Unitary Fermions on the Lattice

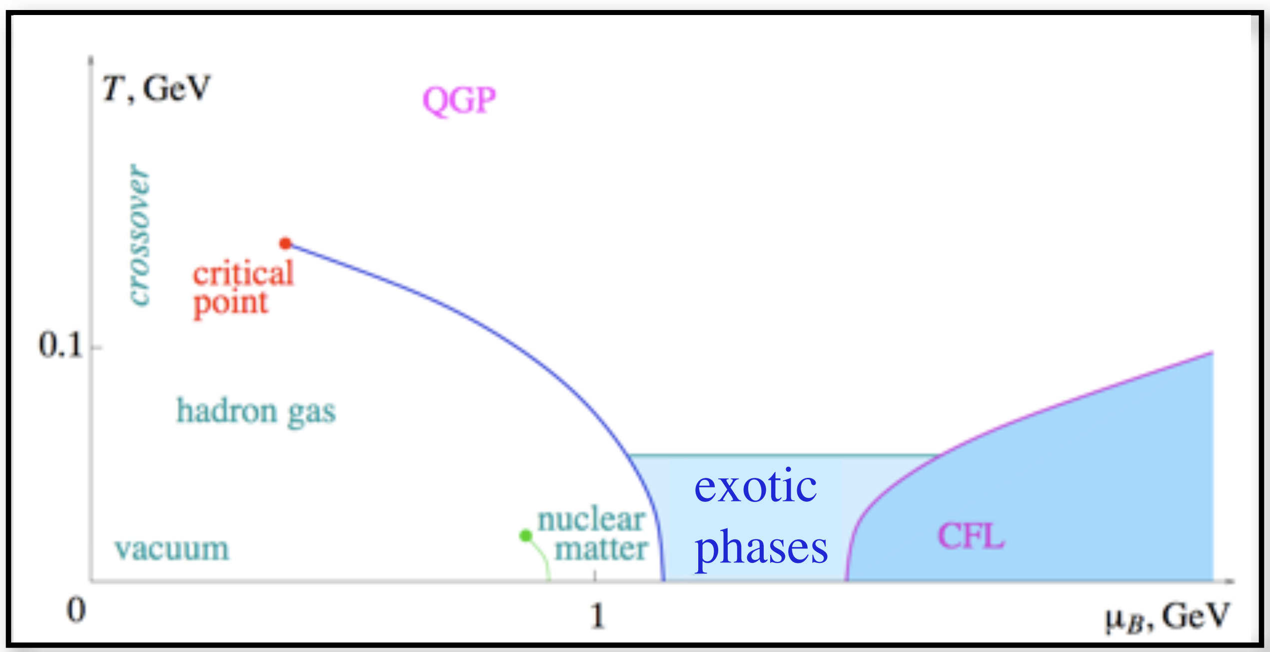
With: Michael Endres, Jong-Wan Lee, Amy Nicholson

Major outstanding problem in LGT: QCD at finite fermion number

Unitary Fermions on the Lattice

With: Michael Endres, Jong-Wan Lee, Amy Nicholson

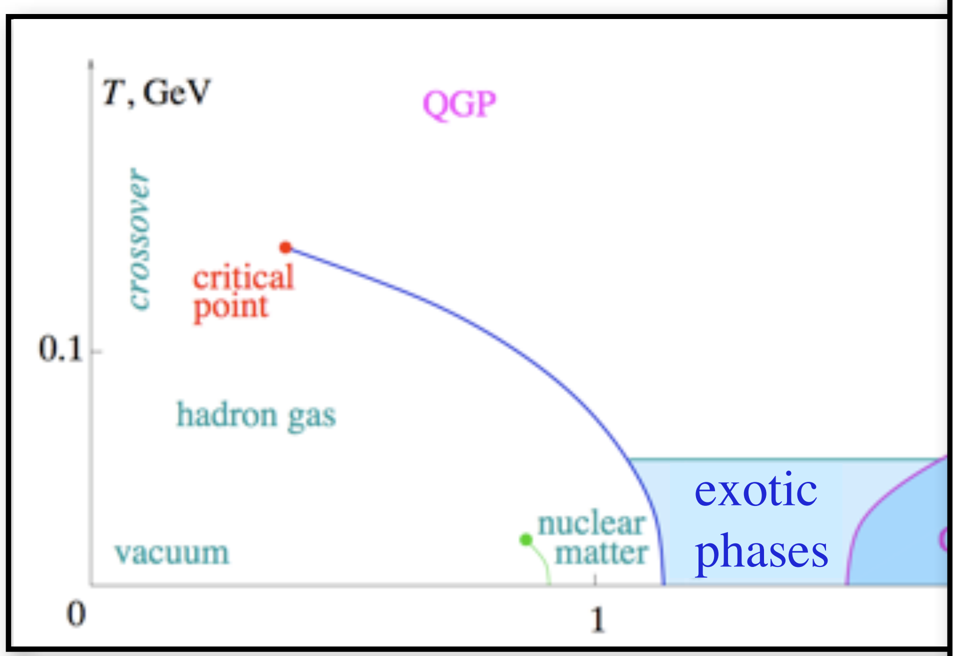
Major outstanding problem in LGT: QCD at finite fermion number

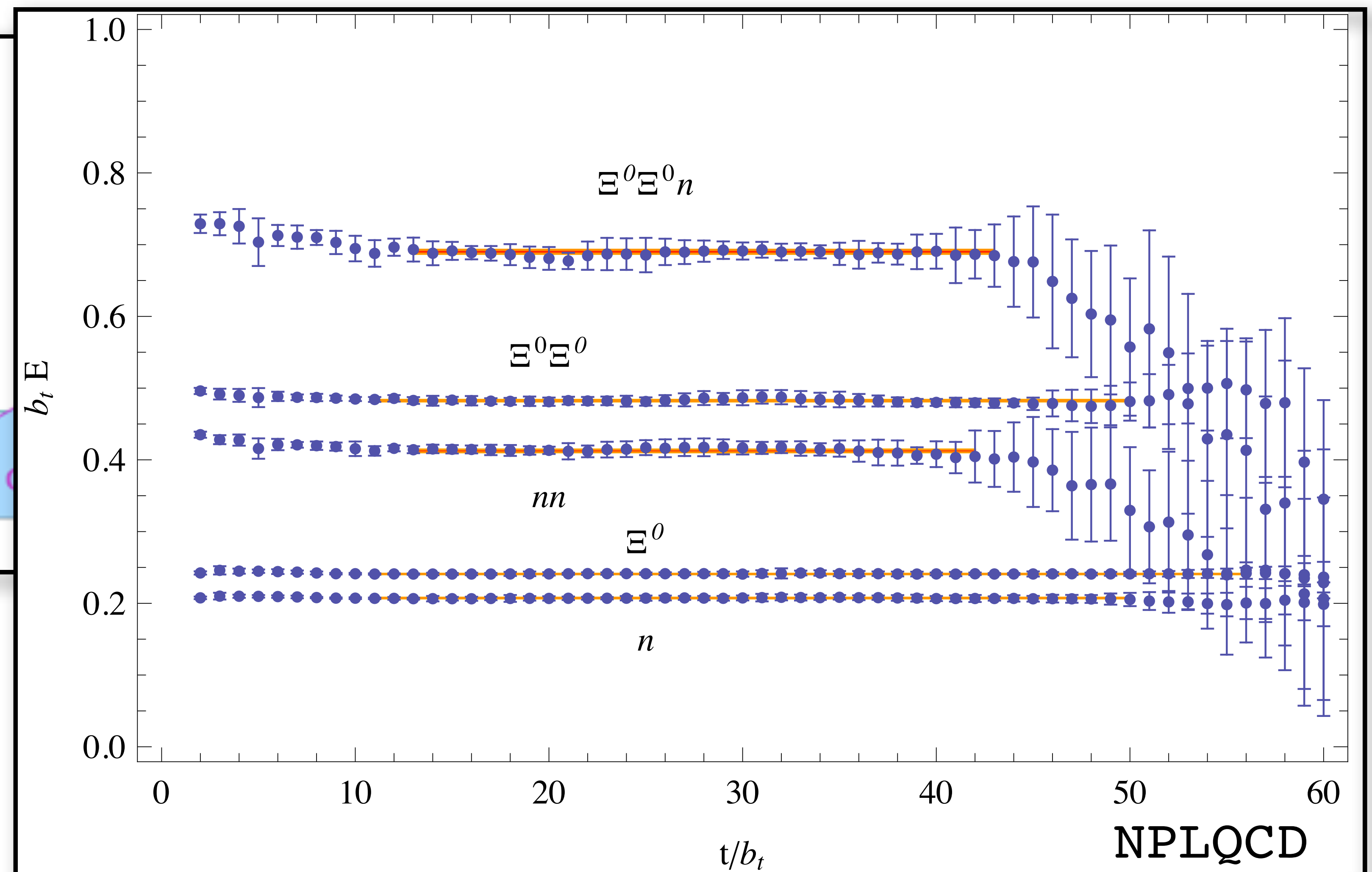


Unitary Fermions on the Lattice

With: Michael Endres, Jong-Wan Lee, Amy Nicholson

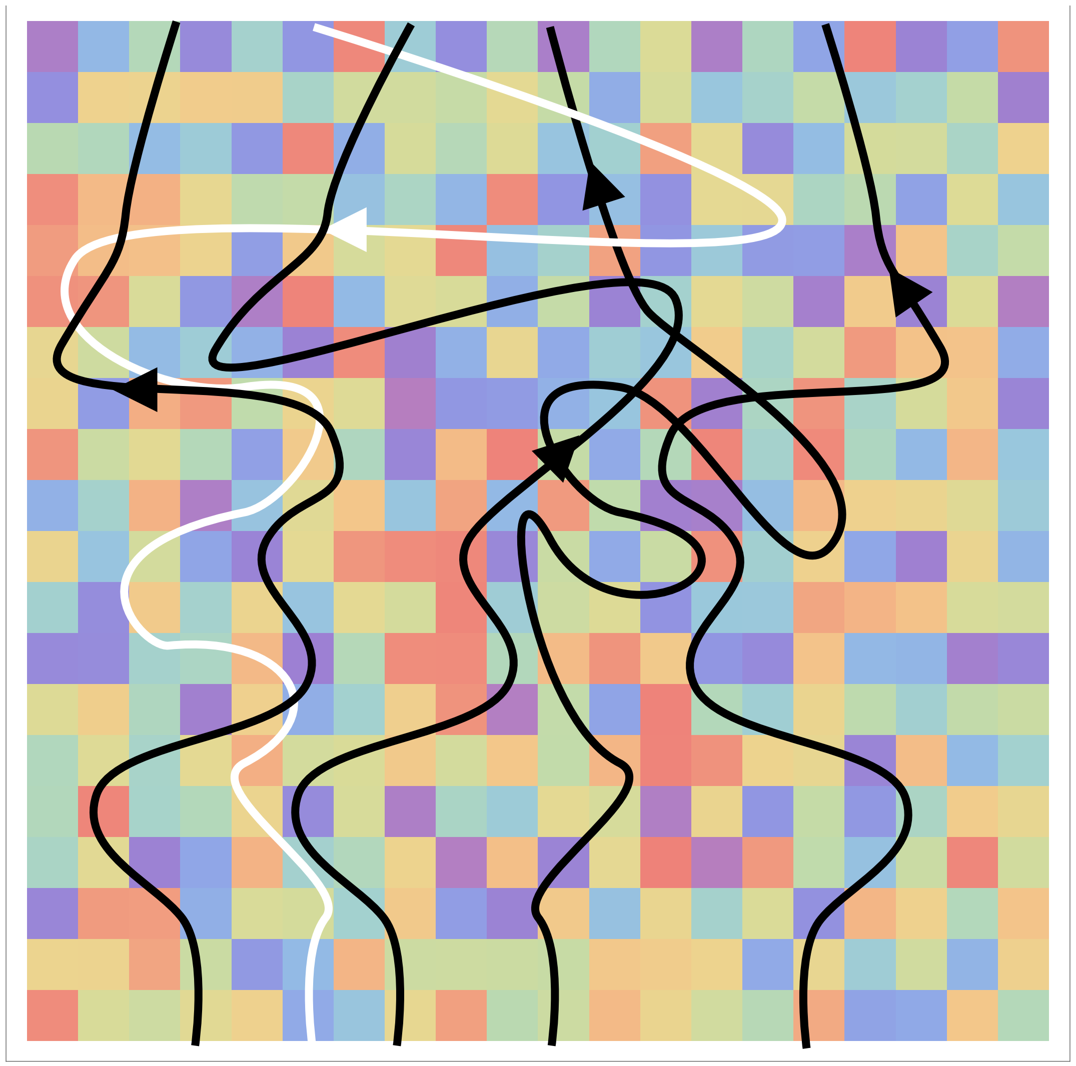
Major outstanding problem in LGT: QCD at finite fermion number





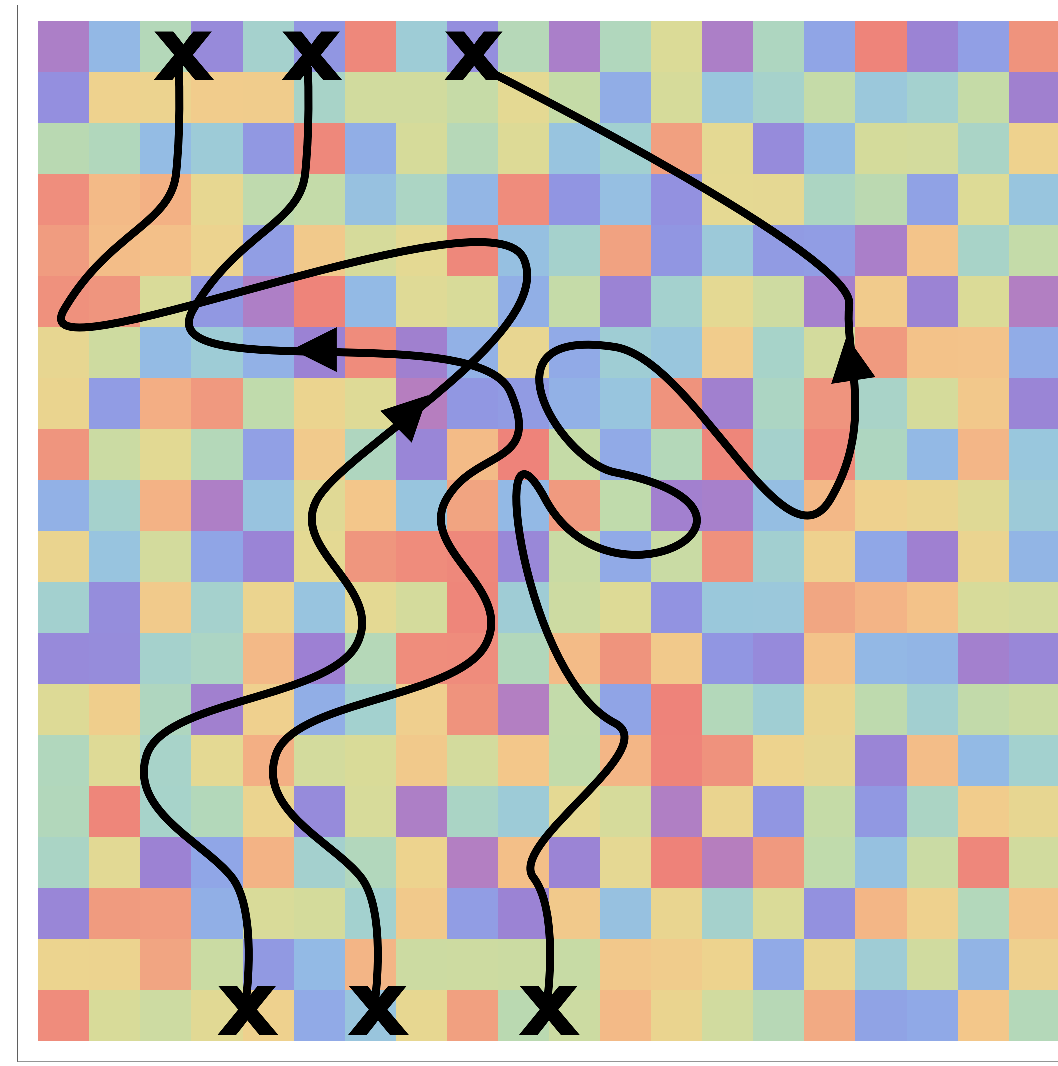
What is the sign problem?

Sign Problem



Grand canonical: exponentially difficult in volume V

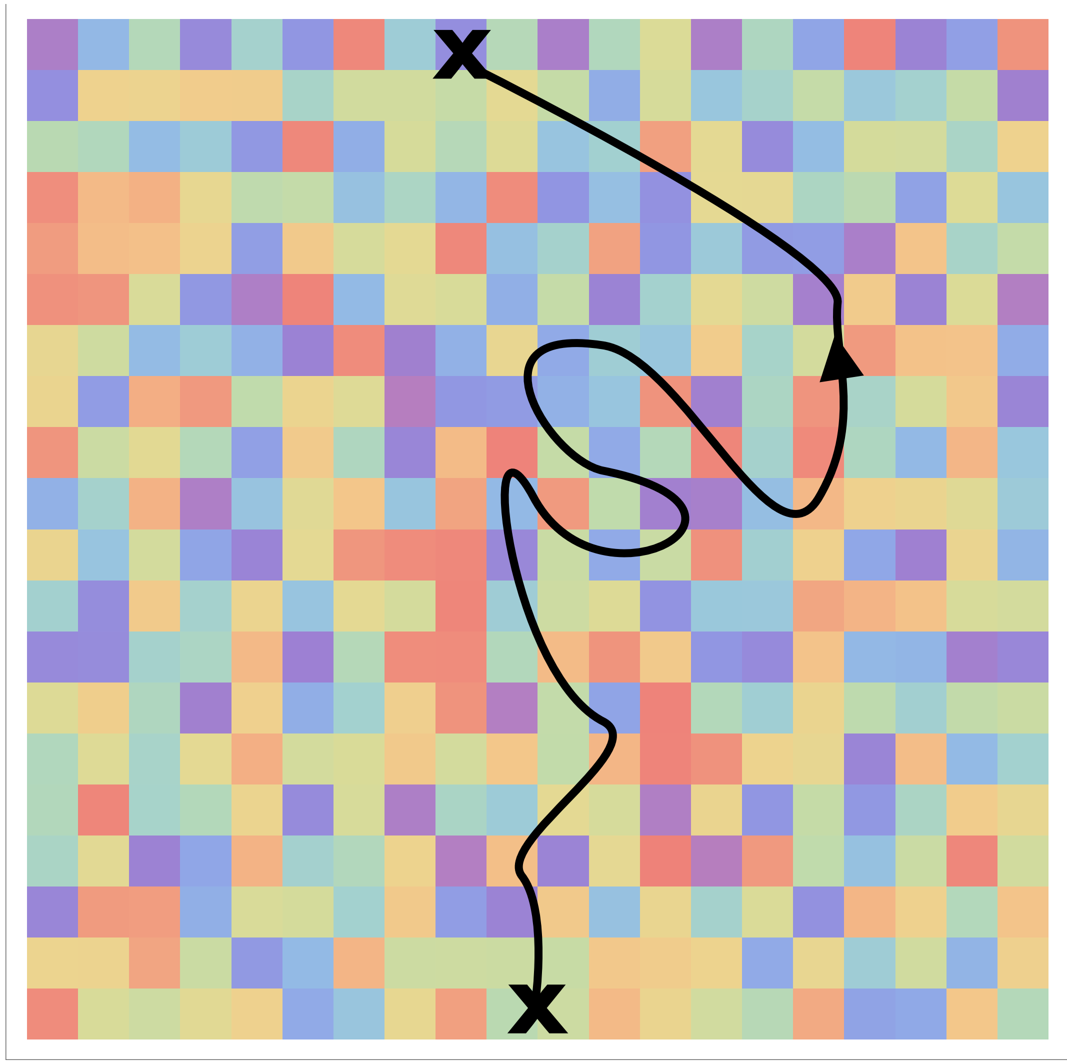
Signal/Noise Problem



Canonical: exponentially bad S/N in Euclidian time t

S/N ~ sign problem:

In background gauge field, quarks don't know about each other.



