The FLAG: lattice results for phaenomenologists

FLAG: Flavianet Lattice Averaging Group

A. VLADIKAS - INFN "Tor Vergata"

CERN, 22 July 2010

The CERN Theory Institute:

Future Directions in Lattice Gauge Theory

Motivation

FLAG: Flavianet Lattice Averaging Group

Prolegomena

- Flavour Physics increasingly important as LHC probes new energies
- precision measurements may lead to signatures of new Physics
- major theoretical limitation: low energy QCD effects in SM are not quantified to a satisfactory precision
- Lattice QCD: a sound, field-theoretic approach, aiming at the computation of these hadronic effects with well controlled (and increasingly decreasing) errors
- Lattice simulations performed by different groups involve different choices both at the level of formalism (lattice actions, number of sea flavours etc.) and at the level of resources (lattice volumes, quark masses etc.)
- often this amounts to making different compromises which in turn introduces different systematic effects
- not all lattice results of a given quantity are directly comparable

Prolegomena

- Aim: answer the question “What is currently the best lattice value for a particular quantity?” in a way which is readily accessible to non-experts
- FLAG: founded in November 2007, operates within the European Network on Flavour Physics (Flavianet)
- FLAG: a group of European lattice and χPT practitioners is making an effort to create a compilation of results on a few quantities, which critically summarize the state of the art
- FLAG members: G.Colangelo (Bern), S.Dürr (Jülich), A.Jüttner (Mainz), L.Lellouch (Marseilles), H.Leutwyler (Bern), V.Lubicz (Rome3), S.Necco (CERN), C.Sachrajda (Southampton), S.Simula (Rome3), T.Vladikas (Rome2), U.Wegner (Bern), H.Wittig (Mainz)
- extrapolation of precise lattice results guided by Chiral Perturbation Theory (χPT) ⇒ close collaboration between lattice and χPT experts

Prolegomena

- First FLAG report limited to important quantities in pion and Kaon Physics
  - Light and strange quark masses
  - decay constants f_K / f_π
  - Kaon decay form factor $f_+(0)$
  - Neutral Kaon oscillation bag parameter B_K
  - SU(2) and SU(3) low energy constants Σ, F, l_3, l_4, l_6, L_4, L_5, L_6, L_8, L_9, L_{10}

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- Quenched results ($N_f = 0$) will not be discussed; $N_f = 2+1$ and $N_f = 2$ are current state of the art
- Results presented in this talk are almost definitive; some minor adjustments are to be expected in the final preprint (to appear soon)

Quality Criteria

FLAG: Flavianet Lattice Averaging Group

Quality Criteria

- a number of criteria have been fixed; these are somewhat subjective and time dependent
- help assess the reliability of a particular simulation without reading the papers!
- this may be oversimplifying, but it is true that phenomenologists tend to take lattice results at face value
- we aim at providing compact information on the quality of a computation
- criteria:

★ systematic error estimated in a satisfactory manner and under control

● a reasonable attempt at estimating systematic error; can be improved

■ no attempt or unsatisfactory attempt at controlling a systematic error

C. Pena, **PoS LAT2006:019,2006** Tucson, Arizona, 23-28 Jul 2006

http://www.physics.utah.edu/lat06/abstracts/sessions/plenary.html

Quality Criteria

- chiral extrapolation:

★ $M_{\pi,\mathrm{min}} < 250$ MeV

● 250 MeV $\lesssim M_{\pi,\mathrm{min}} \lesssim$ 400 MeV

■ 400 MeV $\lesssim M_{\pi,\mathrm{min}}$

NB: at least 3 points requested (otherwise there is a "special mention")

- continuum extrapolation:

★ at least 3 lattice spacings, at least two below 0.1 fm

● 2 or more lattice spacings, at least one below 0.1 fm

■ otherwise

NB: theory should be O(a)-improved; for non-improved theories an extra point is needed for each criterion

Quality Criteria

- finite volume effects:
  - ★ $[M_\pi \; L\,]_{\mathrm{min}} > 4$ or at least 3 volumes
  - 🟡 $[\,M_\pi L\,]_{\mathrm{min}} > 3$ and at least 2 volumes
  - 🟥 otherwise, and in any case if $L < 2$ fm

  NB: p-regime
- renormalization (where applicable):
  - ★ non perturbative
  - 🟡 2-loop perturbation theory
  - 🟥 otherwise
- renormalization group running (where applicable):
  - ★ non perturbative
  - 🟡 otherwise

Quality Criteria

- **Averages**: there are several independent results for some physical quantities; averaging them gives the lattice estimate for this quantity
- which results are dropped from averaging? unless we have a reason for making an exception, we drop data with 🟥
- **Publication status:** only peer-reviewed, published papers are included in the averages
- exception: obvious updates of published results in conference proceedings

  A: published, or plain update of published paper

  P: preprint

  C: conference contribution
- **Flavours**: only deal with physical quantities characterized by light and strange quarks; disregard quenched simulations
- average $N_f = 2$ and $N_f = 3$ results separately

Form factor, decay constants and unitarity

Form factor, decay constants and unitarity

- unitarity: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

- experiment: $|V_{ub}| = 3.93\ (36)\ \cdot 10^{-3}$

- Kaon decays: $|V_{us}|\ f_+(0) = 0.21661\ (47)$

form factor @ zero momentum transfer $K^0 \rightarrow \pi^- \nu l^+$

$$\left|\frac{V_{us} f_K}{V_{ud} f_\pi}\right| = 0.27599\ (59)$$

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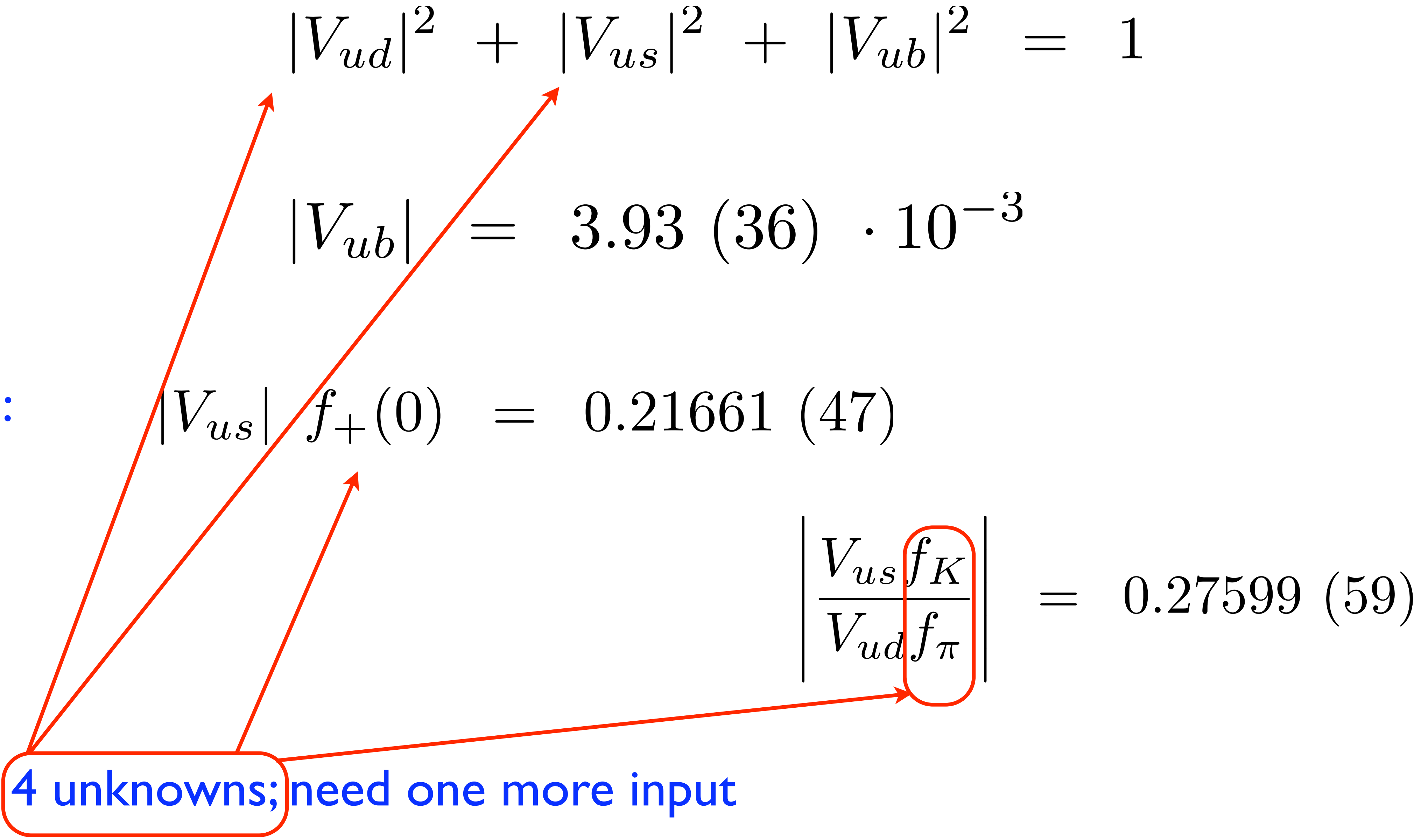
Form factor, decay constants and unitarity



- unitarity: $$|V_{ud}|^2 \;+\; |V_{us}|^2 \;+\; |V_{ub}|^2 \;=\; 1$$
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- e.g. V_{ud} from nuclear β decays or V_{us} from τ decays

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- 3 expressions, 4 unknowns; need one more input
- lattice provides independent determinations of f_K / f_π and $f_+(0)$

Form factor, decay constants and unitarity

$N_f = 2+1$

$N_f = 2$

Collaboration	N_f	publication status	chiral extrapolation	finite volume errors	continuum extrapolation	$f_+(0)$
RBC/UKQCD 07	2+1	A	●	★	■	0.9644(33)(34)(14)
ETM 09A	2	A	●	●	●	0.9560(57)(62)
QCDSF 07	2	C	■	★	■	$0.9647(15)_{stat}$
RBC 06	2	A	■	★	■	0.968(9)(6)
JLQCD 05	2	C	■	★	■	0.967(6)

Table 1: Colour code for the data on $f_+(0)$.

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MILC 09	2+1	P	★	★	★	$1.197(3)(^{+6}_{-13})$
ALVdW 08	2+1	C	★	●	●	1.191(16)(17)
PACS-CS 08, 08B	2+1	A	★	■	■	1.189(20)
BMW 08	2+1	C	★	★	★	1.18(1)(1)
HPQCD/UKQCD 08	2+1	A	★	●	★	1.189(2)(7)
RBC/UKQCD 08	2+1	A	●	★	■	1.205(18)(62)
NPLQCD 06	2+1	A	●	■	■	$1.218(2)(^{+11}_{-24})$
ETM 09	2	A	●	●	★	1.210(6)(15)(9)
QCDSF/UKQCD 07	2	C	●	★	●	1.21(3)

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Form factor, decay constants and unitarity

$$f_+(0) \quad = \quad 0.964\ (3)\ (4) \qquad (N_f = 2+1)$$
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most systematics OK

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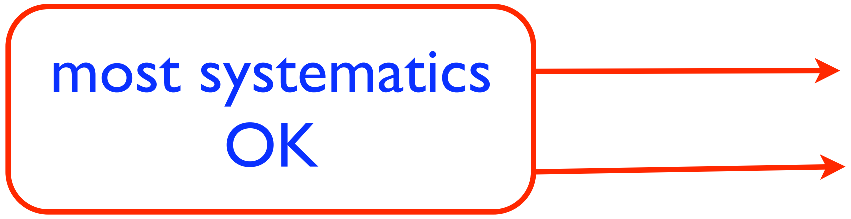
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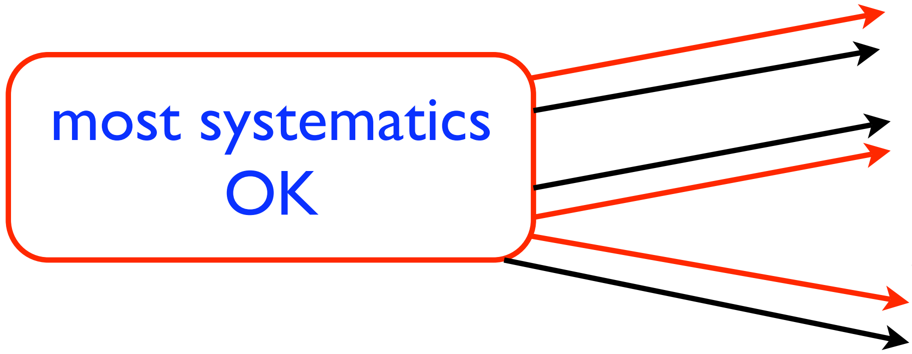
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Best result to-date
(not "our estimate")

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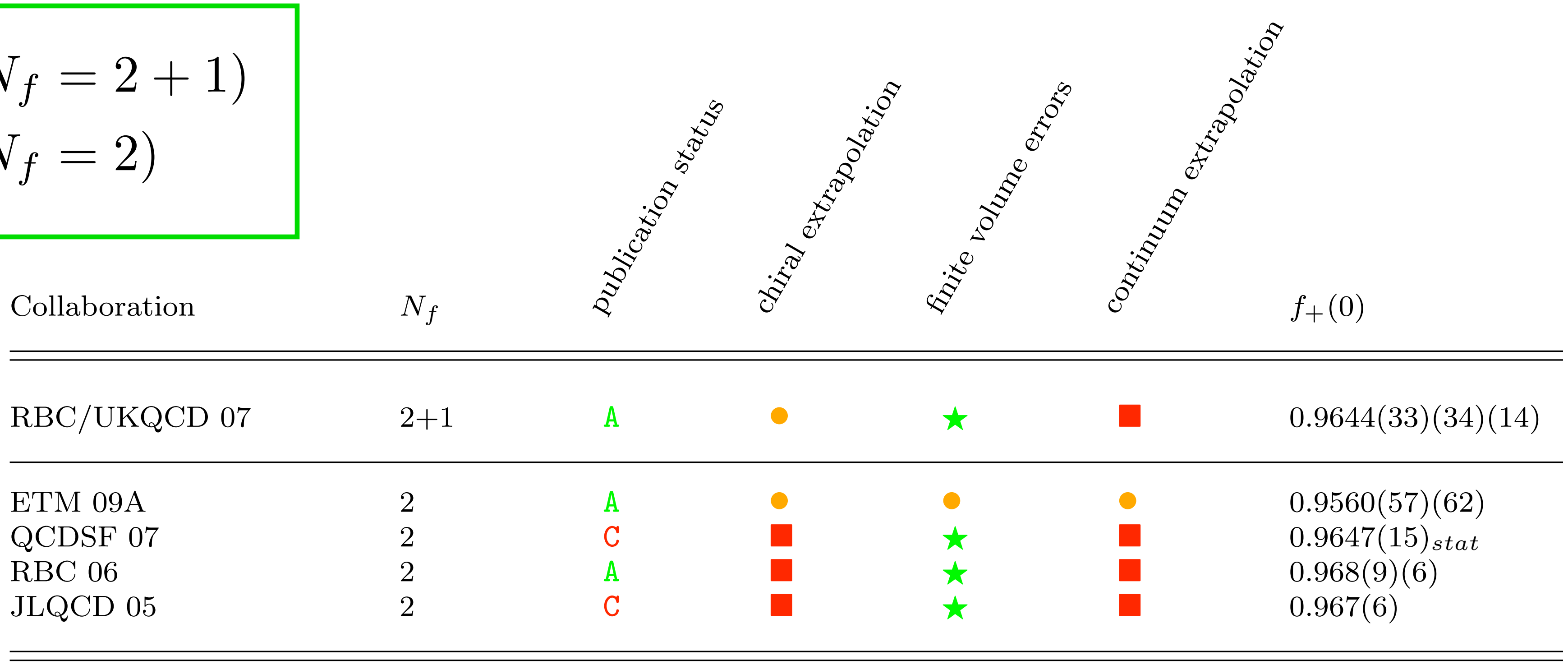

