

Conformal or Walking?
Monte Carlo Renormalization Group studies in
technicolor-inspired models

LGT10 - CERN, July 2010

Anna Hasenfratz
University of Colorado

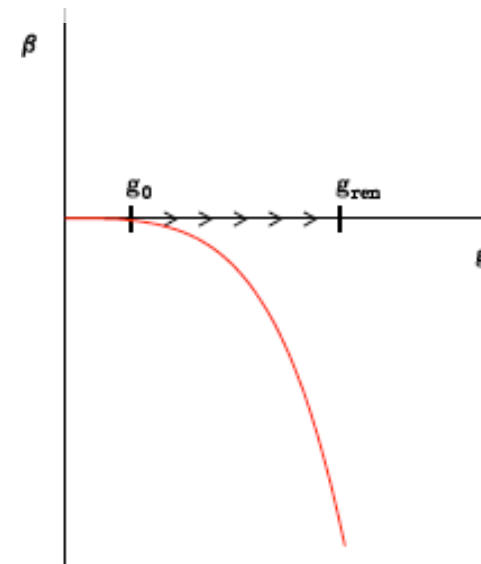
Beyond the Standard Model:

Technicolor-inspired electroweak symmetry breaking

The nature of electroweak symmetry breaking is one of the fundamental issues LHC could reveal. Some of the theoretical models require only a gauge theory with fermions:

Few fermions: QCD like

- Asymptotically free, chirally broken and confining
- Scaled-up QCD Technicolor does not satisfy electroweak precision tests



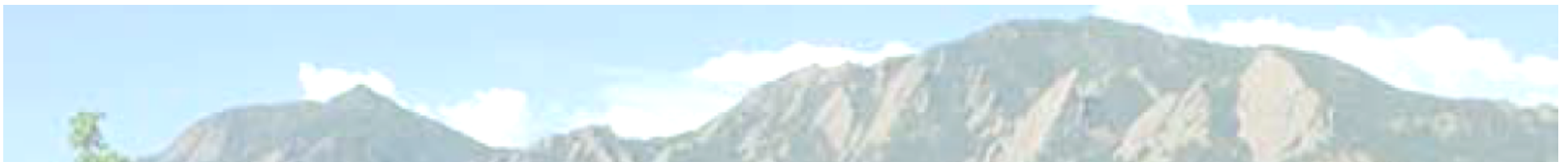
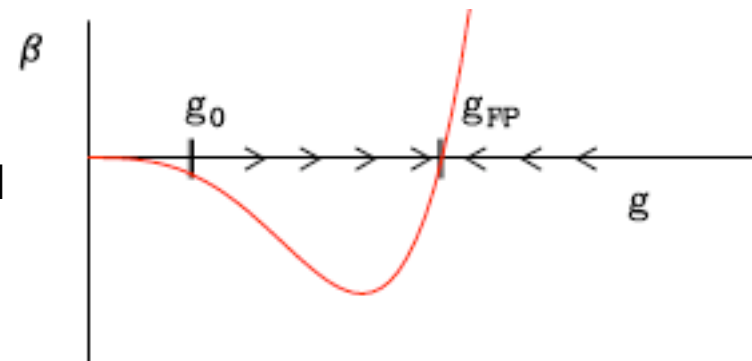
Beyond the Standard Model:

Technicolor-inspired electroweak symmetry breaking

The nature of electroweak symmetry breaking is one of the fundamental issues LHC could reveal. Some of the theoretical models require only a gauge theory with fermions:

More fermions: conformal systems

- Asymptotically free
- The gauge coupling develops an infrared fixed point and becomes an irrelevant operator.
- Unparticles or ?



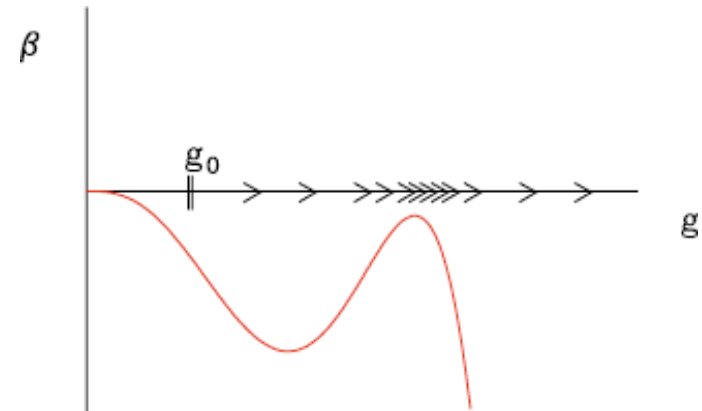
Beyond the Standard Model:

Technicolor-inspired electroweak symmetry breaking

The nature of electroweak symmetry breaking is one of the fundamental issues LHC could reveal. Some of the theoretical models require only a gauge theory with fermions:

Just below the conformal window: walking

- AF, confining, chirally broken
- The gauge coupling is walking
- Best option for technicolor **if** it has a large anomalous mass dimension across a large energy scale



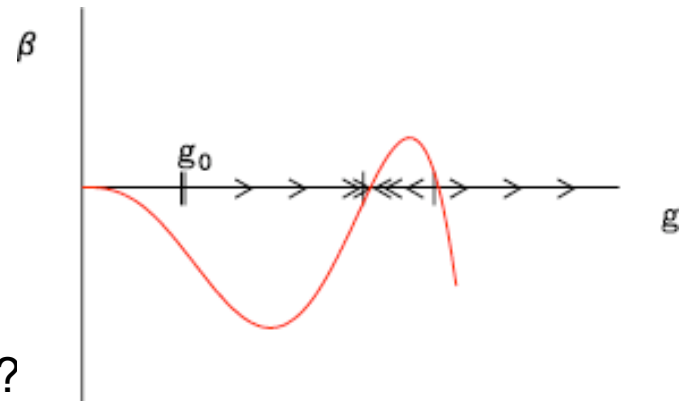
Beyond the Standard Model:

Technicolor-inspired electroweak symmetry breaking

The nature of electroweak symmetry breaking is one of the fundamental issues LHC could reveal. Some of the theoretical models require only a gauge theory with fermions:

Fantasy? or Future?

- AF, conformal at weak coupling
- Second FP (UV) at strong coupling:
What kind of IR physics does it describe?
Which operators are relevant?

