Oblique correction from sextet QCD

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Outline

- Conformality "for all practical purposes"
- Precision electroweak tests for fun and profit
- The lattice calculation – usual overlap miracles
- QCD games to check techniques
- Oblique correction from sextet QCD
- Conclusions

Work based on big program with B. Svetitsky and Y. Shamir

arXiv:1006.3777

Background

- Lots of interest in BSM systems by lattice community
- TD+Svetitsky + Shamir's system – $SU(3)$ gauge group with $N_f = 2$ flavors of sextet fermions
- Lattice discretization – clover fermions
- Weak coupling phase (in finite volume) is chirally restored, deconfined
- No stable $am_q^{AWI} = 0$ in strong coupling
- Don't know if it has an IR fixed point
- Certainly it is a theory whose coupling runs very slowly
- Let's use the fact that it walks to do a calculation

Physics question: What would the S-parameter look like, for nearly conformal dynamics?

Target: "conformal technicolor" or "unparticle" or "hidden valley" fans

Conformality “for all practical purposes”

- Schr̈odinger functional coupling runs really slowly
- In any lattice volume, coupling at shortest and longest distance varies by $\sim 10 - 15$ per cent
- This is slow enough that it’s hard to see it run
- Slow running is approximately no running
- No running means that the quark mass is the relevant operator, $m = 0$ is critical
- This gives several ways to measure the mass anomalous dimension
  - Finite size scaling analysis of correlation length (TD, November)
  - Schrödinger functional running mass (DSS, June)
- Gives a nice measurement of $\gamma_m(g^2)$ which is
  - Unfortunately, fatally low for TC phenomenology ($\gamma_m = 1$ is desired)
  - Still the world’s record for biggest γ_m from a simulation (as of June 2010)

Some pictures:

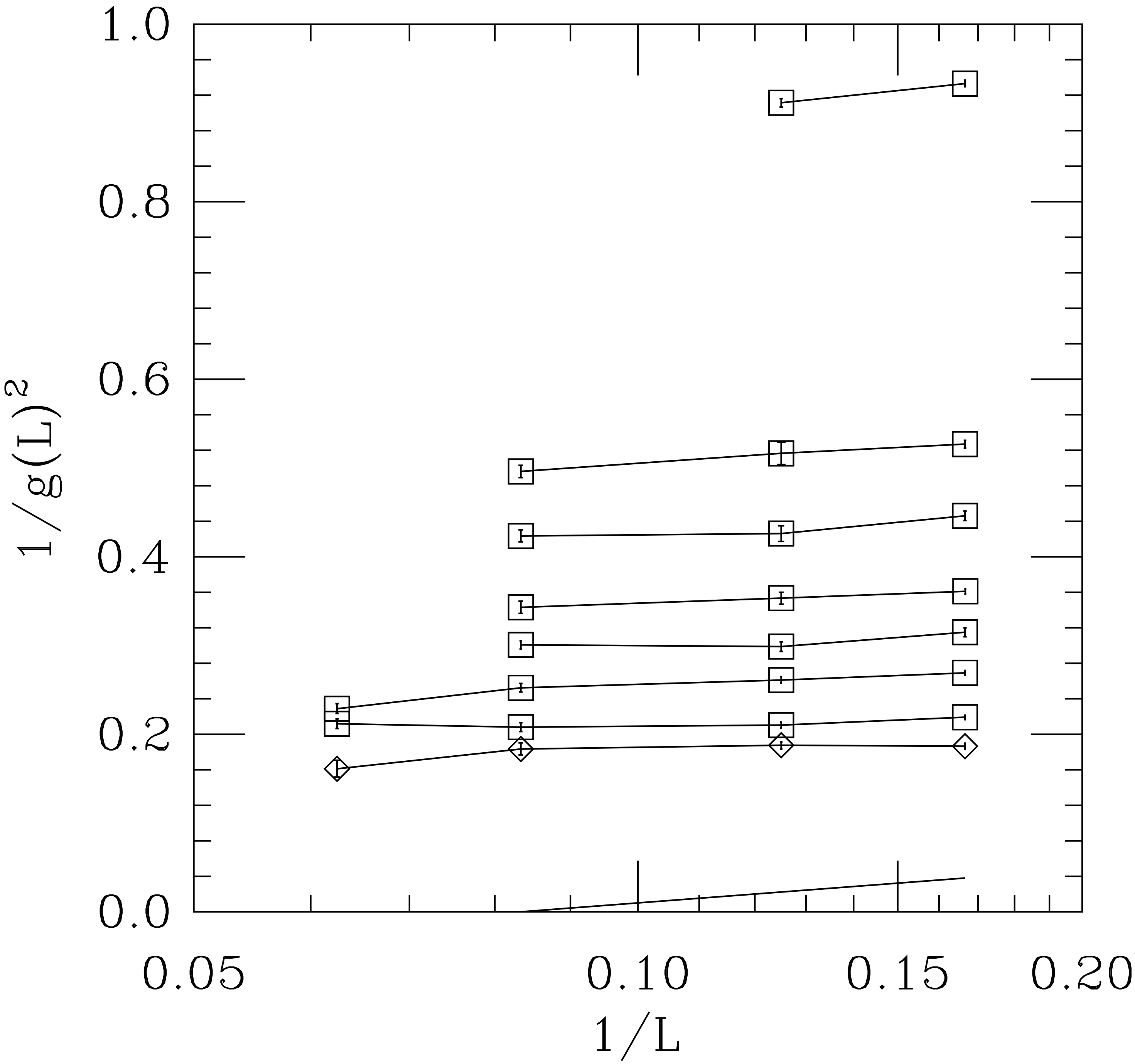


Figure 1: SF coupling $1/g^2$ vs. a/L. Is $1/g^2 = 2b_1/(16\pi^2)\log\ L+$ constant?