Is the quark in a pion?

⇒ big correlation after long time

Is the quark in a baryon?

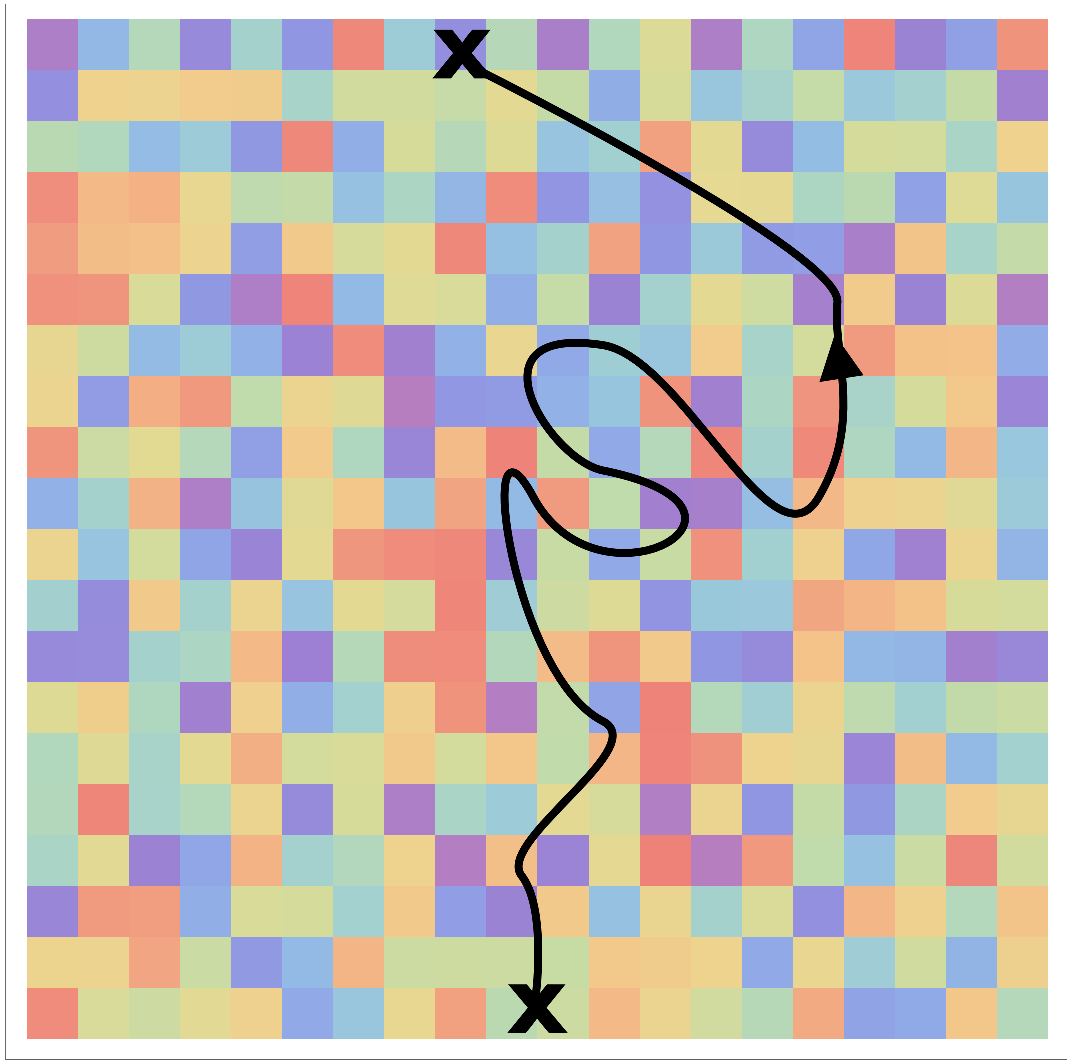
⇒ exponentially smaller correlation

(quarks "weigh more" in a baryon)

What is a quark to do??

S/N ~ sign problem:

In background gauge field, quarks don't know about each other.



Is the quark in a pion?

⇒ big correlation after long time

Is the quark in a baryon?

⇒ exponentially smaller correlation

(quarks "weigh more" in a baryon)

What is a quark to do??

<u>How lattice QCD solves this problem:</u>

- Every quark has long correlation in a background gauge field

  "M_q" ~ $M_\pi/2$

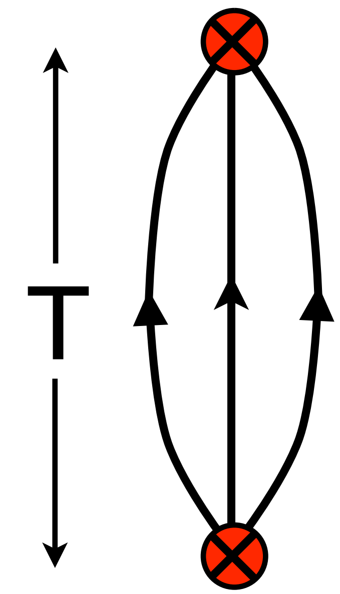
- If the quark is in a baryon, EXPONENTIAL CANCELLATIONS when averaging gauge fields (since $M_B > 3\,M_\pi/2$)

Lepage argument for the signal/noise problem

e.g: measuring the nucleon mass in LQCD:

$$\langle C\rangle = \frac{1}{N}\sum_{\{A\}} C(A) \propto e^{-MT} + \dots$$

nucleon:
lightest 3q state



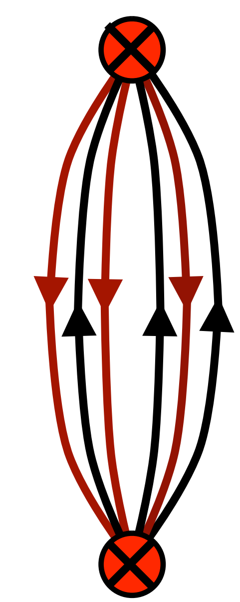
$C(A)$

Dispersion in measurement:

3 anti-quarks
3 quarks

$$\sigma = \langle C^\dagger C\rangle = \frac{1}{N}\sum_{\{A\}} C^\dagger(A) C(A) \propto e^{-3m_\pi T} + \dots$$

3π: lightest 3q+ 3q* state



$C^\dagger(A)C(A)$

$$\frac{\text{signal}}{\text{noise}} \propto \frac{1}{\sqrt{N}} e^{-(M-\frac{3}{2}m_\pi)T}$$

Basically same picture, with caveats:
"N"? Gaussian distribution?

Important to investigate

- systems with many fermions
  - are usual correlator measurements feasible?
  - choice of sources?

- systems with signal to noise problems
  - multi-fermion
  - disconnected diagrams

QCD too hard for playing around: find a simpler nontrivial system to investigate

A physically interesting multi-fermion system that is more tractable: "Unitary Fermions"

<u>Nonrelativistic</u> scattering from a short range interaction:

$$\mathcal{A} = \frac{4\pi}{M}\frac{1}{p\cot\delta - ip}$$

$$p\cot\delta = -\frac{1}{a} + \frac{1}{2}r_0 p^2 + O(p^4)$$

$$p = \sqrt{ME}$$

A physically interesting multi-fermion system that is more tractable: "Unitary Fermions"

Nonrelativistic scattering from a short range interaction:

$$\mathcal{A} = \frac{4\pi}{M}\frac{1}{p\cot\delta - ip}$$

$$p\cot\delta = -\frac{1}{a} + \frac{1}{2}r_0 p^2 + O(p^4)$$

phase shift

$p = \sqrt{ME}$

A physically interesting multi-fermion system that is more tractable: "Unitary Fermions"

Nonrelativistic scattering from a short range interaction:

$$\mathcal{A} = \frac{4\pi}{M}\frac{1}{p\cot\delta - ip}$$

phase shift

$$p = \sqrt{ME}$$

$$p\cot\delta = -\frac{1}{a} + \frac{1}{2}r_0 p^2 + O(p^4)$$

scattering length

A physically interesting multi-fermion system that is more tractable: "Unitary Fermions"

Nonrelativistic scattering from a short range interaction:

$$\mathcal{A} = \frac{4\pi}{M}\frac{1}{p\cot\delta - ip} \qquad p\cot\delta = -\frac{1}{a} + \frac{1}{2}r_0 p^2 + O(p^4)$$

phase shift

$$p = \sqrt{ME}$$

scattering length

effective range

A physically interesting multi-fermion system that is more tractable: "Unitary Fermions"

Nonrelativistic scattering from a short range interaction:

$$\mathcal{A} = \frac{4\pi}{M}\frac{1}{p\cot\delta - ip}$$

$$p\cot\delta = -\frac{1}{a} + \frac{1}{2}r_0 p^2 + O(p^4)$$

phase shift

$p = \sqrt{ME}$

scattering length

effective range

"unitary" fermions: $p\cot\delta = 0$

$$\delta = 90^\circ$$

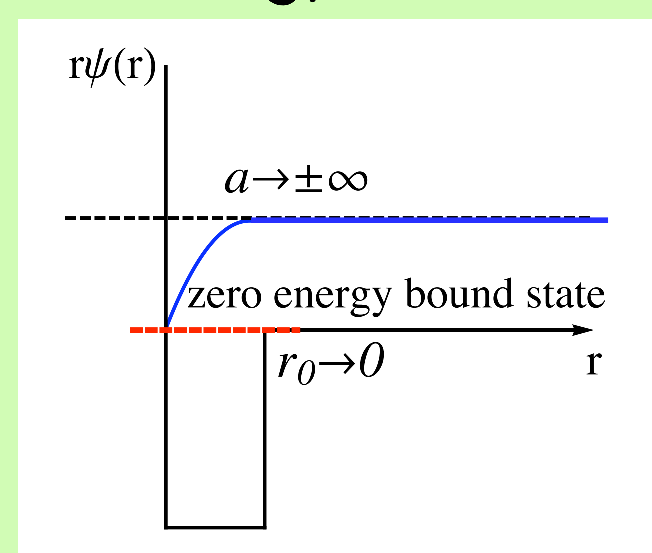
$$a = \infty$$

$$r_0 = 0\ldots$$

A strongly-coupled conformal system

Zero-range potential

Zero-energy bound state

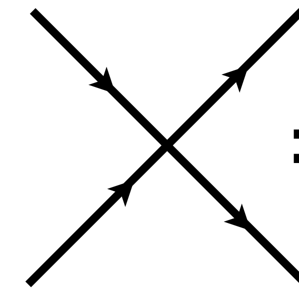


Effective theory:

D.K., M. Savage, M.Wise (1998)

Fermion scattering at very low energy; leading interaction:

$$\mathcal{L}_{\rm EFT} = \frac{C_0}{4} N^\dagger N N^\dagger N + \ldots$$



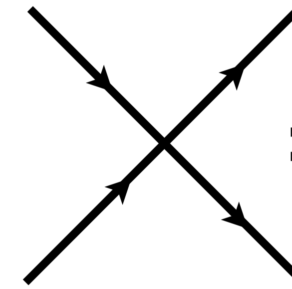
$= iC_0$

Effective theory:

D.K., M. Savage, M.Wise (1998)

Fermion scattering at very low energy; leading interaction:

$$\mathcal{L}_{\rm EFT} = \frac{C_0}{4} N^\dagger N N^\dagger N + \ldots$$



$= iC_0$

Renormalization group: C_0 scales with UV cutoff μ:

$$\widehat{C}_0 \equiv -\frac{M\mu}{4\pi} C_0 = \frac{\mu}{\mu + \frac{1}{a}} \qquad \widehat{\beta} = \mu \frac{\partial \widehat{C}_0}{\partial \mu} = -\widehat{C}_0 \left(\widehat{C}_0 - 1 \right)$$

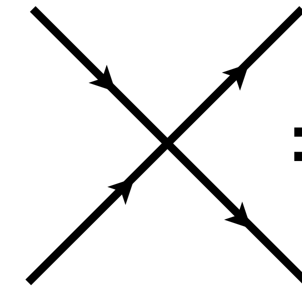
Scattering length

Effective theory:

D.K., M. Savage, M.Wise (1998)

Fermion scattering at very low energy; leading interaction:

$$\mathcal{L}_{\rm EFT} = \frac{C_0}{4} N^\dagger N N^\dagger N + \dots$$

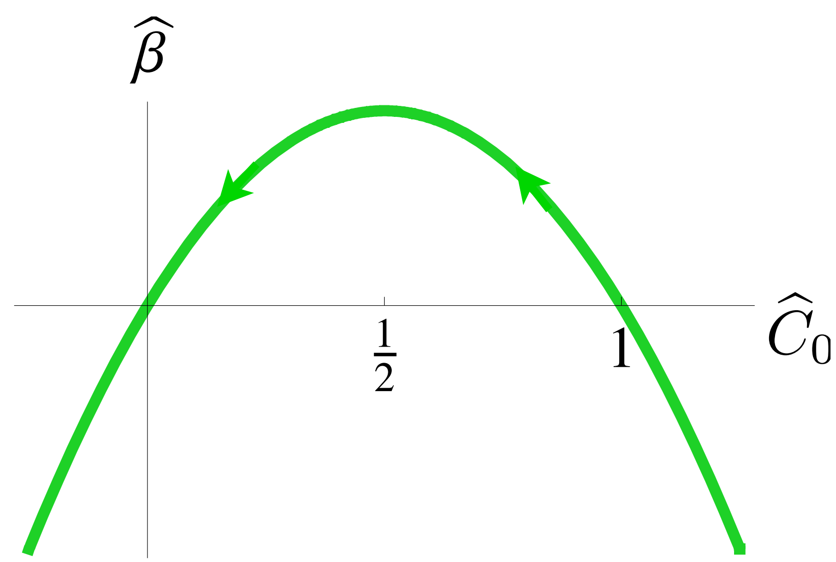


$= iC_0$

Renormalization group: C_0 scales with UV cutoff μ:

$$\widehat{C}_0 \equiv -\frac{M\mu}{4\pi} C_0 = \frac{\mu}{\mu + \frac{1}{a}} \qquad \widehat{\beta} = \mu \frac{\partial \widehat{C}_0}{\partial \mu} = -\widehat{C}_0 \left(\widehat{C}_0 - 1 \right)$$

Scattering length

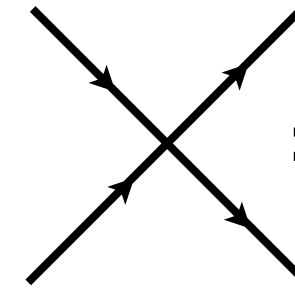


Effective theory:

D.K., M. Savage, M.Wise (1998)

Fermion scattering at very low energy; leading interaction:

$$\mathcal{L}_{\rm EFT} = \frac{C_0}{4} N^\dagger N N^\dagger N + \ldots$$

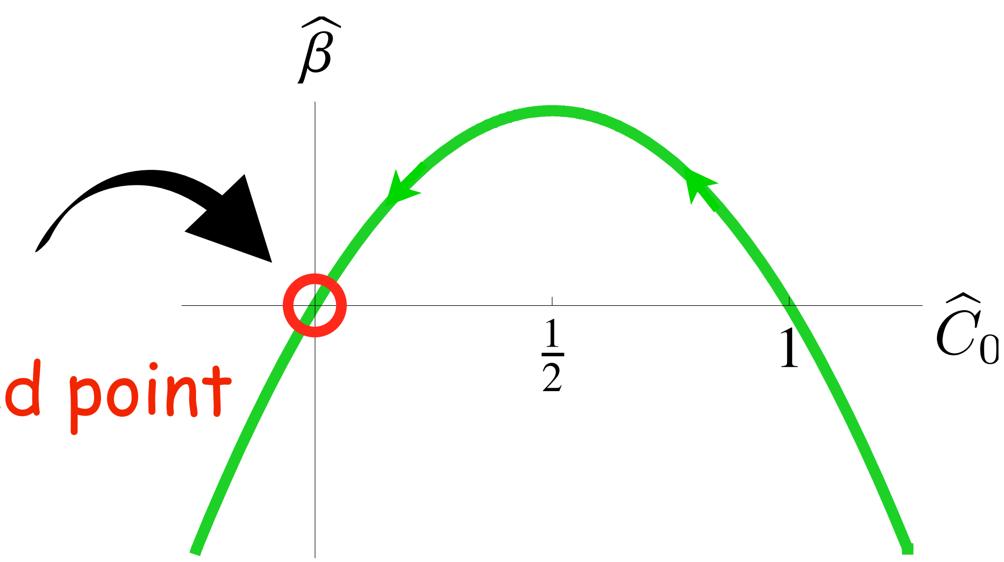


$= iC_0$

Renormalization group: C_0 scales with UV cutoff μ:

$$\widehat{C}_0 \equiv -\frac{M\mu}{4\pi} C_0 = \frac{\mu}{\mu + \frac{1}{a}} \qquad \widehat{\beta} = \mu \frac{\partial \widehat{C}_0}{\partial \mu} = -\widehat{C}_0 \left(\widehat{C}_0 - 1\right)$$

Scattering length



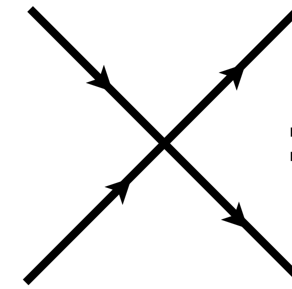
Trivial IR fixed point
free fermions

Effective theory:

D.K., M. Savage, M.Wise (1998)

Fermion scattering at very low energy; leading interaction:

$$\mathcal{L}_{\rm EFT} = \frac{C_0}{4} N^\dagger N N^\dagger N + \ldots$$

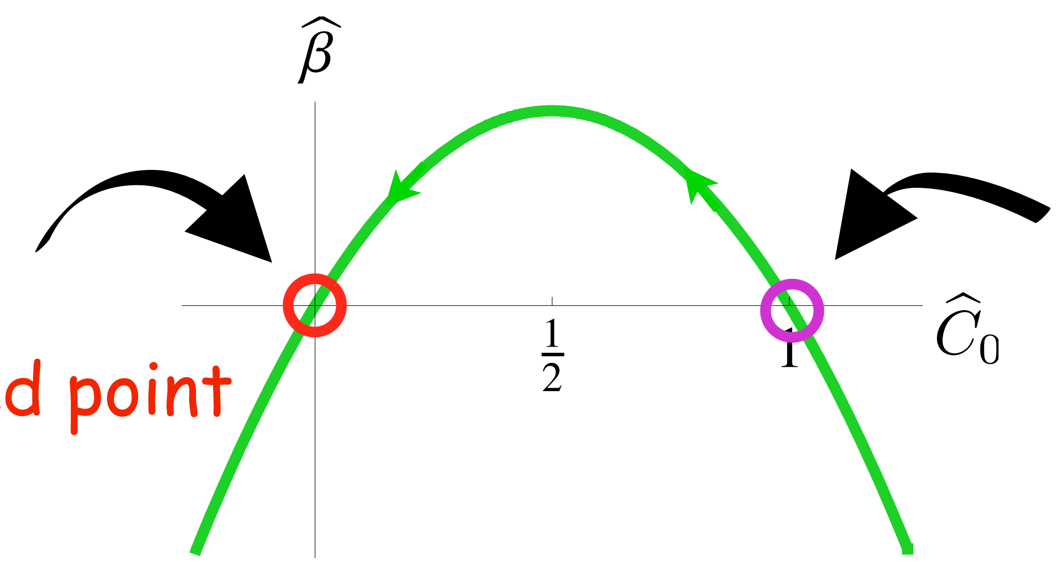


$= iC_0$

Renormalization group: C_0 scales with UV cutoff μ:

$$\widehat{C}_0 \equiv -\frac{M\mu}{4\pi} C_0 = \frac{\mu}{\mu + \frac{1}{a}} \qquad \widehat{\beta} = \mu \frac{\partial \widehat{C}_0}{\partial \mu} = -\widehat{C}_0 \left(\widehat{C}_0 - 1 \right)$$