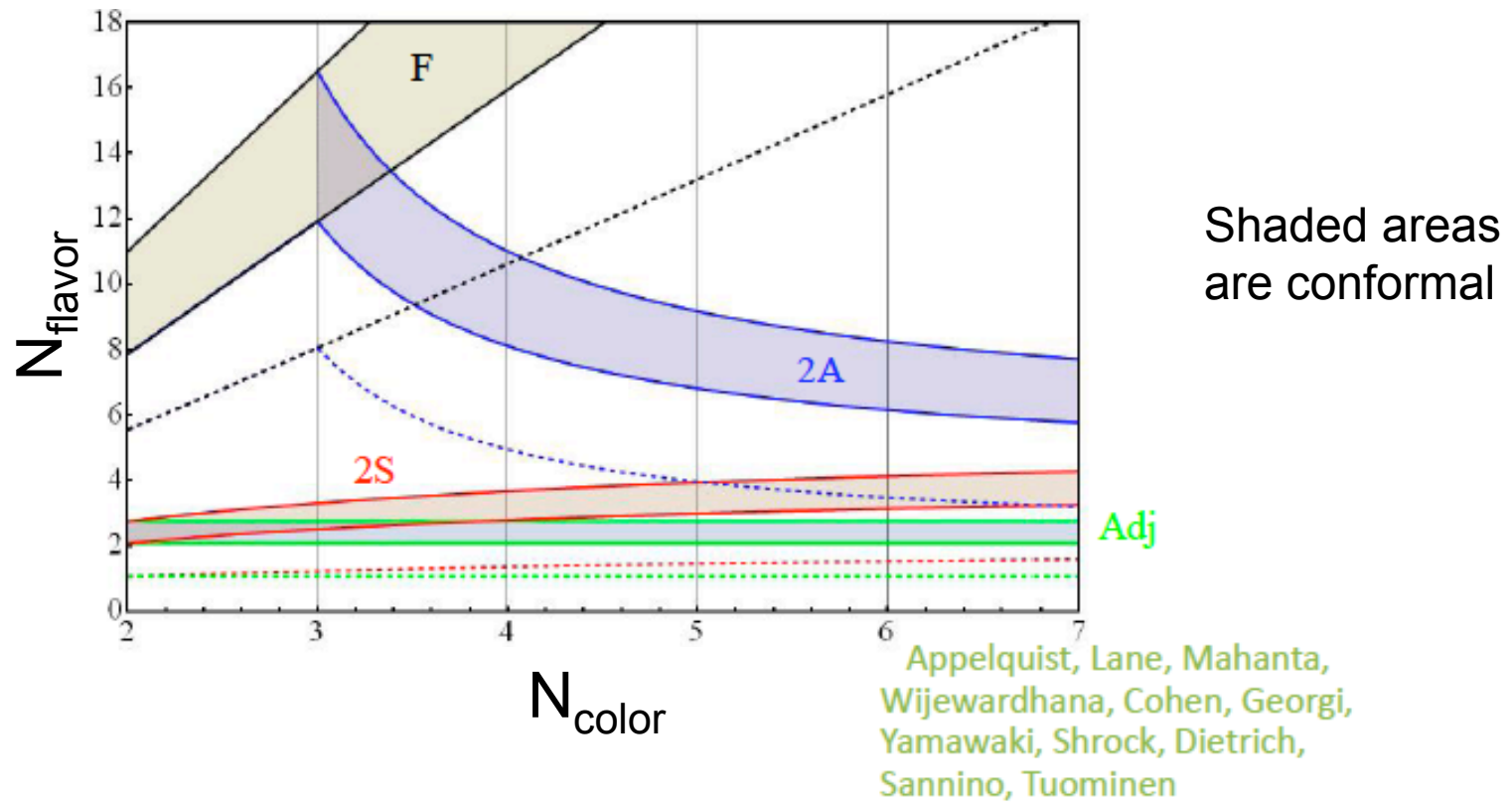


(Kaplan et al)



Roadmap for the conformal window

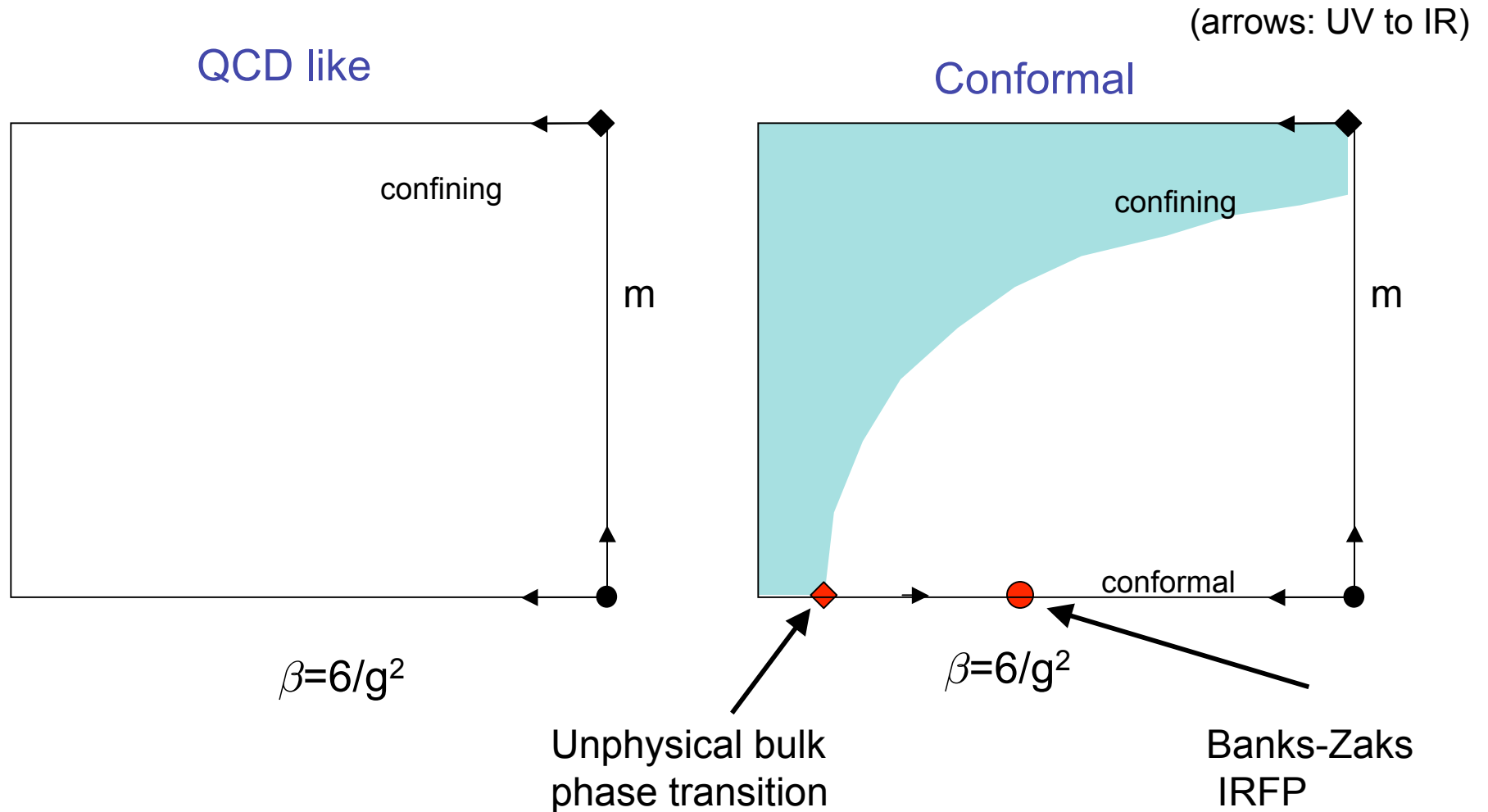
S-D type calculations



Needs non-perturbative verification!

