Kaon Physics from Lattice QCD

CERN Theory Colloquium

July 28, 2010

Norman H. Christ

RBC/UKQCD Collaborations

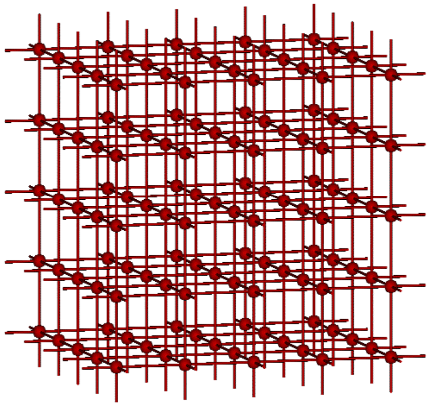
Outline

- Introduction
- RBC/UKQCD physics program
- Domain wall fermions
- Low energy QCD
- $K^0 - \overline{K}^0$ mixing
- $K \rightarrow \pi\,\pi$ decay

Introduction

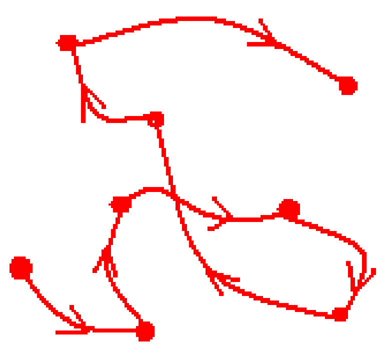
Lattice QCD

- Regulate using a space-time lattice.
- Evaluate Euclidean Feynman path integral numerically.
  - **Precise non-perturbative formulation.**
  - **Potential numerical errors.**



$$\sum_n \langle n | e^{-Ht} \mathcal{O} | n \rangle = \int d[U_\mu(n)] e^{-\mathcal{A}[U]_{\text{gauge}}} \det(D+m) \mathcal{O}[U]$$

$$\det(D+m) = \int d[\phi] d[\phi^*] e^{-\phi^\dagger \frac{1}{(D+m)} \phi}$$



- Evaluate using Monte Carlo methods with hybrid molecular dynamics + Langevin evolution.

Development of Lattice methods

- Introduced by Wilson in 1973
- 1^{st} numerical evaluation by Creutz 1979.
- Driven by spectacular technological progress:

10^7 x

VAX 780 (1984)
1 Mflops (10^6)

BG/P (2008)
10 Tflops (10^{13})

- Matching algorithmic innovation
- Requires large human and computer resources

UKQCD Collaboration

- Edinburgh
  - Rudy Arthur
  - Peter Boyle
  - Luigi del Debbio
  - Nicolas Garron
  - Chris Kelly
  - Tony Kennedy
  - Richard Kenway
  - Chris Maynard
  - Brian Pendleton
  - James Zanotti
- Southampton
  - Dirk Brommel
  - Jonathan Flynn
  - Patrick Fritzsch
  - Elaine Goode
  - Chris Sachrajda

RBC Collaboration

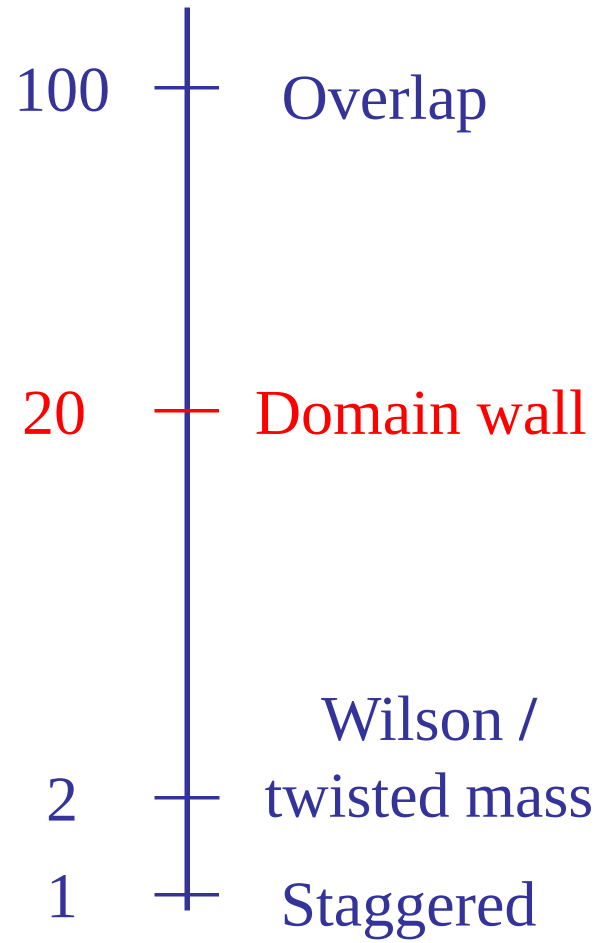
- Columbia
  - Norman Christ
  - Michael Endres
  - Xiao-Yong Jin
  - Matthew Lightman
  - Meifeng Lin (Yale)
  - Qi Liu
  - Robert Mawhinney
  - Hao Peng
  - Dwight Renfrew
  - Hantao Yin

- RBRC
  - Yasumichi Aoki
  - Tom Blum (Connecticut)
  - Saumitra Chowdhury (Connecticut)
  - Chris Dawson (Virginia)
  - Tomomi Ishikawa (Connecticut)
  - Taku Izubuchi (BNL)
  - Christopr Lehner
  - Shigemi Ohta (KEK)
  - Eigo Shintani
  - Ran Zhou (Connecticut)

- BNL
  - Michael Creutz
  - Shinji Ejiri
  - Prasad Hegde
  - Chulwoo Jung
  - Frithjof Karsch
  - Swagato Mukherjee
  - Chuan Miao
  - Peter Petreczky
  - Amarjit Soni
  - Ruth Van de Water
  - Alexander Velytsky
  - Oliver Witzel

RBC – UKQCD
Research Program

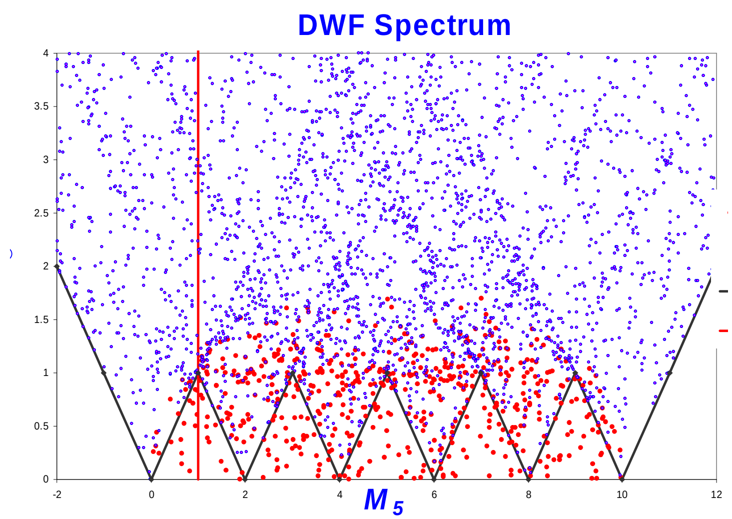
- Use domain wall fermions
- Generate a few large N_f=2+1 ensembles
- Focus on low-energy chiral physics
- Exploit off-shell chiral symmetry to control operator mixing for K^0–$\overline{K}^0$ mixing and $\Delta S = 1$, K decays
- Many other projects
  - Nucleon form factors/structure functions
  - QCD thermodynamics
  - Heavy quark physics
  - Neutron EDM
  - E&M simulations:
  - g_μ-2, ….



Domain Wall Fermions

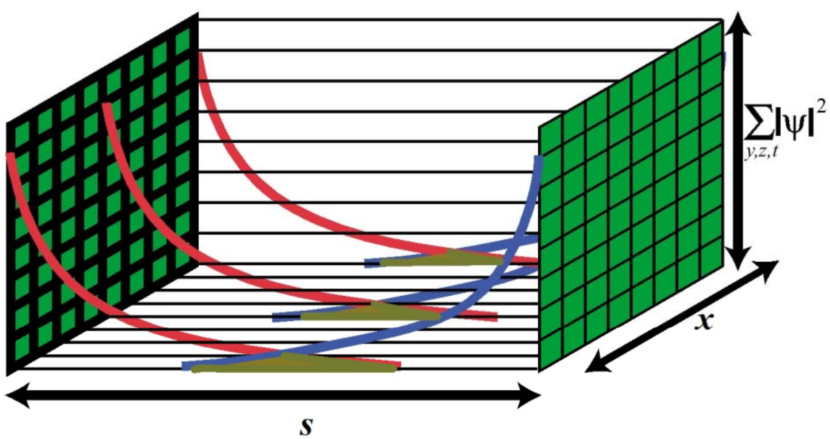
Domain Wall Fermions

- Invented by Kaplan, 1993.
- 5-D theory with 4-D, chiral surface states.
- Typical 5-D extent of 16.
- $L_s \rightarrow \infty$ gives the overlap operator of Neuberger.



