Preliminary results of $\Delta I = 1/2$ and $3/2$, K to $\pi\pi$ Decay Amplitudes from Lattice QCD

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Introduction

We do a direct, brutal force calculation of K to $\pi - \pi$ amplitudes.
Experiment facts:

- $\Delta I = \frac{1}{2}$ rule.

$$\frac{Re(A_0)}{Re(A_2)} = 25.7$$

- Direct CP violation in $K \to \pi\pi$ decays

$$Re(\epsilon'/\epsilon) = (1.65 \pm 0.26) \times 10^{-3}$$

$\sim 16\%$ error
(from PDG 2010 book)

Effective Harmiltonian

$$\langle(\pi\pi)_I|H_w|K^0\rangle = A_I e^{i\delta_I} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us} \sum_{i=1}^{10} [(z_i(\mu) + \tau y_i(\mu))\langle Q_i\rangle_I(\mu)]$$

- Current-Current operators(1,2):

$$Q_2 = (\bar{s}_\alpha d_\beta)_{V-A}(\bar{u}_\beta u_\alpha)_{V-A}$$

- QCD penguin operators(3,4,5,6):

$$Q_6 = (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V+A}$$

- Electroweak penguin operators(7,8,9,10):

$$Q_7 = \frac{3}{2}(\bar{s}_\alpha d_\alpha)_{V-A} \sum_{q=u,d,s} e_q(\bar{q}_\beta q_\beta)_{V+A}$$

Overview of the Steps to get A_0 and A_2

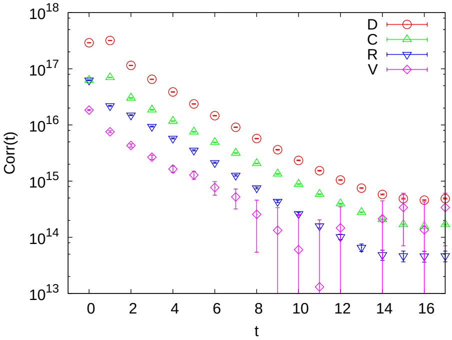
1. Lattice calculation: $Q_i^{lat}(a)$
   - $< \pi\pi(t)|\pi\pi(0) >= Z_{\pi\pi} Z^*_{\pi\pi}(e^{-E_{\pi\pi}t} + e^{-E_{\pi\pi}(T-t)} + C)$
   - $< K|K >= Z_k Z^*_k(e^{-m_k t} + e^{-m_k(T-t)})$
   - $< \pi\pi(t_\pi)|Q_i(t)|K(0) >= Q_i^{lat}(a) Z^*_{\pi\pi} Z_k e^{-E_{\pi\pi}t_\pi} e^{-(m_k - E_{\pi\pi})t}$
2. Renormalization: $Q_i^{cont}(\mu) = Z_{ij}(\mu, a) Q_j^{lat}(a)$
   - RI/MOM scheme vs. $\overline{MS}$ scheme
3. Wilson Coefficients: $z_i(\mu)$ and $y_i(\mu)$
4. Finite volume effect: Lellouch Luscher factor

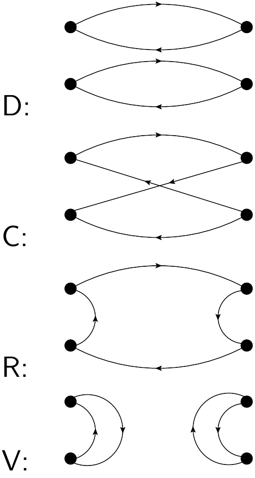
Computational specifics

- Lattice:
  - 2+1 flavor DWF, $m_s = 0.032$, $m_l = 0.01$
  - $16^3 \times 32$ space time volume with $L_s = 16$
  - Box size $L = 1.82 fm$, $m_\pi = 420 MeV$
- $K \to \pi\pi$ set up
  - Partially quenched strange quark $m_s(valence) = 0.066, 0.099, 0.165$
  - Periodic Boundary condition: Total momenum $\vec{P} = 0$, or $2\pi/L\hat{x}$
- Propagators: $D_w(t_{sink}; t_{src})$
  - Coulomb Gauge Fixed Wall Source and Sink
  - Propagators are calculated on all time slices T=32 ($\times$ 12 inversion)
    $\to$ Huge statistics $\times$ 400 configurations
    $\to$ Resolve signal from disconnect diagrams
  - Eigenvector accelerator code, provided by Ran Zhou