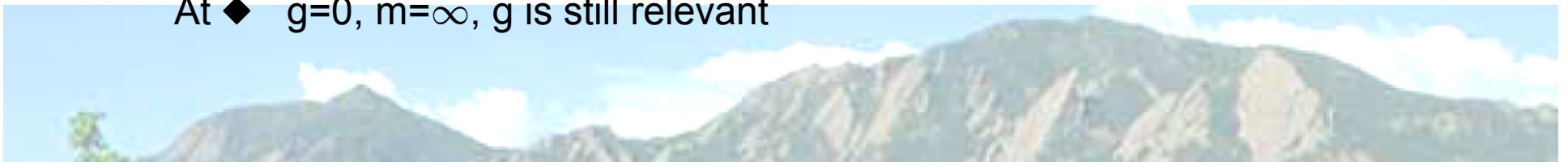


The lattice phase diagram



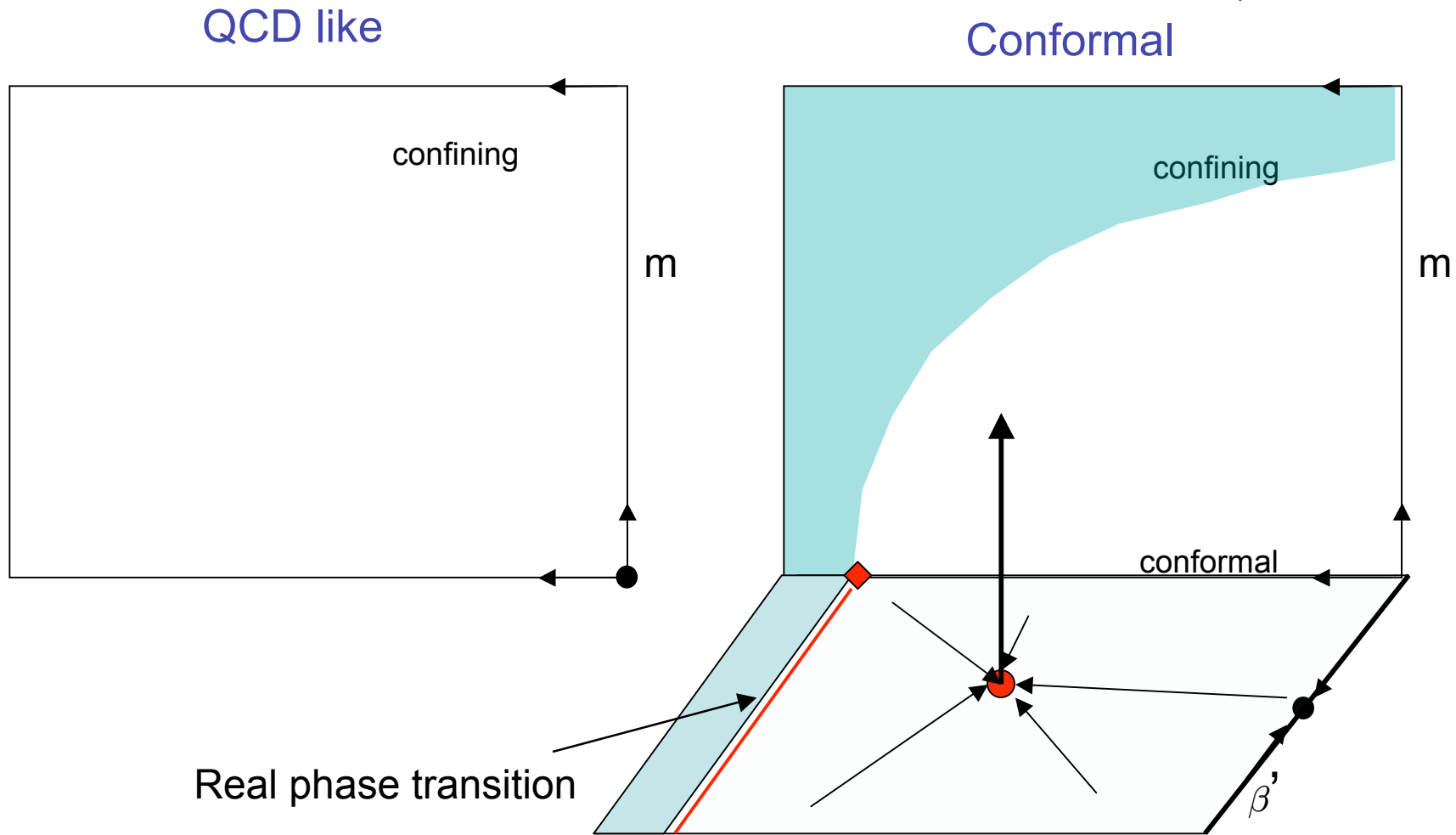
At ● $g=0, m=0$, both couplings are relevant

At ◆ $g=0, m=\infty$, g is still relevant



The lattice phase diagram

(arrows: UV to IR)

