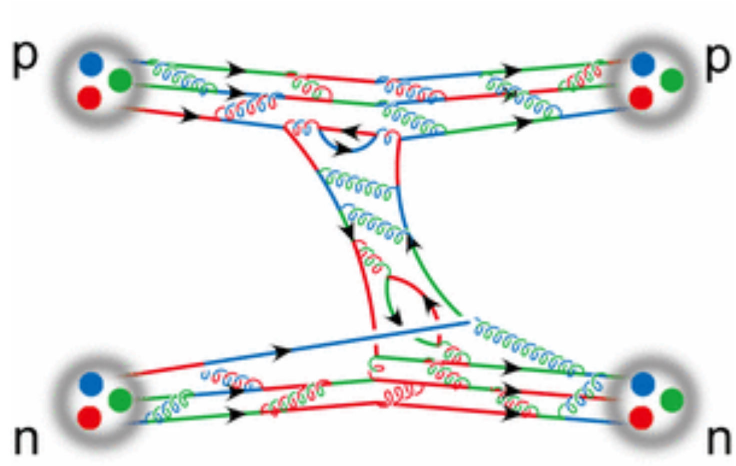
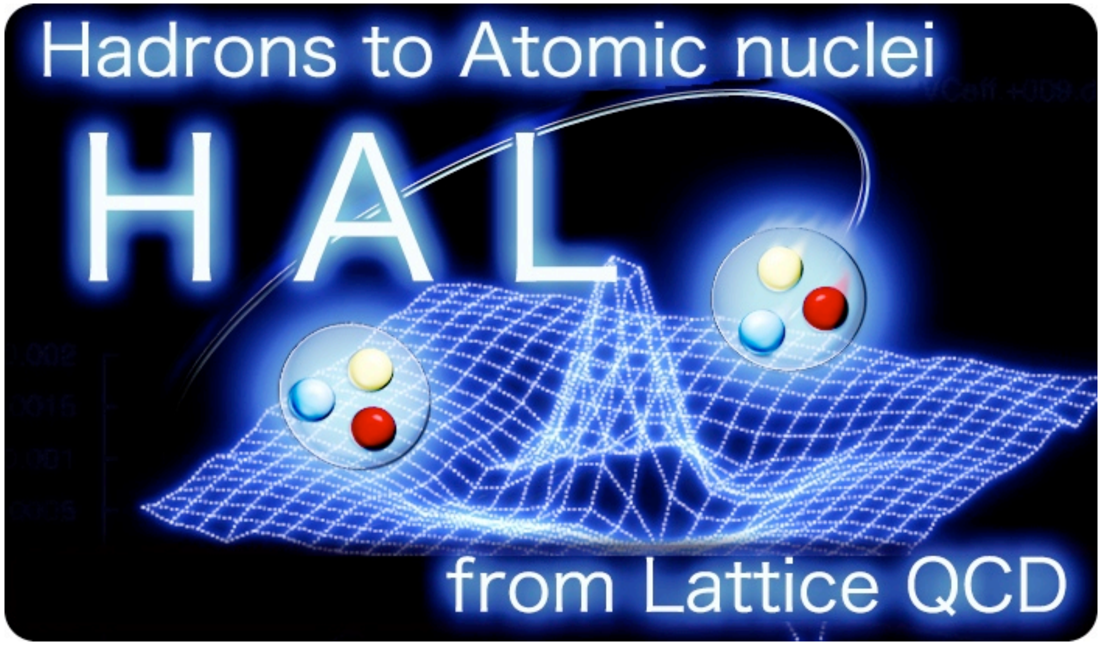
Extraction of hadron interactions from Lattice QCD

Sinya AOKI
University of Tsukuba



CERN Theory Institute
“Future directions in lattice gauge theory-LGT10”
19 July - 13 August, 2010, CERN, Geneva

HAL QCD Collaboration

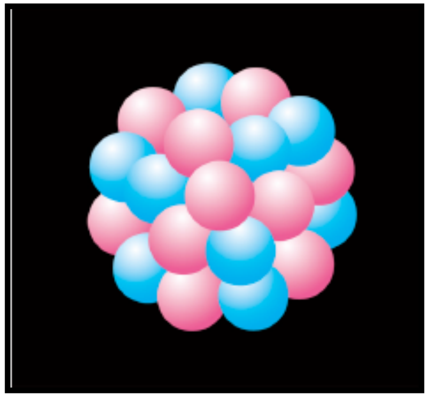


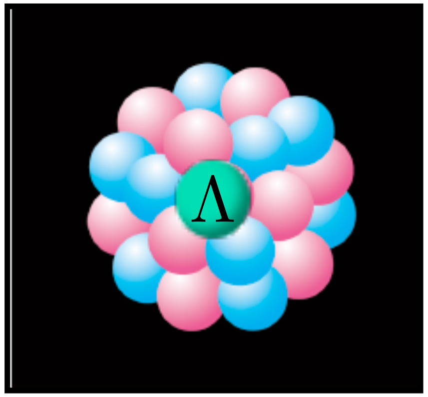
S. Aoki (Tsukuba)
T. Doi (Tsukuba)
T. Hatsuda (Tokyo)
Y. Ikeda (Riken)
T. Inoue (Nihon)
K. Murano (KEK)
H. Nemura (Tohoku)
K. Sasaki (Tsukuba)

1. Motivation

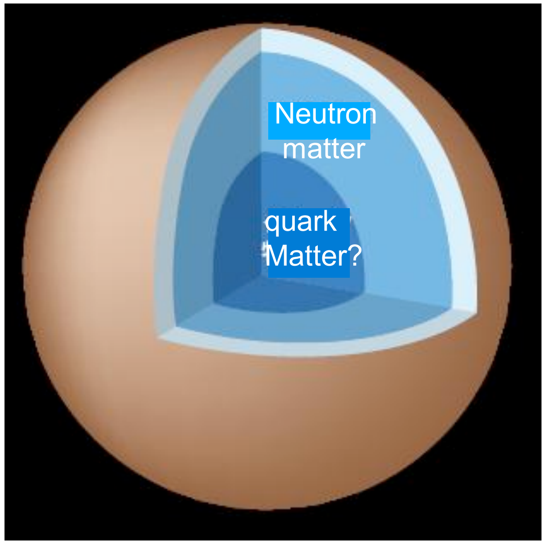
Nuclear force is a basis for understanding ...

- Structure of ordinary and hyper nuclei





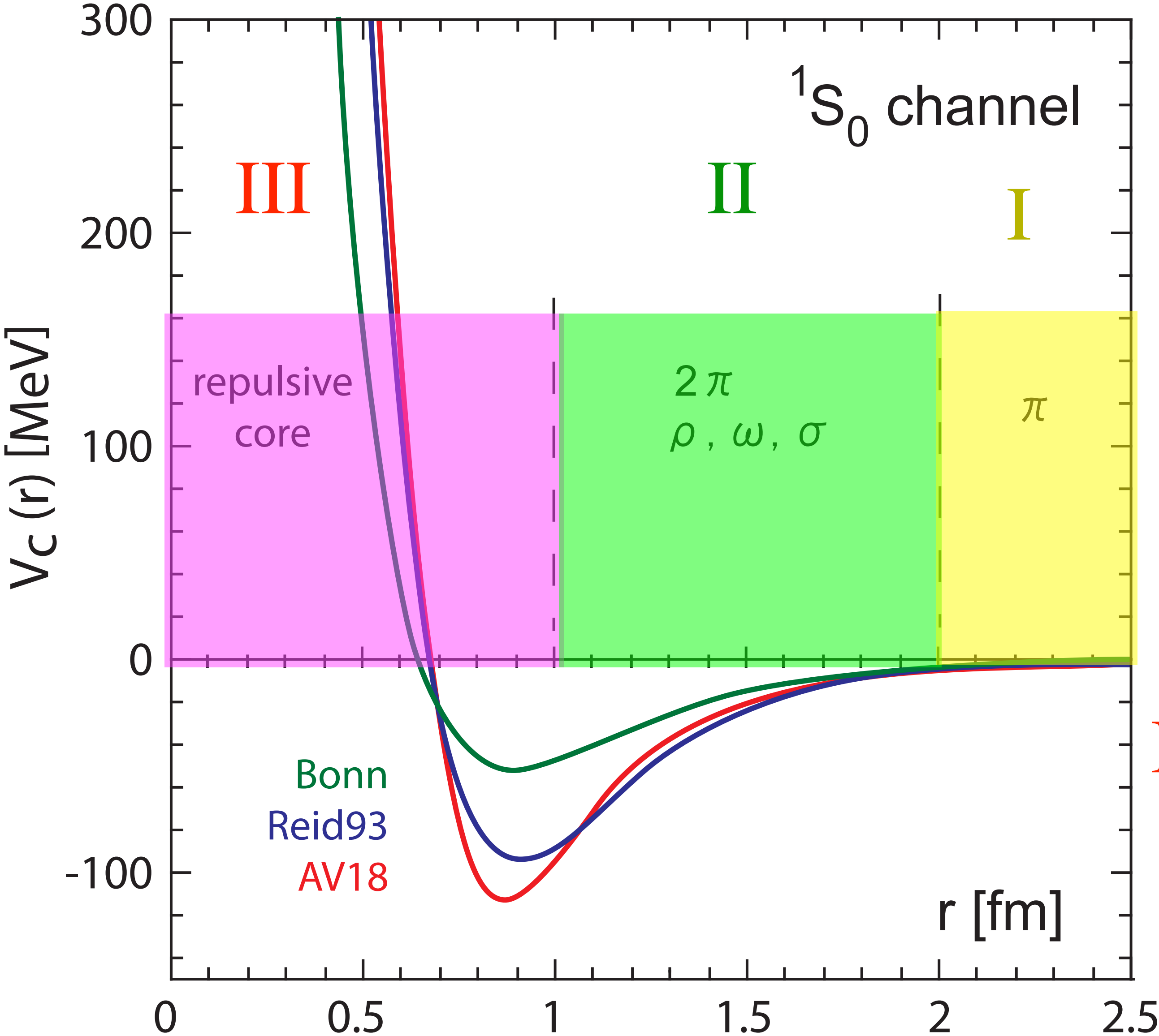
- Structure of neutron star



- Ignition of Type II SuperNova

Phenomenological NN potential

(~40 parameters to fit 5000 phase shift data)



I One-pion exchange

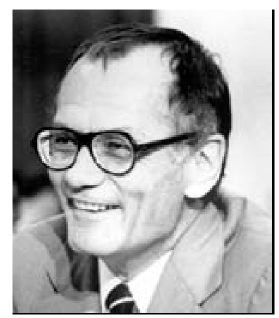
Yiukawa(1935)

II Multi-pions

Taketani et al.(1951)

III Repulsive core

Jastrow(1951)



Plan of my talk

1. Motivation
2. Strategy in (lattice) QCD to extract “potential”
3. More structure: tensor potential
4. Inelastic scattering: octet baryon interactions
    1. Baryon-Baryon interactions in an SU(3) symmetric world
    2. Proposal for S=-2 inelastic scattering
    3. H-dibaryon
5. New method for hadron interactions in lattice QCD
6. Summary and Discussion

2. Strategy in (lattice) QCD to extract "potential"

Challenge to Nambu's statement

"Even now, it is impossible to completely describe nuclear forces beginning with a fundamental equation."

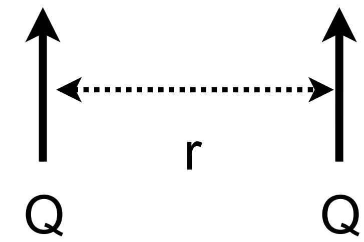
Y. Nambu, "Quarks: Frontiers in Elementary Partcile Physics", World Scientific (1985)

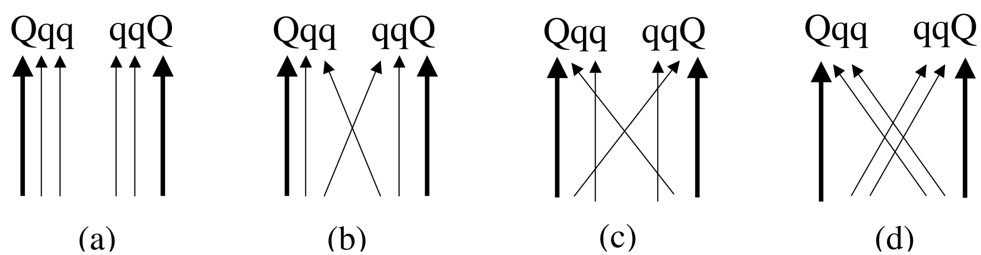
Definition of "Potential" in (lattice) QCD ?

Previous attempt

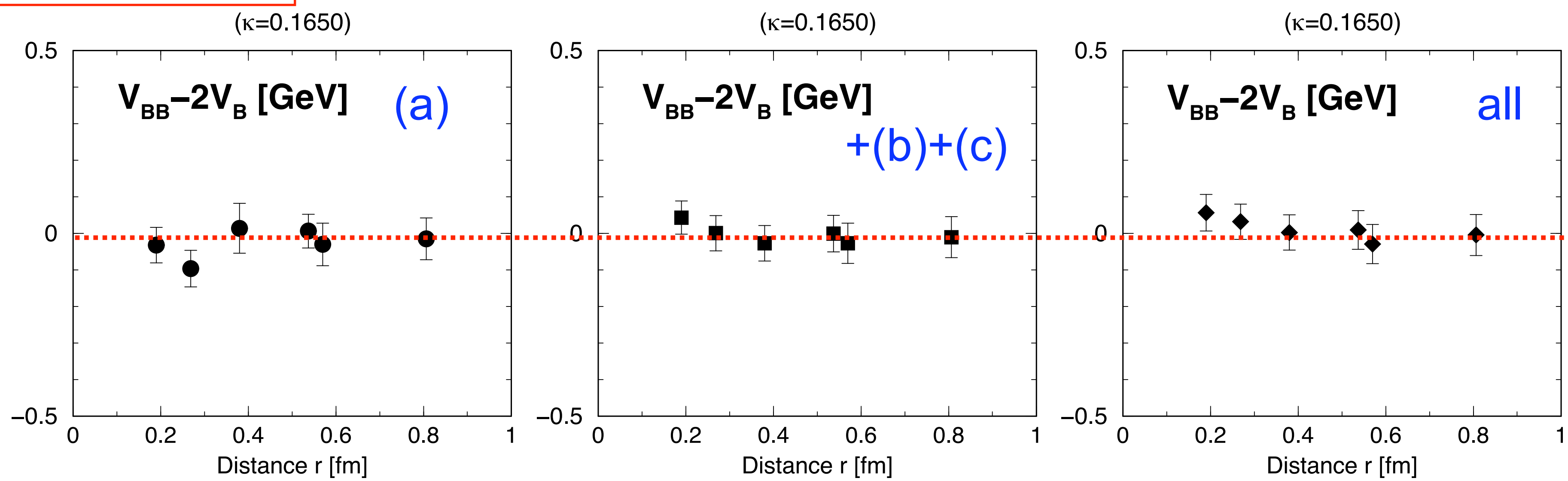
Takahashi-Doi-Suganuma, AIP Conf.Proc. 842,249(2006)

calculate energy of Qqq +Qqq as a function of r between 2Q.
Q:static quark, q: light quark





Quenched result



Almost no dependence on r !

cf. Recent successful result in the strong coupling limit
(deForcrand-Fromm, PRL104(2010)112005)

Quantum Field Theoretical consideration

S.Aoki, LesHouches 2009

- S-matrix below inelastic threshold. Unitarity gives $E < E_{th}$

$$S = e^{2i\delta}$$

- Nambu-Bethe-Salpeter (NBS) Wave function

$$E = 2\sqrt{\mathbf{k}^2 + m_N^2}$$

$$\varphi_E(\mathbf{r}) = \langle 0|N(\mathbf{x}+\mathbf{r},0)N(\mathbf{x},0)|6q,E\rangle$$