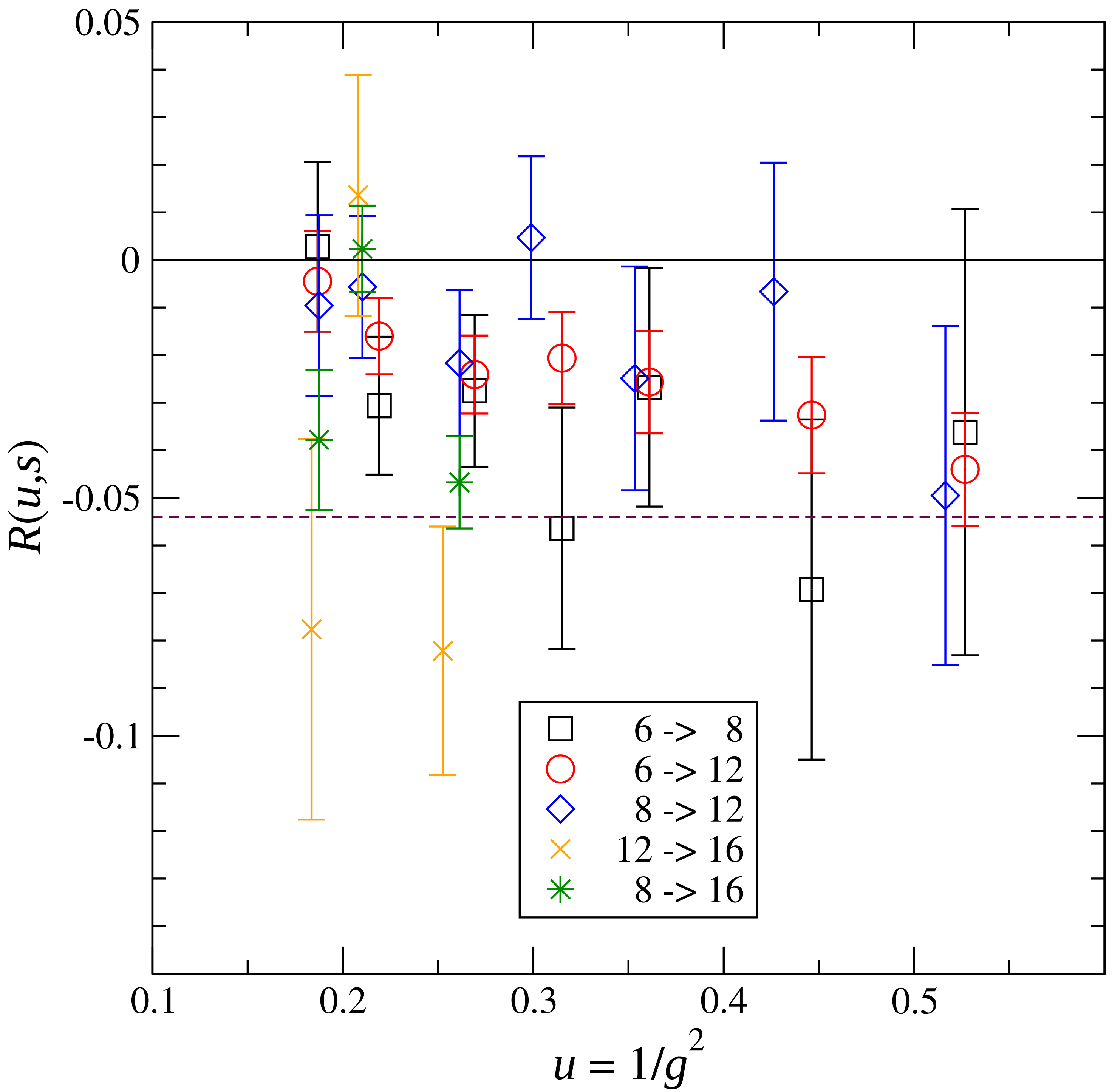


Figure 2: Lattice approximants to the beta function for many scale factors

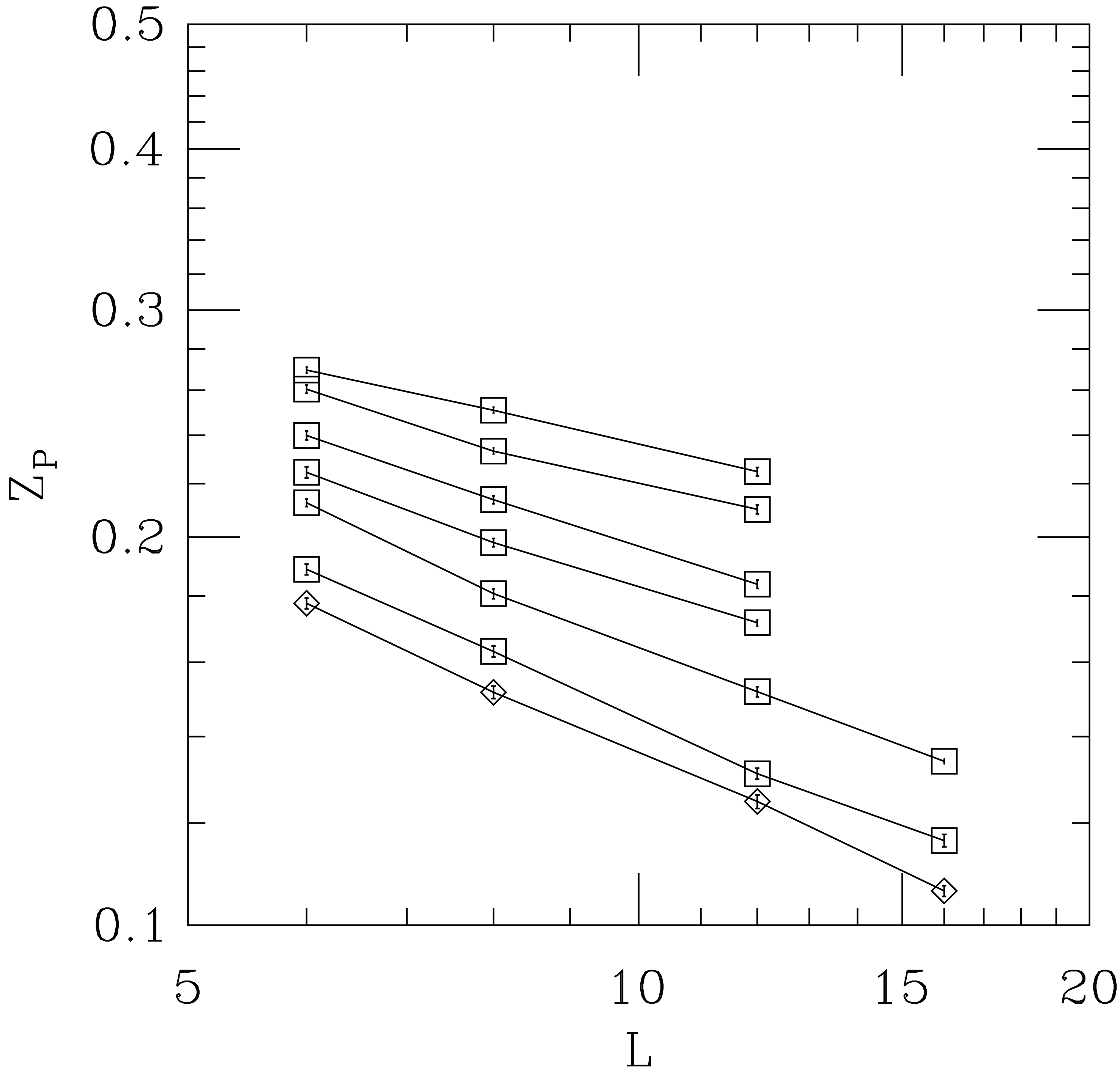


Figure 3: Pseudoscalar renormalization constant Z_P vs. L/a. Is $Z_P \propto L^{-\gamma_m}$?