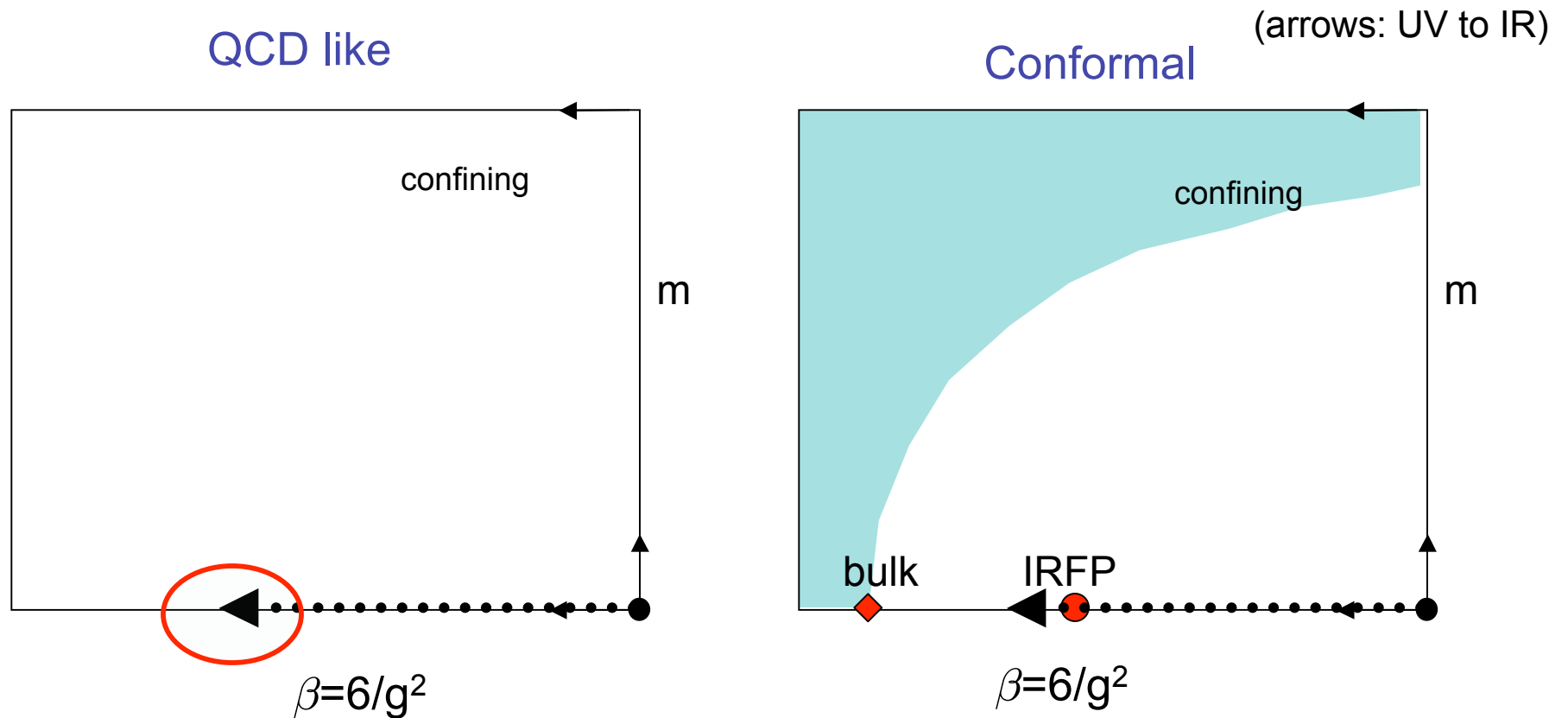


Existence of IRFP is universal;
Its location is RG dependent

At ● IRFP only the mass is relevant



How can we distinguish QCD-like and conformal systems?



Lattice simulations can connect the perturbative FP and strong coupling

- Found IRFP ? Done ✓
- No IRFP? Show that it is confining before a bulk transition is reached

Connecting weak and strong coupling: the bare differential step scaling function

$$s_b(\beta) = \beta - \beta' \quad \text{where} \quad \xi(\beta) = \xi(\beta')/2 \quad (\beta = 2N_c/g_0^2)$$

ξ is the correlation length defined by some physical mass.

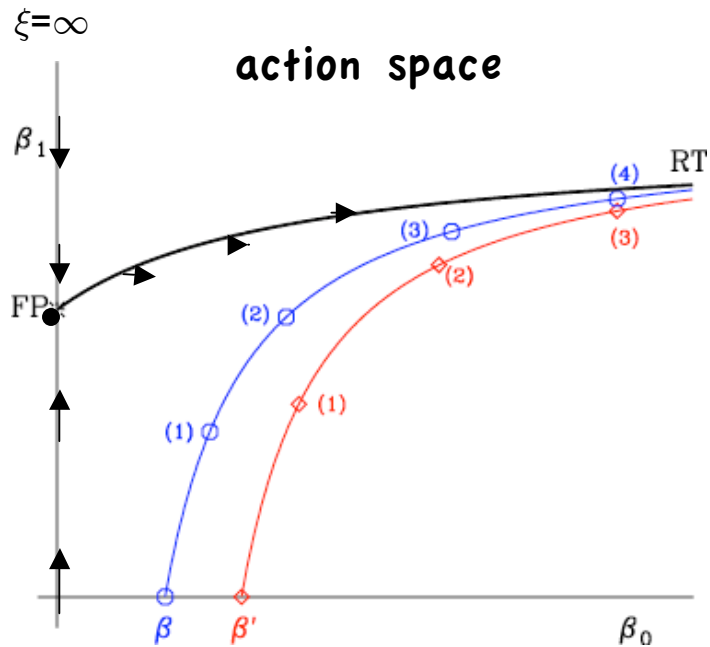
Sensible definition when ξ is finite

s_b is universal only as far as ξ is

- Can be measured directly or
- Through some running coupling(the Schrodinger functional formalism) or
- **Use RG flow** : $s_b(\beta)$ is the “projection” of the RG flow to a lower dimensional coupling space



Step scaling function around a UVFP

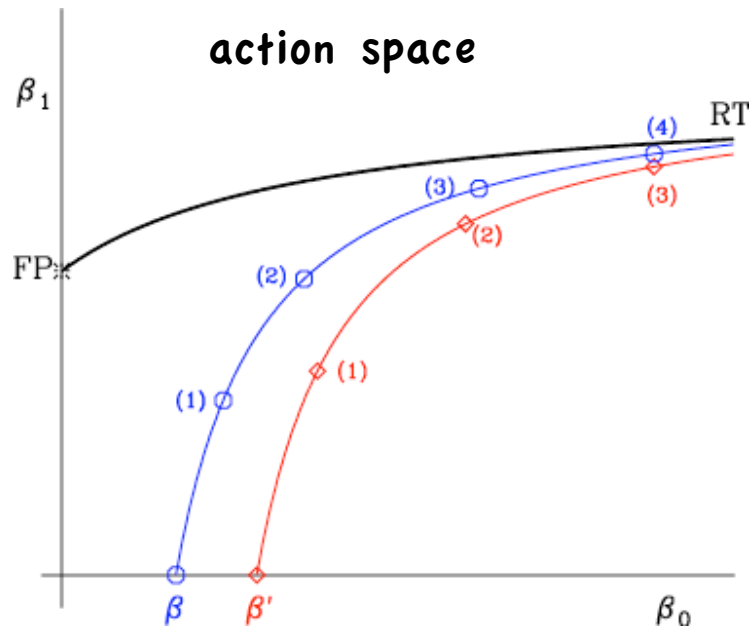


- Do simulations at β and β' ($m=0$)
 - RG block and compare the blocked actions
 - if $S(\beta^{(n)}) = S(\beta'^{(n-1)}) \rightarrow a(\beta) = a(\beta')/2$
- the step scaling function is

$$s_b(\beta) = \lim_{n_b \rightarrow \infty} (\beta - \beta')$$



Calculating $s_b(\beta)$ with MCRG



Two actions are identical if all operator expectations values agree



Match operators (local expectation values) after several blocking steps

- The location of the FP on the critical surface depends on the RG transformation
- Tuning free parameters in the RG transformation can pull the FP and its RT close, reducing systematical errors (optimization)

Along a relevant direction $s_b(\beta)$ is universal (up to lattice artifacts)



The step scaling function in a conformal system

In the chiral limit $\xi = \infty$ everywhere !