QCD eigen-state with energy E and #quark =6

$$N(x) = \varepsilon_{abc} q^a(x) q^b(x) q^c(x): \text{ local operator}$$

off-shell T-matrix

$$\varphi_E(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} + \int \frac{d^3p}{(2\pi)^3} e^{i\mathbf{p}\cdot\mathbf{r}} \frac{E_k + E_p}{8E_p^2} \frac{T(\mathbf{p},-\mathbf{p} \leftarrow \mathbf{k},-\mathbf{k})}{\mathbf{p}^2 - \mathbf{k}^2 - i\epsilon}$$

$$+ \ \mathcal{I}(\mathbf{r})$$

inelastic contribution $\propto O(e^{-\sqrt{E_{th}^2 - E^2}|\mathbf{r}|})$

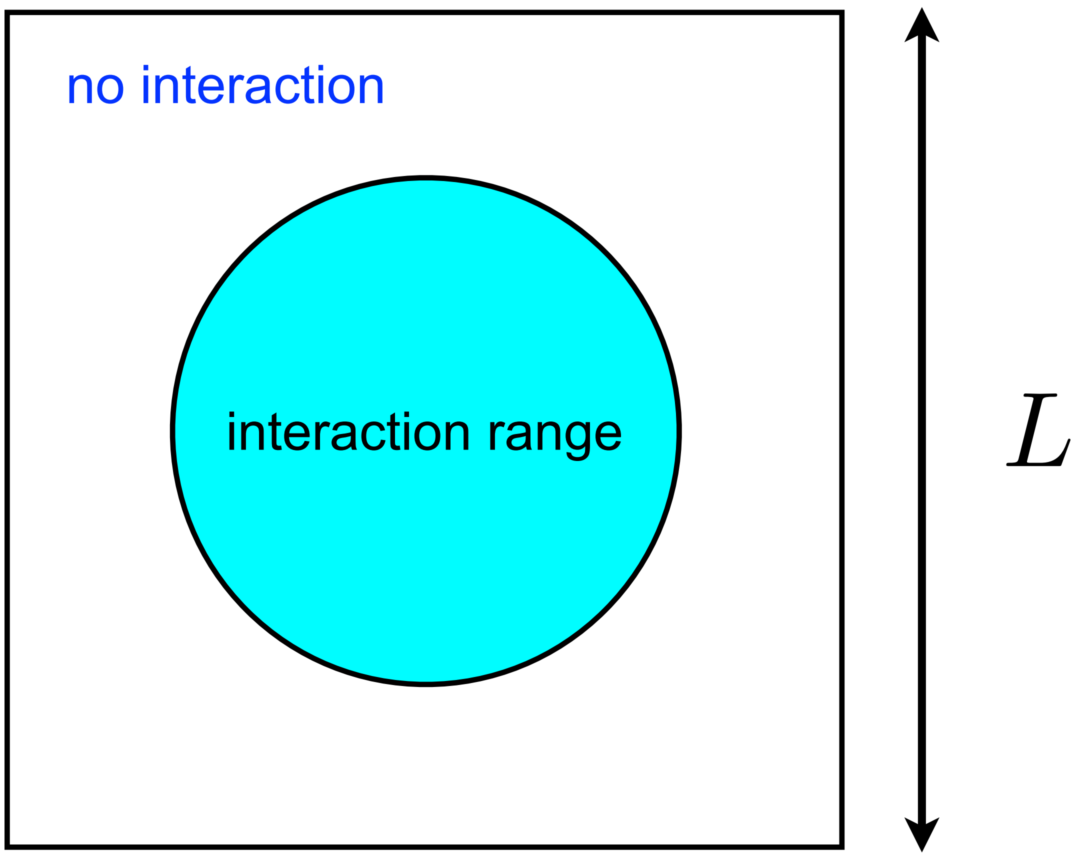
C.-J.D.Lin et al., NPB69(2001) 467
CP-PACS Coll., PRD71 (2005) 094504

Asymptotic behavior $\quad r = |\mathbf{r}| \to \infty$

$$\varphi_E^l(r) \longrightarrow A_l \frac{\sin(kr - l\pi/2 + \delta_l(k))}{kr} \qquad l = 0, 1, 2, \cdots$$

partial wave

$\Rightarrow$ $\delta_l(k)$ is the scattering phase shift



Finite volume

$\Downarrow$

allowed value: k_n^2

$\Downarrow$

Lueshcer's formula

$\delta_l(k_n)$

Systemtic procedure to define the NN potential in lattice QCD

Aoki, Hatsuda & Ishii, PTP123(2010)89

1. Choose your favorite operator: e.g. $N(x) = \varepsilon_{abc} q^a(x) q^b(x) q^c(x)$

2. Measure the NBS amplitude:

$$\varphi_E(\mathbf{r}) = \langle 0 | N(\mathbf{x}+\mathbf{r}, 0) N(\mathbf{x}, 0) | 6q, E \rangle$$

3. Define the non-local potential:

$$\epsilon_k = \frac{\mathbf{k}^2}{2\mu} \qquad H_0 = \frac{-\nabla^2}{2\mu}$$

$$[\epsilon_k - H_0] \varphi_E(\mathbf{x}) = \int d^3y \, U(\mathbf{x}, \mathbf{y}) \varphi_E(\mathbf{y})$$

4. Velocity expansion: $U(\mathbf{x}, \mathbf{y}) = V(\mathbf{x}, \nabla) \delta^3(\mathbf{x} - \mathbf{y})$

$$V(\mathbf{x}, \nabla) = V_0(r) + V_\sigma(r)(\boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma_2}) + V_T(r) S_{12} + V_{\mathrm{LS}}(r) \mathbf{L} \cdot \mathbf{S} + O(\nabla^2)$$

LO LO LO NLO NNLO

spins

tensor operator $S_{12} = \frac{3}{r^2}(\boldsymbol{\sigma_1} \cdot \mathbf{x})(\boldsymbol{\sigma_2} \cdot \mathbf{x}) - (\boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma_2})$

Okubo-Marshak (1958)

5. Calculate observables: phase shift, binding energy etc.

NBS wave function on the lattice

4 point nucleon correlator

$$\mathcal{G}_{\alpha\beta}(\mathbf{x},\mathbf{y},t-t_0;J^P) \equiv \langle 0|n_\beta(\mathbf{y},t)p_\alpha(\mathbf{x},t)\overline{\mathcal{J}}_{pn}(t_0;J^P)|0\rangle$$

$$\mathbf{r}=\mathbf{x}-\mathbf{y} \qquad = \sum_n A_n \langle 0|n_\beta(\mathbf{y},0)p_\alpha(\mathbf{x},0)|E_n\rangle e^{-E_n(t-t_0)}$$

$$\longrightarrow A_0 \psi_{\alpha\beta}(\mathbf{r};J^P) e^{-E_0(t-t_0)}$$

$$A_n = \langle E_n|\overline{\mathcal{J}}_{pn}(0;J^P)|0\rangle$$

Wall source

$$L=0 \qquad \mathcal{J}_{pn}(t_0;J^P) = P^{(s)}_{\beta\alpha}[p^{\rm wall}_\alpha(t_0)n^{\rm wall}_\beta(t_0)] \qquad q(\mathbf{x},t_0)\to q^{\rm wall}(t_0)=\sum_{\mathbf{x}} q(\mathbf{x},t_0)$$

$$(A_1) \qquad (J,J_z)=(s,s_z) \qquad P=+$$

with Coulomb gauge fixing

cubic group

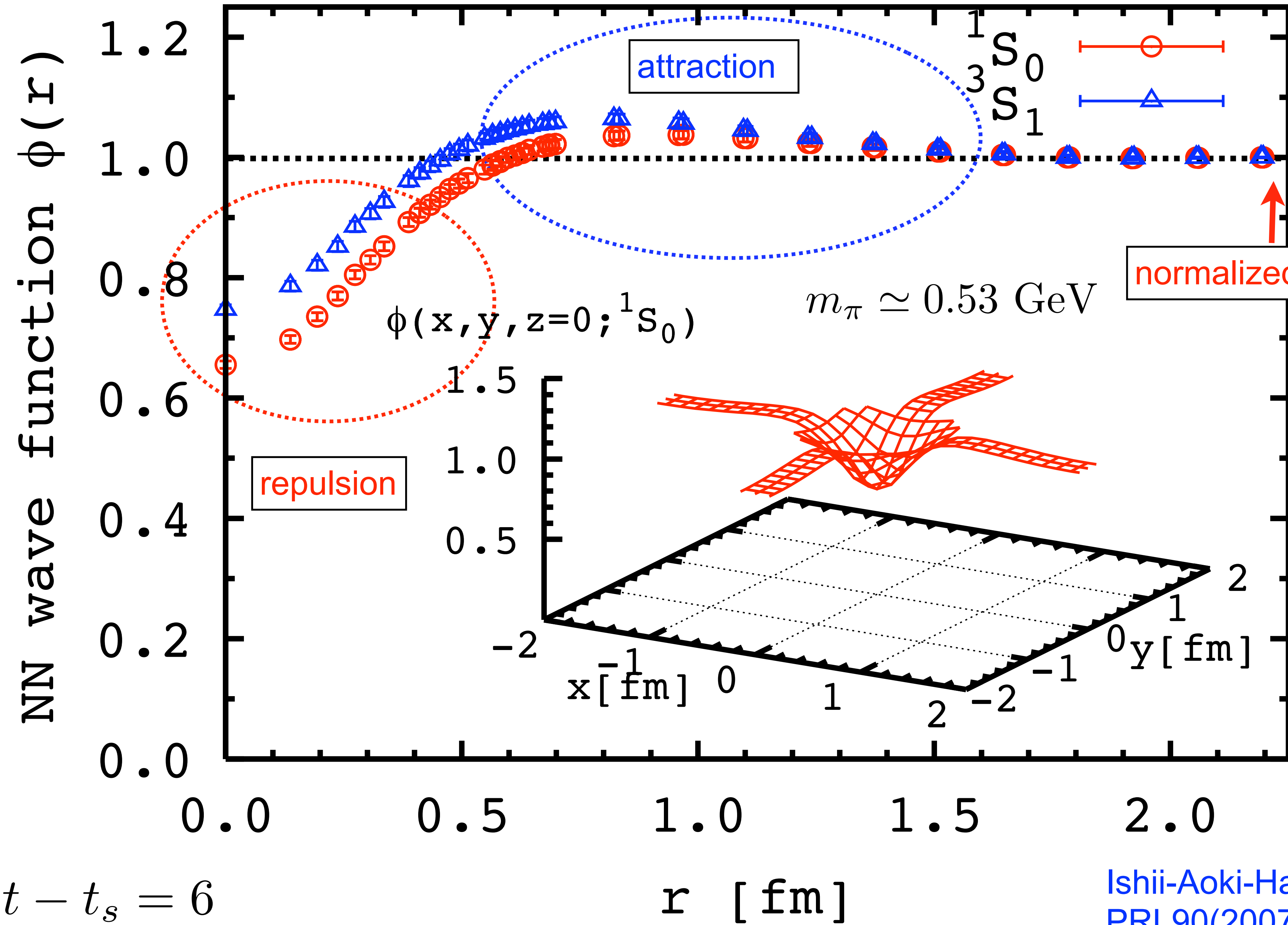
$$\psi(r;{}^1S_0) = P^{(A_1)}P^{(s=0)}\psi(\mathbf{r};0^+) \equiv \frac{1}{24}\sum_{g\in O} P^{(s=0)}_{\beta\alpha}\psi_{\alpha\beta}(g^{-1}\mathbf{r};0^+)$$

$$\psi(r;{}^3S_1) = P^{(A_1)}P^{(s=1)}\psi(\mathbf{r};1^+) \equiv \frac{1}{24}\sum_{g\in O} P^{(s=1)}_{\beta\alpha}\psi_{\alpha\beta}(g^{-1}\mathbf{r};1^+)$$

NN wave function

Quenched QCD

a=0.137fm



$t - t_s = 6$

Ishii-Aoki-Hatsuda,
PRL90(2007)0022001

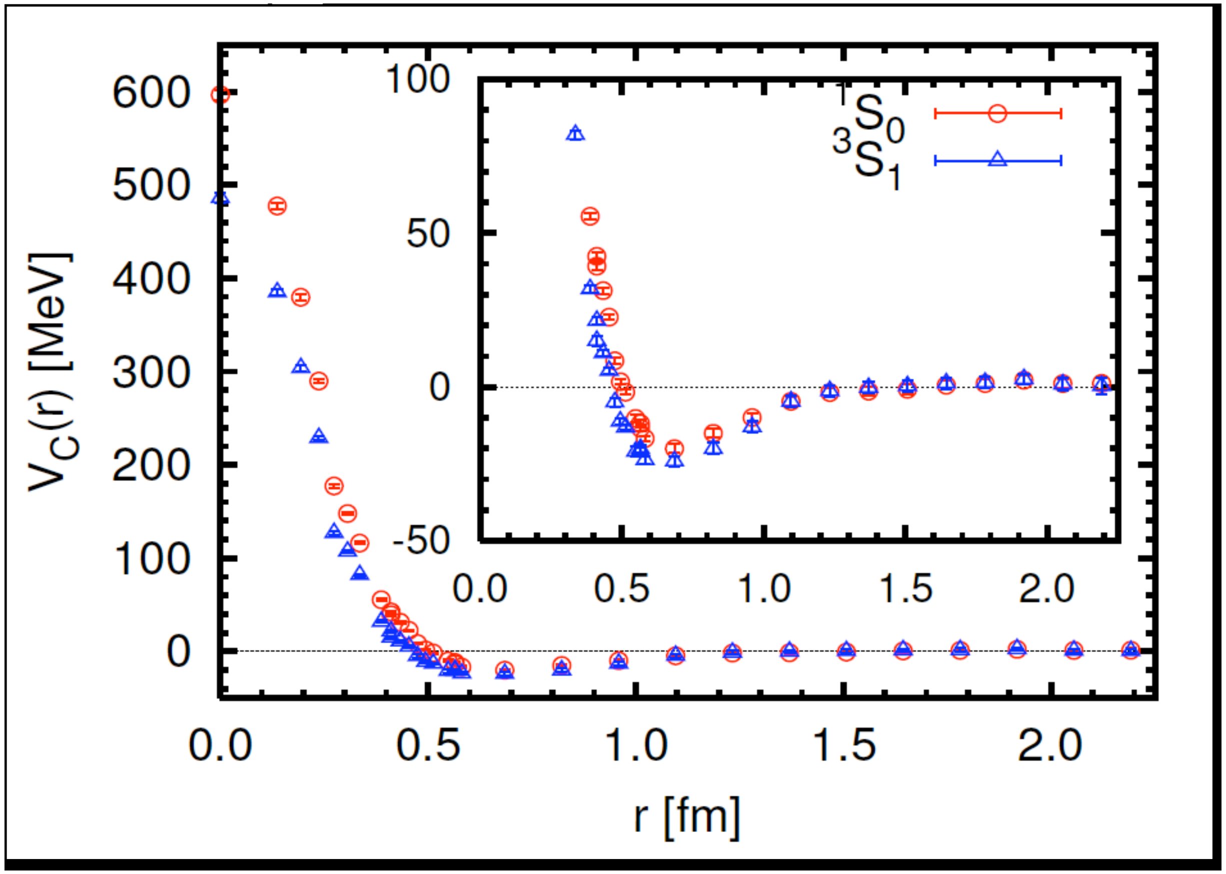
(quenched) potentials

LO (effective) central Potential

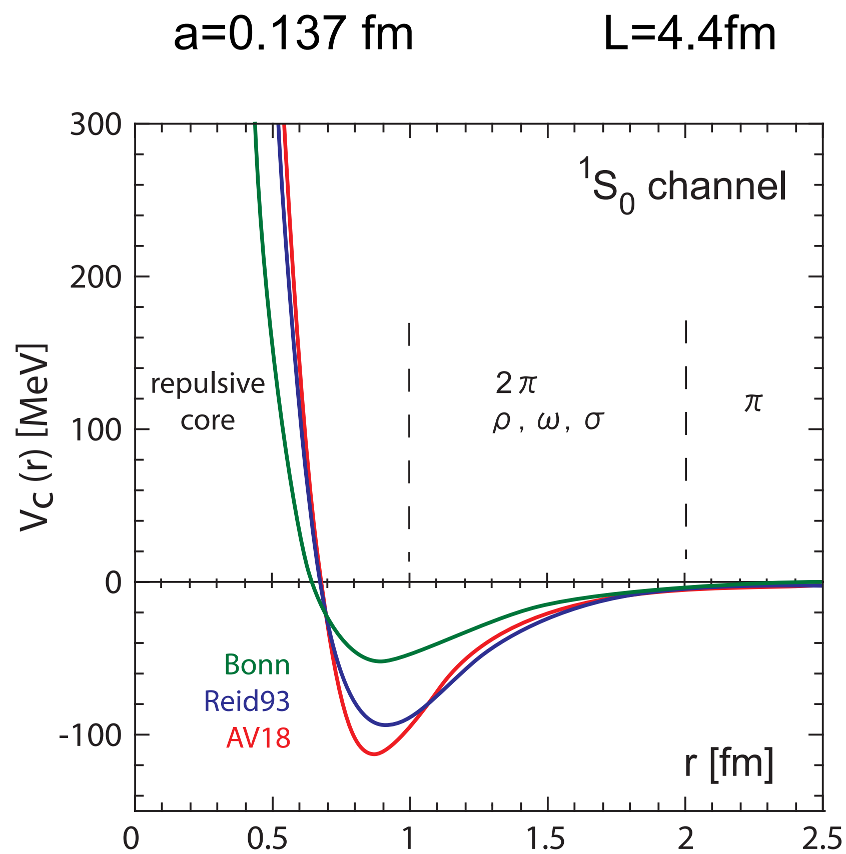
$$V(r;{}^1S_0) = V_0^{(I=1)}(r) + V_\sigma^{(I=1)}(r)$$

$$V(r;{}^3S_1) = V_0^{(I=0)}(r) - 3V_\sigma^{(I=0)}(r)$$

$E \simeq 0$ $\quad m_\pi \simeq 0.53$ GeV



a=0.137 fm L=4.4fm



Qualitative features of NN potential are reproduced !