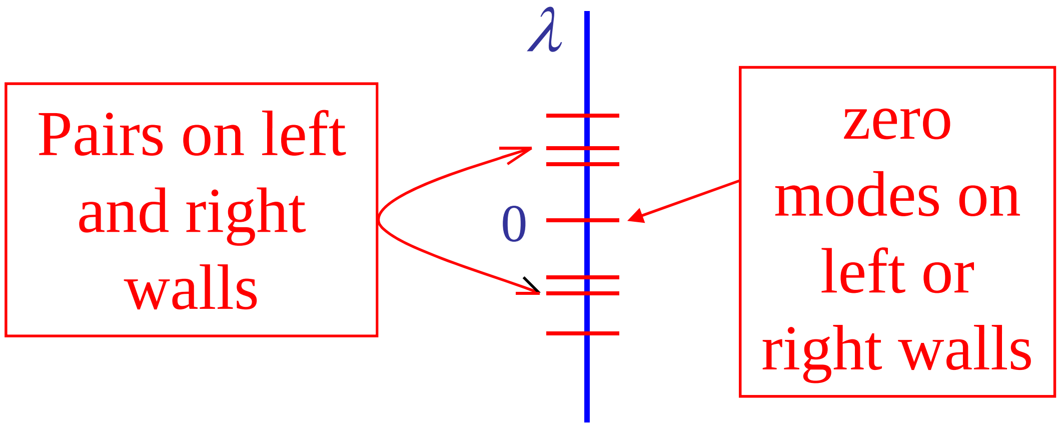
5-D mass

Simulations run at $M_5 = 1.8$



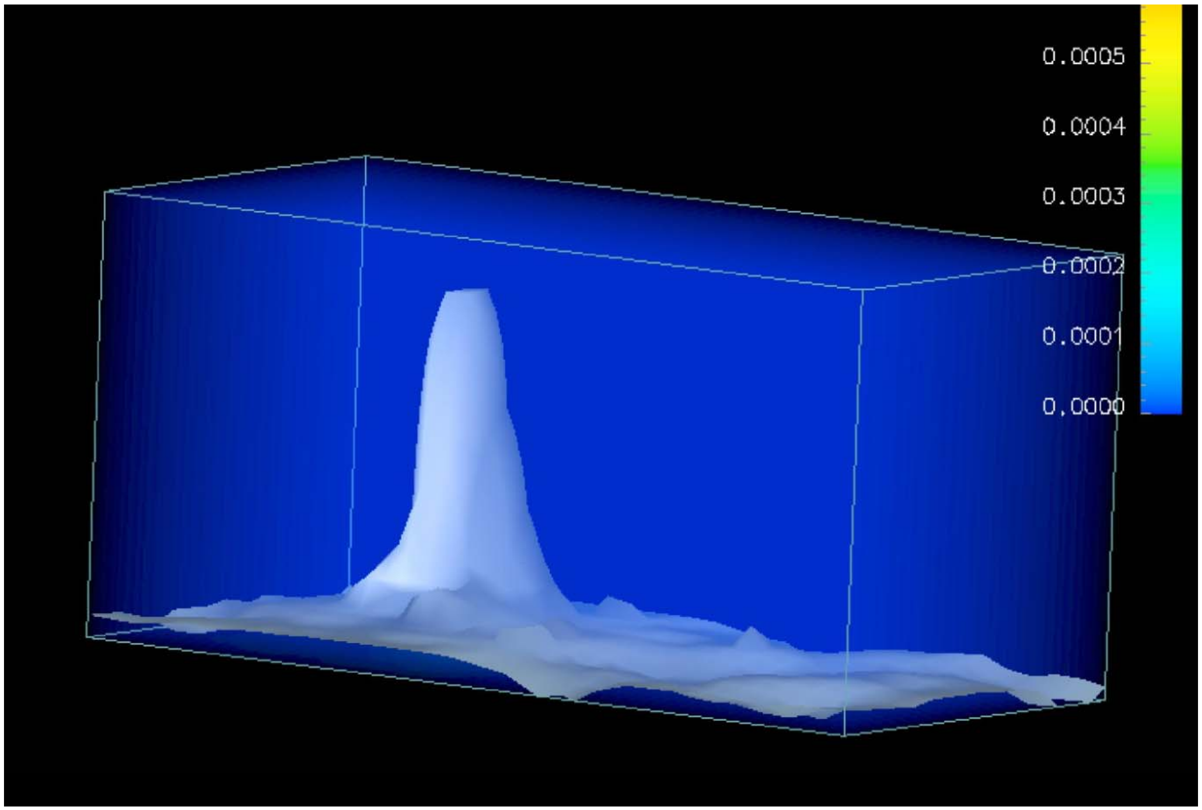
The Ghost of Doubling Problem

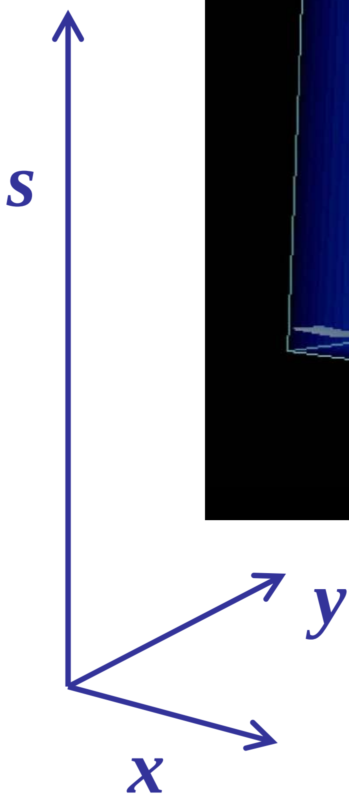
- For the Dirac operator, eigenvalues are paired except for zero modes:



- If the Pontryagin index changes, all modes must mix between left and right walls.
- Tearing the gauge field must violate chirality!

Local chirality violation





Peter Boyle

Lattice Chiral Symmetry Breaking

- For $L_s < \infty$ the right and left states can mix.
- Gives "residual" mass, $m_{\rm res}$, plus higher dimension operators:

$$\mathcal{L}_{\rm eff} = \overline{\psi}\{D^\mu \gamma^\mu + m\}\psi + m_{\rm res}\overline{\psi}\psi + c_{\rm SW}\overline{\psi}\sigma^{\mu\nu}\psi F^{\mu\nu}$$

- Both $m_{\rm res}$ and $c_{\rm SW}$ decrease rapidly as L_s grows or as $g^2 \to 0$:

$$m_{\rm res}(L_s) = c_1 \frac{e^{-\lambda_c L_s}}{L_s} + c_2 \frac{1}{L_s}$$

Standard 5-D states with $\lambda \sim$ lattice cutoff.

Localized states created by changing topology.

Monte Carlo Ensembles

- RBC/UKQCD gauge ensembles:

Volume	$1/a$	L	m_π	Time units	$m_{\rm quark}a$	Gauge Action
24^3 x 64	1.73 GeV	2.7 fm	315 MeV	9000	0.005+0.0032	Iwasaki
			402 MeV	9000	0.01+0.0032	
32^3 x 64	2.28 GeV	2.7 fm	290 MeV	7000	0.004+0.0006	
			350 MeV	8000	0.006+0.0006	
			410 MeV	6000	0.008+0.0006	
32^3 x 64	1.4 GeV	4.5 fm	180 MeV	1000	0.001+0.0018	Iwasaki + DSDR
			250 MeV	1800	0.004+0.0018	

Low Energy QCD

Compare with ChPT

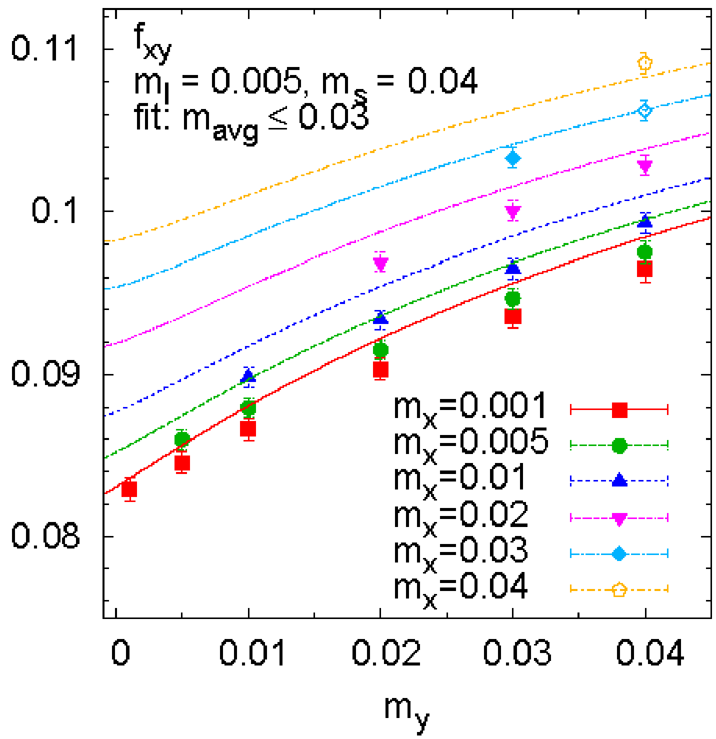
- "Measure" pseudo-scalar decay constant and mass:

  $f_{xy}(m_x,m_y,m_l,m_h)$ and $m_{xy}(m_x,m_y,m_l,m_h)$
  - Valence quark masses: m_x and m_y
  - Sea quark masses: m_l and m_h
- Compare with SU(N_f) x SU(N_f) PQChPT
  - $N_f = 3$ f_0 and $B_0 + L^{(3)}_{4,5,6,8}$
  - $N_f = 2$ f and $B + L^{(2)}_{4,5,6,8}$
- Use $1/a = 1.73$ GeV ensemble. Phys. Rev. D78:114509 (2008)

