Technicolor and conformal window on the lattice

Minimal walking technicolor

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CERN Theory workshop: Future directions in lattice gauge theory

Contents:

- Technicolor & conformal window
- What can lattice do?
- Minimal walking technicolor
- Improvement

Background:

- The Standard Model (with Higgs) is phenomenologically extremely successful.
- However: The Higgs field is special:
  - *it is the linchpin of the standard model: provides the mechanism for the electroweak symmetry breaking*
  - *it has not been found*
  - *it is a scalar*

  $\Rightarrow$ *theoretical problems at very high scales:*
  hierarchy problem, vacuum stability, unitarity bound . . .
- Most BSM models aim to ameliorate these problems by e.g.
  - pairing scalars with fermions (SUSY)
  - introducing a cutoff (extra dimensions)
  - not having scalars at all (Technicolor and many other strongly coupled BSM models)

Chiral symmetry breaking vs. Higgs mechanism

Consider the standard Electroweak symmetry breaking with Higgs and the chiral symmetry breaking (χSB) in QCD:

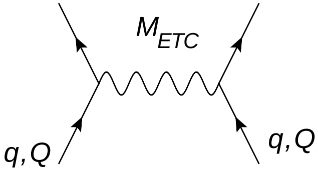
	EWSB	χSB
condensate (Breaks EW):	Higgs vev v	f_π decay constant
Goldstone bosons:	W, Z longitudinal modes	π-mesons
radial excitation:	Higgs particle	scalar meson

Technicolor

- New gauge field (technigauge) + massless fermions (techniquarks) Q.
- Techniquarks have both technicolor and EW charge (like quarks in SM)
- Chiral symmetry breaking in technicolor $\longrightarrow$ Electroweak symmetry breaking
- Scale: $\Lambda_{\rm TC} \approx \Lambda_{\rm EW}$
- After chiral symmetry breaking:
  - $\Rightarrow$ decay constant $f_{\rm TC} \leftrightarrow$ Higgs expectation value v.
  - $\Rightarrow$ scalar $\bar{Q}Q$ -meson $\leftrightarrow$ Higgs
  - $\Rightarrow$ pseudoscalars $\leftrightarrow$ W,Z -longitudinal modes
  - $\Rightarrow$ exotic technihadrons (observable!)
- Describes well the W, Z+Higgs sector (depending on the model, may have too many Goldstone bosons)
- Elegant, "proven" mechanism in the Standard Model
- Does not explain fermion masses (Yukawa). For that, we need additional structure $\rightarrow$ *Extended technicolor*

Extended technicolor

- In addition to the "pure" technicolor, take new massive gauge boson coupling Standard Model fermions and techniquarks: extended technicolor (ETC)



[Eichten,Lane,Holdom,Appelquist,Sannino,Luty. . .]

- $\frac{g^2_{\rm ETC}}{M^2_{\rm ETC}} \bar{Q}Q\bar{q}q \longrightarrow$ fermion mass $m_q \propto \frac{1}{M^2_{\rm ETC}} \langle \bar{Q}Q\rangle_{\rm ETC}$
- $\frac{g^2_{\rm ETC}}{M^2_{\rm ETC}} \bar{q}q\bar{q}q \longrightarrow$ extra FCNC's
- $\frac{g^2_{\rm ETC}}{M^2_{\rm ETC}} \bar{Q}Q\bar{Q}Q \longrightarrow$ explicit χSB in the techniquark sector

$\langle \bar{Q}Q\rangle_{\rm ETC}$: condensate evaluated at the ETC scale.

Extended technicolor

- $\bar{q}q\bar{q}q$ -term leads to unwanted FCNC's. In order to be compatible with precision electroweak tests, we must have

a) $\Lambda_{\rm ETC} \approx M_{\rm ETC} \gtrsim 1000 \times \Lambda_{\rm EW} = \Lambda_{TC}$

b) For EWSB we must have $\langle \bar{Q}Q \rangle_{\rm TC} \propto \Lambda_{\rm TC} \approx \Lambda_{\rm EW}$

c) On the other hand, $\langle \bar{Q}Q \rangle_{\rm ETC} \propto m_q \Lambda^2_{\rm ETC}$ (top quark!)

- Using RG evolution

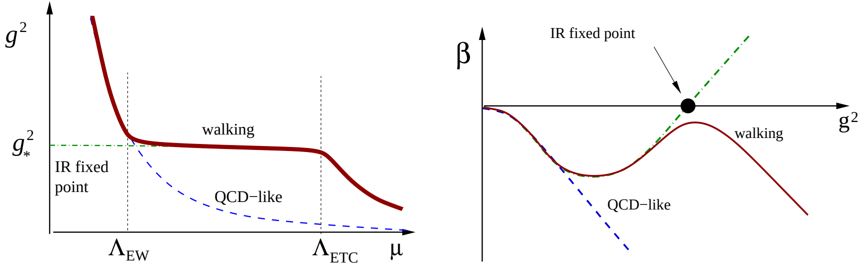
$$\langle \bar{Q}Q \rangle_{\rm ETC} = \langle \bar{Q}Q \rangle_{\rm TC} \exp \left[\int_{\Lambda_{\rm TC}}^{M_{\rm ETC}} \frac{\gamma(g^2)}{\mu} d\mu \right]$$

  where $\gamma(g^2)$ is the mass anomalous dimension.