Ishii-Aoki-Hatsuda, PRL90(2007)0022001

This paper has been selected as one of 21 papers in Nature Research Highlights 2007

Frequently Asked Questions

[Q1] Operator dependence of the potential

[Q2] Energy dependence of the potential

[A1] choice of operator = scheme, cf. running coupling

(N(x), U(x,y)) is a combination to define ovservables

QM: (Φ, U) → observables

QFT: (asymptotic field, vertices) → observables

EFT: (choice of field, vertices) → observables

- local operator = convenient choice for reduction formula

[A2] U(x,y) is E-independent by construction

- non-locality can be determined order by order in velocity expansion (cf. ChPT)

Non-local, E-independent ⟺ Local, E-dependent

$$\left(E + \frac{\nabla^2}{2m}\right) \varphi_E(\mathbf{x}) = \int d^3y \, U(\mathbf{x}, \mathbf{y}) \varphi_E(\mathbf{y}) \qquad V_E(\mathbf{x})\varphi_E(\mathbf{x}) = \left(E + \frac{\nabla^2}{2m}\right) \varphi_E(\mathbf{x})$$

Validity of the velocity expansion of U

Leading Order $$V_C(r) = \frac{(E - H_0)\varphi_E(\mathbf{x})}{\varphi_E(\mathbf{x})}$$ Local potential approximation

E-dependent ⟷ Non-locality

From E-dependence, one may determine higher order terms:

$$V(\mathbf{x}, \nabla) = V_C(r) + V_T(r)S_{12} + V_{\rm LS}(r)\mathbf{L}\cdot\mathbf{S} + \{V_D(r), \nabla^2\} + \cdots$$

Numerical check in quenched QCD

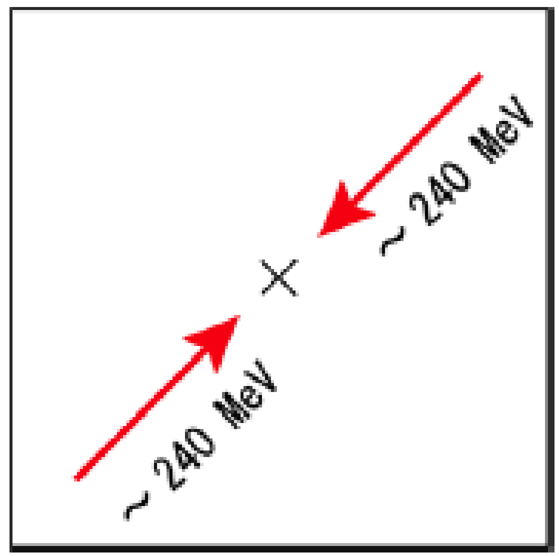
$m_\pi \simeq 0.53$ GeV

a=0.137fm

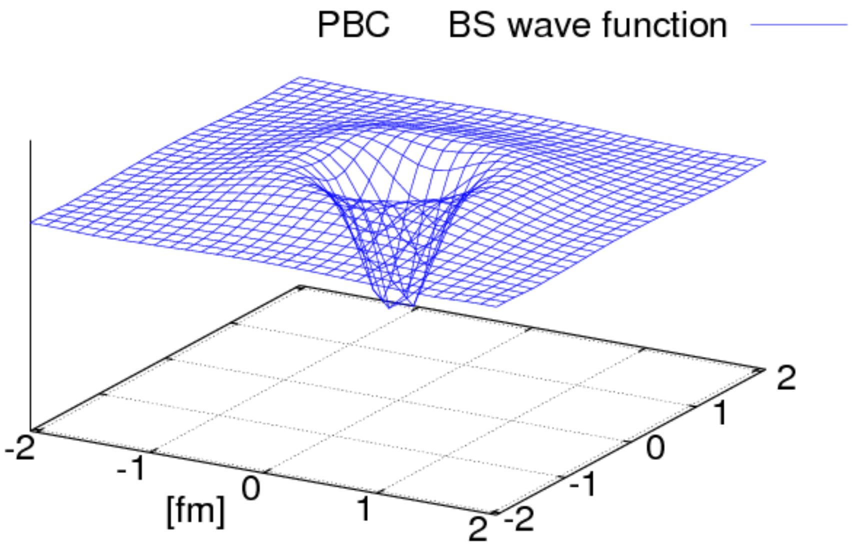
K. Murano, N. Ishii, S. Aoki, T. Hatsuda

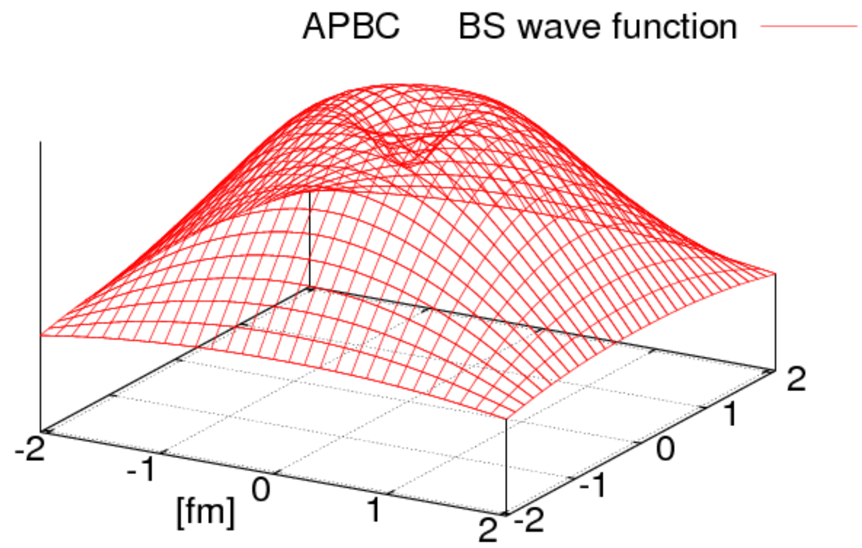
PoS Lattice2009 (2009)126.

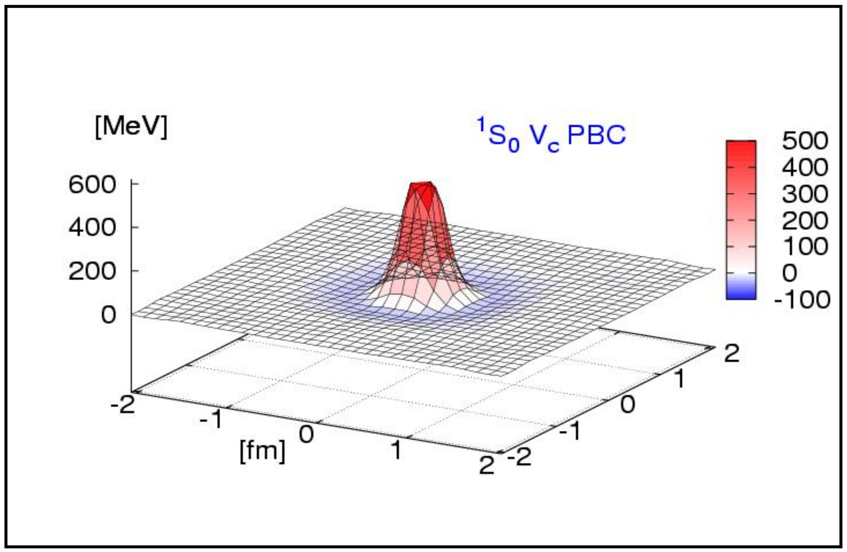
Anti-Periodic B.C.

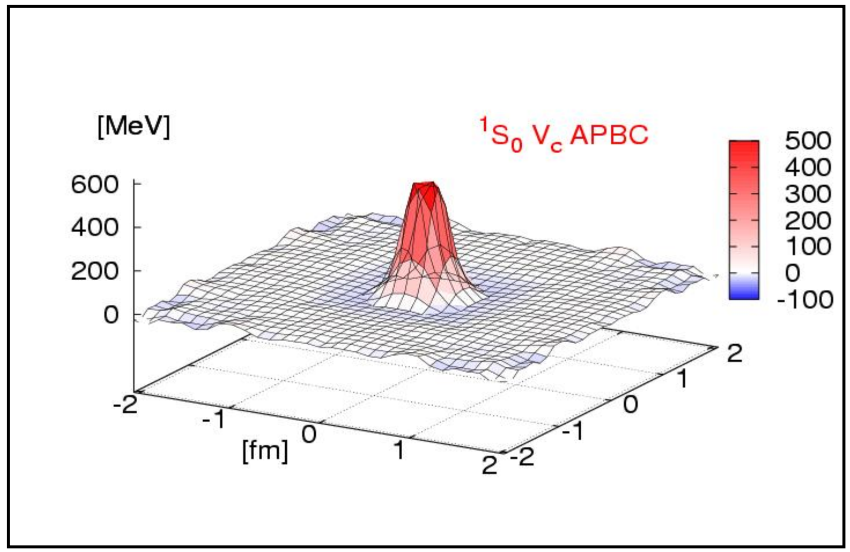


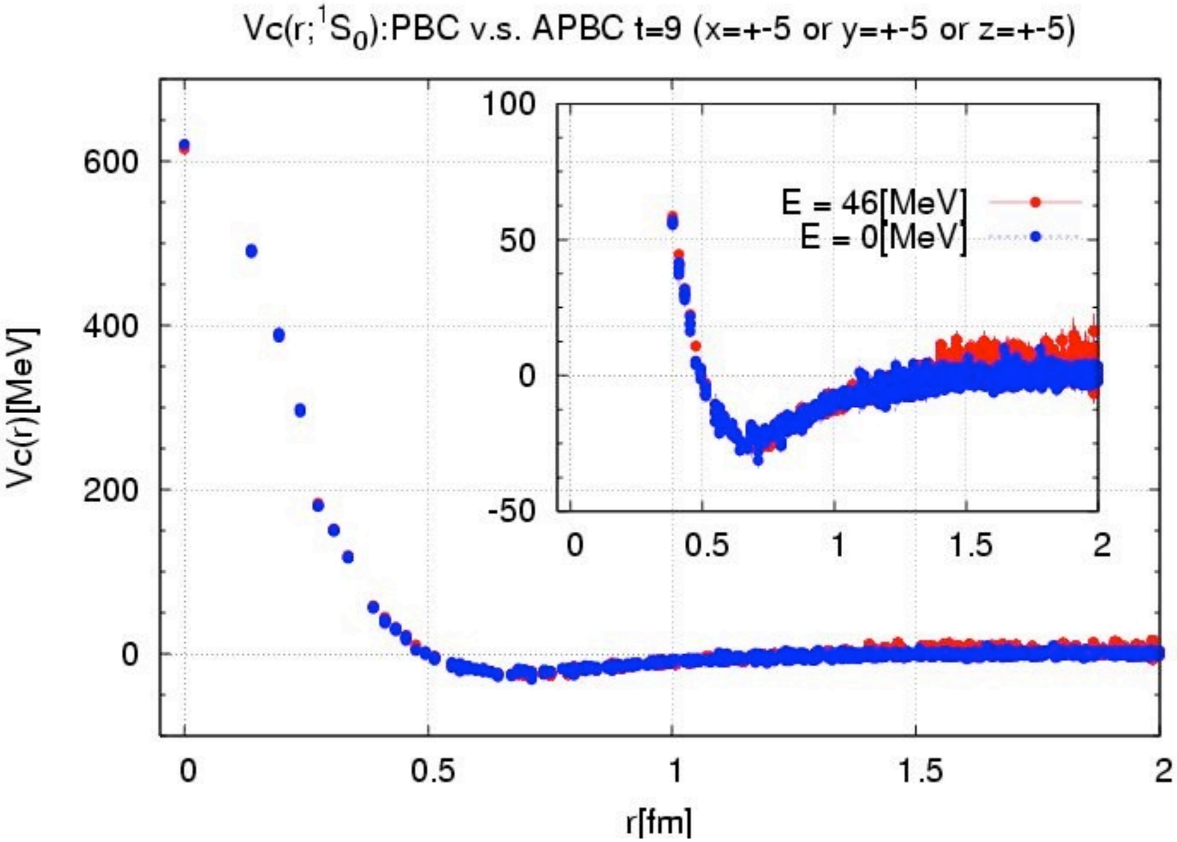
- PBC (E∼0 MeV)
- APBC (E∼46 MeV)









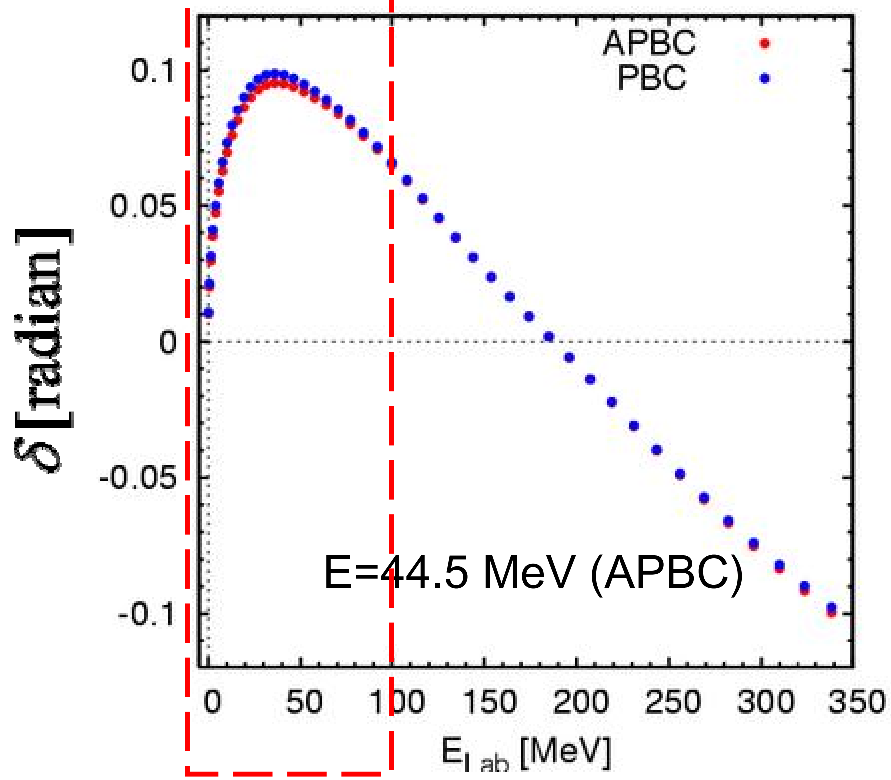


Quenched QCD

$m_\pi \simeq 0.53\ \mathrm{GeV}$

a=0.137fm

phase shifts from potentials



E-dependence of the local potential turns out to be very small at low energy in our choice of wave function.