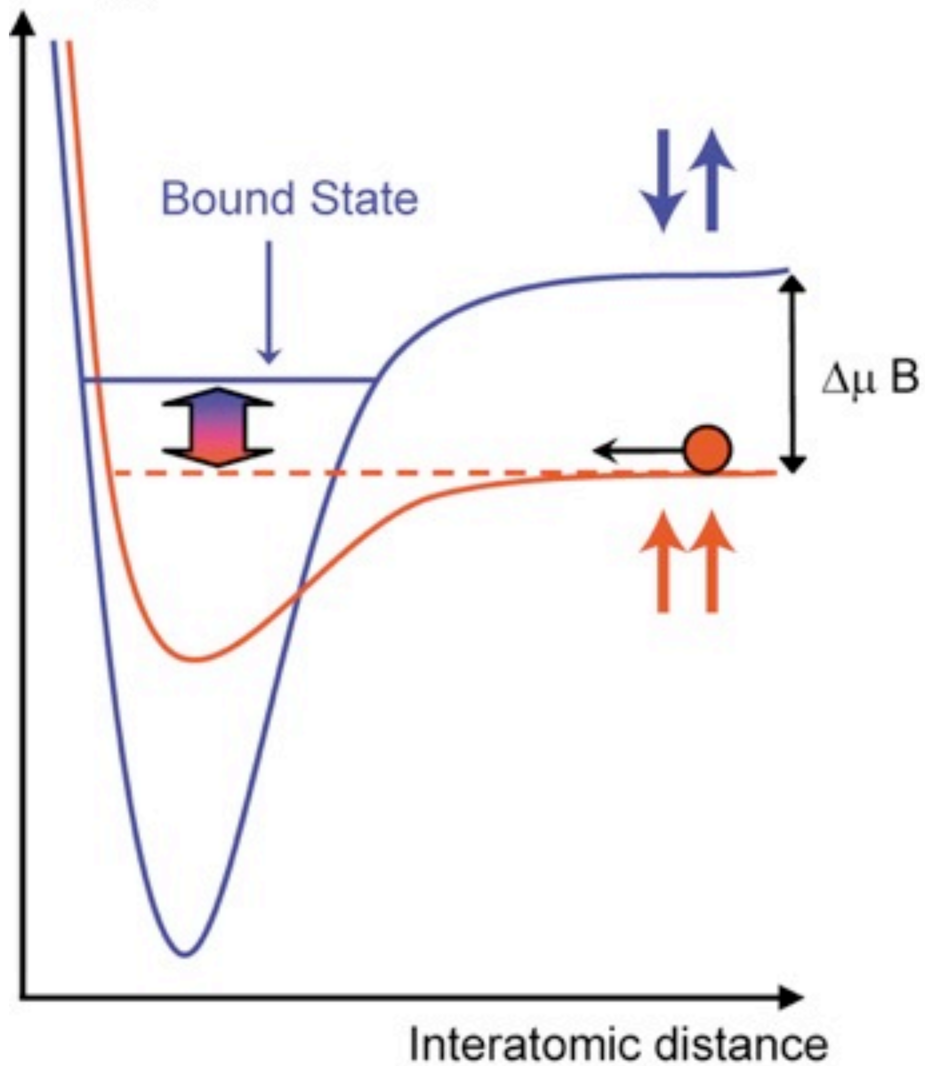
Scattering length

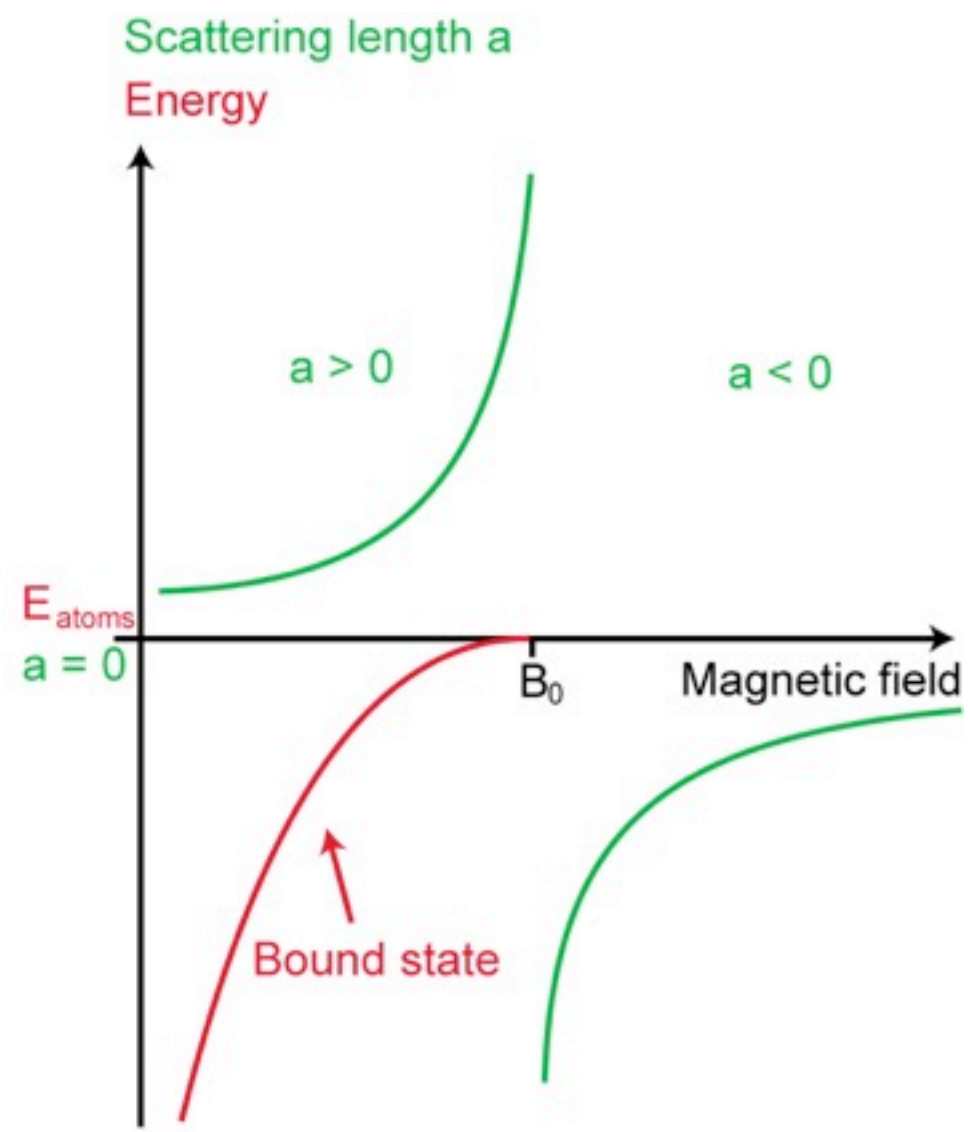


Trivial IR fixed point
free fermions

Nontrivial UV fixed point
Conformal, infinite a
Unitary fermions

Can study universal properties of unitary fermions experimentally with trapped atoms (JILA, MIT, Innsbruck)

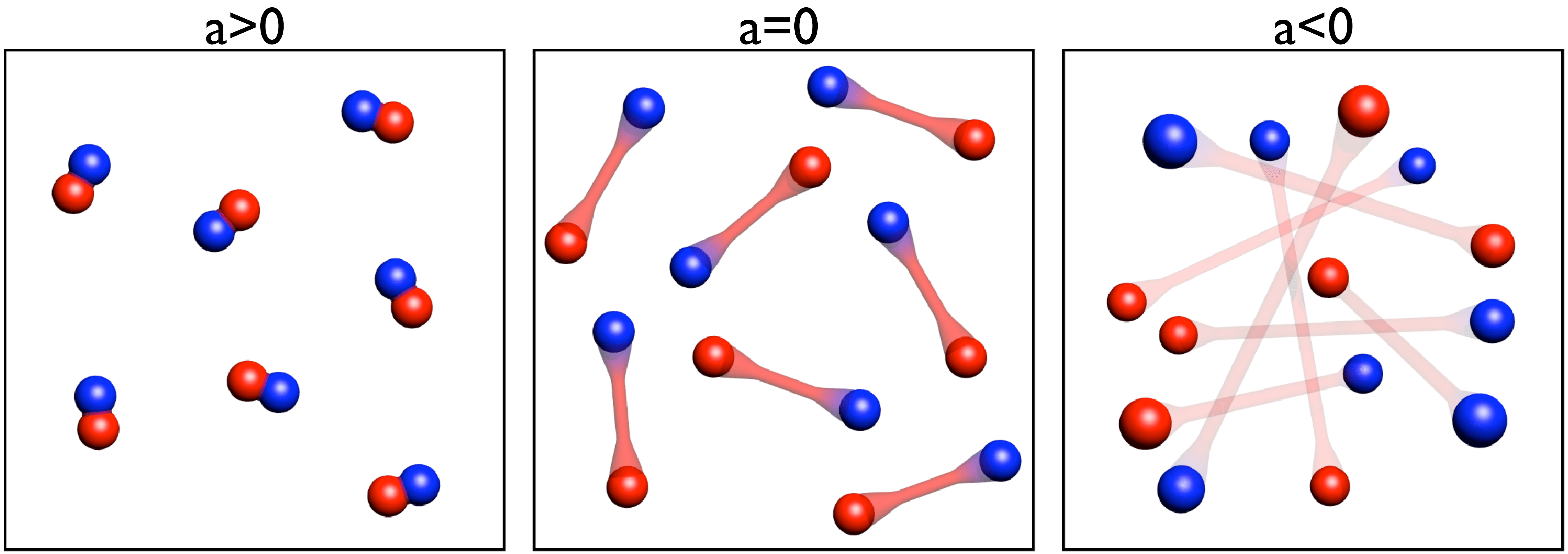




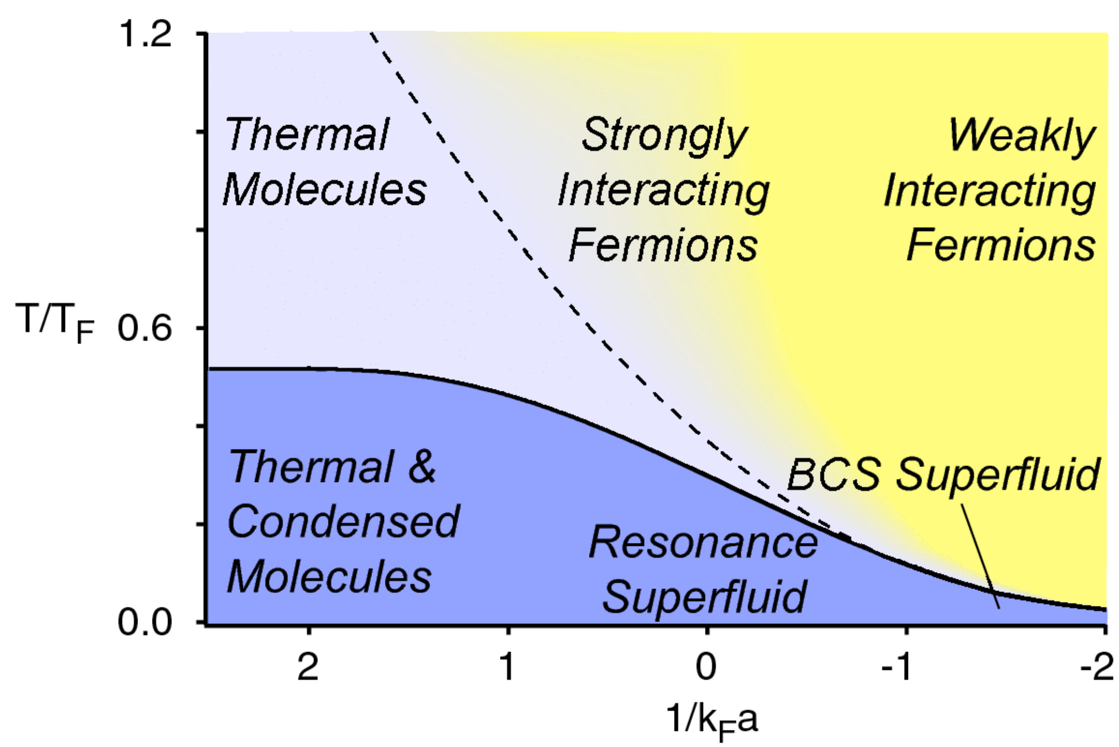
From: Ketterle Lab web page

Feshbach resonance with trapped atoms: tune to unitarity

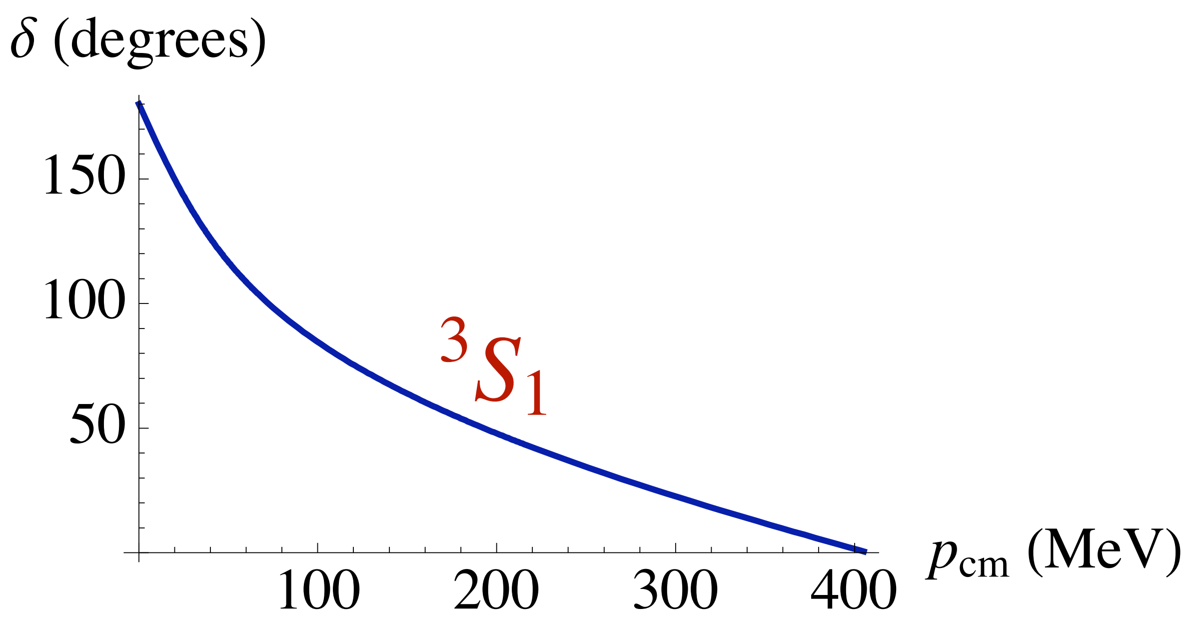
Many-body physics:

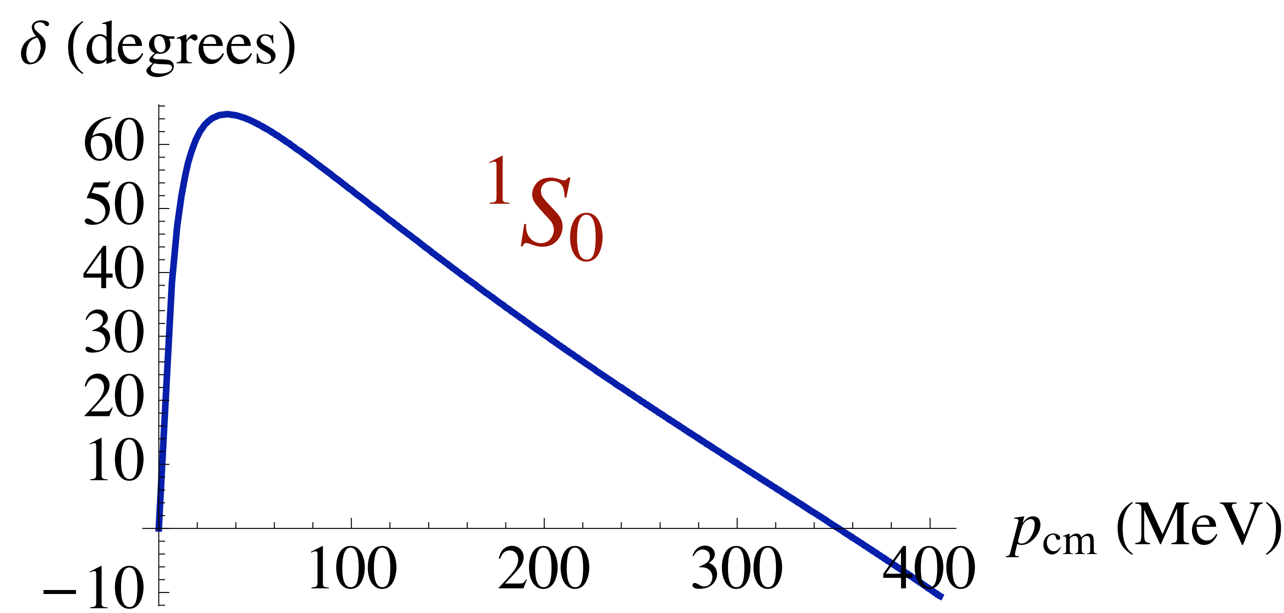


BEC of Molecules | Crossover Superfluid | BCS state



Nucleons are pretty close to being unitary fermions





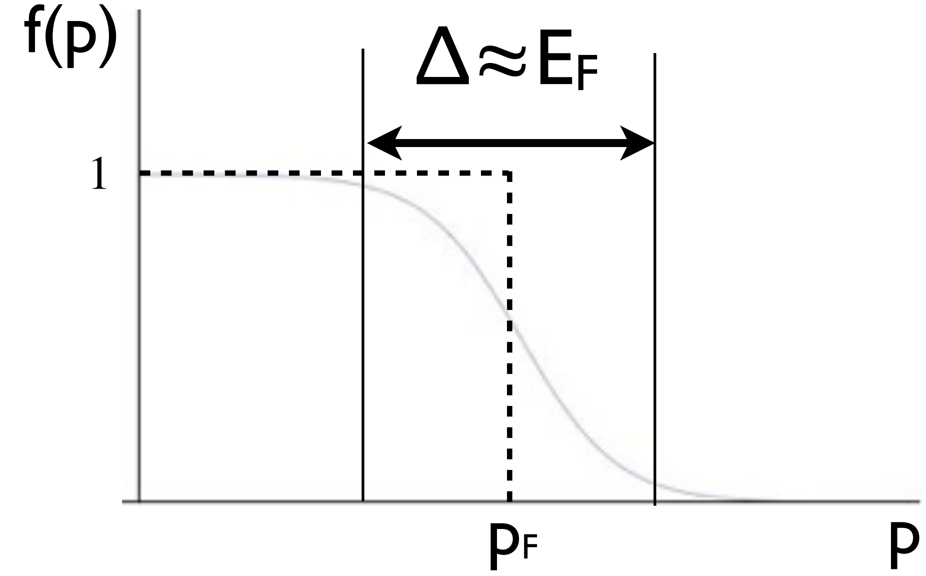
$$a_{^3S_1} = \frac{1}{45\ \mathrm{MeV}}$$

$$a_{^1S_0} = -\frac{1}{8\ \mathrm{MeV}}$$

Compare with pion Compton wavelength:

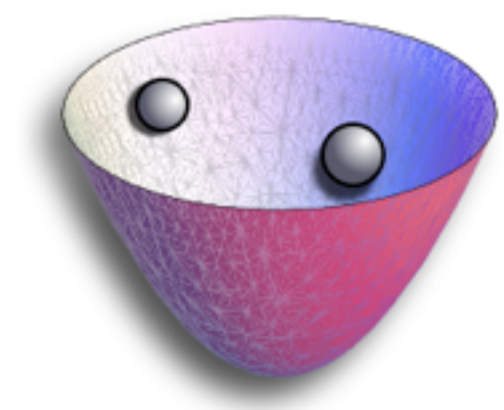
$$\lambda = \frac{1}{m_\pi} = \frac{1}{140\ \mathrm{MeV}}$$

What is interesting to measure on the lattice?