- In weakly coupled theory $\gamma \sim 0$, and $\langle \bar{Q}Q \rangle$ is $\sim$ constant.
- *Thus, it is not possible to satisfy the constraints a), b), c) in a QCD-like theory, where the coupling is large only on a narrow energy range above χSB.*

Walking coupling

- If the coupling *walks*, i.e. if $g^2 \approx g_*^2$ constant over the range from TC to ETC, then $\langle\bar{Q}Q\rangle_{\rm ETC} \approx \left(\frac{\Lambda_{\rm ETC}}{\Lambda_{\rm TC}}\right)^{\gamma(g_*^2)} \langle\bar{Q}Q\rangle_{\rm TC}$ (condensate enhancement)
- $\gamma(g_*^2)$ should be in the interval 1 – 2 in order to satisfy the conditions a)–c) (depends on the details).



- In a walking theory the β-function $\beta = \mu\frac{dg}{d\mu}$ reaches almost zero near g_*^2.
- If the β-function hits zero there is an IR fixed point, where the system becomes *conformal*.

Perturbative β-function

2-loop universal β-function for SU(N_c) gauge theory with N_f fermions:

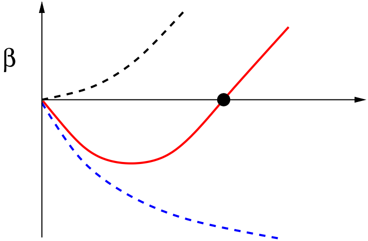
$$\beta(g) = -\mu \frac{dg}{d\mu} = -\beta_0 \frac{g^3}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)^2}$$

where the coefficients are

$$\beta_0 = \frac{11}{3} C_r - \frac{4}{3} T_r N_f, \qquad \beta_1 = \frac{34}{3} C_r^2 - \frac{20}{3} C_r T_r N_f - 4 C_r T_r N_f$$

When N_f is varied, generically 3 different behaviours seen:

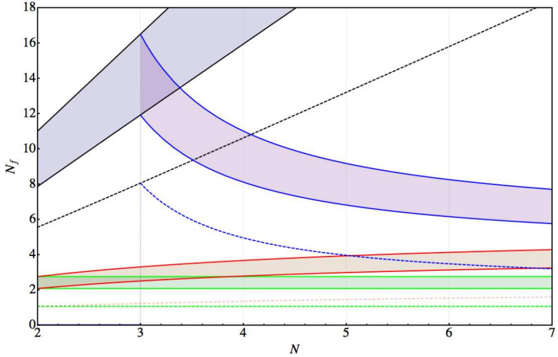
- confinement and χSB at small N_f
- IR fixed point (conformal window) at medium N_f
- Asymptotic freedom lost at large N_f



Conformal window in SU(N)

- Location of the conformal window in SU(N) theory depends of the fermion representation:

fundamental



[Appelquist, Lane, Mahanta, Cohen, Georgi, Yamawaki, Schrock, Sannino, Tuominen, Dietrich]

2-index antisymmetric

2-index symmetric

adjoint

- Upper edge: asymptotic freedom lost
- Lower edge: ladder approximation
- Walking can be found near the lower edge of the conformal window: large coupling, non-perturbative - lattice simulations needed!
- In higher reps it is easier to satisfy EW constraints.

How can lattice help?

Simplify: consider only technicolor + techniquarks.
What questions can be (have been) studied?

- Mass spectrum as a function of techniquark mass m_Q
- $\beta(g^2)$
- $\gamma(g^2)$
- Critical properties in the neighbourhood of the IRFP
- Classification conformal / QCD-like / walking
  (walking not observed yet unambiguously)
- TC contribution to the S,T,U -parameters (Electroweak precision measurements)

What is difficult to study:

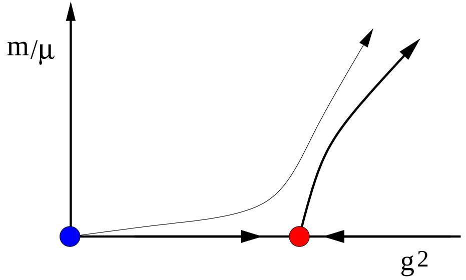
- Coupling to the weak gauge field (chiral)
- Chiral technicolor models
- ETC?