3. More structure:tensor potential

Tensor potential

$$(H_0 + V_C(r) + V_T(r)S_{12})\psi(\mathbf{r};1^+) = E\psi(\mathbf{r};1^+)$$

mixing between ${}^3S_1$ and ${}^3D_1$ through the tensor force

sink

$T_1(\text{spin}) \otimes A_1(L=0) = T_1(J=1)$

$T_1(\text{spin}) \otimes E(L=2) = T_1(J=1) \oplus T_2$

$\longleftarrow$

source

$T_1(\text{spin}) \otimes A_1(L=0) = T_1(J=1)$

$$\psi(\mathbf{r};1^+) = \mathcal{P}\psi(\mathbf{r};1^+) + \mathcal{Q}\psi(\mathbf{r};1^+)$$

$$\mathcal{P}\psi_{\alpha\beta}(\mathbf{r};1^+) = P^{(A_1)}\psi_{\alpha\beta}(\mathbf{r};1^+)$$ "projection" to L=0 ${}^3S_1$

$$\mathcal{Q}\psi_{\alpha\beta}(\mathbf{r};1^+) = (1 - P^{(A_1)})\psi_{\alpha\beta}(\mathbf{r};1^+)$$ "projection" to L=2 ${}^3D_1$

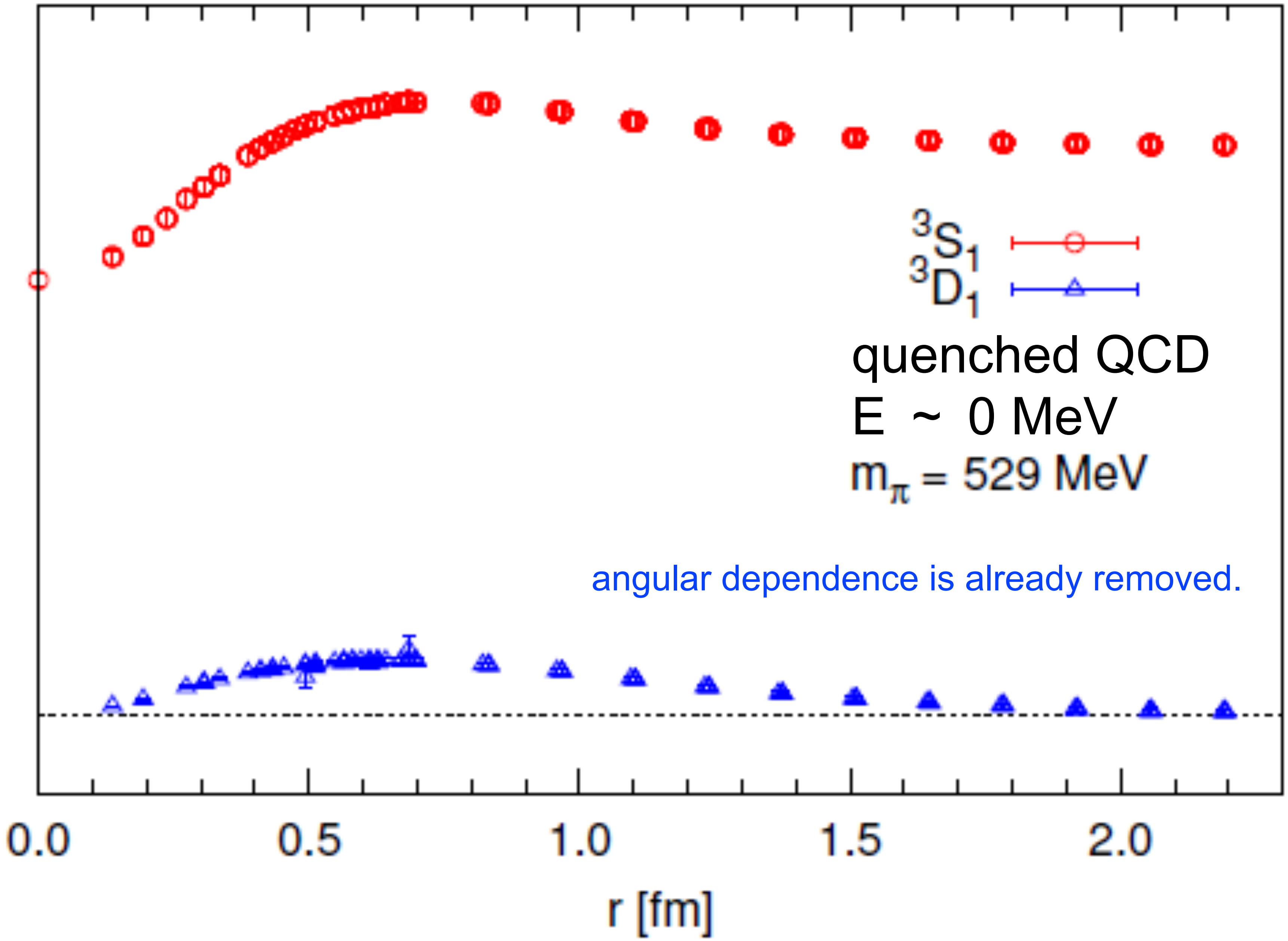
$\Rightarrow$

$$\begin{aligned} H_0[\mathcal{P}\psi](\mathbf{r}) + V_C(r)[\mathcal{P}\psi](\mathbf{r}) + V_T(r)[\mathcal{P}S_{12}\psi](\mathbf{r}) &= E[\mathcal{P}\psi](\mathbf{r}) \\ H_0[\mathcal{Q}\psi](\mathbf{r}) + V_C(r)[\mathcal{Q}\psi](\mathbf{r}) + V_T(r)[\mathcal{Q}S_{12}\psi](\mathbf{r}) &= E[\mathcal{Q}\psi](\mathbf{r}) \end{aligned}$$

Wave functions

Quenched

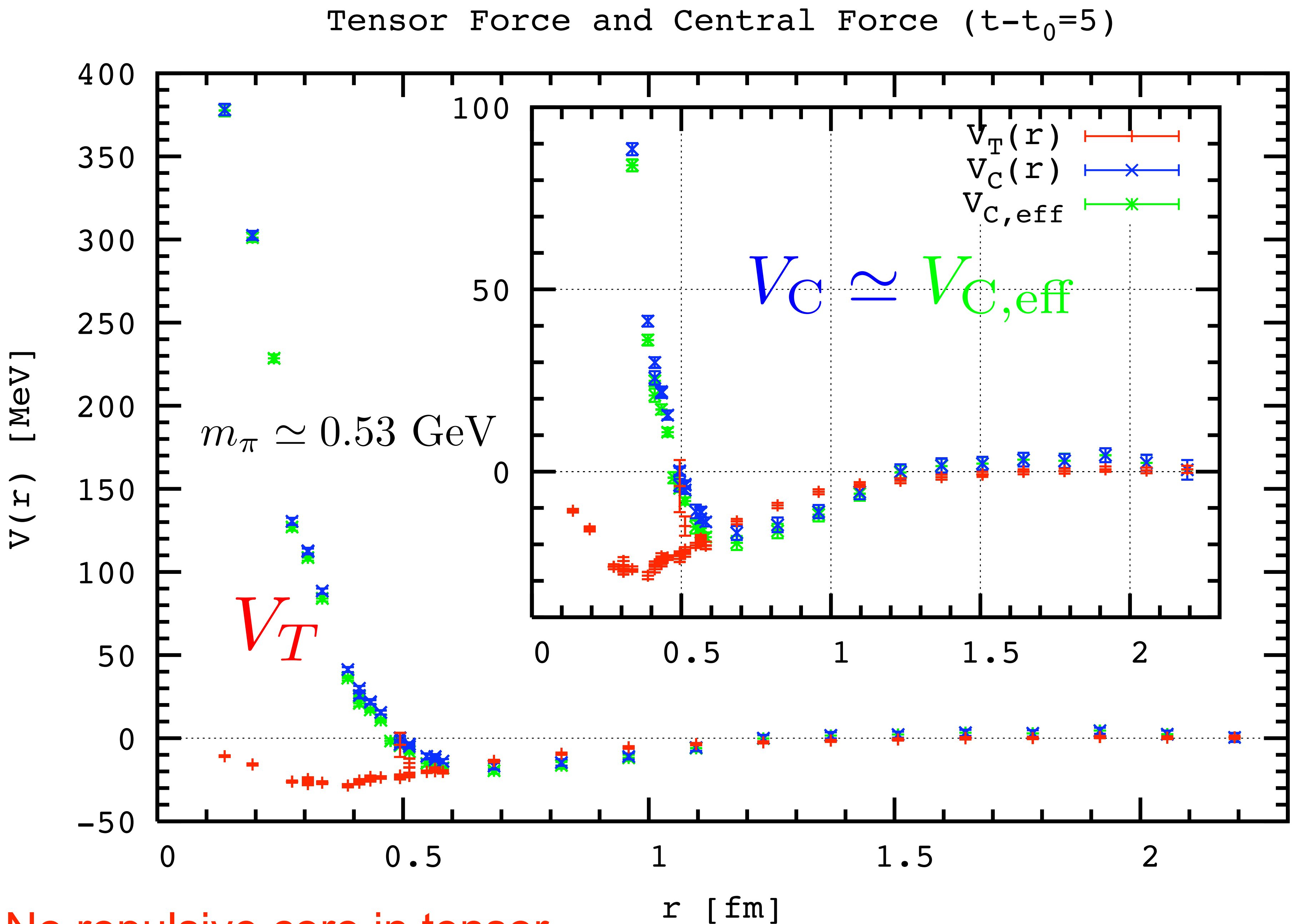
Aoki, Hatsuda, Ishii, PTP 123 (2010)89
arXiv:0909.5585



Potentials

Quenched

Aoki, Hatsuda, Ishii, PTP 123 (2010)89
arXiv:0909.5585

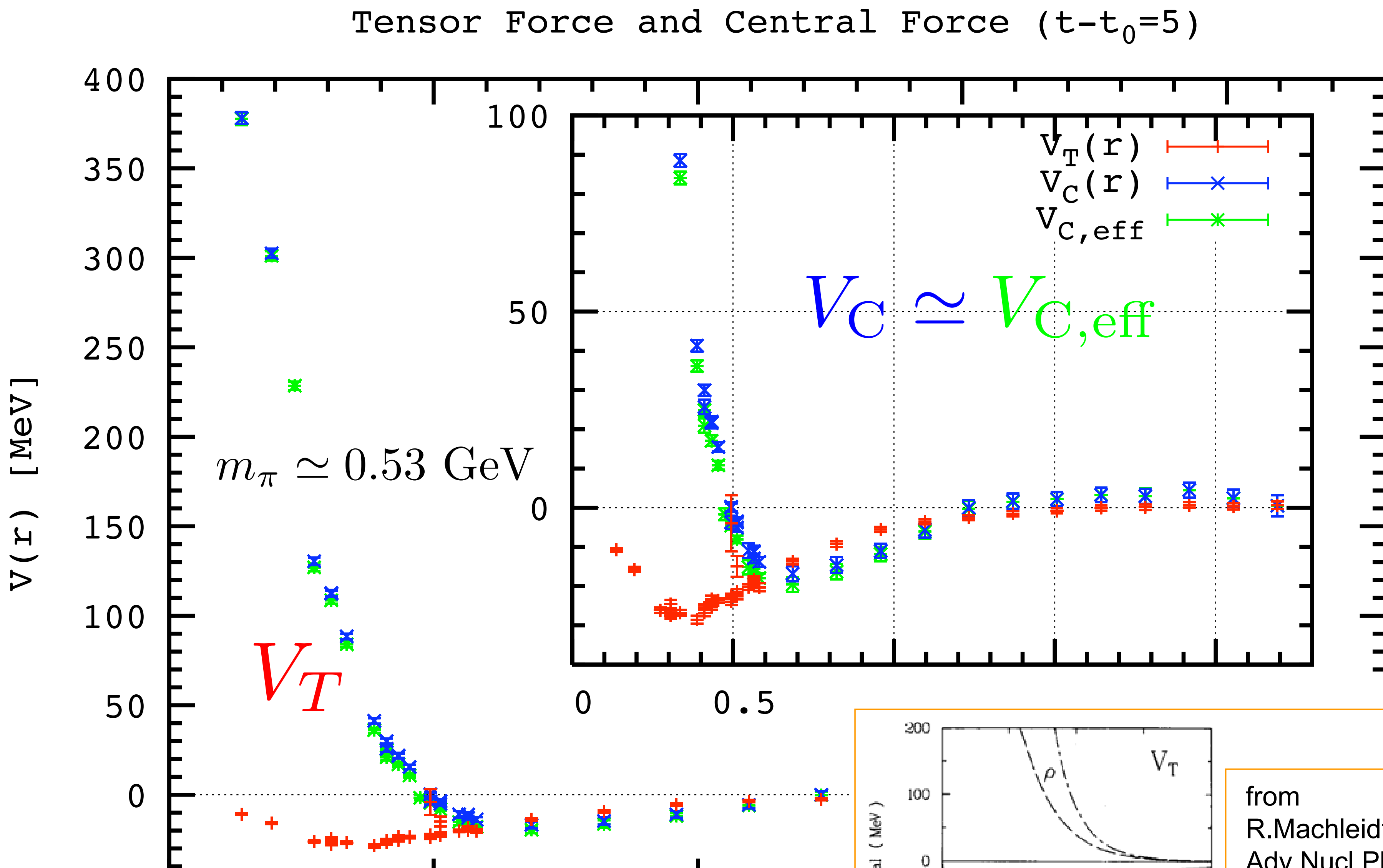


No repulsive core in tensor

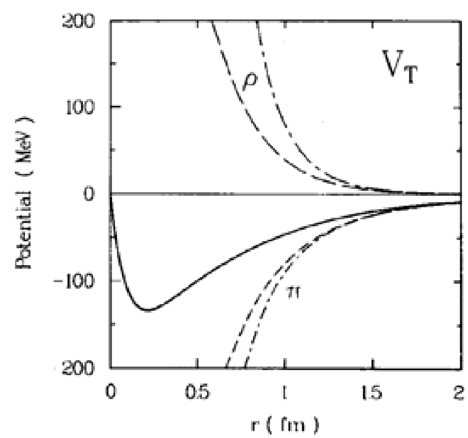
Potentials

Quenched

Aoki, Hatsuda, Ishii, PTP 123 (2010)89
arXiv:0909.5585



No repulsive core in tensor

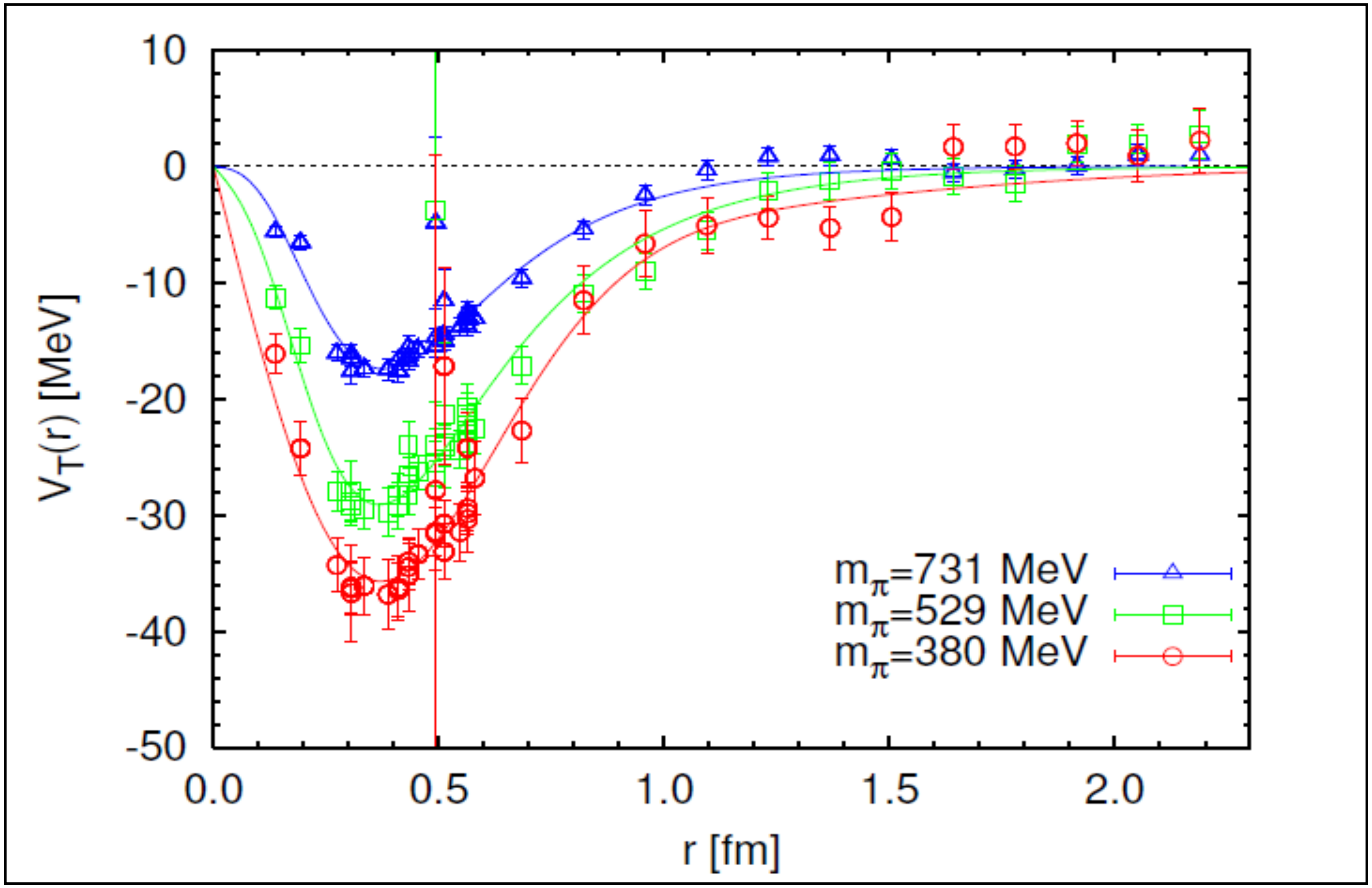


from R.Machleidt, Adv.Nucl.Phys.**19**

Fig. 3.7. The contributions from π and ρ (dashed) to the T = 0 tensor potential. The solid line is the full potential. The dash-dot lines are obtained when the cutoff is omitted.

