# Excited State Spectroscopy from Lattice QCD

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Auspices of the Hadron Spectrum Collaboration

# Spectroscopy

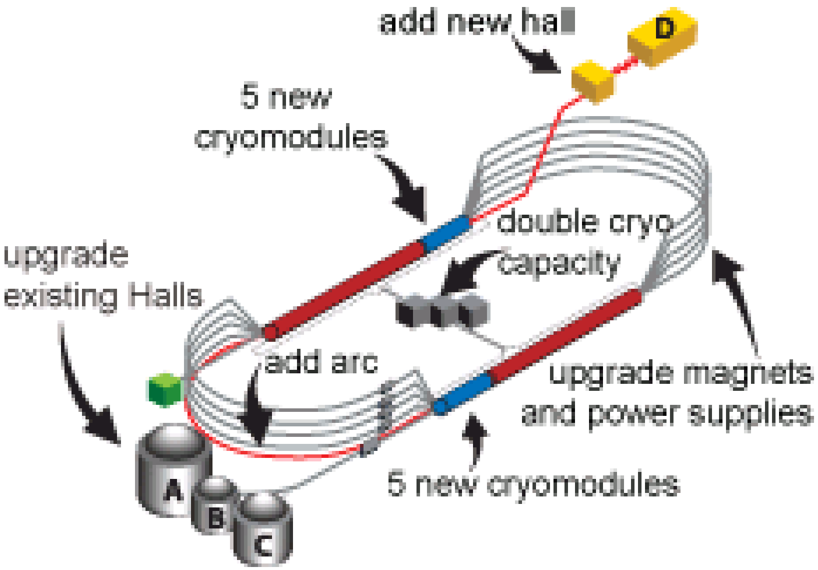
Spectroscopy reveals fundamental aspects of hadronic physics

- Essential degrees of freedom?
- Gluonic excitations in mesons - exotic states of matter?

- New spectroscopy programs world-wide
  - E.g., BES III (Beijing), GSI/Panda (Darmstadt)
  - Crucial complement to 12 GeV program at JLab.
    - Excited nucleon spectroscopy (JLab)
    - JLab GlueX: search for gluonic excitations.

# Nuclear Physics & Jefferson Lab

## JLab undergoing a major upgrade




Future Hall D

- **Lab doubling beam energy to 12GeV**
- **Adding new experimental Hall**

# Excited states: anisotropy+operators+variational

- Anisotropic lattices with $N_f=2+1$ dynamical (clover)fermions
  - Temporal lattice spacing $a_t < a_s$ (spatial lattice spacing)
  - High temporal resolution → Resolve noisy & excited states
  - Major project within USQCD – Hadron Spectrum Collab.

PRD 78 (2008) & PRD 79 (2009)

- Extended operators
  - Sufficient derivatives → nonzero overlap at origin

PRD 72 (2005), PRD 72 (2005), 0907.4516 (PRD), 0909.0200

- Variational method:
  - Matrix of correlators → project onto excited states

PRD 76 (2007), PRD 77 (2008), 0909.0200

# $N_f$=2+1 Anisotropic Clover

$N_f = 2 + 1$ (u,d + s)

Using $a_t m_\Omega$ & $\xi$=3.5 : $a_s = 0.1227(3)$fm, $(a_t)^{-1} \sim 5.640$ GeV

| $L_s$(fm) | 1.96fm | 2.45fm | 2.95fm | 3.93fm | 5.89fm |
|---|---|---|---|---|---|
| $m_\pi$(MeV) | $16^3 \times 128$ | $20^3 \times 128$ | $24^3 \times 128$ | $32^3 \times 256$ | $48^3 \times 384$ |
| 700 | 11k | 11k | | | |
| 520 | 10k | 11k | | | |
| 450 | 11k | 10k | | | |
| 400 | 13k | 13k | 13k | 4k | |
| 230 | | | 12k | 7k | |
| 140 | | | | | X |

0803.3960 & 0810.3588

# Spin and Operator Construction

Gamma matrices and derivatives in circular basis:
Couple to build any J,M via usual CG

$$\mathcal{O}^{JM} \leftarrow (CGC's)_{i,j,k,l} \bar{\psi}\, \vec{\Gamma}_i \times \left[\vec{D}_j \vec{D}_k \ldots \vec{D}_l\right] \psi$$

Only using symmetries of continuum QCD

Construct all possible **operators** up to 3 derivatives
(3 units orbital angular momentum)

Subduce onto lattice irreps: "*remembers*" J

$$\mathcal{O}^{[J]}_{\Lambda\lambda} \leftarrow \sum_M S^{\Lambda\lambda}_{JM} \mathcal{O}^{JM}$$

0905.2160 (PRD), 0909.0200 (PRL), 1004.4930 (PRD)

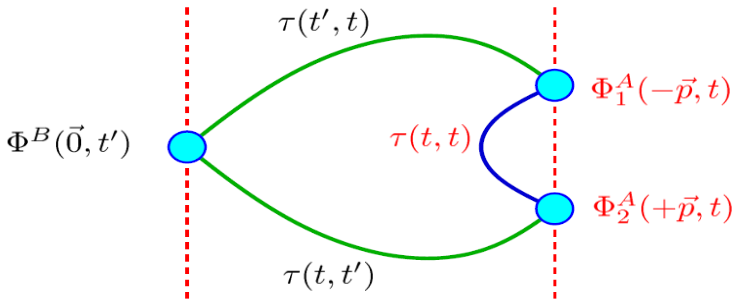
# Distillation

Replace smearing with **low rank** approximation

$$\Box(t) = V(t)V^\dagger(t) \implies \Box_{xy}(t) = \sum_{k=1}^{N} v_x^{(k)}(t) v_y^{(k)\dagger}(t)$$

Matrix elements of $v_k(t)$ $\rightarrow$ propagators, mesons, baryons, etc. Make correlators

$$C_M^{(2)}(t',t) = \mathrm{Tr}\Big[\Phi^B(t')\,\tau(t',t)\,\big\{\Phi_1^A(t)\cdot\tau(t,t)\cdot\Phi_2^A(t)\big\}\,\tau(t,t')\Big]$$




Gauge covariant, mom. conservation (source & sink) $\rightarrow$ reduced “noise”

Stochastic variants (lower cost)

0905.2160 (PRD), 1002.0818, Bulava (thesis 2010)

# Spectrum from variational method

Two-point correlator

$$C(t) = \langle 0 | \Phi'(t)\, \Phi(0) | 0 \rangle$$

$$C(t) = \sum_{\mathfrak{n}} e^{-E_{\mathfrak{n}} t} \langle 0 | \Phi'(0) | \mathfrak{n} \rangle \langle \mathfrak{n} | \Phi(0) | 0 \rangle$$

Matrix of correlators

$$C(t) = \begin{bmatrix} \langle 0|\Phi_1(t)\Phi_1(0)|0\rangle & \langle 0|\Phi_1(t)\Phi_2(0)|0\rangle & \dots \\ \langle 0|\Phi_2(t)\Phi_1(0)|0\rangle & \langle 0|\Phi_2(t)\Phi_2(0)|0\rangle & \dots \\ \vdots & & \ddots \end{bmatrix}$$

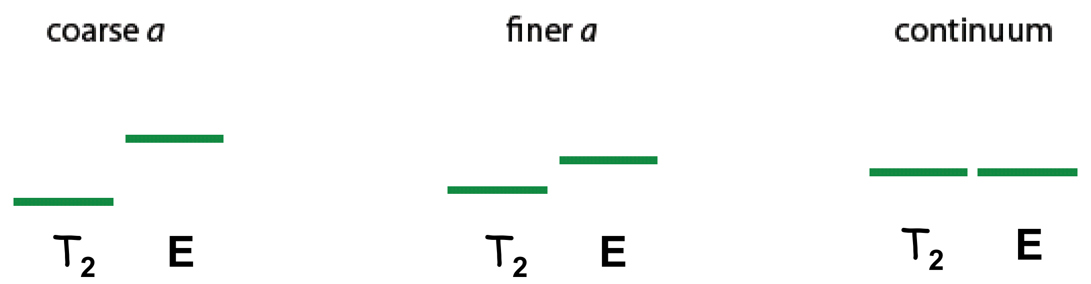
Diagonalize:
- eigenvalues $\rightarrow$ spectrum
- eigenvectors $\rightarrow$ wave function overlaps

Benefit: orthogonality for near degenerate states

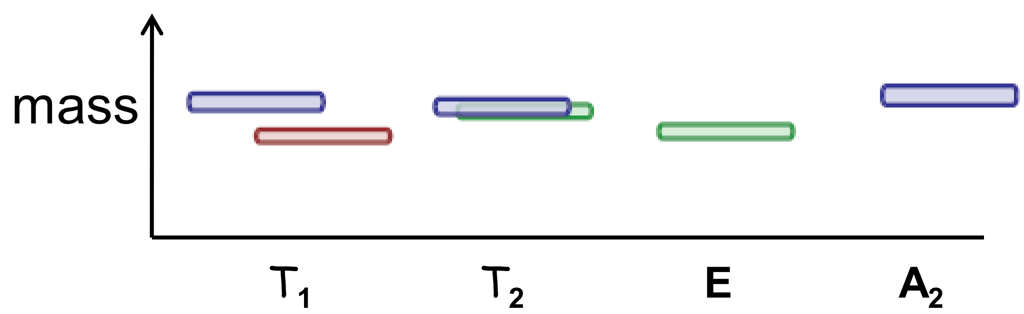
# Determining spin on a cubic lattice?