$s_b(\beta)$ can be defined through the RG flow

$$s_b(\beta) = \beta - \beta' \quad \text{where} \quad S^{(n)}(\beta) = S^{(n-1)}(\beta')$$

$s_b(\beta)$ can be defined through a running coupling as well

$$g^2(\beta;L) = g^2(\beta';L/2) \quad (\beta=2N_c/g_0^2)$$

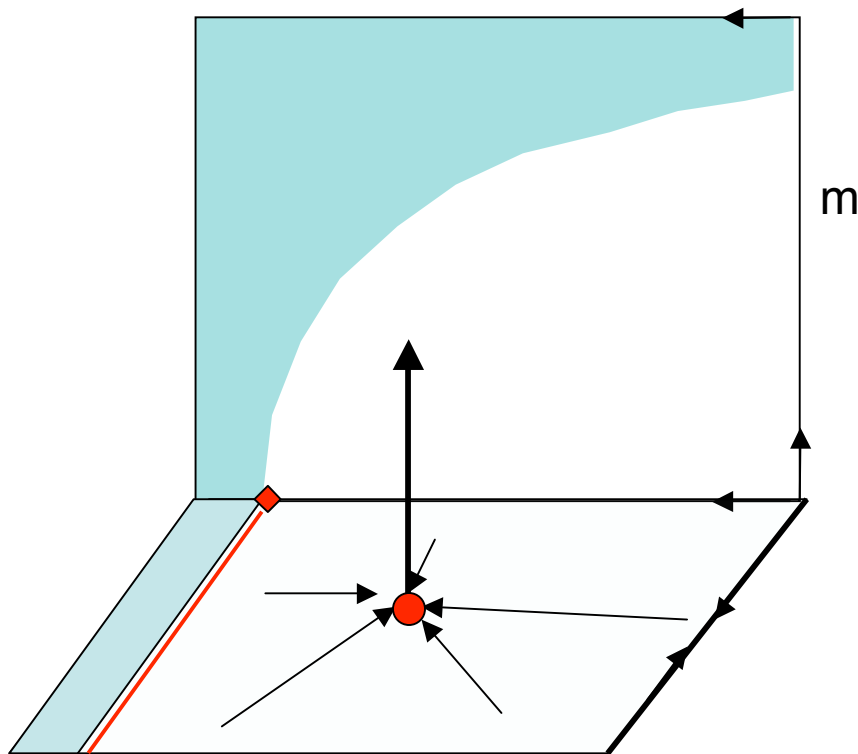
Calculate $g^2(\beta;L)$ using Schrodinger functional or potential or something else



RG flow lines around an IRFP

On the critical surface ($m=0$) around an IRFP the flows converge to the FP when $n_b \rightarrow \infty$

With finite n_b the flow picks up the slowest flowing operator



The location of the IRFP depends on the RG transformation

$s_b(\beta)$ along an irrelevant direction depends on the blocking (scheme dependence)

This could be a signal for non-QCD-like behavior



Matching of 2 relevant operators

- Matching around a FP with 2 relevant operators require tuning of 2 parameters (β and m)

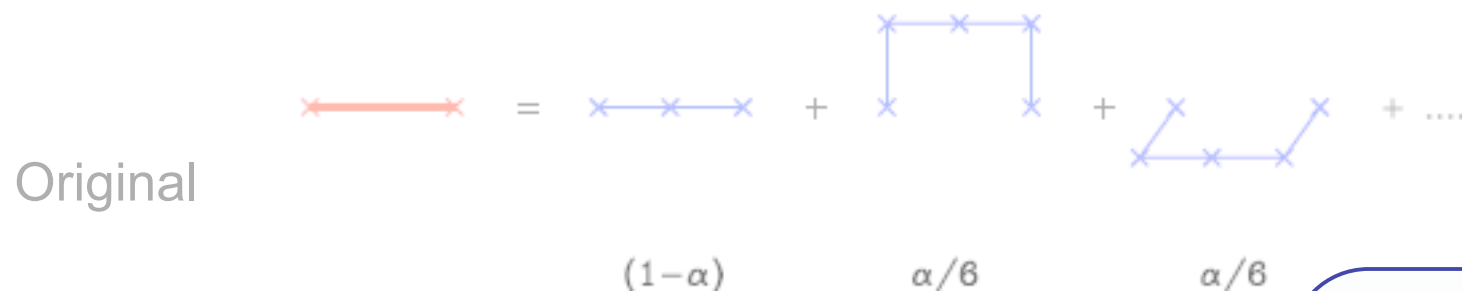
OR

- Set one operator to its critical value ($m=0$) and tune only in the other one (β) $\rightarrow s_b$
- Next fix s_b and tune $m \rightarrow \gamma_m$



The 3 Renormalization Group transformations

A real space block transformation averages out the UV modes leading to the renormalized trajectory that describes **perfect actions**



optimize with α

optimize with α_1
(play with α_2, α_3)

Optimization is essential to pull the FP/RT close to the simulation action!

Summary: 2- lattice matching MCRG

- Works with bare couplings - sufficient to study the phase diagram
- Can be optimized by tuning the free parameter(s) of the RG transformation
- Finite volume effects are largely controlled
- Requires relatively small statistics
- Has a lot of built-in consistency checks
 - compare several blocking levels
 - compare several operators
 - compare different RG transformations



Some results:

- SU(3) pure gauge (test)
- SU(3) gauge + $N_f=8, 16$ and 12 fundamental flavors

All with nHYP smeared staggered fermions (no rooting!)
Wilson plaquette gauge action

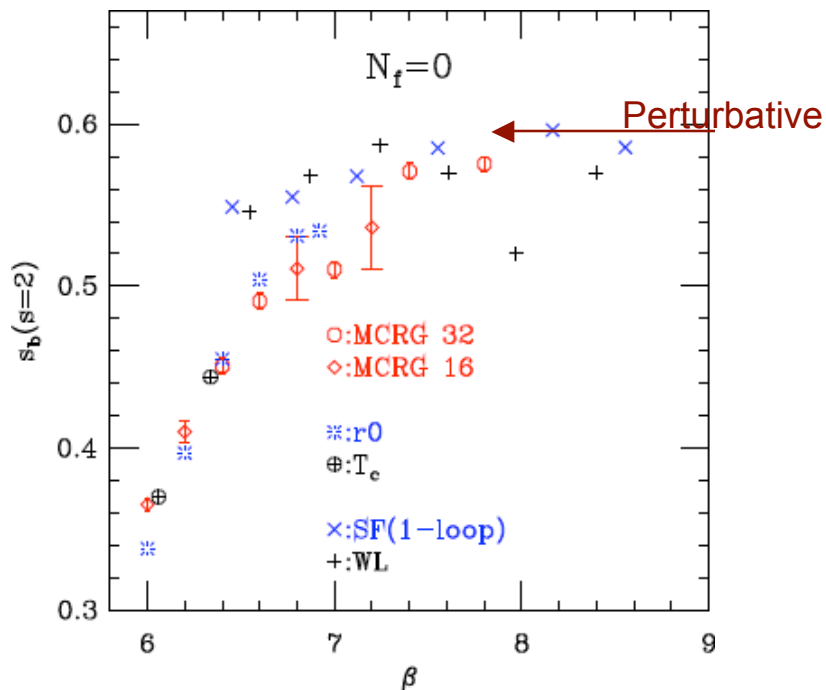
Warning: $s_b > 0$ when $\beta(g) < 0$!