Quark mass dependence

Quenched



- Rapid quark mass dependence of tensor potential
- Evidence of one-pion exchange

Fit function

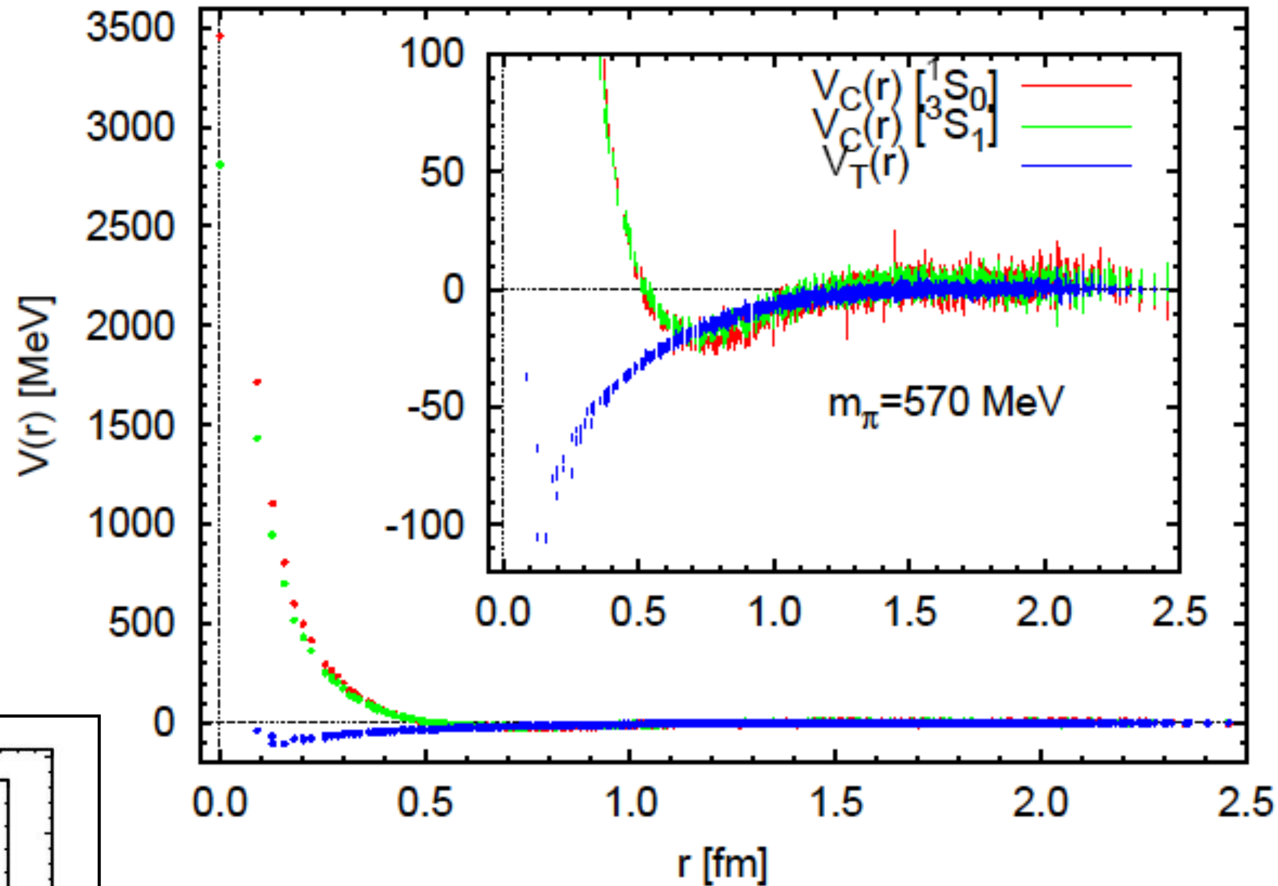
$$V_T(r) = b_1(1-e^{-b_2 r^2})^2\left(1+\frac{3}{m_\rho r}+\frac{3}{(m_\rho r)^2}\right)\frac{e^{-m_\rho r}}{r} + b_3(1-e^{-b_4 r^2})^2\left(1+\frac{3}{m_\pi r}+\frac{3}{(m_\pi r)^2}\right)\frac{e^{-m_\pi r}}{r},$$

Full QCD Calculation

Full QCD

$m_\pi = 570 \text{ MeV}, L = 2.9 \text{ fm}$

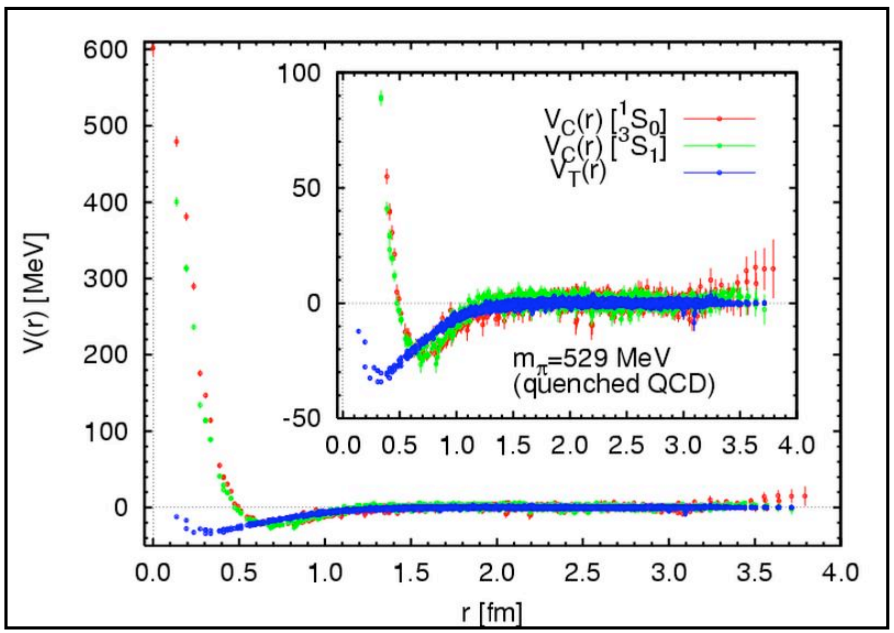
a=0.1fm



Quenched QCD

L=4.4fm

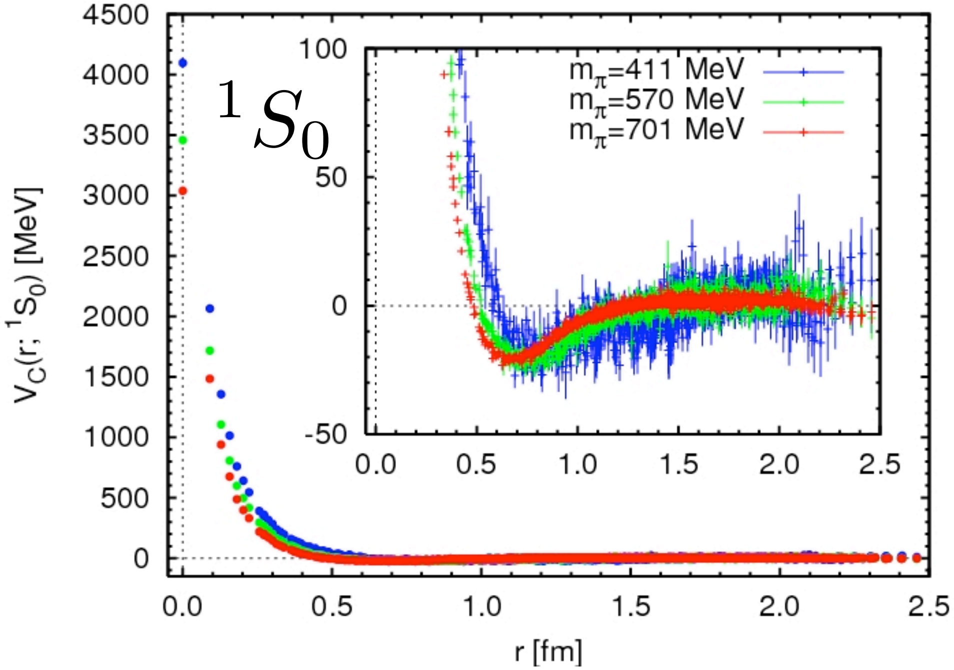
$m_\pi \simeq 0.53 \text{ GeV}$ a=0.137 fm