$\pi\pi$ scattering

$$< 20(t)|20(0) > \; = \; 2(D-C)$$
$$< 00(t)|00(0) > \; = \; 2D+C-6R+3V$$

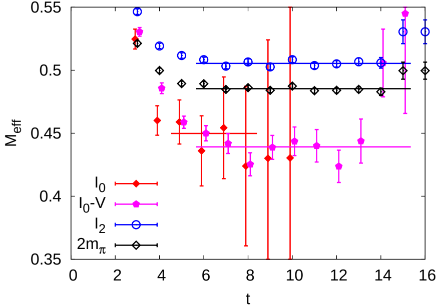




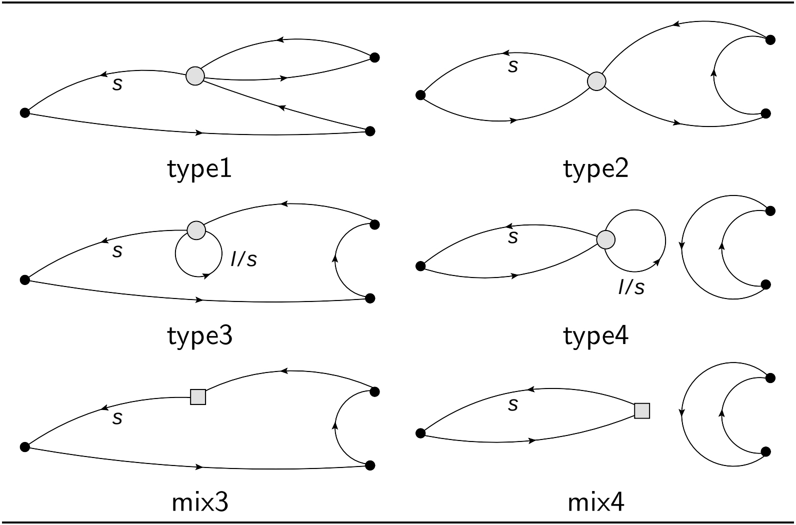
$\pi\pi$ energy

P	E_π	E_{I0}	E_{I0V}	E_{I2}
0	0.24267(68)	0.450(17)	0.4392(59)	0.5054(15)
1	0.4698(35)	0.753(25)	0.6987(87)	0.7382(39)

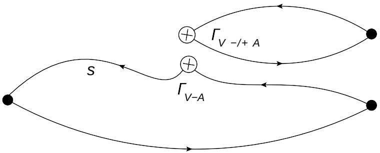
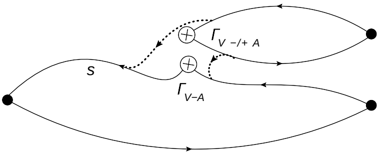
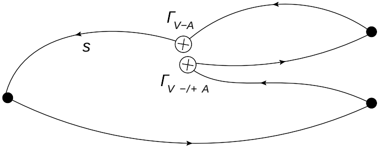
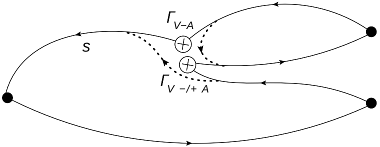
P	$E_k(0)$	$E_k(1)$	$E_k(2)$
0	0.4255(6)	0.5070(6)	0.6453(7)
1	0.5855(16)	0.6485(15)	0.7647(14)



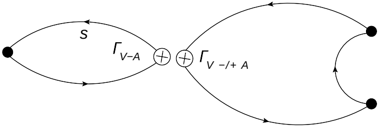
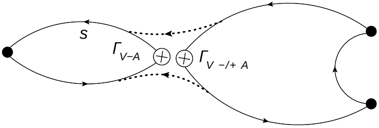
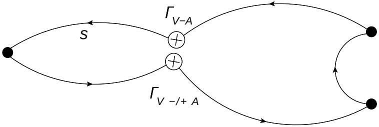
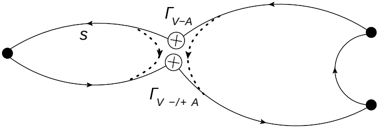
$K^0 \to \pi\pi(I=0)$ contractions