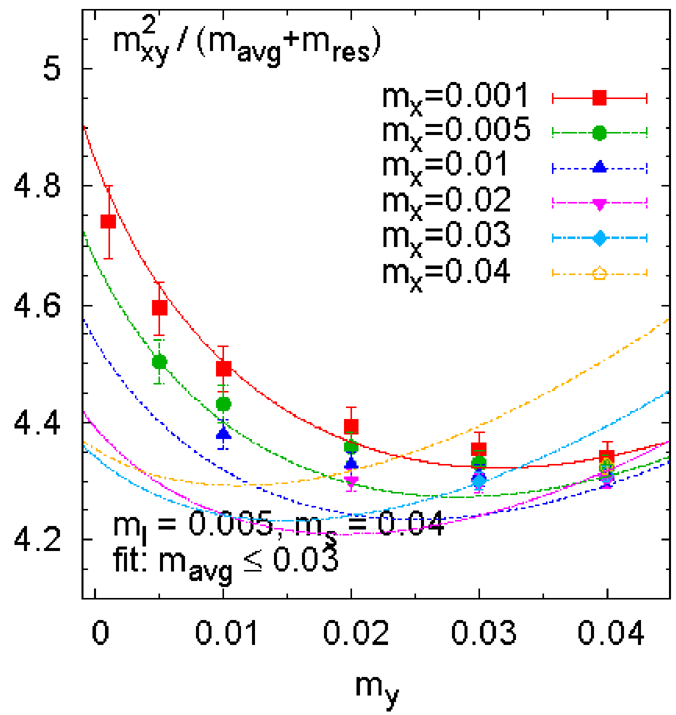
SU(3) x SU(3) fails at m_K

- Attempt to fit m_{xy} and f_{xy} for 242 MeV ≤ m_{xy} ≤ 653 MeV

f_{xy}



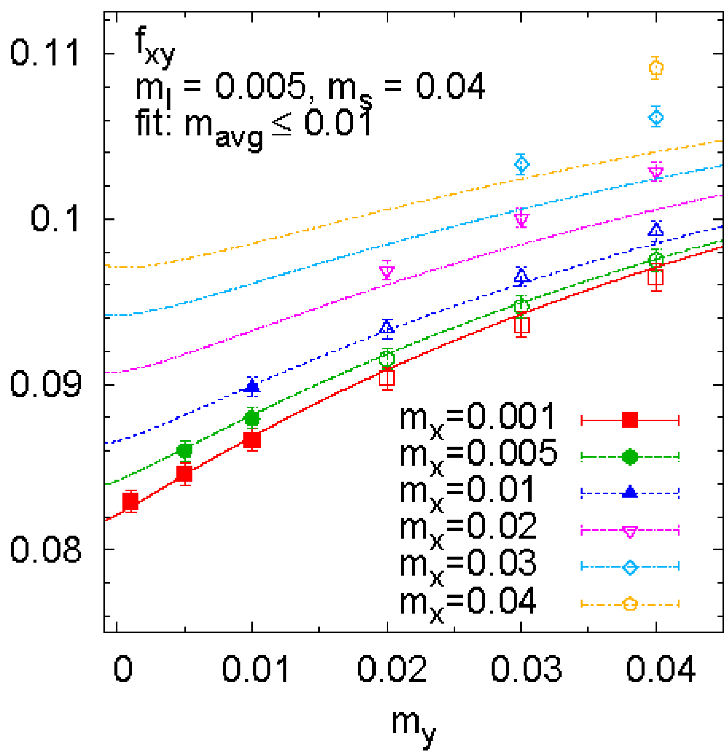
$m_{xy} /(m_{avg}+m_{res})$



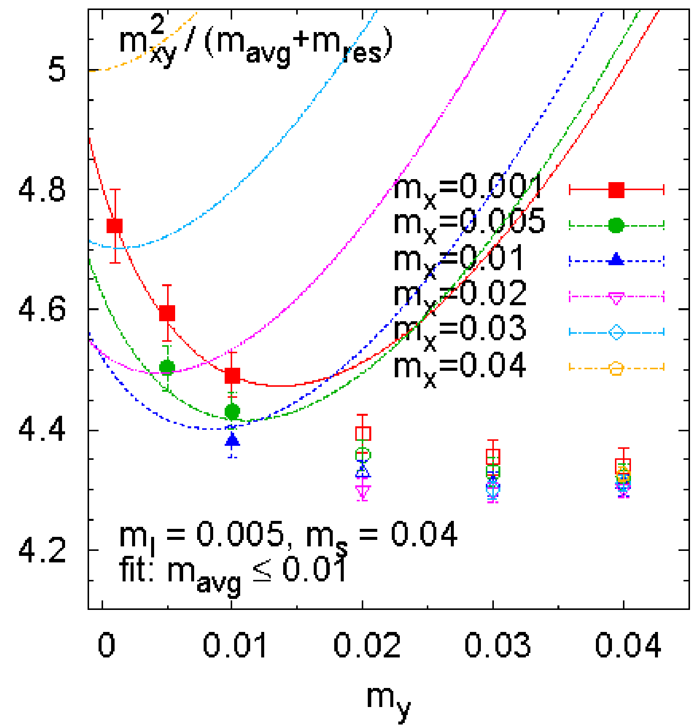
SU(2) x SU(2) is consistent

- Attempt to fit m_{xy} and f_{xy} for 242 MeV ≤ m_{xy} ≤ 414 MeV

f_{xy}

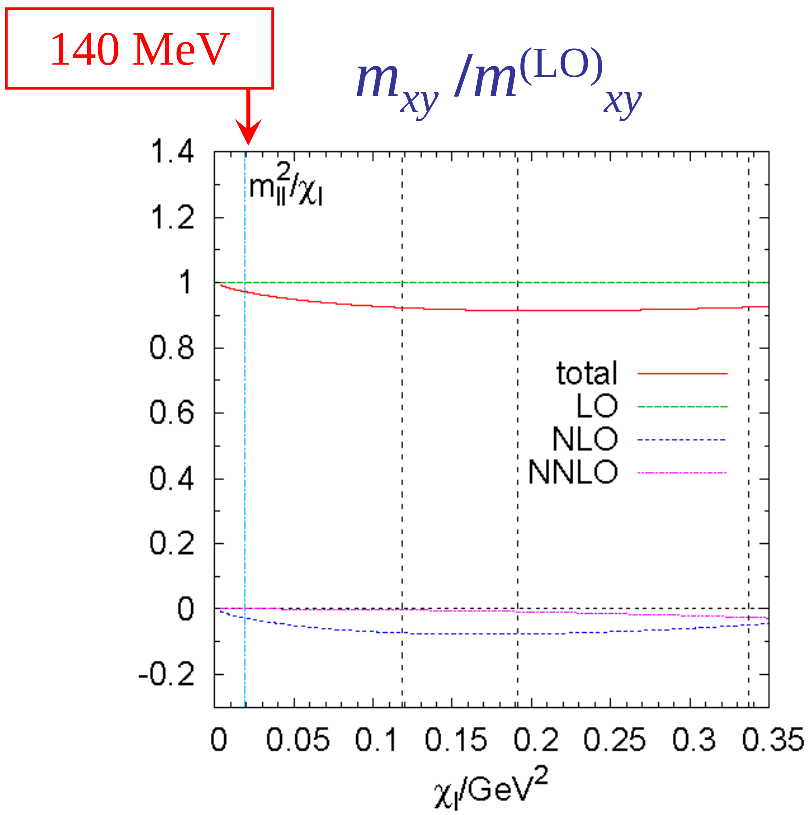


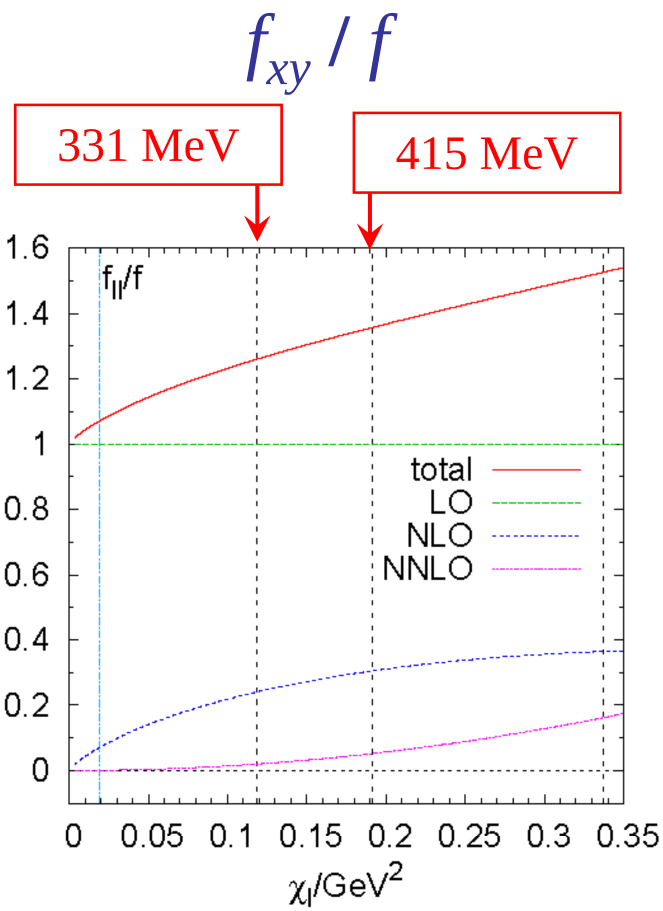
$m_{xy} /(m_{avg}+m_{res})$



Convergence of SU(2) x SU(2)

- Add 3 of 4 NNLO analytic terms; fit for 242 MeV ≤ m_{xy} ≤ 546 MeV



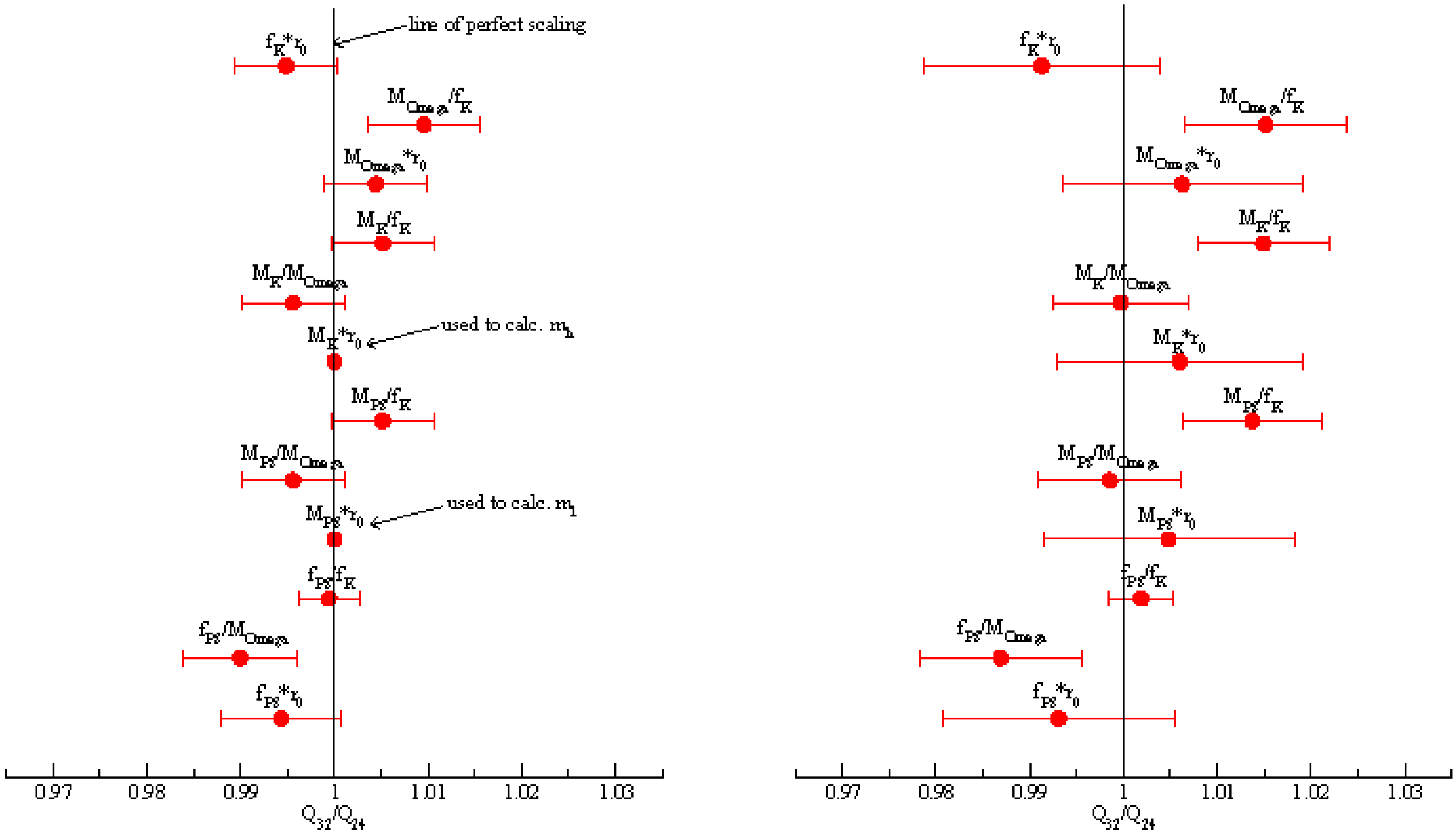


Include 2nd lattice spacing

- Previous discussion based on two 24^3 x 64, $1/a$=1.73 GeV ensembles
- Now include three 32^3 x 64, 1/a=2.28 GeV ensembles
- Extrapolate to the continuum limit:
  - Use m_Ω to set the scale
  - Use m_π and m_K to determine the quark masses
  - Use *scaling trajectories*: fixed m_π/m_Ω and m_K/m_Ω
  - Note: m_l^{RI}/m_h^{RI} will now vary with $O(a^2)$ errors along trajectory
- First examine scaling along unphysical trajectories

Scaling: 1.73 GeV (24^3) – 2.32 GeV (32^3)

(Chris Kelly)