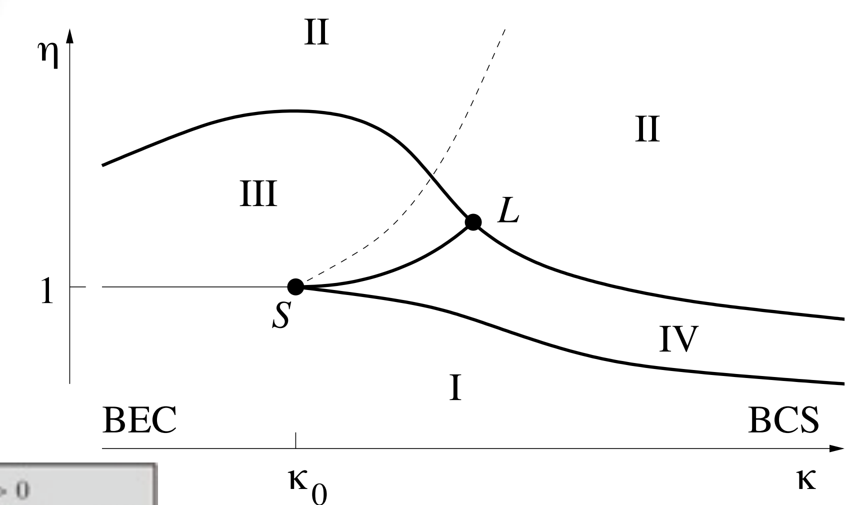


1. Bertsch parameter: as N → ∞, $E_{unitary}/E_{free} = \xi$

2. Pairing gap Δ

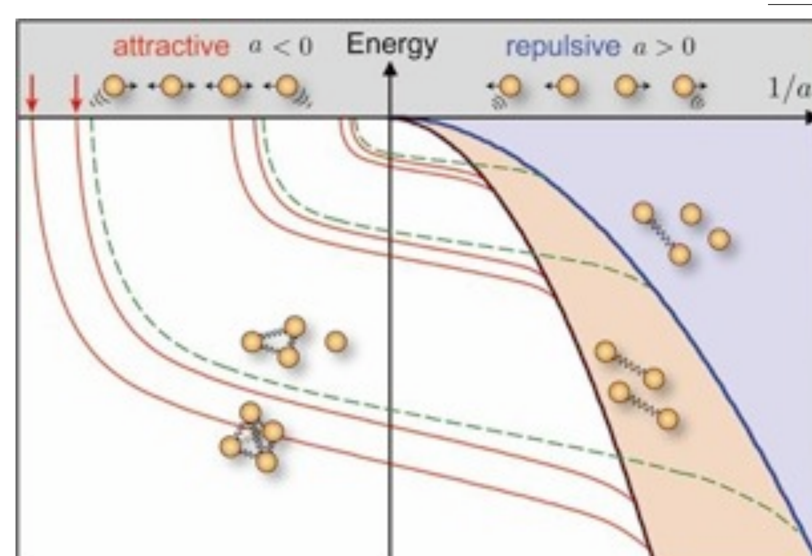


3. Energies of ($N_\uparrow$, $N_\downarrow$) fermions in a harmonic trap yield operator anomalous dimensions (!)



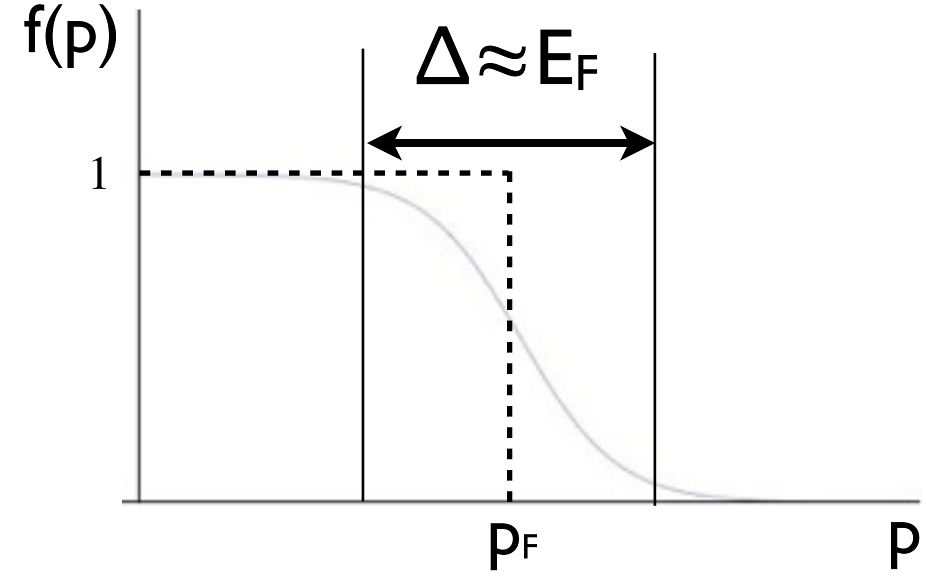
4. Phases at $N_\uparrow \neq N_\downarrow$

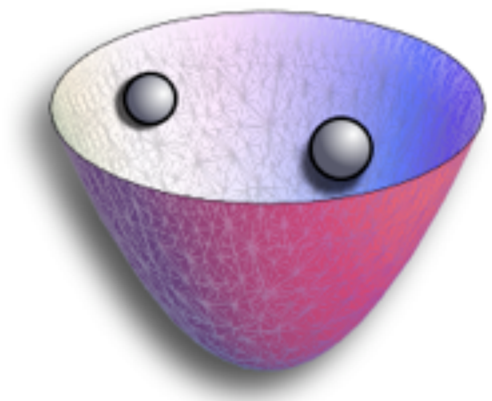
5. More than two species of fermions (Efimov states)

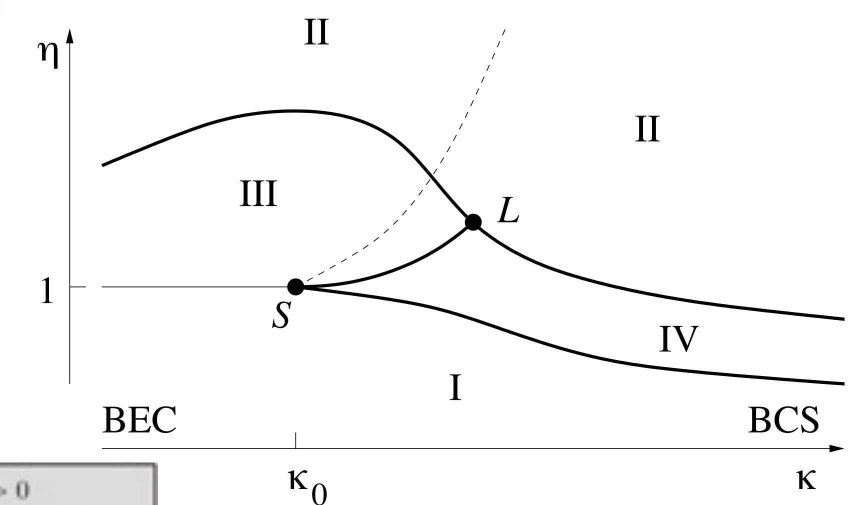


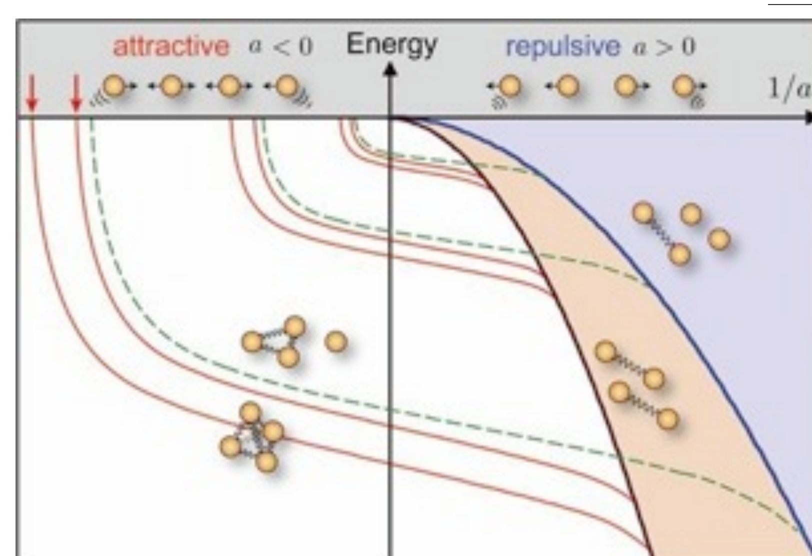
What is interesting to measure on the lattice?

1. Bertsch parameter: as $N \to \infty$, $E_{unitary}/E_{free} = \xi$
2. Pairing gap Δ
3. Energies of $(N_\uparrow, N_\downarrow)$ fermions in a harmonic trap yield operator anomalous dimensions (!)
4. Phases at $N_\uparrow \neq N_\downarrow$
5. More than two species of fermions (Efimov states)









Attractive pairing: multi-fermion system more like pions than baryons in QCD - no obvious sign or signal/noise problem

Goal: high accuracy calculations at large fermion number

To achieve high precision:

- Formulate theory so that quenched theory is exact
  - no closed fermion loops containing interactions
  - rules out grand canonical; work at fixed fermion #
- No bosonic action (compute fermion propagators in random background scalar field)
- Use highly improved fermion propagators
- Unusual statistical analysis for creating mass plots?
- Nontrivial construction of sources

Work in progress: expect 1-2% accuracy for g.s. energy for up to ~100 fermions on 16^3 x 64 lattice

Basic formulation (Chen & Kaplan, 2003):

$$\mathcal{L} = \bar{\psi}\left(\partial_\tau - \frac{\nabla^2}{2M}\right)\psi + C_0(\bar{\psi}\psi)^2$$

C_0 tuned to give infinite scattering length *(defines continuum limit)*

Interaction included by random Z_2 auxiliary field

$$\mathcal{L} \to \bar{\psi}\underbrace{\left(\partial_\tau - \frac{\nabla^2}{2M} + \sqrt{C_0}\phi\right)}_{K(\phi)}\psi$$

$$\mathcal{L} \to \bar{\psi} \underbrace{\left(\partial_\tau - \frac{\nabla^2}{2M} + \sqrt{C_0}\phi\right)}_{K(\phi)} \psi$$

$$K(\phi)_{\tau\tau'} = \begin{pmatrix} D & X(0) & 0 & 0 & \ldots & 0 \\ 0 & D & X(1) & 0 & \ldots & 0 \\ 0 & 0 & D & X(2) & \ldots & 0 \\ 0 & 0 & 0 & D & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & X(T-1) \\ \cancelto{0}{-X(T)} & 0 & 0 & 0 & \ldots & D \end{pmatrix}$$

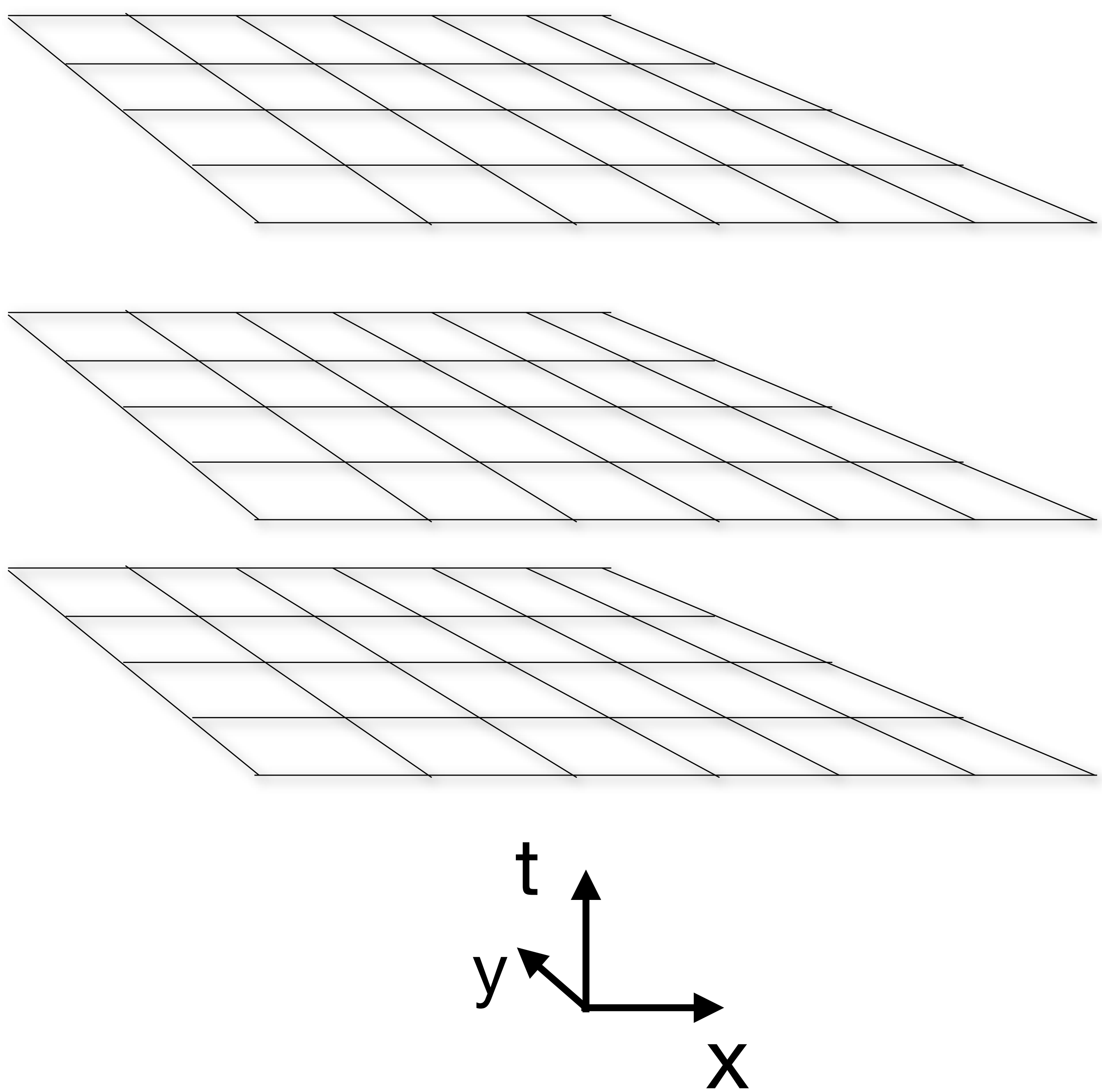
Open B.C.

$$D = 1 - \frac{\nabla^2}{2M} , \quad X = 1 - \sqrt{C_0}\phi(\mathbf{x}, t)$$

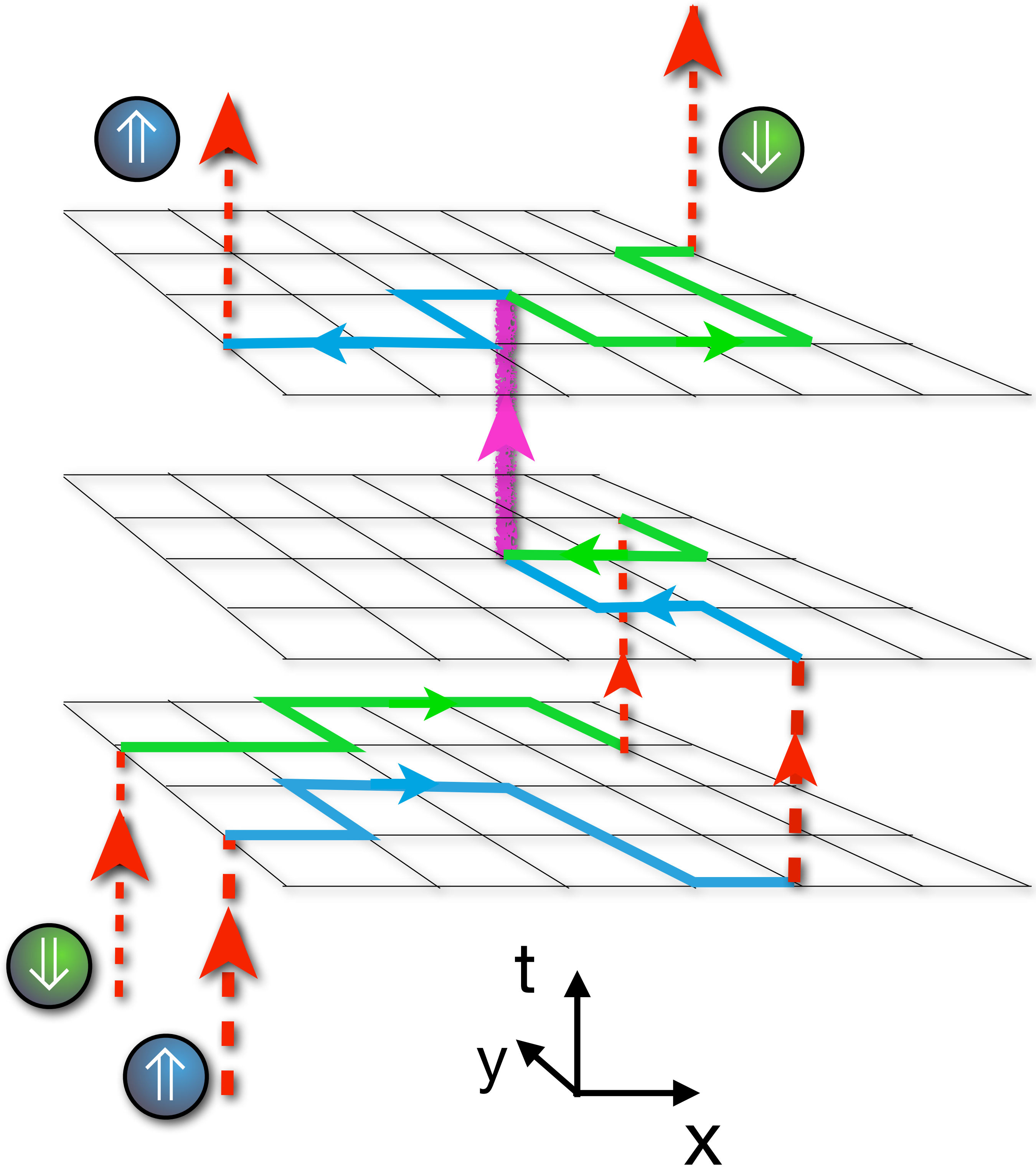
★Fermions propagate only forward in time
★Interactions only on time links
★Open B.C. (incompatible w. grand canonical)