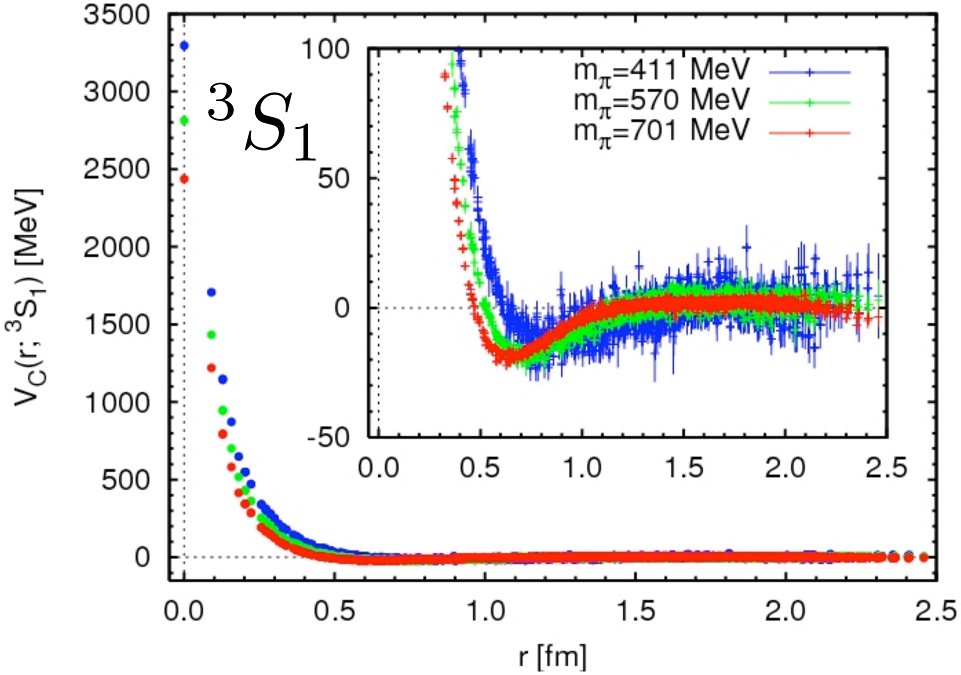


* Large repulsive core than quenched

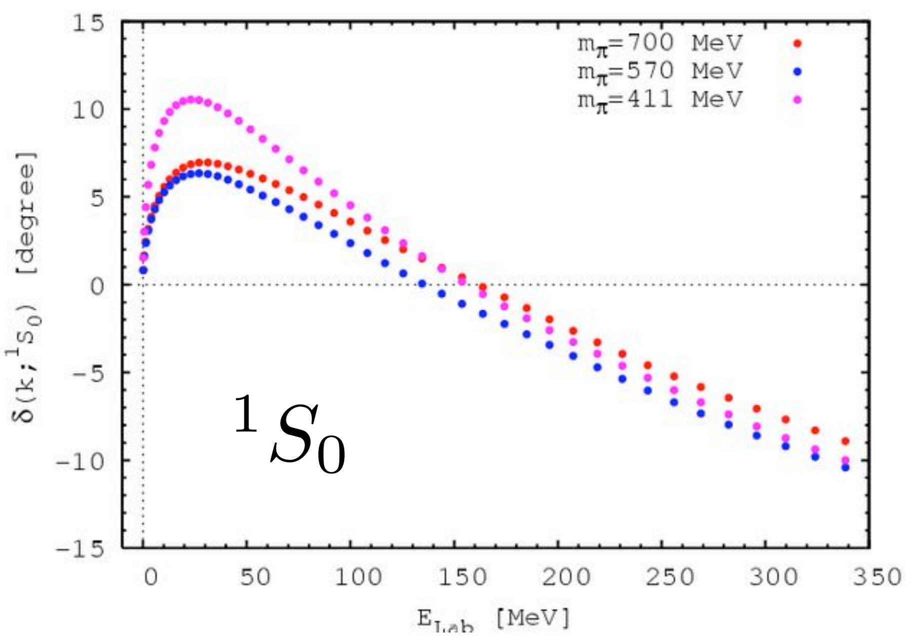
* Large tensor force than quenched

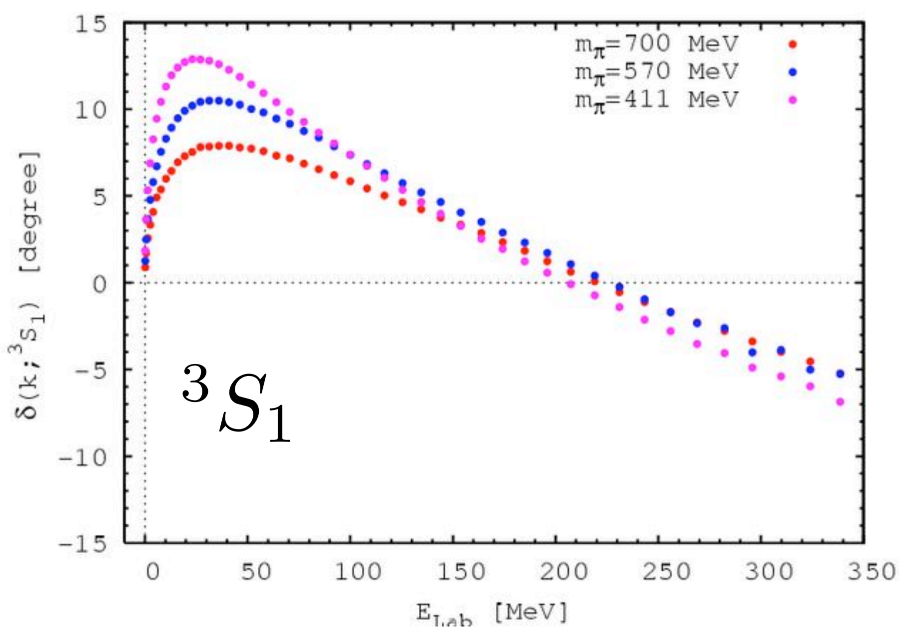
Phase shift from V(r) in full QCD





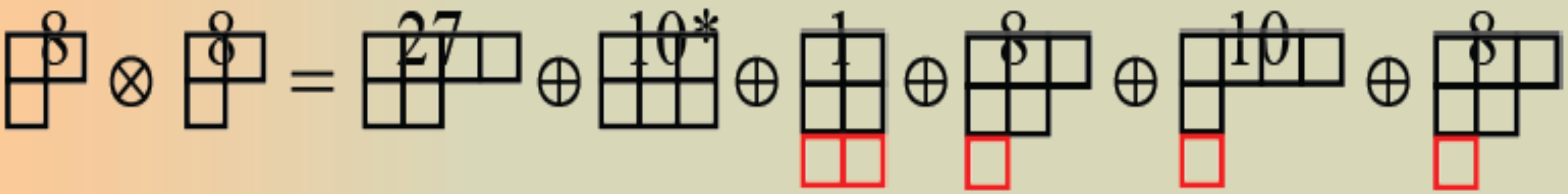
a=0.1 fm, L=2.9 fm

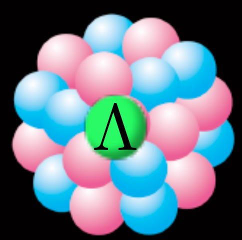


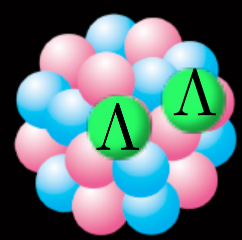


4. Inelastic scattering: octet baryon interactions

Octet Baryon interactions



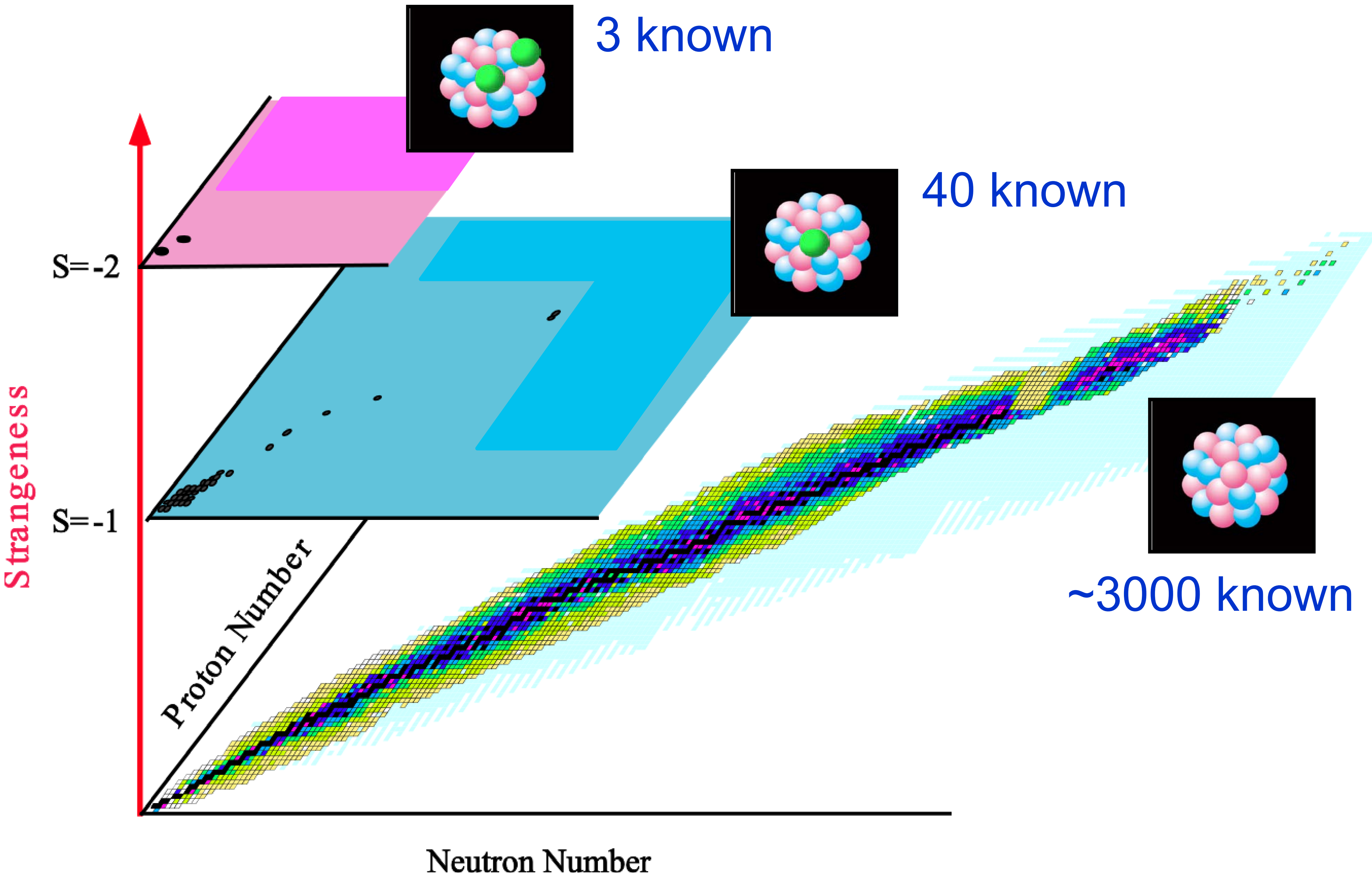




- no phase shift available for YN and YY scattering
- plenty of hyper-nucleus data will be soon available at J-PARC

- prediction from lattice QCD
- difference between NN and YN ?

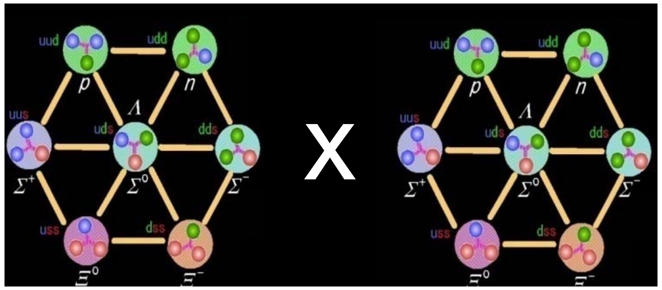
3D Nuclear chart



4-1. Baryon-Baryon interactions in an SU(3) symmetric world

$$m_u = m_d = m_s$$

1. First setup to predict YN, YY interactions not accessible in exp.
2. Origin of the repulsive core (universal or not)



$$8 \times 8 = \underline{27 + 8s + 1} + \underline{10^* + 10 + 8a}$$

Symmetric — Anti-symmetric

6 independent potential in flavor-basis

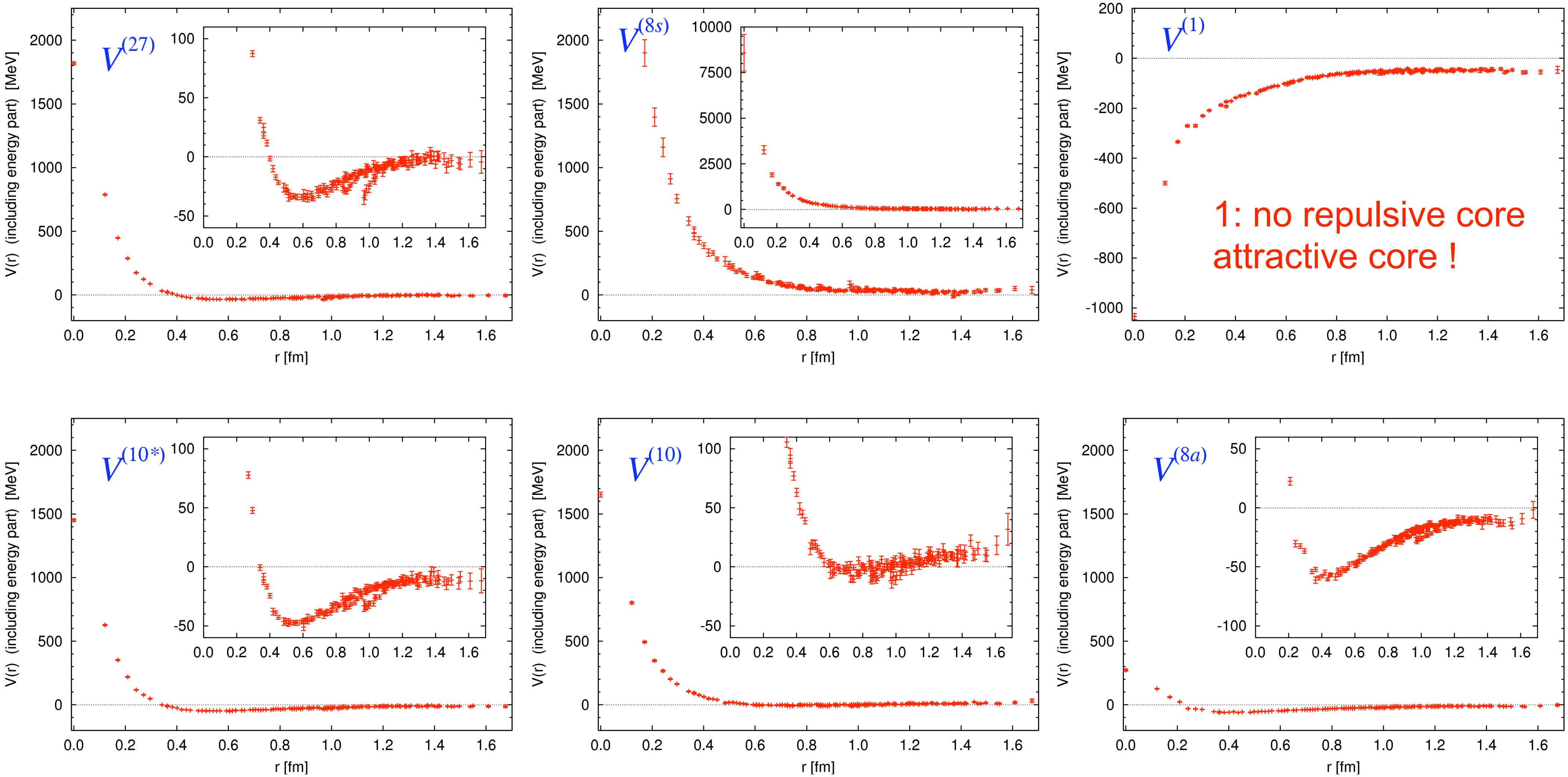
$$V^{(27)}(r),\ V^{(8s)}(r),\ V^{(1)}(r) \longleftarrow {}^1S_0$$

$$V^{(10^*)}(r),\ V^{(10)}(r),\ V^{(8a)}(r) \longleftarrow {}^3S_1$$

Potentials(full QCD)

a=0.12 fm, L=2 fm

$m_{\rm PS} \simeq 840$ MeV

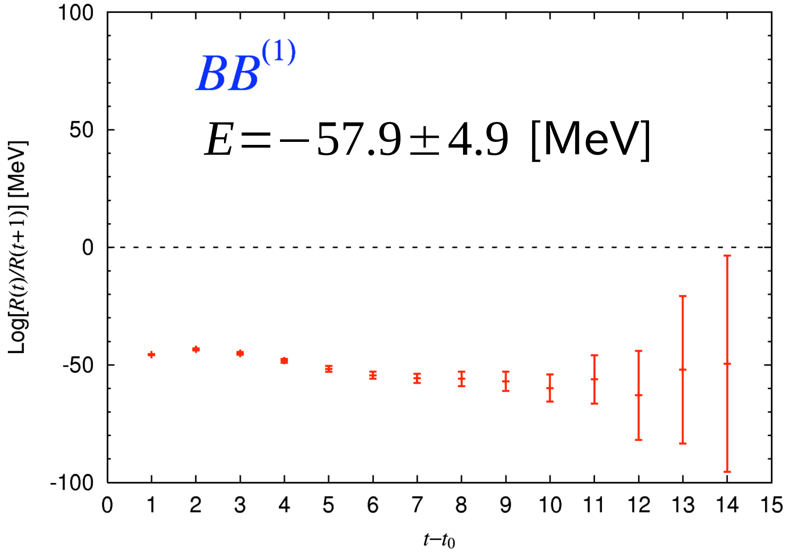


27, 10*: same as before
NN channel

8s, 10: strong repulsive core

8a: week repulsive core,
deep attractive pocket

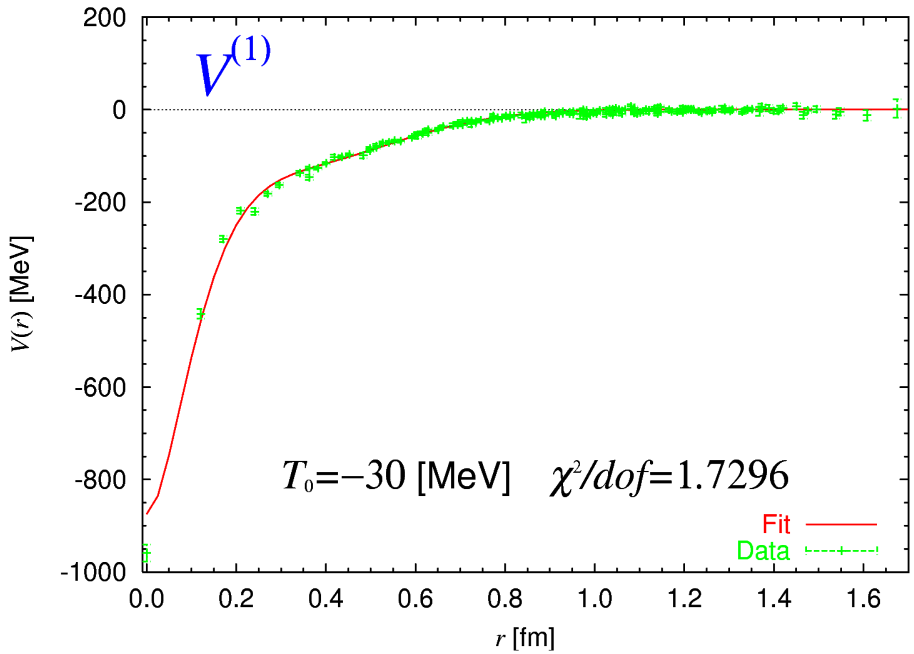
Inoue *et al.* ,HAL QCD Collaboration, arXiv:1007.3559[hep-lat]



Bound state in 1(singlet) channel ? H-dibaryon ?

However, it is difficult to determine E precisely, due to contaminations from excited states.

Singlet potential with a certain value of E



$$V(r) = a_1 e^{-a_2 r^2} + a_3 \left(1 - e^{-a_4 r^2}\right)^2 \left(\frac{e^{-a_5 r}}{r}\right)^2$$

Schroedinger eq. predicts a bound state at E < -30 MeV

E [MeV]	E_0 [MeV]	$\sqrt{\langle r^2 \rangle}$ [fm]
E = −30	−0.018	24.7
E = −35	−0.72	4.1
E = −40	−2.49	2.3

finite size effect is very large on this volume.
(consistent with previous results.)
simulations on larger volume is in progress.

4-2. Proposal for S=-2 In-elastic scattering

$$m_N = 939 \text{ MeV}, \, m_\Lambda = 1116 \text{ MeV}, \, m_\Sigma = 1193 \text{ MeV}, \, m_\Xi = 1318 \text{ MeV}$$

S=-2 System(I=0)

$$M_{\Lambda\Lambda} = 2232 \text{ MeV} < M_{N\Xi} = 2257 \text{ MeV} < M_{\Sigma\Sigma} = 2386 \text{ MeV}$$

The eigen-state of QCD in the finite box is a mixture of them:

$$|S=-2, I=0, E\rangle_L = c_1(L)|\Lambda\Lambda, E\rangle + c_2(L)|\Xi N, E\rangle + c_3(L)|\Sigma\Sigma, E\rangle$$

$$E = 2\sqrt{m_\Lambda^2 + \mathbf{p}_1^2} = \sqrt{m_\Xi^2 + \mathbf{p}_2^2} + \sqrt{m_N^2 + \mathbf{p}_2^2} = 2\sqrt{m_\Sigma^2 + \mathbf{p}_3^2}$$

In this situation, we can not directly extract the scattering phase shift in lattice QCD.

HAL's proposal

Let us consider 2-channel problem for simplicity.