Models studied on the lattice

- SU(2) + $N_f = 2$ adjoint rep: (*Minimal walking technicolor*) conformal
  [Catterall et al; Bursa et al; Hietanen et al]
- SU(3) + $N_f = 2$ 2-index symmetric rep: unclear [de Grand et al; Sinclair and Kogut]
- SU(3) + $N_f = 8$–16 fundamental rep:
  - $N_f = 8$: χSB [Appelquist et al; Deuzeman et al; Fodor et al; Jin et al]
  - $N_f = 9$: χSB [Fodor et al]
  - $N_f = 10$: unclear [Yamada et al]
  - $N_f = 12$: conflicting results [Hasenfratz; Fodor et al; Appelquist et al; Deuzeman et al]
  - $N_f = 16$: conformal [Damgaard et al; Heller; Hasenfratz; Fodor et al]
- SU(2) + fundamental rep fermions:
  - $N_f = 2$: χSB [many]
  - $N_f = 3$: conformal(?) [Iwasaki et al]
  - $N_f = 6$: conformal(?) [Del Debbio et al]
  - $N_f = 8$: conformal [Iwasaki et al]
  - $N_f = 10$: conformal [Karavirta et al (to be published)]

RG flow in conformal case

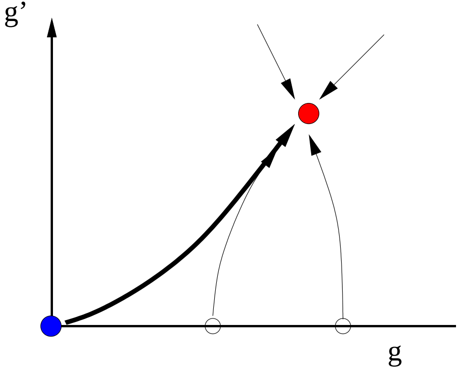
- Relevant parameters at UV: g^2 and m_Q



- m_Q is relevant at the IRFP.
- Scaling near IRFP: correlation length $\xi \approx 1/M \propto (m_Q)^{-1/(1+\gamma)}$
- New UV fixed point at stronger coupling? [Kaplan et al; Lombardo et al; Hasenfratz]

RG flow on the lattice

- Consider $m_Q = 0$ case
- Symanzik: $\mathcal{L} = \mathcal{L}_0 + a\mathcal{L}_1 + \ldots$

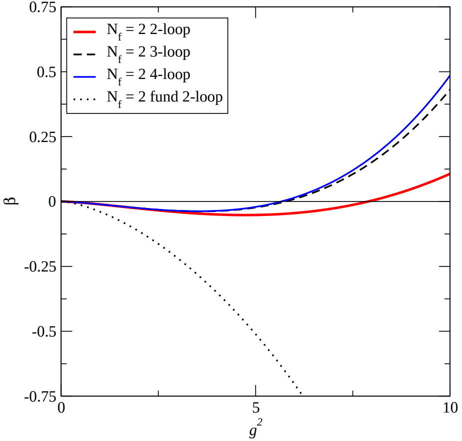


- g: lattice gauge coupling; g' generic irrelevant parameter
- Improved actions go along lines closer to the "perfect" one
- Quality of the lattice action important! The direction of the approach to IRFP can affect critical exponents.

Minimal technicolor

Minimal technicolor

- SU(2) with $N_f = 2$ adjoint representation techniquarks
- Perturbative β-function (compared with fundamental rep):



What is studied?

- Particle spectrum: do we observe chiral symmetry breaking (QCD) or do all modes become massless as $m_q \to 0$ (no χSB, possibly conformal)
- Measure the evolution of the coupling directly using the Schrödinger functional method
- Improvement of the lattice action
- Also studied by [Catterall, Sannino; Del Debbio, Patella, Pica; Bursa et al, Del Debbio et al].
  - Spectrum, mass anomalous exponent γ

Lattice model:

- SU(2) gauge action in fundamental rep.
- massless fermions in adjoint rep.

$$\mathcal{L} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \bar{\psi} i \gamma_\mu D_\mu \psi$$

- On the lattice:
  - gauge fields U in the fundamental rep.
  - For the fermion action, these transformed into adjoint rep

    $$V^{ab} = 2\,\mathrm{Tr}[U^\dagger \lambda^a U \lambda^b]$$

    $a, b = 1, 2, 3.$
  - We use standard Wilson action (these results); now non-perturbatively O(a) improved Wilson-clover action (future results)
  - For comparison, we also do analysis with $N_f = 2$ fundamental quarks