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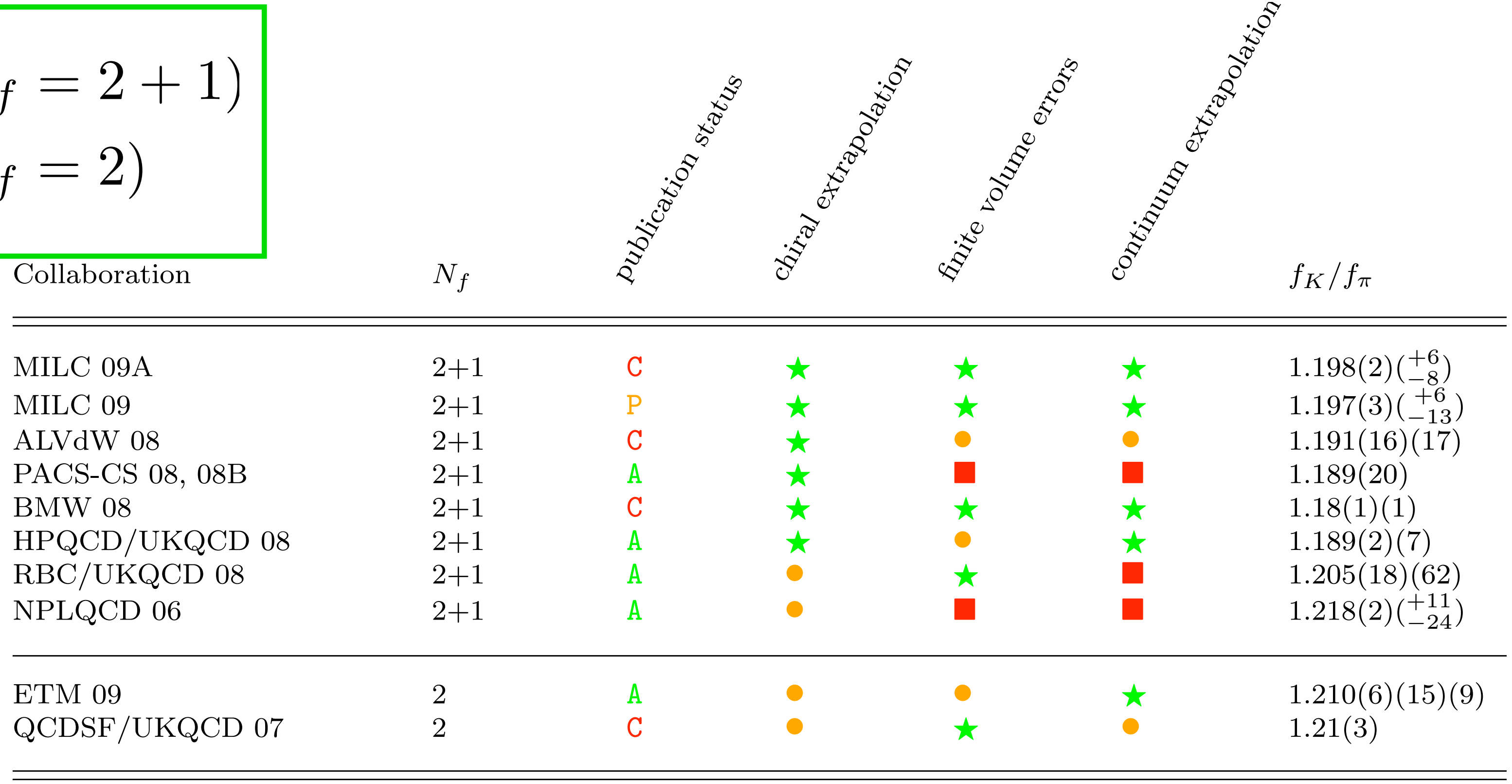

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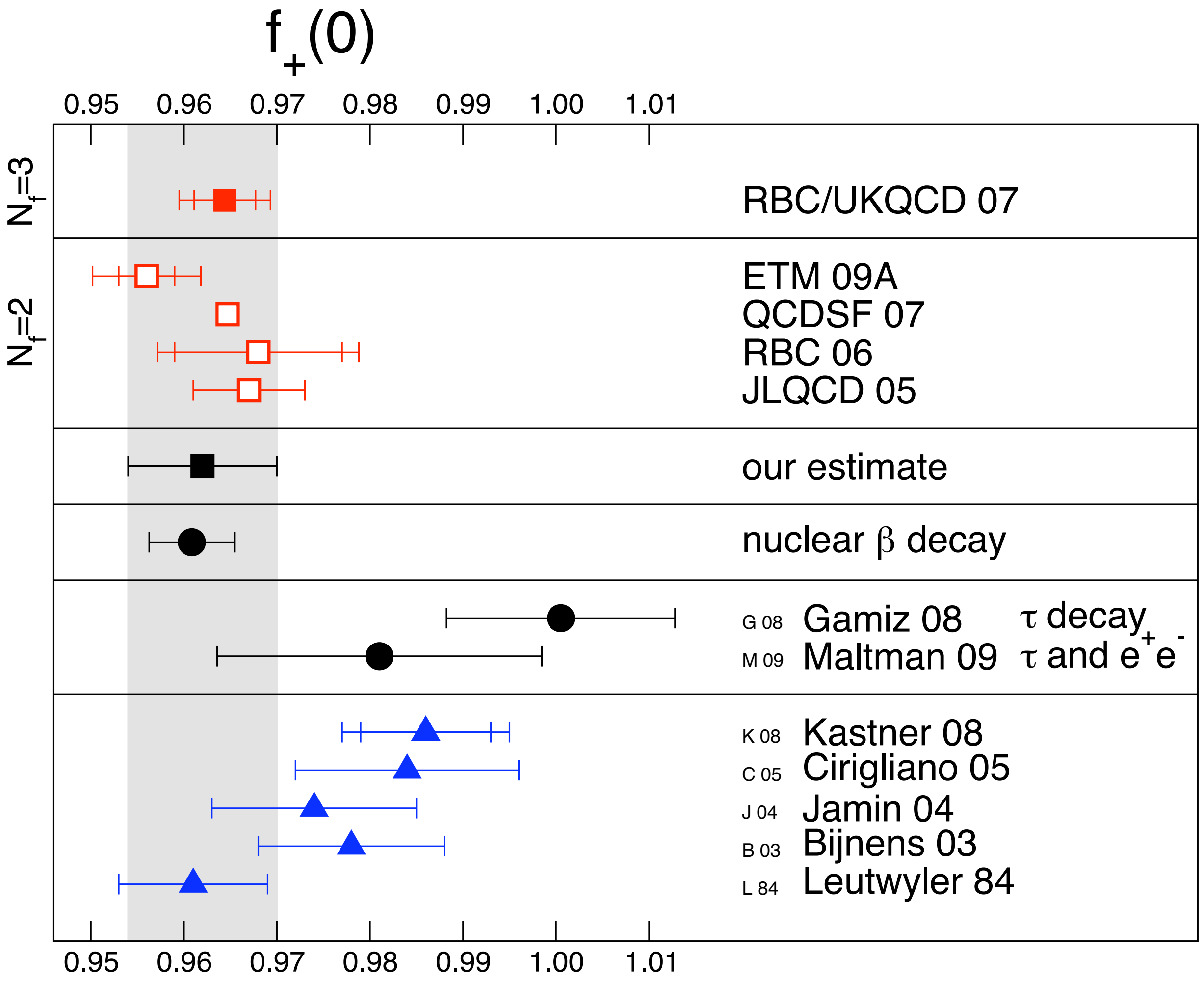
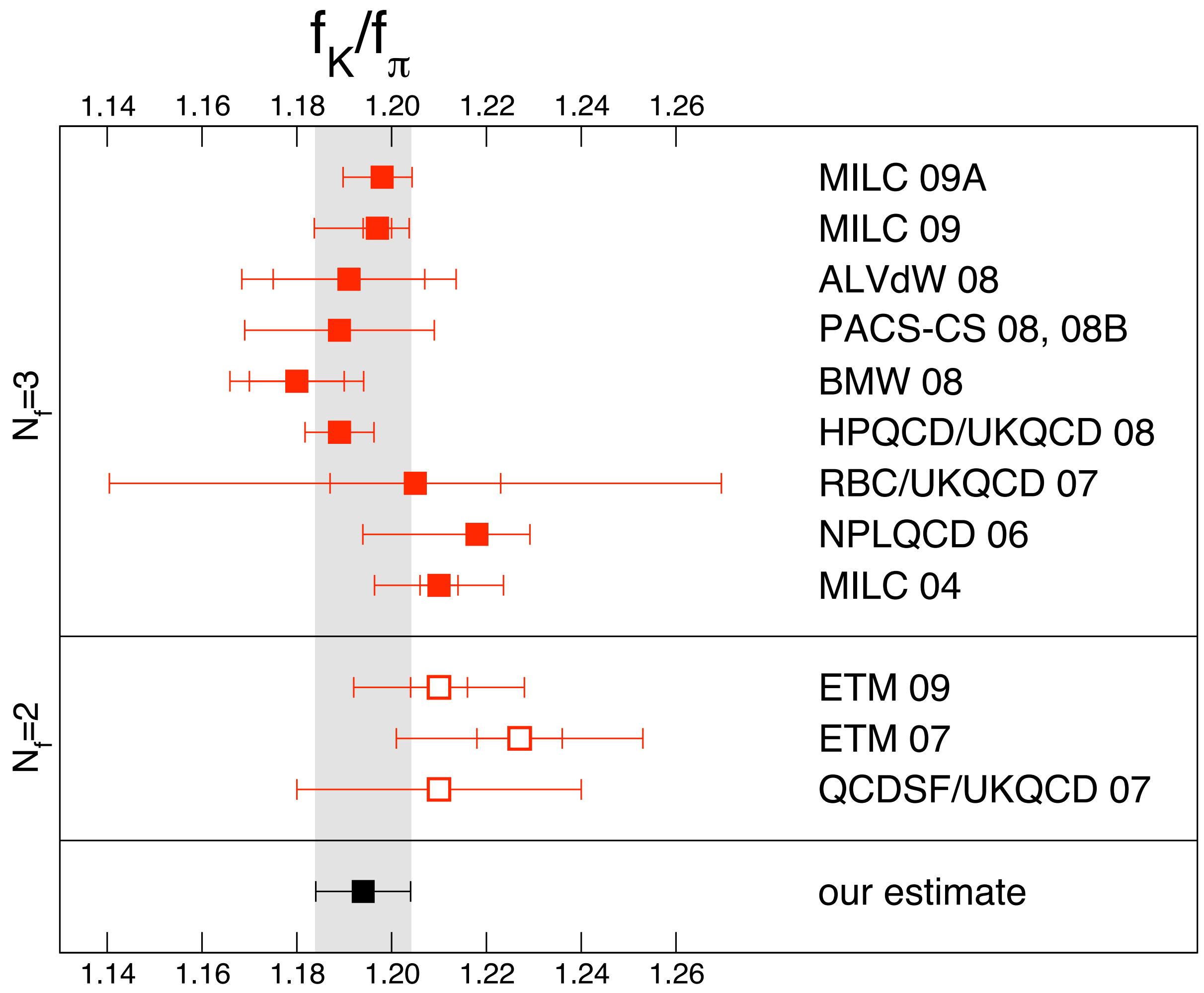




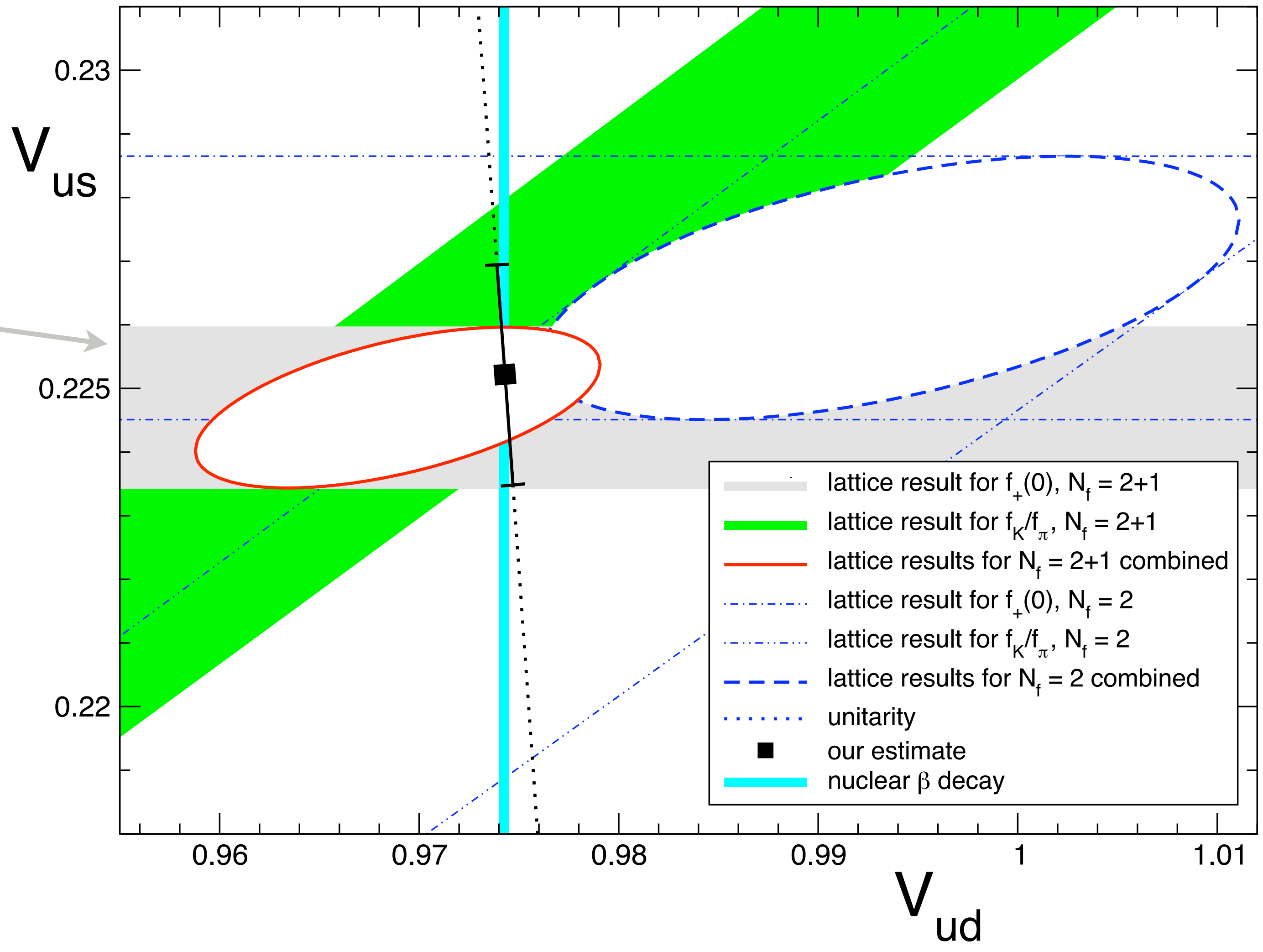
- lattice agrees with nuclear β decay
- disagrees with semi-inclusive τ decay
- “our estimate” explained later
- from χPT:

$$\Delta f \equiv f_+(0) - 1 - f_2 = f_+(0) - 0.977$$

- lattice suggests $\Delta f < 0$
- results from various model estimates vary; Δf sign unclear