} Det[K(φ)] is independent of φ
Quenched = exact

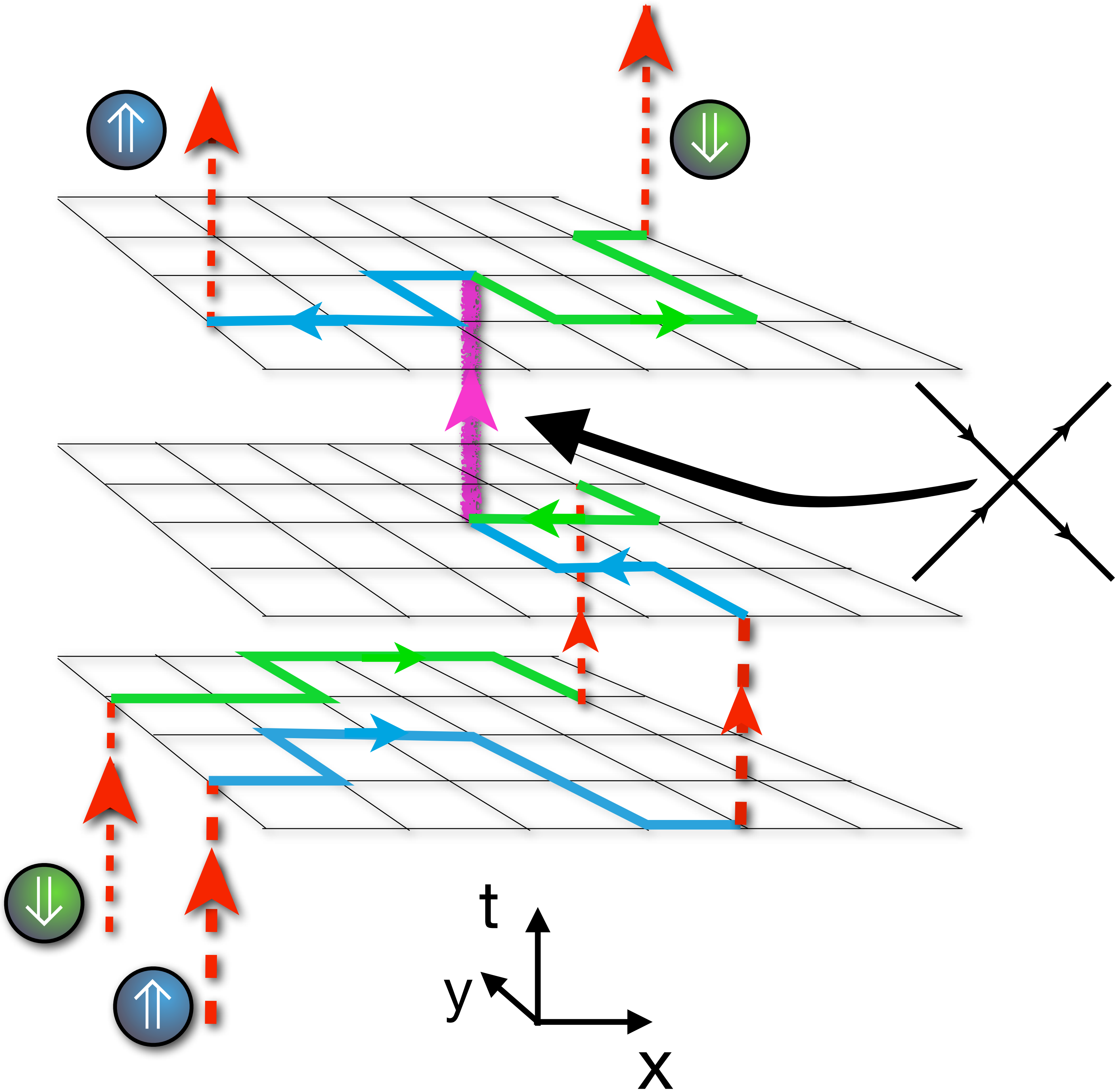
All one does: compute fermion propagators in background φ



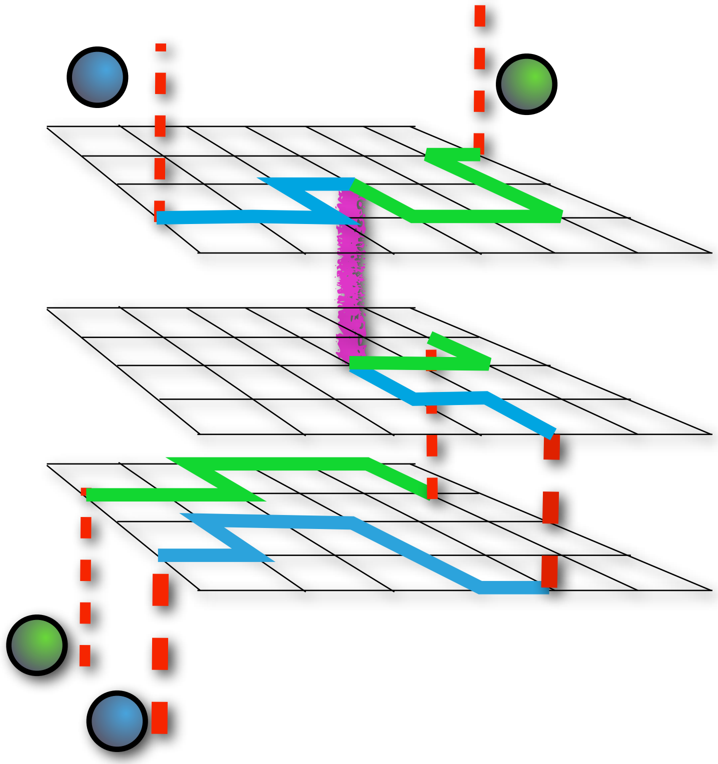
All one does: compute fermion propagators in background φ



All one does: compute fermion propagators in background φ



- Free propagation on spatial slices
- Only forward propagation in time
- Local particle interactions only on time-like links
- No closed fermion loops with interactions = no nontrivial determinant



$$K(\phi)_{\tau\tau'} = \begin{pmatrix} D & X(0) & 0 & 0 & \dots & 0 \\ 0 & D & X(1) & 0 & \dots & 0 \\ 0 & 0 & D & X(2) & \dots & 0 \\ 0 & 0 & 0 & D & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & X(T-1) \\ \cancelto{0}{-X(T)} & 0 & 0 & 0 & \dots & D \end{pmatrix}$$

Single-particle correlator:

$$C_1(\tau,0) = K^{-1}(\tau,0) = D^{-1}X(\tau-1)D^{-1}X(\tau-2)\cdots X(0)D^{-1}$$

$$= D^{-1/2}\left[T_1(\tau-1)T_1(\tau-2)\cdots T_1(0)\right]D^{-1/2}$$

$$T_1 = D^{-1/2}XD^{-1/2}$$ 1-particle transfer matrix

N-particle transfer matrix: $T_N = \left(\otimes T_1\right)^N$

Toward a more perfect fermion action (fermion propagator):

I. 1-particle physics: Improve kinetic energy ∇^2

Free particle: $$T_1 = D^{-1} = \left(1 - \nabla^2/(2M)\right)^{-1}$$

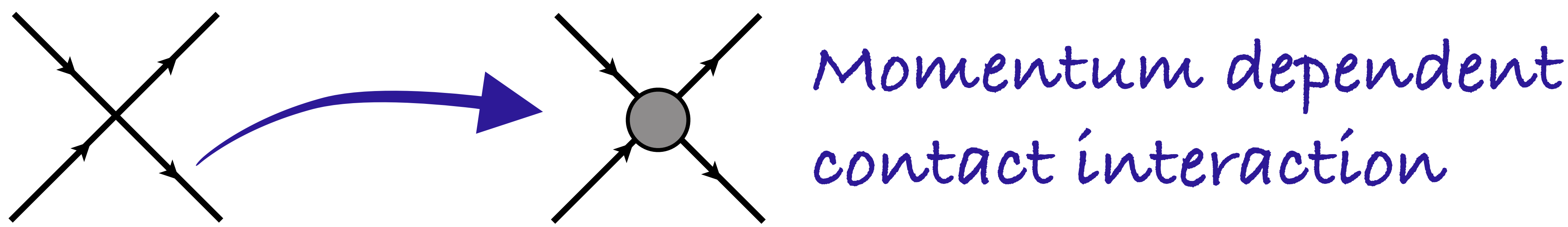
Define ∇ to attain perfect action for $p<\Lambda$:

$$D^{-1} = \begin{cases} e^{-p^2/2M} & p < \Lambda \\ 0 & p \geq \Lambda \end{cases}$$

In practice, take $\Lambda=\pi$ in lattice units *more perfect fermion action continued:*

II. 2-particle physics: improving interaction

Tuning C_0 to infinite scattering length still leaves nonzero pcotδ...need to tune away effective range, etc.



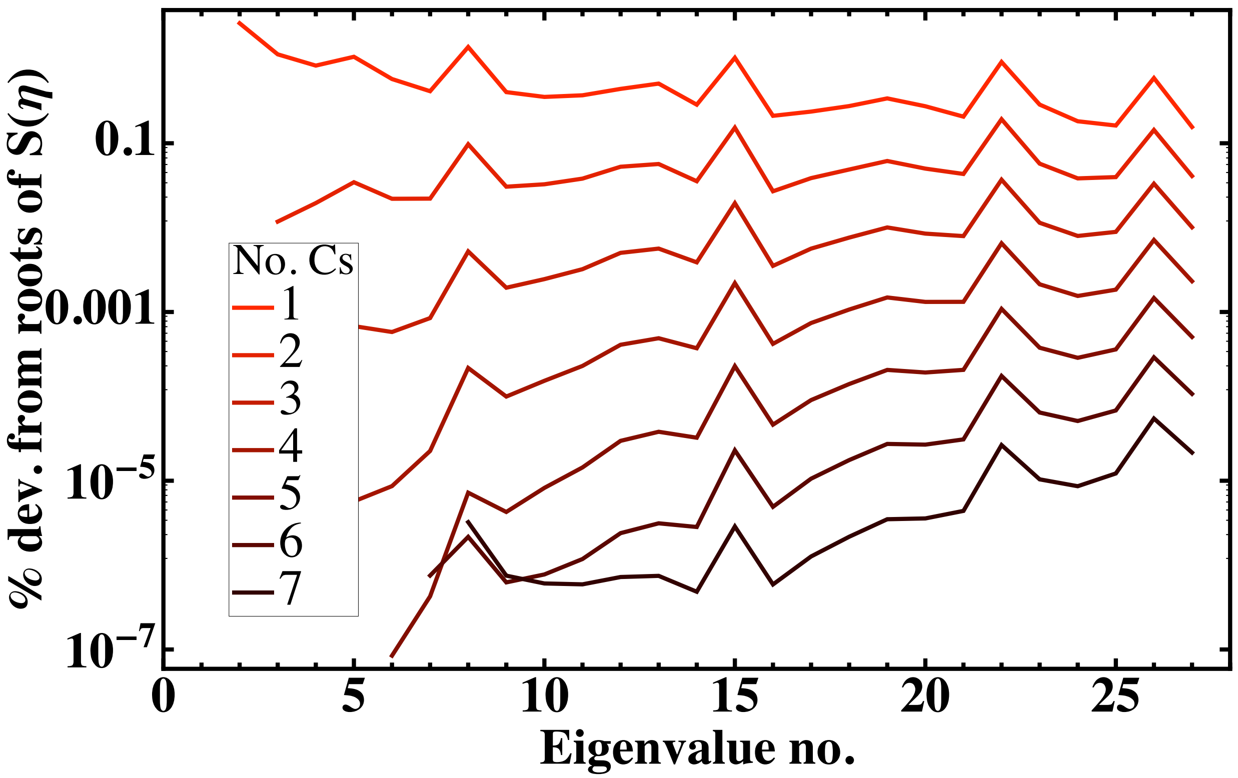
$$X = 1 - \sqrt{C_0}\phi \quad \Rightarrow \quad X = 1 - \sqrt{C(p^2)}\phi$$

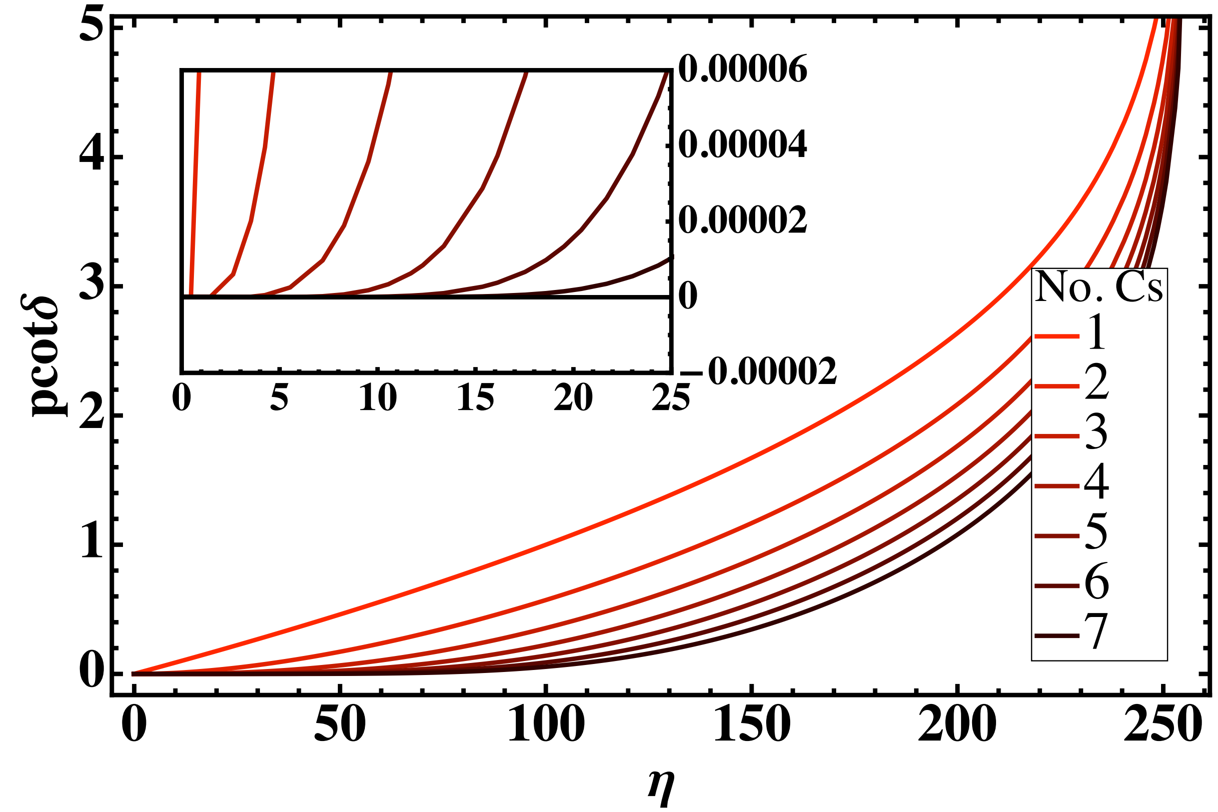
Tuning method:

i. Expand coupling C in set of operators: $C(p^2) = \sum_{n=1}^{N} C_n \mathcal{O}_n(p^2)$

ii. Fit C_n to match first N energy eigenvalues for <u>continuum</u> box L, pcotδ=0 (Lüscher formula)

Out[76]=





$$\eta = \left(\frac{pL}{2\pi}\right)^2$$

(p cot δ) very small after tuning, even for large momenta.

Procedure: *Time evolution of single particle wave functions on random background configurations*

For each configuration:

Initialize N sources at time zero ψ_i^{source} $(i = 1, \ldots, N)$

For each source:

Compute $\psi_i(0) = D^{-1}\psi_i^{source}$

For each time slice τ thereafter:

Generate random auxiliary fields at time slice

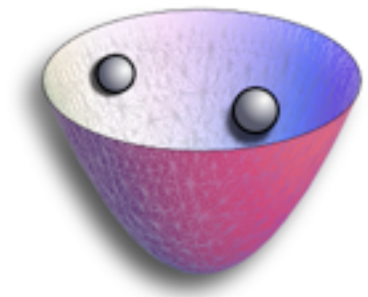
Compute $\psi_i(\tau) = D^{-1}X(\tau)\psi_i(\tau - 1)$ using FFTs

Project propagators onto single/multi-particle sinks

Perform contractions (e.g. Slater determinants) **(Michael Endres)**

How good is our method? How large are discretization & finite volume errors?