Results: lattice phase diagra

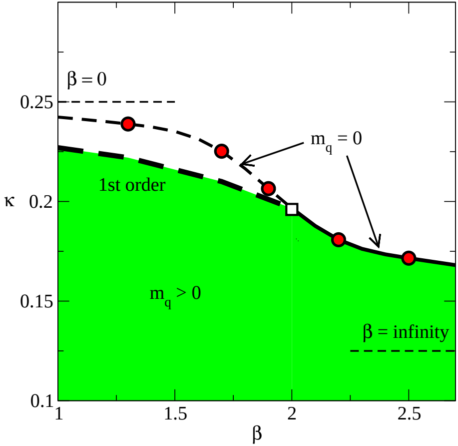
Lattice (Wilson) parameters:

$$\beta_L = \frac{4}{g^2} \qquad \kappa = \frac{1}{8 + 2m_{q,\mathrm{bare}}}$$

Because Wilson action breaks chiral symmetry, quark mass is additively renormalised. Physical quark mass is measured using the axial Ward identity

$$m_Q = \frac{1}{2}\frac{\partial_t V_{\mathrm{PA}}(t = L/2)}{V_{\mathrm{PP}}(t = L/2)}$$

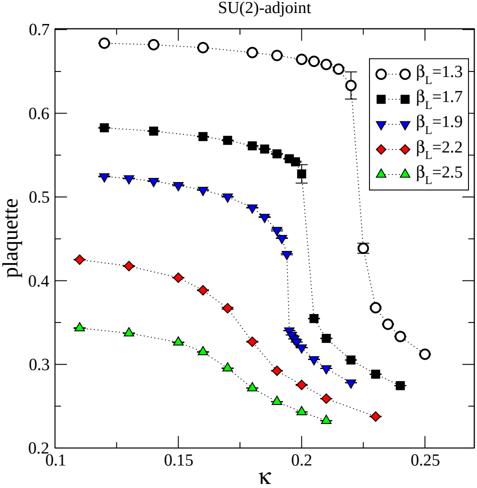
$m_Q(\beta,\kappa) = 0$ defines the critical line $\kappa_c(\beta)$



At $\beta_L \lesssim 2$ there appears a strong 1st order phase transition at $m_Q > 0$, preventing the $m_Q \to 0$ limit.
At $\beta \gtrsim 2$, $m_Q \to 0$ limit is possible. This is the relevant region for us.

1st order transition

The 1st order transition is cleary visible in the plaquette ($\propto F_{\mu\nu}^2$) expectation value (on small volumes)



Origin of the transition?

- Assume IRFP exists.
- At a given volume and at strong enough coupling, the massless fermions prevent the system from becoming confined.
- Make fermions heavier $\rightarrow$ phase transition to confined phase

$\rightarrow$ must not use too heavy quarks! (depends on coupling and system size)

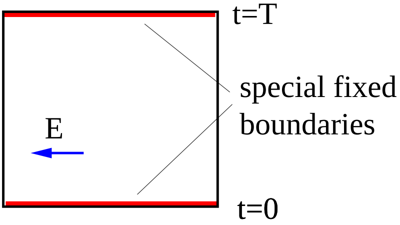
- As system size $L \rightarrow \infty$, only $m_Q = 0$ remains deconfined.
- For us, it is not possible to use the small m_Q-side at strong coupling, because m_Q jumps to negative values. (Lattice artifact $\rightarrow$ improved action)

I Evolution of the coupling

Schrödinger functional: Generate a *background* chromoelectric field using non-trivial fixed boundary conditions, parametrised by a twist angle η

At the classical level, we have

$$\frac{dS_{\rm class.}}{d\eta} = \frac{A}{g^2}$$



where $A(\eta)$ is a known constant.
At the quantum level, we define the coupling through

$$\frac{1}{g^2} = \frac{1}{A}\frac{dS}{d\eta}$$
$$= \text{const.} \times \langle(\text{boundary plaq.})\rangle$$

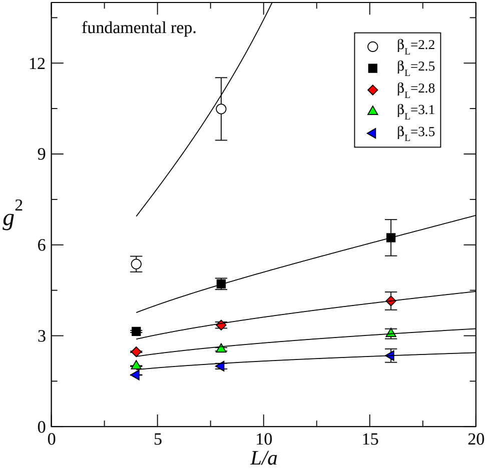
- Evaluates g^2 directly at length scale L, the lattice size
- Can use $m_Q = 0$
- Has been used very succesfully in QCD by the Alpha collaboration

Evolution of the coupling

Measure g^2 at different β_L (lattice spacing a) and lattice sizes

Testing with **fundamental representation**:

- L/a grows, $k \sim a/L$ decreases, $g^2(L)$ increases: *asymptotic freedom*, OK!
- Large $\beta_L \to$ small lattice spacing $\to$ small volume
- Continuous line: coupling evaluated from the 2-loop β-function (integration constant fixed to measurement at $L/a = 16$)
- Not a continuum limit, but shows consistency