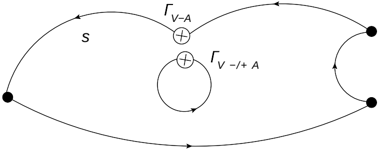
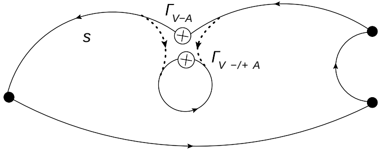
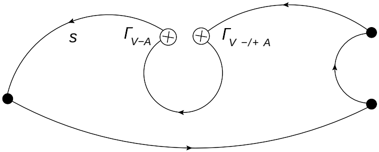
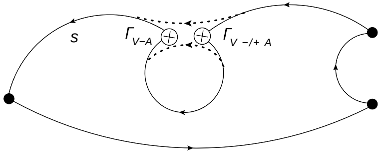
Contraction details

Color unmixed diagrams	Color mixed diagrams
	
1/3	2/4
	
5/7	6/8
⑥ = −①	⑤ = −②

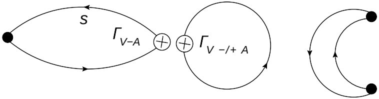
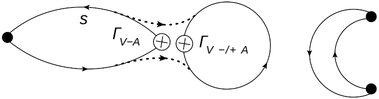
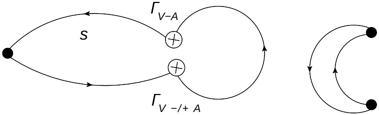
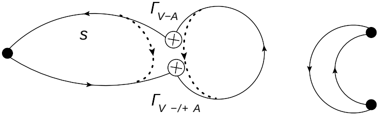
Contraction details

Color unmixed diagrams	Color mixed diagrams
 9/11	 10/12
 13/15	 14/16
(14) $= -$(9)	(13) $= -$(10)

Contraction details

Color unmixed diagrams	Color mixed diagrams
	
17/19/21/23	18/20/22/24
	
25/27/29/31	26/28/30/32

Contraction details

Color unmixed diagrams	Color mixed diagrams
 33/35/37/39	 34/36/42/44
 41/43/45/47	 42/44/46/48

Contraction of Q_2 and Q_6

$$< 00|Q_2|K^0 > \quad = \quad i\frac{1}{\sqrt{3}}\{-\textcircled{2} - 2\cdot\textcircled{6} + 3\cdot\textcircled{10} + 3\cdot\textcircled{18} - 3\cdot\textcircled{34}\}$$

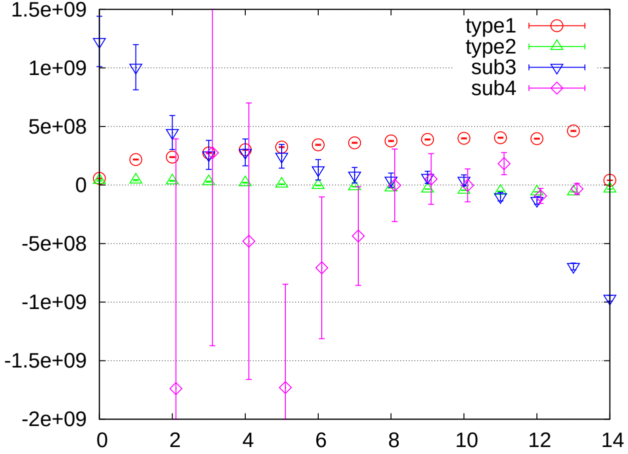
$$< 00|Q_6|K^0 > \quad = \quad i\sqrt{3}\{-\textcircled{8} + 2\cdot\textcircled{12} - \textcircled{16} + 2\cdot\textcircled{20} + \textcircled{24} - \textcircled{28} - \textcircled{32} - 2\cdot\textcircled{36} - \textcircled{40} + \textcircled{44} + \textcircled{48}\}$$

For simplicity, we write the contribution to each operator as

$$\begin{aligned} & < 00(t_\pi)|Q_i(t)|K^0(0) >_{sub} \\ = \quad & < 00(t_\pi)|Q_i(t)|K^0(0) > -\alpha_i < 00(t_\pi)|\bar{s}\gamma_5 d(t)|K^0(0) > \\ = \quad & Type1 + Type2 + Type3 + Type4 - Mix3 - Mix4 \\ = \quad & Type1 + Type2 + (Type3 - Mix3) + (Type4 - Mix4) \\ = \quad & Type1 + Type2 + Sub3 + Sub4 \end{aligned}$$