Benchmark for N≤6 fermions in harmonic trap (accurate Schrödinger eq. calculations)

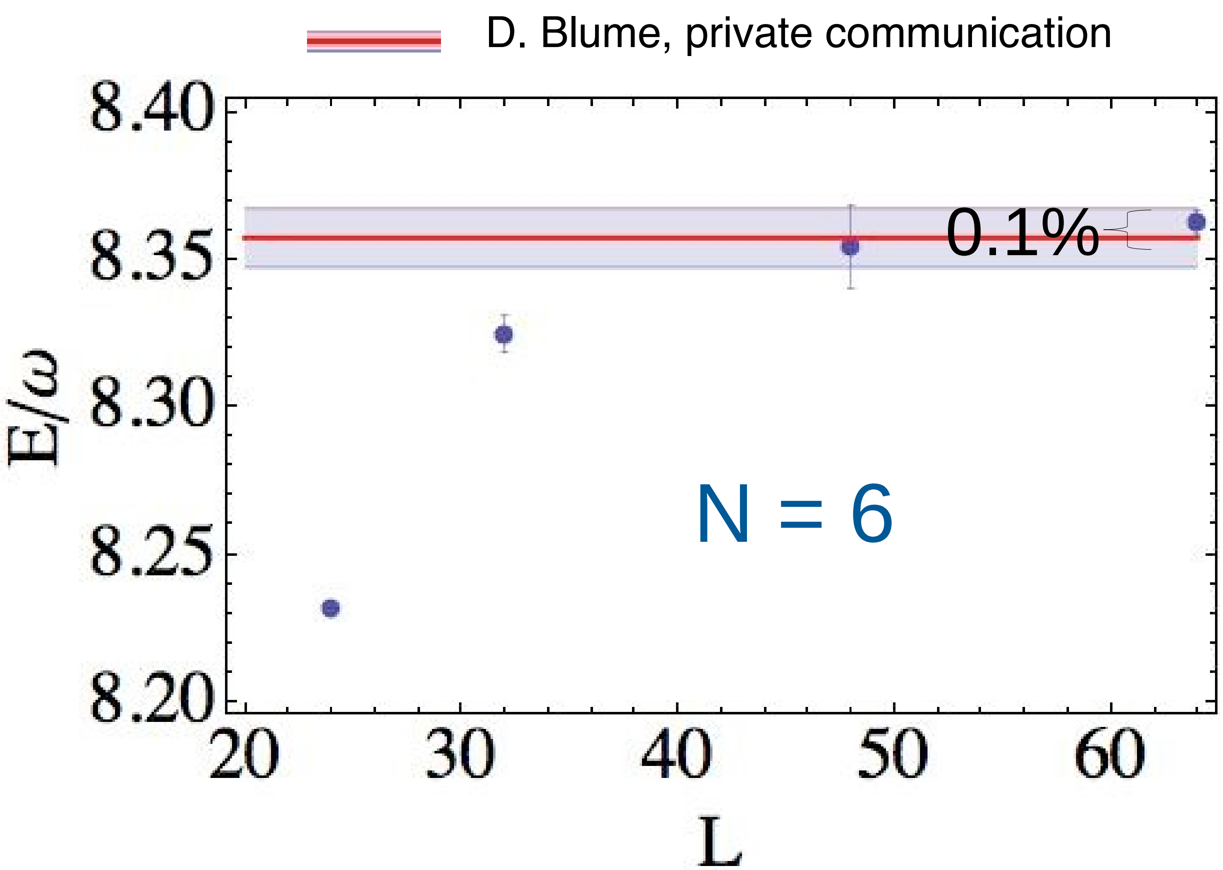


N	This Work	Comparison	% Deviation
3	4.253(2)(4)	4.2727[*]	0.5
4	5.058(1)(1)	5.028(20)[†]	0.6
5	7.513(3)(2)	7.457(10)[‡]	0.8
6	8.338(4)(5)	8.357(10)[‡]	0.2

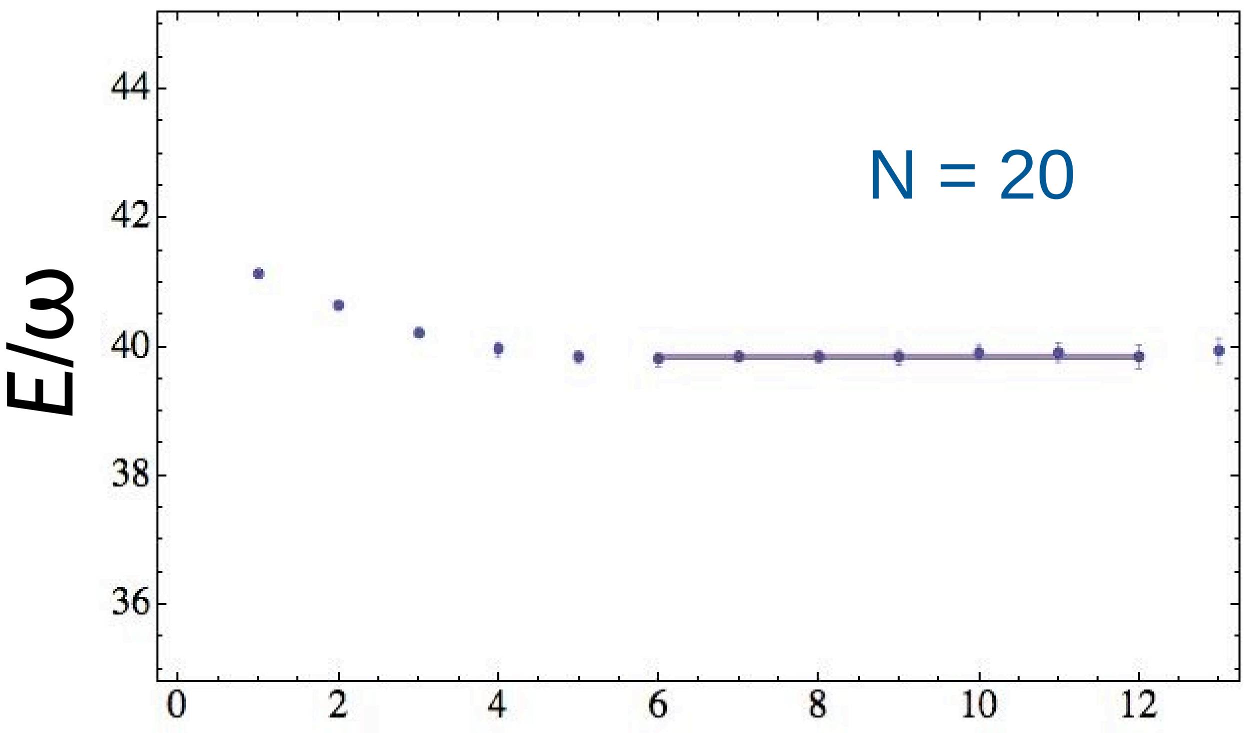
PRELIMINARY

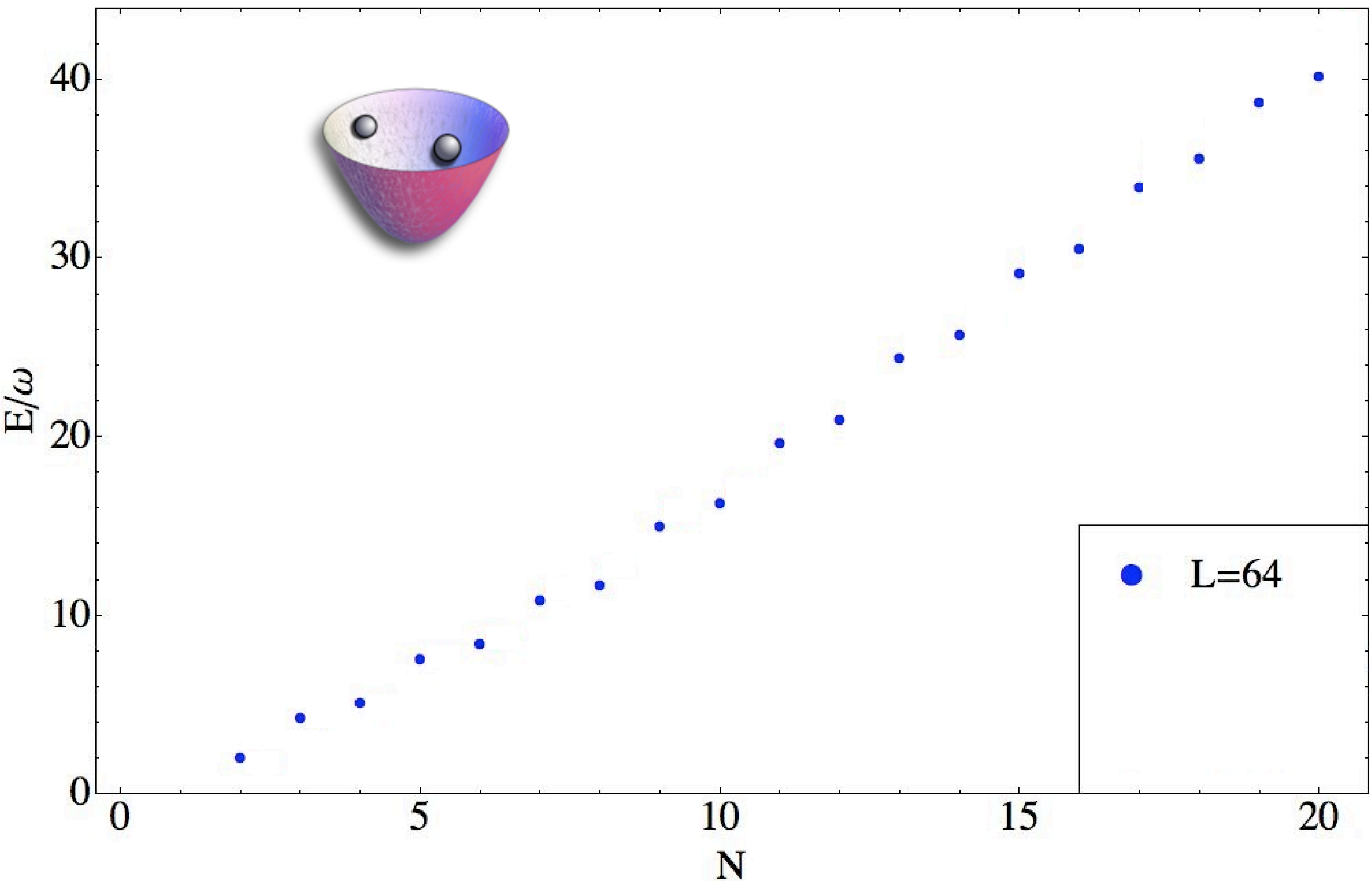
*F. Werner and Y. Castin, Phys. Rev. Lett. **97**. 150401 (2006)
†D. Blume, J. von Stecher, and C. Greene, Phys. Rev. Lett. **99**. 233201 (2007)
‡D. Blume, private communication

Finite volume errors



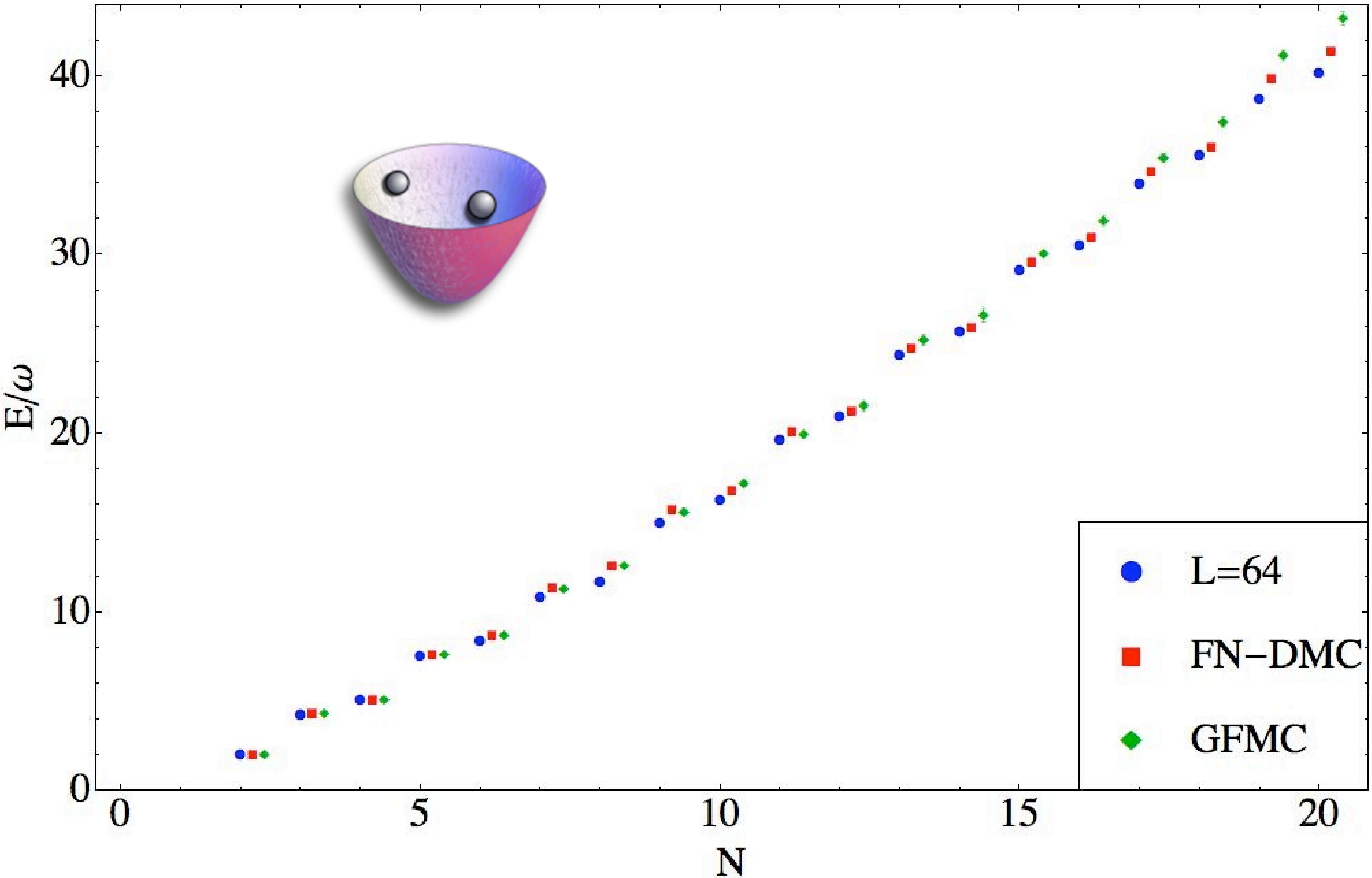
Larger N





40
30
20
10
0
E/ω
0
5
10
15
20
N
L=64

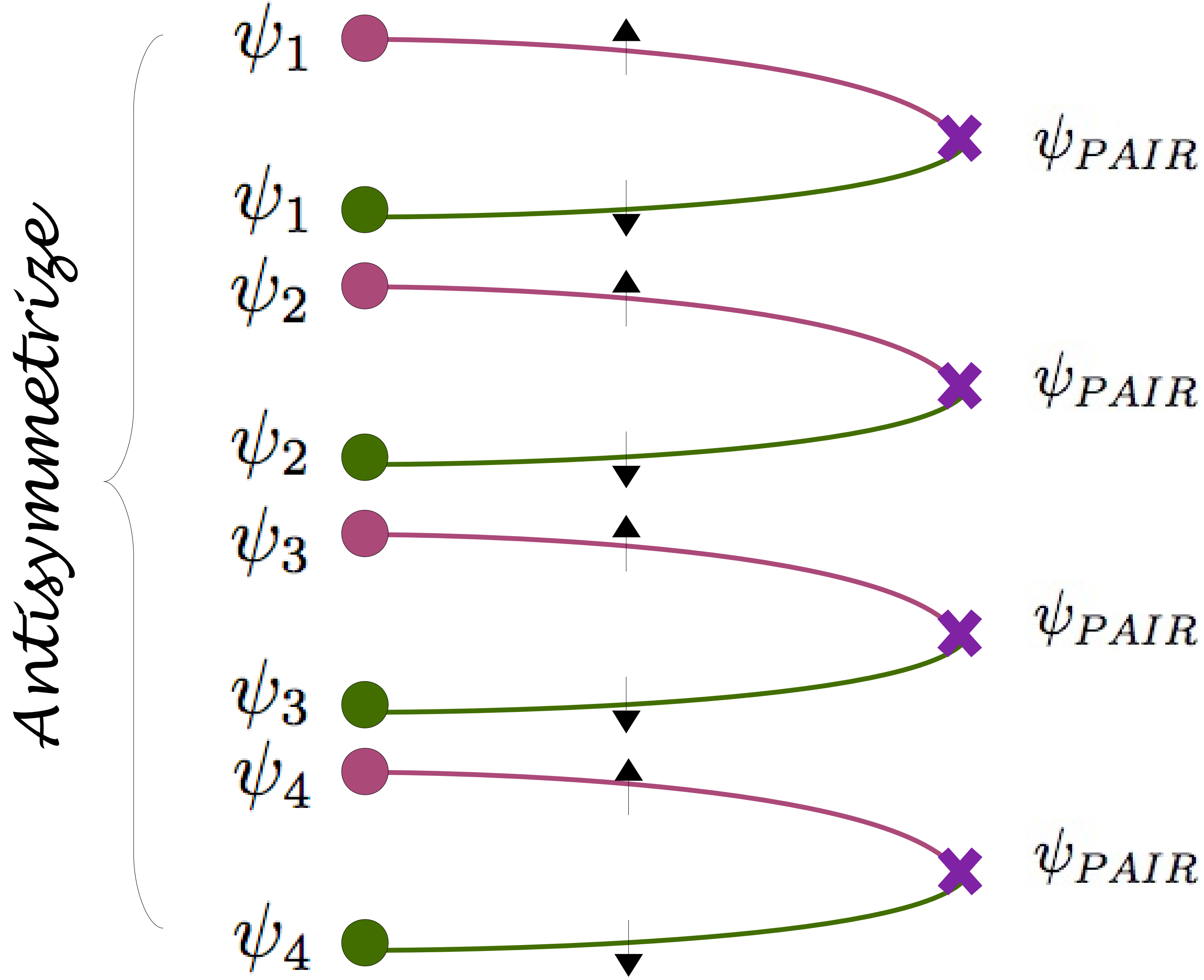
Comparison with earlier Monte Carlo calculations



FN-DMC: D. Blume, J. von Stecher, Chris H. Greene, arXiv:0708.2734

GFMC: S. Y. Chang and G. F. Bertsch, arXiv:physics/0703190

This accuracy is <u>not</u> possible with Slater determinant sources: need to build pairing correlations into source



$$\psi_{PAIR} \propto \frac{e^{-(x^2+y^2)/(2L_0^2)}}{|x-y|}$$

Exact ground state wave function for 2 unitary fermions in SHO

$$C_N(\tau) = \begin{vmatrix} \langle\psi_{PAIR}|(\mathcal{T})^\tau|\psi_1\psi_1\rangle & \langle\psi_{PAIR}|(\mathcal{T})^\tau|\psi_1\psi_2\rangle & \cdots & \langle\psi_{PAIR}|(\mathcal{T})^\tau|\psi_1\psi_{N/2}\rangle \\ \langle\psi_{PAIR}|(\mathcal{T})^\tau|\psi_2\psi_1\rangle & \langle\psi_{PAIR}|(\mathcal{T})^\tau|\psi_2\psi_2\rangle & \cdots & \langle\psi_{PAIR}|(\mathcal{T})^\tau|\psi_2\psi_{N/2}\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle\psi_{PAIR}|(\mathcal{T})^\tau|\psi_{N/2}\psi_1\rangle & \langle\psi_{PAIR}|(\mathcal{T})^\tau|\psi_{N/2}\psi_2\rangle & \cdots & \langle\psi_{PAIR}|(\mathcal{T})^\tau|\psi_{N/2}\psi_{N/2}\rangle \end{vmatrix}$$

N unitary fermions in a box:

compute the Bertsch parameter $\xi = \lim_{N \to \infty} (E_{unitary}/E_{free})$

N unitary fermions in a box:
compute the Bertsch parameter $\xi = \lim_{N\to\infty} (E_{unitary}/E_{free})$