SU(3) pure gauge : test case

The bare step scaling function can be calculated in many ways

- physical observables r_0, T_c
- Schrodinger fn; Wilson loop ratios,
- RG matching: $32^4 \rightarrow 16^4$ and $16^4 \rightarrow 8^4$



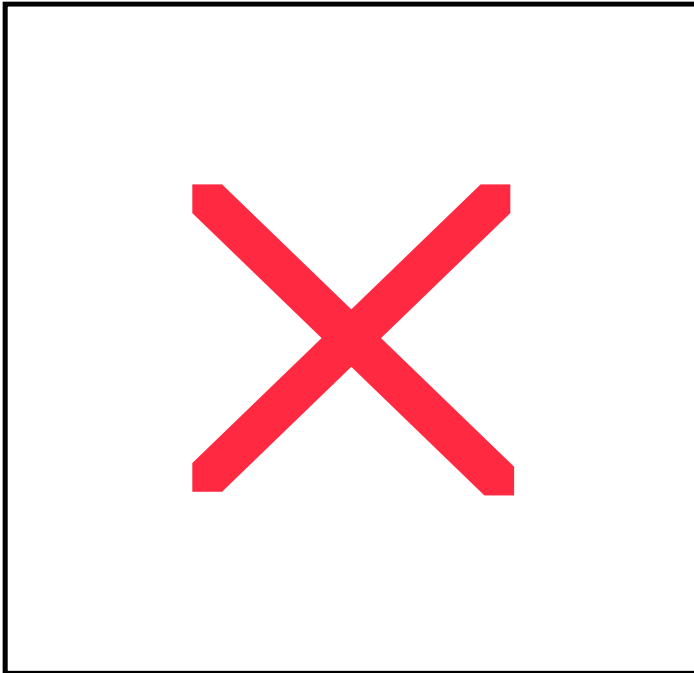
- Good agreement between r_0, T_c and MCRG
- $32^4 \rightarrow 16^4$ and $16^4 \rightarrow 8^4$ are consistent with ~ 0.02 accuracy
- Both SF and MCRG approach the perturbative value
- Since at $\beta=6$ we can test confinement, we know there is no physical IRFP



Compare different RG transformations:

When the flow is governed by a UVFP, $s_b(\beta)$ is universal (up to lattice corrections).

Compare 3 different RG transformations:



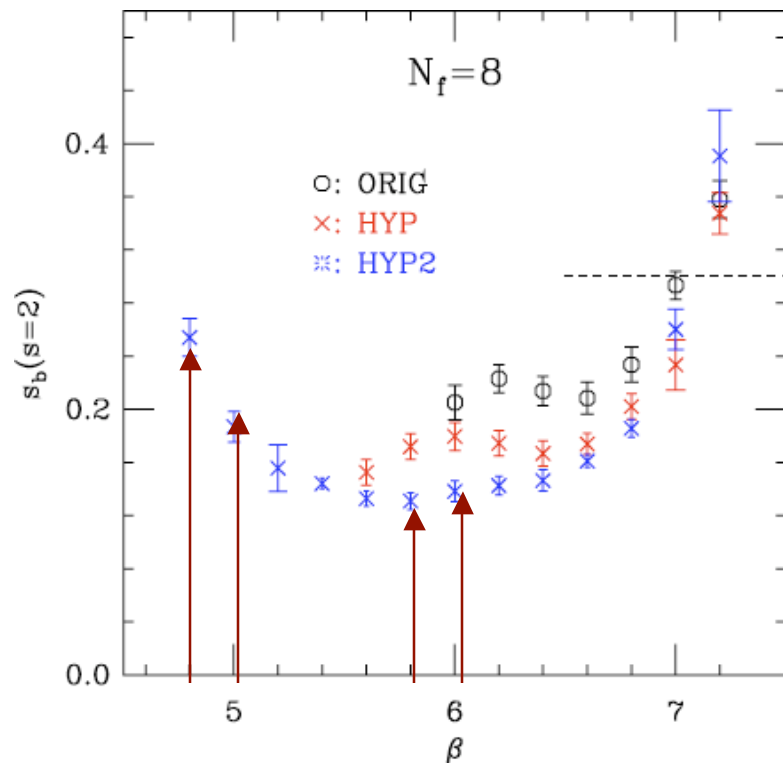
Excellent agreement between the
3 RG blockings
→ attractive region of a UVFP



$N_f=8$ flavors

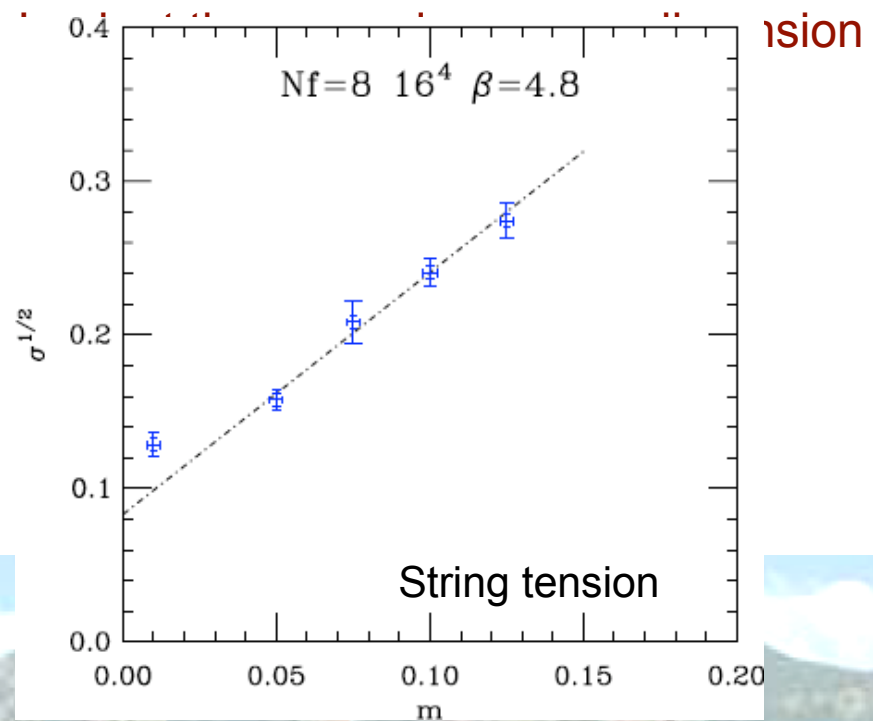
Expected to be QCD-like: analytical & numerical results