The subtraction coefficient can be calculated from $\alpha_i = \frac{<0|Q_i(t)|K^0(0)>}{<0|\bar{s}\gamma_5 d(t)|K^0(0)>}$

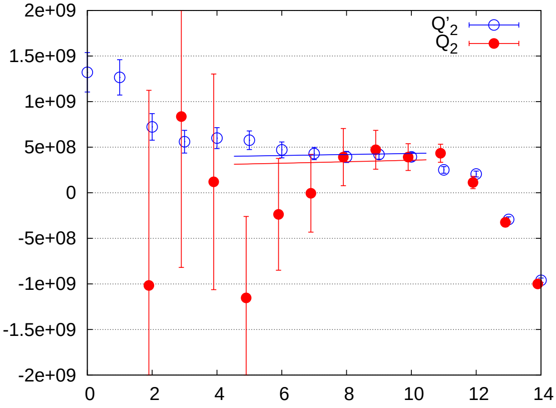
Operator Q_2



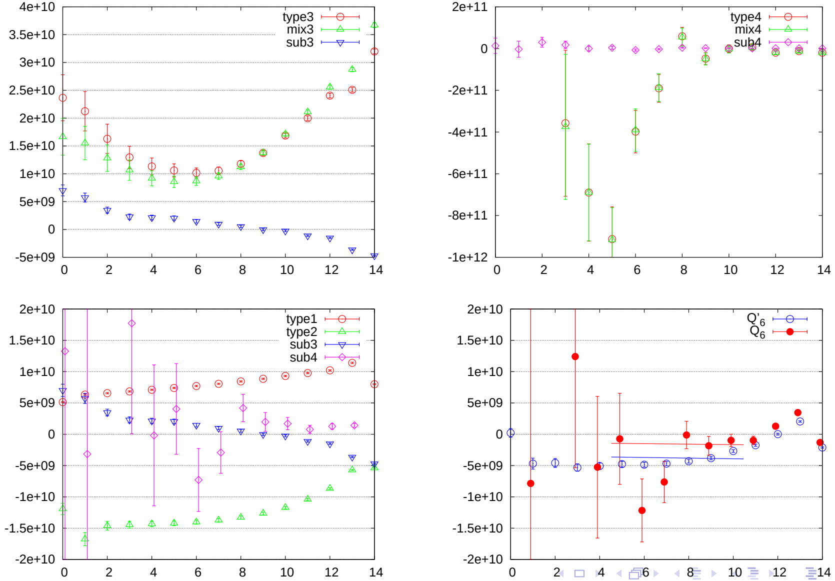
K^0 (t=0) $\Longrightarrow Q_2(t) \Longrightarrow$ $\pi\pi(I=0)$ (t=14)

Operator Q_2

$$< \pi\pi(t_\pi)|Q_i(t)|K(0) >= Q_i^{lat}(a)\, Z^*_{\pi\pi} Z_k e^{-E_{\pi\pi} t_\pi} e^{-(m_k - E_{\pi\pi})t}$$



Operator Q_6


type3
mix3
sub3
4e+10
3.5e+10
3e+10
2.5e+10
2e+10
1.5e+10
1e+10
5e+09
0
-5e+09
0 2 4 6 8 10 12 14
type4
mix4
sub4
2e+11
0
-2e+11
-4e+11
-6e+11
-8e+11
-1e+12
0 2 4 6 8 10 12 14
type1
type2
sub3
sub4
2e+10
1.5e+10
1e+10
5e+09
0
-5e+09
-1e+10
-1.5e+10
-2e+10
0 2 4 6 8 10 12 14
Q'6
Q6
2e+10
1.5e+10
1e+10
5e+09
0
-5e+09
-1e+10
-1.5e+10
-2e+10
0 2 4 6 8 10 12 14