NBS wave functions for 2 channels at 2 values of energy:

$$\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) = \langle 0|\Lambda(\mathbf{x})\Lambda(\mathbf{0})|E_\alpha\rangle$$
$$\Psi_\alpha^{\Xi N}(\mathbf{x}) = \langle 0|\Xi(\mathbf{x})N(\mathbf{0})|E_\alpha\rangle$$

$$\alpha = 1, 2$$

They satisfy

$$(\nabla^2 + \mathbf{p}_\alpha^2)\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) = 0$$
$$(\nabla^2 + \mathbf{q}_\alpha^2)\Psi_\alpha^{\Xi N}(\mathbf{x}) = 0$$

$$|\mathbf{x}| \to \infty$$

We define the "potential" from the coupled channel Schroedinger equation:

$$\left(\frac{\nabla^2}{2\mu_{\Lambda\Lambda}} + \frac{\mathbf{p}_\alpha^2}{2\mu_{\Lambda\Lambda}}\right)\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) = V^{\Lambda\Lambda\leftarrow\Lambda\Lambda}(\mathbf{x})\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) + V^{\Lambda\Lambda\leftarrow\Xi N}(\mathbf{x})\Psi_\alpha^{\Xi N}(\mathbf{x})$$

diagonal   off-diagonal

$$\left(\frac{\nabla^2}{2\mu_{\Xi N}} + \frac{\mathbf{q}_\alpha^2}{2\mu_{\Xi N}}\right)\Psi_\alpha^{\Xi N}(\mathbf{x}) = V^{\Xi N\leftarrow\Lambda\Lambda}(\mathbf{x})\Psi_\alpha^{\Lambda\Lambda}(\mathbf{x}) + V^{\Xi N\leftarrow\Xi N}(\mathbf{x})\Psi_\alpha^{\Xi N}(\mathbf{x})$$

off-diagonal   diagonal

μ: reduced mass

$$\begin{pmatrix} (E_1 - H_0^X)\Psi_1^X(\mathbf{x}) \\ (E_2 - H_0^X)\Psi_2^X(\mathbf{x}) \end{pmatrix} = \begin{pmatrix} \Psi_1^X(\mathbf{x}) & \Psi_1^Y(\mathbf{x}) \\ \Psi_2^X(\mathbf{x}) & \Psi_2^Y(\mathbf{x}) \end{pmatrix} \begin{pmatrix} V^{X\leftarrow X}(\mathbf{x}) \\ V^{X\leftarrow Y}(\mathbf{x}) \end{pmatrix}$$

$X \neq Y$

$$E_\alpha = \frac{\mathbf{p}_\alpha^2}{2\mu_{\Lambda\Lambda}}, \quad \frac{\mathbf{q}_\alpha^2}{2\mu_{\Xi N}}$$

$X, Y = \Lambda\Lambda$ or ΞN

$$\Rightarrow \begin{pmatrix} V^{X\leftarrow X}(\mathbf{x}) \\ V^{X\leftarrow Y}(\mathbf{x}) \end{pmatrix} = \begin{pmatrix} \Psi_1^X(\mathbf{x}) & \Psi_1^Y(\mathbf{x}) \\ \Psi_2^X(\mathbf{x}) & \Psi_2^Y(\mathbf{x}) \end{pmatrix}^{-1} \begin{pmatrix} (E_1 - H_0^X)\Psi_1^X(\mathbf{x}) \\ (E_2 - H_0^X)\Psi_2^X(\mathbf{x}) \end{pmatrix}$$

Using the potentials:

$$\begin{pmatrix} V^{\Lambda\Lambda\leftarrow\Lambda\Lambda}(\mathbf{x}) & V^{\Xi N\leftarrow\Lambda\Lambda}(\mathbf{x}) \\ V^{\Lambda\Lambda\leftarrow\Xi N}(\mathbf{x}) & V^{\Xi N\leftarrow\Xi N}(\mathbf{x}) \end{pmatrix}$$

we solve the coupled channel Schroedinger equation in the infinite volume with an appropriate boundary condition.

For example, we take the incomming $\Lambda\Lambda$ state by hand.

In this way, we can avoid the mixture of several "in"-states.

$$|S=-2, I=0, E\rangle_L = c_1(L)|\Lambda\Lambda, E\rangle + c_2(L)|\Xi N, E\rangle + c_3(L)|\Sigma\Sigma, E\rangle$$

Lattice is a tool to extract the interaction kernel ("T-matrix" or "potential").

Preliminary results from HAL QCD Collaboration

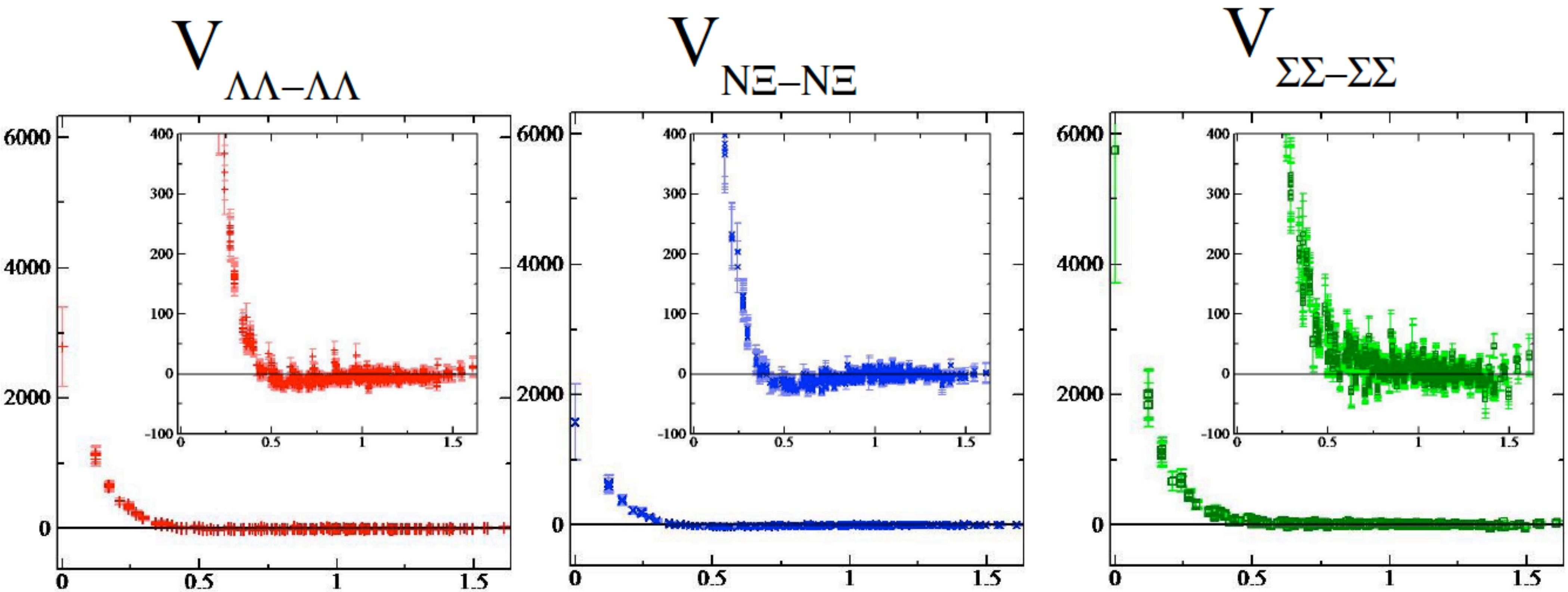
Sasaki for HAL QCD Collaboration

2+1 flavor full QCD

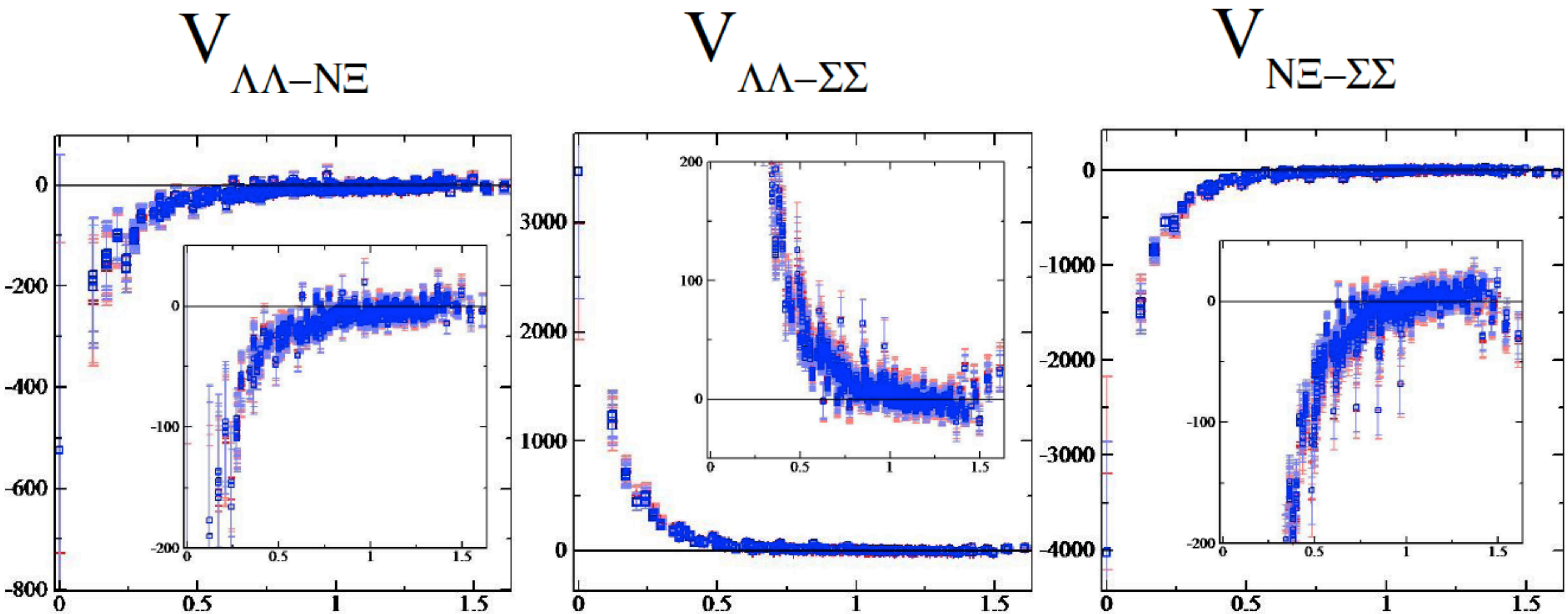
a=0.1 fm, L=2.9 fm

$m_\pi \simeq 870 \text{ MeV}$

Diagonal part of potential matrix



Non-diagonal part of potential matrix



$$V_{A-B} \simeq V_{B-A}$$

Hermiticity ! (non-trivial check)

4-3. H-dibaryon

1. S=-2 singlet state become the bound state in flavor SU(3) limit.

2. In the real world (s is heavier than u,d), some resonance appears above ΛΛ but below ΞN threshold.

3. We can check this scenario using the lattice QCD.

   3.1. The potential in SU(3) limit

   3.2. The 3 x 3 potential matrix in real world

4. Trial demonstration:

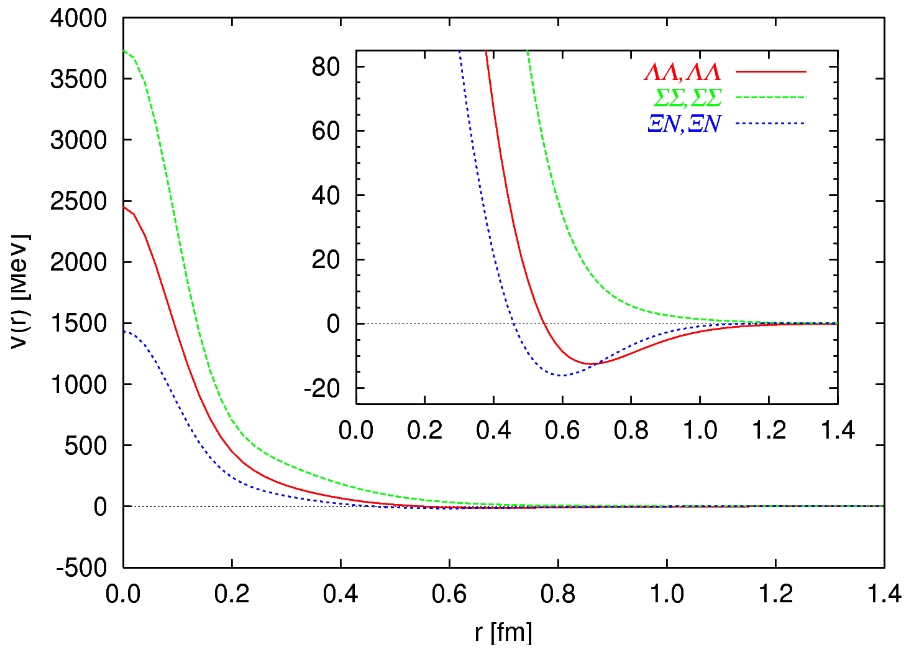
   Inoue for HAL QCD Collaboration

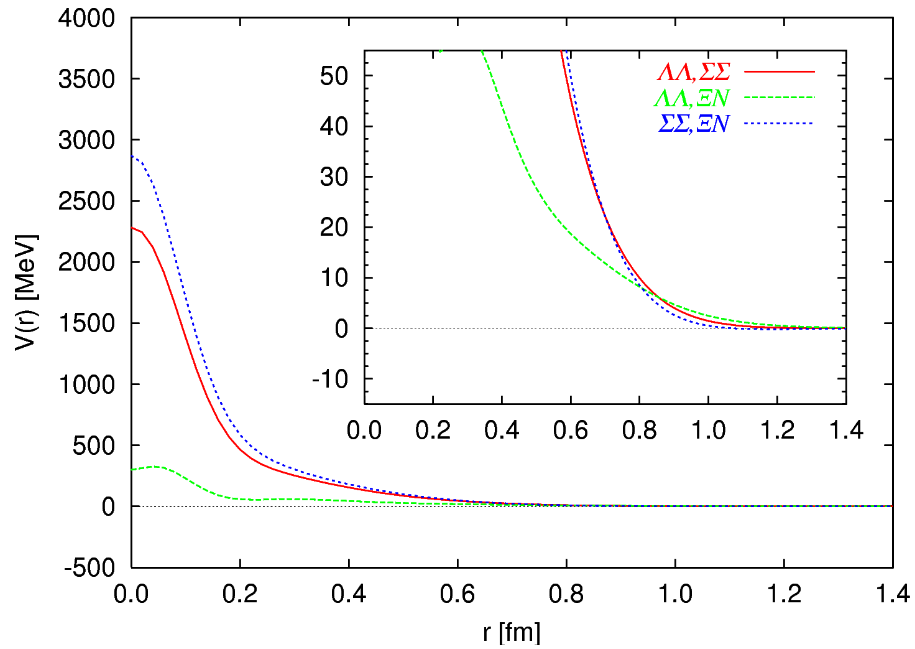
   4.1. Use potential in SU(3) limit

   4.2. Introduce only mass difference from 2+1 simulation

Potentials in particle basis in SU(3) limit

$$\begin{pmatrix} \Lambda\Lambda \\ \Sigma\Sigma \\ \Xi N \end{pmatrix} = U \begin{pmatrix} |27\rangle \\ |8\rangle \\ |1\rangle \end{pmatrix}, \quad U \begin{pmatrix} V^{(27)} & & \\ & V^{(8)} & \\ & & V^{(1)} \end{pmatrix} U^t \longrightarrow \begin{pmatrix} V^{\Lambda\Lambda} & V^{\Lambda\Lambda}_{\Sigma\Sigma} & V^{\Lambda\Lambda}_{\Xi N} \\ & V^{\Sigma\Sigma} & V^{\Sigma\Sigma}_{\Xi N} \\ & & V^{\Xi N} \end{pmatrix}$$