Form factor, decay constants and unitarity

- use: $|V_{us}|\ f_+(0)\ =\ 0.21661\ (47)$
- N_f = 3 result of $f_+(0)$ gives:



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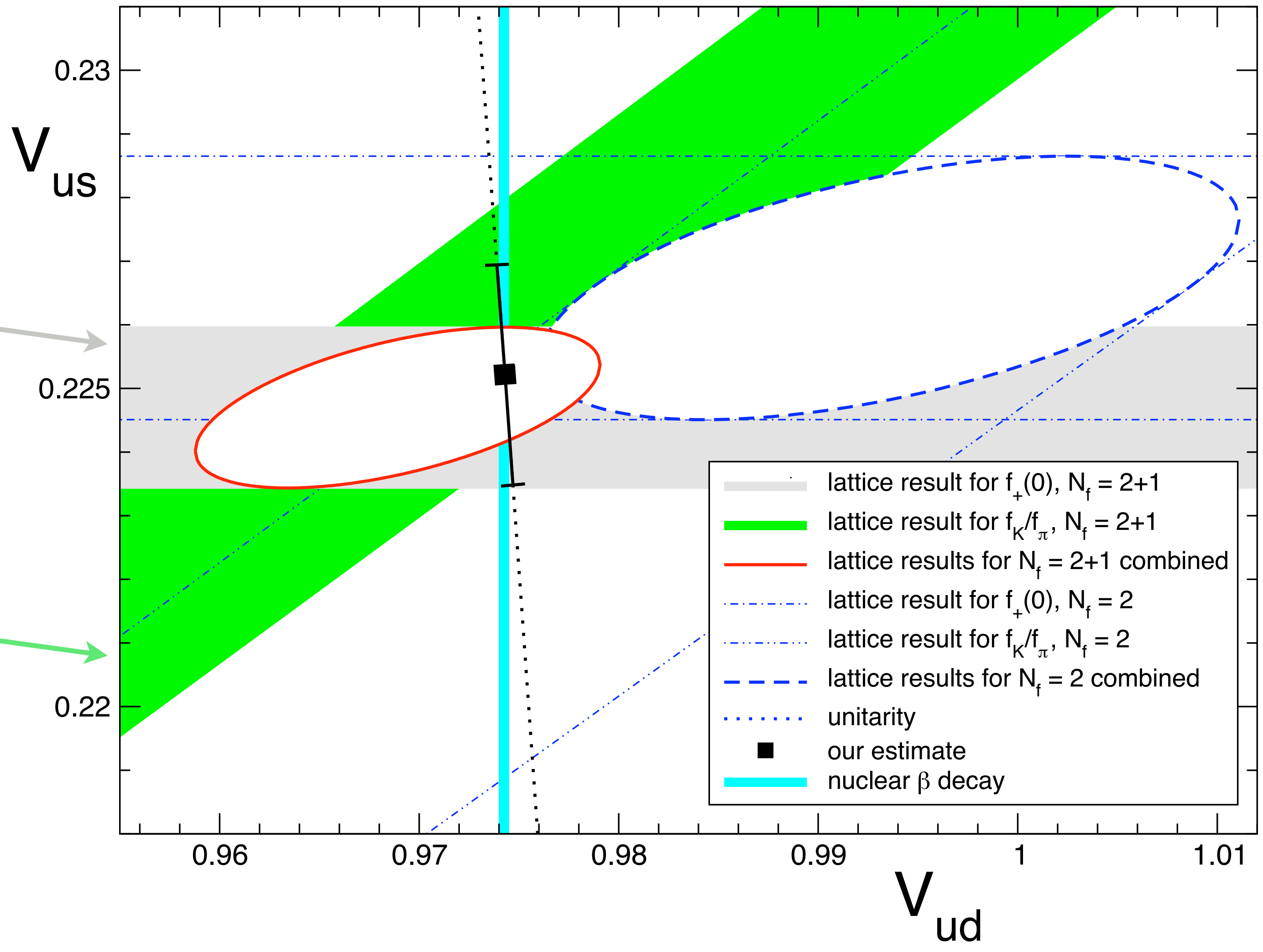

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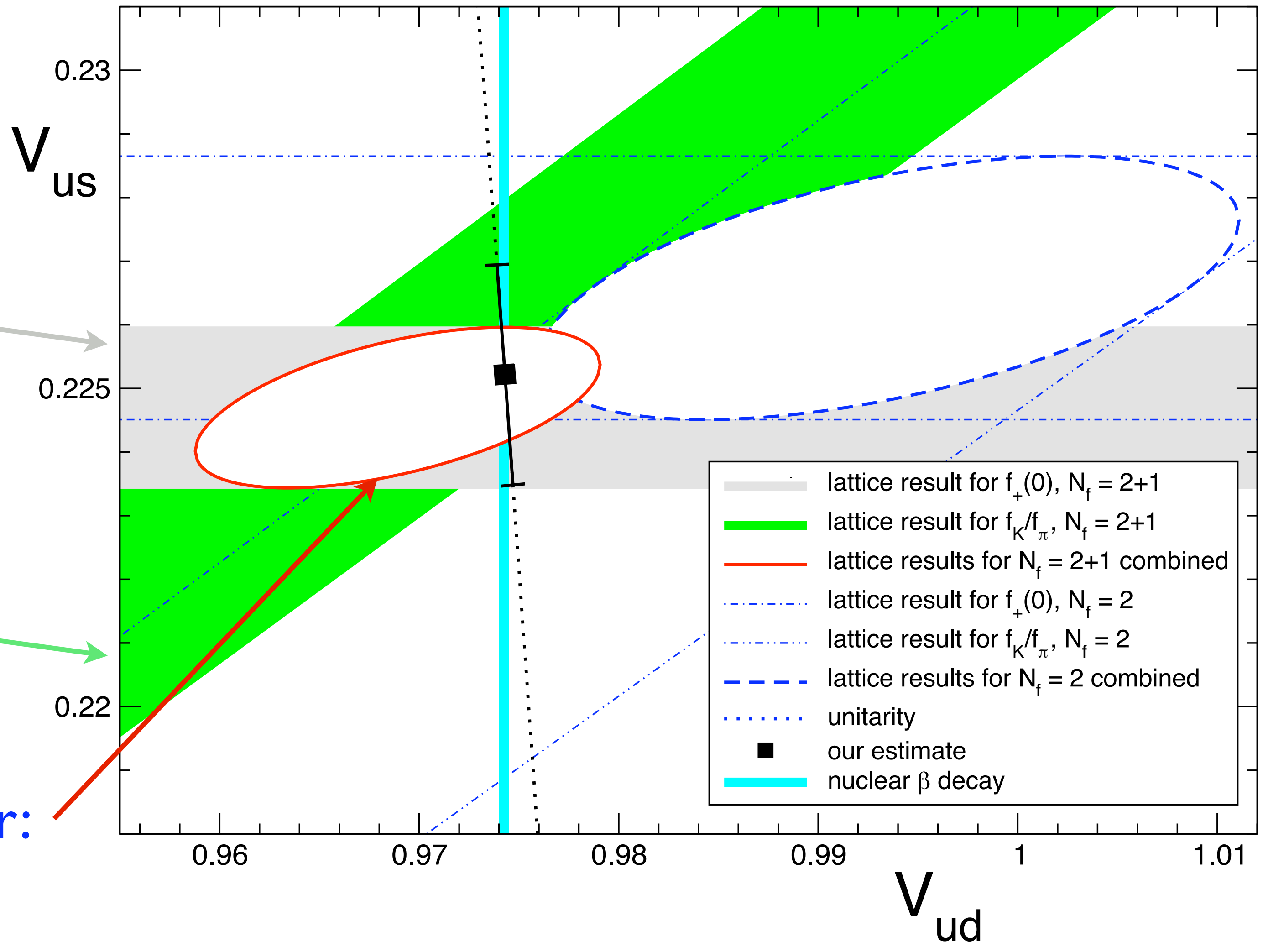
treating these two results as independent measurements gives the 68% likelihood contour:



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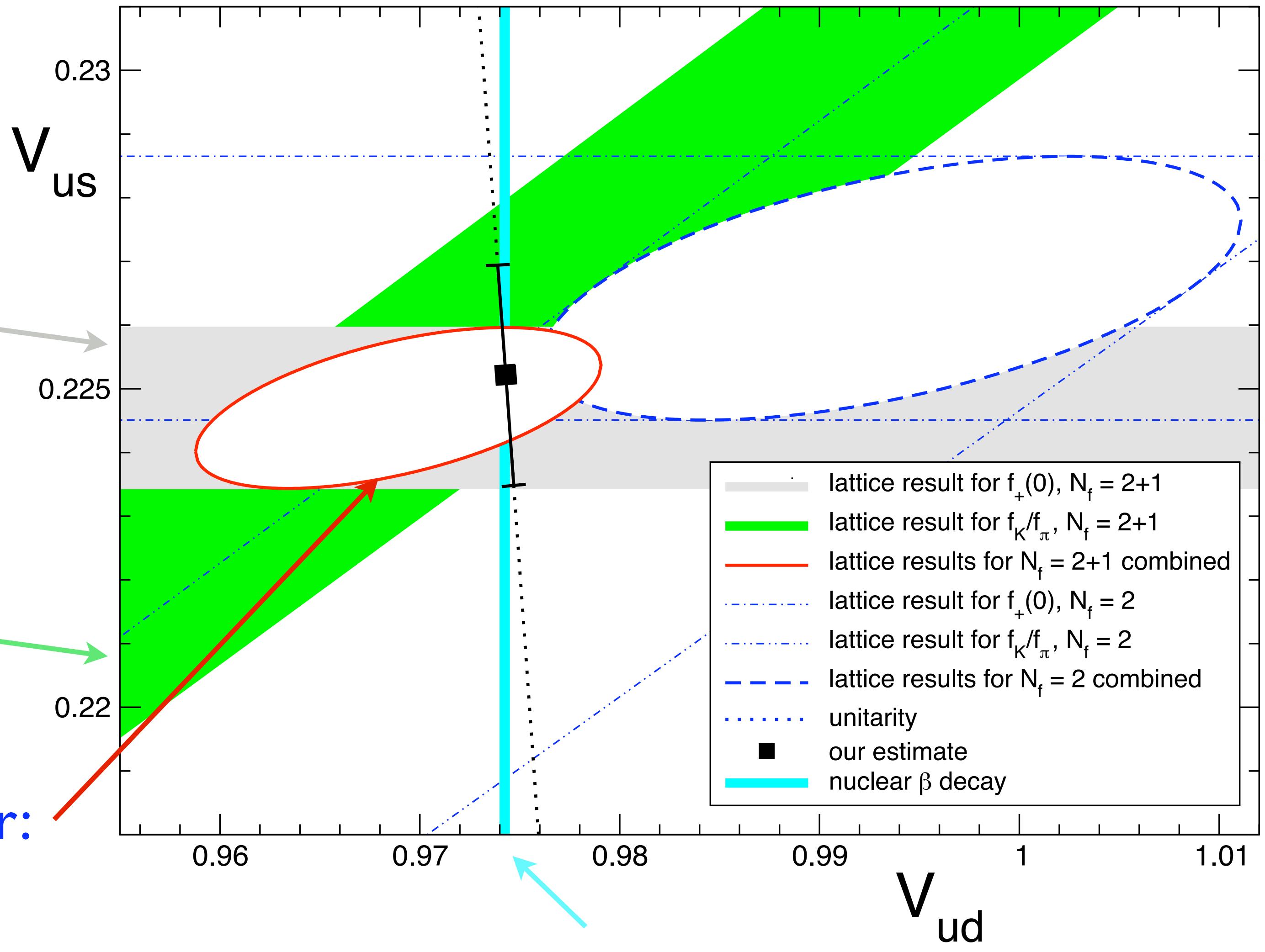

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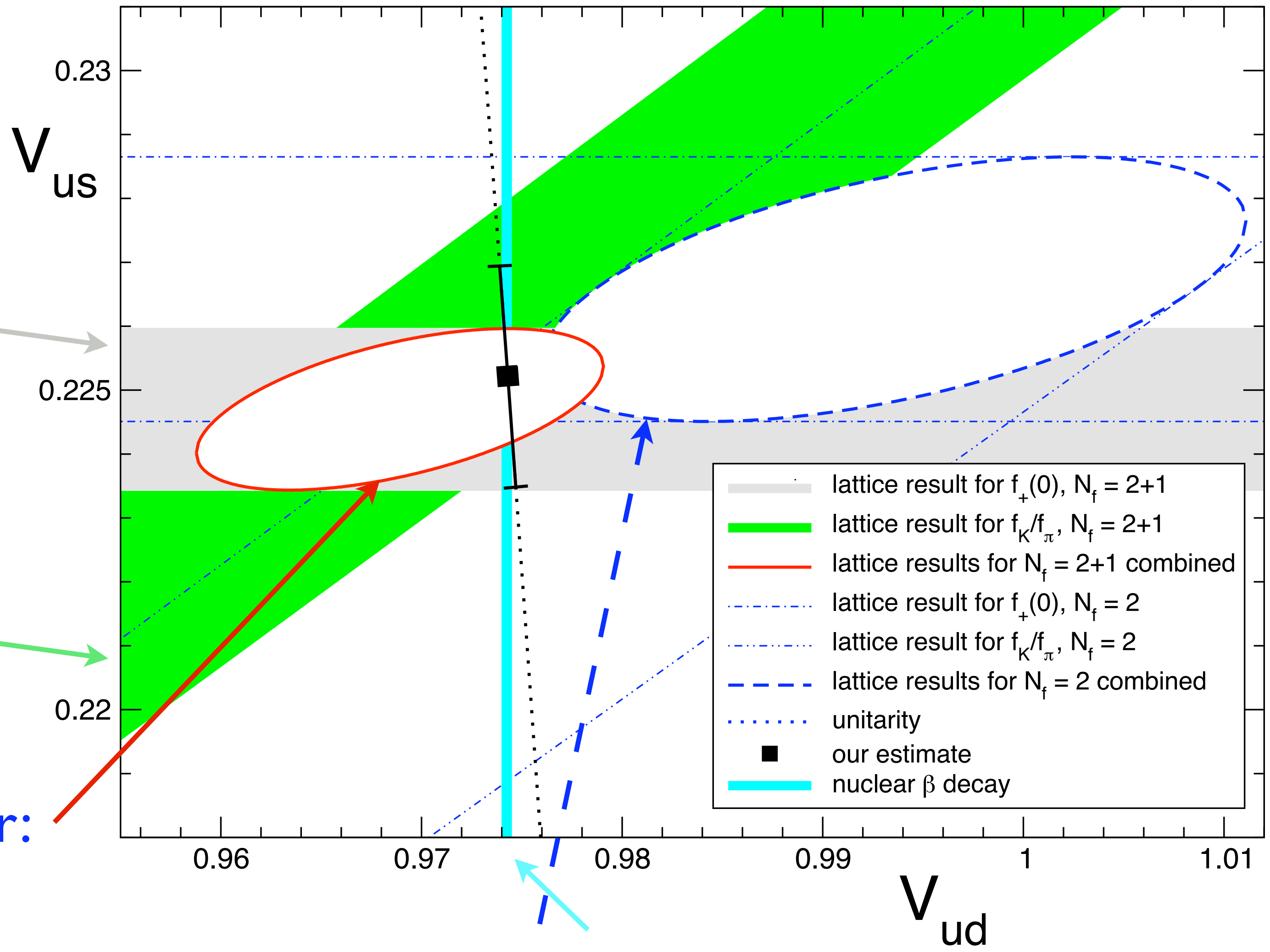