Fitting Results

i	$Q_i'(a)$	$Q_i(a)$	% to $Re(A_0)$	% to $Im(A_0)$
1	-6.5(38)e-03	-4(12)e-03	8.5	0
2	1.75(14)e-02	1.37(52)e-02	91.6	0
3	1.0(10)e-02	1.2(33)e-02	0.003	6.8
4	3.39(80)e-02	2.9(27)e-02	0.50	-61.1
5	-5.04(91)e-02	-1.7(30)e-02	-0.03	-2.7
6	-1.59(10)e-01	-6.4(40)e-02	-0.37	141.2
7	1.435(44)e-01	1.16(12)e-01	0.02	-0.48
8	4.42(11)e-01	3.49(24)e-01	-0.14	9.6
9	-1.50(29)e-02	-1.0(10)e-02	-0.0003	5.8
10	9.2(29)e-03	6.6(97)e-03	0003	0.82

Q_i' means the result without the fully disconnected graph (no type4 graph).

Fitting range [5:10]

Combine everything together

$$A_I = \frac{G_F}{\sqrt{2}} V_{ud} V_{us} \sum_{i=1}^{10} \{(z_i(2.15) + \tau y_i(2.15)) Z_{ij}(2.15, a) \langle Q_j \rangle_I(a)\}$$

1. Wilson Coefficients $z_i(2.15GeV)$ and $y_i(2.15GeV)$.
2. Renormalization factor $Z_{ij}(2.15GeV, a^{-1} = 1.73GeV)$

   Taken from Our group's previous paper(Shu Li).
3. Finite Volumn effect (and normalization of states on lattice)
   - Lellouch Luscher factor

     $$|A|^2 = 8\pi\gamma^2(\frac{E_{\pi\pi}}{p})^3\{p\frac{\partial\delta(p)}{\partial p} + q\frac{\partial\phi(q)}{\partial q}\}|M|^2 = F^2|M|^2$$

   - Free field limit ($\vec{P} = 0$ case)

     $$|A|^2 = 4(2m_\pi)^2 m_K L^3 |M|^2 = F_f^2 |M|^2$$

These are derived by assuming that $< p|p' >= 2p_0(2\pi)^3\delta(\vec(p) - \vec(p'))$, and Particle states in finite volume are normalized to unity.

$Re(A_0)$ and $Im(A_0)$

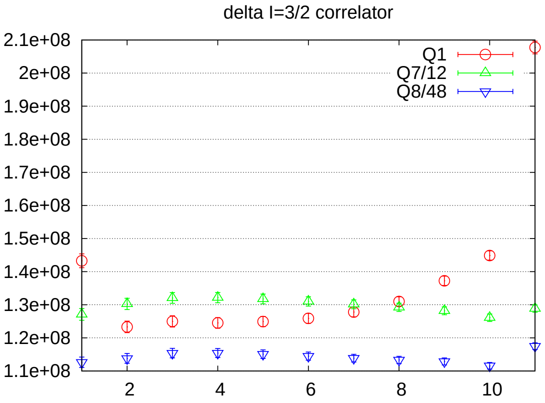
Notice that $E_{I=0} = 0.450(17)$ and $E_{I=0vout} = 0.4392(59)$

m_K	F_f	$Re(A_0')(GeV)$	$Re(A_0)(GeV)$	$Im(A_0')(GeV)$	$Im(A_0)(GeV)$
0.4255(6)	40.5	$53.5(2.9)e^{-8}$	$40(11)e^{-8}$	$-87.8(7.4)e^{-12}$	$-30(28)e^{-12}$
0.5070(6)	44.2	$61.5(3.4)e^{-8}$	$50(14)e^{-8}$	$-95.7(7.8)e^{-12}$	$-68(38)e^{-12}$
on shell	-	$54.8(3.0)e^{-8}$	$43(12)e^{-8}$	$-89.2(7.5)e^{-12}$	$-41(31)e^{-12}$

From $t_\pi - t_K = 14$, and fitting range [5:10]

Used Free field normalization of states.
For I=0, it is very difficult to apply Lellouch Luscher factor here given the small volume. Numerically, $\partial\phi(q)/\partial q$ becomes divergent at $q^2 = -0.06639$ which correspond to $E_{I0} = 0.441$. Luscher's derivation requires that the Interaction range $R < L/2$. If we plug in $E_{I0} = 0.450$, we'll get $F = 90 \approx 2F_f$.

$\Delta I = 3/2\ K \to \pi\pi$ correlators.



