Testing universality and automatic O(a) improvement in massless lattice QCD with Wilson quarks

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- O(a) effects in the Schrödinger functional
- The chirally rotated Schödinger functional
- Lattice implementation
- Relations from universality
- Lattice set-up and simulation results

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Motivation: a typical renormalisation problem

A typical calculation of a hadronic matrix element such as B_K can be split in 4 independent parts:

$$\langle h_f | O^{\overline{\rm MS}}(\mu = 1/L_{\rm min}) | h_i \rangle =$$

$$\underbrace{Z_{\overline{\rm MS},{\rm SF}}\left(\bar{g}^2(L_{\rm min})\right)}_{\text{1-loop PT}} \times \underbrace{U(L_{\rm min}, L_{\rm max})}_{\text{scale evolution in CL}} \times \underbrace{\lim_{a\to 0} \left\{ Z(g_0^2, a/L_{\rm max}) \underbrace{\langle h_f | O(a) | h_i \rangle}_{\text{bare matrix element}} \right\}}_{\text{renormalized matrix element at scale } 1/L_{\rm max}}$$

The factor

$$U(L_{\rm min}, L_{\rm max}) = \lim_{a\to 0} \frac{Z(g_0^2, a/L_{\rm min})}{Z(g_0^2, a/L_{\rm max})}$$

is constructed <u>in the continuum limit</u> from the step-scaling functions;

- may use different regularisation from calculation of bare matrix element;
- use Wilson quarks or staggered quarks (computationally cheap).

O(a) effects in SF schemes

- The time boundaries induce O(a) effects; can be cancelled by 2 boundary counterterms:

$$a \int_{x_0=0,T} \mathrm{d}^3\mathbf{x}\ \mathrm{tr}\,\{F_{0k}F_{0k}\}, \qquad a \int_{x_0=0,T} \mathrm{d}^3\mathbf{x}\ \bar{\psi}\gamma_0 D_0 \psi.$$

- Wilson fermions: addditional O(a) effects are cancelled by bulk O(a) counterterms $\propto c_{\mathrm{sw}}, c_{\mathrm{A}}, \ldots$.

$\Rightarrow$ these are needed despite automatic O(a) improvement of massless Wilson quarks (Frezzotti & Rossi '03)!

- The boundary O(a) terms can be monitored/controlled; it would be nice if the mechanism of automatic O(a) improvement in the bulk could be made compatible with the SF!

Mechanism of automatic O(a) improvement (Frezzotti & Rossi '03)

In the absence of boundaries, automatic O(a) improvement follows from chiral symmetry of the massless continuum action:

$$\psi \to \gamma_5 \psi \qquad \bar{\psi} \to -\bar{\psi}\gamma_5,$$

Observables can be decomposed

$$O = O_+ + O_-, \qquad O_\pm[\psi,\bar{\psi}] \to O_\pm[\gamma_5\psi, -\bar{\psi}\gamma_5] = \pm O_\pm[\psi,\bar{\psi}]$$

One then shows that

$$\langle O_+ \rangle = \langle O_+ \rangle^{\mathrm{cont}} + \mathrm{O}(a^2), \qquad \langle O_- \rangle = \mathrm{O}(a),$$

Thus O(a) effect are localised in γ_5-odd observables/correlation functions & can be avoided!

- With SF boundary conditions: standard argument of automatic O(a) improvement fails:

$$\gamma_5 P_\pm = P_\mp \gamma_5, \qquad P_\pm = \tfrac{1}{2}(1 \pm \gamma_0)$$

$\Rightarrow$ no way to define γ_5-even or γ_5-odd correlation functions!

- Possible solution: Give a flavour structure to the γ_5-transformation, for $N_\mathrm{f} = 2$:

$$\psi \rightarrow \gamma_5 \tau^1 \psi, \qquad \bar{\psi} \rightarrow -\bar{\psi}\gamma_5 \tau^1,$$

  and change quark boundary projectors, such that they commute with $\gamma_5\tau^1$, e.g.

$$\mathcal{P}_\pm = \tfrac{1}{2}(1 \pm \gamma_0 \tau^3), \qquad \tilde{Q}_\pm = \tfrac{1}{2}(1 \pm i\gamma_0\gamma_5\tau^3)$$

$$\Rightarrow [\mathcal{P}_\pm, \gamma_5\tau^1] = 0 = [\tilde{Q}_\pm, \gamma_5\tau^1]$$

- Here: focus on projectors $\tilde{Q}_\pm$; these arise naturally when chirally rotating the standard SF b.c.'s!

The Schrödinger functional and chiral rotations

In correlation functions derived from the Schrödinger functional,

$$\langle O[\psi,\bar{\psi}]\rangle_{(P_+)} = \mathcal{Z}^{-1}\int D[\psi,\bar{\psi},A]O[\psi,\bar{\psi}]\mathrm{e}^{-S}$$

the fermion fields ψ and $\overline{\psi}$ satisfy homogeneous Dirichlet boundary conditions with projectors $P_\pm = \frac{1}{2}(1\pm\gamma_0)$,

$$\begin{aligned} P_+\psi(x)|_{x_0=0} &= 0, & \qquad P_-\psi(x)|_{x_0=T} &= 0, \\ \overline{\psi}(x)P_-|_{x_0=0} &= 0, & \qquad \overline{\psi}(x)P_+|_{x_0=T} &= 0. \end{aligned}$$

Perform a chiral field rotation,

$$\psi \rightarrow \exp(i\alpha\gamma_5\tau^3/2)\psi, \qquad \overline{\psi} \rightarrow \overline{\psi}\exp(i\alpha\gamma_5\tau^3/2),$$

the rotated fields satisfy chirally rotated boundary conditions

$$P_+(\alpha)\psi(x)|_{x_0=0} = 0, \qquad P_-(\alpha)\psi(x)|_{x_0=T} = 0,$$

$$\overline{\psi}(x)\gamma_0 P_-(\alpha)|_{x_0=0} = 0, \qquad \overline{\psi}(x)\gamma_0 P_+(\alpha)|_{x_0=T} = 0,$$

with the projectors

$$P_\pm(\alpha) = \tfrac{1}{2}\left[1 \pm \gamma_0 \exp(i\alpha\gamma_5\tau^3)\right],$$

Setting $\alpha = \pi/2$:

$$P_\pm(\pi/2) \equiv \tilde{Q}_\pm = \tfrac{1}{2}(1 \pm i\gamma_0\gamma_5\tau^3) = \mathrm{diag}(Q_+, Q_-), \qquad Q_\pm = \tfrac{1}{2}(1 \pm i\gamma_0\gamma_5)$$

A change of variables in the functional integral leads to:

$$\langle O[\psi,\bar{\psi}]\rangle_{(P_\pm)} = \langle \tilde{O}[\psi,\bar{\psi}]\rangle_{(P_\pm(\alpha))}$$

with
$$\tilde{O}[\psi,\bar{\psi}] = O\left[\exp(i\alpha\gamma_5\tau^3/2)\psi, \bar{\psi}\exp(i\alpha\gamma_5\tau^3/2\right]$$

NB: All mass terms are set to zero, otherwise one also needs to rotate masses covariantly ($\rightarrow$ twisted mass QCD)

A note on symmetries

- In QCD the flavour vector transformation are identified as those leaving the quark mass term invariant. In massless QCD we are free to pick a convention.