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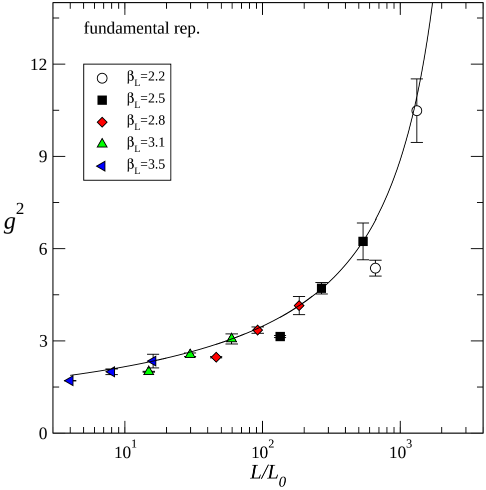
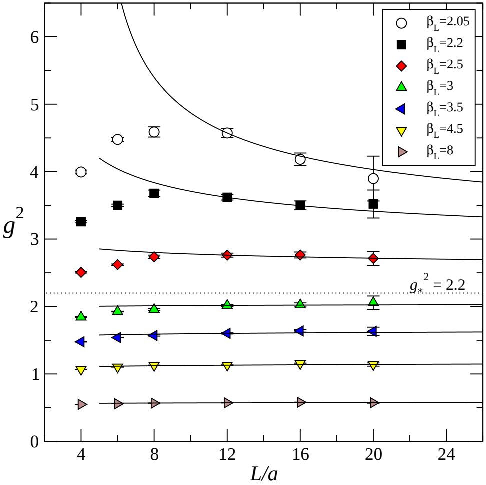


Evolution of the coupling

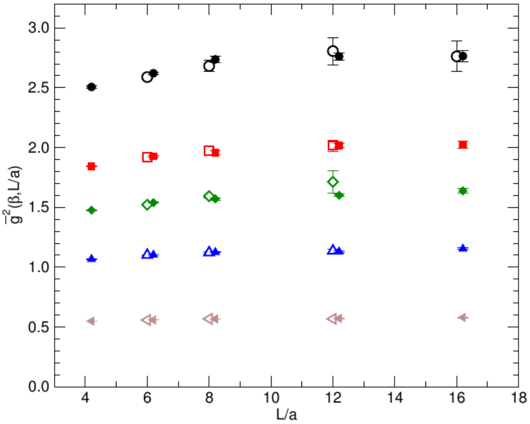
In **adjoint representation**:

- At small $g^2(L)$: increases with L (asymptotic freedom)
- At large $g^2(L)$: *decreases* as L increases
  $\Rightarrow \beta$-function positive here!
- As $L/a \to \infty$, apparently $g^2(L) \to g_*^2 \approx 2 \ldots 3$.
  $\Rightarrow$ conformal behaviour!?
- In continuum, curves must be monotonic: *cutoff effects at small L/a.*
- Continuous line: coupling evaluated with fitted β-function ansatz (to be described)



Evolution of the coupling

Data agrees with [Del Debbio et al]



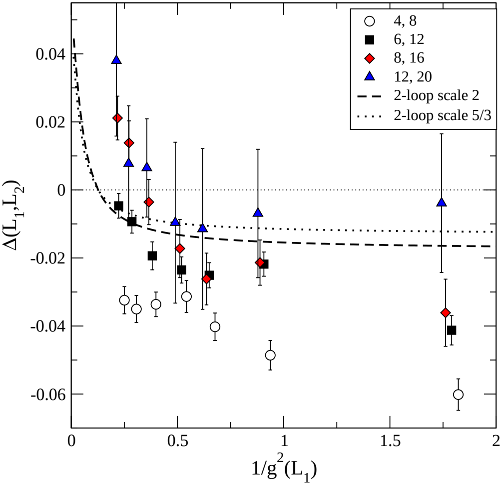
Step scaling function

Step scaling function

$$B_2(L) = \frac{1}{g^2(2L/a)} - \frac{1}{g^2(L/a)}$$

shown as a function of $1/g^2(L/a)$, for $(L, 2L) = (4,8), (6,12), (8,16)$.