Zero momentum results (420MeV pion mass)

$Re(A_2)$ and $Im(A_2)$, Used Lellouch Luscher factor

m_K	E_{I2}	F	$Re(A_2)(GeV)$	$Im(A_2)(GeV)$
0.5070(6)	0.5054(15)	36.8(2)	$7.629(64)e^{-8}$	$-1.102(11)e^{-12}$

From $t_\pi - t_K$=12, fitting range [5:7] to get more accurate result.

Compare with A_0:

$Re(A_0')$	$Re(A_0)$	$Im(A_0')$	$Im(A_0)$
$54.8(3.0)e^{-8}$	$43(12)e^{-8}$	$-89.2(7.5)e^{-12}$	$-41(31)e^{-12}$

At this highly unphysical kinematics point: $m_\pi = 420MeV$

- $\frac{Re(A_0): K^0(778) \to \pi(420)\pi(420)}{Re(A_2): K^0(874) \to \pi(420)\pi(420)} \sim 6$ (or 13 using the LL factor)
- $\epsilon' = ie^{i(\delta_2-\delta_0)} \frac{1}{\sqrt{2}} \frac{Re(A_2)}{Re(A_0)} [\frac{Im(A_2)}{Re(A_2)} - \frac{Im(A_0)}{Re(A_0)}] \sim ie^{i(\delta_2-\delta_0)} 10(9)e^{-6}$

Non zero total Momemtum, 420MeV pion, $\vec{P} = 679MeV$

$Re(A_2)$ and $Im(A_2)$

E_K	E_{I2}	F	$Re(A_2)(GeV)$	$Im(A_2)(GeV)$
0.6485(15)	0.7382(39)	43.5(3)	$11.11(11)e^{-8}$	$-0.744(16)e^{-12}$
0.7647(14)	0.7382(39)	43.5(3)	$12.23(12)e^{-8}$	$-0.604(14)e^{-12}$

$Re(A_0)$ and $Im(A_0)$ Barely see a signal even without the fully disconnected graph. Lellouch Luscher factor are calculated($p \neq 0$).

E_K	E_{I0}	F	$Re(A'_0)(GeV)$	$Im(A'_0)(GeV)$
0.6485(15)	0.6987(87)	38(1)	$44(16)e^{-8}$	$-91(30)e^{-12}$
0.7647(14)	0.6987(87)	38(1)	$36(12)e^{-8}$	$-57(24)e^{-12}$

From $t_\pi - t_K = 12$, fitting range [5:8]

Conclusion

We did a trial direct $K \to \pi\pi$ for both $\Delta I = 1/2$ and $3/2$ calculation.

- Pros
  - A direct calculation is possible.
  - Divergent subtraction is needed and shown.
  - 25% statistical errors for $Re(A_0)$
  - Compare results from 100 confs and 400 confs. Error does scale like $1/\sqrt{N}$
- Cons
  - $16^3 \times 32$ lattice
  - $m_\pi = 420 MeV$
  - Zero momentum or too big error

Future

We need huge ammount of statistics.

- Computer: Faster machine ~ 100. (comes late this year)
- Algorithm: Deflation, EigenCG, multigrid, other CG ~ 10 ?

Toward physical case

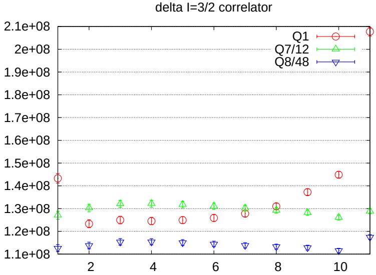
- $32^3 \times 64$ lattice
- $m_\pi = 140$MeV: Better signal for the disconnected graph because signal decrease slower while the noisy is a constant.
- Big volume: Means more statistics
- How to include momentum: G-parity or Periodic Boundary condition?

How much is it more difficult? for each configuration, it is $\sim 32 * 5 * 2 = 3200$ times more difficult.

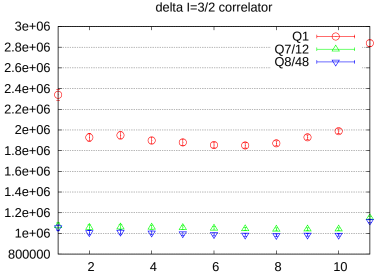
Thank you!

Back Up

$\Delta I = 3/2\ K \to \pi\pi$ correlators. Both cases are close to on shell.



p=0, s=1



p=1, s=2