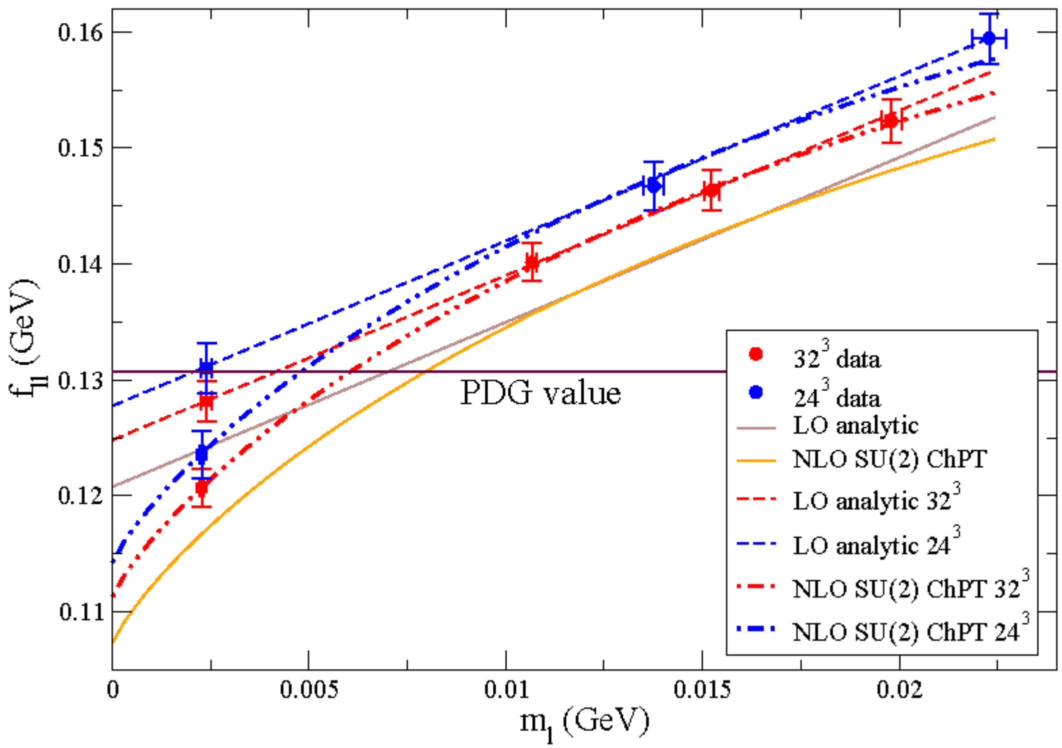


(c) $m_l^{24} = 0.005,\ m_h^{24} = 0.04$ (d) $m_l^{24} = 0.01,\ m_h^{24} = 0.04$

arXiv:0911.1309

Pseudo scalar decay constant

- Compare NLO ChPT and analytic fits



- Average ChPT and analytic: $f_\pi = 122.2\ (2)_{stat}(5)_{sys}$ MeV

Summary of preliminary results

f_π (MeV)	$122\ (2)_{stat}(5)_{sys}$	130.7(0.37) [expt]
f_K (MeV)	$147(2)_{stat}(4)_{sys}$	159.8(1.5) [expt]
f_π / f_K	$1.208(8)_{stat}(27)_{sys}$	1.223(12) [expt]
m_{ud} (MeV)	$3.65(20)_{stat}(13)_{sys}(8)_{ren}$	3.39(15) [HPQCD]
m_s (MeV)	$97.3(1.4)_{stat}(0.2)_{sys}(2.1)_{ren}$	92.2(3) [HPQCD]
r_0 (fm)	$0.4864\ (81)_{stat}(2)_{fv}(2)_\chi$	0.462(11)(4) [MILC]
r_1 (fm)	$0.3331\ (59)_{stat}(2)_{fv}(2)_\chi$	0.3117(6)(+12/-31) [MILC]
$\Sigma_{MS}^{1/3}$(MeV)	$251(4)_{stat}(2)_{ren}$	

Kaon weak matrix elements

Weak Interaction Matrix Elements

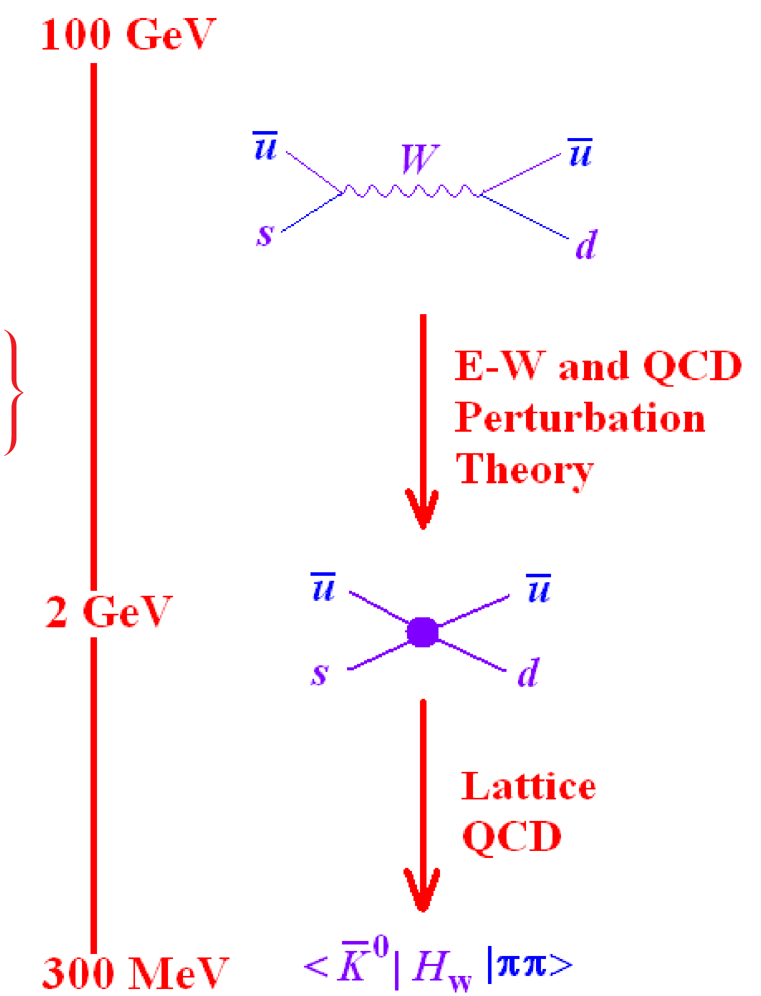
- K^0 - $\overline{K}^0$ mixing
  - CP violation: ε
- $K^0 \rightarrow \pi\,\pi$ decays
  - $\Delta I = 3/2,\ I_{\pi\pi} = 2$
  - $\Delta I = 1/2,\ I_{\pi\pi} = 0$
  - CP violation: ε'

Low Energy Effective Theory

- Represent weak interactions by local four-quark Lagrangian

$$\mathcal{H}^{(\Delta S=1)} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \left\{ \sum_{i=1}^{10} \left[z_i(\mu) - \frac{V_{td}}{V_{us}^*} \frac{V_{ts}^*}{V_{ud}} y_i(\mu) \right] Q_i \right\}$$

- $V_{qq'}$ – CKM matrix elements
- z_i and y_i – Wilson Coefficients
- Q_i – four-quark operators



$K^0 - \bar{K}^0$ mixing

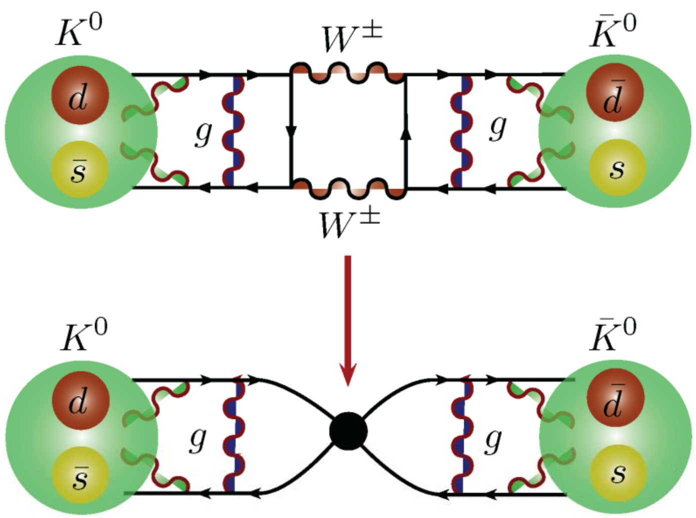
Indirect CP Violation

- CP violating phase of K^0 - $\overline{K^0}$ mixing amplitude specified by the CP odd parameter ε

$$\epsilon = \hat{B}_K \, \mathrm{Im}\lambda_t \, \frac{G_F^2 f_K^2 m_K M_W^2}{12\sqrt{2}\pi^2 \Delta M_K} \left\{ \mathrm{Re}\lambda_c \left[\eta_1 S_0(x_c) - \eta_3 S_0(x_c, x_t) \right] - \mathrm{Re}\lambda_t \, \eta_2 S_0(x_t) \right\} \exp(i\pi/4)$$

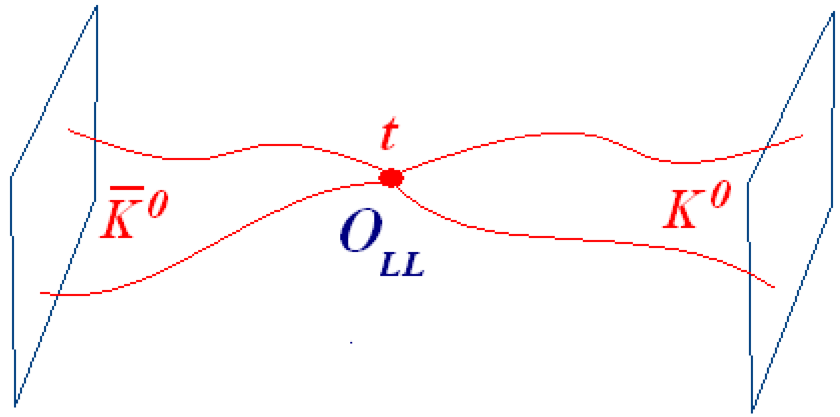
$$\langle \overline{K}^0 | Q^{(\Delta S=2)}(\mu) | K^0 | \rangle \equiv \frac{8}{3} B_K(\mu) f_K^2 m_K^2$$

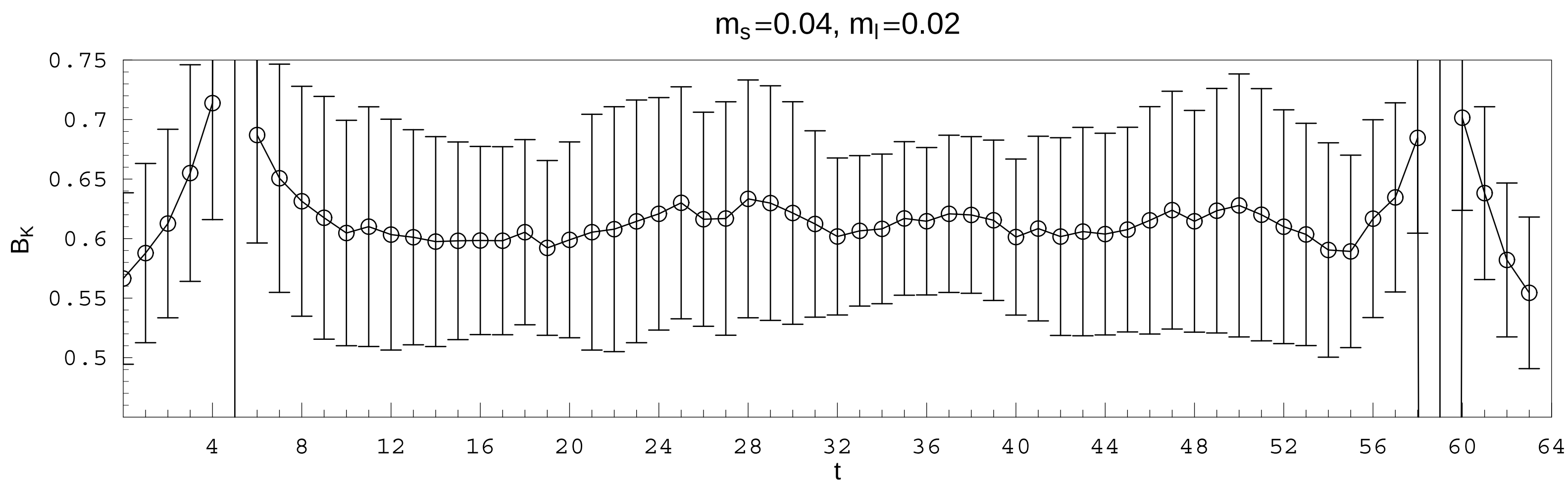
- $\lambda_k = V_{kd} \, V_{ks}{}^*$
- The matrix element B_K which can be only computed from lattice QCD.