Compare the different RG transformations ($m \approx 0$)



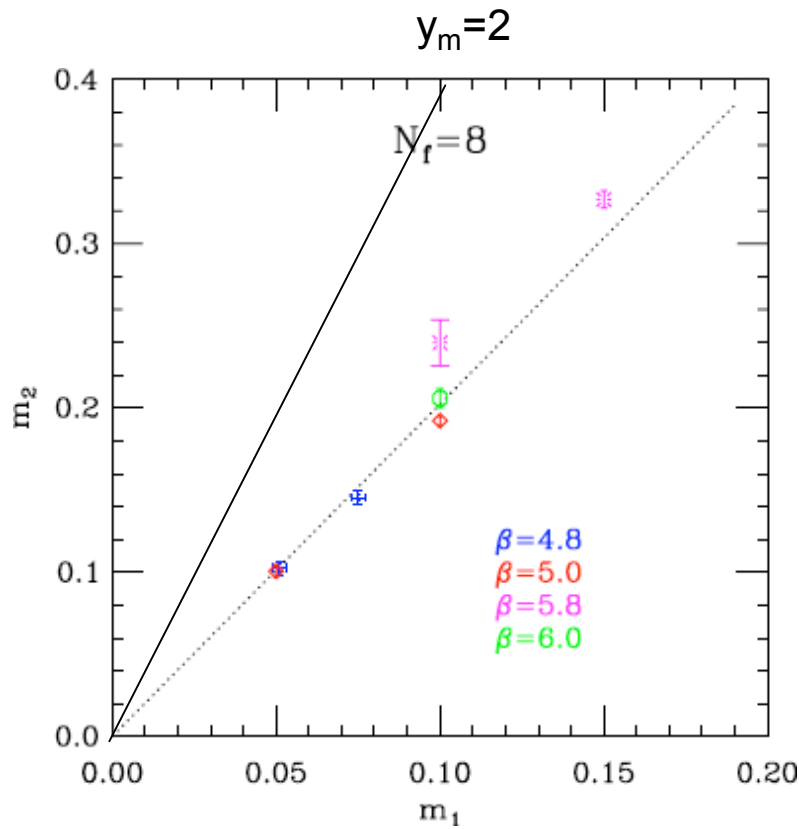
$s_b > 0$ everywhere - no IRFP

Is it confining?



$N_f=8$ flavors, anomalous mass

4 different couplings ($\beta=4.8, 5.0, 5.8, 6.0$), optimal RG from $m=0$ data



$$m_2 = m_1 2^{-1/y_m}$$
$$\gamma_m = y_m^{-1}$$

All 4 β values predict similar value

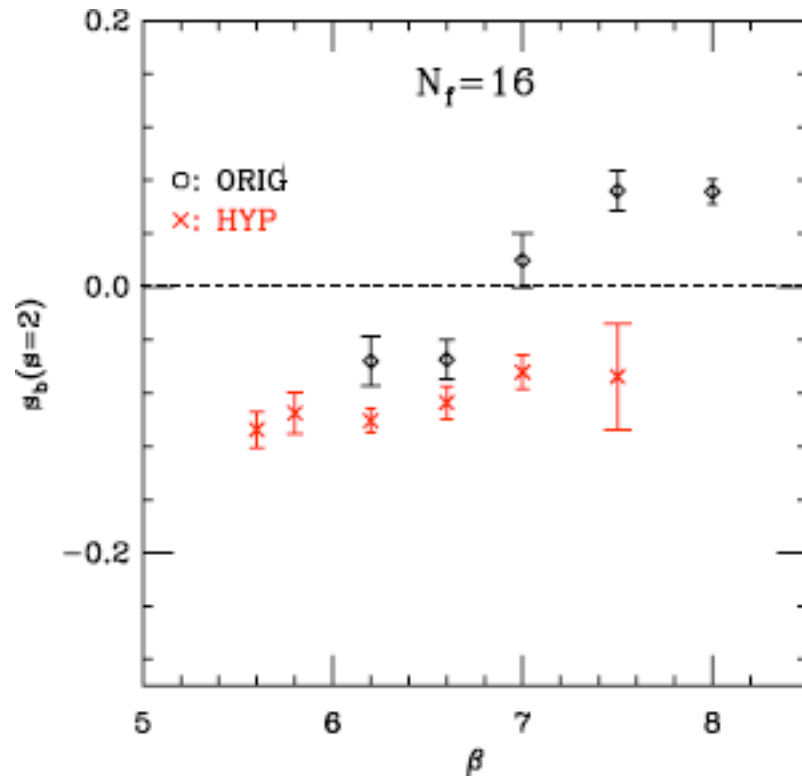
$$\gamma_m = 0.02(5)$$

close to free field exponent



$N_f=16$ flavors

$16^4 \rightarrow 8^4$ MCRG



ORIG blocking shows $s_b(\beta)=0$ around $\beta=7.0$

HYP blocking has an IRFP around $\beta=10.0$

Different block transformations predict different $s_b(\beta)=0$ but they both show a positive RG β function



$N_f=12$ flavors

Some history:

- The analytic works predicts $N_f=12$ is just above the conformal window
- Yale group found an IRFP at fairly strong coupling, using Schrodinger functional method, unimproved action
- Groningen/INF group identified a bulk phase transition characteristic to a conformal system and claim chiral symmetry at weaker coupling
- Two groups (San Diego and Columbia) have studied the spectrum of the model with improved and unimproved actions. Both see QCD-like behavior, though at stronger gauge couplings.

If $N_f=12$ is conformal,

Could the spectral measurements be in the strong coupling phase?

