Recent Results in Precision Heavy Quark Physics

John Bulava*, Micheal Donnellan, Rainer Sommer

NIC, DESY Zeuthen
Zeuthen, DE

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CERN LGT Institute

DESY ALPHA Collaboration NIC CLS based SFB TR9

Outline

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Motivation

- Leading-Order Heavy-Meson Chiral Lagrangian (heavy field part):

$$\mathcal{L}_{\mathrm{heavy}} = -\,\mathrm{Tr}[\overline{H}(iv\cdot D)H] + \hat{g}\ \mathrm{Tr}[\overline{H}H\gamma_\mu A^\mu \gamma_5]$$

- Also define:

$$\langle B(p)\pi(q)|B_i^*(p')\rangle = -g_{B^*B\pi}(q^2)\ q\cdot\eta_i(p')$$

- At tree level in the leading order effective theory,

$$\lim_{q^2\to m_\pi^2}\ g_{B^*B\pi}(q^2) = \frac{2m_B}{f}\hat{g}$$

- The coupling $\hat{g}$ is needed to constrain the chiral behavior of other quantities, such as f_B. See talks by B. Blossier and N. Garron at LATTICE2010

Calculational Technique

- LSZ reduction + PCAC Relation:

$$\langle B(p)\pi(q)|B_i^*(p')\rangle = \frac{1}{f_\pi}\frac{m_\pi^2 - q^2}{m_\pi^2}\langle B(p)|q^\mu A_\mu(q)|B_i^*(p+q)\rangle$$

- Parametrize in terms of Lorentz invariant quantities:

$$\langle B(p)|A_\mu(q)|B_i^*(p+q)\rangle = (\eta_i)_\mu F_1(q^2) + (\eta_i \cdot q)(2p+q)_\mu F_2(q^2) + (\eta_i \cdot q)q_\mu F_3(q^2).$$

- Calculate this 3-pt correlation function on the lattice at $q^2 = 0$ and use

$$g_{B^*B\pi}(0) = -\frac{1}{f_\pi}F_1(0).$$

Previous Results

Ref.	N_f	m_π (GeV)	a(fm)	$g_{B^* B\pi}$
de Divitiis, et al. '98	0	-	.18	.42(4)(8)
Abada, et al. '02	0	-	.07	.69(18)
Negishi, et al. '07	0	-	.06	.517(16)
Becirevic, et al. '09	2	.45-1	.09-.05	.44(3)(7)
Ohki, et al. '08	2+1	.5-1	.22-.16	.516(60)

- No continuum extrapolations
- Relatively low statistics, large pion masses
- Our goal: $\sim 5-10$ percent accuracy, with all sources of systematic error controlled, for $N_f = 0$ and $N_f = 2$.

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Matrix elements from correlation functions: standard method

- Define:

$$C^{3pt}_{mn}(t_f - t', t' - t_i) = \langle \mathcal{O}_m(t_f) A(t') \bar{\mathcal{O}}_n(t_i) \rangle$$
$$C^{2pt}_{mn}(t_f - t_i) = \langle \mathcal{O}_m(t_f) \bar{\mathcal{O}}_n(t_i) \rangle$$

- Difficult to keep $(t_f - t')$ and $(t' - t_i)$ large simultaneaously, as $(t_f - t_i)$ cannot be too large (signal-to-noise decays exponentially).

- Furthermore, excited state corrections are $(t = t_f - t_i)$

$$\frac{C^{3pt}_{mn}(t/2, t/2)}{\sqrt{C^{2pt}_{mm}(t) C^{2pt}_{nn}(t)}} \rightarrow_{t\rightarrow\infty} \mathcal{M} + \mathcal{O}(e^{-(t/2)(E_1 - E_0)})$$

Matrix elements from correlation functions: improved method 1

- Reduce excited state contamination via a summed insertion (Maiani, et al. '87):

$$D_{mn}(t_f - t_i) = \sum_{t'} C^{3pt}_{mn}(t_f - t', t' - t_i)$$

- Corrections in the summed 3-pt correlation function:

$$\partial_t \left[\frac{D_{mn}(t)}{\sqrt{C^{2pt}_{mm}(t) C^{2pt}_{nn}(t)}} \right] \rightarrow_{t\rightarrow\infty} \mathcal{M} + \mathcal{O}(e^{-t(E_1 - E_0)})$$

- Now only $t = (t_f - t_i)$ is required to be large!