Indirect CP Violation

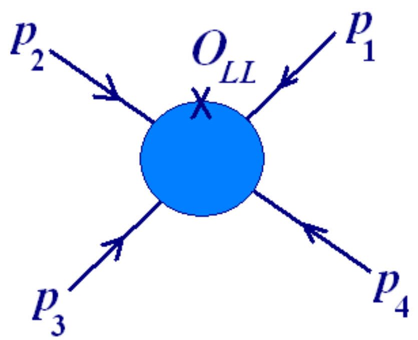
- O_{LL} matrix element:
  - K^0 on right
  - $\bar{K}^0$ on left
  - Operator at t

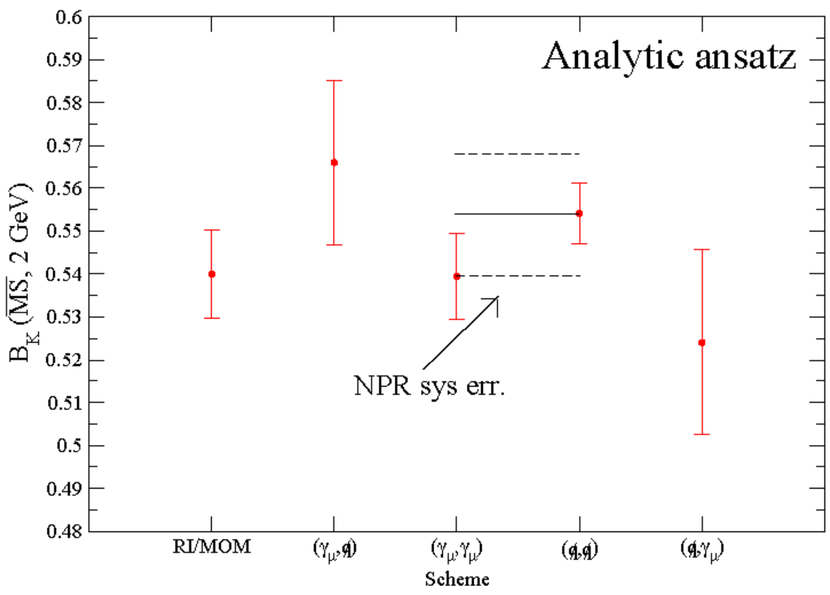




Operator normalization: Z_{BK}

- Use RI/MOM renormalization scheme (Rome/Southampton)
  - Fix to Landau gauge
  - Evaluate off-shell Green's functions
  - Impose momentum-space normalization condition: $\Gamma_{abcd}\,\Lambda_{abcd}(p_1, p_2, p_3, p_4)\,|_{\mu^2} = 1$
  - Use non-exceptional momenta
- Try 4 choices for Γ_{abcd} and chose the two that agree best with perturbative running:

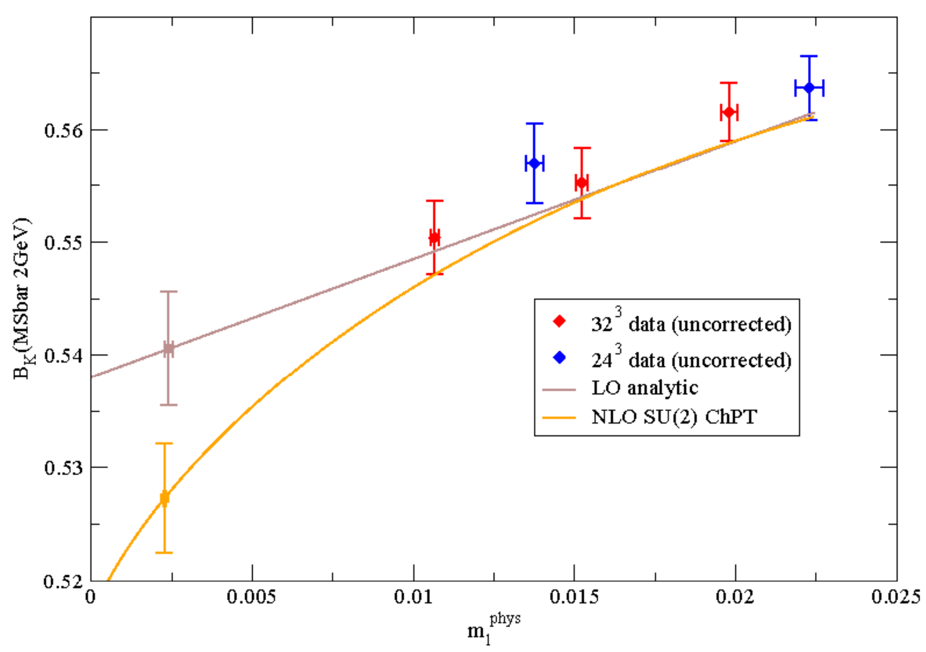




Continuum results

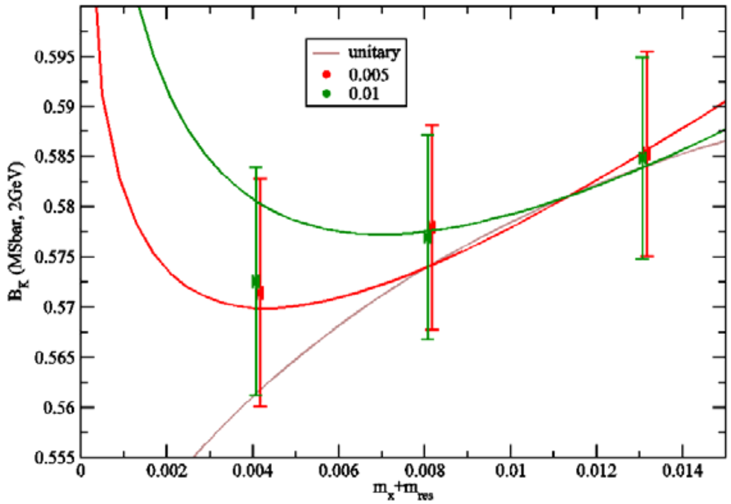
- Results from two lattice spacings
  - Small, 1-2% , $O(a^2)$ errors
  - $B_K = 0.524(10)_{\text{stat}}(28)_{\text{sys}}$ [PRL, 2008]

    → $B_K = 0.546(7)_{\text{stat}}(16)_{\chi}\ (3)_{\text{FV}}\ (14)_{\text{ren}}$ [preliminary]

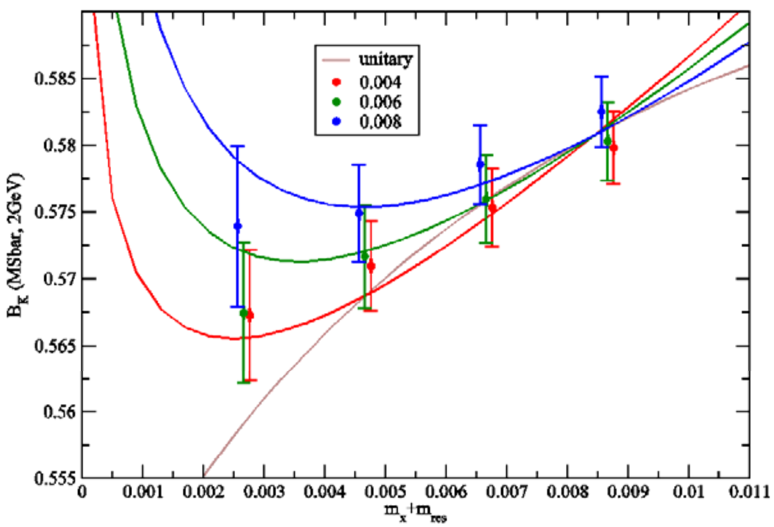


Case for ChPT weakens $24^3 \rightarrow 32^3$

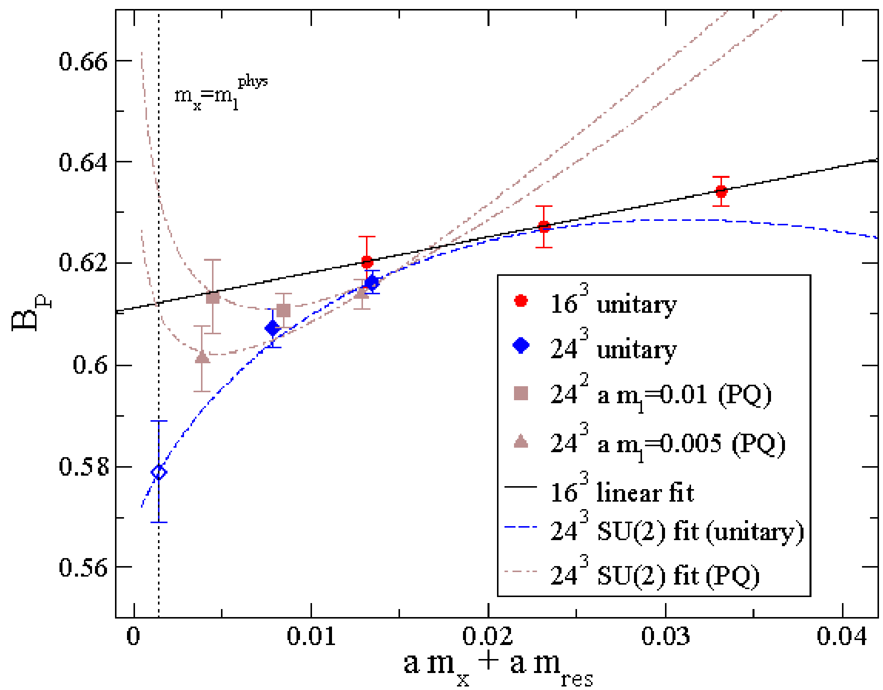
New 24^3



New 32^3



Old 24^3

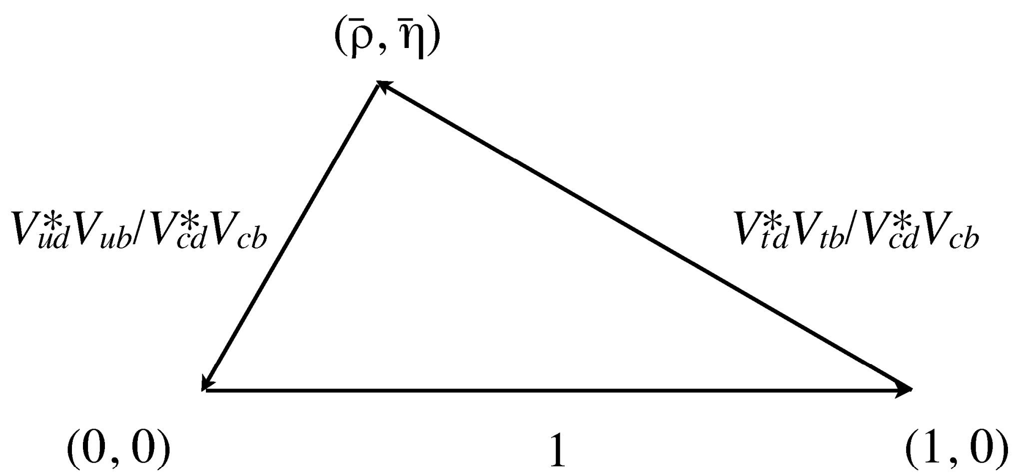


Comparison with Expt.