- $N_f = 3$ lattice data consistent with nuclear beta decay prediction of V_{ud}:
- $N_f = 2$ lattice data consistent with $N_f = 3$ data within errors (just!!):

Form factor, decay constants and unitarity

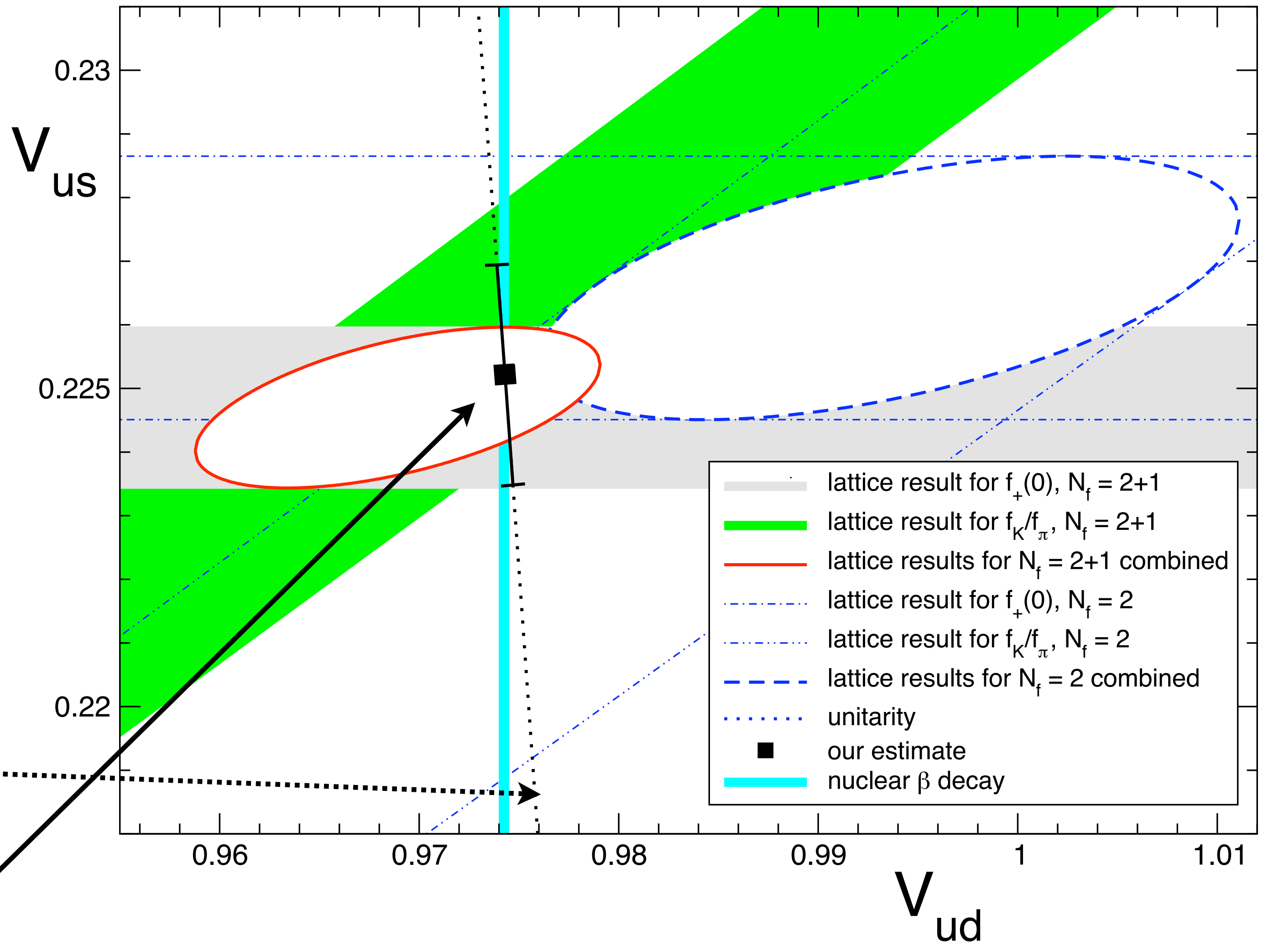
- unitarity:

$$|V_{ud}|^2 \; + \; |V_{us}|^2 \; + \; |V_{ub}|^2 \;\; = \;\; 1$$

- ... & experiment:

$$|V_{ub}| \;\; = \;\; 3.93\;(36) \;\; \cdot 10^{-3}$$

- ... imply this constraint:



- ... which agrees very well with "our best estimate" lattice result (obtained as will be explained shortly)

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- which combine with $N_f = 3$ lattice results of $f_+(0)$ and f_K/f_π to give $|V_{us}|$ and $|V_{ud}|$

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- which combine with $N_f = 3$ lattice results of $f_+(0)$ and f_K/f_π to give $|V_{us}|$ and $|V_{ud}|$
- take $|V_{ub}|$ from experiment; the unitarity constraint is well satisfied:

$$\boxed{|V_{ud}|^2 \ + \ |V_{us}|^2 \ + \ |V_{ub}|^2 \ = \ 0.989\ (20) \qquad N_f = 2+1}$$

Form factor, decay constants and unitarity

- Test of Standard Model: relax unitarity constraint and test it!

- from Kaon decays we have:

$$|V_{us}| \;\; f_+(0) \;\; = \;\; 0.21661 \; (47) \qquad\qquad \left|\frac{V_{us} f_K}{V_{ud} f_\pi}\right| \;\; = \;\; 0.27599 \; (59)$$

- which combine with $N_f = 3$ lattice results of $f_+(0)$ and f_K/f_π to give $|V_{us}|$ and $|V_{ud}|$
- take $|V_{ub}|$ from experiment; the unitarity constraint is well satisfied:

$$\boxed{|V_{ud}|^2 \;+\; |V_{us}|^2 \;+\; |V_{ub}|^2 \;\;=\;\; 0.989 \; (20) \qquad N_f = 2+1}$$

- now use V_{ud} from β decays and f_+ (0) from $N_f = 3$ lattice:

$$\boxed{|V_{ud}|^2 \;+\; |V_{us}|^2 \;+\; |V_{ub}|^2 \;\;=\;\; 0.9997 \; (7)}$$

Form factor, decay constants and unitarity

- Test of Standard Model: relax unitarity constraint and test it!

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- now use V_{ud} from β decays and f_K / f_π from $N_f = 3$ lattice:

$$\boxed{|V_{ud}|^2 \ + \ |V_{us}|^2 \ + \ |V_{ub}|^2 \ = \ 1.0002 \ (10)}$$

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- $N_f = 2$ 1.038(35) - OKish

- now use V_{ud} from β decays and $f_+(0)$ from $N_f = 3$ lattice:

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Form factor, decay constants and unitarity

- Test of Standard Model: relax unitarity constraint and test it!

- from Kaon decays we have:

$$|V_{us}|\ f_+(0) \ = \ 0.21661\ (47) \qquad \left|\frac{V_{us}f_K}{V_{ud}f_\pi}\right| \ = \ 0.27599\ (59)$$

- which combine with *N*$_f$ = *3* lattice results of *f*$_+$(0) and *f*$_K$/*f*$_\pi$ to give |V$_{us}$| and |V$_{ud}$|
- take |V$_{ub}$| from experiment; the unitarity constraint is well satisfied:

$$|V_{ud}|^2 \ + \ |V_{us}|^2 \ + \ |V_{ub}|^2 \ = \ 0.989\ (20) \qquad N_f = 2+1$$

- *N*$_f$ = 2 1.038(35) - OKish

- now use V$_{ud}$ from β decays and *f*$_+$ (0) from *N*$_f$ = *3* lattice:

$$|V_{ud}|^2 \ + \ |V_{us}|^2 \ + \ |V_{ub}|^2 \ = \ 0.9997\ (7)$$

- *N*$_f$ = 2 1.0005(10) - OK

- now use V$_{ud}$ from β decays and *f*$_K$ / *f*$_\pi$ from *N*$_f$ = *3* lattice:

$$|V_{ud}|^2 \ + \ |V_{us}|^2 \ + \ |V_{ub}|^2 \ = \ 1.0002\ (10)$$

Form factor, decay constants and unitarity

- Test of Standard Model: relax unitarity constraint and test it!

- from Kaon decays we have:

$$|V_{us}|\ f_+(0) \ = \ 0.21661\ (47) \qquad \left|\frac{V_{us} f_K}{V_{ud} f_\pi}\right| \ = \ 0.27599\ (59)$$

- which combine with $N_f = 3$ lattice results of $f_+(0)$ and f_K/f_π to give $|V_{us}|$ and $|V_{ud}|$
- take $|V_{ub}|$ from experiment; the unitarity constraint is well satisfied:

$$|V_{ud}|^2 \ + \ |V_{us}|^2 \ + \ |V_{ub}|^2 \ = \ 0.989\ (20) \qquad N_f = 2+1$$

- $N_f = 2$ 1.038(35) - OKish

- now use V_{ud} from β decays and $f_+(0)$ from $N_f = 3$ lattice:

$$|V_{ud}|^2 \ + \ |V_{us}|^2 \ + \ |V_{ub}|^2 \ = \ 0.9997\ (7)$$

- $N_f = 2$ 1.0005(10) - OK

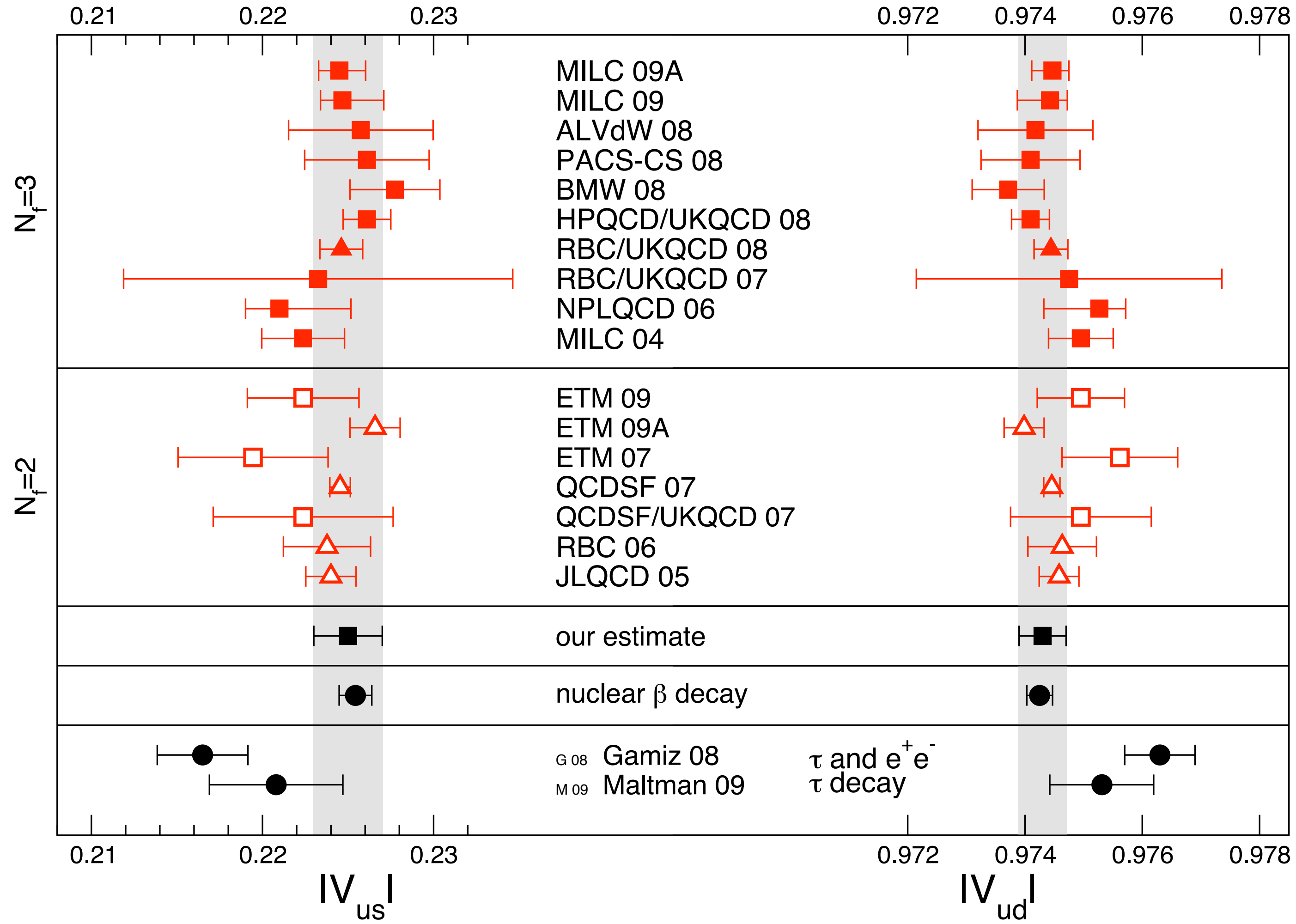
- now use V_{ud} from β decays and f_K / f_π from $N_f = 3$ lattice:

- $N_f = 2$ 0.9986(16) - OK

$$|V_{ud}|^2 \ + \ |V_{us}|^2 \ + \ |V_{ub}|^2 \ = \ 1.0002\ (10)$$

Form factor, decay constants and unitarity

- Analysis based on Standard Model:
  - unitarity: $$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$
  - experiment: $$|V_{ub}| = 3.93\ (36)\ \cdot 10^{-3}$$
  - Kaon decays: $$|V_{us}|\ f_+(0) = 0.21661\ (47) \qquad \left|\frac{V_{us} f_K}{V_{ud} f_\pi}\right| = 0.27599\ (59)$$
- 3 expressions, 4 unknowns $f_+(0)$, f_K/f_π, $|V_{us}|$, $|V_{ud}|$; one input determines three quantities