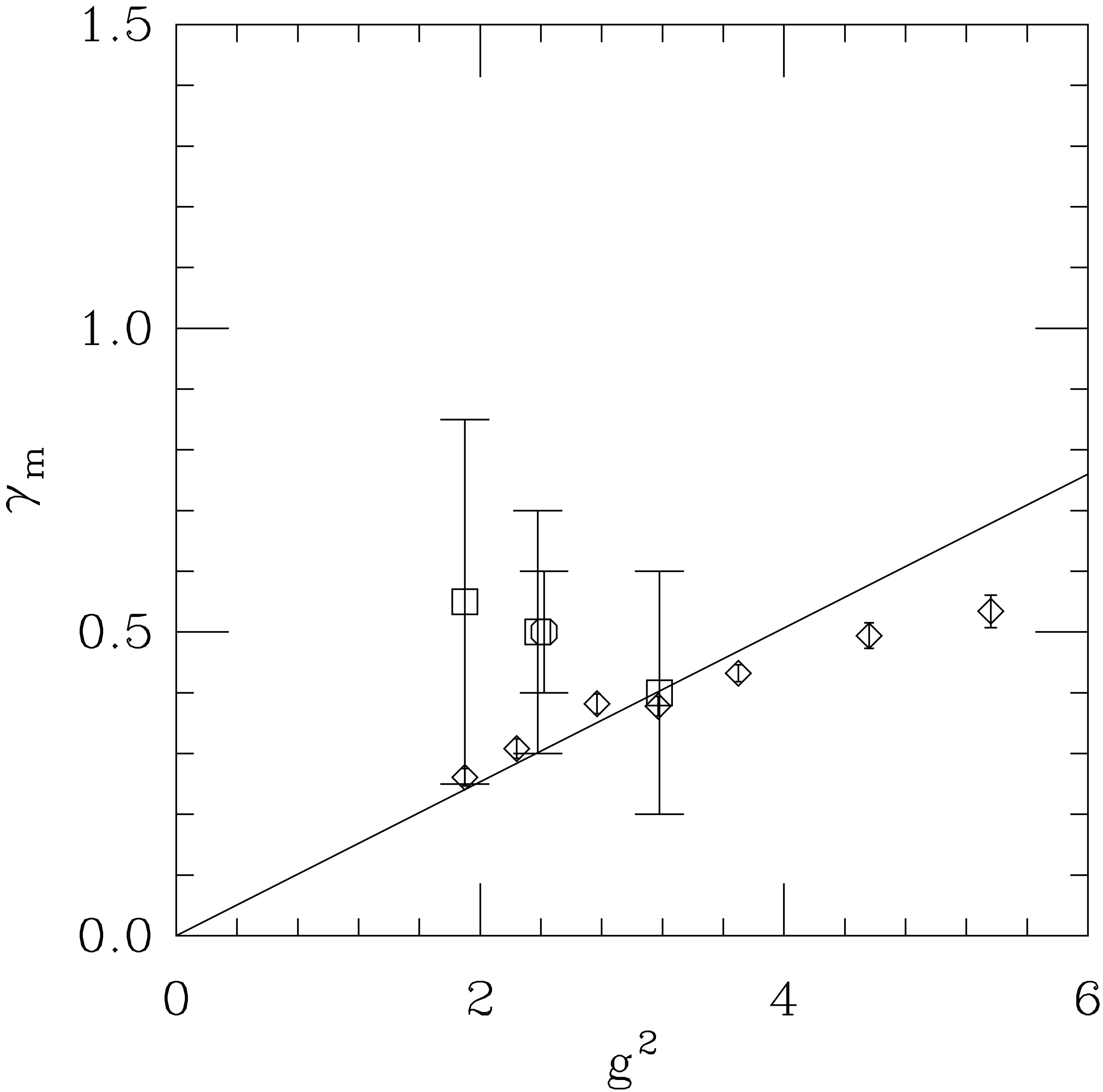


Figure 4: Mass anomalous dimension γ_m vs. g^2, the $L = 6a$ SF coupling.

Precision EW Background

"Oblique correction" to gauge boson vacuum polarization given by current - current correlator

$$\begin{aligned} \Pi_{\mu\nu}(q) &= \int d^4q \exp(iqx) \langle J_\mu^L(x) J_\nu^R(0) \rangle \\ &\equiv (q^2\delta_{\mu\nu} - q_\mu q_\nu)\Pi_T^{LR}(q^2) + q_\mu q_\nu \Pi_L^{LR}(q^2).. \end{aligned} \quad (1)$$

Peskin Takeuchi S-parameter is $S = 16\pi d(q^2\Pi_T^{LR}(q^2))/dq^2$ at small q^2,

Lattice problems

- Lattice dirt in
$$\Pi_{\mu\nu}(q) = P_{\mu\nu}^T(q)\Pi^T(q) + P_{\mu\nu}^L(q)\Pi_L(q) + \dots \quad (2)$$
- Extra quadratic divergence if lattice currents aren't conserved
- Nonlocal currents have contact terms
- Different Z_V and Z_A

Valence overlap fermions cure the last three!

Wonderful overlap miracles

V and A currents related by Ward identity so

- $Z_V = Z_A$
- quadratic term cancels
- use local currents (actually "improved" ones)

Decompose with $\bar{q}_\mu = (2/a)\sin q_\mu a/2)$

$$\Pi_{\mu\nu}(q) = P^T_{\mu\nu}(q)\Pi_T(q) + P^L_{\mu\nu}(q)\Pi_L(q) \tag{3}$$

where

$$P^T_{\mu\nu}(q) = \bar{q}^2\delta_{\mu\nu} - \bar{q}_\mu\bar{q}_\nu \tag{4}$$

and

$$P^L_{\mu\nu}(q) = \bar{q}_\mu\bar{q}_\nu \tag{5}$$

Check decomposition with JLQCD operator

$$\Delta_J(q) = \sum_{\mu\nu} \bar{q}_\mu\bar{q}_\nu(\frac{1}{\bar{q}^2} - \frac{\bar{q}_\nu}{\sum_\lambda \bar{q}^3_\lambda})\Pi^J_{\mu\nu}(q) \tag{6}$$

Force fit to trial decomposition, P^T and P^L are projectors...q by q

QCD fun & games

Did this for quenched and $N_f = 2$ QCD. All looks "normal."

Pause for physics: resonance dominance (basically large N_c) writes

$$\Pi_T^{LR}(q^2) = \sum_V \frac{f_V^2 M_V^2}{q^2 + M_V^2} - \sum_A \frac{f_A^2 M_A^2}{q^2 + M_A^2} - \frac{f_\pi^2}{q^2}. \tag{7}$$

Weinberg sum rules:

$$\sum_V f_V^2 M_V^2 - \sum_A f_A^2 M_A^2 - f_\pi^2 = 0 \tag{8}$$

$$\sum_V f_V^2 M_V^4 - \sum_A f_A^2 M_A^4 = 0 \tag{9}$$

Usual additional approximation: saturate with lowest resonances, π, ρ, a_1 (5 parameters)

I can

- Measure all 5 in a simulation – this doesn't reproduce WSR's – or data
- Measure f_π, m_ρ, f_ρ, use WSR's to compute f_{a_1}, m_{a_1} – looks like data