- Is the CKM matrix unitary?
- Check orthogonality of 1st and 3rd columns:

$$\boldsymbol{V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb} = 0}$$

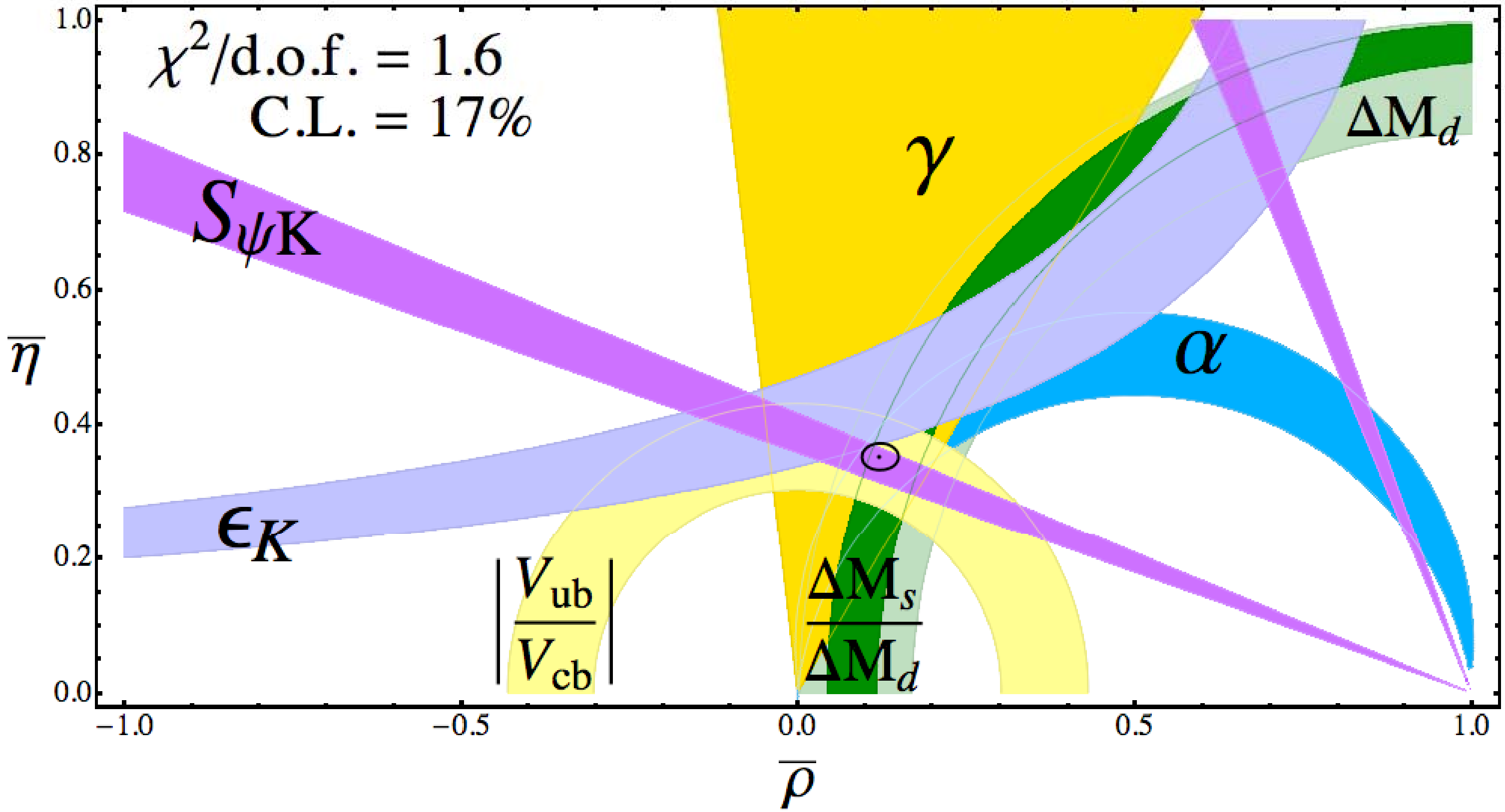
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



Comparison with Expt.

- Some tension between ε_K and other constraints:

Laiho, Lunghi, Van de Water, arXiv:0910.2928



Decrease the quark mass!

Decrease the quark mass

- Finite m_{quark} a larger problem than finite a
- Smaller m_π → larger volume → larger a

$$m_{\text{res}}(L_s) = c_1 \frac{e^{-\lambda_c L_s}}{L_s} + c_2 \frac{1}{L_s}$$

- Improve the action to suppress dislocations

Grows at stronger coupling and larger a.

Iwasaki + DSDR Action

- Use determinant ratio:

$$\frac{\det\{(D^{4D}_{\text{Wilson}} + M_5 - i\,\varepsilon_f\,\gamma^5)^\dagger\,(D^{4D}_{\text{Wilson}} + M_5 + i\,\varepsilon_f\,\gamma^5)\}}{\det\{(D^{4D}_{\text{Wilson}} + M_5 - i\,\varepsilon_b\,\gamma^5)^\dagger\,(D^{4D}_{\text{Wilson}} + M_5 + i\,\varepsilon_b\,\gamma^5)\}} = \prod_i \frac{\lambda_i^2 + \varepsilon_f^2}{\lambda_i^2 + \varepsilon_b^2}$$

  - $\mathbf{det\{}\boldsymbol{D^{4D}_{\text{Wilson}} + M_5}\mathbf{\}}$: Vranas GapDWF
  - $\boldsymbol{\varepsilon_b}$: JLQCD reduction of short distance disturbance.
  - $\boldsymbol{\varepsilon_f} \neq 0$: Allows topology change.
- Calculation begun in June 2009 ($\varepsilon_f = 0.02$ $\varepsilon_b = 0.5$)

Present status

$a \approx 0.14$ fm, $1/a \approx 1.4$ GeV, $m_{\rm res} \approx 0.0018$ (4 MeV)

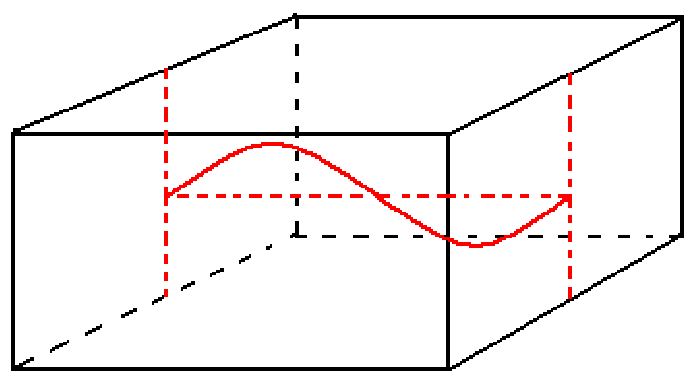
Volume	m_π (MeV)	m_l	Time units	$L\,m_\pi$
32^3 x 64	180	0.001	1000	4.2
32^3 x 64	250	0.0042	1800	5.7

- $L = 4.5$ fm
- Scaling errors $\leq$ 5%
- Partially quenched: $m_l = 0.0001$ → $\boldsymbol{m_\pi} = 145$ MeV
- Determine effect of lighter quarks on f_π , f_K , B_K ….
- **Study QCD Thermodynamics**
- **Study $K \to \pi\,\pi$ decays**

$K \to \pi\pi$ Decays

Calculate π - π final state directly

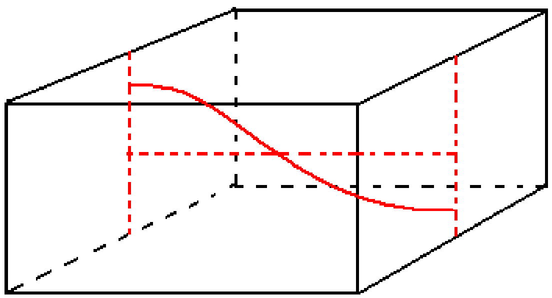
- SU(3) ChPT failure:
  Abandon $<K|H_W|\pi>$ & $<K|H_W|0>$ $\rightarrow$ $<K|H_W|\pi\pi>$
- Maiani-Testa theorem (1990):
  - Euclidean space methods use e^{-Ht} to project onto lowest energy state
  - For π - π state this state will have zero relative momentum
- Lellouch-Luscher method (2000):
  - Use finite-volume quantization
  - Adjust volume so 1st or 2nd excited state has correct p
  - Correctly include π - π interactions
  - Extra finite-volume normalization factor.

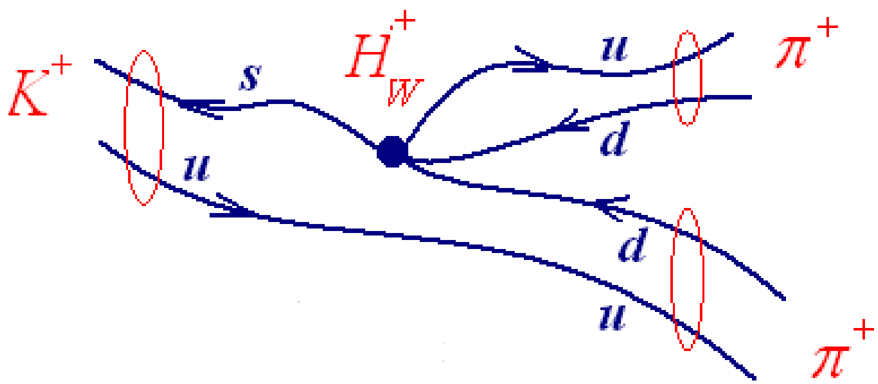


$p = 2\pi/L$

Δ I = 3/2 $K \rightarrow \pi\,\pi$

- I = 2 final state has no vacuum overlap.
- Use twisted boundary conditions (Changhoan Kim, hep-lat/0210003).
- I = 2 quantum number must be carried by four I=1/2 valence quarks.
  - Twist only valence quarks Sachrajda and Villadoro (hep-lat/0411033).
  - Safe to use slightly different valence and sea quark masses.





$\Delta I = 3/2\ \ K \rightarrow \pi\pi$

(Matthew Lightman and Elaine Goode)

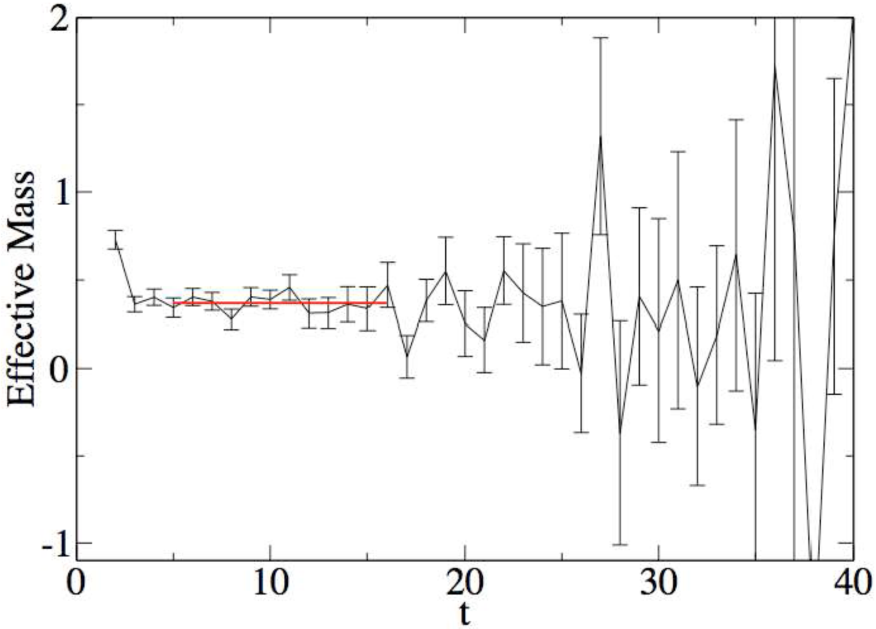
- Use 4.5 fm DSDR DWF ensembles.
  - m_π = 250 and 180 MeV
  - $1/a$ = 1.4 GeV
  - Finite a errors ≤ 5%.
- Use physical valence light quark mass.
  - Sea quark mass dependence of I=2, $K \rightarrow \pi\pi$ expected to be very small
  - m_{sea} = 0.008→0.004, < 3% (Lightman, arXiv:0906.1847 [hep-lat])
- Use anti-periodic boundary condition in two space directions (47 *configurations – preliminary*!)
  - $\boldsymbol{m_\pi}$ = 145.6(5) MeV
  - $\boldsymbol{m_K}$ = 519(2)
  - $\boldsymbol{E_{\pi\pi}}$ = 516(9) MeV