■ data from f_K/f_π □

▲ data from $f_+(0)$ △

$|V_{us}|$, $|V_{ud}|$ results consistent from f_K/f_π and from $f_+(0)$

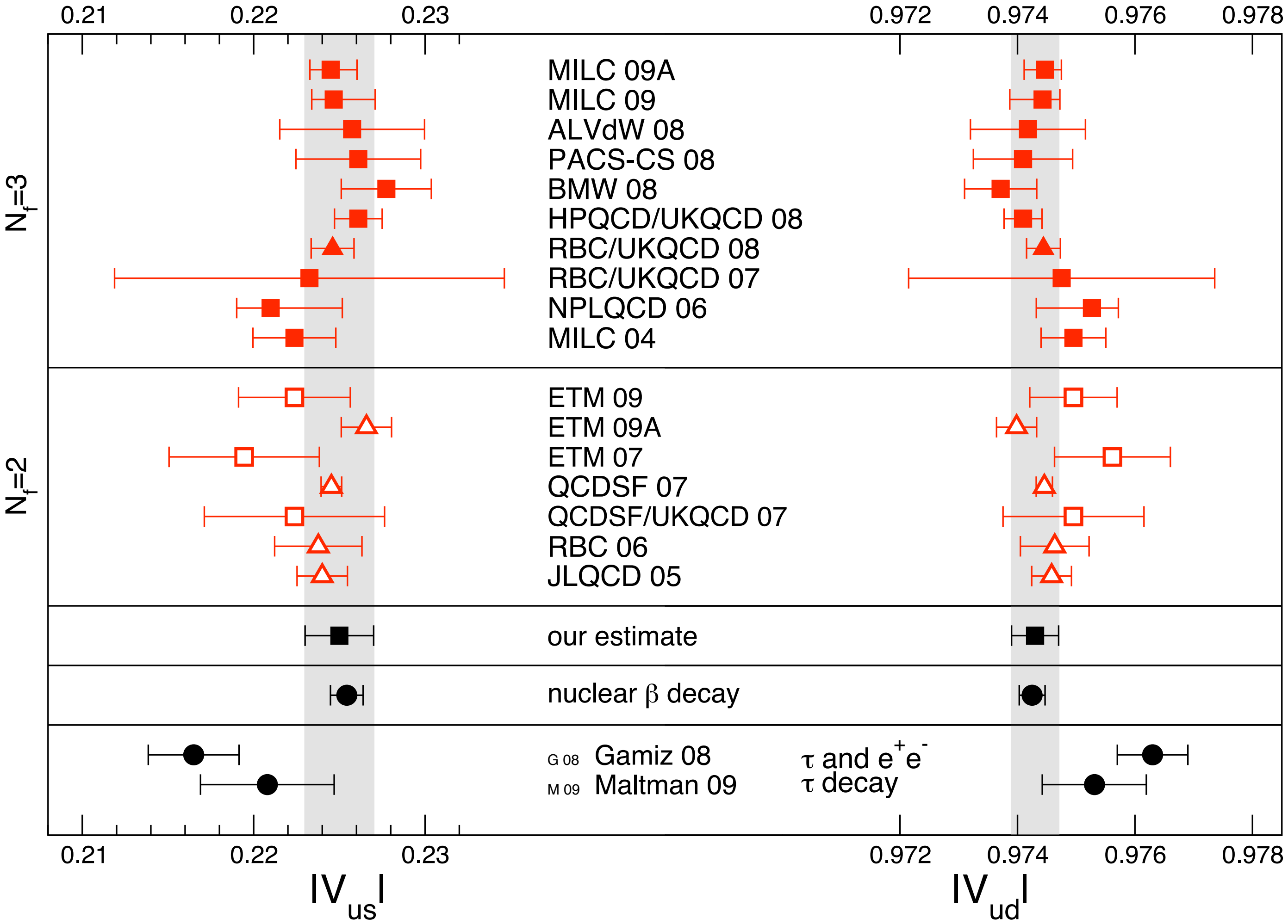
"our estimates" obtained by combining the "chosen" lattice results

Form factor, decay constants and unitarity

- Analysis based on Standard Model:

	$\lvert V_{us}\rvert$	$\lvert V_{ud}\rvert$	$f_+(0)$	f_K/f_π
$N_f = 2+1$	$0.2251(11)$	$0.97433(24)$	$0.9626(43)$	$1.1944(61)$
$N_f = 2$	$0.2253(17)$	$0.97428(40)$	$0.9608(73)$	$1.1934(98)$
our estimate	$0.225(2)$	$0.9743(4)$	$0.962(8)$	$1.194(10)$

Table 1: Final results for the analysis of the lattice data within the Standard Model



- combine data from direct f_K/f_π measurements with f_K/f_π results obtained from direct $f_+(0)$ measurements, to get **best f_K/f_π result** at a given N_f
- vice versus get **best f_K/f_π result**
- extremely close agreement between N_f=2 and N_f=2+1 results; take biggest uncertainty into account to obtain “our estimate”

ΔS=2 transitions: B_K

ΔS=2 transitions: ε_K

indirect CP-violation

$$\epsilon_K = \frac{\mathcal{A}[K_L \to (\pi\pi)_{I=0}]}{\mathcal{A}[K_S \to (\pi\pi)_{I=0}]} = [2.282(17) \times 10^{-3}] \exp(i\pi/4)$$

can also be expressed in terms of K^0 - K^0 mixing
dominant EW process is FCNC (2 W exchange)

$$|\epsilon_K| \approx C_\epsilon \, \hat{B}_K \, \mathrm{Im}\{V_{td}^* V_{ts}\} \, \{\mathrm{Re}\{V_{cd}^* V_{cs}\}[\eta_1 \, S_0(x_c) \; - \; \eta_3 \, S_0(x_c, x_t)] - \mathrm{Re}\{V_{td}^* V_{ts}\}\eta_2 \, S_0(x_t)]\}$$

long distance NP

Put in NLO PT + Cabibbo angle + A + $m_{c,t}$:

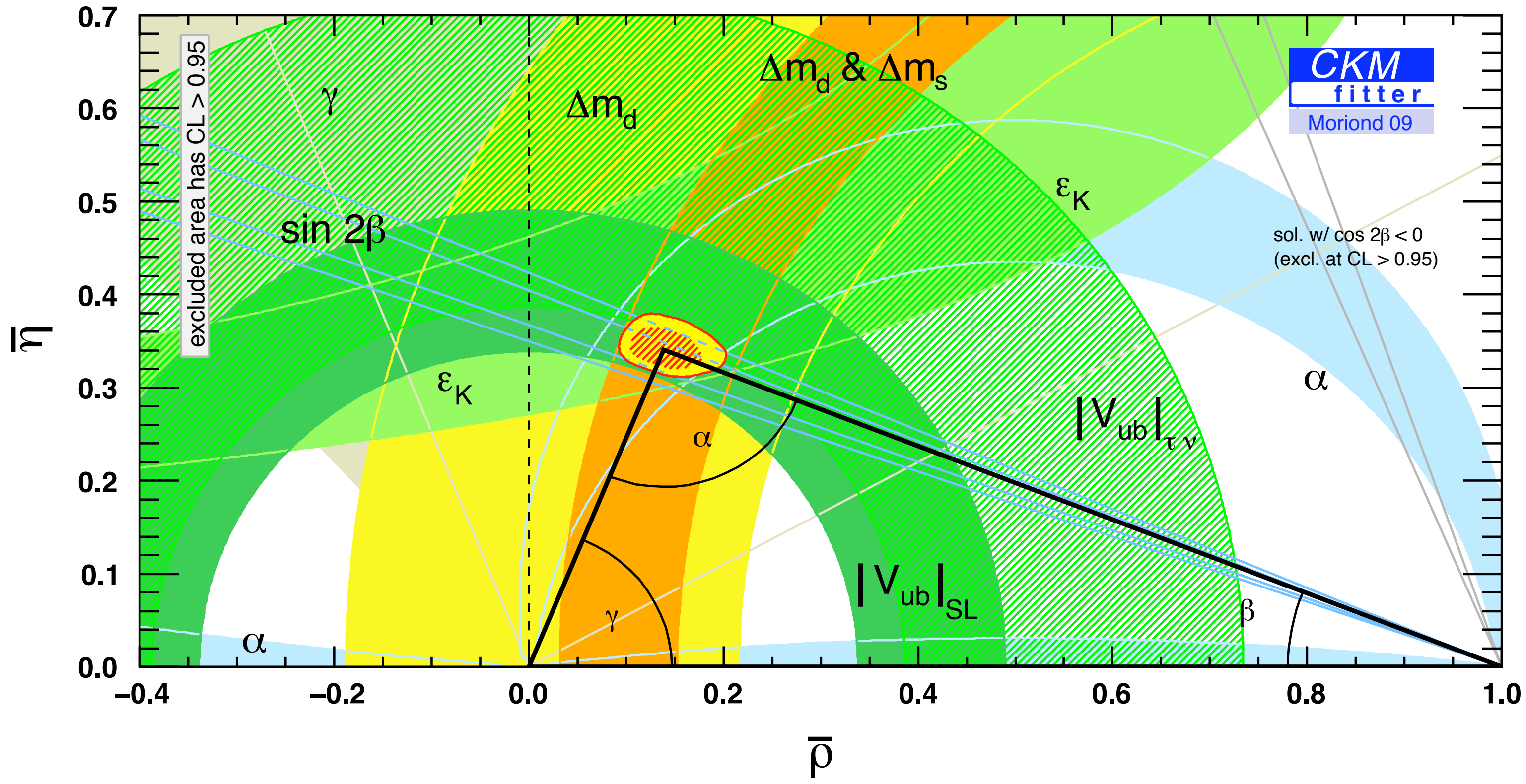
$$\boxed{\hat{B}_K = \frac{\langle \bar{K}^0 | \hat{O}^{\Delta S=2} | K^0 \rangle}{\frac{8}{3} F_K^2 \, m_K^2}}$$

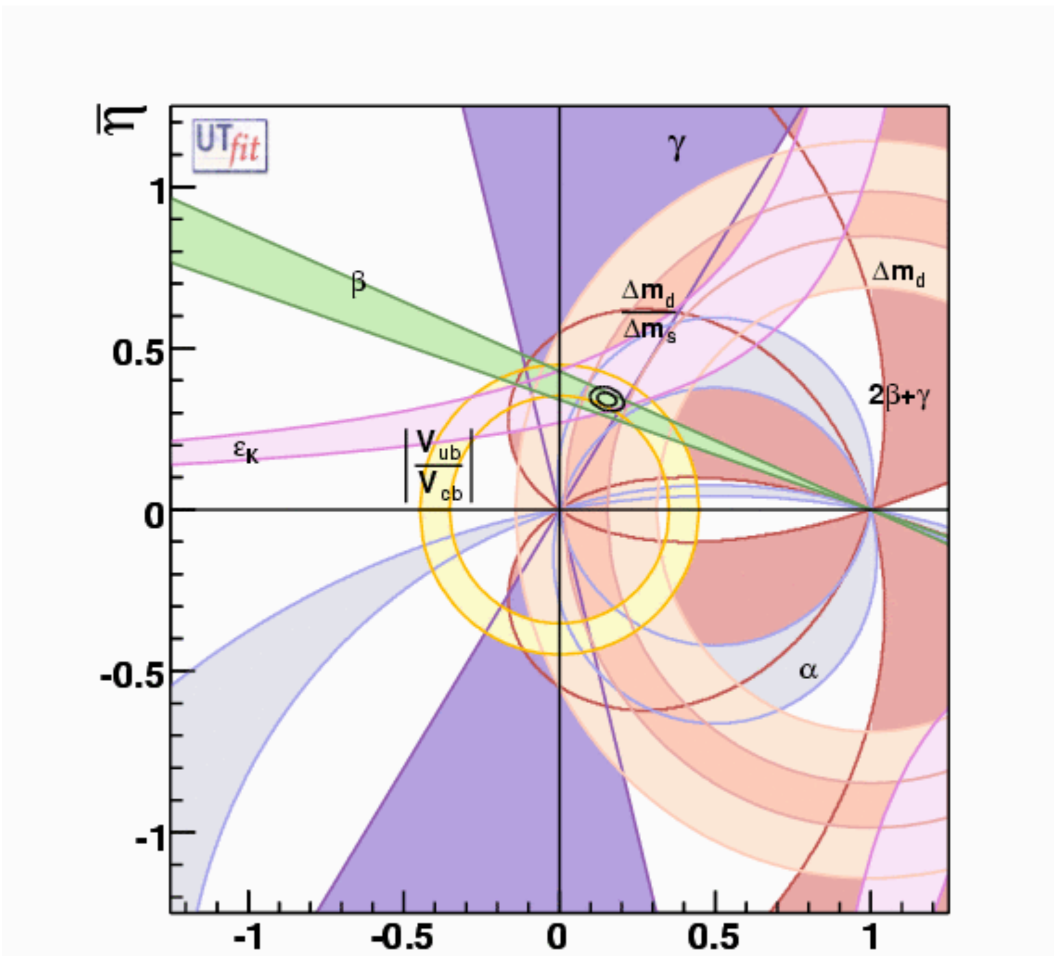
$$\bar{\eta}(1.4 - \bar{\rho}) \, \hat{B}_K \approx 0.40$$

hyperbola

ΔS=2 transitions: B_K

$$|\epsilon_K| = \frac{\mathcal{A}(K_L \to (\pi\pi)_{I=0})}{\mathcal{A}(K_S \to (\pi\pi)_{I=0})} \overset{\text{exp}}{=} [2.282(17) \times 10^{-3}]\, e^{i\pi/4}$$





ΔS=2 transitions: B_K

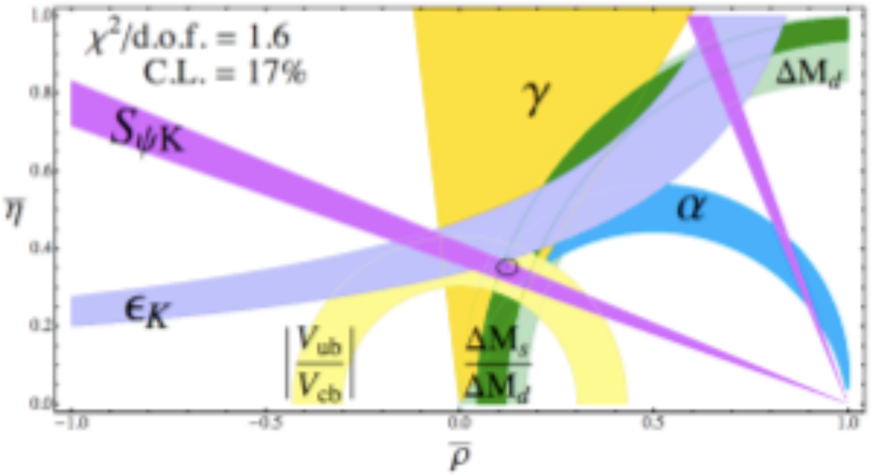


Figure 8: Global fit of the CKM unitarity triangle [14] . The current fit is consistent with the Standard Model at the 23% level. The constraints from ε_K, $|V_{ub}|/|V_{cb}|$, $\Delta M_s/\Delta M_d$, and ΔM_d are all limited by theoretical uncertainties from lattice QCD.