Matrix elements from correlation functions: improved method 2

- FURTHER reduce excited state contamination by using the GEVP(Blossier, et al. '09; Luscher and Wolff '90 ; Michael et al. '85):

$$C^{2pt}(t)v_n(t,t_0) = \lambda_n(t,t_0)C^{2pt}(t_0)v_n(t,t_0)$$

- Build up a basis of 8 operators of different Gaussian smearing radii $(0 - .7\mathrm{fm})$.

$$\psi_l^k(x) = (1 + \kappa_G a^2\Delta)^{R_k}\psi_l(x)$$

$$r_{phys,k} \approx 2\sqrt{\kappa_G R_k}a$$

- Construct Optimized ratio:

$$R(t,t_0) = \partial_t\Big[\frac{\hat{D}(t,t_0)}{\hat{C}^{2pt}(t,t_0)}\Big]$$

$$\hat{D}(t,t_0) = v_0^\dagger(t,t_0)D(t)v_0(t,t_0)$$

Stochastic Evaluation

- Stochastic All-to-all (with time dilution) for the light quark prop.

$$M_\ell^{-1}(x,y) = \langle \sum_d \phi^{(d)}(x)\eta^{(d)\dagger}(y)\rangle_\eta$$

$$\phi^{(d)}(x) = M_\ell^{-1}(x,y)\eta^{(d)}(y)$$

- 3pt function is straightforward to evaluate using a sequential source. ($\mathcal{O}_m = \bar{q}\Gamma_m q$).

$$\langle \mathcal{O}_m(t_f)A_k(t)\bar{\mathcal{O}}_n(t_i)\rangle =$$
$$\langle \sum_d \psi_k^{(d)}(x_f)\Gamma_m M_{stat}^{-1}(x_f,x_i)\Gamma_n^\dagger \eta^{(d)\dagger}(x_i)\rangle_\eta$$

$$\psi_k^{(d)}(x) = M_\ell^{-1}(x,y)\gamma_k\gamma_5\phi^{(d)}(y)$$

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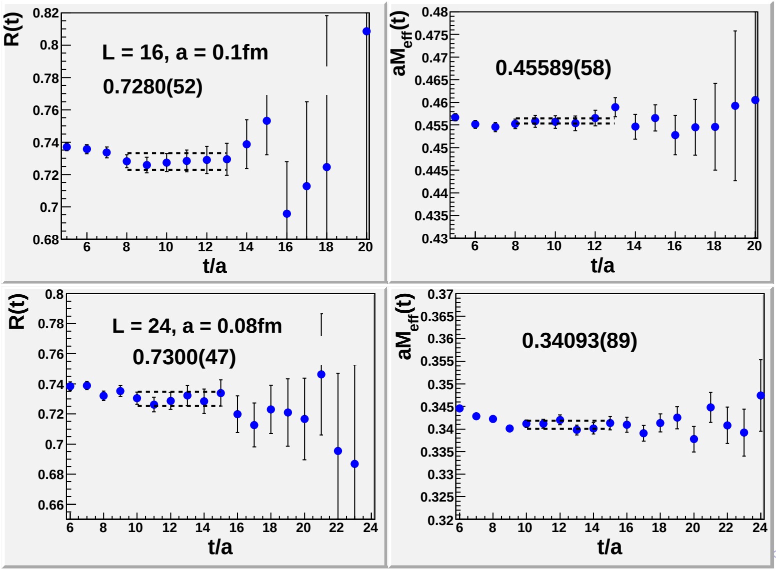
$N_f = 0$ Results : A test of the method

- Wilson gauge action, Non-pert. Improved Clover valence quarks. $m_q \approx m_s$.

	N_{conf}	N_{noise}
$16^3 \times 32$, $a \approx 0.10$ fm	100	200
$24^3 \times 48$, $a \approx 0.08$ fm	100	48
$32^3 \times 64$, $a \approx 0.05$ fm	100	32

- Two discretizations of the static quark action (HYP1, HYP2) to further assess discretization errors.
- Fix a physical distance for $t_0 = .3\mathrm{fm}$ and $t_{min} = .65\mathrm{fm}$, take the plateau average of $R(t, t_0)$ over a suitable range.
- Demand that fit range of 3pt function corresponds with plateau region of 2pt function.

$N_f = 0$ PLATEAUX

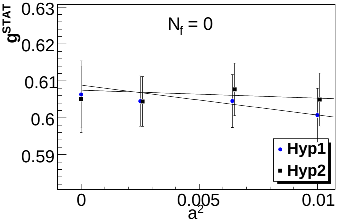


$N_f = 0$ continuum extrapolation

- Renormalization:

$$(A_\mu)_R = Z_A(1 + b_A a m_q)(A_\mu + a c_A \partial_\mu P)$$

- At $\mathbf{p} = 0$, c_A term does not contribute to A_i. b_A taken from 1-loop perturbation theory, Z_A from (Lüscher et al. '96).



- Hyp1: $g^{STAT} = 0.6063(91)$
- Hyp2: $g^{STAT} = 0.6050(90)$
- Error dominated by error on Z_A.
- Continuum limit is rather flat.

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Gauge Ensembles

- CLS Ensembles: Wilson Gauge action, $N_f = 2$ Non-pert. imp. Clover Fermions, DD-HMC algorithm (M. Lüscher).