- $\exists$ IR fixed point
- Large finite L/a effects
- Proper continuum limit too noisy to be practical
- Use only $L/a \geq 12$ in subsequent analysis

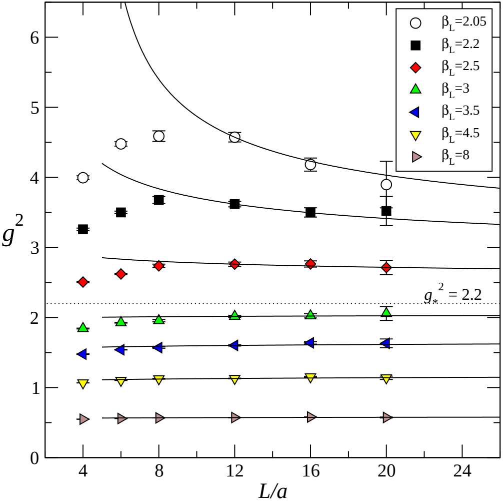


β-function

Assuming that the lattice effects on the large-volume data are small, we can describe the features of the β-function by fitting an ansatz:

$$\beta = -L\frac{dg}{dL} = -b_1 g^3 - b_2 g^5 - b_3 g^\delta$$

Here b_1, b_2 are perturbative constants and b_3 and δ are fit parameters. (Parametrising the location of the fixed point and the slope of the β-function there). The ansatz is fitted to the data at $L/a = 12, 16, 20$:



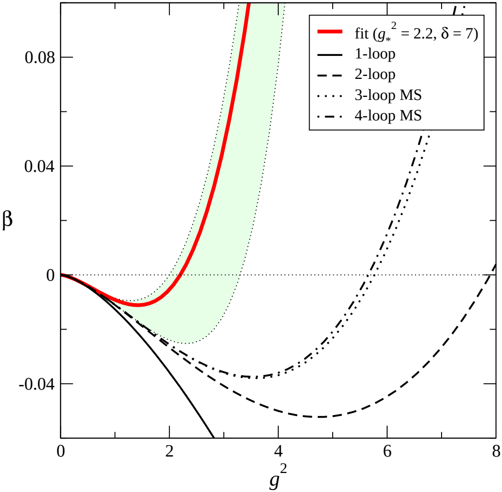
β-function

Fit result:

$$\beta = -L\frac{dg}{dL} = -b_1 g^3 - b_2 g^5 - b_3 g^\delta$$

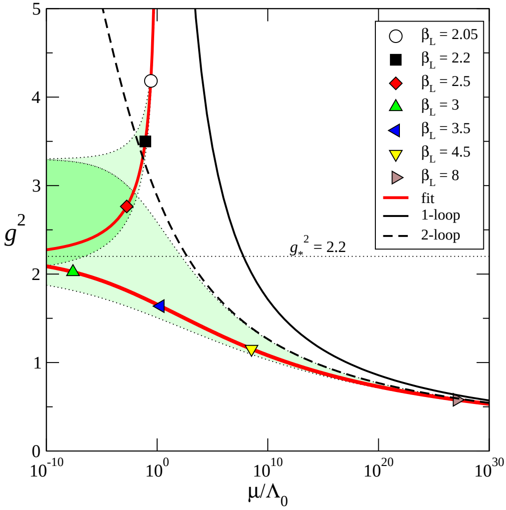
FP is at substantially smaller coupling than indicated by 2-loop P.T.

In MS-schema, β-function is known to 4-loop order: [Ritbergen, Vermaseren, Larin]
Not directly comparable to lattice (beyond 2 loops), because of different schema! But quantifies perturbative uncertainty.



Coupling constant

Integrating the β-function we obtain the coupling:
Asymptotically free branch at $g^2 < g_*^2$, non-free branch at $g^2 > g_*^2$.
Error bands are large
Need to do better!



I Particle spectrum

SU(2) + fundamental quarks:

- 2-quark and quark-antiquark states $\bar{q}q$, qq (degenerate except in isoscalar channel)
- glueballs

SU(2) + adjoint quarks:

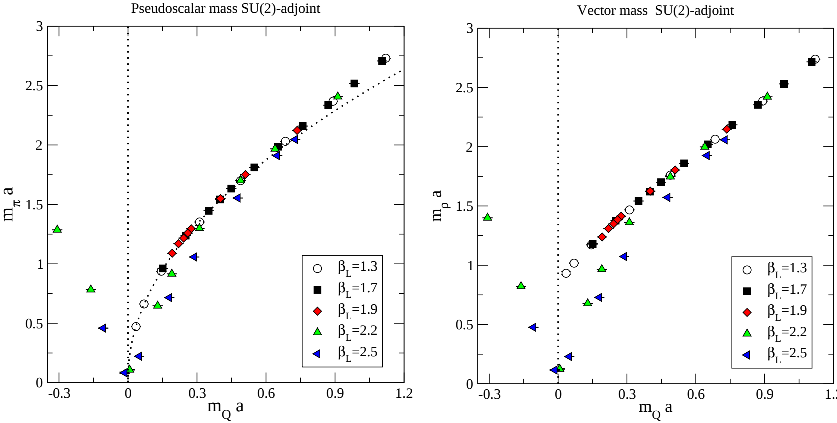
- $\bar{Q}Q$, QQ 2-quark states – "π", "ρ" ...
- QQQ "proton"
- Qg quark-gluon state
- glueballs

What to expect from the spectrum:

- If QCD-like χSB: as $m_Q a \to 0$,
  - $m_\pi \propto m_Q^{1/2}$
  - other states have finite mass.
- If IR fixed conformal point: $m_Q a \to 0$, all states become massless with the same exponent.
- If walking behaviour: at high energy $\sim$ conformal, at small χSB.