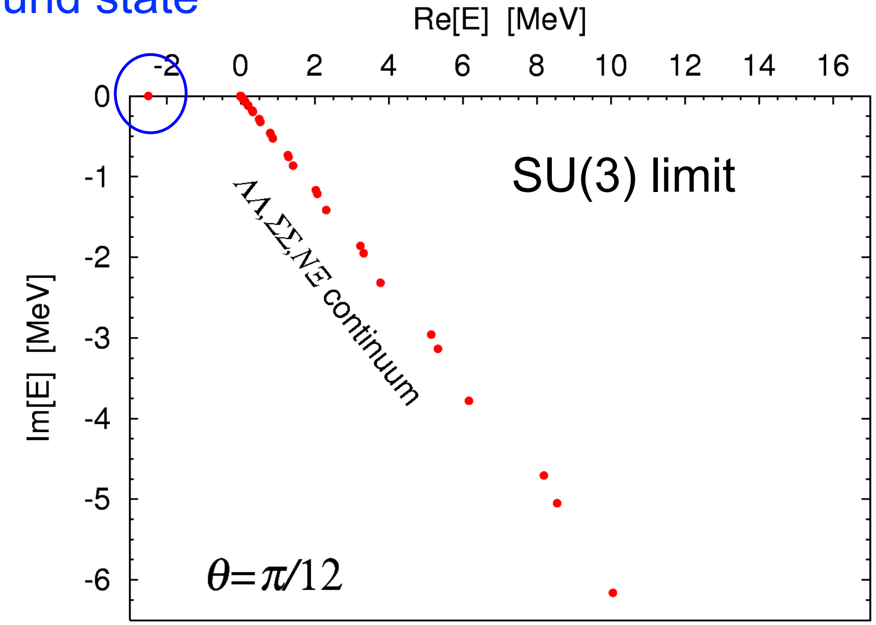


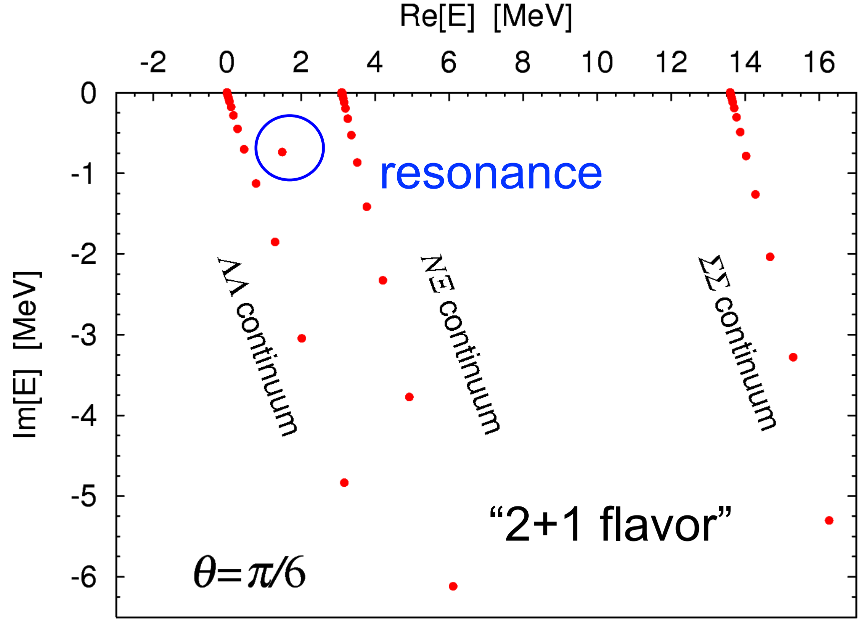
where $T_0^{(1)} = -25$, $T_0^{(8)} = 25$, $T_0^{(27)} = -5$ [MeV] are used

2

$S = -2, I = 0, {}^1S_0$ scattering $\quad E^{(1)} = -40$ MeV

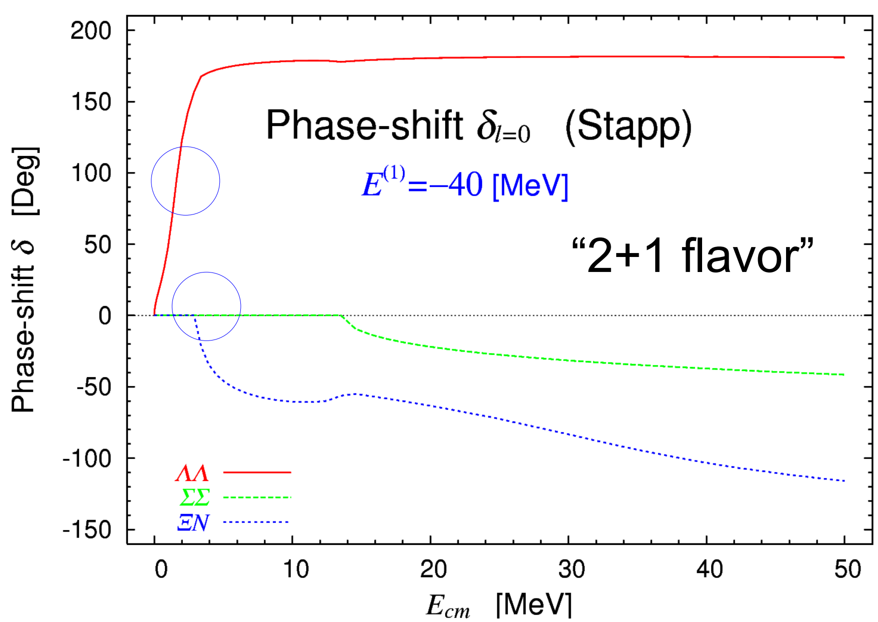
bound state

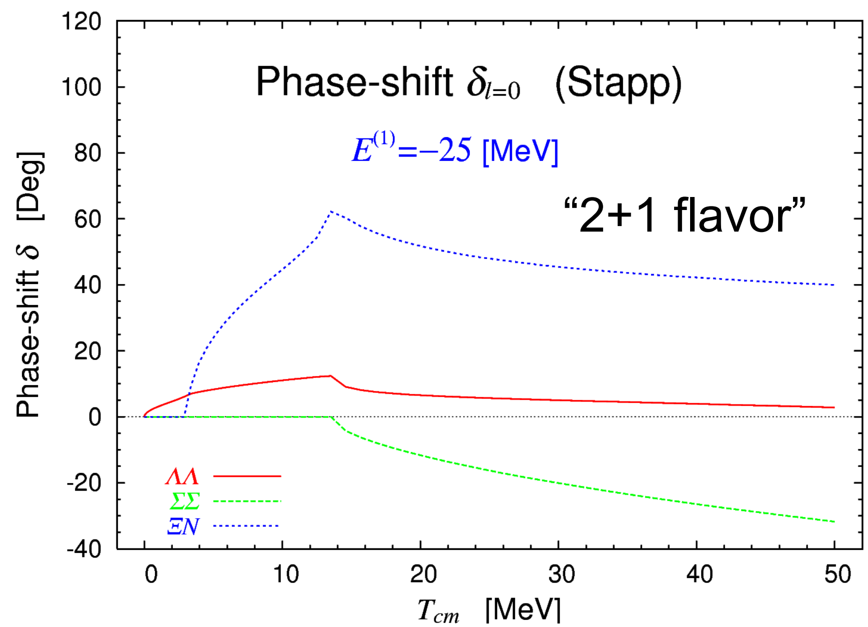




resonance

no resonance





5. New method for hadron interactions in lattice QCD

Inelastic scattering II: particle production

$E \geq E_{th} = 2m_N + m_\pi$

elastic scattering $NN \leftarrow NN$

NBS wave function

$$\varphi_E(\mathbf{r}) = e^{i\mathbf{k}\cdot\mathbf{r}} + \int \frac{d^3p}{(2\pi)^3} e^{i\mathbf{p}\cdot\mathbf{r}} \frac{E_k + E_p}{8E_p^2} \frac{T(\mathbf{p}, -\mathbf{p} \leftarrow \mathbf{k}, -\mathbf{k})}{\mathbf{p}^2 - \mathbf{k}^2 - i\epsilon}$$
$$+ \mathcal{I}(\mathbf{r})$$

inelastic contribution $NN\pi \leftarrow NN$ $\propto e^{i\mathbf{q}\cdot\mathbf{r}}$ $|\mathbf{q}| = O(E - E_{th})$

Consider additional NBS wave function

$$\varphi_{E,\pi}(\mathbf{r}, \mathbf{y}) = \langle 0 | N(\mathbf{r} + \mathbf{x}, 0) \pi(\mathbf{y} + \mathbf{x}, 0) N(\mathbf{x}, 0) | 6q, E\rangle$$

Note that

$$|6q, E\rangle = c_1 |NN, E\rangle_{\rm in} + c_2 |NN\pi, E\rangle_{\rm in} + \cdots$$

Coupled channel equations

$$
\begin{aligned}
(E-H_0)\varphi_E(\mathbf{x}) &= \int d^3y\, U_{11}(\mathbf{x};\mathbf{y})\varphi_E(\mathbf{y}) + \int d^3y d^3z\, U_{12}(\mathbf{x};\mathbf{y},\mathbf{z})\varphi_{E,\pi}(\mathbf{y},\mathbf{z}) \\
(E-H_0)\varphi_{E,\pi}(\mathbf{x},\mathbf{y}) &= \int d^3z\, U_{21}(\mathbf{x},\mathbf{y};\mathbf{z})\varphi_E(\mathbf{z}) + \int d^3z d^3w\, U_{22}(\mathbf{x},\mathbf{y};\mathbf{z},\mathbf{w})\varphi_{E,\pi}(\mathbf{z},\mathbf{w})
\end{aligned}
$$

Velocity expansion at LO, two values of E

$i = 1, 2$

$$
\begin{aligned}
(E_i-H_0)\varphi_{E_i}(\mathbf{x}) &= V_{11}(\mathbf{x})\varphi_{E_i}(\mathbf{x}) + V_{12}(\mathbf{x},\mathbf{x})\varphi_{E_i,\pi}(\mathbf{x},\mathbf{x}) \\
(E_i-H_0)\varphi_{E_i,\pi}(\mathbf{x},\mathbf{y}) &= V_{21}(\mathbf{x},\mathbf{y})\varphi_{E_i}(\mathbf{x}) + V_{22}(\mathbf{x},\mathbf{y})\varphi_{E_i,\pi}(\mathbf{x},\mathbf{y})
\end{aligned}
$$

$V_{11}(\mathbf{x}) : NN \leftarrow NN$ $\quad$ $V_{12}(\mathbf{x},\mathbf{x}) : NN \leftarrow NN\pi$

$V_{21}(\mathbf{x},\mathbf{y}) : NN\pi \leftarrow NN$ $\quad$ $V_{22}(\mathbf{x},\mathbf{y}) : NN\pi \leftarrow NN\pi$

Solve Schroedinger equation with these potentials and a specific B.C.

General prescription

- Consider a QCD eiegnstate with given quantum numbers Q and energy E.
- Take all possible combinations with Q of stable particles whose threshold is below or near E.

$$\text{ex. } Q = 6q : \ NN, NN\pi, NN\pi\pi, NNK^+K^-, NN\bar{N}N, \cdots$$

- Calculate NBS wave functions for all combinations.
- Extract coupled-channel potentials in a finite volume.
- Solve Schroedinger equation with these potentials in the infinite volume with a suitable B.C. to obtain physical observables.

In practice, of course, final states more than 2 particles are very difficult to deal with.

6. Summary and Discussion

Summary

- Potentials from NBS wave function are useful tools to extract hadron interactions in lattice QCD. Finite size effect is smaller and quark mass dependence is milder than the phase shift.
  - Velocity expansion is needed. Validity can be checked.(Murano)
  - Combined with Schroedinger equation in the infinite box. Rotational symmetry is recovered.
  - NN, tensor force; NY,YY (Nemura); SU(3) limit (Inoue)

    Nemura-Ishii-Aoki-Hatsuda, PLB673(2009)136.

    Inoue *et al.*(HAL QCD), arXiv:1007.3559.
  - Others: N-η_c (Kawanai-Sasaki), p-K^+ (Ikeda)

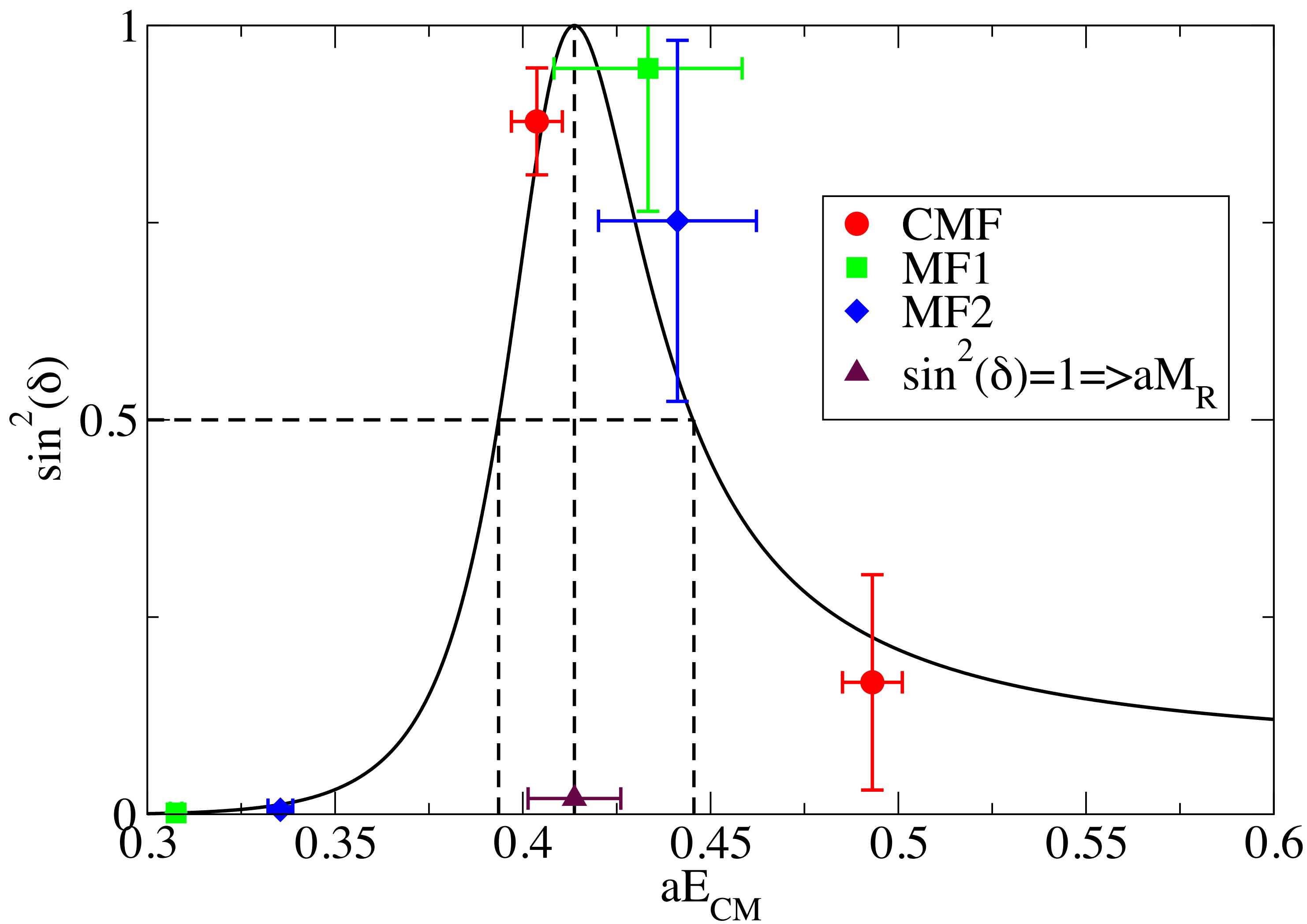
    Ikeda *et al.*(HAL QCD), arXiv:1002.2309.
- Inelastic scattering can also be analysed in terms of coupled channel "potentials".
  - ΛΛ scattering (Sasaki), H-dibaryon as a resonance

Applications and extensions

- unstabel particle as a resonace
  - ρ meson, Δ, Roper etc.
  - exotic: penta-quark (Ikeda), X, Y etc.
- Parity odd part of potentials, LS force (Murano, Ishii)
- 3-Baryon forces : NNN (Doi) , BBB-> Neutron star
- Theoretical understanding of the repulsive core
  - OPE analysis + pQCD+RG Aoki-Balog-Weisz, JHEP05(2010)008(Nf=2); arXiv:1007.4117 (Nf=3).
  - AdS/QCD Hashimoto-Iizuka-Yi, arXiv:1003.4988
- Weak decay ?

$\pi^+\pi^-$ scattering (ρ meson width)

Finite volume method



ETMC: Feng-Jansen-Renner, PLB684(2010)