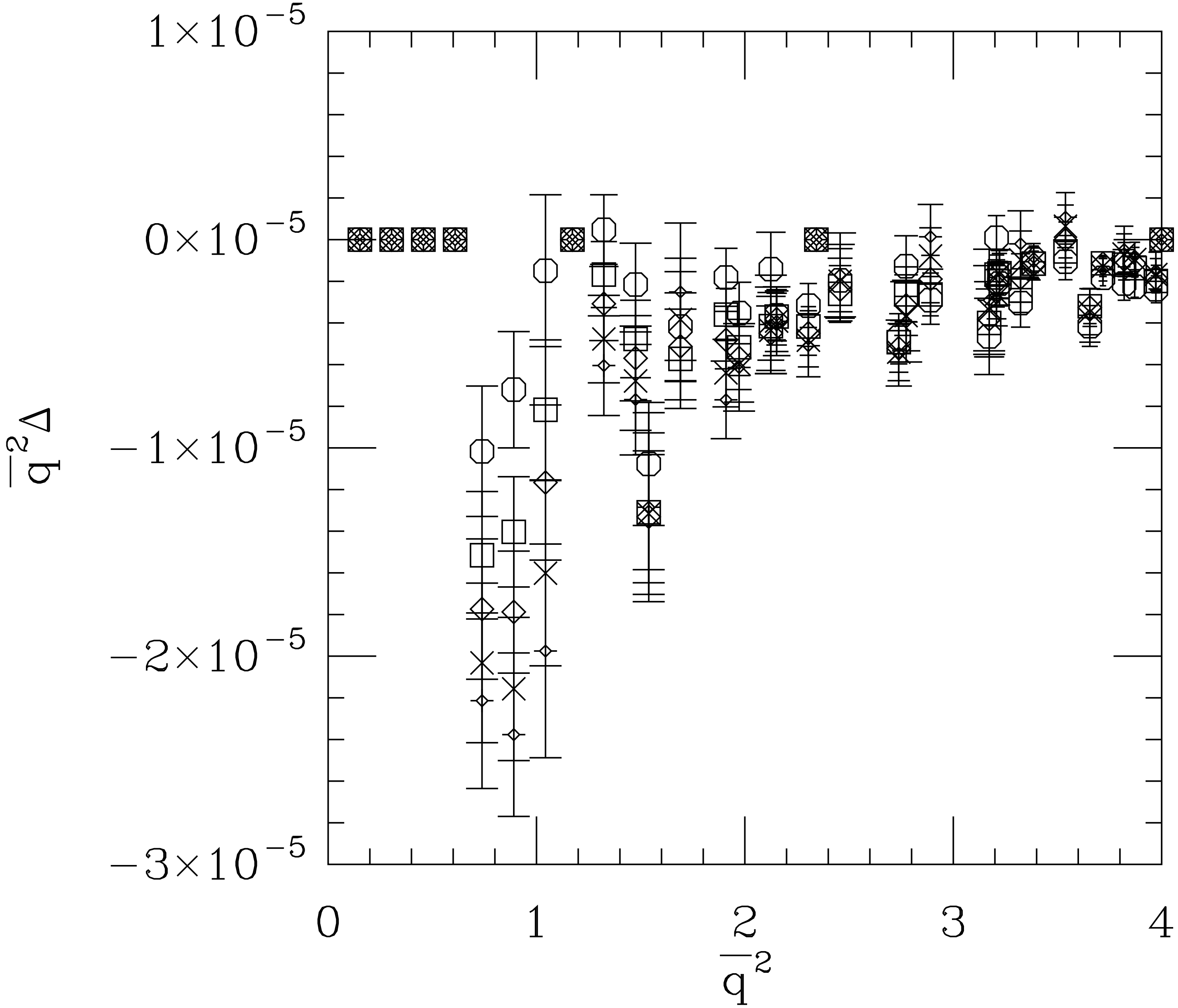


Figure 5: JLQCD parameter for quenched QCD

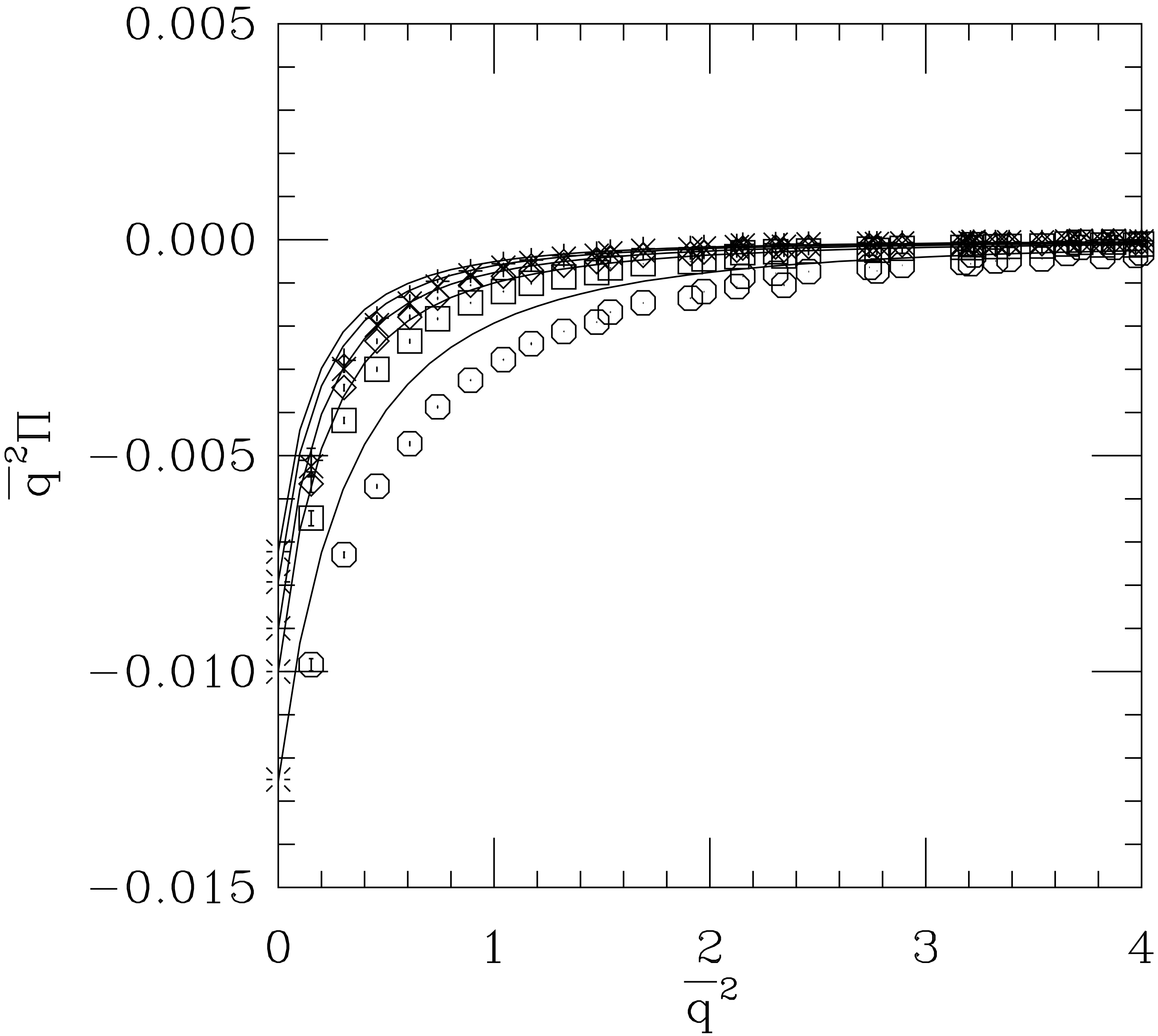


Figure 6: Π_{LR}^{T} for quenched QCD – lines are low state model

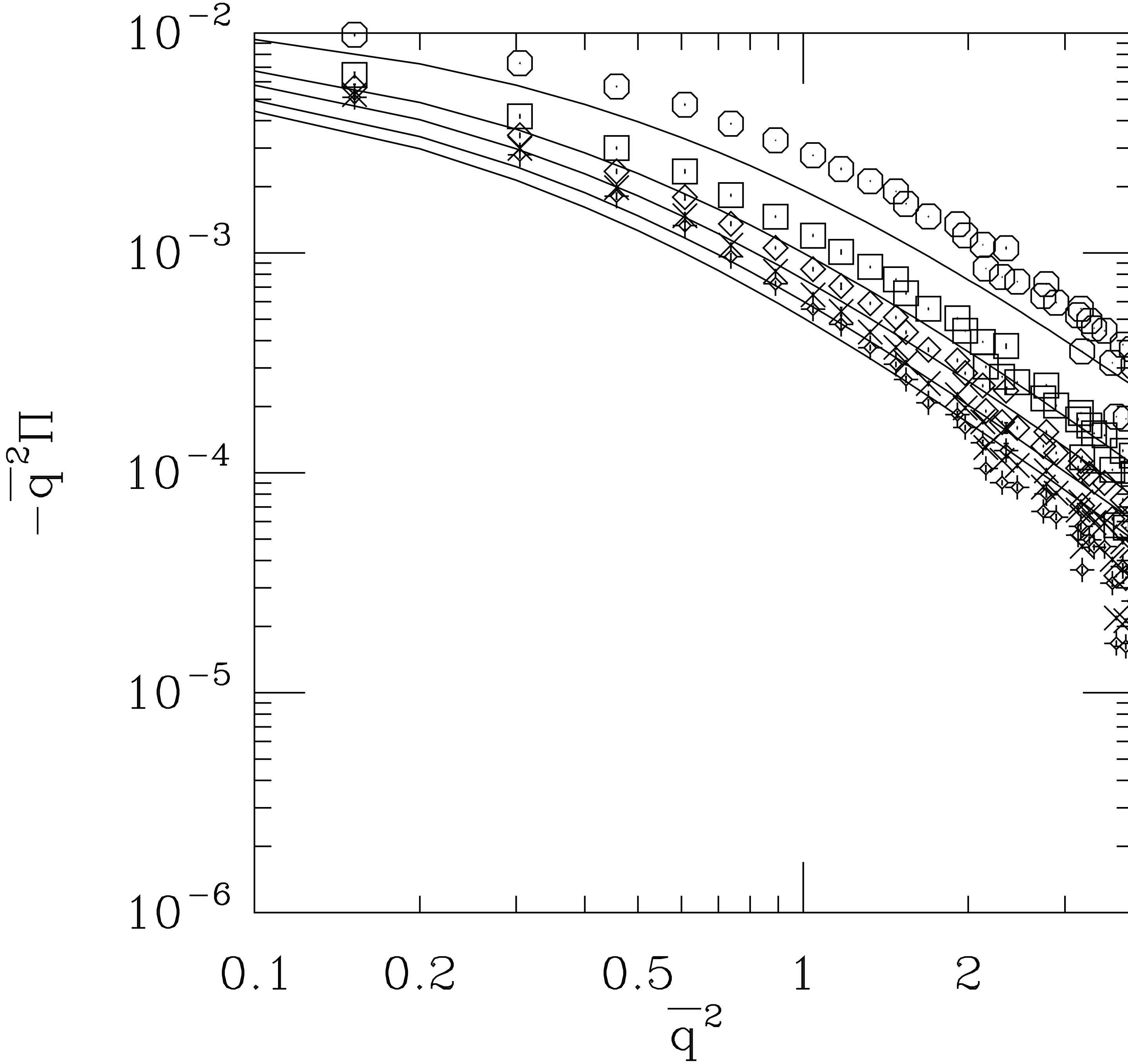


Figure 7: Π^T_{LR} for quenched QCD – log-log plot – lines are low state model

L_{10} and Δm_π^2

With "lowest mass dominance" plus Weinberg sum rules one can predict