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ΔS=2 transitions: B_K

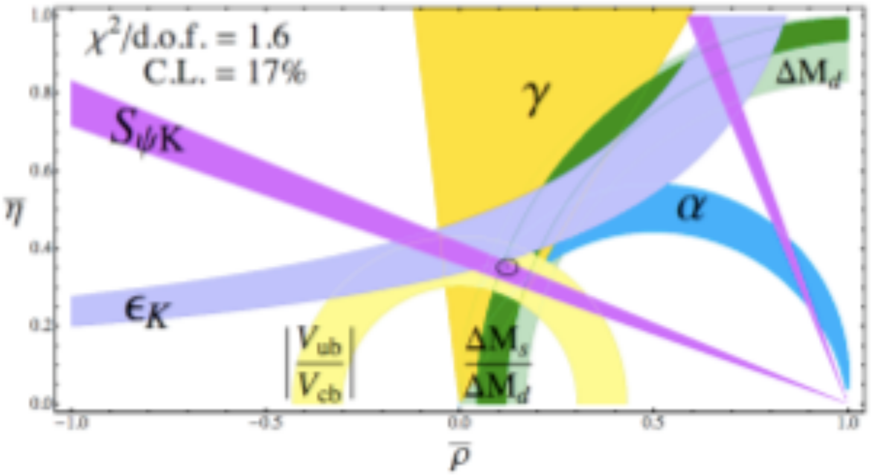


Figure 8: Global fit of the CKM unitarity triangle [14] . The current fit is consistent with the Standard Model at the 23% level. The constraints from ϵ_K, $|V_{ub}|/|V_{cb|}$, $\Delta M_s/\Delta M_d$, and ΔM_d are all limited by theoretical uncertainties from lattice QCD.

Van de Water PoS(LAT2009)014

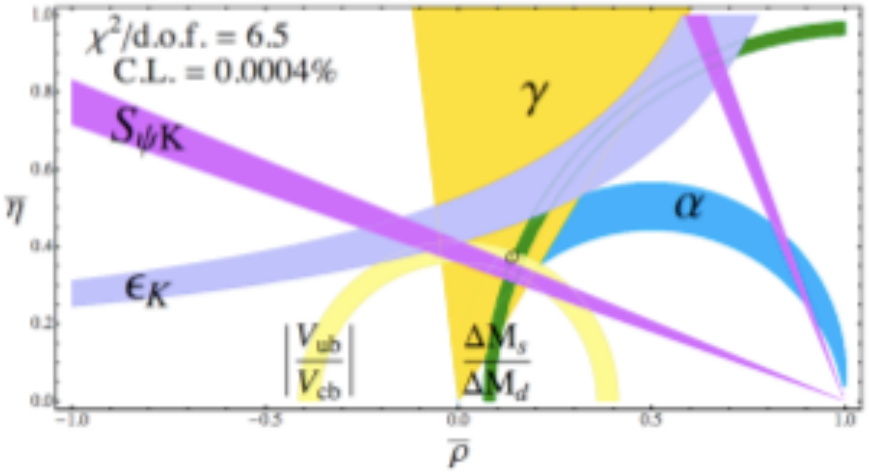


Figure 9: Potential impact of future lattice determinations on the global unitarity triangle fit. If the theoretical errors in all of the lattice QCD inputs are reduced to 1% with the central values fixed, the fit would no longer be consistent with Standard Model expectations. Figure courtesy of E. Lunghi.

ΔS=2 transitions: B_K

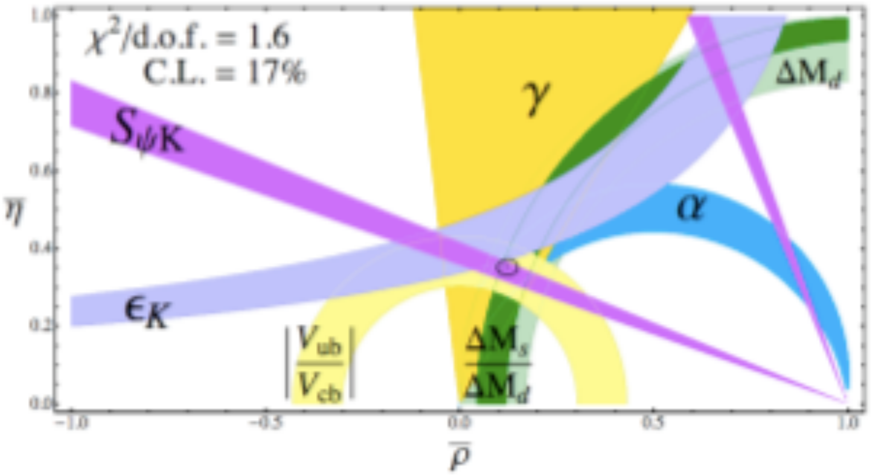


Figure 8: Global fit of the CKM unitarity triangle [14] . The current fit is consistent with the Standard Model at the 23% level. The constraints from ε_K, $|V_{ub}|/|V_{cb}|$, $\Delta M_s/\Delta M_d$, and ΔM_d are all limited by theoretical uncertainties from lattice QCD.

Dominant error in the ε_K band comes from $|V_{cb}|$ (8%), while B_K has a 4-6% error

Laiho et al., Phys.Rev. D81 (2010) 034503

Van de Water PoS(LAT2009)014

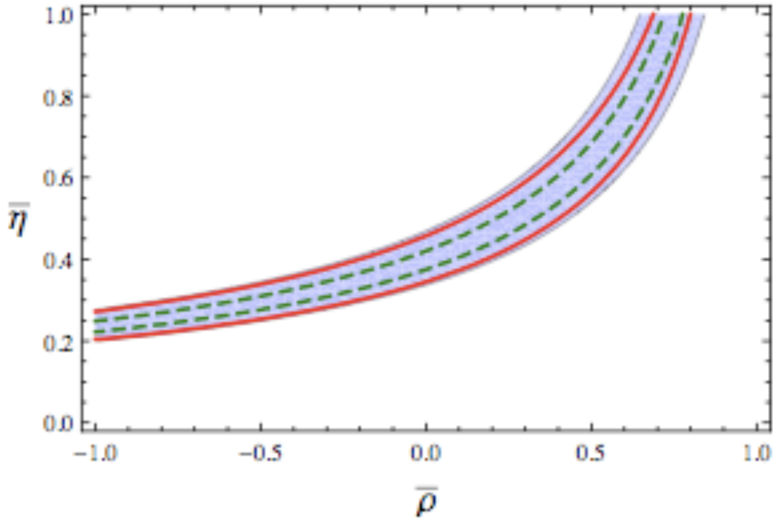
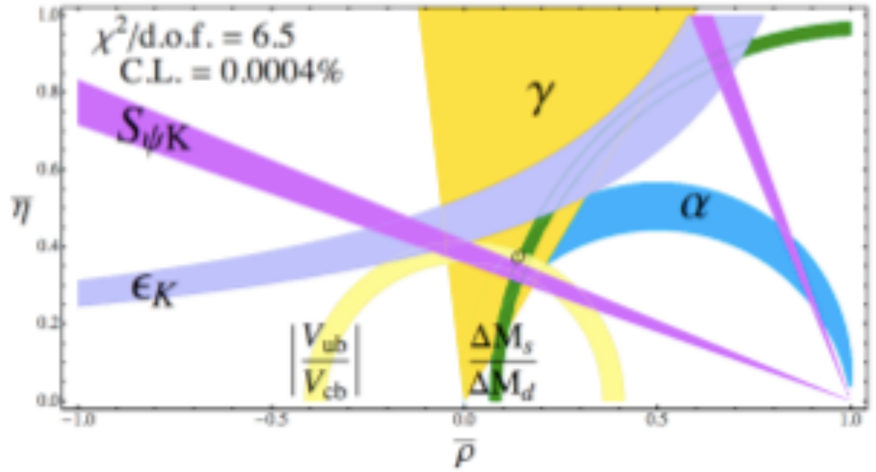


Figure 7: Contributions of $|V_{cb}|$ (solid red line) and $\hat{B}_K$ (dashed green line) to the uncertainty in the ε_K band. The errors introduced by the remaining inputs to the ε_K band are negligible. Figure from Ref. [14].



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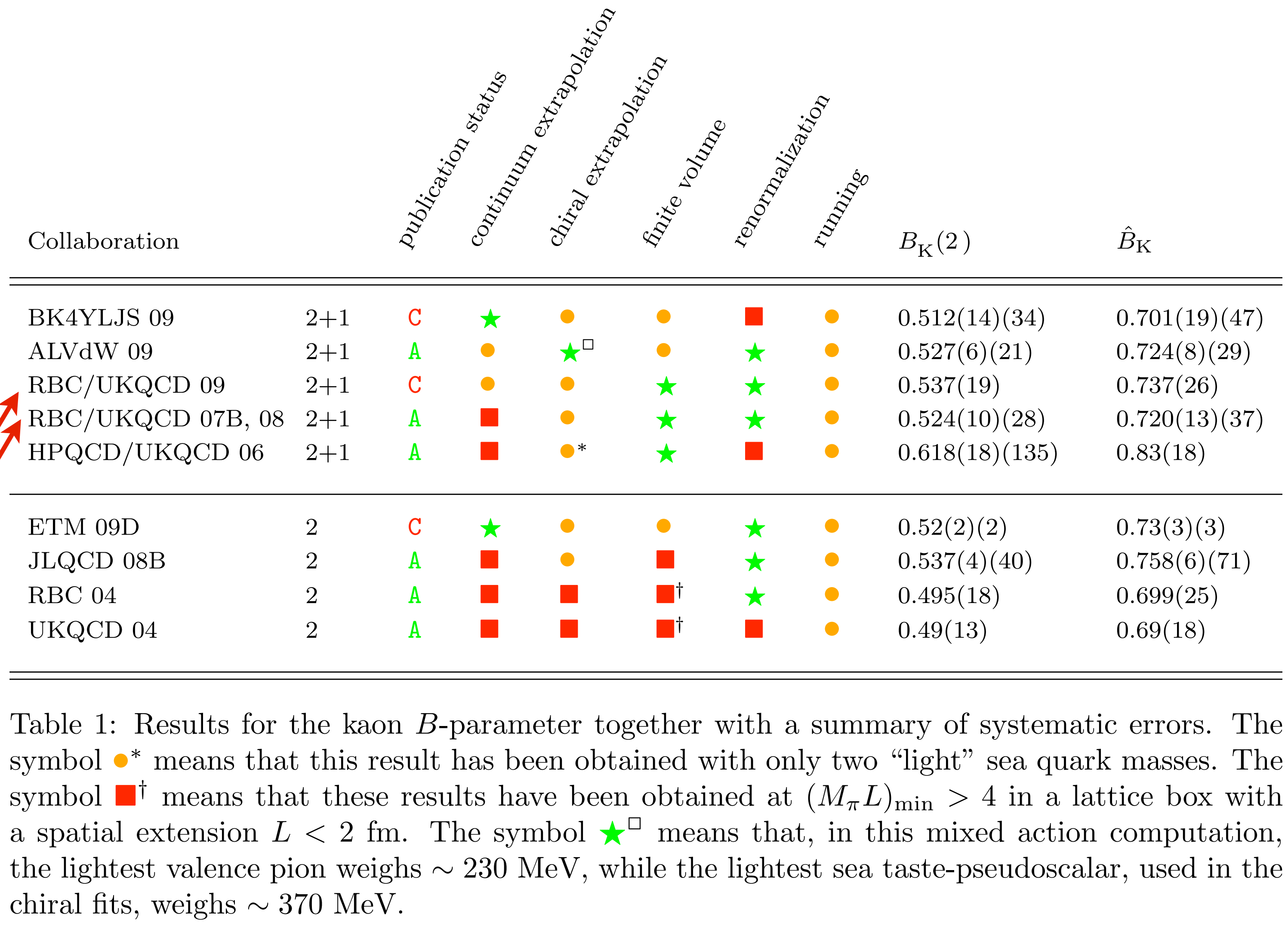
ΔS=2 transitions: B_K

$N_f = 2+1$

$N_f = 2$



Collaboration		publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	running	$B_K(2\,)$	$\hat{B}_K$
BK4YLJS 09	2+1	C	★	●	●	■	●	0.512(14)(34)	0.701(19)(47)
ALVdW 09	2+1	A	●	★$^{\square}$	●	★	●	0.527(6)(21)	0.724(8)(29)
RBC/UKQCD 09	2+1	C	●	●	★	★	●	0.537(19)	0.737(26)
RBC/UKQCD 07B, 08	2+1	A	■	●	★	★	●	0.524(10)(28)	0.720(13)(37)
HPQCD/UKQCD 06	2+1	A	■	●*	★	■	●	0.618(18)(135)	0.83(18)
ETM 09D	2	C	★	●	●	★	●	0.52(2)(2)	0.73(3)(3)
JLQCD 08B	2	A	■	●	■	★	●	0.537(4)(40)	0.758(6)(71)
RBC 04	2	A	■	■	■†	★	●	0.495(18)	0.699(25)
UKQCD 04	2	A	■	■	■†	■	●	0.49(13)	0.69(18)

Table 1: Results for the kaon B-parameter together with a summary of systematic errors. The symbol ●* means that this result has been obtained with only two "light" sea quark masses. The symbol ■† means that these results have been obtained at $(M_\pi L)_{\min} > 4$ in a lattice box with a spatial extension $L < 2$ fm. The symbol ★$^{\square}$ means that, in this mixed action computation, the lightest valence pion weighs ~ 230 MeV, while the lightest sea taste-pseudoscalar, used in the chiral fits, weighs ~ 370 MeV.