- Convention used here: standard SF boundary conditions are parity & isospin invariant, this defines our "physical basis"

⇒ Symmetries take a non-standard form in the chirally rotated SF (χSF), as illustrated by the Ward identities:

$$\begin{aligned}\langle \delta_{\rm A}^a O\rangle_{(P_\pm)} &= \tfrac{1}{2}\int \mathrm{d}^3 z\,\langle\left[\bar\zeta(\mathbf{z})\gamma_5\tau^a\zeta(\mathbf{z}) + \bar\zeta'(\mathbf{z})\gamma_5\tau^a\zeta'(\mathbf{z})\right] O\rangle_{(P_\pm)} \\ \langle \delta_{\rm V}^a O\rangle_{(P_\pm)} &= 0\end{aligned}$$

$$\begin{aligned}\langle \delta_{\rm A}^a O\rangle_{(\tilde Q_\pm)} &= \tfrac{i}{2}\delta^{3a}\int \mathrm{d}^3 z\,\langle\left[\bar\zeta(\mathbf{z})\zeta(\mathbf{z}) + \bar\zeta'(\mathbf{z})\zeta'(\mathbf{z})\right] O\rangle_{(\tilde Q_\pm)} \\ \langle \delta_{\rm V}^a O\rangle_{(\tilde Q_\pm)} &= \tfrac{i}{2}\varepsilon^{3ab}\int \mathrm{d}^3 z\,\langle\left[\bar\zeta(\mathbf{z})\gamma_5\tau^b\zeta(\mathbf{z}) - \bar\zeta'(\mathbf{z})\gamma_5\tau^b\zeta'(\mathbf{z})\right] O\rangle_{(\tilde Q_\pm)}\end{aligned}$$

Lattice implementation of rotated SF boundary conditions

An orbifold procedure yields the structure of the lattice Dirac operator in the free theory:

$$
\begin{aligned}
S_f[U,\psi,\bar{\psi}] &= a^4 \sum_x \bar{\psi}(x)(\mathcal{D}_W + m_0)\psi(x) \\
a\mathcal{D}_W \psi(x) &= -U(x,0)P_-\psi(x+a\hat{\mathbf{0}}) + K\psi(x) - U(x-a\hat{\mathbf{0}})^\dagger P_+\psi(x-a\hat{\mathbf{0}}). \\
K\psi(x) &= \left(1 + am_0 + \tfrac{1}{2}\sum_{k=1}^{3}\left\{a(\nabla_k + \nabla_k^*)\gamma_k - a^2\nabla_k^*\nabla_k\right\}\right)\psi(x) \\
&\quad + \delta_{x_0,0} i\gamma_5\tau^3 P_-\psi(x) + \delta_{x_0,T} i\gamma_5\ \tau^3 P_+\psi(x).
\end{aligned}
$$

The lattice symmetries imply a list of possible $d=3$ counterterms:

$$
\begin{aligned}
O_1 &= \bar{\psi}\gamma_0 Q_+\psi - \bar{\psi}\gamma_0 Q_-\psi = \bar{\psi} i\gamma_5\tau^3\psi, \\
O_2 &= \bar{\psi}\tilde{Q}_+\psi, \\
O_3 &= \bar{\psi}\tilde{Q}_-\psi,
\end{aligned}
$$

List of dimension-4 counterterms:

$$\begin{aligned}
O_4 &= \bar{\psi}\,\tilde{Q}_+\gamma_k D_k\psi - \bar{\psi}\,\overleftarrow{D}_k\gamma_k\tilde{Q}_+\psi, \\
O_5 &= \bar{\psi}\,\tilde{Q}_-\gamma_k D_k\psi - \bar{\psi}\,\overleftarrow{D}_k\gamma_k\tilde{Q}_-\psi, \\
O_6 &= \bar{\psi}\,\tilde{Q}_+\gamma_0 D_0\psi - \bar{\psi}\,\overleftarrow{D}_0\gamma_0\tilde{Q}_+\psi, \\
O_7 &= \bar{\psi}\,\tilde{Q}_-\gamma_0 D_0\psi - \bar{\psi}\,\overleftarrow{D}_0\gamma_0\tilde{Q}_-\psi, \\
O_8 &= \bar{\psi}\,\tilde{Q}_+ D_0\psi + \bar{\psi}\,\overleftarrow{D}_0\tilde{Q}_+\psi, \\
O_9 &= \bar{\psi}\,\tilde{Q}_- D_0\psi + \bar{\psi}\,\overleftarrow{D}_0\tilde{Q}_-\psi, \\
O_{10} &= \bar{\psi}\,\tilde{Q}_+\gamma_0\gamma_k D_k\psi + \bar{\psi}\,\overleftarrow{D}_k\gamma_k\gamma_0\tilde{Q}_+\psi, \\
O_{11} &= \bar{\psi}\,\tilde{Q}_-\gamma_0\gamma_k D_k\psi + \bar{\psi}\,\overleftarrow{D}_k\gamma_k\gamma_0\tilde{Q}_-\psi,
\end{aligned}$$

Equations of motion:

$$O_4 + O_6 = 0, \qquad O_5 + O_7 = 0, \qquad O_8 + O_{10} = 0, \qquad O_9 + O_{11} = 0.$$

Total derivative:

$$O_{10} - O_{11} = \partial_k\left(\bar{\psi}\,\gamma_k i\gamma_5\tau^3\psi\right),$$

So there are 11 operators and 5 relations $\Rightarrow$ 6 counterterms!