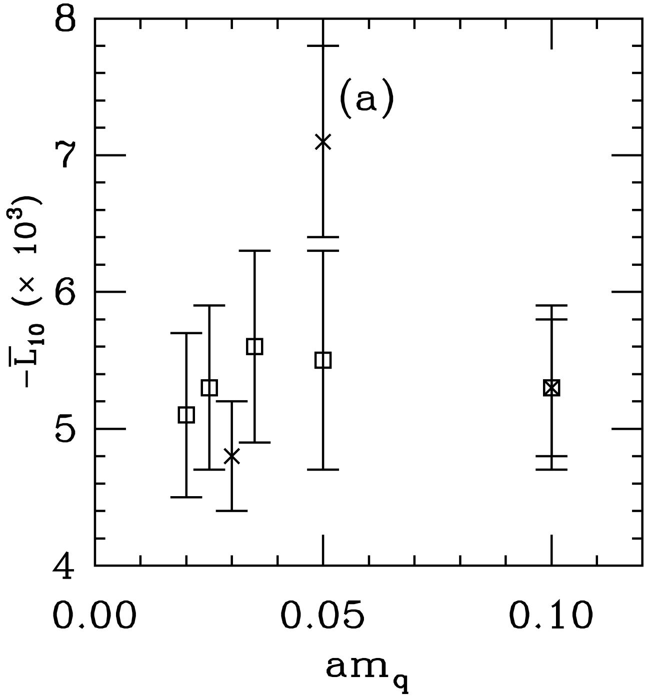
$$L_{10} = f_\rho^2 - f_{a_1}^2 \tag{10}$$

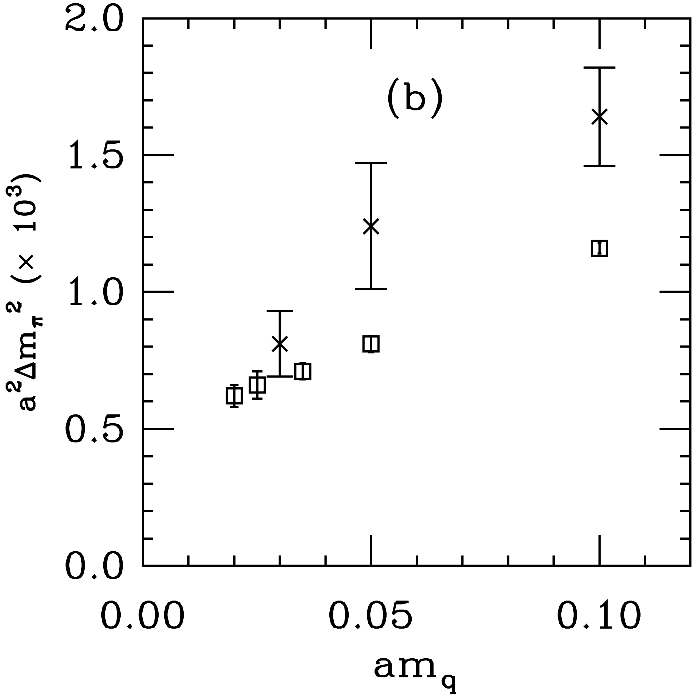
Das, Guralnik, Mathur, Low, Young sum rule – just do the integral

$$\Delta m_\pi^2 = \int dq^2 \Pi_{LR}^T(q^2) = \frac{3\alpha}{4\pi} \frac{1}{(\frac{1}{m_\rho^2} - \frac{1}{m_{a_1}^2})} \log \frac{m_{a_1}^2}{m_\rho^2} \tag{11}$$

Works pretty well, $L_1 0 \sim 5 \times 10^{-3}$, $\Delta m_\pi^2 = 1100$ MeV.