RBC/UKQCD (domain wall): $m_{\pi(\mathrm{val})} \sim 240$ MeV; $m_{\pi(\mathrm{sea})} \sim 290$ MeV

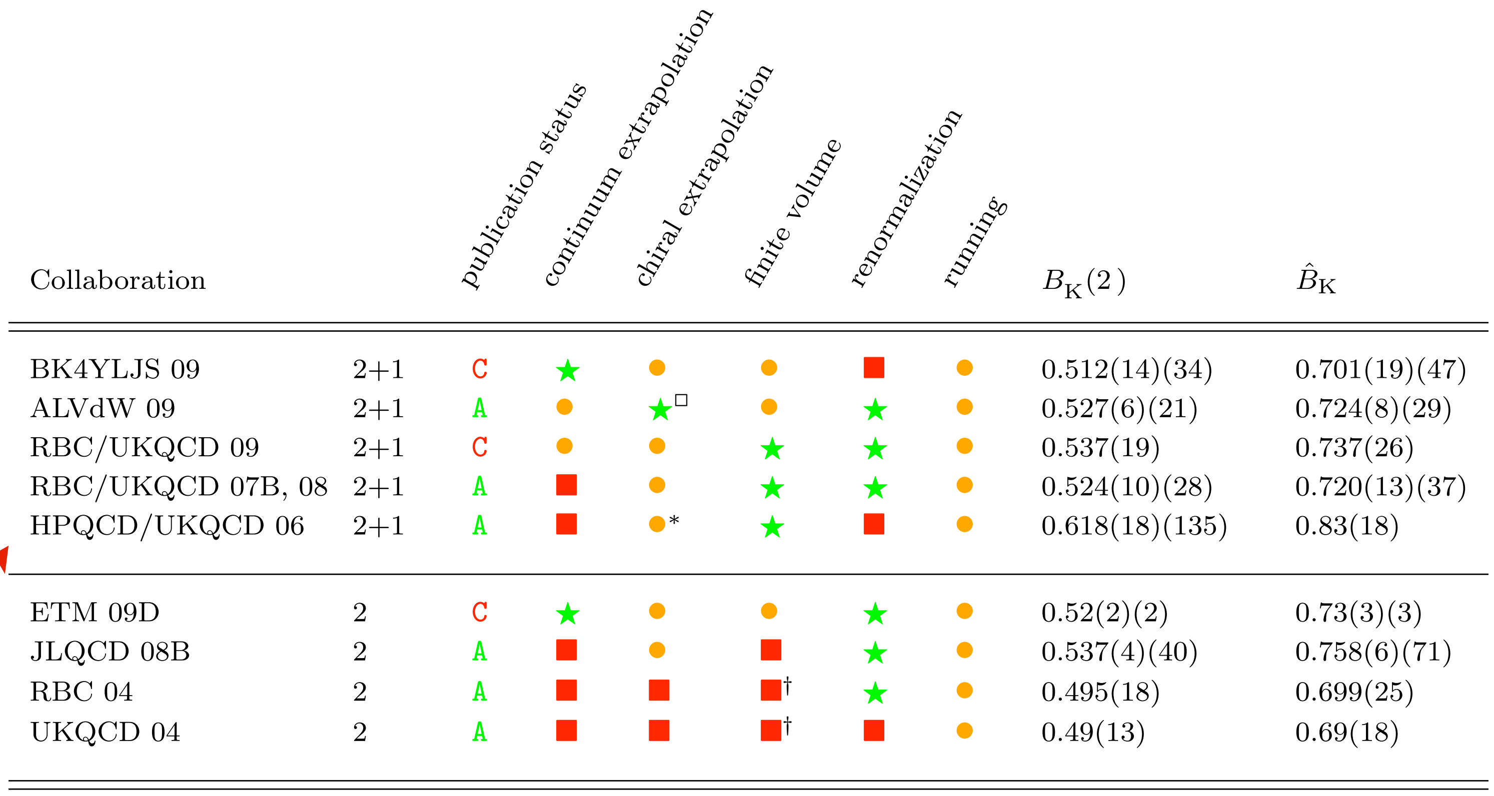
NB: NP renormalization

BUT: single coarse lattice ($a \sim 0.11$ fm) update reports preliminary result on second a

ΔS=2 transitions: B_K

$N_f = 2+1$

$N_f = 2$



Collaboration		publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	running	$B_K(2)$	$\hat{B}_K$
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ALVdW 09	2+1	A	●	★$^\square$	●	★	●	0.527(6)(21)	0.724(8)(29)
RBC/UKQCD 09	2+1	C	●	●	★	★	●	0.537(19)	0.737(26)
RBC/UKQCD 07B, 08	2+1	A	■	●	★	★	●	0.524(10)(28)	0.720(13)(37)
HPQCD/UKQCD 06	2+1	A	■	●*	★	■	●	0.618(18)(135)	0.83(18)
ETM 09D	2	C	★	●	●	★	●	0.52(2)(2)	0.73(3)(3)
JLQCD 08B	2	A	■	●	■	★	●	0.537(4)(40)	0.758(6)(71)
RBC 04	2	A	■	■	■†	★	●	0.495(18)	0.699(25)
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HPQCD/UKQCD (staggered): $m_{\pi(\rm val)} \sim 360$ MeV

BUT: 1-loop PT renormalization is main source of systematic error

BUT: single coarse lattice ($a \sim 0.11$ fm)

ΔS=2 transitions: B_K

N_f = 2+1

N_f = 2

Collaboration		publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	running	$B_K(2)$	$\hat{B}_K$
BK4YLJS 09	2+1	C	★	●	●	■	●	0.512(14)(34)	0.701(19)(47)
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RBC/UKQCD 09	2+1	C	●	●	★	★	●	0.537(19)	0.737(26)
RBC/UKQCD 07B, 08	2+1	A	■	●	★	★	●	0.524(10)(28)	0.720(13)(37)
HPQCD/UKQCD 06	2+1	A	■	●*	★	■	●	0.618(18)(135)	0.83(18)
ETM 09D	2	C	★	●	●	★	●	0.52(2)(2)	0.73(3)(3)
JLQCD 08B	2	A	■	●	■	★	●	0.537(4)(40)	0.758(6)(71)
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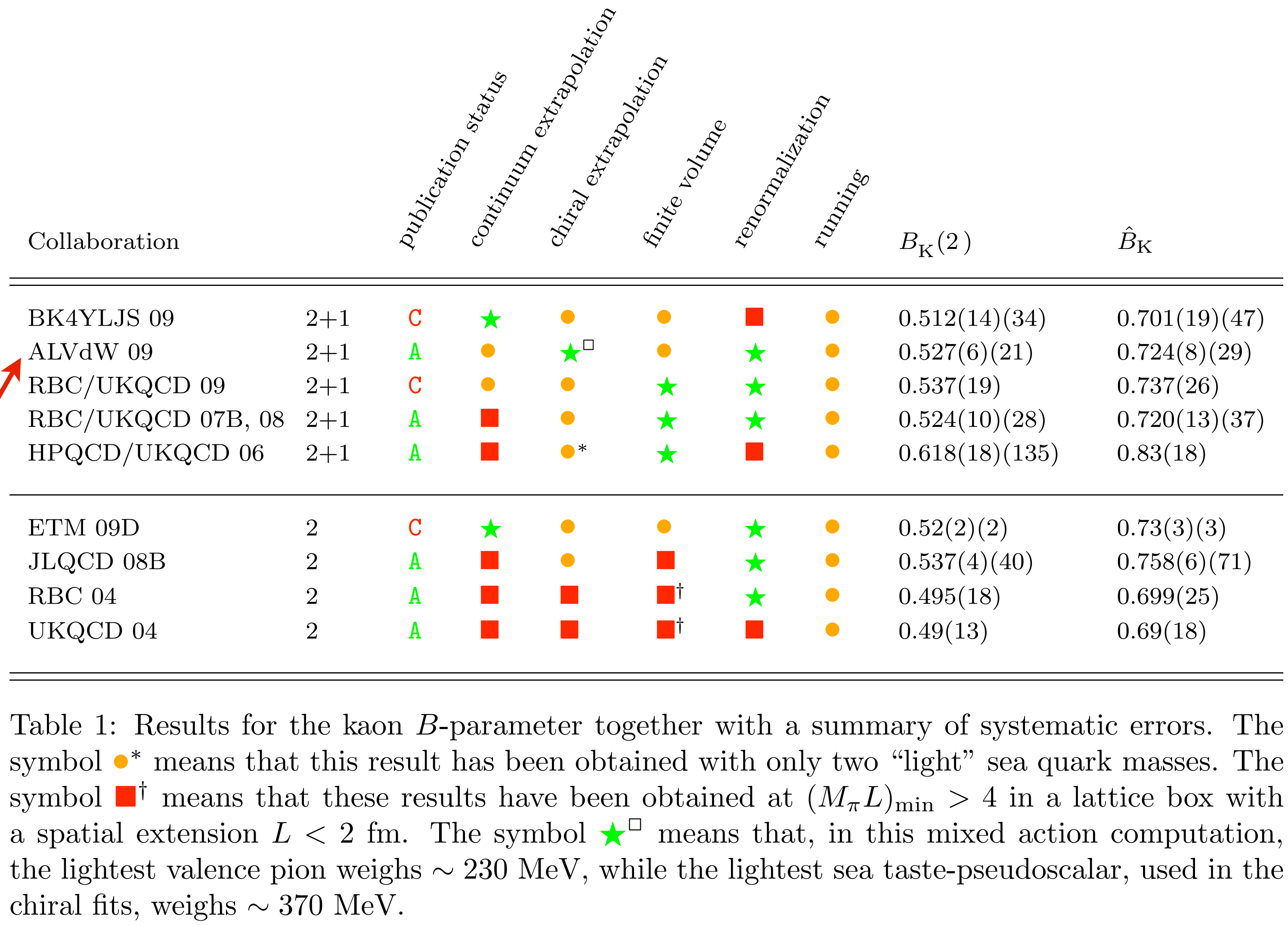
BK4YLJS(harpe) (staggered): preliminary

BUT: 1-loop PT renormalization is main source of systematic error

ΔS=2 transitions: B_K

$N_f = 2+1$

$N_f = 2$



Collaboration		publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	running	$B_K(2)$	$\hat{B}_K$
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RBC/UKQCD 09	2+1	C	●	●	★	★	●	0.537(19)	0.737(26)
RBC/UKQCD 07B, 08	2+1	A	■	●	★	★	●	0.524(10)(28)	0.720(13)(37)
HPQCD/UKQCD 06	2+1	A	■	●*	★	■	●	0.618(18)(135)	0.83(18)
ETM 09D	2	C	★	●	●	★	●	0.52(2)(2)	0.73(3)(3)
JLQCD 08B	2	A	■	●	■	★	●	0.537(4)(40)	0.758(6)(71)
RBC 04	2	A	■	■	■†	★	●	0.495(18)	0.699(25)
UKQCD 04	2	A	■	■	■†	■	●	0.49(13)	0.69(18)

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ALVdW(Aubin et al.) (mixed action; staggered sea, domain wall valence) with

$m_{\pi(\mathrm{val})} \sim 270$ MeV $m_{\pi(\mathrm{sea})} \sim 370$ MeV

two lattices ($a \sim 0.09$ fm, 0.12 fm)

NP renormalization

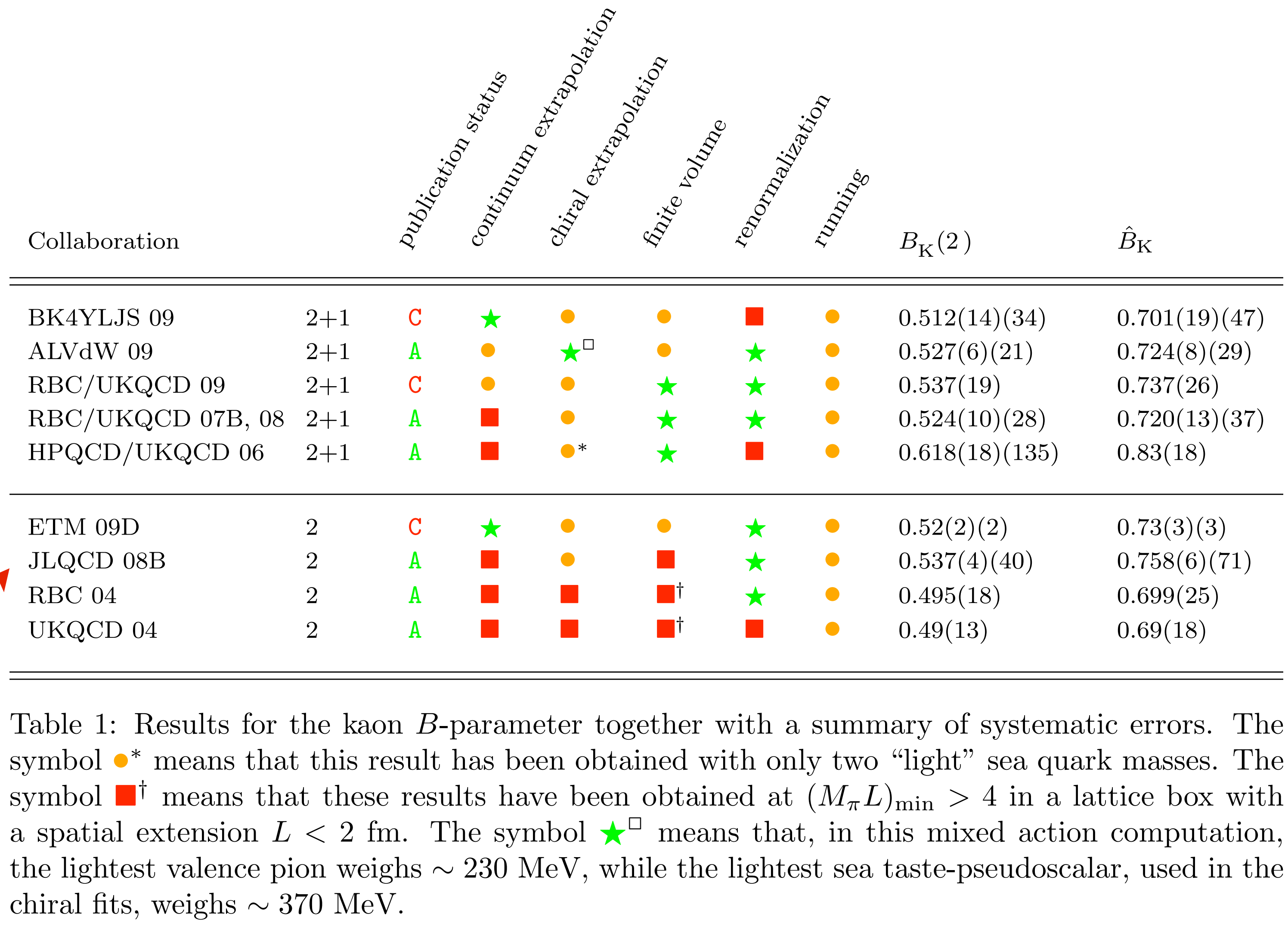
NB: main source of systematic error when renormalizing/matching/running from bare to $\overline{MS}$

This is the "best result to date" quoted by FLAG

ΔS=2 transitions: B_K

N_f = 2+1

N_f = 2



Collaboration		publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	running	$B_K(2)$	$\hat{B}_K$
BK4YLJS 09	2+1	C	★	●	●	■	●	0.512(14)(34)	0.701(19)(47)
ALVdW 09	2+1	A	●	★□	●	★	●	0.527(6)(21)	0.724(8)(29)
RBC/UKQCD 09	2+1	C	●	●	★	★	●	0.537(19)	0.737(26)
RBC/UKQCD 07B, 08	2+1	A	■	●	★	★	●	0.524(10)(28)	0.720(13)(37)
HPQCD/UKQCD 06	2+1	A	■	●*	★	■	●	0.618(18)(135)	0.83(18)
ETM 09D	2	C	★	●	●	★	●	0.52(2)(2)	0.73(3)(3)
JLQCD 08B	2	A	■	●	■	★	●	0.537(4)(40)	0.758(6)(71)
RBC 04	2	A	■	■	■†	★	●	0.495(18)	0.699(25)
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JLQCD(overlap) with $m_\pi \sim$ 290 MeV and $m_\pi L \sim 2.75$ (too small, as overlap is costly!!!)

one coarse lattice ($a \sim 0.118$ fm)

ΔS=2 transitions: B_K

$N_f = 2+1$

$N_f = 2$

Collaboration		publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	running	$B_K(2)$	$\hat{B}_K$
BK4YLJS 09	2+1	C	★	●	●	■	●	0.512(14)(34)	0.701(19)(47)
ALVdW 09	2+1	A	●	★$^\square$	●	★	●	0.527(6)(21)	0.724(8)(29)
RBC/UKQCD 09	2+1	C	●	●	★	★	●	0.537(19)	0.737(26)
RBC/UKQCD 07B, 08	2+1	A	■	●	★	★	●	0.524(10)(28)	0.720(13)(37)
HPQCD/UKQCD 06	2+1	A	■	●*	★	■	●	0.618(18)(135)	0.83(18)
ETM 09D	2	C	★	●	●	★	●	0.52(2)(2)	0.73(3)(3)
JLQCD 08B	2	A	■	●	■	★	●	0.537(4)(40)	0.758(6)(71)
RBC 04	2	A	■	■	■†	★	●	0.495(18)	0.699(25)
UKQCD 04	2	A	■	■	■†	■	●	0.49(13)	0.69(18)