Implementation of counterterms

- Replace $\mathcal{D}_W \to \mathcal{D}_W + \delta\mathcal{D}_W$,

$$
\begin{aligned}
\delta\mathcal{D}_W\psi(x) &= (\delta_{x_0,0} + \delta_{x_0,T}) \Big[(z_f - 1) + (d_s - 1)\, a\mathbf{D}_s\Big]\psi(x). \\
\mathbf{D}_s &= \tfrac{1}{2}(\nabla_k + \nabla_k^*)\gamma_k.
\end{aligned}
$$

- Parameterize the basic fermionic 2-point functions, e.g.

$$
\begin{aligned}
[\psi(x)\bar{\zeta}(\mathbf{y})]_F &= S(x; a, \mathbf{y})U_0(0, \mathbf{y})^\dagger \left(1 - \bar{d}_s a\overleftarrow{\mathbf{D}}_s\right) \tilde{Q}_-\,, \\
[\zeta(\mathbf{x})\bar{\zeta}(\mathbf{y})]_F &= \tilde{Q}_- \left(1 + \bar{d}_s a\mathbf{D}_s\right) \Big[\bar{S}(0, \mathbf{x}; 0, \mathbf{y}) - (\tilde{z}_f + \tilde{d}_s a\mathbf{D}_s)\delta(\mathbf{x} - \mathbf{y})\Big] \left(1 - \bar{d}_s a\overleftarrow{\mathbf{D}}_s\right) \tilde{Q}_{-} \\
[\zeta(\mathbf{x})\bar{\zeta}'(\mathbf{y})]_F &= \tilde{Q}_- \left(1 + \bar{d}_s a\mathbf{D}_s\right) \bar{S}(0, \mathbf{x}; T, \mathbf{y}) \left(1 - \bar{d}_s a\overleftarrow{\mathbf{D}}_s\right) \tilde{Q}_+\,,
\end{aligned}
$$

- Renormalize the quark boundary fields:

$$
\zeta_{\mathrm{R}} = Z_\zeta \zeta, \qquad \bar{\zeta}_{\mathrm{R}} = Z_\zeta \bar{\zeta}, \qquad \zeta'_{\mathrm{R}} = Z_\zeta \zeta', \qquad \bar{\zeta}'_{\mathrm{R}} = Z_\zeta \bar{\zeta}',
$$

Some observations

- The constants z_f, $\tilde{z}_f$ are finite (scale independent) and must be tuned to restore parity & flavour symmetry

- The constant $\bar{d}_s$ parametrizes the $\gamma_5\tau^1$-odd counterterm O_{10} and can usually be ignored.

- The constants $\tilde{z}_f$ and $\tilde{d}_s$ only occur in correlation functions where disconnected diagrams contribute; can be avoided by appropriate flavour assignments.

- Comparison to standard SF: in practice,

$$\{Z_\zeta, c_\mathrm{t}, \tilde{c}_\mathrm{t}, c_\mathrm{sw}, c_\mathrm{A}, \ldots\} \qquad \longleftrightarrow \qquad \{Z_\zeta, c_\mathrm{t}, d_s, z_f\}$$

  i.e. automatic O(a) improvement is obtained by tuning z_f.

- Tree level values: $Z_\zeta = 1$, $z_f = 1$, $d_s = \frac{1}{2}$, $\tilde{d}_s = 1$, $\bar{d}_s = 0$,

SF correlation functions

Standard SF correlators: use pseudo-scalar and vector boundary sources

$$\mathcal{O}_5^{f_1 f_2} = a^6 \sum_{\mathbf{y},\mathbf{z}} \bar{\zeta}_{f_1}(\mathbf{y})\gamma_5 P_- \zeta_{f_2}(\mathbf{z}) \qquad \mathcal{O}_k^{f_1 f_2} = a^6 \sum_{\mathbf{y},\mathbf{z}} \bar{\zeta}_{f_1}(\mathbf{y})\gamma_k P_- \zeta_{f_2}(\mathbf{z})$$

consider all possible 2-point functions with quark bilinear operators $X^{f_1 f_2}(x)$ ($X = A_0, P, V_0, S$) and $Y_k^{f_1 f_2}(x)$ ($Y_k = V_k, T_{0k}, A_k, \tilde{T}_{0k}$):

$$f_{\mathrm{X}}(x_0) = -\tfrac{1}{2}\langle X^{f_1 f_2}(x)\mathcal{O}_5^{f_2 f_1}\rangle_{(P_\pm)}, \qquad k_{\mathrm{Y}}(x_0) = -\tfrac{1}{6}\sum_{k=1}^{3}\langle Y_k^{f_1 f_2}(x)\mathcal{O}_k^{f_2 f_1}\rangle_{(P_\pm)},$$

Boundary-to-boundary correlators:

$$f_1 = -\tfrac{1}{2}\langle \mathcal{O}_5^{f_1 f_2}\mathcal{O}_5^{\prime f_2 f_1}\rangle_{(P_\pm)}, \qquad k_1 = -\tfrac{1}{6}\sum_{k=1}^{3}\langle \mathcal{O}_k^{f_1 f_2}\mathcal{O}_k^{\prime f_2 f_1}\rangle_{(P_\pm)}$$

χSF correlation functions

Main difference: two-point functions now depend on the flavour indices:

$$g_{\mathrm{X}}^{f_1 f_2}(x_0)_\pm = -\tfrac{1}{2}\langle X^{f_1 f_2}(x)\mathcal{Q}_{5,\pm}^{f_2 f_1}\rangle_{(\tilde{Q}_\pm)} \qquad l_{\mathrm{Y}}^{f_1 f_2}(x_0)_\pm = -\tfrac{1}{6}\sum_{k=1}^{3}\langle Y_k^{f_1 f_2}(x)\mathcal{Q}_k^{f_2 f_1}\rangle_{(\tilde{Q}_\pm)}$$

Boundary-to-boundary correlators:

$$g_1^{f_1 f_2} = -\tfrac{1}{2}\langle \mathcal{Q}_5^{f_1 f_2}\mathcal{Q}_5^{\prime\, f_2 f_1}\rangle_{(\tilde{Q}_\pm)}, \qquad l_1^{f_1 f_2} = -\tfrac{1}{6}\sum_{k=1}^{3}\langle \mathcal{Q}_k^{f_1 f_2}\mathcal{Q}_k^{\prime\, f_2 f_1}\rangle_{(\tilde{Q}_\pm)}$$

Boundary sources:

$$\mathcal{Q}_5^{uu'} = a^6\sum_{\mathbf{y},\mathbf{z}}\bar{\zeta}_u(\mathbf{y})\gamma_0\gamma_5 Q_-\zeta_{u'}(\mathbf{z}), \qquad \mathcal{Q}_5^{ud} = a^6\sum_{\mathbf{y},\mathbf{z}}\bar{\zeta}_u(\mathbf{y})\gamma_5 Q_+\zeta_d(\mathbf{z})$$

$$\mathcal{Q}_k^{uu'} = a^6\sum_{\mathbf{y},\mathbf{z}}\bar{\zeta}_u(\mathbf{y})\gamma_k Q_-\zeta_{u'}(\mathbf{z}), \qquad \mathcal{Q}_k^{ud} = a^6\sum_{\mathbf{y},\mathbf{z}}\bar{\zeta}_u(\mathbf{y})\gamma_0\gamma_k Q_+\zeta_d(\mathbf{z})$$

Notation: $\zeta(\mathbf{x}) \leftrightarrow \psi(x)|_{x_0=0}$ and $\bar{\zeta}(\mathbf{x}) \leftrightarrow \bar{\psi}(x)|_{x_0=0}$ (translated to the lattice)