Spin reducible on lattice

$$\text{spin } 2_{\ (5)} \rightarrow T2_{\ (3)} + E_{\ (2)}$$




Might be dynamical degeneracies




Spin 1, 2, 3, or 4 ?

# Spin reduction & (re)identification

Variational solution:

$$C_{ij}(t) = \langle 0|\phi_i(t)\phi_j(0)|0\rangle \rightarrow \sum_\alpha Z_i^{(\alpha)} Z_j^{(\alpha)*} e^{-m_\alpha t}$$

Continuum

$$\langle 0 \mid \mathcal{O} \mid 2^-(\vec{0}, r)\rangle = Z$$

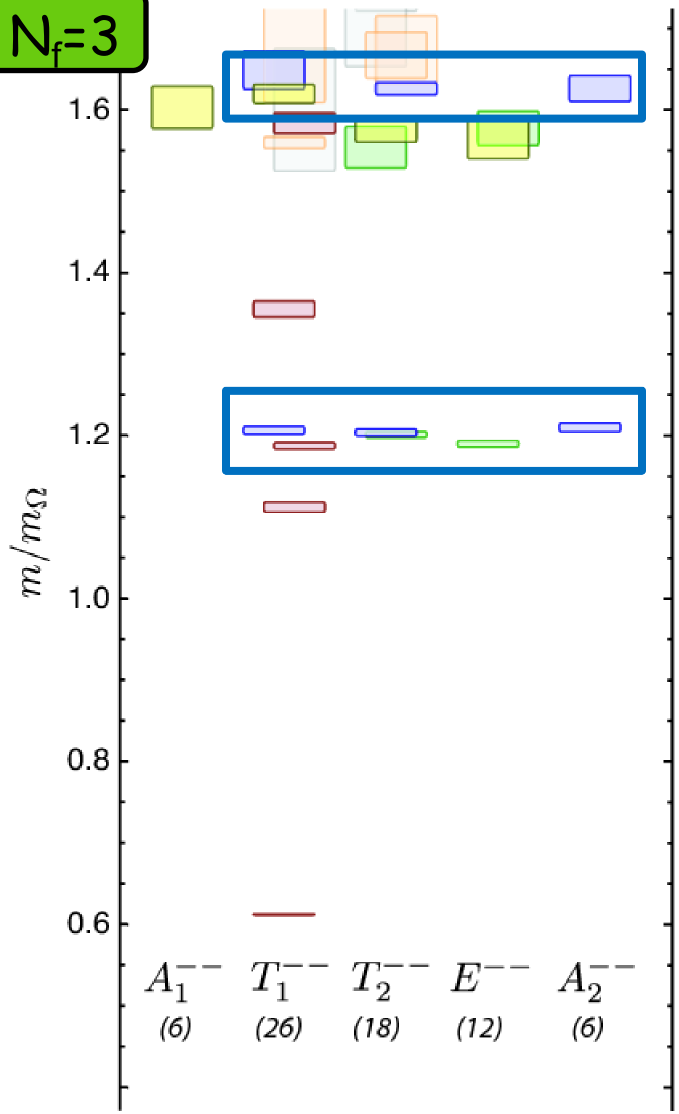
Lattice

$$\begin{aligned}\langle 0 \mid \mathcal{O}_{T_2} \mid 2^-(\vec{0}, r)\rangle &= Z_{T_2} \\ \langle 0 \mid \mathcal{O}_{E} \mid 2^-(\vec{0}, r)\rangle &= Z_{E}\end{aligned}$$