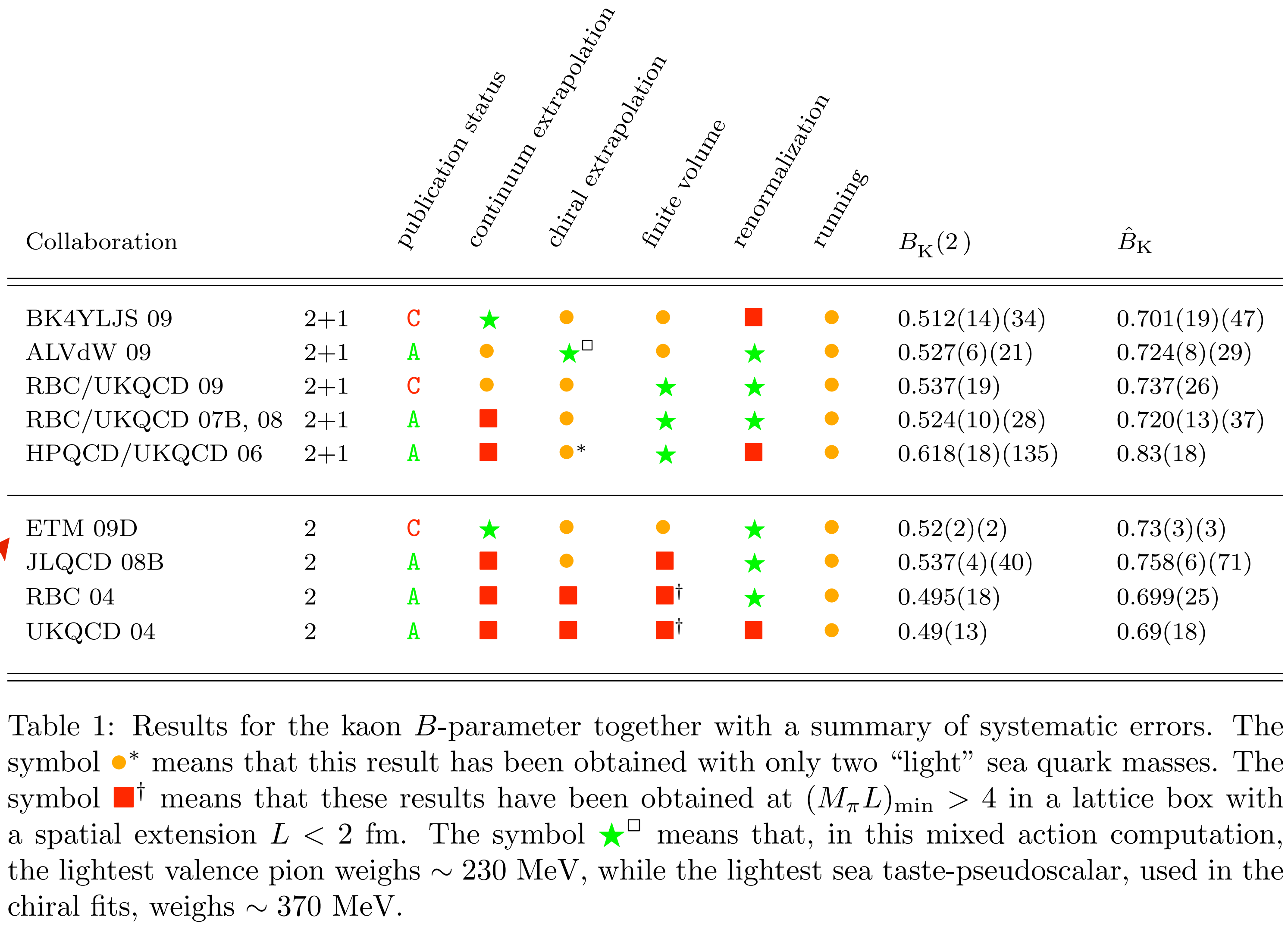
Table 1: Results for the kaon B-parameter together with a summary of systematic errors. The symbol ●* means that this result has been obtained with only two "light" sea quark masses. The symbol ■† means that these results have been obtained at $(M_\pi L)_{\min} > 4$ in a lattice box with a spatial extension $L < 2$ fm. The symbol ★$^\square$ means that, in this mixed action computation, the lightest valence pion weighs ~ 230 MeV, while the lightest sea taste-pseudoscalar, used in the chiral fits, weighs ~ 370 MeV.

RBC (domain wall) with $m_\pi \sim$ 490 MeV much too heavy!!

ΔS=2 transitions: B_K

N_f = 2+1

N_f = 2



Collaboration		publication status	continuum extrapolation	chiral extrapolation	finite volume	renormalization	running	$B_K(2)$	$\hat{B}_K$
BK4YLJS 09	2+1	C	★	●	●	■	●	0.512(14)(34)	0.701(19)(47)
ALVdW 09	2+1	A	●	★$^\square$	●	★	●	0.527(6)(21)	0.724(8)(29)
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HPQCD/UKQCD 06	2+1	A	■	●*	★	■	●	0.618(18)(135)	0.83(18)
ETM 09D	2	C	★	●	●	★	●	0.52(2)(2)	0.73(3)(3)
JLQCD 08B	2	A	■	●	■	★	●	0.537(4)(40)	0.758(6)(71)
RBC 04	2	A	■	■	■†	★	●	0.495(18)	0.699(25)
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ETM(Wilson-twisted) with $m_\pi \sim$ 270 MeV - 400 MeV (depending on a)

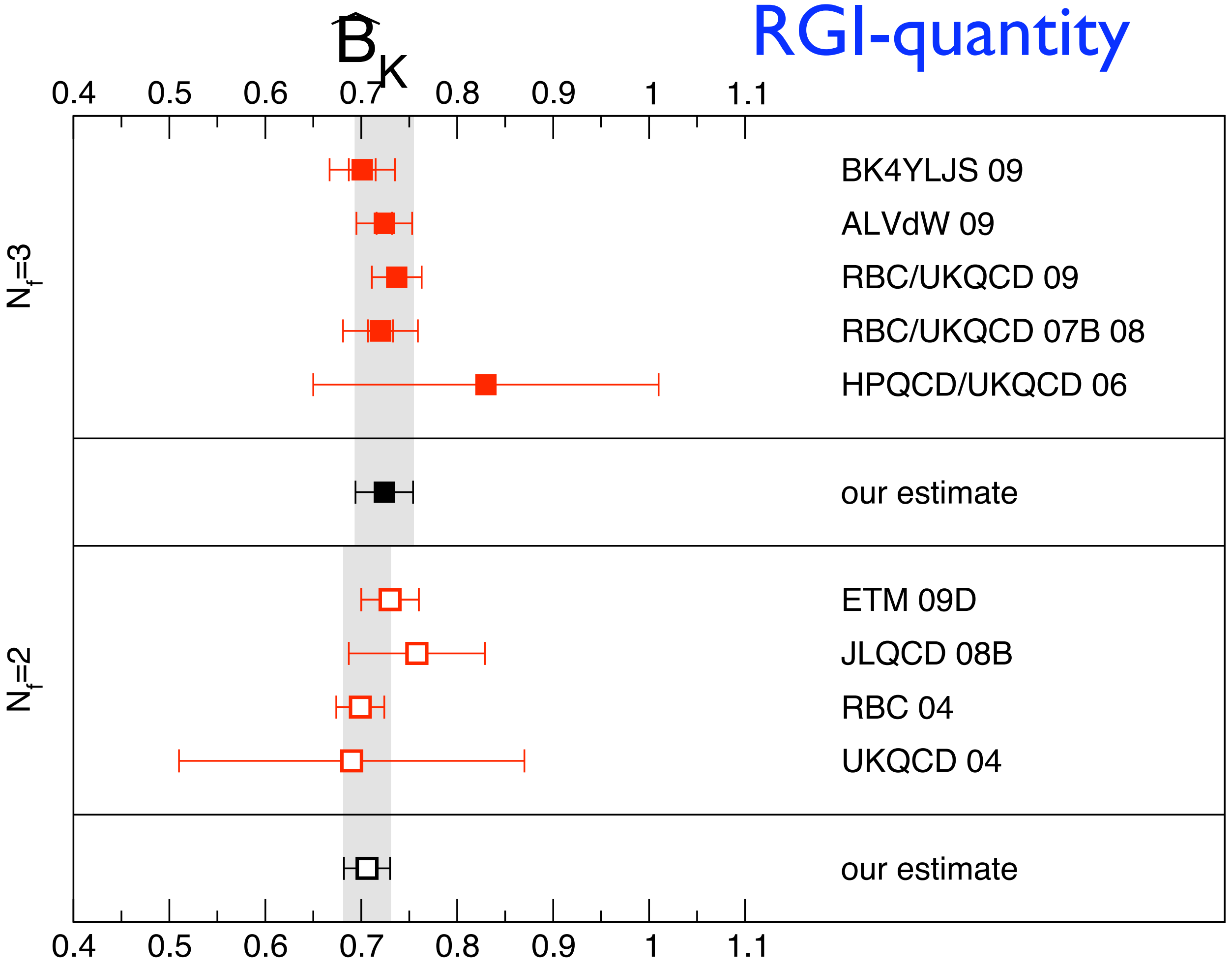
three lattices ($a \sim$ 0.1 fm, 0.085 fm, 0.065 fm)

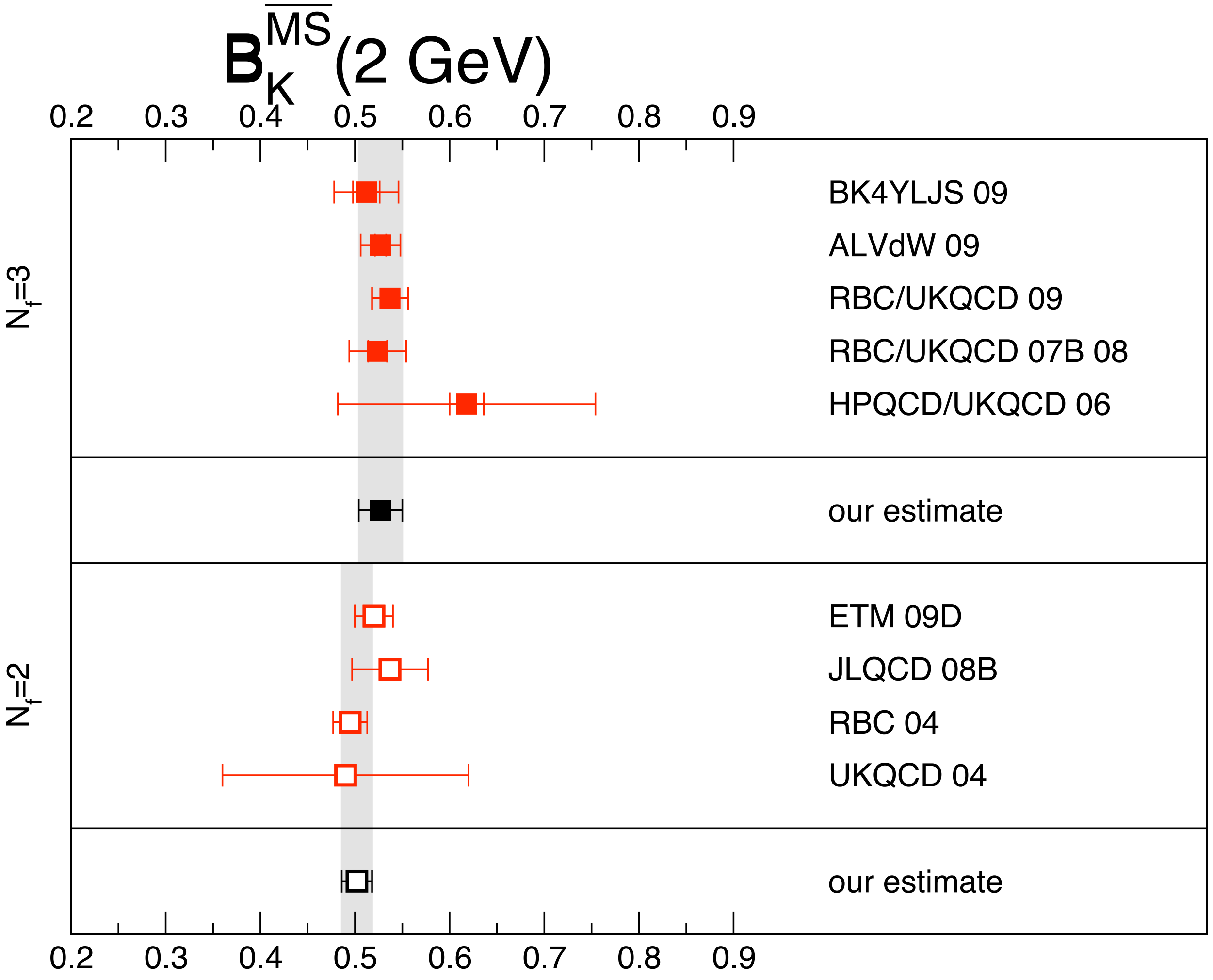
best result @ two flavours

BUT: still unpublished

ΔS=2 transitions: B_K

RGI-quantity





$$B_K^{\overline{\mathrm{MS}},\mathrm{NDR}}(2\mathrm{GeV}) = 0.527(23)$$
$$\hat{B}_K = 0.724(30) \qquad N_f = 2+1$$

$$B_K^{\overline{\mathrm{MS}},\mathrm{NDR}}(2\mathrm{GeV}) = 0.502(16)$$
$$\hat{B}_K = 0.706(24) \qquad N_f = 2$$

lots of work still to be done

NB: preliminary!! It includes only JLQCD & RBC; they will be replaced by ETM, once published

ΔS=2 transitions: B_K

NB: situation much better in quenched approximation (still...)

Collaboration	$B_{\mathrm{K}}(2\,)$	$\hat{B}_{\mathrm{K}}$
ALPHA	0.534(52)	0.74(7)
CP-PACS	0.565(6)	0.782(9)
ALPHA	0.532(25)	0.73(3)
JLQCD	0.628(42)	0.86(6)

Table 1: Quenched (= 0) results for the B-parameter B_{K} from various collaborations.

NB: quenched results agree with our best estimate

$$\boxed{\begin{aligned} B_K^{\overline{\mathrm{MS}},\mathrm{NDR}}(2\mathrm{GeV}) &= 0.527(23) \\ \hat{B}_K &= 0.724(30) \qquad N_f = 2+1 \end{aligned}}$$

Conclusions

- Lattice results are rapidly becoming more accurate and reliable, as control of systematic errors has increased.

- The $N_f = 2$ era is still an active topic, but $N_f > 2$ results are occupying centre stage.

- These positive developments are due to increased computer power, better algorithms etc.

- BUT: it is fair to acknowledge that the biggest stride has been the control of chirality on the lattice:
  - actions with better chiral properties (Ginsparg-Wilson, tmQCD,...)
  - lighter pions
  - better (more dedicated) χPT

- A high precision confirmation of unitarity is provided by lattice data

Conclusions

- **Future**:
- Periodic updates of data (biannual?) and include bottom, QCD coupling...
- Abandon Eurocentrism:
  - representatives of more lattice groups from Japan and US, as well as other communities will hopefully join in
  - 2-3 alternatives may mushroom out (cf. UTfit - CKMfitter paradigm)

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  - not a representative effort of, say, the US lattice community (this effort is still in its infancy...)