Dictionary: standard ↔ rotated SF correlation functions

- Perform a chiral rotation by $\alpha = \pi/2$; 4 possibilities: $f_1 f_2 = uu', dd', ud, du$; assume existence of a second doublet (u', d') $\Rightarrow$ no disconnected diagrams

- Non-vanishing correlations functions with pseudo-scalar source

$$f_{\rm A} = g_{\rm A}^{uu'} = -ig_{\rm V}^{ud}, \qquad f_{\rm P} = ig_{\rm S}^{uu'} = g_{\rm P}^{ud}, \qquad f_1 = g_1^{uu'} = g_1^{ud}$$

- Non-vanishing correlations functions with vector source

$$k_{\rm V} = l_{\rm V}^{uu'} = -il_{\rm A}^{ud} \qquad k_{\rm T} = il_{\tilde{\rm T}}^{uu'} = l_{\rm T}^{ud} \qquad k_1 = l_1^{uu'} = l_1^{ud}$$

- All other correlation functions vanish by parity or flavour symmetries!

$$\begin{aligned} f_{\rm S} = ig_{\rm P}^{uu'} = g_{\rm S}^{ud} = \quad 0 \quad = f_{\rm V} = g_{\rm V}^{uu'} = -ig_{\rm A}^{ud}, \\ k_{\rm A} = l_{\rm A}^{uu'} = -il_{\rm V}^{ud} = \quad 0 \quad = k_{\tilde{\rm T}} = il_{\rm T}^{uu'} = l_{\tilde{\rm T}}^{ud} \end{aligned}$$

Rôle of z_f and boundary conditions

- Counterterm $\propto z_f$: renormalisation of α in the boundary projectors $P_\pm(\alpha)$ away from $P_\pm(\pi/2) \equiv \tilde{Q}_\pm$.

- Tuning of z_f restores the $\gamma_5\tau^1$-symmetry which is necessary for automatic O(a) improvement. This symmetry is a flavour symmetry in our conventions.

$\Rightarrow$ Tuning conditions for z_f: require any $\gamma_5\tau^1$-odd quantity (e.g. g_{A}^{ud}, $l_{\mathrm{T}}^{uu'}$,...) to vanish exactly; different tuning conditions: expect O(a) differences in tuned values z_f^*

- Boundary quark fields are defined as in the standard SF:

$$\zeta(\mathbf{x}) \quad \leftrightarrow \quad U(0,\mathbf{x};0)\tilde{Q}_-\psi(a,\mathbf{x}), \qquad \bar{\zeta}(\mathbf{x}) \leftrightarrow \bar{\psi}(a,\mathbf{x})\tilde{Q}_-U(x,0)^\dagger,$$

To test the Dirichlet boundary conditions we reverse the projectors $\tilde{Q}_- \rightarrow \tilde{Q}_+$. For comparison we do the same in the standard SF.

Non-perturbative quenched study

- Quenched simulations with Wilson quarks with and without O(a) improvement à la Sheikholeslami-Wohlert

- DDHMC code + SF boundary conditions, both standard and rotated

- Lattice sizes $(L/a)^3 \times (T/a)$ with $T = L$ and

$$L/a = 8, 12, 16, 24, 32$$

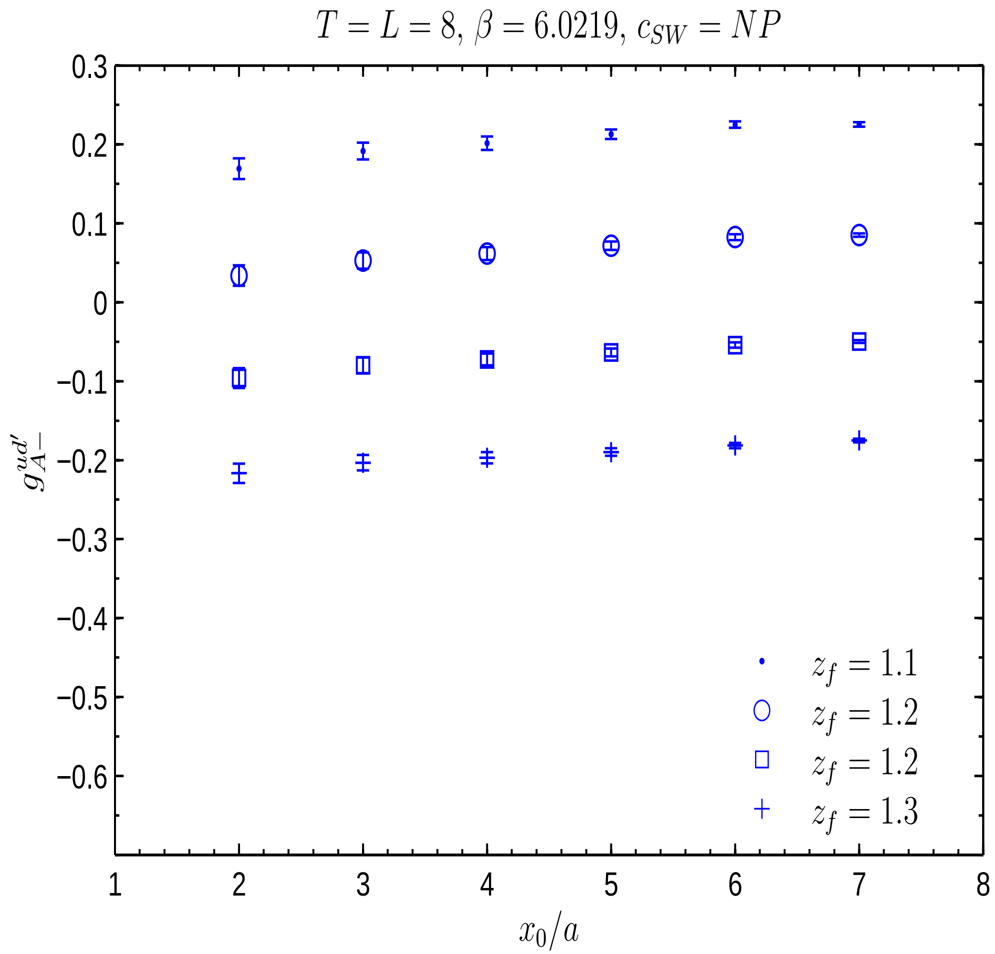
  keeping L/r_0 constant $\Rightarrow a = 0.025 - 0.1\,\mathrm{fm}$