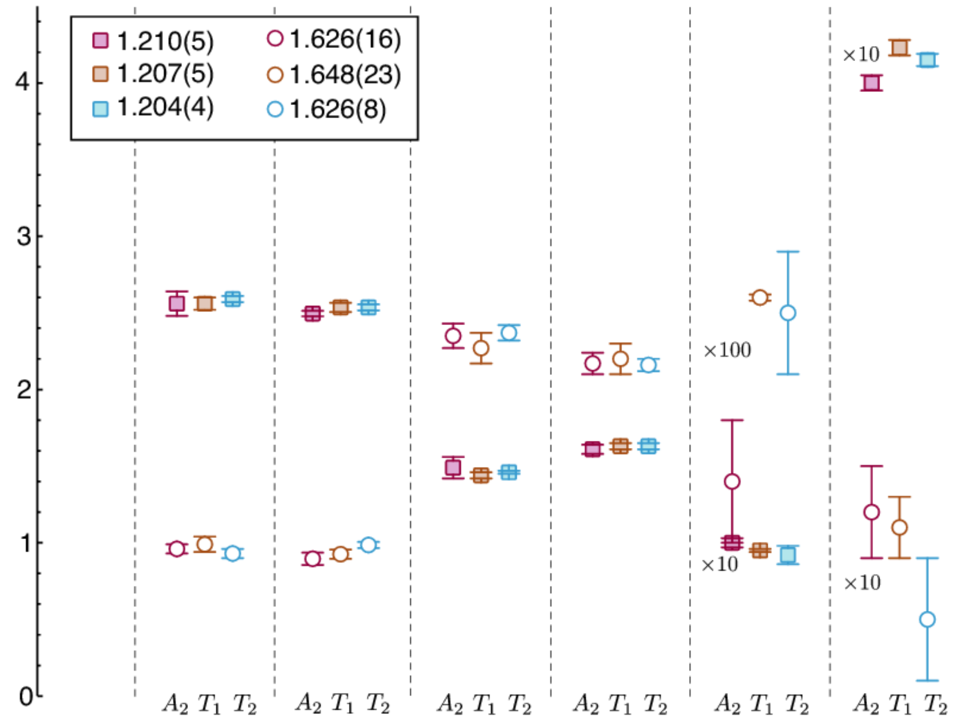
Method: Check if converse is true

# Spin (re)identification

$N_f=3$

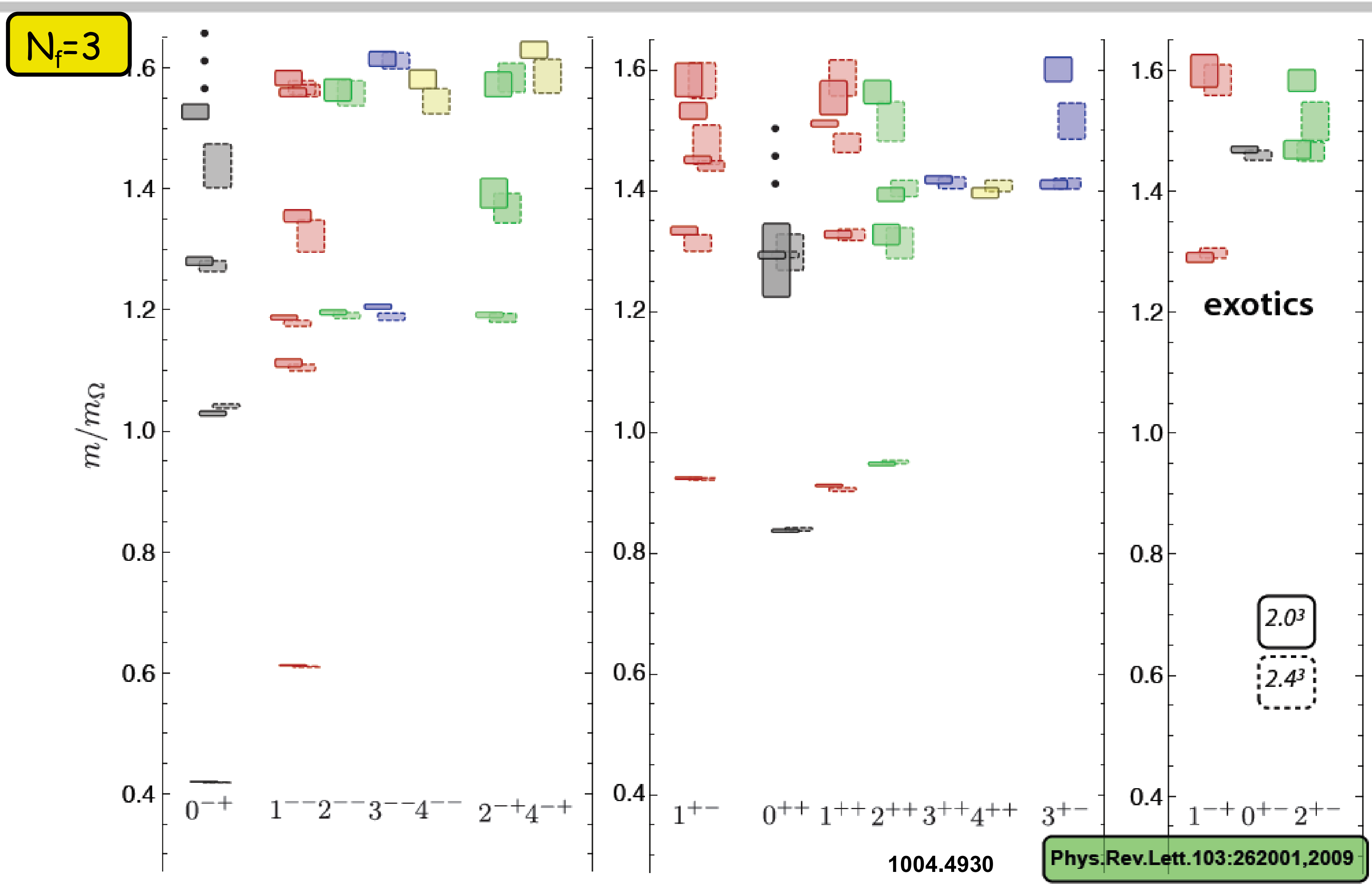



Z values: spin 3




Well identified: suggests small rotational breaking

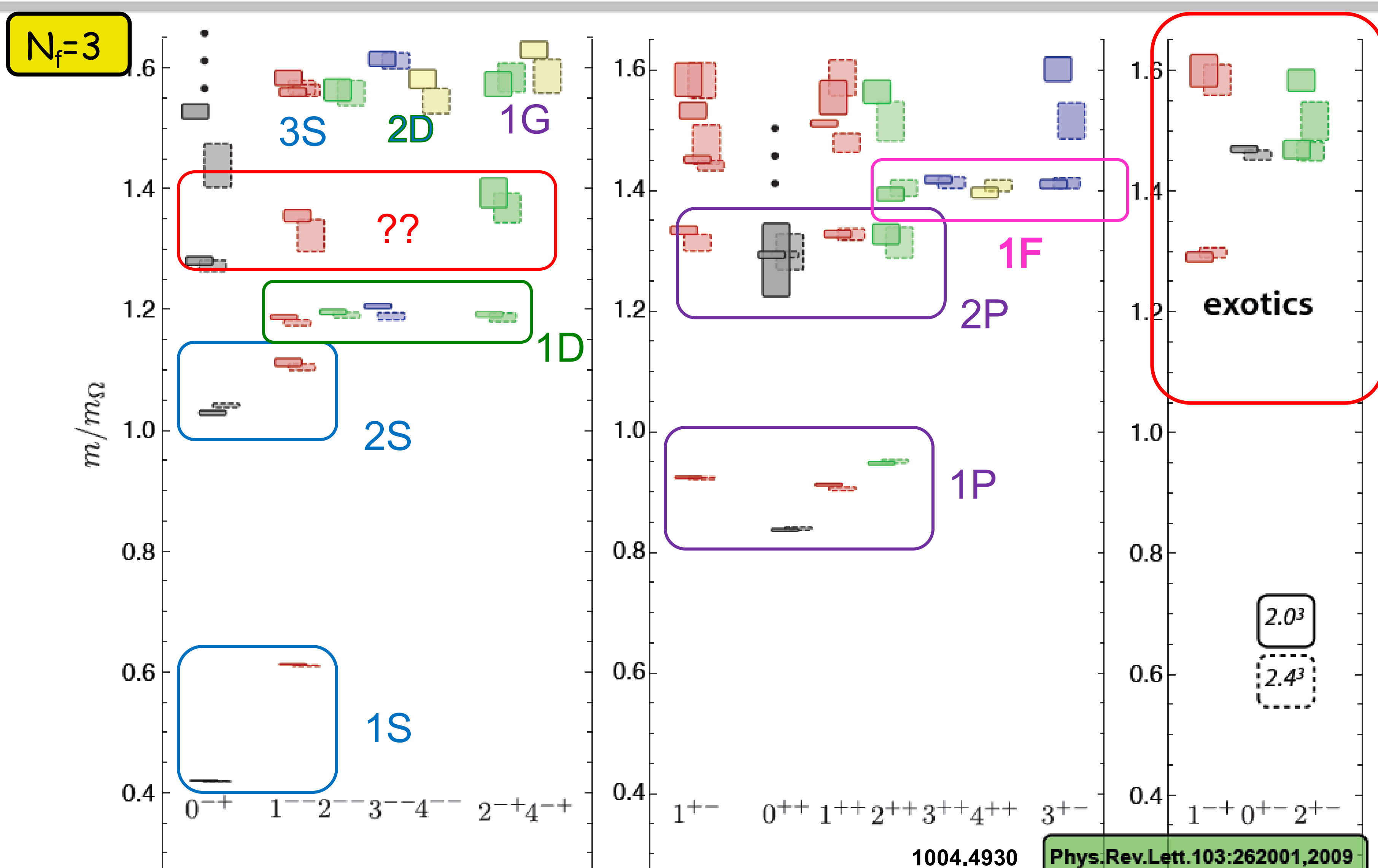
# Isovector Meson Spectrum




1004.4930

Phys.Rev.Lett.103:262001,2009

# Isovector Meson Spectrum



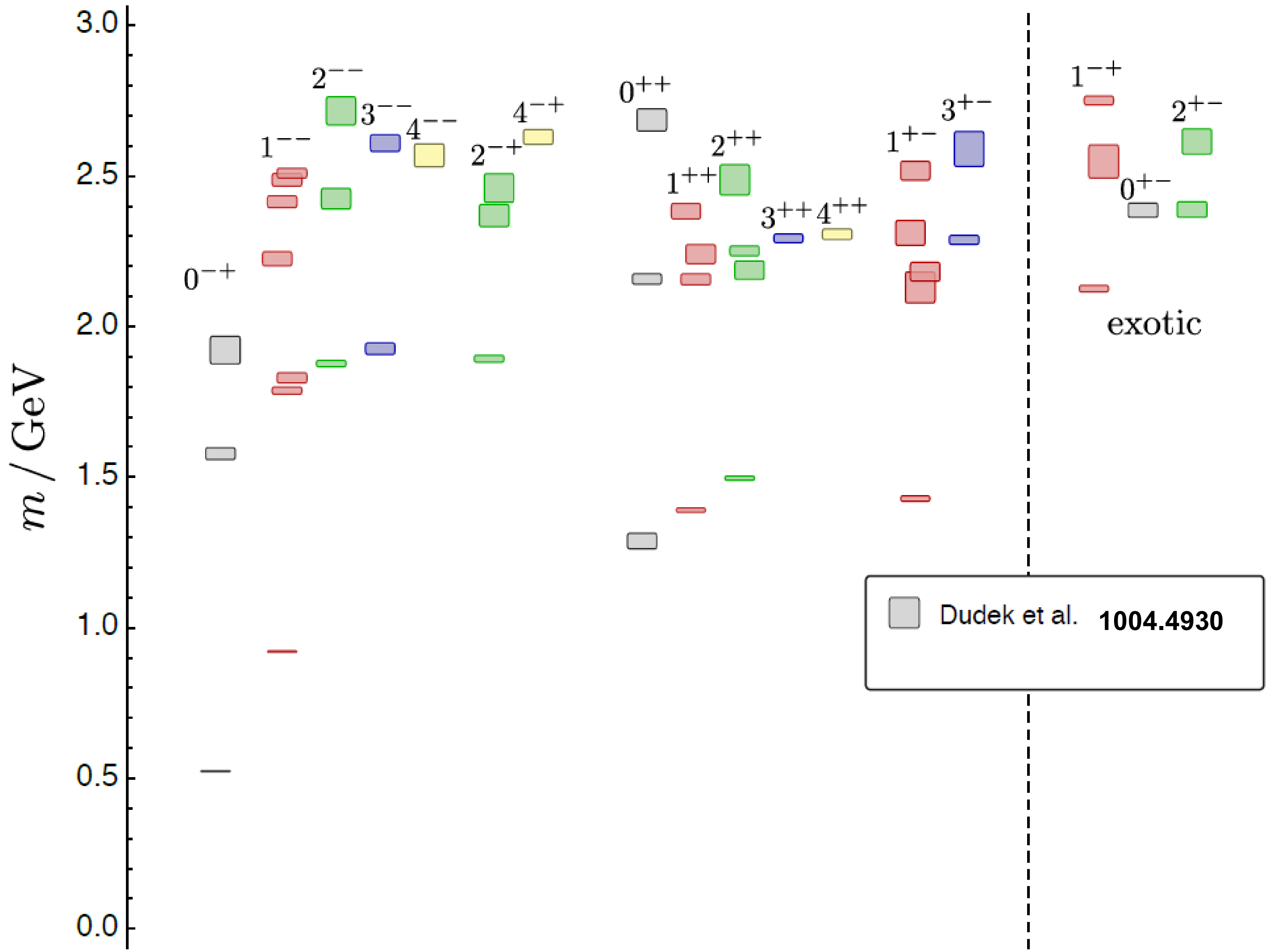

1004.4930

Phys.Rev.Lett.103:262001,2009

# Isovector Meson Spectrum

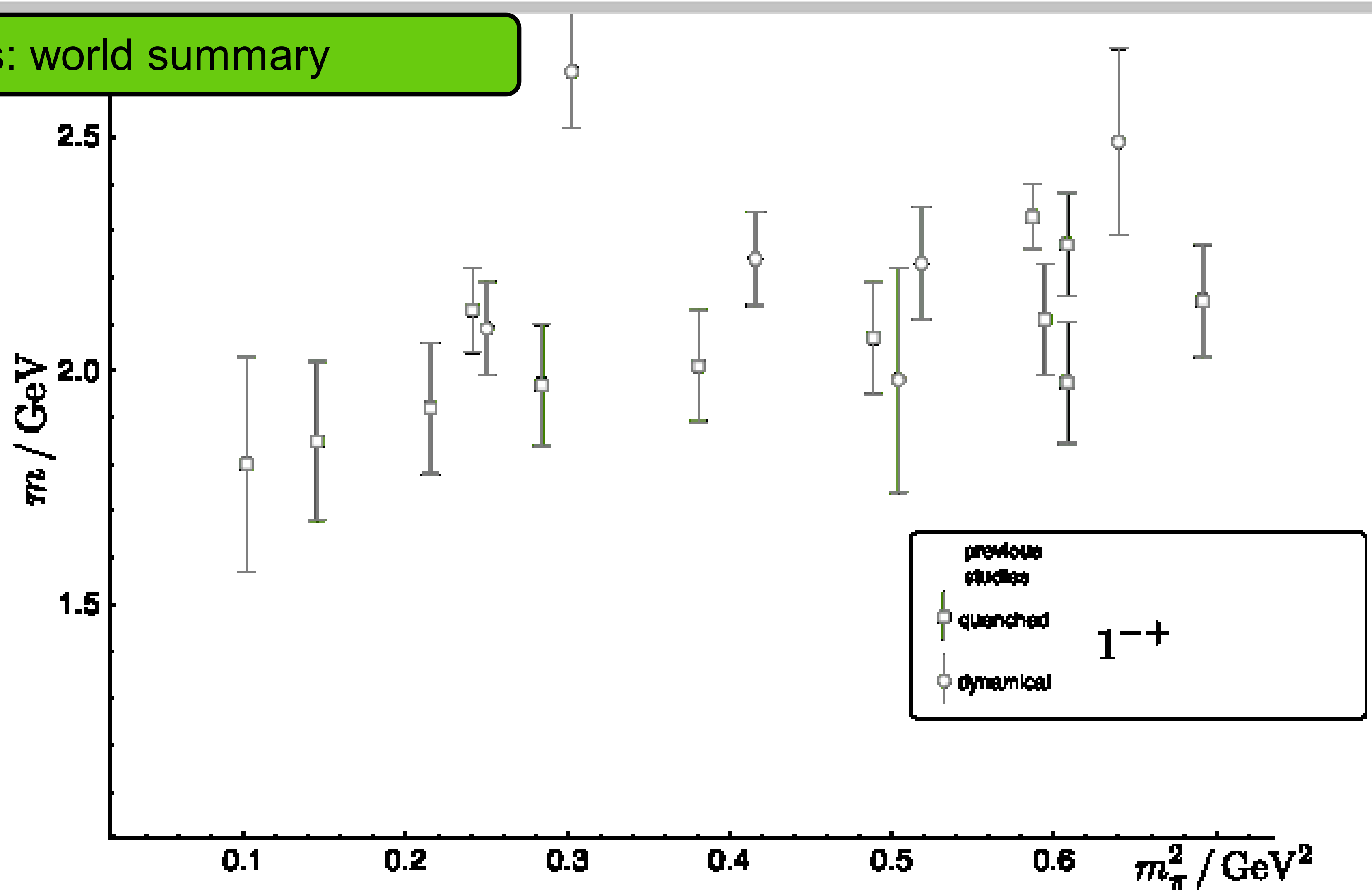
$N_f$=2+1, $m_\pi$ ~ 520MeV, L~2fm

Similar to Nf=3

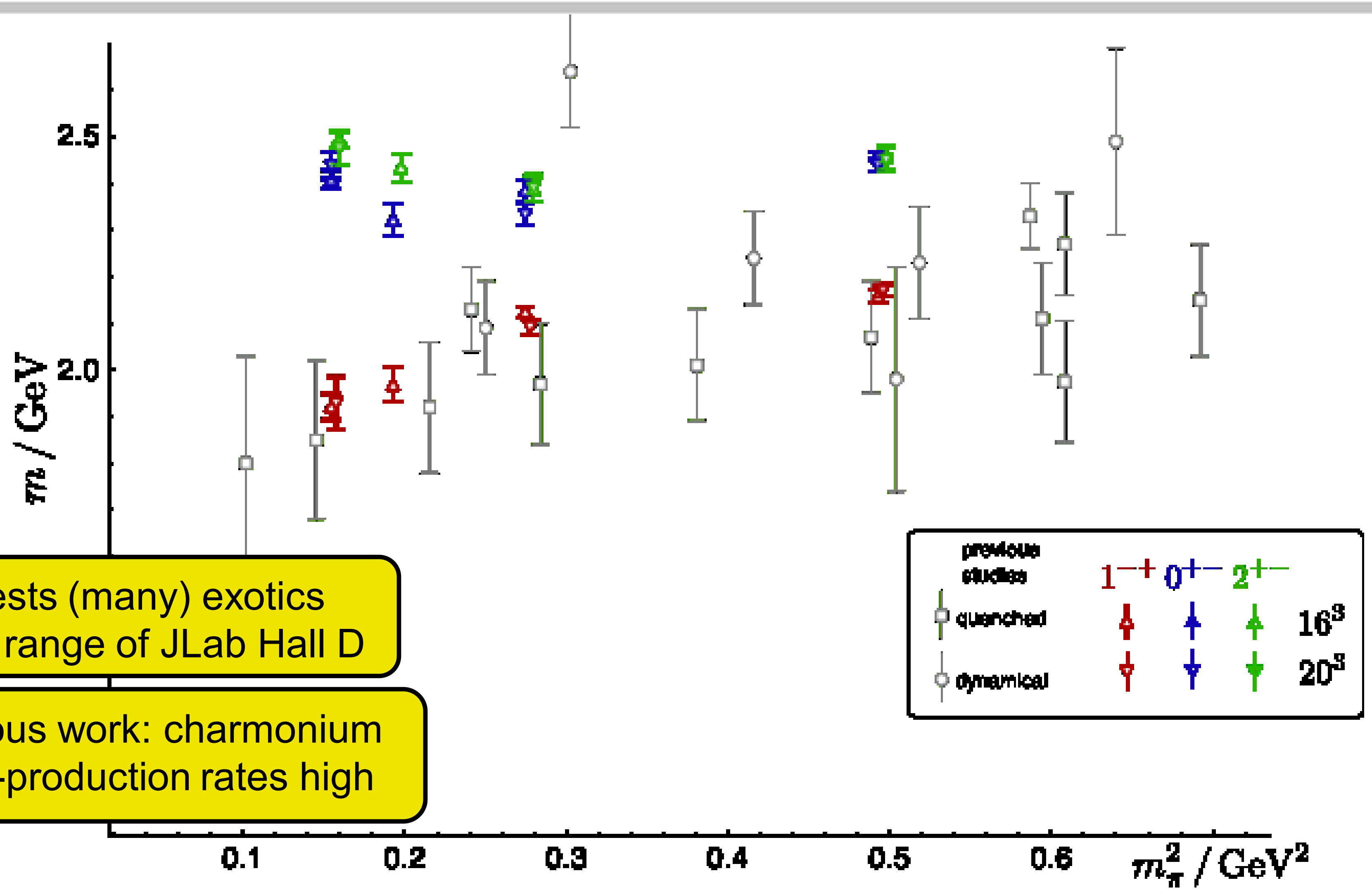