This is just fooling around, to show $\Pi_{LR}^T(q^2)$ looks ordinary


(a)
$-\bar{L}_{10}$ (× 10^3)
am_q
8
7
6
5
4
0.00
0.05
0.10


(b)
$a^2\Delta m_\pi^2$ (× 10^3)
am_q
2.0
1.5
1.0
0.5
0.0
0.00
0.05
0.10

S-parameter for our conformal theory

Ran valence sextet overlap fermions on a set of 16^4 dynamical sextet clovers at one parameter value

$(am_q^{AWI} = 0.04$, $\gamma_m = 0.35)$

What I found:

- It doesn't look anything like QCD (but why should it?)
- $\Pi^T_{LR}(q^2)$ vanishes at large q^2, vanishes as $m^2 \to 0$
- Saturation with lowest states fails!
  - Open any strong coupling BSM review
  - Everyone assumes low state saturation
  - Even true for papers with "conformal" in the title
- Finite differences for S-parameter – power law scaling

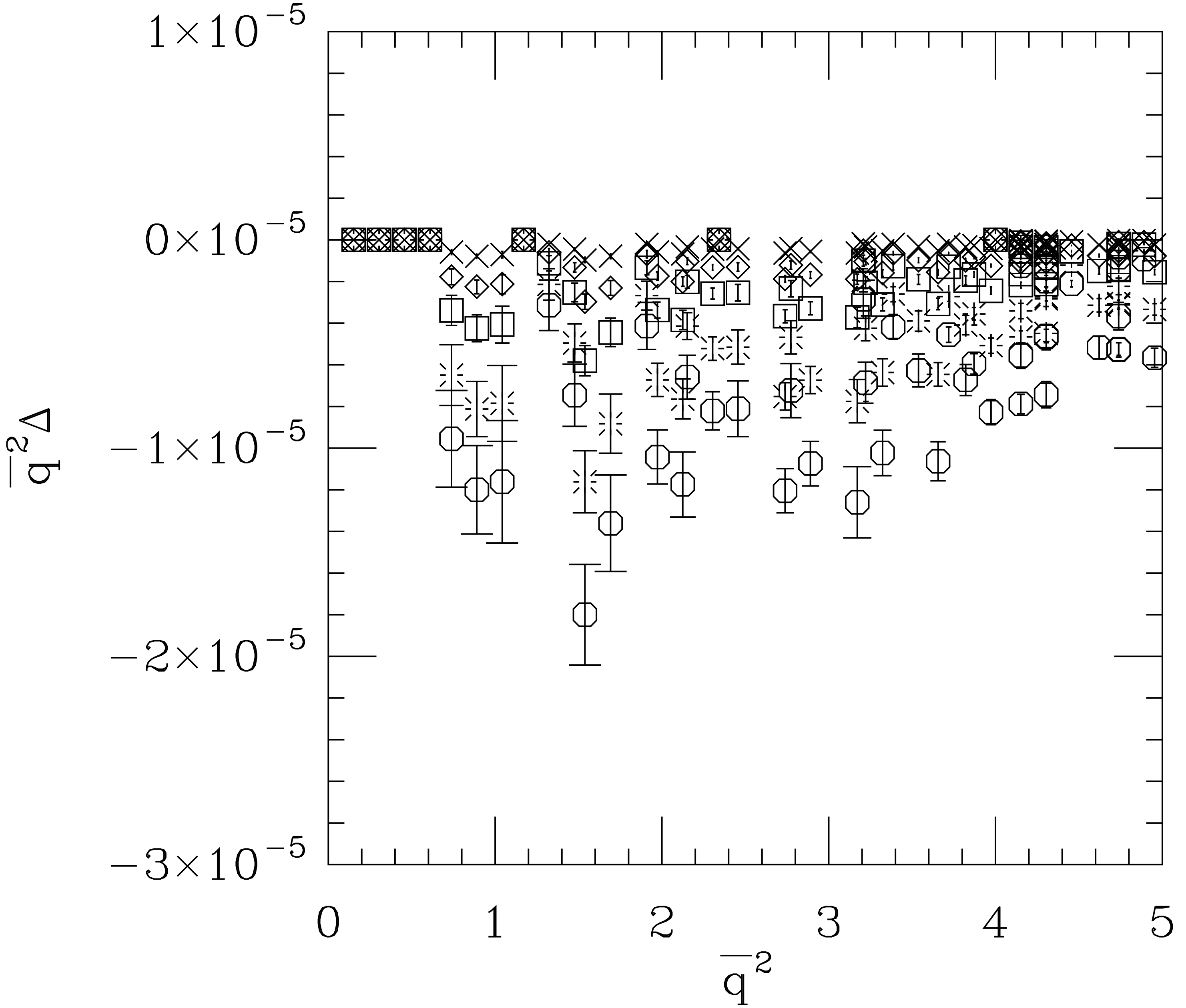


Figure 8: JLQCD parameter for sextet QCD

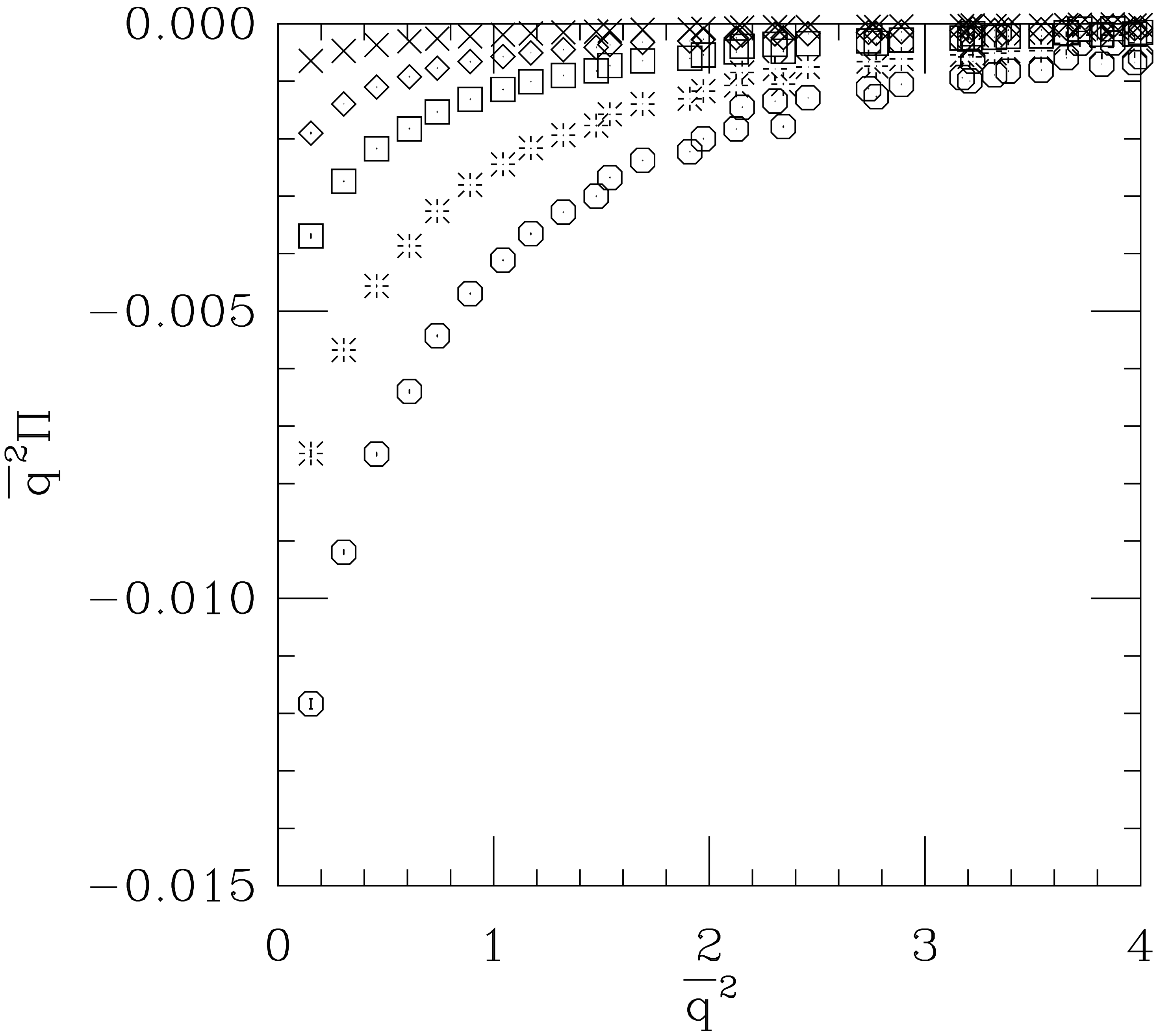


Figure 9: Π_{LR}^{T} for sextet QCD

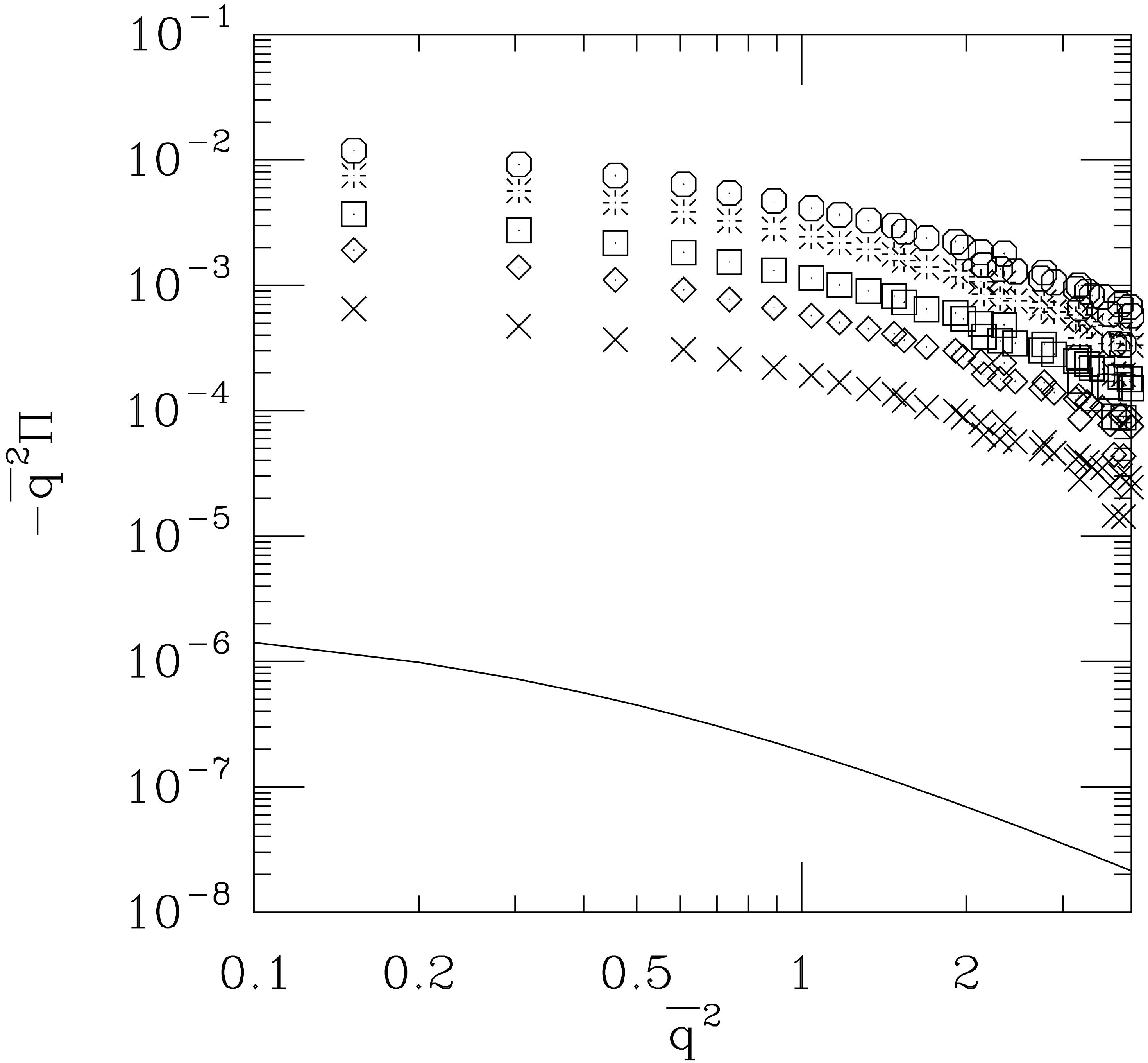


Figure 10: Π^T_{LR} for sextet QCD – log-log plot plus low state formula at one mass