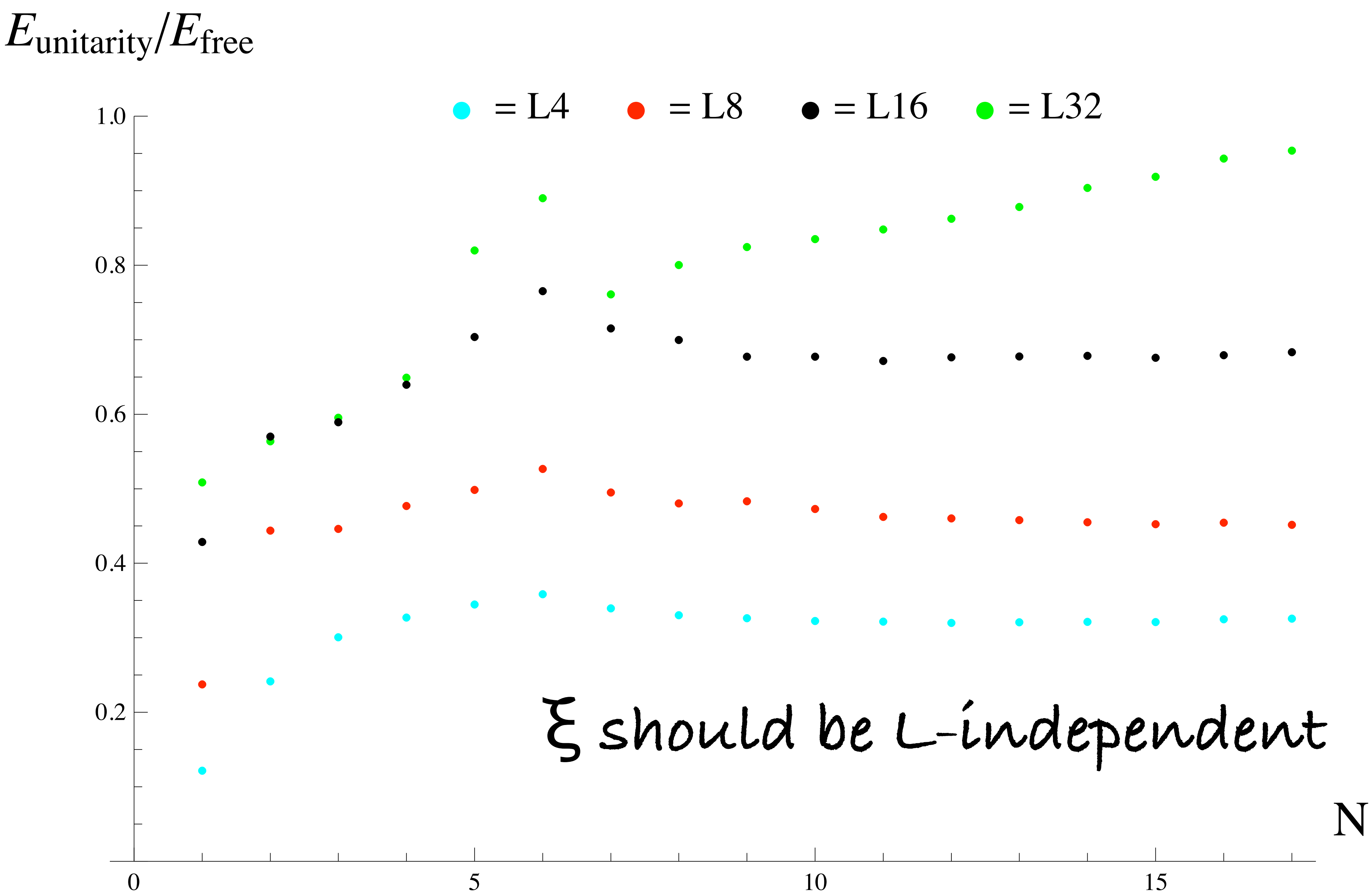
Get <u>nonsense</u> if one does not build correlations into source!
Results from creating an antisymmetrized product of free fermions:

N unitary fermions in a box:

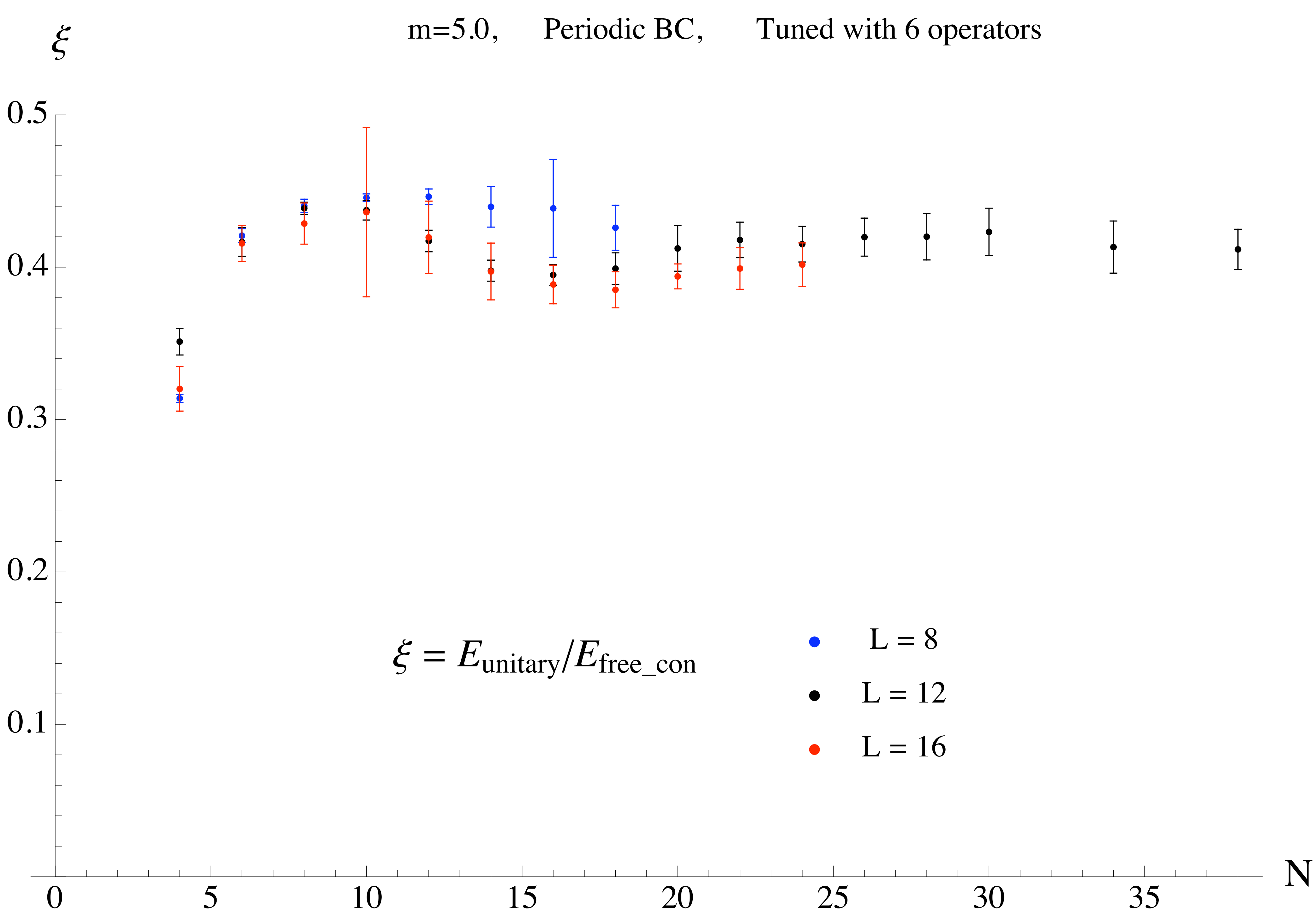
compute the Bertsch parameter $\xi = \lim_{N\to\infty} (E_{unitary}/E_{free})$

Get <u>nonsense</u> if one does not build correlations into source!

Results from creating an antisymmetrized product of free fermions:



With pairing correlations built into source, no significant volume dependence for ξ



What about the signal/noise problem?

What about the signal/noise problem?

The theory at finite μ and $N_\uparrow = N_\downarrow$ has real det: no sign problem = no S/N problem?

What about the signal/noise problem?

The theory at finite μ and $N_\uparrow = N_\downarrow$ has real det:
no sign problem = no S/N problem?

Lepage argument naively suggests a huge S/N problem!
Squaring correlator is like doubling # of fermion species...that system has no ground state in continuum (Efimov states, bounded below by the lattice cutoff)

What about the signal/noise problem?

The theory at finite μ and $N_\uparrow = N_\downarrow$ has real det:
no sign problem = no S/N problem?

Lepage argument naively suggests a huge S/N problem! Squaring correlator is like doubling # of fermion species...that system has no ground state in continuum (Efimov states, bounded below by the lattice cutoff)

What do we see? A significant but not disastrous S/N problem due to non-Gaussian distribution of correlators

Plot of the distribution of the 4-particle correlator $C_4(t)$ in an ensemble of 10^5 random φ fields for t=1,...,64

Movie doesn't reproduce in pdf...see Cdist.mov

Very long tail; gets worse with time.

Plot of the distribution of the log of the 4-particle correlator $C_4(t)$ in an ensemble of random φ fields for t=1,...,64

Movie doesn't reproduce in pdf...see LogCdist.mov

Red = Gaussian fit... very good!

Conclusion: correlators obey **Log-Normal** distribution. Why??

Plot of the distribution of the log of the 4-particle correlator $C_4(t)$ in an ensemble of random φ fields for t=1,...,64

Movie doesn't reproduce in pdf...see LogCdist.mov

Red = Gaussian fit... very good!

Conclusion: correlators obey **Log-Normal** distribution. Why??

Caveat: am now discussing 1-week-old ideas!

Log-normal distribution naturally arise in products of *positive* random numbers:

$$x = \prod_i c_i \equiv \prod_i e^{\xi_i} = e^{\sum_i \xi_i} \equiv e^y$$

Log-normal distribution naturally arise in products of *positive* random numbers:

$$x = \prod_i c_i \equiv \prod_i e^{\xi_i} = e^{\sum_i \xi_i} \equiv e^y$$

Gaussian distributed

$$\mathcal{P}(y = \ln x) = \mathcal{N} e^{-(y-\mu)^2/2\sigma^2}$$

Log-normal distribution naturally arise in products of *positive* random numbers:

$$x = \prod_i c_i \equiv \prod_i e^{\xi_i} = e^{\sum_i \xi_i} \equiv e^y$$

Gaussian distributed

$$\mathcal{P}(y = \ln x) = \mathcal{N} e^{-(y-\mu)^2/2\sigma^2}$$

$$\mathcal{P}(x) = \frac{\mathcal{N}}{x} e^{-(\ln x - \mu)^2/2\sigma^2}$$

Log-normal distribution

Log-normal distribution naturally arise in products of *positive* random numbers:

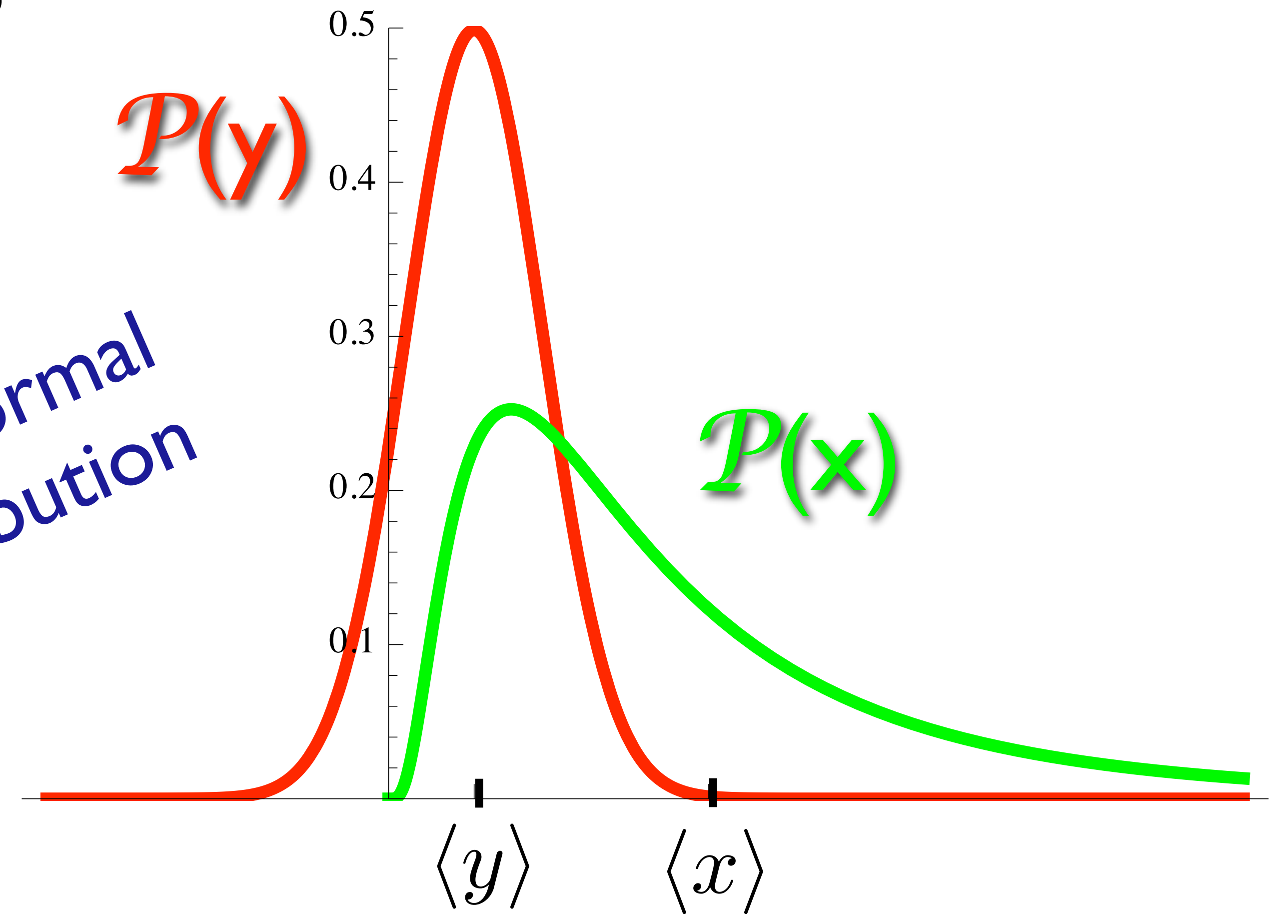
$$x = \prod_i c_i \equiv \prod_i e^{\xi_i} = e^{\sum_i \xi_i} \equiv e^y$$

Gaussian distributed

$$\mathcal{P}(y = \ln x) = \mathcal{N} e^{-(y-\mu)^2/2\sigma^2}$$

$$\mathcal{P}(x) = \frac{\mathcal{N}}{x} e^{-(\ln x - \mu)^2/2\sigma^2}$$

Log-normal distribution



Log-normal distribution naturally arise in products of *positive* random numbers:

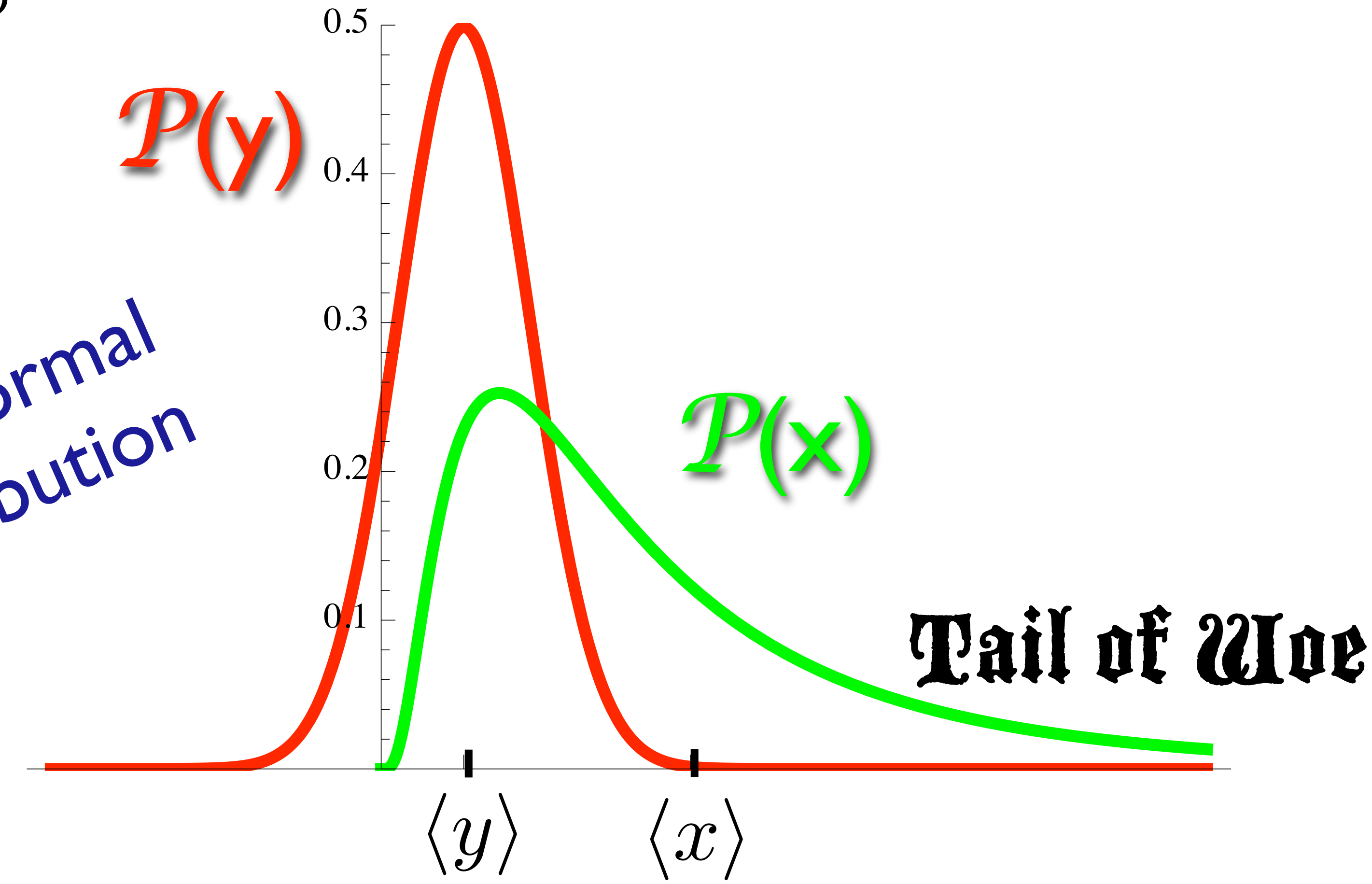
$$x = \prod_i c_i \equiv \prod_i e^{\xi_i} = e^{\sum_i \xi_i} \equiv e^y$$

Gaussian distributed

$$\mathcal{P}(y = \ln x) = \mathcal{N} e^{-(y-\mu)^2/2\sigma^2}$$

$$\mathcal{P}(x) = \frac{\mathcal{N}}{x} e^{-(\ln x - \mu)^2/2\sigma^2}$$

Log-normal distribution



Log-normal distribution naturally arise in products of *positive* random numbers:

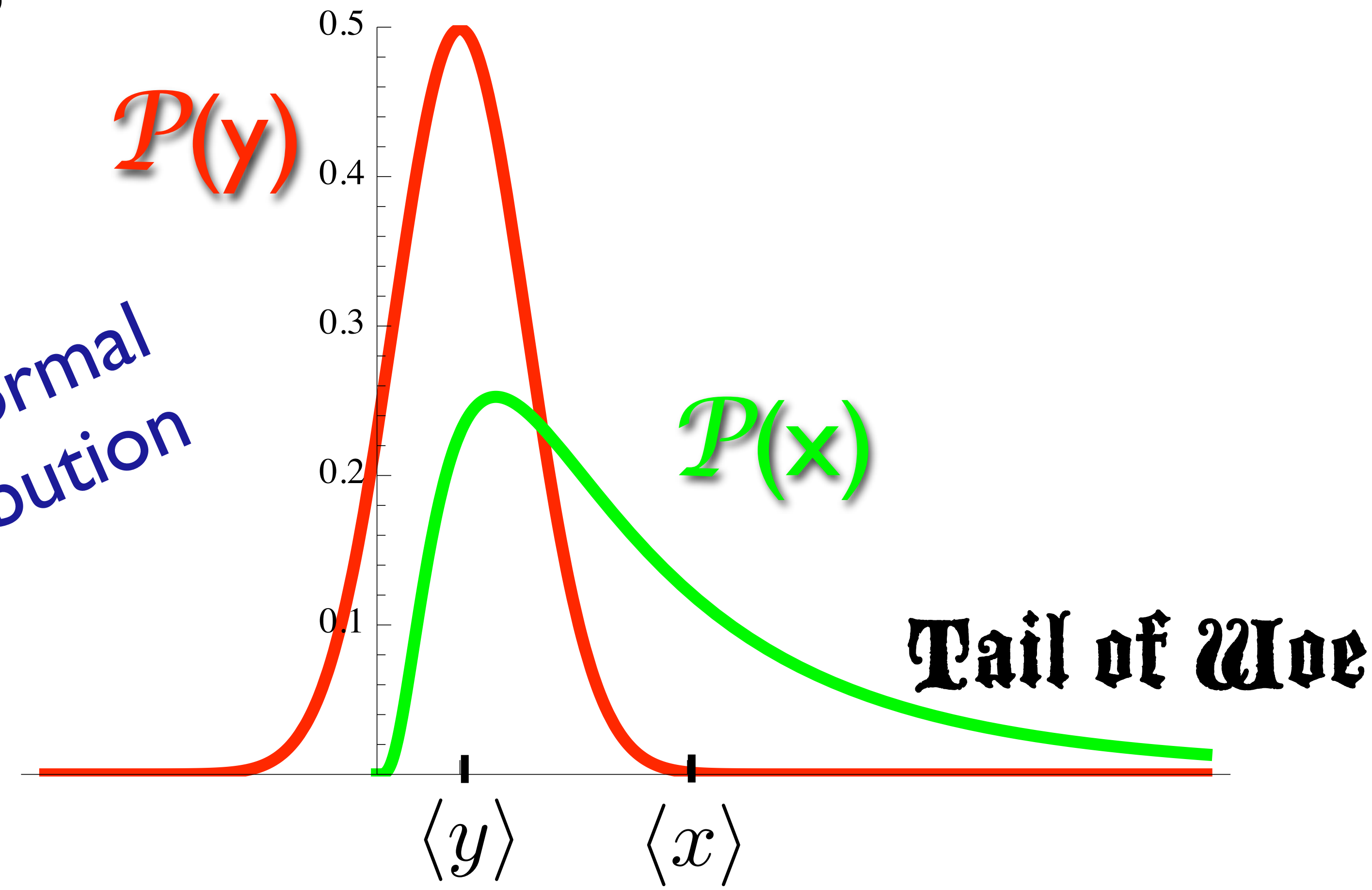
$$x = \prod_i c_i \equiv \prod_i e^{\xi_i} = e^{\sum_i \xi_i} \equiv e^y$$