# Exotic matter

Exotics: world summary



# Exotic matter




Suggests (many) exotics within range of JLab Hall D

Previous work: charmonium photo-production rates high

**Current work: (strong) decays**

# Vector isoscalars (I=0)

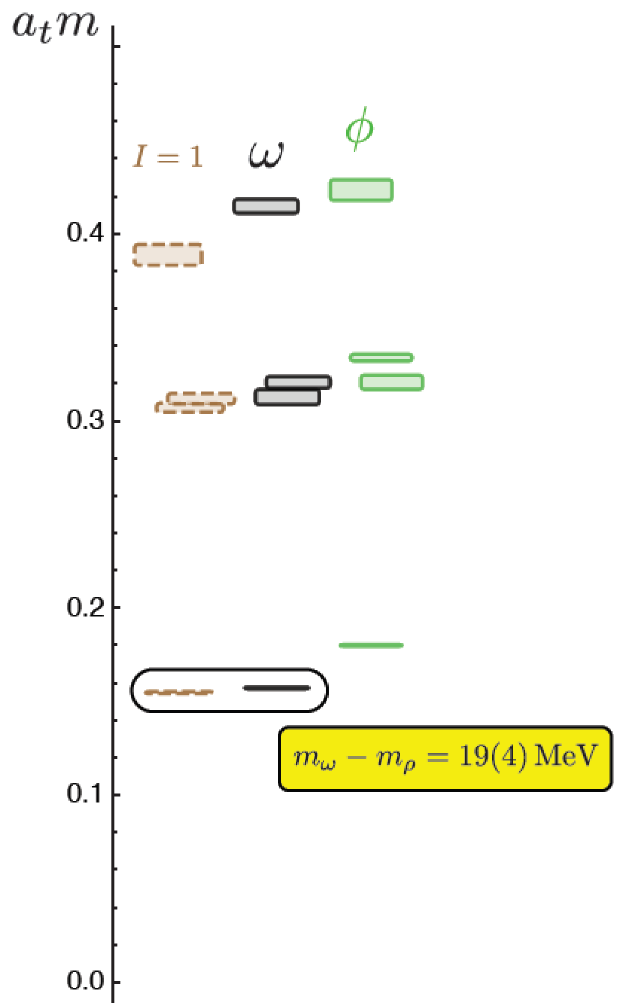
light-strange ($ls$) basis

$$\mathcal{O}_l^{\Gamma} = \frac{1}{\sqrt{2}}\left(\bar{u}\boldsymbol{\Gamma}\mathbf{u} + \bar{\mathbf{d}}\boldsymbol{\Gamma}\mathbf{d}\right)$$

$$\mathcal{O}_s^{\Gamma} = \bar{s}\boldsymbol{\Gamma}\mathbf{s}$$

I = 0
Must include all disconnected diagrams

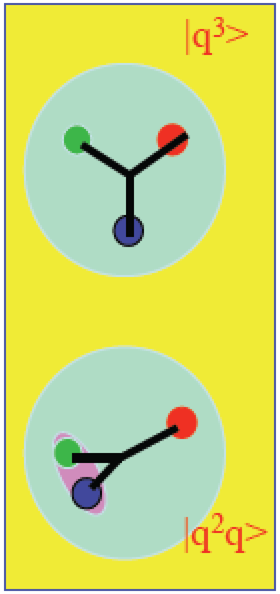