$\Delta I = 3/2 \;\; K \rightarrow \pi\pi$

(Matthew Lightman and Elaine Goode)

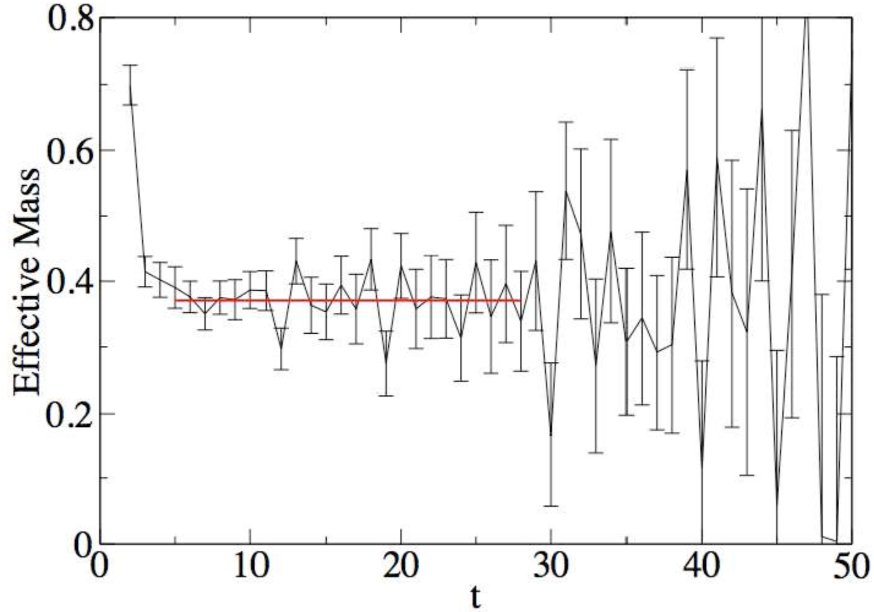
$\pi\pi$ and K effective mass: $m_{\text{eff}}(t) = \ln\big(C(t) \,/\, C(t+1) \big)$

$\pi\pi$ ($\mathrm{p} = \sqrt{2}\pi / L$)



$E_{\pi\pi}$ = 516(9) MeV

K



m_K = 519(2) MeV

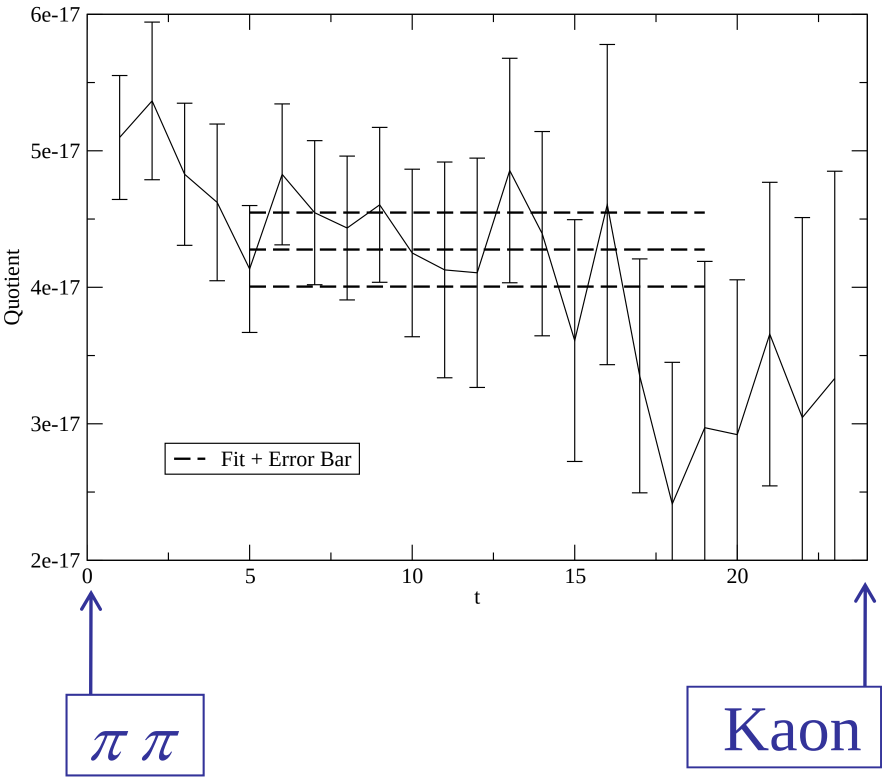
$< \pi\,\pi \,|\, O^{(27,1)} \,|\, K >$ from 47 configurations

(Matthew Lightman and Elaine Goode)

Quantity	This Calculation	Physical
m_π	145.6(5) MeV	139.6 MeV
m_K	519(2) MeV	493.7 MeV
$E_{\pi\pi}(p_\pi \approx 0)$	294(1) MeV	-
$E_{\pi\pi}(p_\pi \approx \sqrt{2}\pi/L)$	516(9) MeV	493.7 MeV
$E_{\pi\pi}(p_\pi \approx \sqrt{2}\pi/L) - m_K$	-2.7(8.3) MeV	0 MeV

$O^{(27,1)}$	0.000945(56)
$O^{(8,8)}$	0.0192(11)
$O^{(8,8)\mathrm{m}}$	0.0641(38)

$O^{(27,1)}$ Amplitude



Determine physical A_2

(Matthew Lightman and Elaine Goode)

- Recall $\langle \pi\pi(I=2)|\mathcal{L}_W(0)|K\rangle = A_2 e^{i\delta_2}$

$$A_2 = \frac{\sqrt{3}}{2\sqrt{2}} \frac{1}{\pi q_\pi} \sqrt{\frac{\partial \phi}{\partial q_\pi} + \frac{\partial \delta}{\partial q_\pi}} L^{3/2} a^{-3} G_F V_{ud} V_{us} \sqrt{m_K} E_{\pi\pi} \times \sum_{i,j} C_i(\mu) Z_{ij}(\mu) \langle \pi\pi | Q_j | K\rangle$$

- Re(A_2) dominated by single operator $O^{(27,1)}$.
- Determine Lellouch-Luscher factor.

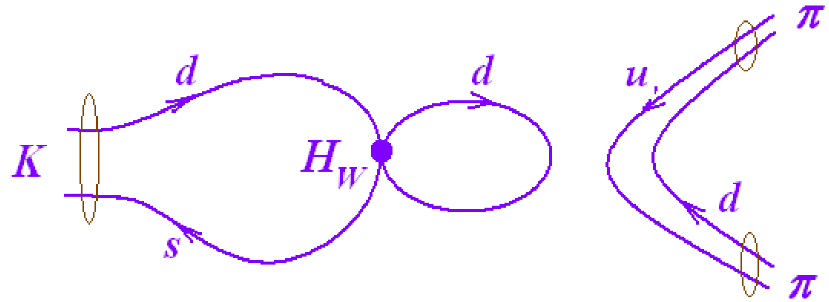
$$\frac{\partial \phi}{\partial q_\pi} = 5.141 \qquad \frac{\partial \delta}{\partial q_\pi} = 0.305$$

- **Re(A_2) = 1.56(7)$_{\text{stat}}$(25)$_{\text{sys}}$ 10^{-8} GeV** [Expt: 1.5 10^{-8} GeV]
- Im(A_2) equally easy, awaits NPR Z factors.

Δ I = 1/2

$\Delta I = 1/2 \; K \rightarrow \pi\pi$
(Qi Liu)

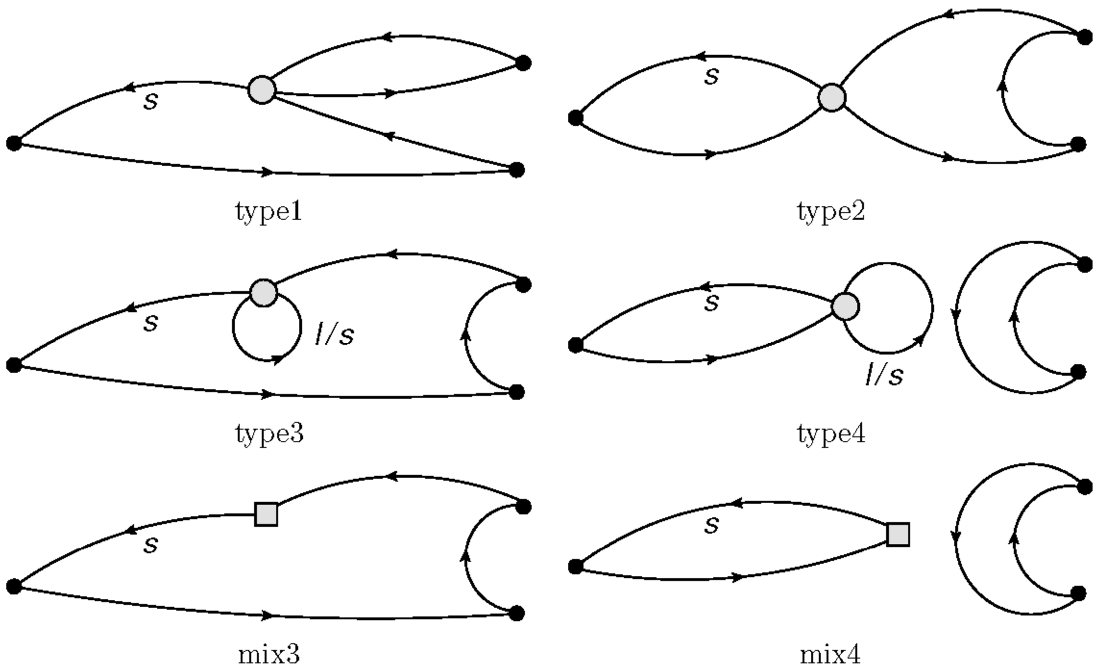
- Made much more difficult by disconnected diagrams:



- Experiment on 16^3 x 32 ensembles.
- $1/a$ = 1.73 GeV, m_π = 420 MeV, L = 1.8 fm
- Start with 4000 time units, measure on every 10.
- Adjust valence strange mass for on-shell, threshold kinematics ($\pi\pi$ state is unitary)