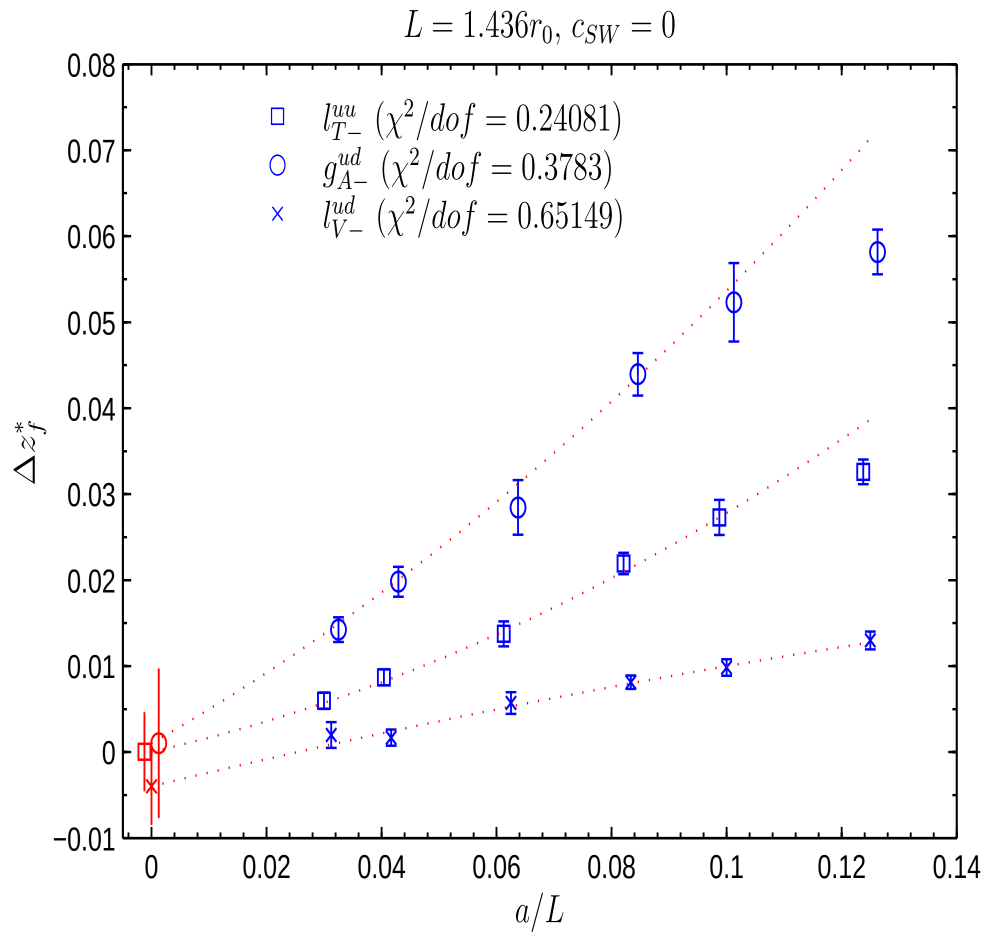
- m_{critical} is taken from the PCAC relation in the standard SF

A laundry list

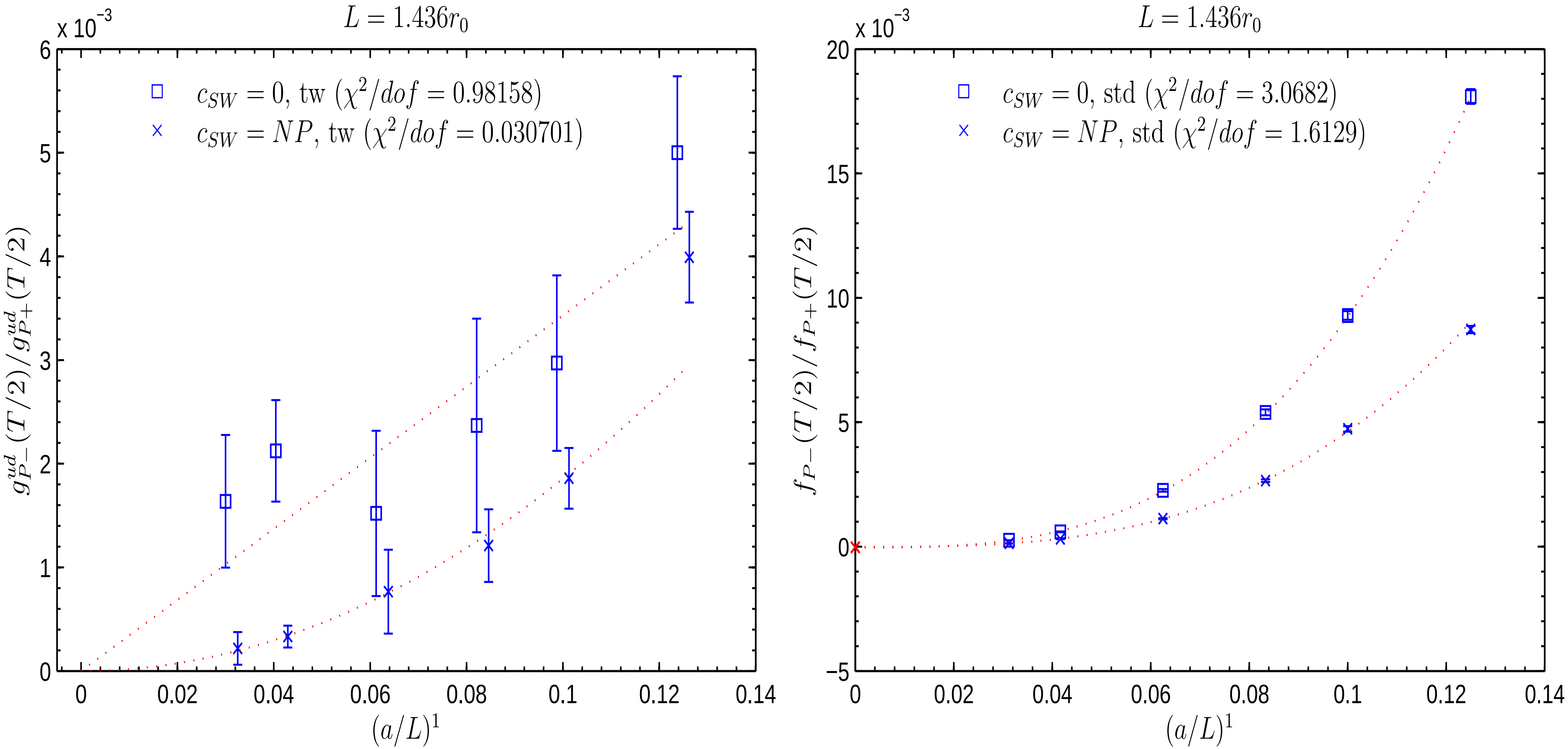
- How difficult is the tuning of z_f?
- check that the Dirichlet boundary conditions are indeed obtained as expected.
- check universality between rotated and standard SF correlation functions
- check that automatic O(a) improvement works out as expected: $\gamma_5\tau^1$-odd correlators should be of O(a).
- Use universality to determine finite ratios of renormalization constants