$N_f$=2+1, $m_\pi$ ~ 400MeV, L~2fm

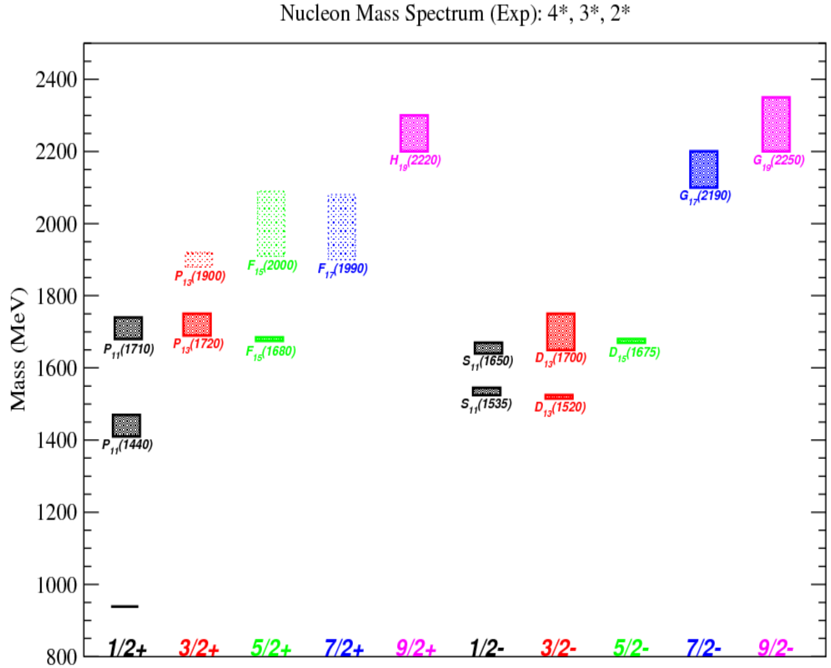


# Baryon Spectrum

"Missing resonance problem"

- What are collective modes?
- What is the structure of the states?
  - Major focus of (and motivation for) JLab Hall B
  - Not resolved experimentally @ 6GeV



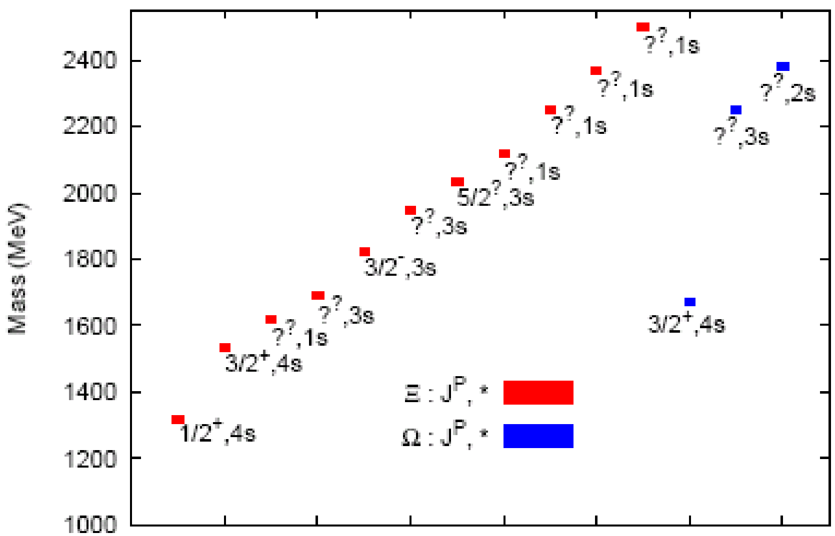

Nucleon spectrum

PDG uncertainty on B-W mass

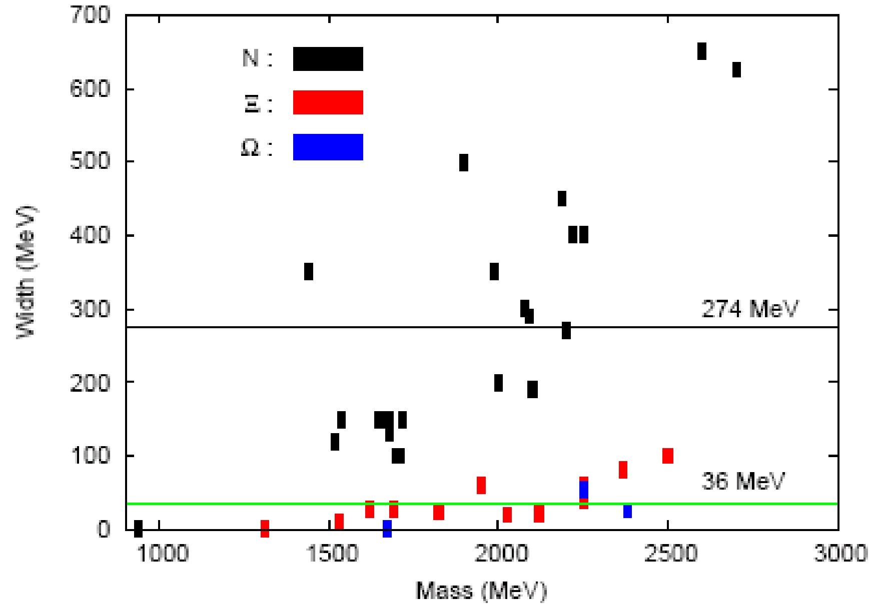
# Strange Quark Baryons

## Strange quark baryon spectrum poorly known

$\Xi$ & $\Omega$: unknown spin & parities




Widths are small




## Future:

- Narrow widths: easy(er) to extract (?)