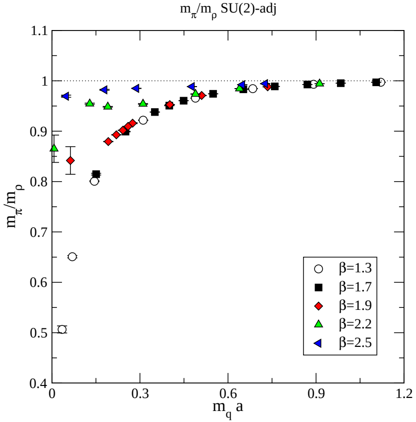
Results

Pseudoscalar ("π") and vector ("ρ") masses



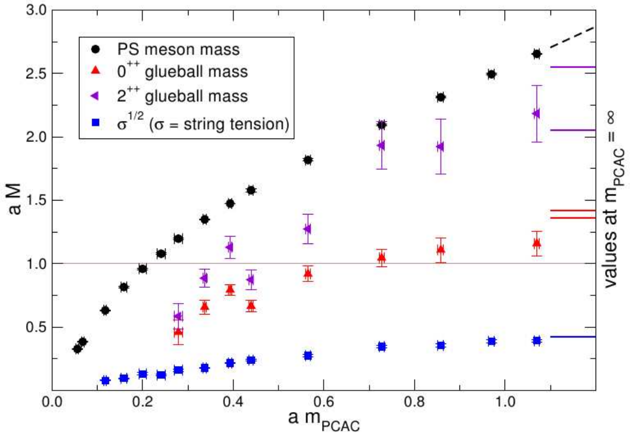
At small β, looks like χSB: too large m_Q, wrong side of the transition! However, we cannot go to $m_Q \to 0$ because of the 1st order transition.
At large β masses $\to 0$?

Mass ratio m_π / m_ρ



At large β looks like $\sim$ massless, possibly conformal.

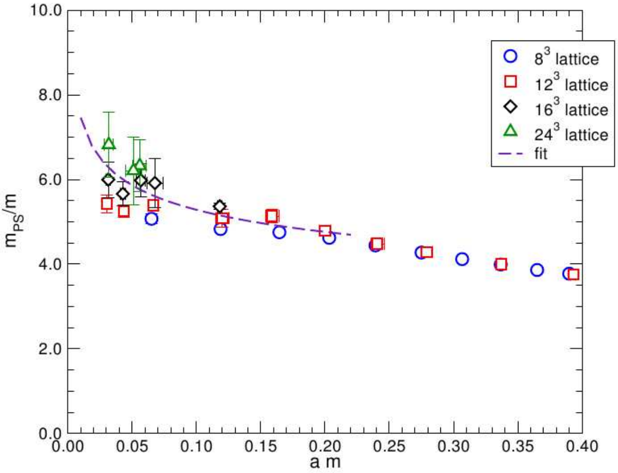
Mass spectrum at $\beta_L = 2.25$



[Del Debbio et al]

Spectrum becomes massless, inverted hierarchy when compared with QCD [Miransky]

Testing scaling of the mass: γ

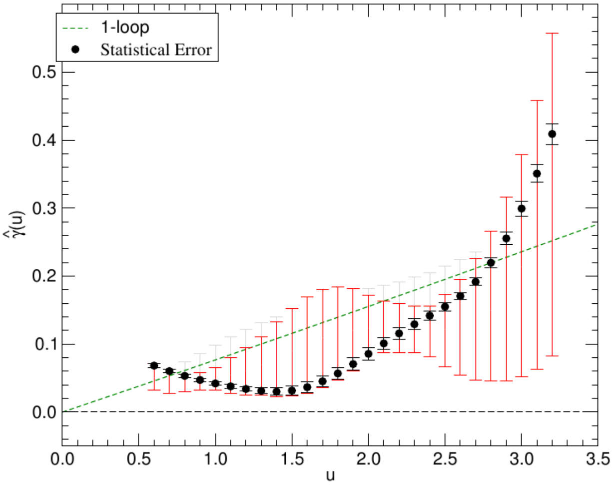


$$\log m_{\rm PS} = \frac{1}{1+\gamma}\log m + C = 0.85 \log m + C$$

$\gamma \sim 0.2$

[Del Debbio et al]

Measurement of γ



[Bursa et al]

$\gamma \lesssim 0.5$ near IRFP

What does the results imply?