Tuning of z_f (see also ETMC (J. Gonzalez-Lopez et al), 09)

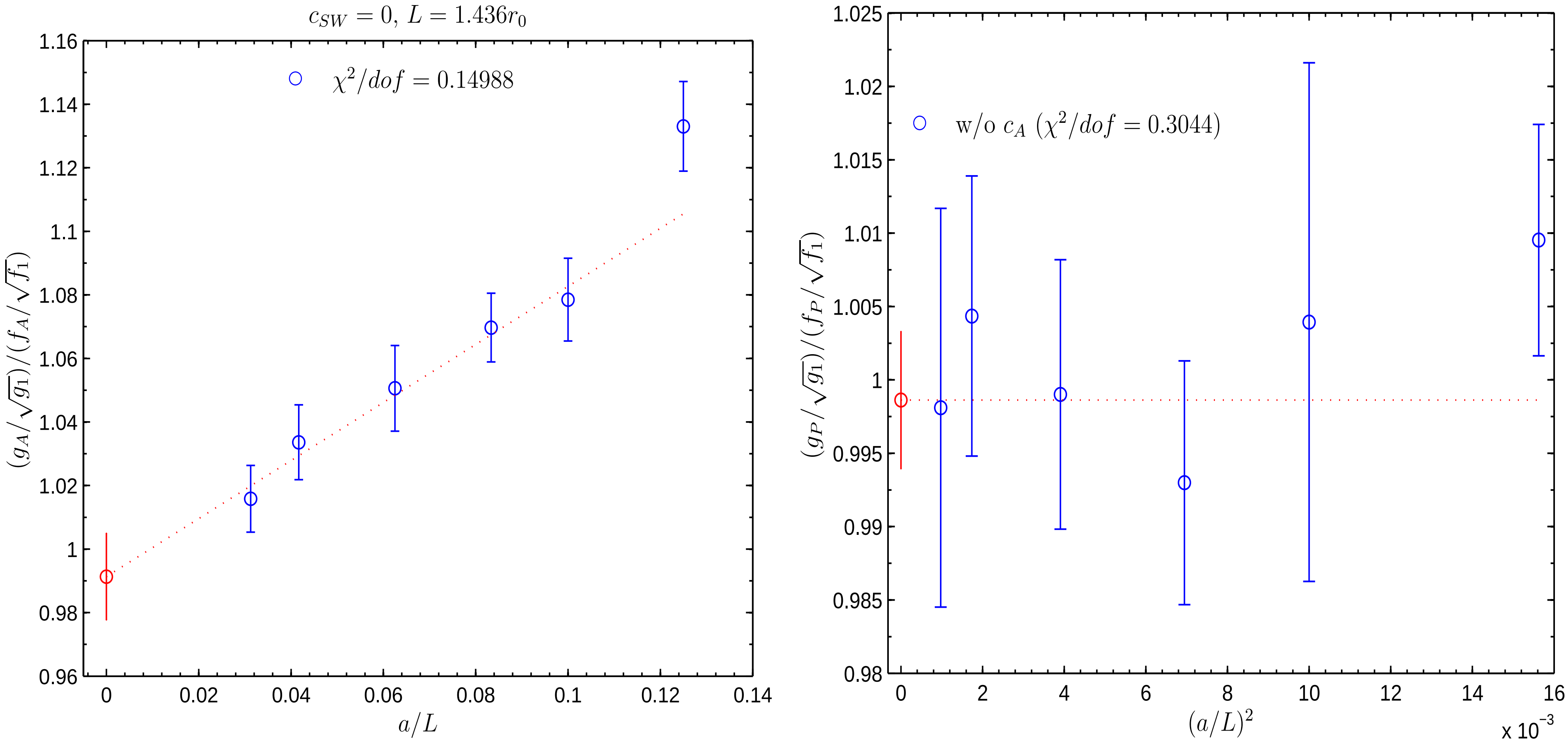




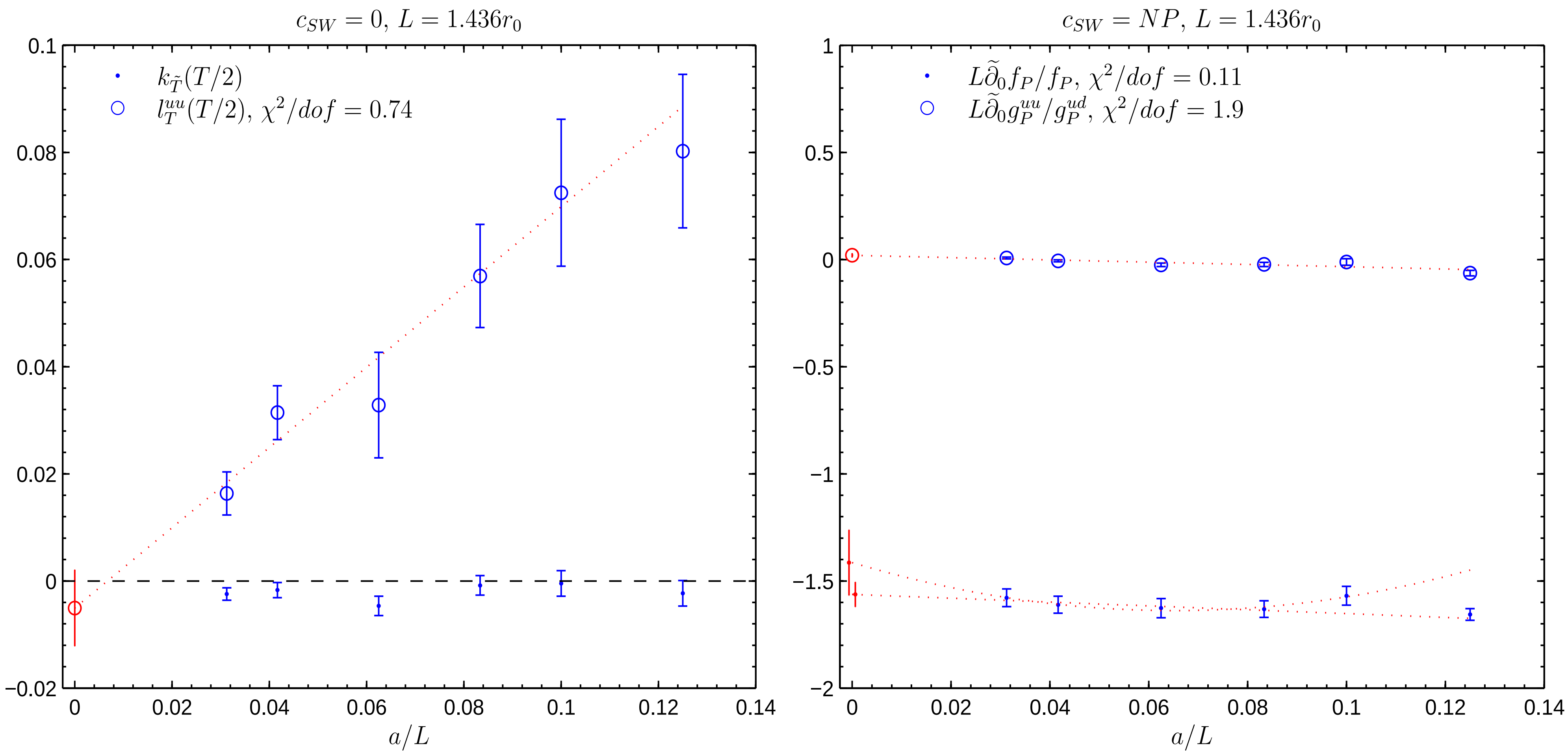
Check of Dirichlet boundary conditions



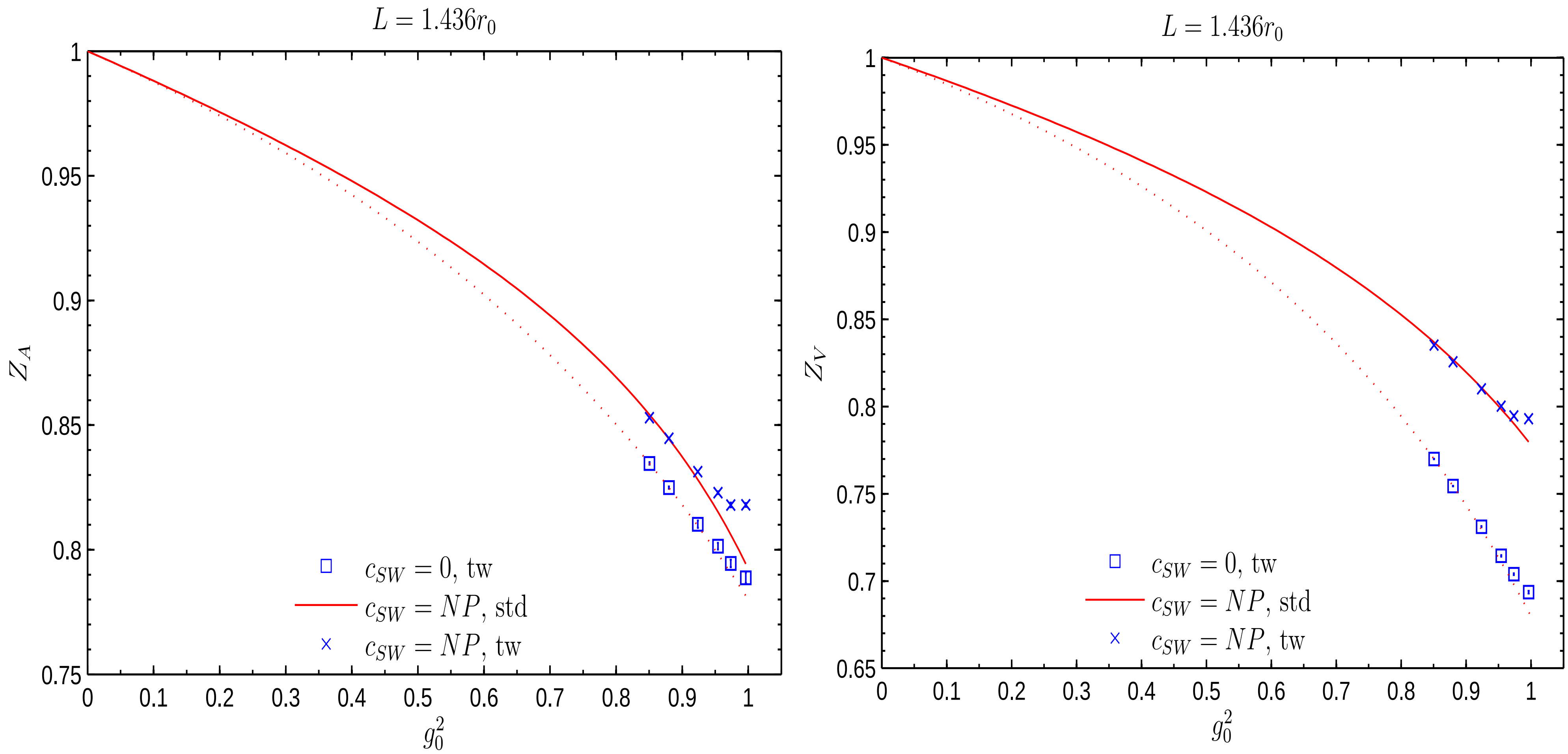
Universality checks



Check of automatic O(a) improvement

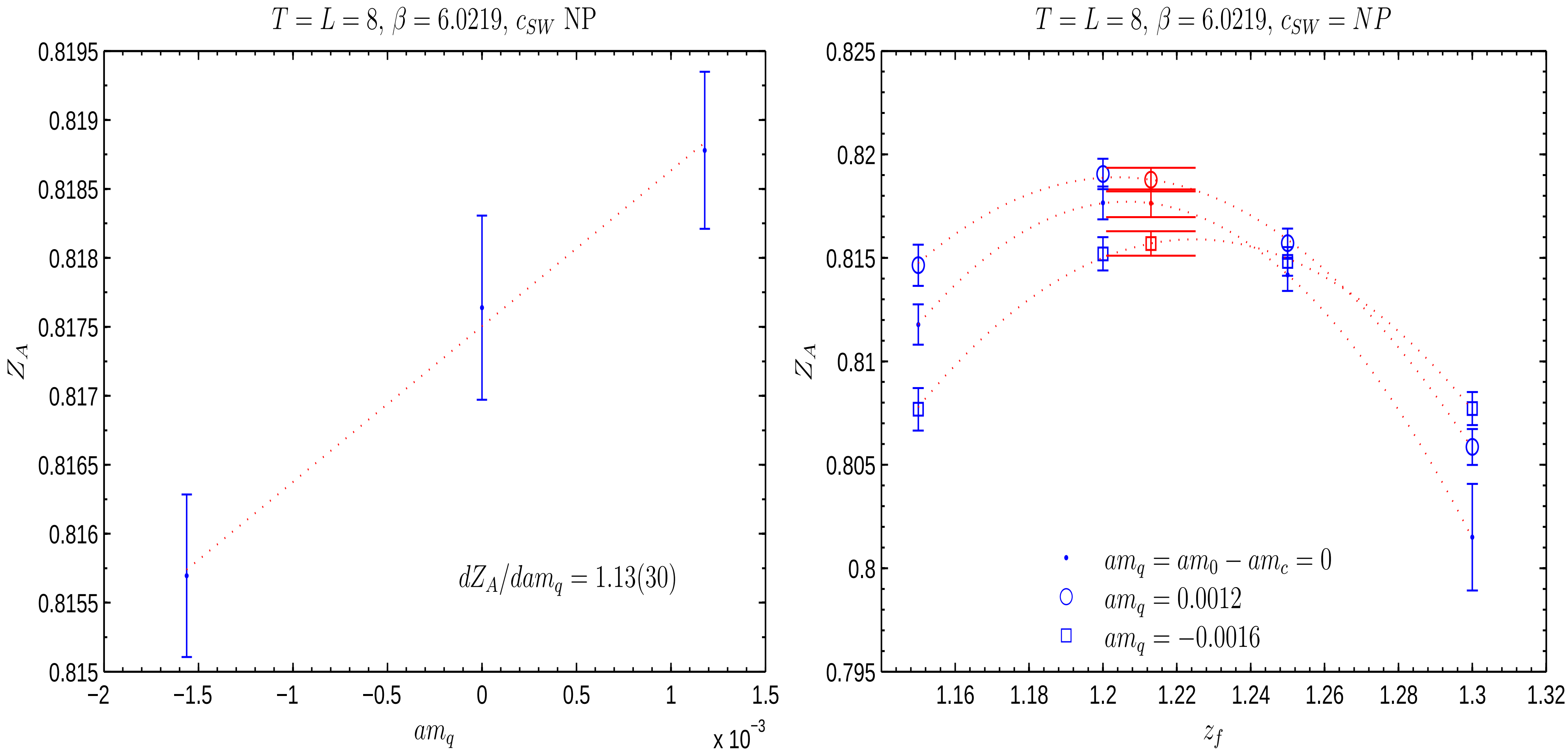


Determination of $Z_{\mathrm{A,V}}$

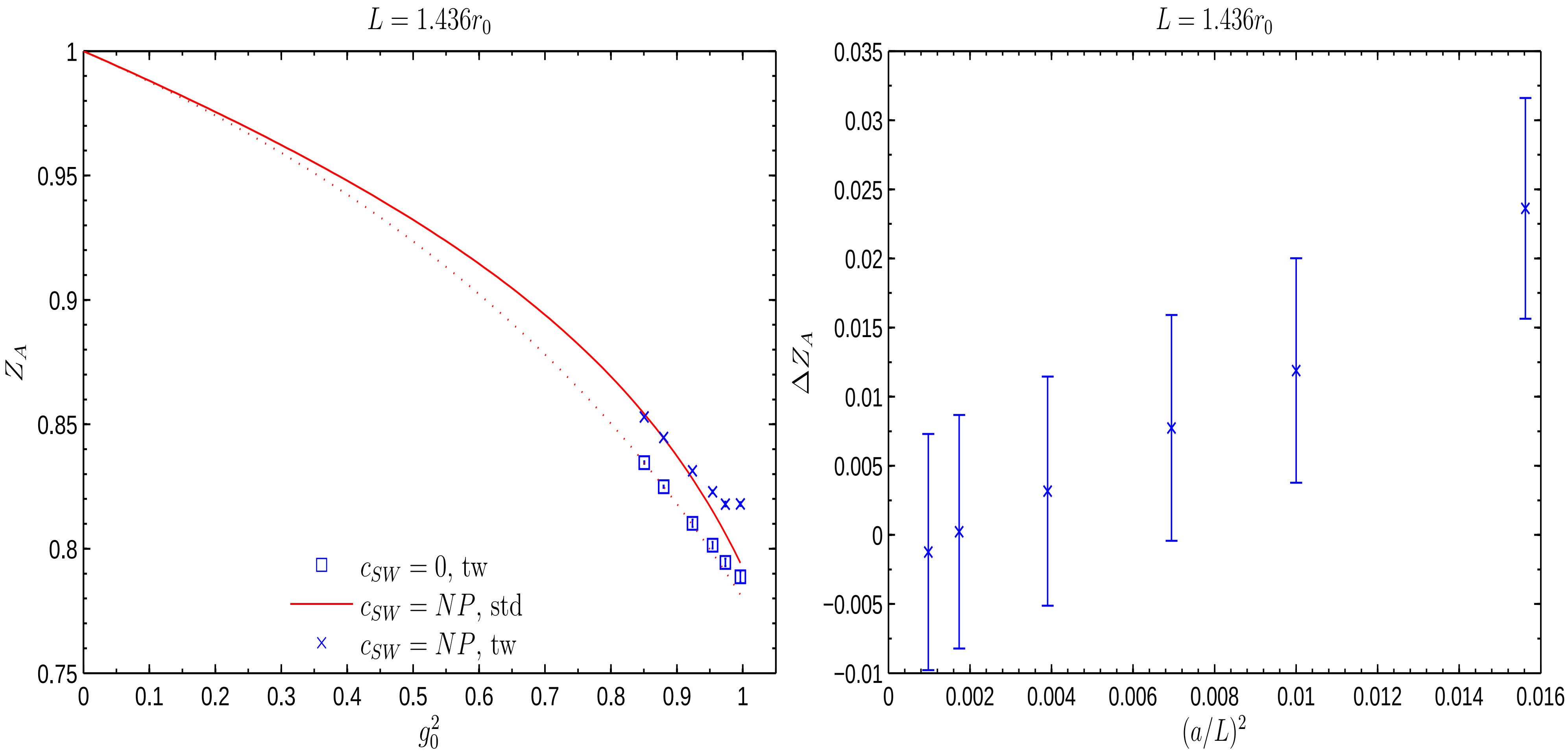


$Z_{\mathrm{A}} = i g_{\tilde{V}}(T/2)/g_A(T/2)$ and $Z_{\mathrm{V}} = g_{\tilde{V}}(T/2)/g_V(T/2)$

m_0- and z_f-dependence of Z_A



O(a^2) uncertainty in Z_{A}



Conclusions and Outlook

- Successful implementation of chirally rotated SF b.c.'s for Wilson quarks
- Tuning of the dimension-3 counterterm coefficient z_f straightforward and almost orthogonal to the tuning of m_0.
- Achievement: bulk O(a) improvement of massless standard or partially improved Wilson quarks

  $\Rightarrow$ Z-factors in SF schemes are O(a) improved by tuning the boundary O(a) counterterms ($c_{\rm t}$ and $d_s \Leftrightarrow \tilde{c}_{\rm t}$;

  interesting for 4-quark operators, higher twist operators,. . .
- Applications to Technicolor-inspired models, avoids determination of $c_{\rm sw}$.
- New methods to determine finite renormalisation constants $Z_{\rm A}$, $Z_{\rm V}$, $Z_{\rm P}/Z_{\rm S}, \ldots$ and improvement coefficients $c_{\rm A}$, $c_{\rm V}$, $c_{\rm sw}, \ldots$).