# Light quark baryons in SU(6)

Conventional non-relativistic construction:

6 quark states in SU(6)

$$u_\uparrow, u_\downarrow, d_\uparrow, d_\downarrow, s_\uparrow, s_\downarrow$$

$$SU(6) \supseteq SU(3)_{\mathrm{Flavor}} \otimes SU(2)_{\mathrm{Spin}}$$

Baryons

$$\mathbf{6} \otimes \mathbf{6} \otimes \mathbf{6} = \mathbf{56}_S \oplus \mathbf{70}_{MS} \oplus \mathbf{70}_{MA} \oplus \mathbf{20}_A$$

$$\begin{aligned} \text{Symmetric} :& (\mathbf{10},\mathbf{4}) + (\mathbf{8},\mathbf{2}) = \mathbf{56} \\ \text{Mixed} :& (\mathbf{10},\mathbf{2}) + (\mathbf{8},\mathbf{4}) + (\mathbf{8},\mathbf{2}) + (\mathbf{1},\mathbf{2}) = \mathbf{70} \\ \text{Antisymmetric} :& (\mathbf{8},\mathbf{2}) + (\mathbf{1},\mathbf{4}) = \mathbf{20} \end{aligned}$$

# Relativistic operator construction: SU(12)

Relativistic construction: 3 Flavors with upper/lower components

$$SU(12) \subseteq SU(3)_{\rm Flavor} \otimes \underbrace{\left[ SU(2)_{\rm Upper/lower} \otimes SU(2)_{\rm Spin} \right]}_{\text{Dirac}}$$

Times space (derivatives)

$$\mathrm{Op}_S \leftarrow \mathrm{Derivative}_M \otimes \left[ \mathrm{Flavor}_M \otimes \mathrm{Dirac}_M \right]_M$$

Color contraction is Antisymmetric $\rightarrow$ Totally antisymmetric operators

More operators than SU(6): mixes orbital ang. momentum & Dirac spin

# Orbital angular momentum via derivatives

Couple derivatives onto single-site spinors:
Enough D's – build any J,M

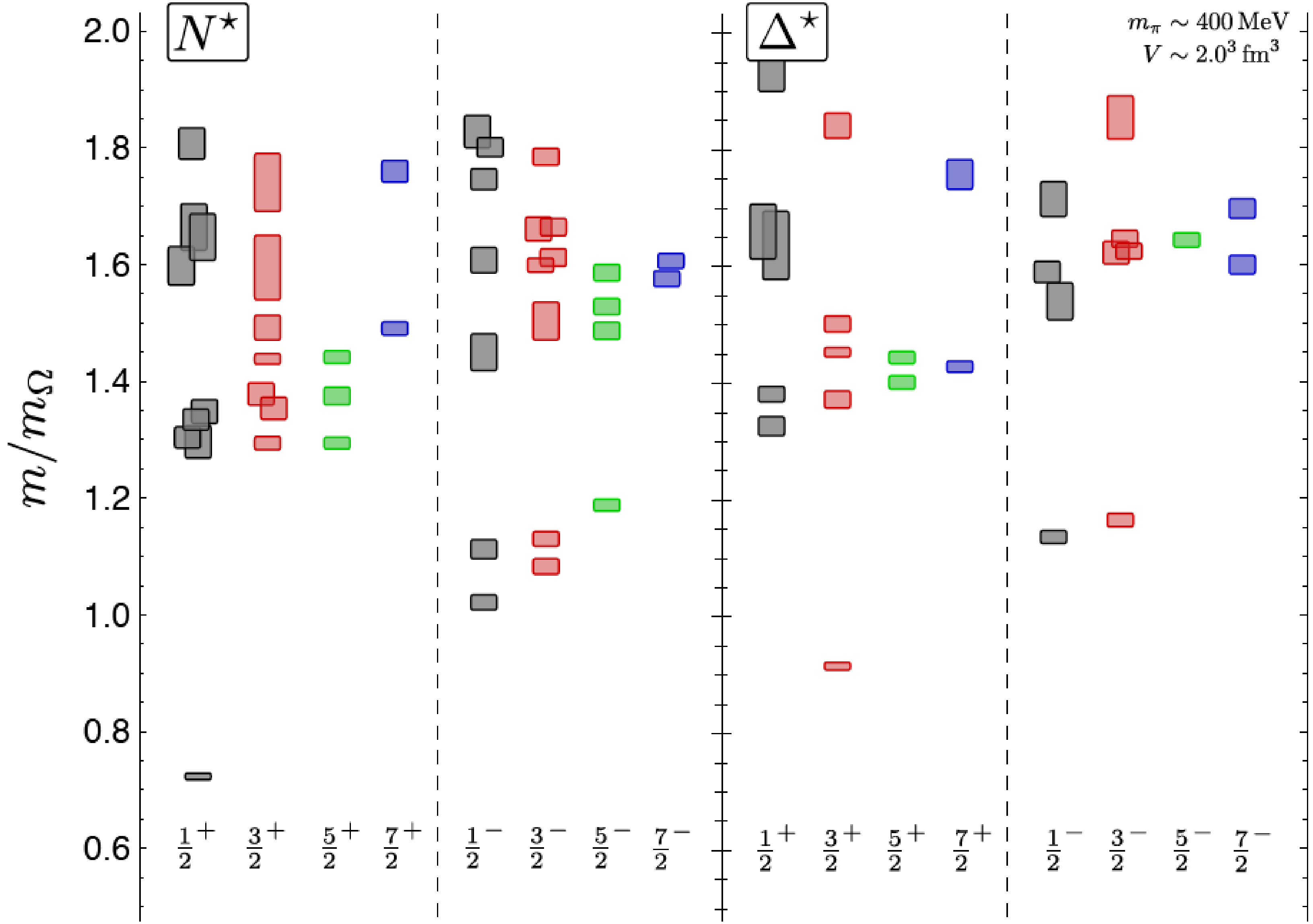
$$\mathcal{O}^{JM} \leftarrow (CGC's)_{i,j,k} \left[\vec{D}_M\right]_i \left[\vec{D}_M\right]_j [\Psi_M]_k$$

Only using symmetries of continuum QCD

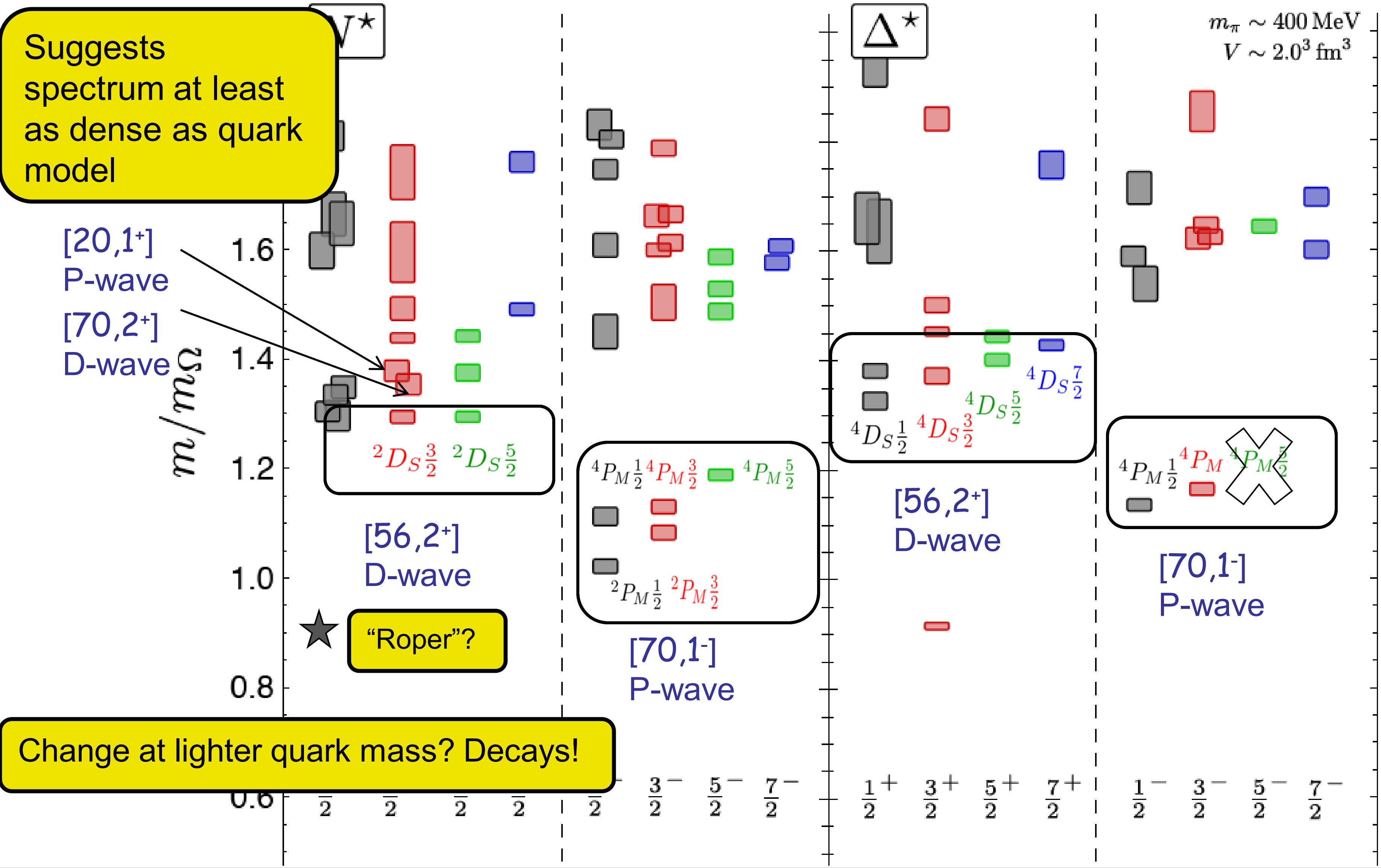
Use all possible **operators** up to 2 derivatives (2 units orbital angular momentum)