the S-parameter

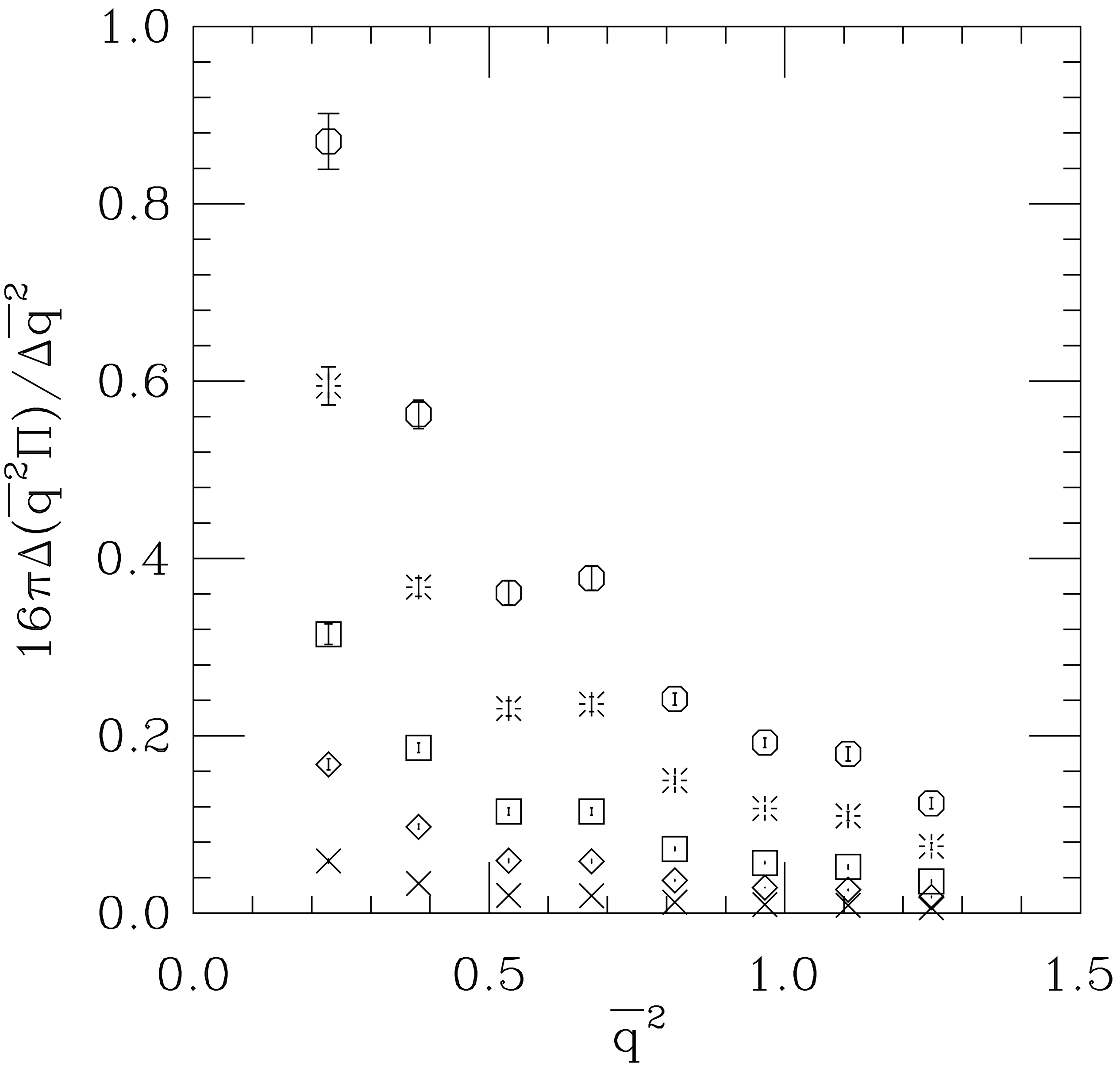


Figure 11: Numerically differentiated "$S-$parameter" for sextet QCD – log-log plot

the S-parameter vs q^2/m^2

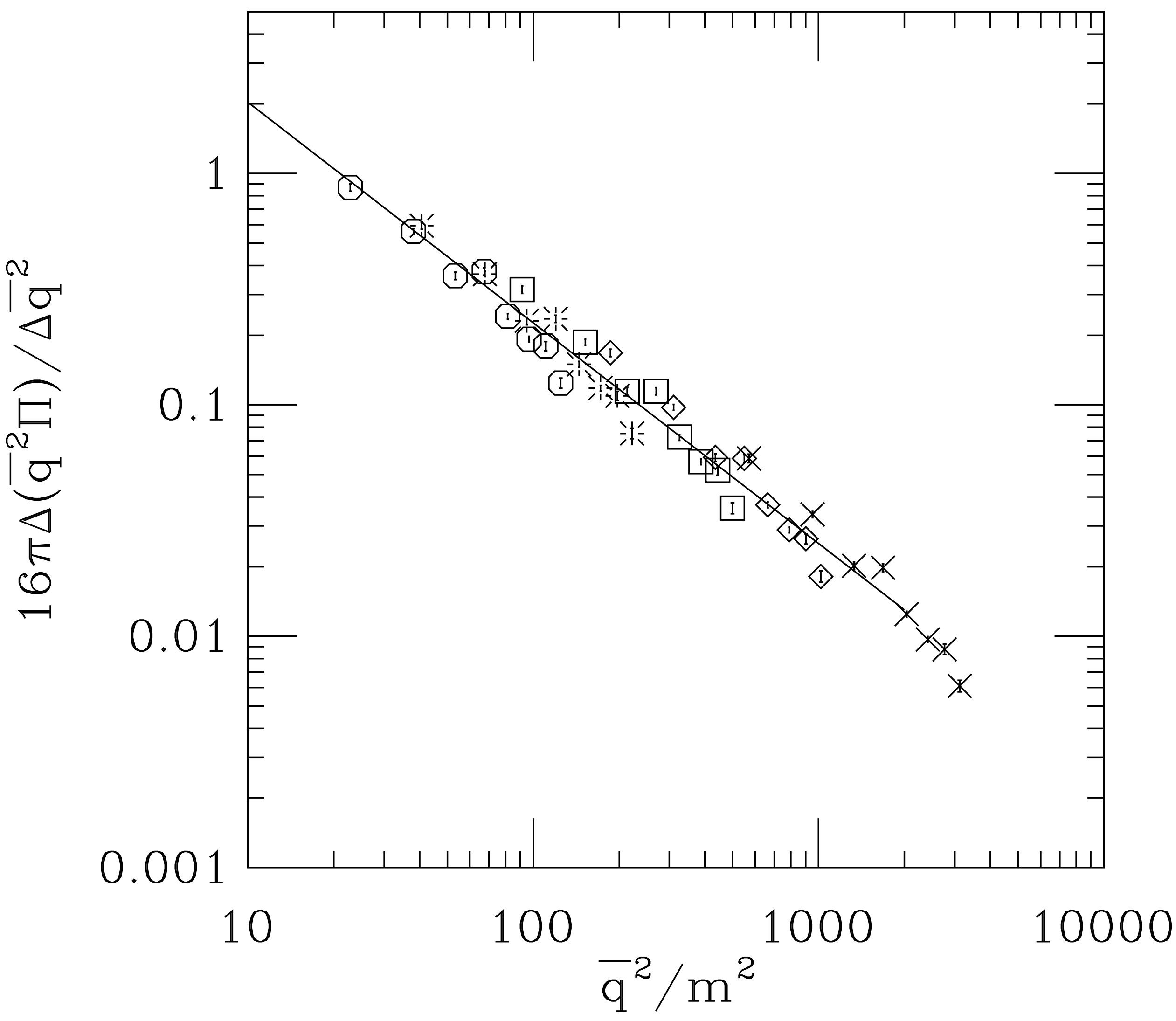


Figure 12: Numerically differentiated "$S-$parameter" for sextet QCD – log-log plot

Conclusions

- A fun calculation – and things like this will be needed for lattice BSM pheno
- Obviously, not the most wonderful system, but you use what you have
- Not much literature on precision EW for (near) conformal theories
  - $\Pi_T^{LR}(q^2) \rightarrow 0$ as $m_q \rightarrow 0$ – no surprise
  - Power law scaling – maybe no surprise if you believe in conformality
  - "Low resonance dominance" failed. I was surprised!
- Same techniques will work for any TC candidate – if anyone can find one!