- Clear hints of a conformal infrared fixed point at $g_*^2 \approx 2 - -3$ in SF schema.
- Exponent $\gamma(g_*^2) \ll 1$. Too small for technicolor.
- However: the results have been obtained independently, but using similar lattice action. Unknown systematics!
- Strong systematic effects have been observed e.g. in simulations of SU(3) with 2 sextet fermions.
- *Must have very careful control of the systematics and the continuum limit*

How to do it better?

- Need to have better control of the continuum limit: *improved actions*
- Wilson fermions have large $O(a)$ cutoff-effects. These are cancelled by adding a irrelevant "clover term" with a fine-tuned coefficient $c_{\rm SW}$.
- In Schrödinger functional schema also boundary term improvement must be computed

Schrödinger functional scheme action

$$S_i = S_u + \delta S_V + \delta S_{G,b} + \delta S_{F,b}$$

$$\begin{aligned}
\delta S_V &= \tfrac{ia^5}{4} c_{sw} \textstyle\sum_{x_0=a}^{L-a} \sum_{\vec{x}} \bar{\psi}(x) \sigma_{\mu\nu} \hat{F}_{\mu\nu}(x) \psi(x) \\
\delta S_{G,b} &= \tfrac{1}{2g_0^2}(c_s - 1) \textstyle\sum_{p_s} \mathrm{Tr}[1 - U(p_s)] \\
&\quad + \tfrac{1}{g_0^2}(c_t - 1) \textstyle\sum_{p_t} \mathrm{Tr}[1 - U(p_t)] \\
\delta S_{F,b} &= a^4(\tilde{c}_s - 1) \textstyle\sum_{\vec{x}} [\hat{O}_s(\vec{x}) + \hat{O}'_s(\vec{x})] \\
&\quad + a^4(\tilde{c}_t - 1) \textstyle\sum_{\vec{x}} [\hat{O}_t(\vec{x}) - \hat{O}'_t(\vec{x})]
\end{aligned}$$

Improvement

- The clover coefficient $c_{\rm SW}$ is determined non-perturbatively
- The boundary coefficients c_t, $\tilde{c}_t$ perturbatively
- c_s, $\tilde{c}_s$ are not needed

We obtain

$\tilde{c}_t = 1 - 0.0135(1) \times C_R g_0^2 + O(g_0^4)$ [Karavirta et al, for fundamental rep Lüscher, Weisz]

Write $c_t = 1 + g_0^2(c_t^{(1,0)} + N_F * c_t^{(1,1)}) + O(g_0^4)$

- $c_t^{(1,0)} = -0.0543(5)$ (for SU(2)) [Lüscher et al.]
  depends only on gauge
- $c_t^{(1,1)}$ is

N	*Fundamental*	*Adjoint*	*Sextet*
2	0.0192(2)	0.075(1)	
3	0.0192(4)	0.113(1)	0.0946(9)
4	0.0192(5)		

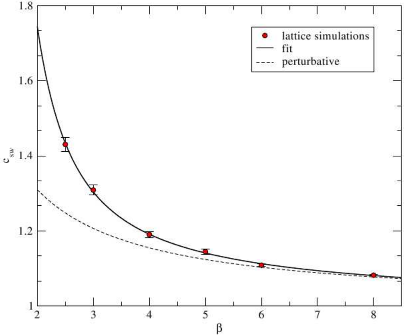
[Fundamental $N = 2, 3$ Sint et al, others Karavirta et al]

These are in agreement with $c_t^{(1,1)} = 0.019141 \times (2T_R)$

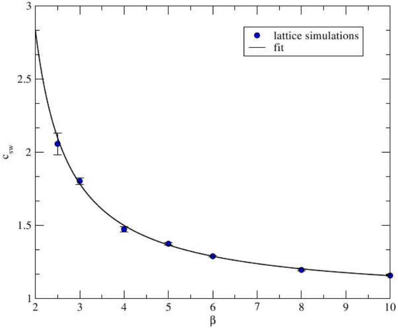
Improvement

c_{SW} coefficient: [Mykkänen et al]

$N_f = 2$ fundamental



$N_f = 2$ adjoint



Conclusions

- Lattice technicolor and conformality:
  - Lot of work has been done, signs of IRFP found in several theories.
  - No clear sign of proper walking found in any (massless) theory
  - Anomalous dimensions appear to be small
  - Methods still under development; e.g. there are 5–6 methods used to measure the evolution of the coupling. Good for cross-checking, but no golden method known
  - Need to live at strong coupling: use an action which minimizes lattice effects there. Improvement!
  - There is a haze over the results due to unknown systematics. More careful analysis!
- SU(2)+2 adjoint fermions:
  - appears to have an IR fixed point.
  - No χSB, no "walking".
  - Small γ
  - Results need checking!
- In the future:
  - Better systematics, careful analysis
  - Deformations (mass, additional couplings)
  - EW precision test
  - Mass spectrum, phenomenology
  - **Lattice can exclude models from contention!**