0905.2160 (PRD), 0909.0200 (PRL), 1004.4930

# Nucleon & Delta Spectrum



# Nucleon & Delta Spectrum



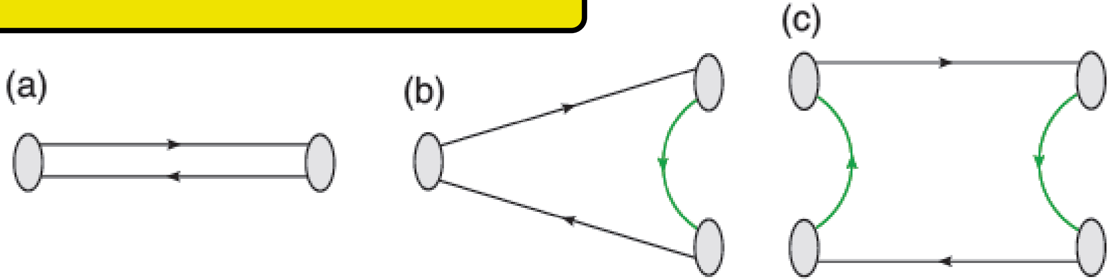
# Hadronic decays

Current spectrum calculations:
no evidence of multi-particle levels

Plot the non-interacting meson levels as a guide

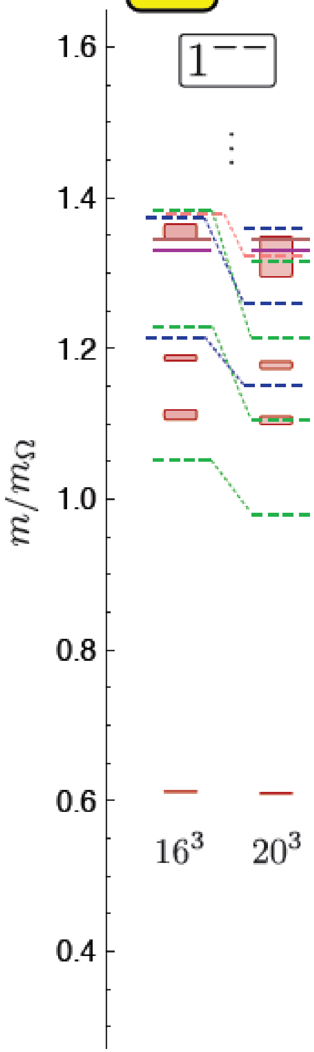
$\left|A(\vec{p})B(-\vec{p})\right\rangle \qquad m_{AB} = \sqrt{m_A^2 + \vec{p}^2} + \sqrt{m_B^2 + \vec{p}^2}$

Require multi-particle operators
- (lattice) helicity construction
- annihilation diagrams




Extract δ(E) at discrete E

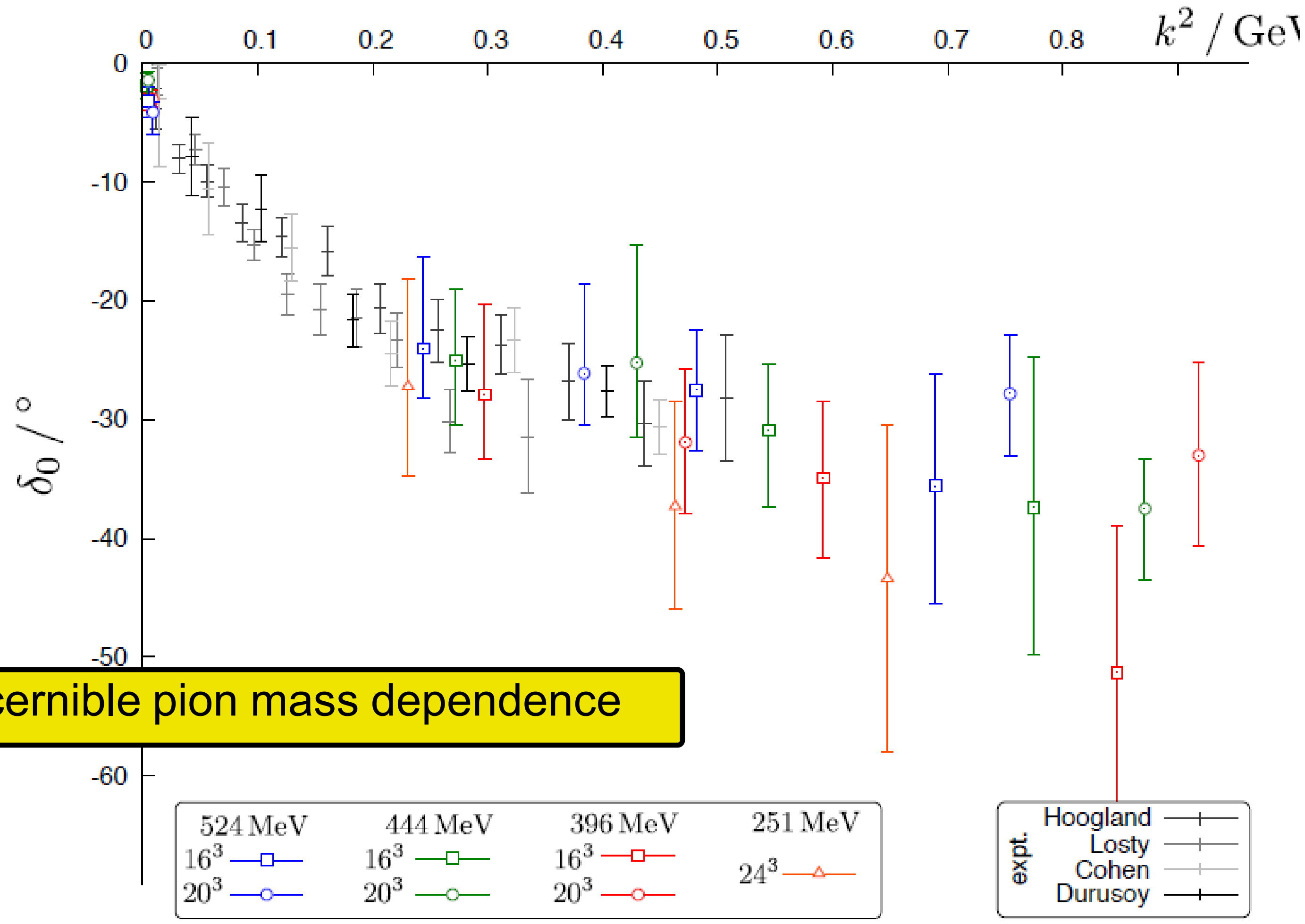
e.g.