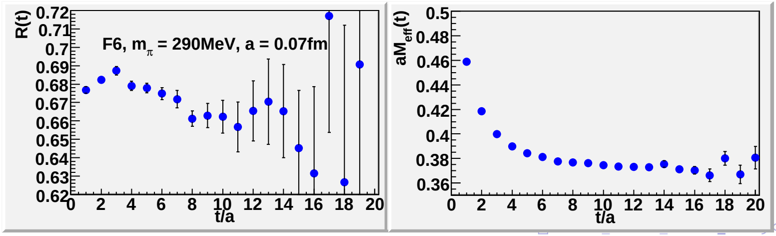
- Ensembles used here:

	Name	$L(fm)$	m_π (MeV)	$m_\pi L$	N_{conf}	N_{noise}
$\beta = 5.2$, $a \approx 0.08$ fm	A1	2.6	≈600	9.0	100	8
	A4	2.6	≈365	4.8	105	8
$\beta = 5.3$, $a \approx 0.07$ fm	E4	2.3	≈560	6.2	157	16
	E5	2.3	≈420	4.7	400	4
	F6	3.4	≈290	5.0	339	2
	F7	3.4	≈250	4.2	301	4
$\beta = 5.5$, $a \approx 0.05$ fm	M5	1.6	≈365	3.0	258	8
	N5	2.4	≈365	5.3	0	0

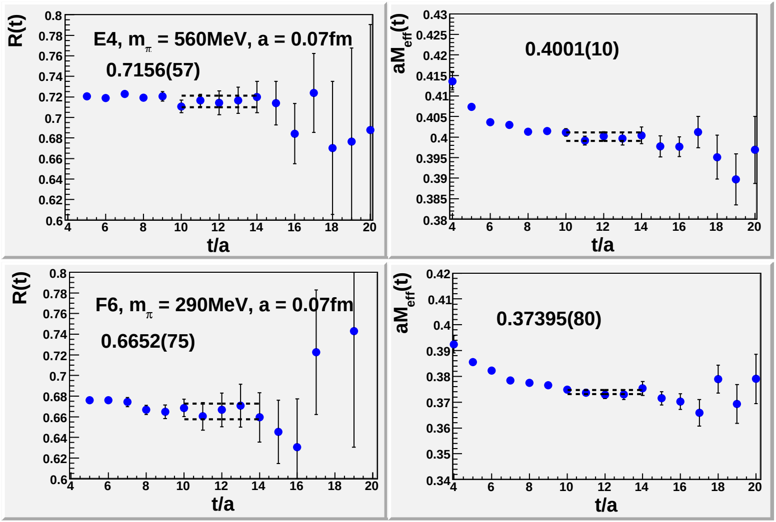
- Z_A taken from (Della Morte et al. '08), b_A taken from 1-loop perturbation theory.

Warning

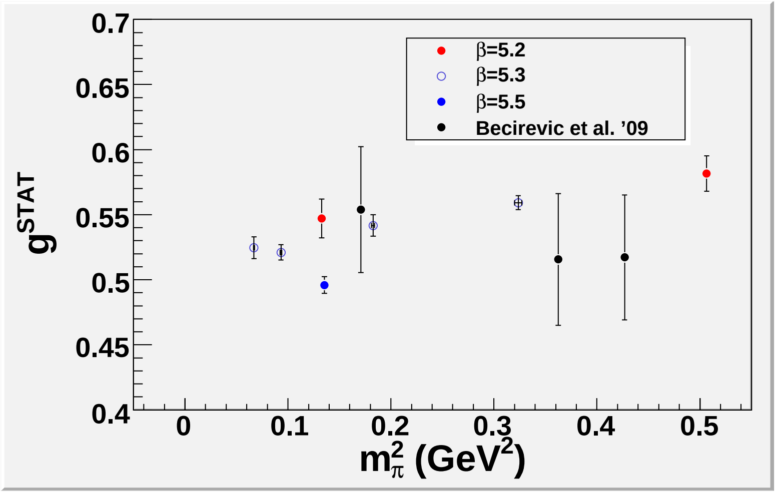
- Excited state contamination in $R(t)$ (and other ratios) can be more complicated than effective masses.
- F6 ensemble, Single interpolating field (no GEVP).
- With less statistics, may be tempting to fit $t = 2 - 7$.
- Does NOT coincide with plateau region of 2p function.
- Dangerous, and in this case incorrect.



$N_f = 2$ Results


E4, m_π = 560MeV, a = 0.07fm
0.7156(57)
R(t)
t/a
0.4001(10)
aM_eff(t)
t/a
F6, m_π = 290MeV, a = 0.07fm
0.6652(75)
R(t)
t/a
0.37395(80)
aM_eff(t)
t/a

Preliminary $N_f = 2$ results



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Future Plans

- Additional $\beta = 5.2, 5.5$ ensembles, increased statistics.
- Take the continuum limit at $m_\pi \approx 365\mathrm{MeV}$, using three lattice spacings.
- Determine explicitly the form of the excited state corrections in the GEVP version of $R(t)$.
- Investigate ratios for use in systems with $E_f \neq E_i$. For example, see talk of B. Knippschild (U. of Mainz) at LATTICE2010.

Conclusions

- Lattice spacing dependence of $N_f = 0$ $\hat{g}$ seems mild.
- Precision determination of $\hat{g}$ for $N_f = 2$ is nearly completed.
- Matrix elements should be computed for t in the plateau region of the 2pt function
- This summed insertion is directly applicable to e.g. B_k, B_B.
- Applicable to e.g. $B \to \pi \ell \nu$ with some modifications.