If $N_f=12$ is QCD-like,

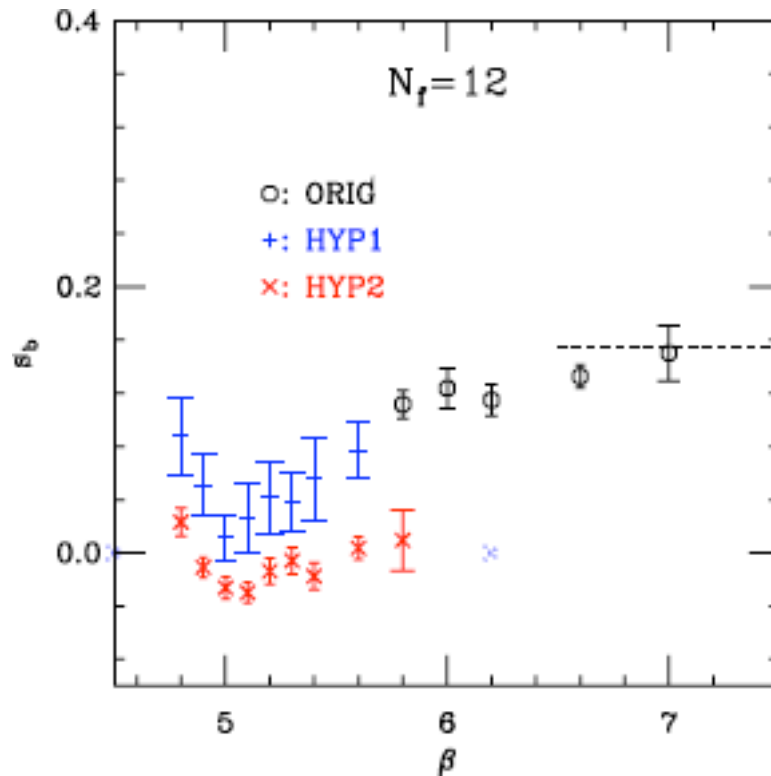
The unimproved actions used with Schrodinger functional could be unreliable

We expect a universal result from all actions. The existence of the conformal phase near $g=0$ is universal, even if the locations of the phase transitions, fixed points are not.



$N_f=12$ flavors with MCRG

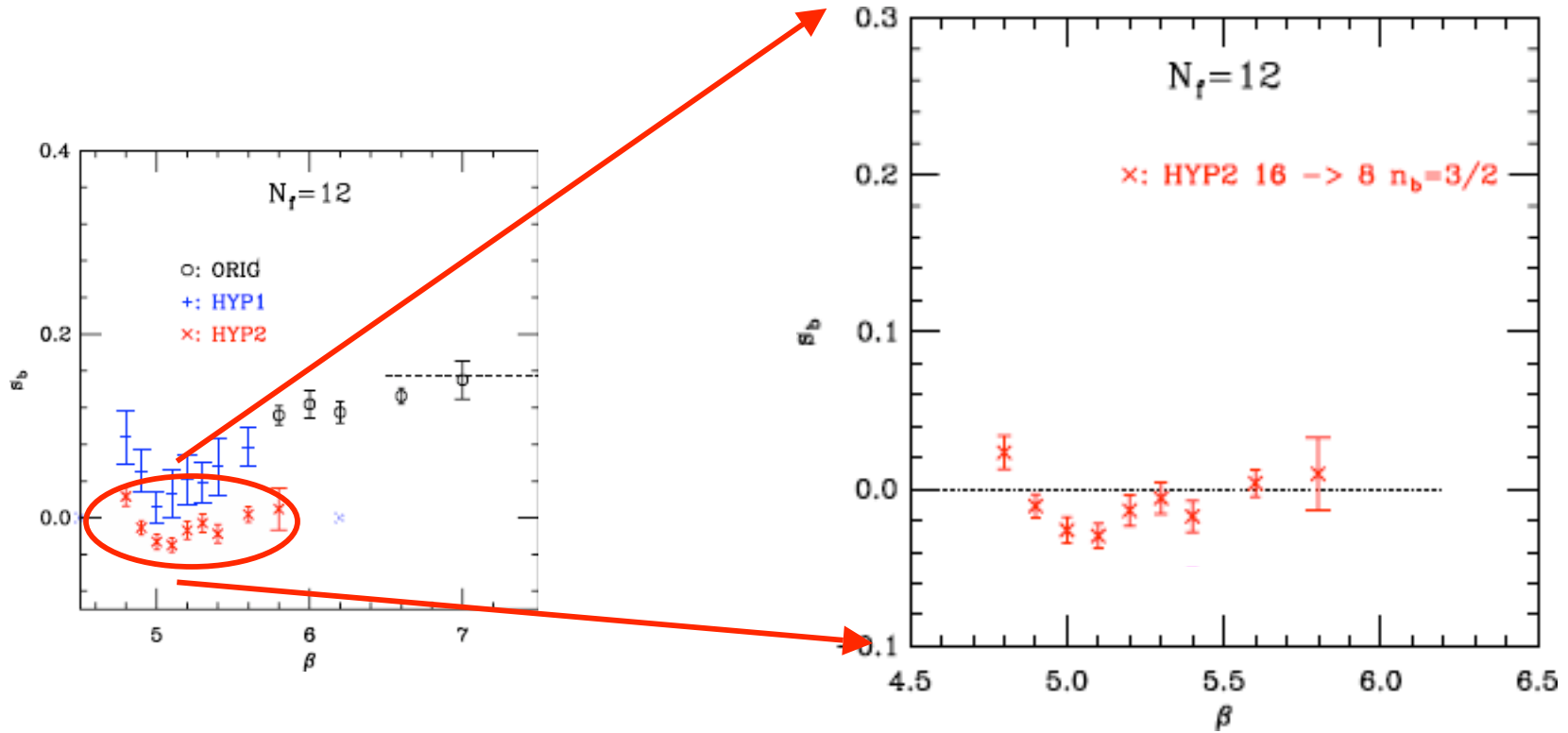
Use the same techniques as before; $16^4 \rightarrow 8^4$, $m=0.0025$ or 0.01



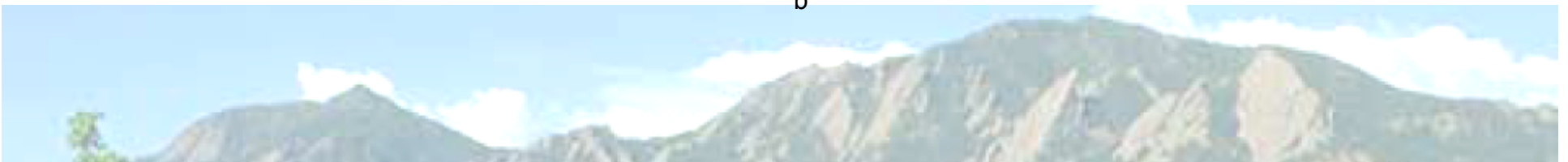
- Orig/HYP blockings predicts different $s_b(\beta)$ functions
- HYP2 hovers around 0 -- likely IRFP
- String tension vanishes at $\beta=4.4$ on 16^4 volumes, but lattice artifacts are large and the volume is small



Look at closer



- There is no phase transition at the second zero
- $32^4 \rightarrow 16^4$ matching shows large
 - finite volume
 - finite n_b effects



Summary: $N_f=12$ flavors

- It is a difficult system



Conclusion

MCRG is an effective alternative method to study the phase structure and scaling properties of lattice QFT's

- The method is very universal, straightforward to implement for any other system
- Can be used to predict anomalous mass dimensions as well

$N_f=0-8, 16$ as expected. $N_f=12$ is difficult

What is next?

- Could the different groups come up with a consistent picture for $N_f=12$?
- SU(2) gauge, other fermion representations can be studied the same way
- Maybe it is time to go beyond fermion-gauge systems