# Phase Shifts: demonstration

$\pi\pi$ isospin=2

Extract $\delta_0(E)$ at discrete E

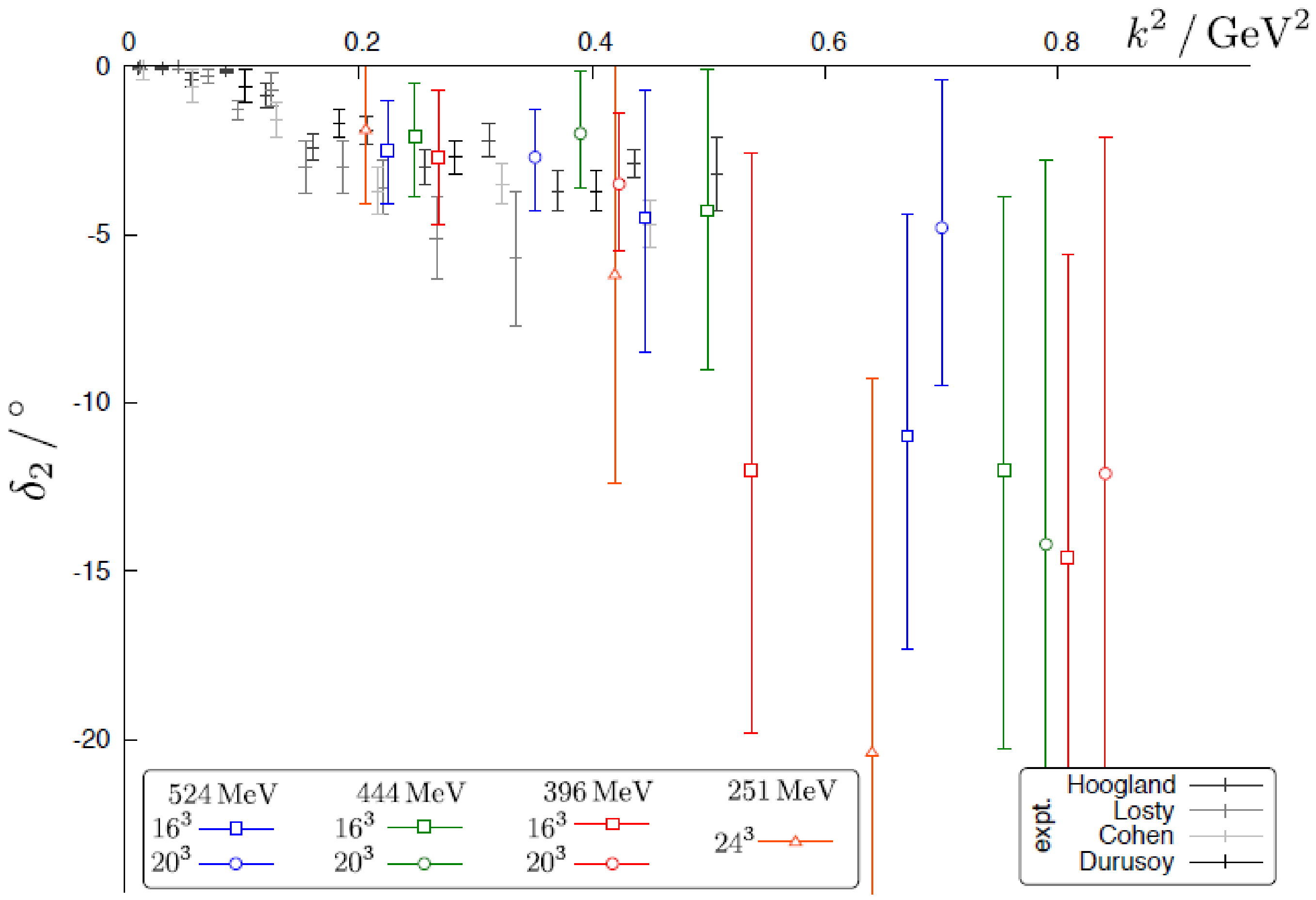



No discernible pion mass dependence

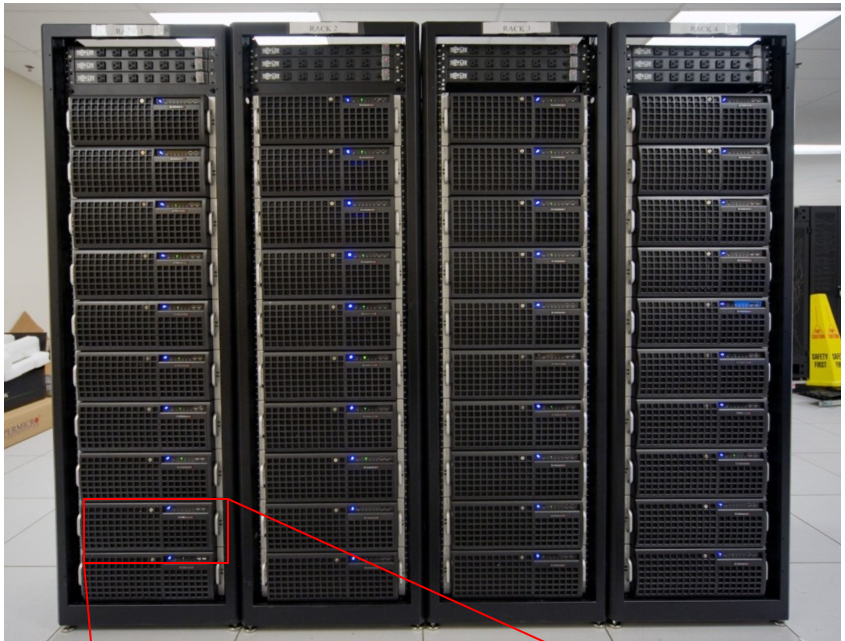
# Phase Shifts: demonstration

$\pi\pi$ isospin=2

$\delta_2(E)$



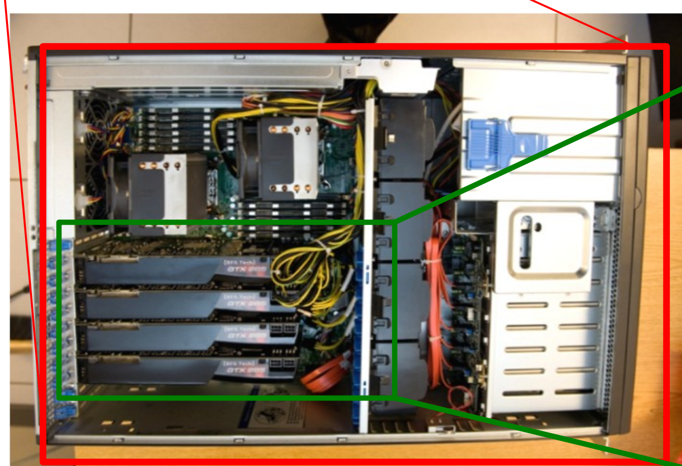
# Hardware: JLab GPU Clusters



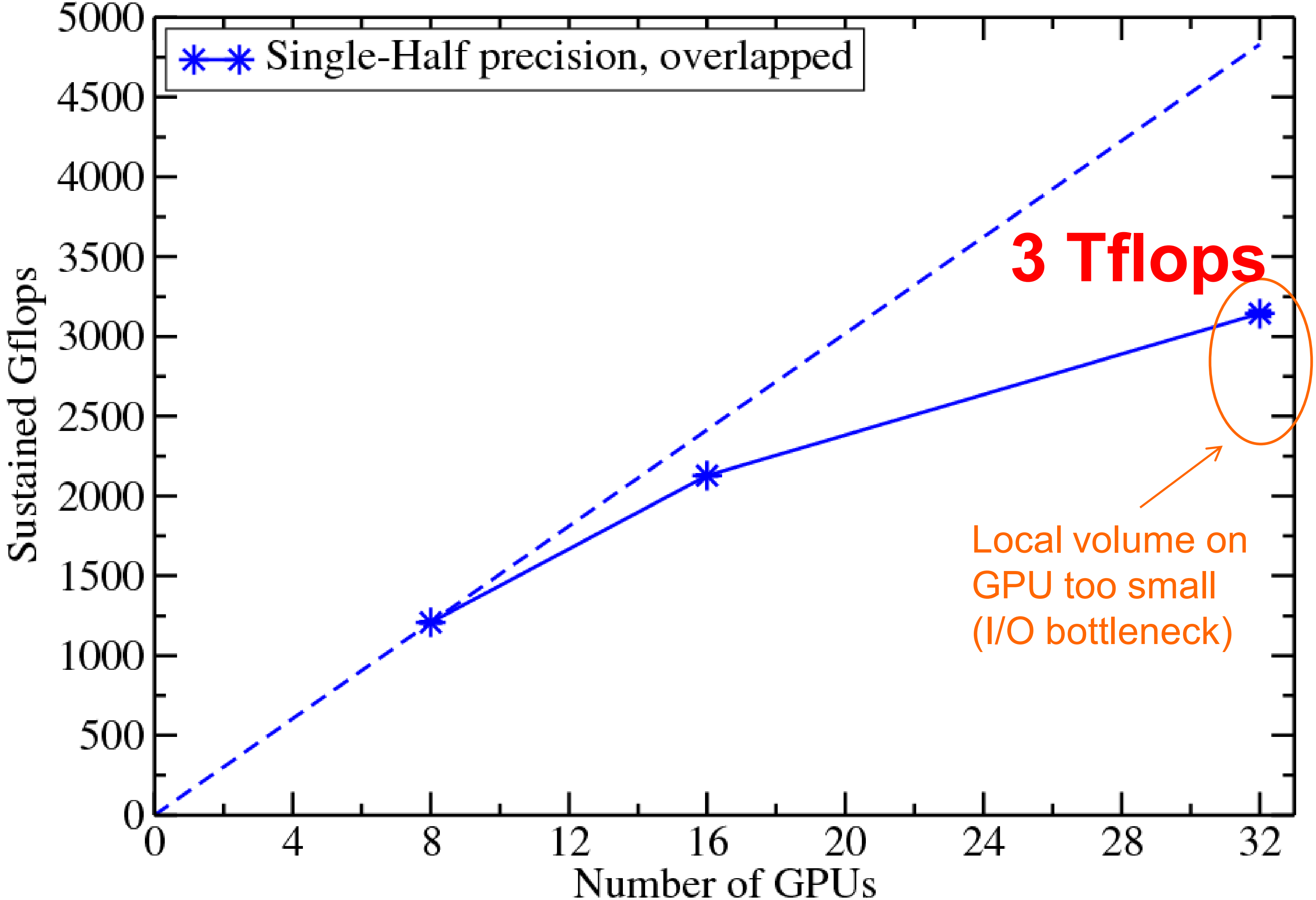
GPU clusters: ~530 cards

Quads
- 2.4 GHz Nehalem
- 48 GB memory / node
- 117 nodes x 4 GPUs -> 468 GPUs

Singles
- 2.4 GHz Nehalem
- 24 GB memory / node
- 64 nodes x 1 GPU -> 64 GPUs



# Inverter Strong Scaling: $V=32^3 \times 256$



# Prospects

- Strong effort in excited state spectroscopy
  - New operator & correlator constructions $\rightarrow$ high lying states
  - Finite volume extraction of resonance parameters – promising
  - Progress! Still much more to do
- Initial results for excited state spectrum:
  - Suggests baryon spectrum at least as dense as quark model
  - Suggests multiple exotic mesons within range of Jlab's Hall D
- Resonance determination:
  - Start at heavy masses: have some "elastic scattering"
  - Use larger volumes & smaller pion masses ($m_\pi$ ~230MeV)
  - Now: multi-particle operators & annihilation diagrams (gpu-s)
  - Need multi-channel finite-volume analysis for (in)elastic scattering
- Future:
  - Transition FF-s, photo-couplings (0803.3020, 0902.2214)