

\Rightarrow Integrals of the form

($\vec{v} = 0$ in partonic CMS, no \vec{q} in denominator)

$$\int d^D q \prod_i d^D r_i \frac{\{(\vec{k}_- \cdot \vec{r}_i), (\vec{q} \cdot \vec{r}_i)\}}{F(q^0, (k_- \cdot q), r_i^0, (\vec{r}_i + \vec{r}_j)^2)} = 0$$

Vanish since no external potential 3-momentum available!

Potential corrections:

- 2nd Coulomb correction
- NLO Coulomb potentials:

$$\tilde{V}_{\text{C}}^{(1)}(\mathbf{p}, \mathbf{q}) = \frac{D_{R_\alpha} \alpha_s^2}{\mathbf{q}^2} \left(a_1 - \beta_0 \ln \frac{\mathbf{q}^2}{\mu^2} \right)$$

- Non-Coulomb potential:

$$\tilde{V}_{\text{nC}}^{(1)}(\mathbf{p}, \mathbf{q}) = \frac{4\pi D_{R_\alpha} \alpha_s}{\mathbf{q}^2} \left[\frac{\pi \alpha_s |\mathbf{q}|}{4m} \left(\frac{D_{R_\alpha}}{2} + C_A \right) + \frac{\mathbf{p}^2}{m^2} + \frac{\mathbf{q}^2}{m^2} v_{\text{spin}} \right],$$

($v_{\text{spin}} = 0$ (singlet); $-2/3$ (triplet))

Corrections to cross section:

$$\Delta \hat{\sigma}_{\text{nC}} = \hat{\sigma}^{(0)} \alpha_s^2 \ln \beta \left[-2D_{R_\alpha}^2 (1 + v_{\text{spin}}) + D_{R_\alpha} C_A \right]$$

(extracted from Beneke, Signer, Smirnov 99, Pineda, Signer 06)