Gaussian distributed

$$\mathcal{P}(y = \ln x) = \mathcal{N} e^{-(y-\mu)^2/2\sigma^2}$$

$$\mathcal{P}(x) = \frac{\mathcal{N}}{x} e^{-(\ln x - \mu)^2/2\sigma^2}$$

Log-normal distribution



$$\langle y \rangle = \mu$$

$$\langle y^2 \rangle = \mu^2 + \sigma^2$$

Measure

Log-normal distribution naturally arise in products of *positive* random numbers:

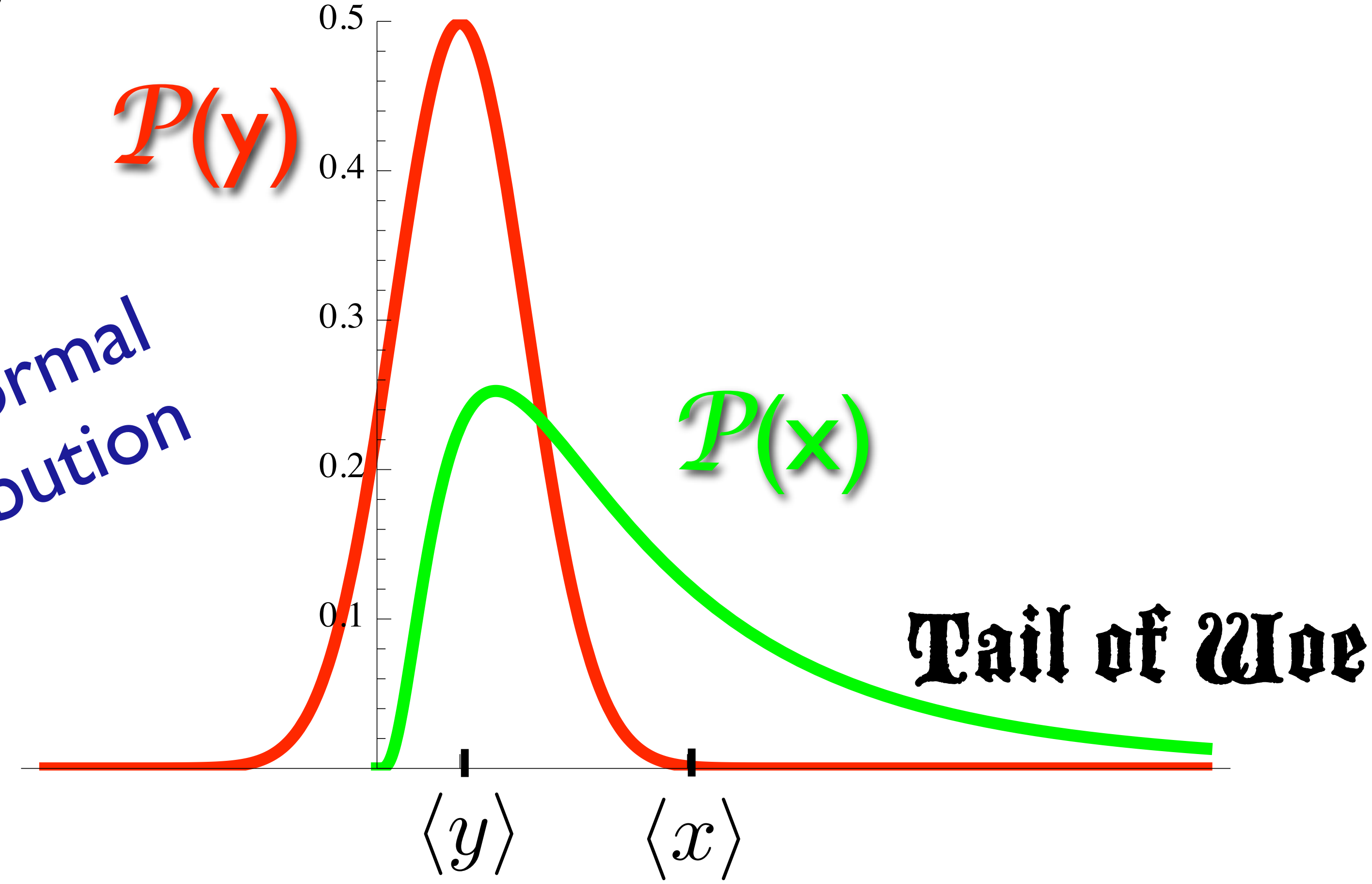
$$x = \prod_i c_i \equiv \prod_i e^{\xi_i} = e^{\sum_i \xi_i} \equiv e^y$$

Gaussian distributed

$$\mathcal{P}(y = \ln x) = \mathcal{N} e^{-(y-\mu)^2/2\sigma^2}$$

$$\mathcal{P}(x) = \frac{\mathcal{N}}{x} e^{-(\ln x - \mu)^2/2\sigma^2}$$

Log-normal distribution



$$\langle y \rangle = \mu$$

$$\langle y^2 \rangle = \mu^2 + \sigma^2$$

Measure

$$\langle x \rangle = e^{\mu^2 + \sigma^2/2}$$

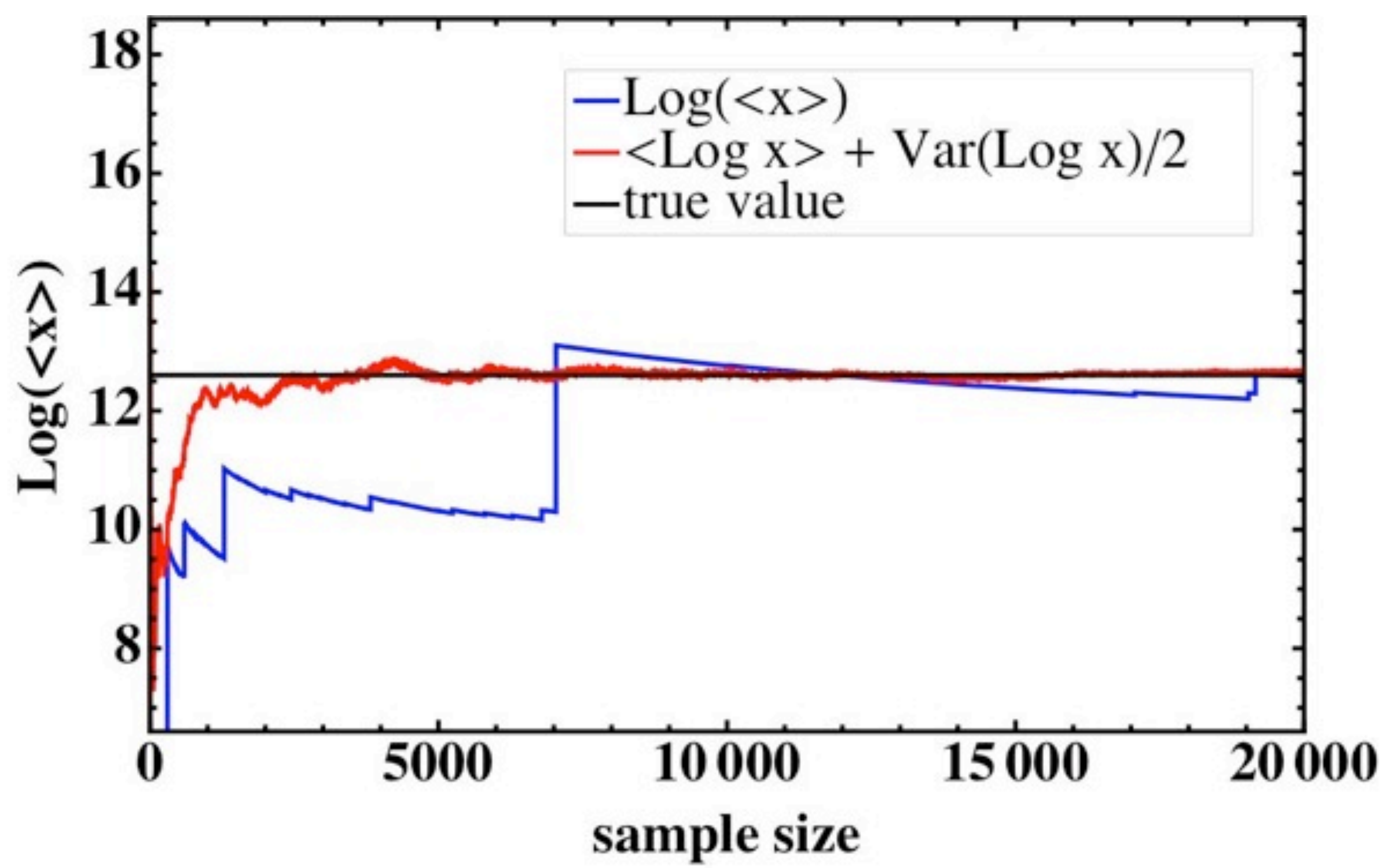
Construct

In our case: $x = \text{correlator} = \prod_i T(\phi_i)$

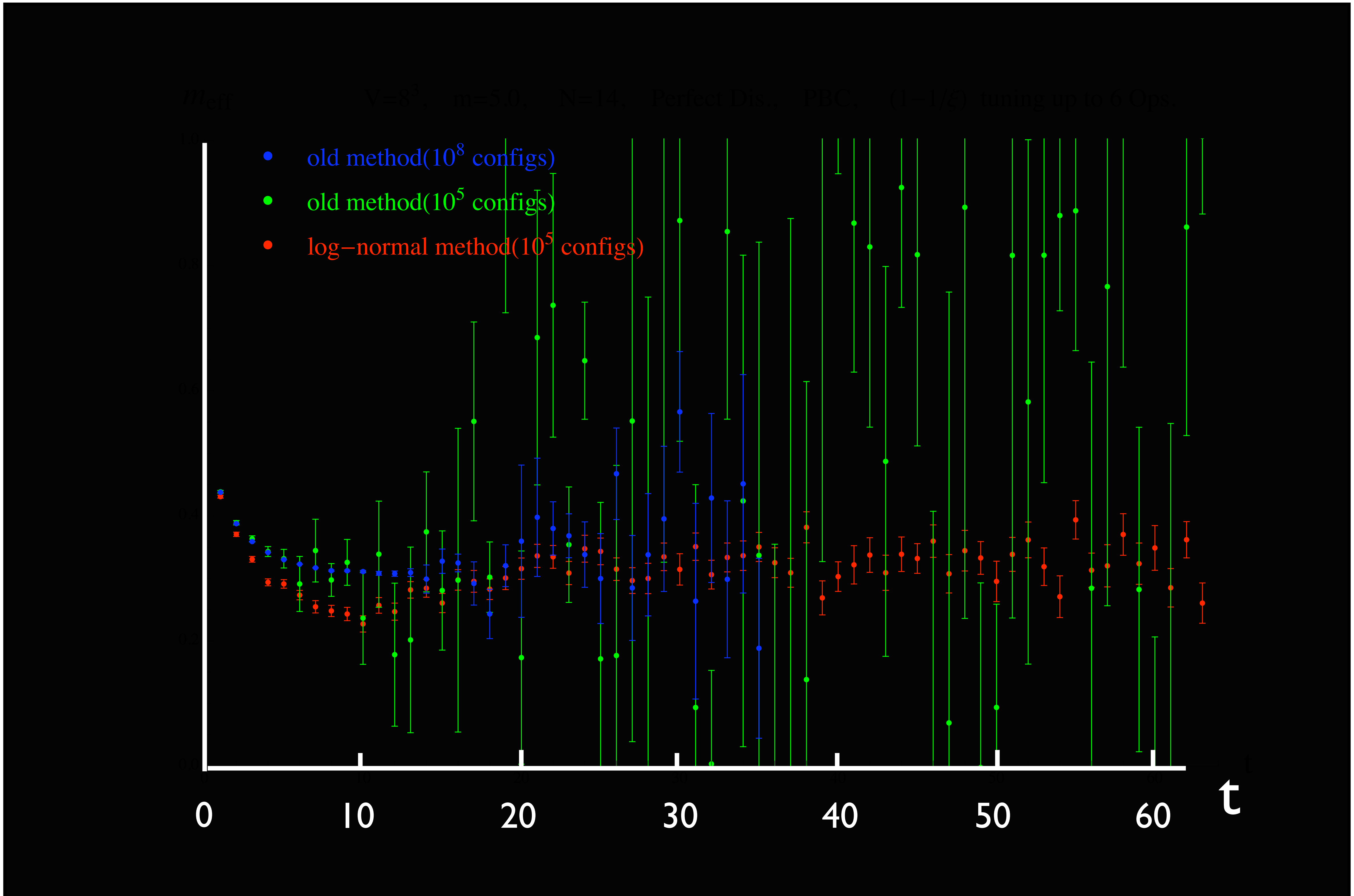
Need to compute: $E = -\frac{1}{t}\ln\langle x\rangle$

Avoid distribution tail by computing $\langle \ln x\rangle\,, \quad \langle \ln^2 x\rangle$
then constructing E.

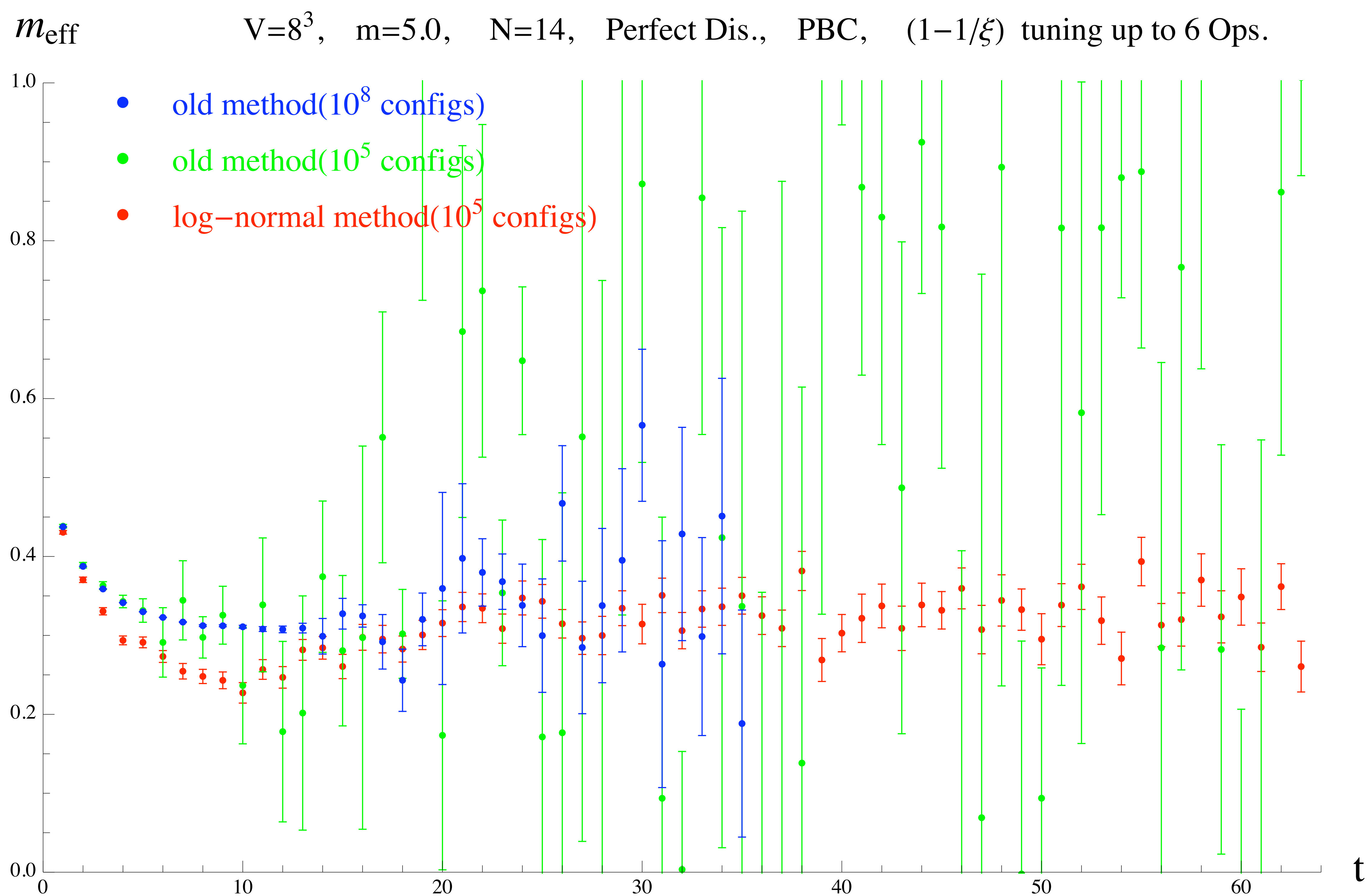
Sampling a variable with
a log-normal distribution



Effective mass plot, N=14 fermions, 8^3x64



Effective mass plot, N=14 fermions, 8^3x64



Conclusions:

- We are on track to achieve 1% measurements for an physically interesting system with up to ~100 fermions
- Interested in extending to non-unitary fermions: lattice EFT for nuclei?
- Learned some lessons:
  - With large number of particles, very difficult to find a ground state you don't already understand!
  - In noisy systems, it pays to examine the raw probability distribution!☆

Wild speculation of the week

Other noisy systems: Perhaps a single hadron correlator C, sampling many "uncorrelated-enough" gauge links, is driven to a fixed-point, non-Gaussian probability distribution $\mathcal{P}^*(C)$ with long tail, such as Log-Normal distribution we are finding

Perhaps can determine the few parameters describing $\mathcal{P}^*(C)$ and hence $\langle C \rangle$ without having to sample tail??

Baryons? Glueballs? Disconnected diagrams?