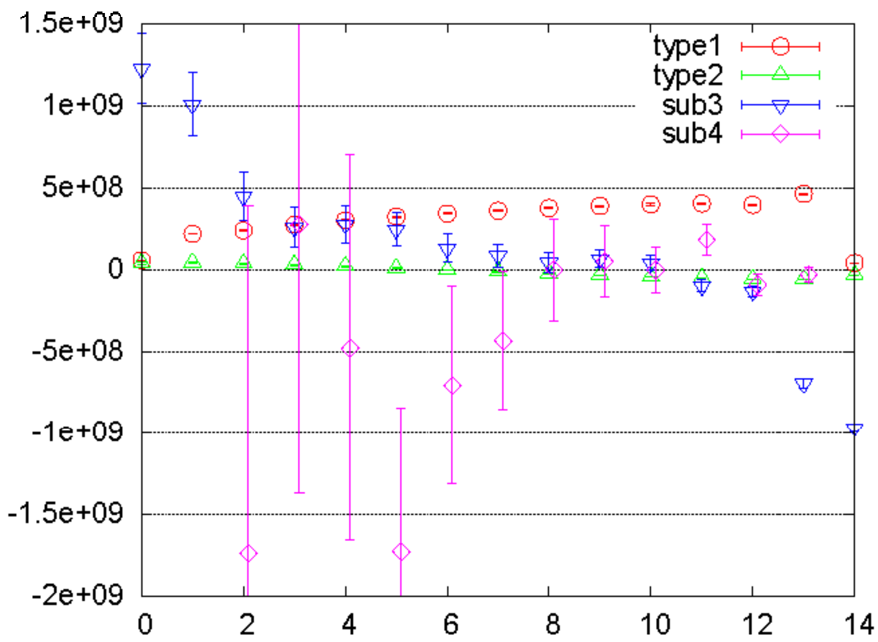
$\Delta I = 1/2\ K \rightarrow \pi\pi$
(Qi Liu)

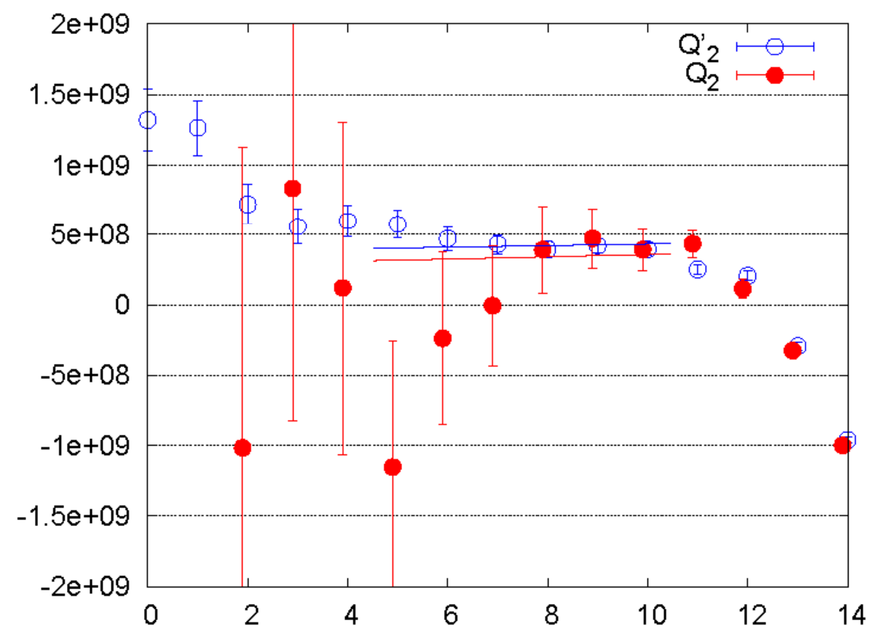
- Code 50 different contractions
- For each of 400 configurations invert with source at each of 32 times.
- Use Ran Zhou's deflation code



$\Delta I = 1/2$ $K \rightarrow \pi\pi$
(Qi Liu)

- Results for Q2, largest part of Re(A_0):

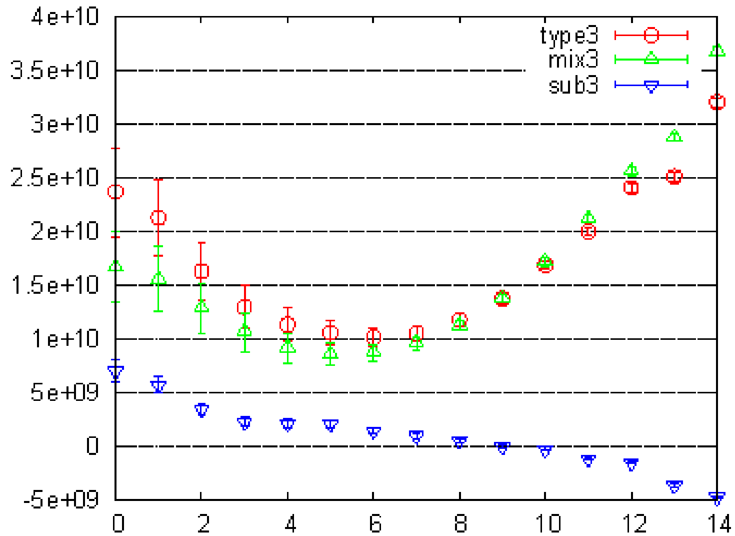


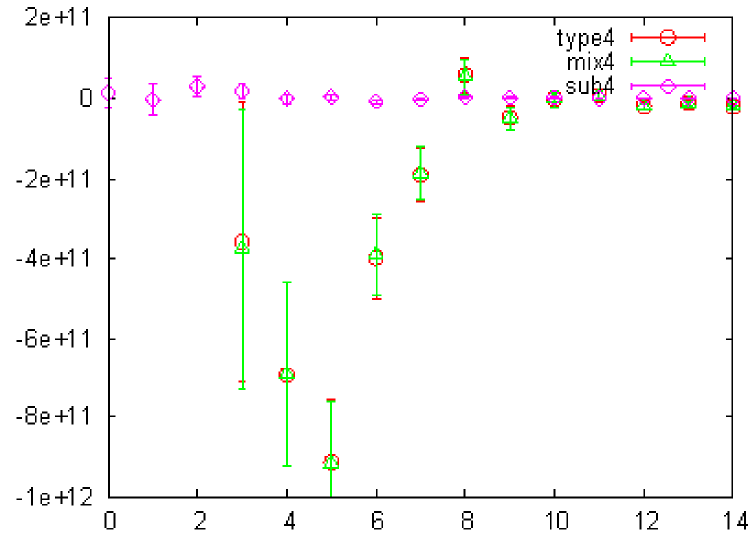


- Re (A_0) = 43(12) 10^{-8}
- Recall, p = 0, m_π = 420 MeV!

$\Delta I = 1/2\ \ K \rightarrow \pi\pi$
(Qi Liu)

- Removing quadratic divergence from Q_6 :

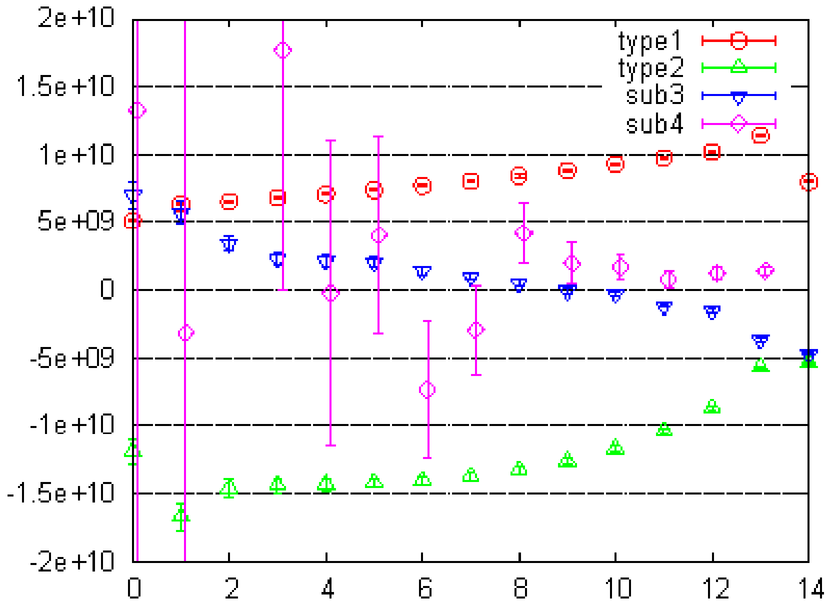


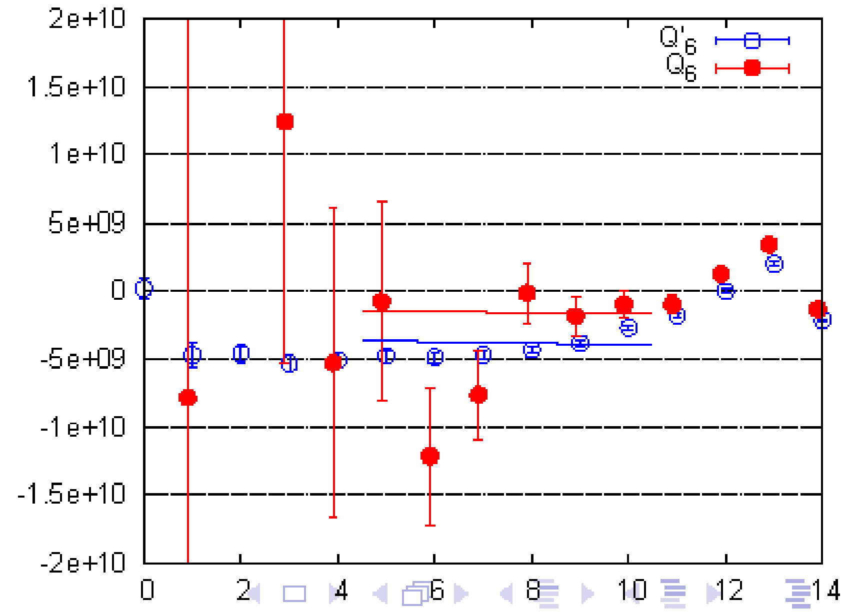


- Subtraction is needed even though it vanishes on-shell
- Off-shell excited states contributions are large.

$\Delta I = 1/2$ $K \rightarrow \pi\pi$
(Qi Liu)

- Results for Q_6, largest part of $\mathrm{im}(A_0)$:





- No result – errors are too large.
- If Q_6' (without disconnected part) is physical then need 4 x statistics?

Future prospects

- Re (A_2) and Im (A_2) known soon to 10%
  - 48^3 x 64 will allow unitary pions
  - Second lattice spacing → 2-3% error

  1 Tf yr → 10 Tf yr
- Re (A_0) and Im (A_0) with physical kinematics

Δ I=½ rule: Re(A_0)/Re(A_2)

$$\epsilon' = \frac{ie^{i(\delta_2-\delta_0)}}{\sqrt{2}} \frac{\mathrm{Re}A_2}{\mathrm{Re}A_0} \left[\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} \right]$$

	Factor	Pflops yr
2000 configurations	1	1.40
$5^2 \cdot 3$ statistics for p != 0	75	105
Benefit of 2x time slices	0.5	52.50
Benefit of split sources	0.25	13.13
Gain from lighter kaon mass	0.53	3.69
Reduced precision	0.75	2.77
Benefit of large volume $16^3 \rightarrow 32^3$	0.13	0.35
Deflation	0.3	**0.10**

Summary

- Low energy QCD with DWF
  - SU(3) x SU(3) ChPT fails at m_K
  - SU(2) x SU(2) ChPT consistent $240 \leq m_\pi \leq 420$ MeV but not compelling – a linear ansatz fits the data better
- Discretization errors small ≤ 1%.
- Finite quark mass errors more important ~ 8%
- $K^0 - \overline{K}^0$ mixing in "tension" with standard model.
- $K \rightarrow \pi\,\pi$ decay
  - Δ I = 3/2 now possible
  - ΔI = 1/2 in two years?